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A Study on Transportation Problem, Transshipment Problem, Assignment Problem and Supply Chain Management

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CERTIFICATE

This is to certify that **Ms. Mansi Suryakant Gaglani** has worked under my guidance for the award of degree of Doctor of Philosophy in Statistics (Operations Research) under Faculty of Science on the topic entitled **“A Study on Transportation Problem, Transshipment Problem, Assignment Problem and Supply Chain Management”.**

We further certify that the work has not been submitted either partially or fully to any other University/Institute for the award of any degree.

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DECLARATION

I hereby declare that the thesis carried out by me at the Department of Statistics, Saurashtra University, Rajkot, Gujarat, India submitted to the Faculty of Science, Saurashtra University, Rajkot and the work incorporated in the present thesis is original and has not been submitted to any University/Institution for the award of the degree.

I am hearty thankful to Dr. D. K. Ghosh who provided me his guidance fully and allow me to submit my research paper in different journals.

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PAPER SUBMITTED FOR PUBLICATIONS

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2. Ghosh, D. K. & Mansi, S. G. (2011). A new alternate method of Transportation Problem, Journal of Sankhya Vigyan (under review).

3. Ghosh, D. K. & Mansi, S. G. (2011). A new alternate method of Transshipment Problem, Journal of Opsearch (under review).

4. Ghosh, D. K. & Mansi, S. G. (2011). A new alternate method of Assignment Problem, European Journal of Operations Research (under review).

CHAPTER 1

INTRODUCTION

1.1 Operations Research

Operations Research, Operational Research or simply O.R. is the use of Mathematical models, Statistics and algorithms to aid in decision-making. It is most often used to analyze complex real-world systems, typically with the goal of improving or optimizing performance. It is one form of the applied mathematics. Operations Research is an interdisciplinary branch of applied mathematics and formal science that uses methods such as mathematical modeling, statistics, and algorithms to arrive at optimal or near optimal solutions to complex problems. It is typically concerned with optimizing the maxima (for an example, profit, assembly line performance, crop yield, bandwidth, etc) or minima (for an example, loss, risk, etc.) of some objective function. Operations research helps the management to achieve its goals using scientific methods.

As per history of Operations Research, it is claimed that Charles Babbage (1791-1871) is the "father of operations research" because his research into the cost of transportation and sorting of mail led to England's universal "Penny Post" in 1840 and studies into the dynamical behavior of railway vehicles in defense of the GWR's broad gauge. The modern field of operations research arose during World War II. Scientists in the United Kingdom including Patrick Blackett, Cecil Gordon, C. H. Waddington, Owen Wansbrough-Jones and Frank Yates and George Dantzig (United States) looked for ways to make better decisions in such areas as logistics and training schedules. After the war it began to be applied to similar problems in industry.

The terms operations research and management sciences are often used synonymously. When a distinction is drawn, management science generally implies a closer relationship to the problems of business management. The field is closely related to Industrial engineering, but takes more of an engineering point of view. Industrial engineers typically consider Operations Research (OR) techniques to be a major part of their toolset. Some of the primary tools used by operations researchers are statistics, optimization, probability theory, queuing theory, game theory, graph theory, decision analysis and simulation techniques. Because of the computational nature of these fields, OR also has ties to computer science, and hence operations researchers use custom-written and off-the-shelf software.

Operations research is distinguished by its frequent use to examine an entire management information system, rather than concentrating only on specific elements (though this is often done as well). An operations researcher faced with a new problem that is expected to determine which techniques are most appropriate for the given nature of the system, the goals for improvement, and constraints on time and computing power. For this and other reasons, the human element of OR is vital. Like any other tools, OR techniques cannot solve problems by themselves. The operations research analyst has a wide variety of methods available for problem solving. For mathematical programming models there are optimization techniques appropriate for almost every type of problem, although some problems may be difficult to solve. For models that incorporate statistical variability there are methods such as probability analysis and simulation that estimate statistics for output parameters. In most cases the methods are implemented in computer programs. It is important that at least some member of an OR study team be aware of the tools available and be knowledgeable concerning their capabilities and limitations.

Along the history, is frequent to find collaboration among scientists and armies officer with the same objective, ruling the optimal decision in battle. In fact that many experts considered the start of Operational Research in the III century

B.C. during the II Punic War with analysis and solution that Arquimedes named for the defense of the city of Syracuse, besieged for Romans. Enter his inventions would find the catapult, and a system of mirrors that was setting to fire the enemy boats by focusing them with the Sun's rays.

Leornado DaVinci (1503), being an engineer took part in the war against Prisa because he knew the techniques to accomplish bombardments, to construct ships, armored vehicles, cannons, catapults and another warlike machine. Another antecedent of use of Operational Research by F.W. Lanchester, who made a mathematical study about the ballistic potency of opponents and hence he developed from a system of equations differential and then called it Lanchester's Square Law that can be available to determine the outcome of a military battle. Thomas Edison made use of Operational Research by contributing in the antisubmarine war, with his greats ideas, like shields against torpedo for the ships. From the mathematical point of view many mathematicians, in centuries XVII and XVIII, like Newton, Leibnitz, Bernoulli and Lagrange etc. worked in obtaining maximum and minimum conditions of certain functions. Mathematical French Jean Baptiste and Joseph Fourier sketched methods of present-day Linear Programming. During later years of the century XVIII, Gaspar Monge laid down the precedents of the Graphical Method thanks to his development of Descriptive Geometry. Janos Von Neumann (1944) published his work called "Theory of Games", that provided the basic concept of Linear Programming to Mathematicians. Neumann (1947) viewed the similitude among the linear programming problems and the matrix theory that developed by himself. Russian mathematician Kantorovich (1939) in association with another Dutchman mathematician Koopmans (1939) developed the mathematical theory called "Linear programming". This investigation rewarded them with the Nobel Prize.

In the late years 30, George Joseph Stigler presented a particular problem known as special diet optimal or more commonly known as problem of the diet that happened because the worry of the USA army to guarantee some nutritionals requests to the lower cost for his troops. It was solved with a heuristic method which solutions only differ in some centimes against the solution contributed years later by the Simplex Method. During the years 1941 and 1942, Kantorovich and Koopmans studied independently about the Transport Problem for first time. Initially this type of problems for solving Transport Problem was called problem of Koopmans-Kantorovich. For his solution, they used geometric methods that are related to Minkowski's theory of convexity. But it does not considered that has been born a new science called Operations Research until the II World War, during battle of England, where Deutsche Air Force, that is the Luftwaffe, was submitting the Britishers to a hard air raid, since these had an little aerial capability, although experimented in the Combat. The British government looking for some method to defend his country convoked several scientists of various disciplines for try to resolve the problem to get the peak of benefit of radars that they had. Thanks to his work for determining the optimal localization of antennas and further they got the best distribution of signals to double the effectiveness of the system of aerial defense. To notice the range of this new discipline, England created another groups of the same nature in order to obtain optimal results in the dispute. Just like United States (USA), when joined the War in 1942, creating the project SCOOP (Scientific Computation of Optimum Programs), where George Bernard Dantzig (1947) developed the method of Simplex algorithm.

During the Cold War, the old Soviet Union (USSR) Plan Marshall wanted to control the terrestrial communications including routes fluvial from Berlin. In order to avoid the rendition of the city, and his submission to be a part of the deutsche communist zone, England and United States decided supplying the city, or else by means of escorted convoys (that would be able to give rise to new confrontations) or by means of airlift, breaking or avoiding in any event the blockage from Berlin. Second option was chosen starting the Luftbrücke (airlift) at June 25, 1948. This went another from the problems in which worked by the SCOOP group, in December of that same year, could carry 4500 daily tons, and after studies of Research Operations optimized the supplying to get to the 8000~9000 daily tons in March of 1949. This cipher was the same that would have been transported for terrestrial means, for that the Soviet decided to suspend the blockage at May 12, 1949. After Second World War the order of United States' resources (USA) (energy, armaments, and all kind of supplies) took opportune to accomplish it by models of optimization, resolved intervening linear programming. At the same time, that the doctrine of Operations Research is being developed, the techniques of computation and computers are also developing, thanks them the time of resolution of the problems decreased.

The first result of these techniques was given at the year 1952, when a SEAC computer was used by National Bureau of Standers in way to obtain the problem´s solution. The success at the resolution time was so encouraging that was immediately used for all kind of military problems, like determining the optimal height which should fly the planes to locate the enemy submarines, monetary founds management for logistics and armament, including to determine the depth which should send the charges to reach the enemy submarines in way to cause the casualties' bigger number that was translated in a increase in five times in Air Force's efficacy. During the 50's and 60's decade, grows the interest and developing of Operational Research, due to its application in the space of commerce and the industry. Take for example, the problem of the calculation of the optimal transporting plan of sand of construction to the works of edification of the city of Moscow which had 10 origins points and 230 destinies. To resolve it, Strena computer was used and that took 10 days in the month of June of 1958 and such solution contributed a reduction of the 11 % of the expenses in relation to original costs.

