

1. ABSTRACT

The first aim of this thesis is to give a unified and self-contained treatment of a number of known results related to the theory of herds. This gives us the technical tools to deal with the second aim of our work which is to obtain some new results about herds and coherds in the pure categorical setting.

A *herd* at this level is a pretorsor $\tau : Q \rightarrow QPQ$ satisfying associativity and unitality properties with respect to a formal dual structure $\mathbb{M} = (\mathbb{A}, \mathbb{B}, P, Q, \sigma^A, \sigma^B)$ where \mathbb{A} and \mathbb{B} are monads over two different categories \mathcal{A} and \mathcal{B} respectively, P and Q are bimodule functors with respect to the monads and σ^A and σ^B are functorial morphisms satisfying linearity and compatibility conditions. In particular we study the special class of tame herds defined by the following extra assumptions.

A formal dual structure \mathbb{M} is called a *Morita context* if it satisfies also the balanced conditions of compatibility with the bimodule structures. Moreover, we will say that a Morita context \mathbb{M} is *tame* if the lifted functorial morphisms ${}_{AB}\sigma_{BA}^A : {}_A Q {}_B B P_A \rightarrow \text{Id}_{\mathbb{A}\mathcal{A}}$ and ${}_{BA}\sigma_{AB}^B : {}_B P_{AA} Q_B \rightarrow \text{Id}_{\mathbb{B}\mathcal{B}}$ are isomorphisms so that the lifted functors ${}_A Q_B : {}_B \mathcal{B} \rightarrow {}_A \mathcal{A}$ and ${}_B P_A : {}_A \mathcal{A} \rightarrow {}_B \mathcal{B}$ yield a category equivalence.

We can prove that such tame herds yield a correspondence with Galois functors.

A formal dual structure \mathbb{M} is called *regular* whenever $(\mathcal{A}, u_A) = \text{Equ}_{\text{Fun}}(u_A A, A u_A)$ and $(\mathcal{B}, u_B) = \text{Equ}_{\text{Fun}}(u_B B, B u_B)$ and the herd related to this formal dual structure is also called regular.

Under the regularity assumption starting with a herd, one can construct a coherd.

Similarly, one can define the dual notions of copretorsor, *coherd* and (regular) formal codual structure. Then, conversely, beginning with a coherd over a regular formal codual structure, a herd can be obtained.

Furthermore, we analyze what happens starting with a regular formal dual structure and a herd, constructing a formal codual structure and a coherd and then computing a new formal dual structure. Under the extra assumption that the starting herd is tame, the final formal dual structure computed comes out to be closely related to the starting one.

A few examples illustrate the theory we develop about herds and coherds.

In the last part of this thesis we investigate the bicategory of balanced bimodule functors which are one of the most useful tools in this work and it is inspired by the balanced bimodules in the classical sense.