Previously, this problems were presented in a discipline knew as Research Companies or Analysis Companies that did not have so effective methods like the developed during Second World War (for example the Método Símplex). There are many applications of Operations Research in War which we can imagine with problems like nutrition of cattle raising, distribution of fields of cultivation in agriculture, goods transportation, location, personnel's distribution, networking problems, queue problems and graphics, etc.

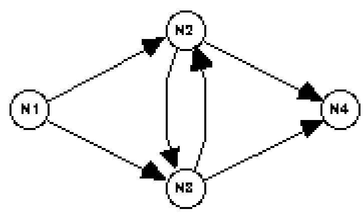
Operations Research contains the following different topics for solving different types of problems.

1. ***Add Teach*** The *Add Teach* add-in allows the user to install and remove add-ins in the *Teach OR* collection without using the *Add-in* command of the *Tools* menu. With the add-in installed, *Teach* appears on the main Excel menu. Selecting the *Add Teach* item presents a dialog that installs or removes add-ins with a click of the button. The add-in also opens demonstration workbooks that illustrate the operations of the *Teach OR* collection.
2. ***Linear Programming*** We provide three units to demonstrate and teach linear programming solution algorithms. Primal Simplex Demonstrations are implemented using *Flash* to illustrate basic concepts of the primal simplex technique. The Teach Linear Programming Add-in implements three different algorithms for solving linear programming models.
3. ***Network Flow Programming*** We provide five units to demonstrate and teach network flow programming solution algorithms. The Teach Network Add-in implements the network primal simplex method for both pure and generalized minimum cost flow problems. A graphical demonstration using *Flash* illustrates and contrasts algorithms for finding the minimal spanning tree and shortest path tree. The Transportation primal simplex method is implemented in the Teach Transportation Add-in. A graphical demonstration using *Flash* illustrates the network primal simplex method.
4. ***Integer Programming*** The Teach IP Add-in implements three methods for solving linear integer programming problems. The Add-in provides demonstrations and hands-on practice for the branch and bound method, the cutting plane method and Benders' algorithm.
5. ***Nonlinear Optimization*:** The Teach NLP Add-in demonstrates direct search algorithms for solving nonlinear optimization problems.

6. ***Dynamic Programming*:** The Teach Dynamic Programming Add-in has features that allow almost any system appropriate for dynamic programming to be modeled and solved. The program includes both backward recursion and reaching.

Here we discuss the detail about the Network Flow Programming.

Network Flow Programming



The term network flow program describes a type of model that is a special case of the more general linear program. The class of network flow programs includes such problems as the transportation problem, the assignment problem, the shortest path problem, the maximum flow problem, the pure minimum cost flow problem and the generalized minimum cost flow problem. It is an important class because many aspects of actual situations are readily recognized as networks and the representation of the model is much more compact than the general linear program. When a situation can be entirely modeled as a network, very efficient algorithms exist for the solution of the optimization problem which is many times more efficient than linear programming in the utilization of computer time and space resources.

The methods for network flow programming are below.

1. Transportation Problem
2. Transshipment Problem
3. Assignment Problem
4. Shortest Path Problem
5. Maximum Flow Problem

1.2 Transportation Problem

Transportation problem is one of the subclasses of Linear Programming Problem(LPP) in which the objective is to transport various quantities of a single homogeneous commodity that are initially stored at various origins to different destinations in such a way that the total transportation cost is minimum. To achieve this objective we must know the amount and location of available supplies and the quantities demanded. In addition, we must know the costs that result from transporting one unit of commodity from various origins to various destinations.

1.2.1 Mathematical Formulation of the Transportation Problem

A Transportation Problem can be stated mathematically as a Linear Programming Problem as below:

$$\boldsymbol{Minimize\ (Maximize)\ Total\ CostZ} = \sum_{i=1}^{m}\sum_{j=1}^{n} \boldsymbol{c_{ij}\, x_{ij}}$$

Subject to the constraints

$\sum_{j=1}^{n} x_{ij} = a_i,$ i=1,2,…,m (supply constraints)

$\sum_{i=1}^{m} x_{ij} = b_j,$ j=1,2,…,n (demand constraints)

$x_{ij} \geq 0$ for all i and j

Where, a_i = quantity of commodity available at origin i

b_j= quantity of commodity demanded at destination j

c_{ij}= cost of transporting one unit of commodity from i^{th} origin to j^{th} destination

x_{ij} = quantity transported from i^{th} origin to j^{th} destination

1.3 Transshipment Problem

In the transportation problems, it was assumed that a source (factory, plant etc.) acts only as a shipper of the goods and a destination (market etc.) acts only as receiver of the goods. We shall now consider the broader class of transportation problem, called as transshipment problem, which allow for the shipment of goods both from one source to another, and from one destination point to another. Thus, the possibility of transshipment-the goods produced/available at one source and destined for some destination point may reach there via other sources and/or destinations and transshipped at these points. This is obviously a more realistic statement of the distribution problem faced by a business/industrial house. For example, a multi-plant firm may find it necessary to send some goods from one plant to another in order to meet the substantial increase in the demand in the second marker. The second plant here would act both as a source and a destination and there is no real distinction between source and destination.

A transportation problem can be converted into a transshipment problem by relaxing the restrictions on the receiving and sending the units on the origins and destinations respectively. An m-origin, n-destination, transportation problem, when expressed as a transshipment problem: with m+n origins and an equal number of destinations. With minor modifications, this problem can be solved using the transportation method. In a transportation problem, shipment of commodity takes place among sources and destinations. But instead of direct shipments to destinations, the commodity can be transported to a particular destination through one or more intermediate or trans-shipment points. Each of these points in turn supply to other points. Thus, when the shipments pass from destination to destination and from source to source, we have a trans-shipment problem.

In a transportation problem, shipments are allowed only between source-sink pairs. In many applications, this assumption is too strong. For example, it is often the case that shipments may be allowed between sources and between sinks. Moreover, there may also exist points through which units of a product can be transshipped from a source to a sink. Models with these additional features are called transshipment problems. Interestingly, it turns out that any given transshipment problem can be converted easily into an equivalent transportation

problem. The availability of such a conversion procedure significantly broadens the applicability of algorithm for solving transportation problem.
Since Transshipment problem is a particular case of Transportation problem and hence the mathematical format of the Transshipment problem is similar to date of Transportation problem.

1.4 Assignment Problem

Imagine, if in a printing press there is one machine and one operator so as to operate. Immediately a question arrays how would you employ the worker? Your immediate answer will be, the available operator will operate the machine. Again suppose there are two machines in the press and two operators are engaged at different rates to operate them. Which operator should operate which machine for maximizing profit? Similarly, if there are n machines available and n persons are engaged at different rates to operate them. Which operator should be assigned to which machine to ensure maximum efficiency? While answering the above questions we have to think about the interest of the press, so we have to find such an assignment by which the press gets maximum profit on minimum investment. Such problems are known as "assignment problems".

1.4.1 Mathematical Formulation of the Assignment Problem

A assignment problem can be stated mathematically as a Linear Programming Problem as below:
The objective function is to,

Minimize (Maximize) Z = $\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij}$

Subject to the constraints,

$$\sum_{j=1}^{n} x_{ij} = 1,\ for\ all\ i\ (resource\ availability)$$

$$\sum_{i=1}^{n} x_{ij} = 1,\ for\ all\ j\ (job\ requirement)$$

Where, x_{ij}=0 or 1 and c_{ij} represents the cost of assignment from resource i to activity j.

1.5 Travelling Salesman Problem

The Traveling Salesman Problem (TSP) is a problem in combinatorial optimization studied in operations research and theoretical computer science. Given a list of cities and their pair wise distances, the task is to find a shortest possible tour that visits each city exactly once.

The problem was first formulated as a mathematical problem by Menger **(**1930) and is one of the most intensively studied problems in optimization. It is used as a benchmark for many optimization methods. Even though the problem is computationally difficult, a large number of heuristics and exact methods are known, so that some instances with tens of thousands of cities can be solved.

1.6 SUPPLY CHAIN MANAGEMENT

There seems to be a universal agreement on what a supply chain is? A supply chain is to be a network of autonomous or semi-autonomous business entities which is collectively responsible for procurement, manufacturing and distribution activities associated with one or more families of related products.

A supply chain is a network of facilities that procure raw materials, transform them into intermediate goods and then final products. Finally deliver the products to customers through a distribution system.

A supply chain is a network of facilities and distribution options that performs the functions of procurement of materials, transformation of these materials into intermediate and finished products, and the distribution of these finished products to customers.

1.7 Definitions of Some Terminology

The following terms are to be defined with reference to the transportation problem, assignment problem and transshipment problem.

Feasible Solution (F. S.)

A set of non-negative allocations $x_{ij} \geq 0$, which satisfies the row and column restrictions is known as feasible solution.

Basic (Initial) Feasible Solution (B. F.S.)

A feasible solution to an m-origin and n-destination problem is said to be basic feasible solution if the number of positive allocations are (m+n–1). If the number of allocations in a basic feasible solutions are less than (m+n–1), it is called degenerate basic feasible solution (DBFS) (otherwise non-degenerate).

Optimal Solution

A feasible solution (not necessarily basic) is said to be optimal if it minimizes the total cost.

CHAPTER 2
REVIEW OF LITERATURE

2.1 Introduction

In this chapter, we discussed the work done by Many scientist/statistician so far in Transportation Problem, Transshipment Problem, Assignment Problem, Travelling salesman problem and Supply Chain Management till yet.

2.2 Transportation Problem

Ji and Chu (2002) have discussed Dual-Matrix Approach Method to solve the Transportation Problem as an alternative to the Stepping Stone Method. The approach considers the dual of the Transportation Model instead of the primal and then obtains the optimal solution of the dual using Matrix operations hence it is called dual matrix approach. In this method, the unit transportation cost is generally indicated on the North-East Corner in each cell. This problem can also be expressed as a linear programming model as follows.

Minimize total cost Z= $\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$

Subject to $\sum_{j=1}^{n} x_{ij} \le a_i$ for i=1, 2,…, m (2.1)

$\sum_{i=1}^{m} x_{ij} \ge b_j$ for j=1,2,…,n (2.2)

$x_{ij} \ge 0$

Here, all a_i and b_j are assumed to be positive and cost c_{ij} are non-negative.

In usual balanced transportation problem, the condition $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ must hold true. If this condition is not met then a dummy origin or destination is generally introduced to make the problem balanced in order to use Stepping Stone Method. However the dual matrix approach introduced by Ji and chu (2002) does not required that a transportation problem to be balanced. This method can be used both for balanced and unbalanced transportation problem. Because of this reason (2.1) is represented as ≤ and (2.2) as ≥ instead of = for both cases. This is one of advantage in Ji and Chu (2002) approach over the Stepping Stone method. The dual matrix approach is similar to that of Stepping Stone Method where first find an initial feasible solution and then get next improved solution by assessing all non basic cells until the optimal solution found.

Adlakha and Kowalski (1999, 2006) suggested an alternative solution algorithm for solving certain TP based on the theory of absolute point. Recently Adalkha and Kowalski (2009) presented various rules governing load distribution for alternate optimal solution in Transportation Problem. The load assignment for an alternate optimal solution is left mostly on the decision of the practitioner. They illustrated the structure of alternate solution in a transportation problem using the shadow price. They also provided a systematic analysis for allocating loads to obtain and alternate optimal solution.

For this purpose they consider the reduced the SP matrix after deleting the rows/columns related to the cells fix due to absolute structure of TP. Here they are interested to determine the minimum amount of load X_{ij}. To determine this amount, analyze every rows and columns of the reduced optimal SP matrix to determine a loaded cell, say, (s, t) where $\sum_{i \neq s} a_i < b_t$ or $\sum_{j \neq l} b_j < a_s$. After identifying cell s_t, the value of X_{st} is set as following.

$X_{st} = b_t - \sum_{i \neq s} a_i$ or $X_{st} = a_s - \sum_{j \neq l} b_j$.

Many problems like multi-commodity transportation problem, transportation problem with different kind of vehicles, multi-stage transportation problem and transportation problem with capacity limit are an extension of the classical transportation problem considering the additional special condition. Solving such problems many optimization techniques are used like dynamic programming, linear programming and heuristic approaches etc. Brezina et. al. (2010) developed a method for solving multi-stage transportation problem with capacity limit that reflects limits of transported materials quantity. They also developed algorithm to find optimal solution. Further they discussed efficiency of presented algorithm depends on selection of algorithm used to obtain the starting solution (Author used VAM).

2.3 Transshipment problem

Orden (1956) introduced the concept and application of Transshipment Problem. He extended the concept of original transportation problem so as to include the possibility that is using the concept of Transshipment Problem. In other words, He argued that any shipping or receiving point is also permitted to act as an intermediate point. In fact, the transshipment technique is used to find the shortest route from one point to another point representing the network diagram.

Rhody (1963) considered Transshipment model as reduced matrix model. He discussed the Transshipment of pork from various places and then Transshipment of finished product. He studied the shipment of interregional competitive position of the hog-pork industry in the United States for finding the optimal location and its Transshipment.

King and Logan (1964) developed two alternative models namely (1) Raw Product – Final Product spatial equilibrium model and (2) Modified Transshipment Model. In this model, they discussed simultaneously the costs of shipping raw materials, processing and shipping final product. This problem is related with the location and size of the California cattle slaughtering plants given the location and quantity of slaughter animals and the final product demand. In fact, they studied based on optimal location, number and size of processing plant.

Judge et al. (1965) formulated the Transshipment model into general linear programming model. They developed model based on interregional Transshipment. They developed the formulation of Transshipment Problem and then converted it into linear programming model. So as to find optimal location and optimal number of live stock. They applied interregional model of transshipment to the live stock industry.

Garg and Prakash (1985) studied time minimizing Transshipment model. In their study, it is seen that how the optimal time can be achieved while transshipping the goods from different origins to different destinations.

However Herer and Tzur (2001) discussed dynamic Transshipment Problem. In the standard form, the Transshipment problem is basically a linear minimum cost network through problem. For such types of optimization problems, a number of effective solutions are available in the literature since many years.

Recently Khurana and Arora (2011) observed that the Transshipment problem is basically a linear minimum cost network problem optimization technique is required with different constraints. Hence they developed Transshipment problem model with mixed constraints.

In this method, we have discussed a simple and alternate method for solving Transshipment Problem which gives an optimal solution.

2.4 Assignment Problem

Konig (1931) developed and introduced a method for solving an Assignment Problem. He gave his name as Hungarian Method (because he was a Hungarian Mathematician). He developed the method as an efficient method of finding the optimal solution without having to make a direct comparison of every solution. His method works on the principal of reducing the given cost matrix to a matrix of opportunity cost. Opportunity costs show the relative penalties associated with assigning resource to an activity as opposed to making the best or least cost assignment. He further said that if we can reduced the cost matrix to the extent of having at least one zero in each row and each column then it will be possible to make an optimal assignment where opportunity cost are zero. We have discussed an algorithm of Hungarian Method for obtaining an optimal solution of an optimal solution of an Assignment Problem is chapter 5.

The Hungarian method is a combinatorial optimization algorithm which solves the assignment problem in polynomial time and which anticipated later primal-dual methods. Kuhn (1955) further developed the assignment problem which has been as "Hungarian method" because the algorithm was largely based on the earlier works of two Hungarian mathematicians: Dénes Kőnig and Jenő Egerváry.

James Munkres (1957) reviewed the algorithm and observed that it is (strongly) polynomial. Since then the algorithm has been known also as Kuhn–Munkres algorithm or Munkres assignment algorithm. The time complexity of the original algorithm was $O(n^4)$, however Edmonds and Karp, and independently Tomizawa noticed that it can be modified to achieve an $O(n^3)$ running time. Ford and Fulkerson extended the method to general transportation problems. In 2006, it was discovered that Carl Gustav Jacobi had solved the assignment problem in the 19th century, and published posthumously in 1890 in Latin.

Thompson (1981) discussed a Recursive method for solving assignment problems which is a polynomially bounded non simplex method for solving assignment problems. This method begins by finding the optimal solution for a problem defined from the first row and they finding the optimum for a problem defined from rows one, two and so on until it solves the problem consisting of all rows. Hence it is a dimensional expanding rather than an improvement method. It has been shown that the row duals are non-increasing and the column duals non-decreasing. However this work was also published online in 2001.

Li and Smith (1995) have studied facility layout and location problems with stochastic congestion in the traffic circulation system. They developed an algorithm for Quadratic Assignment Problems. The algorithm for quadratic assignment problem is in fact a heuristic algorithm which they used for solving the complex problems for in traffic circulation system. The advantage of their algorithm was that the algorithm can be used for solving large scale Quadratic Assignment Problems with reasonable computing times and efficient performance. They claimed that their method is straight forward approach.

Ji et. al. (1997) discussed a new algorithm for the assignment problem which they also called an alternative to the Hungarian Method. There assignment algorithm is based on a 2n*2n matrix where operations are performed on the matrix until an optimal solution is found.

2.5 Travelling Salesman Problem

Hamilton (1856) studied the Travelling Salesman Problem by finding Pak and Circuits on the dodecahedral graph, which satisfied certain conditions like adjacency condition etc. Hamilton (1856) also introduced Icosian game which was marketed in 1859. This game was based on dodecahedral graph of the adjacency condition.

Menger (1930) studied Hamiltonian Path, in which he has discussed messenger problem. In messenger problem, he has discussed how to solve a postman problem as well as many travelers problem. Further he carried out to find shortest path joining all of a finite set of points where pair wise distances are known. However his work was unnoticed until and unless his book " the Travelling Salesman Problem" was published during 1931-32 in journal. So Monger was the first person who introduced the name "Travelling Salesman Problem".

2.6 Supply Chain Management

Oliver and Webber (1982) introduced the term Supply Chain Management. They further developed the Supply Chain Management system to express the need to integrate the key business processes from end user through original suppliers. Being those provides products, services and information that add values for Customers and other stake holders. The basic idea behind Supply

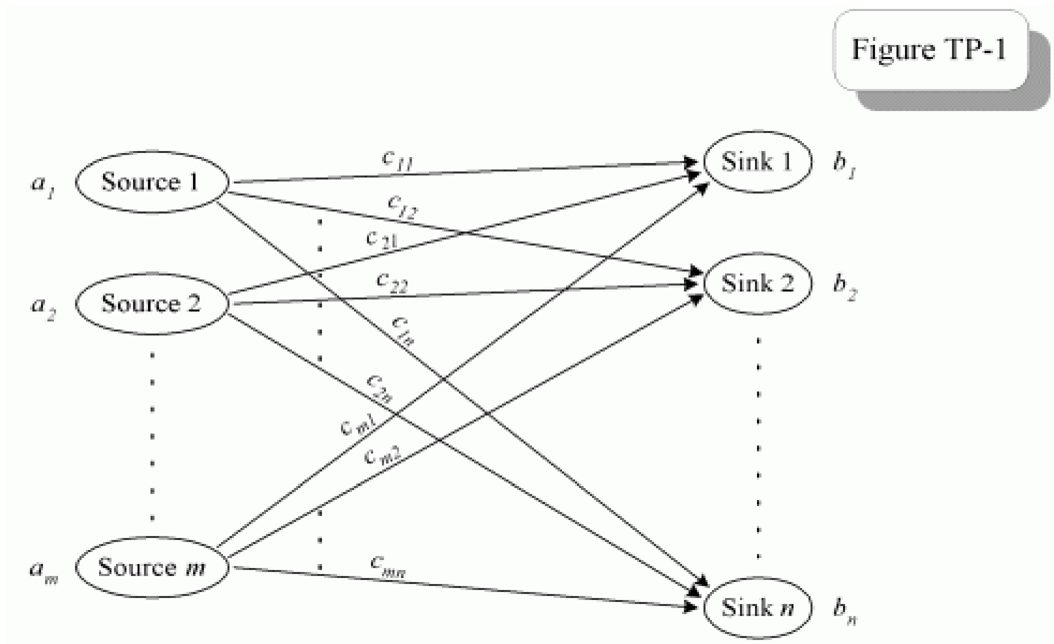
Chain Management is that companies and collaboration involved themselves in a Supply Chain by exchanging information regarding marketing fluctuation and production capability.

CHAPTER 3
NEW ALTERNATE METHODS OF TRANSPORTATION PROBLEM

3.1 Introduction

The transportation problem and cycle canceling methods are classical in optimization. The usual attributions are to the 1940's and later. However, Tolsto (1930) was a pioneer in operations research and hence wrote a book on transportation planning which was published by the National Commissariat of Transportation of the Soviet Union, an article called Methods of ending the minimal total kilometrage in cargo-transportation planning in space, in which he studied the transportation problem and described a number of solution approaches, including the, now well-known, idea that an optimum solution does not have any negative-cost cycle in its residual graph. He might have been the first to observe that the cycle condition is necessary for optimality. Moreover, he assumed, but did not explicitly state or prove, the fact that checking the cycle condition is also sufficient for optimality.

The transportation problem is concerned with finding an optimal distribution plan for a single commodity. A given supply of the commodity is available at a number of sources, there is a specified demand for the commodity at each of a number of destinations, and the transportation cost between each source-destination pair is known. In the simplest case, the unit transportation cost is constant. The problem is to find the optimal distribution plan for transporting the products from sources to destinations that minimizes the total transportation cost. This can be seen in Figure 1.



Here sources indicated the place from where transportation will begin, destinations indicates the place where the product has to be arrived and c_{ij} indicated the transportation cost transporting from source to destination and Sink denotes the destination.

There are various types of transportation models and the simplest of them was first presented by Hitchcock (1941). It was further developed by Koopmans (1949) and Dantzig (1951). Several extensions of transportation model and methods have been subsequently developed.

Transportation Problem (TP) is based on supply and demand of commodities transported from several sources to the different destinations. The sources from which we need to transport refer the supply while the destination where commodities arrive referred the demand. It has been seen that on many occasion, the decision problem can also be formatting as TP. In general we try to minimize total transportation cost for the commodities transporting from source to destination.

There are two types of Transportation Problem namely (1) Balanced Transportation Problem and (2) Unbalanced Transportation Problem.

Definition of Balanced Transportation Problem: A Transportation Problem is said to be balanced transportation problem if total number of supply is same as total number of demand.

Definition of Unbalanced Transportation Problem: A Transportation Problem is said to be unbalanced transportation problem if total number of supply is not same as total number of demand.

TP can also be formulated as linear programming problem that can be solved using either dual simplex or Big M method. Sometimes this can also be solved using interior approach method. However it is difficult to get the solution using all this method. There are many methods for solving TP. Vogel's method gives approximate solution while MODI and Stepping Stone (SS) method are considered as a standard technique for obtaining to optimal solution. Since decade these two methods are being used for solving TP. Goyal (1984) improving VAM for the Unbalanced Transportation Problem, Ramakrishnan (1988) discussed some improvement to Goyal's Modified Vogel's Approximation method for Unbalanced Transportation Problem. Moreover Sultan (1988) and Sultan and Goyal (1988) studied initial basic feasible solution and resolution of degeneracy in Transportation Problem. Few researchers have tried to give their alternate method for over coming major obstacles over MODI and SS method. Adlakha and Kowalski (1999, 2006) suggested an alternative solution algorithm for solving certain TP based on the theory of absolute point. Ji and Chu (2002) discussed a new approach so called Dual Matrix Approach to solve the Transportation Problem which gives also an optimal solution. Recently Adlakha and Kowalski (2009) suggested a systematic analysis for allocating loads to obtain an alternate optimal solution. However study on alternate optimal solution is limited in the literature of TP. In this chapter we have tried an attempt to provide two alternate algorithms for solving TP. It seems that the methods discussed by us in this chapter are simple and a state forward. We observed that for certain TP, our method gives the optimal solution. However for another certain TP, it gives the near to optimal solution. In this chapter we have discussed only balanced transportation problem for minimization case however these two methods can also be used for maximization case. Moreover, we may also use these two methods for unbalanced transportation problem for minimization and maximization case.

3.2 Mathematical Statement of the Problem

The classical transportation problem can be stated mathematically as follows:

Let a_i denotes quantity of product available at origin i, b_j denotes quantity of product required at destination j, C_{ij} denotes the cost of transporting one unit of product from source/origin i to destination j and x_{ij} denotes the quantity transported from origin i to destination j.

Assumptions: $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$

This means that the total quantity available at the origins is precisely equal to the total amount required at the destinations. This type of problem is known as balanced transportation problem. When they are not equal, the problem is called unbalanced transportation problem. Unbalanced transportation problems are then converted into balanced transportation problem using the dummy variables.

3.2.1 Standard form of Transportation Problem as L. P. Problem

Here the transportation problem can be stated as a linear programming problem as:

Minimise total cost Z= $\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$

Subject to $\sum_{j=1}^{n} x_{ij} = a_i$ for i=1, 2,…, m

$$\sum_{i=1}^{m} x_{ij} = b_j \qquad \text{for } j=1,2,\ldots,n$$

and $x_{ij} \geq 0$ for all i=1, 2,…, m and j=1,2,…,n

The transportation model can also be portrayed in a tabular form by means of a transportation table, shown in Table 3.1.

Table 3.1 Transportation Table

Origin(i)	Destination(j)				Supply(a_i)
	1	2	…	n	
1	$\boxed{x_{11}}$ c_{11}	$\boxed{x_{12}}$ c_{12}		$\boxed{x_{1n}}$ c_{1n}	a_1
2	$\boxed{x_{21}}$ c_{21}	$\boxed{x_{22}}$ c_{22}		$\boxed{x_{2n}}$ c_{2n}	a_2
…	…	…	…	…	…
M	$\boxed{x_{m1}}$ c_{m1}	$\boxed{x_{m2}}$ c_{m2}		$\boxed{x_{mn}}$ c_{mn}	a_m
Demand(b_j)	b_1	b_2	…	b_n	$\sum a_i = \sum b_j$

The number of constraints in transportation table is (m+n), where m denotes the number of rows and n denotes the number of columns. The number of variables required for forming a basis is one less, i.e. (m+n-1). This is so, because there are only (m+n-1) independent variables in the solution basis. In other words, with values of any (m+n-1) independent variables being given, the remaining would automatically be determined on the basis of those values. Also, considering the conditions of feasibility and non-negativity, the numbers of basic variables representing transportation routes that are utilized are equal to (m+n-1) where all other variables are non-basic, or zero, representing the unutilized routes. It means that a basic feasible solution of a transportation problem has exactly (m+n-1) positive components in comparison to the (m + n) positive components required for a basic feasible solution in respect of a general linear programming problem in which there are (m + n) structural constraints to satisfy.

3.3 Solution of the Transportation Problem

A transportation problem can be solved by two methods, using (a) Simplex Method and (b) Transportation Method. We shall illustrate this with the help of an example.

Example 3.3.1 A firm owns facilities at six places. It has manufacturing plants at places A, B and C with daily production of 50, 40 and 60 units respectively. At point D, E and F it has three warehouses with daily demands of 20, 95 and 35 units respectively. Per unit shipping costs are given in the following table. If the firm wants to minimize its total transportation cost, how should it route its products?

Table 3.2

		Warehouse		
		D	E	F
Plant	A	6	4	1
	B	3	8	7
	C	4	4	2

(a) Simplex Method

The given problem can be expressed as an LPP as follows:

Let x_{ij} represent the number of units shipped from plant i to warehouse j. Let Z representing the total cost, it can state the problem as follows.

The objective function is to,

$$\textbf{Minimise } Z = 6x_{11}+4x_{12}+1x_{13}+3x_{21}+8x_{22}+7x_{23}+4x_{31}+4x_{32}+2x_{33}$$

Subject to constrains:

$x_{11}+x_{12}+x_{13}=50$ $x_{11}+x_{21}+x_{31}=20$

$x_{21}+x_{22}+x_{23}=40$ $x_{12}+x_{22}+x_{32}=95$

$x_{31}+x_{32}+x_{33}=60$ $x_{13}+x_{23}+x_{33}=35$

$x_{ij}\geq 0$ for $i=1,2,3$ and $j=1,2,3$

Using Simplex method, the solution is going to be very lengthy and a cumbersome process because of the involvement of a large number of decision and artificial variables. Hence, for an alternate solution, procedure called the transportation method which is an efficient one that yields results faster and with less computational effort.

(b) Transportation Method

The transportation method consists of the following three steps.

1. Obtaining an initial solution, that is to say making an initial assignment in such a way that a basic feasible solution is obtained.
2. Ascertaining whether it is optimal or not, by determining opportunity costs associated with the empty cells, and if the solution is not optimal.
3. Revising the solution until an optimal solution is obtained.

3.3.1 Methods for Obtaining Basic Feasible Solution for Transportation Problem

The first step in using the transportation method is to obtain a feasible solution, namely, the one that satisfies the rim requirements (i.e. the requirements of demand and supply). The initial feasible solution can be obtained by several methods. The commonly used are

(I). North – west Corner Method

(II). Least Cost Method (LCM)

(III). Vogel's Approximation Method (VAM)

(I) North-West corner method (NWCM)

The North West corner rule is a method for computing a basic feasible solution of a transportation problem where the basic variables are selected from the North – West corner (i.e., top left corner).

Steps

1. Select the north west (upper left-hand) corner cell of the transportation table and allocate as many units as possible equal to the minimum between available supply and demand requirements, i.e., min (s_1, d_1).
2. Adjust the supply and demand numbers in the respective rows and columns allocation.
3. If the supply for the first row is exhausted then move down to the first cell in the second row.
4. If the demand for the first cell is satisfied then move horizontally to the next cell in the second column.
5. If for any cell supply equals demand then the next allocation can be made in cell either in the next row or column.
6. Continue the procedure until the total available quantity is fully allocated to the cells as required.

Table 3.3 Basic Feasible Solution Using North-West Corner Method of Example 3.3.1

From \ To	D	E	F	Supply
A	**20** 6	**30** 4	1	50
B	3	**40** 8	7	40
C	4	**25** 4	**35** 2	60
Demand	20	95	35	**150**

Total Cost: (6*20) + (4*30) + (8*40) + (4*25) + (2*35) = **Rs. 730**

This routing of the units meets all the rim requirements and entails 5 (=m+n-1 = 3+3-1) shipments as there are 5 occupied cells; It involves a total cost of Rs. 730.

(II) Least Cost Method (LCM)

Matrix minimum method is a method for computing a basic feasible solution of a transportation problem where the basic variables are chosen according to the unit cost of transportation.

Steps

1. Identify the box having minimum unit transportation cost (c_{ij}).
2. If there are two or more minimum costs, select the row and the column corresponding to the lower numbered row.
3. If they appear in the same row, select the lower numbered column.
4. Choose the value of the corresponding x_{ij} as much as possible subject to the capacity and requirement constraints.
5. If demand is satisfied, delete that column.
6. If supply is exhausted, delete that row.
7. Repeat steps 1-6 until all restrictions are satisfied.

Table 3.4 Basic Feasible Solution Using Least Cost Method of Example 3.3.1

To / From	D	E	F	Supply
A	6	**15** 4	**35** 1	50
B	**20** 3	**20** 8	7	40
C	4	**60** 4	2	60
Demand	20	95	35	**150**

Total Cost: 3*20 + 4*15 + 8*20 +4*60 + 1*35 = **Rs. 555**

This routing of the units meets all the rim requirements and entails 5 (=m+n-1 = 3+3-1) shipments as there are 5 occupied cells; It involves a total cost of Rs. 555.

(III) Vogel's Approximation Method (VAM)

The Vogel approximation method is an iterative procedure for computing a basic feasible solution of the transportation problem.

Steps

1. Identify the boxes having minimum and next to minimum transportation cost in each row and write the difference (penalty) along the side of the table against the corresponding row.
2. Identify the boxes having minimum and next to minimum transportation cost in each column and write the difference (penalty) against the corresponding column
3. Identify the maximum penalty. If it is along the side of the table, make maximum allotment to the box having minimum cost of transportation in that row. If it is below the table, make maximum allotment to the box having minimum cost of transportation in that column.
4. If the penalties corresponding to two or more rows or columns are equal, select the top most row and the extreme left column.

Table 3.5 Basic Feasible Solution Using Vogel's Method of Example 3.3.1

To / From	D	E	F	Supply	Iteration	
					I	II
A	6	$\boxed{15}$ 4	$\boxed{35}$ 1	50	3	**3**
B	$\boxed{20}$ 3	$\boxed{20}$ 8	7	40	**4**	1
C	4	$\boxed{60}$ 4	2	60	2	2
Demand	20	95	35	**150**		
I	1	0	1			
II	-	0	1			

Total Cost: 3*20 + 4*15 + 8*20 +4*60 + 1*35 = **Rs. 555**

This routing of the units meets all the rim requirements and entails 5 (=m+n-1 = 3+3-1) shipments as there are 5 occupied cells; It involves a total cost of Rs. 555.

3.3.2 Test for Optimality

Once an initial solution is obtained, the next step is to check its optimality. An optimal solution is one where there is no other set of transportation routes (allocations) that will further reduce the total transportation cost. Thus, we have to evaluate each unoccupied cell (represents unused route) in the transportation table in terms of an opportunity of reducing total transportation cost.

An unoccupied cell with the largest negative opportunity cost is selected to include in the new set of transportation routes (allocations). This is also known as an incoming variable. The outgoing variable in the current solution is the occupied cell (basic variable) in the unique closed path (loop) whose allocation will become zero first as more units are allocated to the unoccupied cell with largest negative opportunity cost. Such an exchange reduces total transportation cost. The process is continued until there is no negative opportunity cost. That is, the current solution cannot be improved further. This is the optimal solution.

An efficient technique called the modified distribution (MODI) method (also called u-v method).

Now we discuss MODI method which gives optimal solution and is shown in 3.3.2.1.

3.3.2.1 Modified Distribution (MODI) Method

Steps

1. Determine an initial basic feasible solution using any one of the three given methods which are namely, North West Corner Method, Least Cost Method and Vogel Approximation Method.
2. Determine the values of dual variables, u_i and v_j, using $u_i + v_j = c_{ij}$
3. Compute the opportunity cost using $d_{ij} = c_{ij} – (u_i + v_j)$ from unoccupied cell.

4. Check the sign of each opportunity cost (d_{ij}). If the opportunity costs of all the unoccupied cells are either positive or zero, the given solution is the optimum solution. On the other hand, if one or more unoccupied cell has negative opportunity cost, the given solution is not an optimum solution and further savings in transportation cost are possible.
5. Select the unoccupied cell with the smallest negative opportunity cost as the cell to be included in the next solution.
6. Draw a closed path or loop for the unoccupied cell selected in the previous step. Please note that the right angle turn in this path is permitted only at occupied cells and at the original unoccupied cell.
7. Assign alternate plus and minus signs at the unoccupied cells on the corner points of the closed path with a plus sign at the cell being evaluated.
8. Determine the maximum number of units that should be shipped to this unoccupied cell. The smallest value with a negative position on the closed path indicates the number of units that can be shipped to the entering cell. Now, add this quantity to all the cells on the corner points of the closed path marked with plus signs and subtract it from those cells marked with minus signs. In this way an unoccupied cell becomes an occupied cell.
9. Repeat the whole procedure until an optimum solution is obtained.

Table 3.6 Optimal Solution Using MODI Method of Example 3.3.1

From \ To	D	E	F	Supply	u_i
A	6	$\boxed{15}$ 4	$\boxed{35}$ 1	50	$u_1=0$
B	$\boxed{20}$ 3	$\boxed{20}$ 8	7	40	$u_2=4$
C	4	$\boxed{60}$ 4	2	60	$u_3=0$
Demand	20	95	35	**150**	
v_j	$v_1=-1$	$v_2=4$	$v_3=1$		

Total Cost: 3*20 + 4*15 + 8*20 +4*60 + 1*35 = **Rs. 555**

This routing of the units meets all the rim requirements and entails 5 (=m+n-1 = 3+3-1) shipments as there are 5 occupied cells; It involves a total cost of Rs. 555.

3.4 NEW ALTERNATE METHOD FOR SOLVING TRANSPORTATION PROBLEM

So far three general methods for solving transportation methods are available in literature which is already discussed. These methods give only initial feasible solution. However here we discuss two new alterative methods which give Initial feasible solution as well as optimal or nearly optimal solution. Apart from above three methods, other two methods called MODI method and Stepping Stone method give the optimal solution. But to get the optimal solution, first of all we have to find initial solution from either of three methods discussed. However the methods discussed in this chapter gives initial as well as either optimal solution or near to optimal solution. In other sense we can say that if we apply any one of the two methods, it gives either initial feasible solution as well as optimal solution or near to optimal solution.

3.4.1 Algorithm for Solving Transportation Problem Using New Method

Following are the steps for solving Transportation Problem

Step 1 Select the first row (source) and verify which column (destination) has minimum unit cost. Write that source under column 1 and corresponding destination under column 2. Continue this process for each source. However if any source has more than one same minimum value in different destination then write all these destination under column 2.

Step 2 Select those rows under column-1 which have unique destination. For example, under column-1, sources are O_1, O_2, O_3 have minimum unit cost which represents the destination D_1, D_1, D_3 written under column 2. Here D_3 is unique and hence allocate cell (O_3, D_3) a minimum of demand and supply. For an example if corresponding to that cell supply is 8, and demand is 6, then allocate a value 6 for that cell. However, if destinations are not unique then follow step 3. Next delete that row/column where supply/demand exhausted.

Step 3 If destination under column-2 is not unique then select those sources where destinations are identical. Next find the difference between minimum and next minimum unit cost for all those sources where destinations are identical.

Step 4 Check the source which has maximum difference. Select that source and allocate a minimum of supply and demand to the corresponding destination. Delete that row/column where supply/demand exhausted.

Remark 1 For two or more than two sources, if the maximum difference happens to be same then in that case, find the difference between minimum and next to next minimum unit cost for those sources and select the source having maximum difference. Allocate a minimum of supply and demand to that cell. Next delete that row/column where supply/demand exhausted.

Step 5 Repeat steps 3 and 4 for remaining sources and destinations till (m+n-1) cells are allocated.

Step 6 Total cost is calculated as sum of the product of cost and corresponding allocated value of supply/ demand. That is,

Total cost = $\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} x_{ij}$

3.4.2 Numerical Examples

In this section we present a detailed example to illustrated the steps of the proposed alternate method.

Example 3.4.2.1 Let us consider the Example 3.3.1.

A firm owns facilities at six places. It has manufacturing plants at places A, B and C with daily production of 50, 40 and 60 units respectively. At point D, E and F it has three warehouses with daily demands of 20, 95 and 35 units respectively. Per unit shipping costs are given in the following table. If the firm wants to minimize its total transportation cost, how should it route its products?

Table 3.7

		Warehouse		
		D	E	F
Plant	A	6	4	1
	B	3	8	7
	C	4	4	2

Solution

Step 1 The minimum cost value for the corresponding sources A,B,C are 1, 3 and 2 which represents the destination F, D and F respectively which is shown in Table 3.8.

Table 3.8

Column 1	Column 2
A	F
B	**D**
C	F

Step 2 Here the destination D is unique for source B and allocate the cell (B, D) min (20, 40) =20. This is shown in Table 3.9.

Table 3.9

To / From	D	E	F	Supply
A	6	4	1	50
B	$\boxed{20}$ 3	8	7	40
C	4	4	2	60
Demand	20	95	35	**150**

Step 3 Delete column D as for this destination demand is exhausted and adjust supply as (40-20) =20. Next the minimum cost value for the corresponding sources A,B,C are 1, 7 and 2 which represents the destination F, F and F respectively which is shown in Table 3.10.

Table 3.10

Column 1	Column 2
A	F
B	F
C	F

Here the destinations are not unique because sources A, B, C have identical destination F. so we find the difference between minimum and next minimum unit cost for the sources A, B and C. The differences are 3, 1 and 2 respectively for the sources A, B and C.

Step 4: Here the maximum difference is 3 which represents source A. Now allocate the cell (A, F), min (50, 35) = 35 which is shown Table 3.11.

Table 3.11

From \ To	E	F	Supply
A	4	$\boxed{35}$ 1	50
B	8	7	20
C	4	2	60
Demand	95	35	**150**

Step 5: Delete column F as demand is exhausted. Next adjust supply as (50-35) =15. Next the minimum unit cost for the corresponding sources A,B and C are 4, 8 and 4 which represents the destination E, E and E respectively which is shown in Table 3.12.

Table 3.12

Column 1	Column 2
A	E
B	E
C	E

Here the source A, B, C have identical destination E, so we must find minimum difference. However only one column remain and hence minimum difference can not be obtained. So allocate the remaining supply 15, 20 and 60 to cells (A, E) (B, E) and (C, E) which is shown in Table 3.13.

Table 3.13

To / From	E	Supply
A	15 (cost 4)	15
B	20 (cost 8)	20
C	60 (cost 4)	60
Demand	95	**150**

Step 6: Here (3+3-1) =5 cells are allocated and hence we got our feasible solution. Next we calculate total cost as some of the product of cost and its corresponding allocated value of supply/demand which is shown in Table 3.14.

Table 3.14 Basic Feasible Solution using new method

To / From	D	E	F	Supply
A	6	15 (cost 4)	35 (cost 1)	50
B	20 (cost 3)	20 (cost 8)	7	40
C	4	60 (cost 4)	2	60
Demand	20	95	35	**150**

Total cost: (15*4) + (35*1) + (20*3) + (20*8) + (60*4) = **555**

This is a basic feasible solution. The solutions obtained using NCM, LCM, VAM and MODI/SSM is 730, 555, 555 and 555 respectively. Hence the basic feasible solution obtained from new method is **optimal solution**.

Result Our solution is same as that of optimal solution obtained by using LCM, VAM and MODI/Stepping stone method. Thus our method also gives optimal solution.

Example 3.4.2.2 A company has factories at F_1, F_2 and F_3 which supply to warehouses at W_1, W_2 and W_3. Weekly factory capacities are 200, 160 and 90 units, respectively. Weekly warehouse requirement are 180, 120 and 150 units, respectively. Unit shipping costs (in rupees) are as follows:

Table 3.15

	W_1	W_2	W_3	Supply
F_1	16	20	12	200
F_2	14	8	18	160
F_3	26	24	16	90
Demand	180	120	150	450

Determine the optimal distribution for this company to minimize total shipping cost.

Solution Here first of all we will obtain the basic feasible solution using NWCM, LCM, VAM and MODI shown in Table 3.16, Table 3.17, Table 3.18 and Table 3.19 respectively and then using the new alternate method discussed in this chapter and this shown in Table 3.20.

3.16 Basic feasible solution using North-West Corner Method

To / From	W1	W2	W3	Supply
F1	180 / 16	20 / 20	12	200
F2	14	100 / 8	60 / 18	160
F3	26	24	90 / 16	90
Demand	180	120	150	**450**

Total Cost: (180*16) + (20*20) + (100*8) + (60*18) + (90*16) = **6600**

3.17 Basic feasible solution using Least Cost Method

To / From	W1	W2	W3	Supply
F1	50 / 16	20	150 / 12	200
F2	40 / 14	120 / 8	18	160
F3	90 / 26	24	16	90
Demand	180	120	150	**450**

Total cost: (50*16) + (150*12) + (40*14) + (120*8) + (90*26) = **6460**

3.18 Basic feasible solution using Vogel's Approximation Method

To / From	W1	W2	W3	Supply	Iteration		
F1	140 / 16	20	60 / 12	200	4	4	4
F2	40 / 14	120 / 8	18	160	6	4	4
F3	26	24	90 / 16	90	8	**10**	**-**
Demand	180	120	150	**450**			
Iteration	2	**12**	4				
	2	-	4				
	2	-	**6**				

Total cost: (140*16) + (60*12) + (40*14) + (120*8) + (90*16) = **5920**

3.19 Optimal Solution using MODI Method

To / From	W1	W2	W3	Supply	u_i
F1	140 / 16	20	60 / 12	200	u_1= 12
F2	40 / 14	120 / 8	18	160	u_2=10
F3	26	24	90 / 16	90	u_3= 16
Demand	180	120	150	**450**	
v_j	v_1= 4	v_2= -2	v_3=0		

Total Cost= (140*16) + (60*12) + (40*14) + (120*8) + (90*16) = **5920**

Now following algorithm 3.4.1, we solve example 3.4.2.2 using new alternate method and obtained the initial feasible solution which is shown in Table 3.20.

Table 3.20 showing the solution of Example 3.4.2.2 using new alternate method

To / From	W1	W2	W3	Supply
F1	140 / 16	20	60 / 12	200
F2	40 / 14	120 / 8	18	160
F3	26	24	90 / 16	90
Demand	180	120	150	**450**

Total cost= (140*16) + (60*12) + (40*14) + (120*8) + (90*16) = **5920**

This is an initial feasible solution. The solutions obtained from NCM, LCM, VAM and MODI/SSM is 6600, 6460, 5920 and 5920 respectively. Hence the basic initial feasible solution obtained from new method is **optimal solution**.

Result Our solution is same as that of optimal solution obtained by using VAM and MODI/Stepping stone method. Thus our method also gives optimal solution.

Now we illustrate some more numerical examples using new alternate method.

Example 3.4.2.3 The following table shows on the availability of supply to each warehouse and the requirement of each market with transportation cost (in rupees) from each warehouse to each market. In market demands are 300, 200 and 200 units while the warehouse has supply for 100, 300 and 300 units.

Table 3.21

	K1	K2	K3	Supply
R1	5	4	3	100
R2	8	4	3	300
R3	9	7	5	300
Demand	300	200	200	700

Determine the total cost for transporting from warehouse to market.

Solution: Now following algorithm 3.4.1, we solve example 3.4.2.3 using new alternate method and obtained the Basic feasible solution which is shown in Table 3.22.

Table 3.22 showing the solution of Example 3.4.2.3 using new alternate method

	K1	K2	K3	Supply
R1	$\boxed{100}$ 5	4	3	100
R2	$\boxed{100}$ 8	$\boxed{200}$ 4	3	300
R3	$\boxed{100}$ 9	7	$\boxed{200}$ 5	300
Demand	300	200	200	700

Total cost= (100*5) + (100*8) + (200*4) + (100*9) + (200*5) = 500+800+800+900+1000= 4000.

This is a basic initial feasible solution. The solutions obtained from NCM, LCM, VAM and MODI/SSM is 4200, 4100, 3900 and 3900 respectively. Hence the basic initial feasible solution obtained from new method is **near to optimal solution.**

Result Our solution is least than that of NWCM and LCM, while more than VAM and MODI/Stepping stone method. Thus our method gives near to optimal solution.

Example 3.4.2.4 Determine an initial feasible solution to the following transportation problem where O_i and D_j represent i^{th} origin and j^{th} destination, respectively.

Table 3.23

		Destination				
		D_1	D_2	D_3	D_4	Supply
Origin	O_1	6	4	1	5	14
	O_2	8	9	2	7	16
	O_3	4	3	6	2	5
	Demand	6	10	15	4	35

Solution Now following algorithm 3.4.1, we solve example 3.4.2.4 using new alternate method and obtained the Basic feasible solution which is shown in Table 3.24.

Table 3.24 showing the solution of Example 3.4.2.4 using new alternate method

	D1	D2	D3	D4	Supply
O1	$\boxed{5}$ 6	$\boxed{9}$ 4	1	5	14
O2	$\boxed{1}$ 8	9	$\boxed{15}$ 2	7	16
O3	4	$\boxed{1}$ 3	6	$\boxed{4}$ 2	5
Demand	6	10	15	4	35

Total cost= (6*5) + (9*4) + (1*8) + (15*2) + (1*3) + (4*2) = 30+36+8+30+3+8 = 115.

This is a basic feasible solution. The solutions obtained from NCM, LCM, VAM and MODI/SSM is 128, 156, 114 and 114 respectively. Hence the basic feasible solution obtained from new method is **near to optimal solution**.

Result Our solution is least than that of NWCM and LCM, while more than VAM and MODI/Stepping stone method. Thus our method gives near to optimal solution.

Example 3.4.2.5 The following table shows all the necessary information on the availability of supply to each warehouse, the requirement of each market and unit transportation cost (in Rs) from each warehouse to each market.

Table 3.25

		Market				Supply
		P	Q	R	S	
Warehouse	A	6	3	5	4	22
	B	5	9	2	7	15
	C	5	7	8	6	8
Demand		7	12	17	9	45

Determine minimum cost value for this transportation problem.

Solution Now following algorithm 3.4.1, we solve example 3.4.2.5 using new alternate method and obtained the Basic feasible solution which is shown in Table 3.26.

Table 3.26 showing the solution of Example 3.4.2.5 using new alternate method

		Market				Supply
		P	Q	R	S	
Warehouse	A	6	[12] 3	[2] 5	[8] 4	22
	B	5	9	[15] 2	7	15
	C	[7] 5	7	8	[1] 6	8
Demand		7	12	17	9	45

Total cost= (12*3)+(2*5)+(8*4)+(15*2)+(7*5)+(1*6) = 149

This is a basic feasible solution. The solutions obtained from NCM, LCM, VAM and MODI/SSM is 176, 150, 149 and 149 respectively. Hence the basic initial feasible solution obtained from new method is **optimal solution**.

Result The solution obtained for example 3.4.2.5 using new alternate method is same as that of optimal solution obtained using VAM and MODI/Stepping stone method. Thus the method discussed in this chapter also gives same optimal solution.

3.5 ANOTHER NEW ALTERNATE METHOD (MINIMUM DEMAND-SUPPLY METHOD) FOR SOLVING TRANSPORTATION PROBLEM

In section 3.4 we discussed a new alternate method for solving a TP based on unique activity. Next we discussed another new alternate method which is based on minimum demand-supply techniques. For this purpose, we explain algorithm for solving Transportation Problem in 3.5.1.

3.5.1 Algorithm for solving Transportation Problem using alternate method:

Following are the steps for solving Transportation Problem

Step1 Formulate the problem and set up in the matrix form. The formulation of TP is similar to that of LPP. So objective function is the total transportation cost and constraints are the supply and demand available at each source and destination respectively.

Step 2 Select that row/column where supply/demand is minimum. Find the minimum cost value in that respective row/column. Allocate minimum of supply demand to that cell.

Step 3 Adjust the supply/demand accordingly.

Step 4 Delete that row/column where supply/demand is exhausted.

Step 5 Continue steps 1 to step 3 till (m+n-1) cells are allocate.

Step 6 Total cost is calculated as sum of the product of cost and corresponding assigned value of supply / demand. That is, Total cost = $\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} x_{ij}$

Remark The new alternate method discussed in this section is optimal for certain TP while it gives initial feasible solution only for another certain TP.

3.5.2 Numerical Examples

Example 3.5.2.1 A firm owns facilities at six places. It has manufacturing plants at places A, B and C with daily production of 50, 40 and 60 units respectively. At point D, E and F it has three warehouses with daily demands of 20, 95 and 35 units respectively. Per unit shipping costs are given in the following table. If the firm wants to minimize its total transportation cost, how should it route its products?

Table 3.27

		Warehouse		
		D	E	F
Plant	A	6	4	1
	B	3	8	7
	C	4	4	2

Solution

Step 1 General transportation matrix is shown in Table 3.28.

Table 3.28

To / From	D	E	F	Supply
A	6	4	1	50
B	3	8	7	40
C	4	4	2	60
Demand	20	95	35	**150**

Step 2 In example 3.5.2.1, among supply and demand, minimum is demand which represents column D. In column D, the minimum unit cost is in cell (B, D). Corresponding to this cell demand is 20 and supply is 40. So allocate min (20, 40) =20 to cell (B, D).

Step 3 For row B is adjusted as 40-20=20, which is shown in Table 3.29.

Table 3.29

To / From	D	E	F	Supply
A	6	4	1	50
B	$\boxed{20}$ 3	8	7	40-20=20
C	4	4	2	60
Demand	20	95	35	**150**

Step 4 Since demand in column D is exhausted and hence delete column D.

Step 5 Next among supply and demand, minimum is supply which represents row B. In row B, the minimum unit cost is in cell (B, F). Corresponding to this cell demand is 35 and supply is 20. So allocate min (20, 35) =20 to cell (B, F). For column F is adjusted as 35-20=15, which is shown in Table 3.30.

Table 3.30

From \ To	E	F	Supply
A	4	1	50
B	8	$\boxed{20}$ 7	20
C	4	2	60
Demand	95	35-20=15	**150**

Step 5 Since supply in row B is exhausted and hence delete row B. Next among supply and demand, minimum is demand which represents column F. In column F, the minimum cost value is in cell (A, F). Corresponding to this cell demand is 15 and supply is 50. So allocate min (15, 50) =15 to cell (A, F). Now row A is adjusted as 50-15=35, which is shown in Table 3.31.

Table 3.31

From \ To	E	F	Supply
A	4	$\boxed{15}$ 1	50-15=35
C	4	2	60
Demand	95	15	**150**

Step 5 Since supply in column F is exhausted and hence delete column F. Next among supply and demand, minimum is supply which represents row A. In row A, the minimum cost value is in cell (A, E). Corresponding to this cell demand is

95 and supply is 35. So allocate min (95, 35) =35 to cell (A, E). Now column E is adjusted as 95-35=60, which is shown in Table 3.32.

Table 3.32

From \ To	E	Supply
A	35 4	35
C	4	60
Demand	95-35=60	**150**

Step 5 Here only one cell C is remains so allocate min (60, 60) =60 to cell (C, E). The final allocated supply and demand is shown in Table 3.33.

Table 3.33 Basic feasible solution using another new method

From \ To	D	E	F	Supply
A	6	35 4	15 1	50
B	20 3	8	20 7	40 20
C	4	60 4	2	60
Demand	20	95	35 15	**150**

In Table 3.33, (3+3-1) =5 cells are allocate and hence we got our feasible solution.

Step 5 Total cost: (35*4) + (15*1) + (20*3) + (20*7) + (60*4) = 595

The solution obtained in our method is less than NWCM but grater than LCM, VAM and MODI. Hence it is better than NWC method and gives **near to optimal solution.**

Example 3.5.2.2 A company has factories at F_1, F_2 and F_3 which supply to warehouses at W_1, W_2 and W_3. Weekly factory capacities are 200, 160 and 90 units, respectively. Weekly warehouse requirement are 180, 120 and 150 units, respectively. Unit shipping costs (in rupees) are as follows:

Table 3.34

		Warehouses			
		W_1	W_2	W_3	Supply
Factory	F_1	16	20	12	200
	F_2	14	8	18	160
	F_3	26	24	16	90
	Demand	180	120	150	450

Determine the optimal distribution for this company to minimize total shipping cost.

Solution

Step 1 General transportation matrix is shown in Table 3.35

Table 3.35

To / From	W_1	W_2	W_3	Supply
F_1	16	20	12	200
F_2	14	8	18	160
F_3	26	24	16	90
Demand	180	120	150	**450**

Step 2 In example 3.5.2.2, among supply and demand, minimum is supply which represents row F_3. In row F_3, the minimum cost value is in cell (F_3, W_3). Corresponding to this cell demand is 150 and supply is 90. So allocate min (150, 90) =90 to cell (F_3, W_3).

Step 3 For column W_3 is adjusted as 150-90=60, which is shown in Table 3.36.

Table 3.36

To / From	W_1	W_2	W_3	Supply
F_1	16	20	12	200
F_2	14	8	18	160
F_3	26	24	$\boxed{90}$ 16	90
Demand	180	120	150-90=60	**450**

Step 4 Since supply in row F_3 is exhausted and hence delete row F_3.

Step 5 Next among supply and demand, minimum is demand which represents column W_3. In column W_3, the minimum cost value is in cell (F_1, W_3). Corresponding to this cell demand is 60 and supply is 200. So allocate min (60, 200) =60 to cell (F_1, W_3). For row F_1 is adjusted as 200-60=140, which is shown in Table 3.37.

Table 3.37

To / From	W_1	W_2	W_3	Supply
F_1	16	20	$\boxed{60}$ 12	200-60=140
F_2	14	8	18	160
Demand	180	120	60	**450**

Step 5 Since demand in column W_3 is exhausted and hence delete column W_3. Next among supply and demand, minimum is demand which represents column W_2. In column W_2, the minimum cost value is in cell (F_2, W_2). Corresponding to this cell demand is 120 and supply is 160. So allocate min (120, 160) =120 to cell (F_2, W_2). Now row F_2 is adjusted as 160-120=40, which is shown in Table 3.38.

Table 3.38

To / From	W_1	W_2	Supply
F_1	16	20	140
F_2	14	$\boxed{120}$ 8	160-120=40
Demand	180	120	**450**

Step 5 Since demand in column W_2 is exhausted and hence delete column W_2. Next among supply and demand, minimum is supply which represents row F_2. In row F_2, the minimum cost value is in cell (F_2, W_1). Corresponding to this cell demand is 180 and supply is 40. So allocate min (180, 40) =40 to cell (F_2, W_1). Now column W_1 is adjusted as 180-40=140, which is shown in Table 3.39.

Table 3.39

To / From	W_1	Supply
F_1	16	140
F_2	$\boxed{40}$ 14	40
Demand	180-40=140	**450**

Step 5 Here only one cell (F_1, W_1) is remains so allocate min (140, 140) =140. The final allocated supply and demand is shown in Table 3.40.

Table 3.40 Basic feasible solution using another new method

To / From	W1	W2	W3	Supply
F1	$\boxed{140}$ 16	20	$\boxed{60}$ 12	200
F2	$\boxed{40}$ 14	$\boxed{120}$ 8	18	160
F3	26	24	$\boxed{90}$ 16	90
Demand	180	120	150	**450**

In Table 3.40, (3+3-1) =5 cells are allocate and hence we achieved feasible solution.

Total cost= (140*16) + (60*12) +(40*14)+(120*8)+(90*16) = **5920**

This is a basic initial feasible solution. The solutions obtained from NCM, LCM, VAM and MODI/SSM is 6600, 6460, 5920 and 5920 respectively. Hence the basic initial feasible solution obtained from new method is **optimal solution**.

Result: Our solution is same as that of optimal solution obtained by using VAM and MODI/Stepping stone method. Thus our method also gives optimal solution.

Remark We have solved example 3.4.2.1(3.5.2.1) using both the **new alternate methods** discussed in 3.4 and 3.5. However we got the same optimal solution from both the methods. This proves that both methods give **optimal solution** for certain TP.

Example 3.5.2.3: The following table shows on the availability of supply to each warehouse and the requirement of each market with transportation cost (in rupees) from each warehouse to each market. In market demands are 300, 200 and 200 units while the warehouse has supply for 100, 300 and 300 units.

Table 3.38

	K1	K2	K3
R1	5	4	3
R2	8	4	3
R3	9	7	5

Determine the total cost for transporting from warehouse to market.

Solution Now following algorithm 3.5.1, we solve example 3.5.2.3 (using another new alternate method) and obtained the Basic feasible solution which is shown in Table 3.39.

Table 3.39 Basic feasible solution using another new method

	K1	K2	K3	Supply
R1	5	4	$\boxed{100}$ 3	100
R2	8	$\boxed{200}$ 4	$\boxed{100}$ 3	300
R3	$\boxed{300}$ 9	7	5	300
Demand	300	200	200	700

Total cost: (100*3) + (200*4) + (100*3) + (300*9) = 300+800+300+2700 = 4100

This value is same as obtained from LCM while better than NWCM but more than VAM/MODI method. This solution is happened to be **near to optimal solution** as we get directly from new alternate method.

Example 3.5.2.4 Determine an initial feasible solution to the following transportation problem where O_i and D_j represent i^{th} origin and j^{th} destination, respectively.

Table 3.40

	D_1	D_2	D_3	D_4	Supply
O_1	6	4	1	5	14
O_2	8	9	2	7	16
O_3	4	3	6	2	5
Demand	6	10	15	4	35

Solution Now following algorithm 3.5.1, we solve example 3.5.2.4 (using another new alternate method) and obtained the Basic feasible solution which is shown in Table 3.41.

Table 3.41 Basic feasible solution using another new method

	D1	D2	D3	D4	Supply
O1	$\boxed{6}$ 6	4	$\boxed{8}$ 1	5	14
O2	8	$\boxed{9}$ 9	$\boxed{7}$ 2	7	16
O3	4	$\boxed{1}$ 3	6	$\boxed{4}$ 2	5
Demand	6	10	15	4	35

Minimum cost: (6*6)+(8*1)+(9*9)+(7*2)+(1*3)+(4*2)= 36+8+81+14+3+8 = 150

This value is least than obtained from LCM but more than NWCM, VAM and MODI method. This solution is happened to be **near to optimal solution** as we get directly from new alternate method.

Example 3.5.2.5 The following table shows all the necessary information on the availability of supply to each warehouse, the requirement of each market and unit transportation cost (in Rs) from each warehouse to each market.

Table 3.42

		Market				Supply
		P	Q	R	S	
Warehouse	A	6	3	5	4	22
	B	5	9	2	7	15
	C	5	7	8	6	8
Demand		7	12	17	9	45

Determine minimum cost value for this transportation problem.

Solution Now following algorithm 3.5.1, we solve example 3.5.2.5 (using another new alternate method) and obtained the Basic feasible solution which is shown in Table 3.43.

Table 3.43 Basic feasible solution using another new method

		Market				Supply
		P	Q	R	S	
Warehouse	A	6	$\boxed{12}$ 3	$\boxed{2}$ 5	$\boxed{8}$ 4	22
	B	5	9	$\boxed{15}$ 2	7	15
	C	$\boxed{7}$ 5	7	8	$\boxed{1}$ 6	8
Demand		7	12	17	9	45

Total cost= (12*3)+(2*5)+(8*4)+(15*2)+(7*5)+(1*6) = 149

This is a basic initial feasible solution. The solutions obtained from NCM, LCM, VAM and MODI/SSM is 176, 150, 149 and 149 respectively. Hence the basic initial feasible solution obtained from new method is **optimal solution**.

Table 3.44 shows comparison of total cost of transportation problem obtained from various methods

Examples	Method 1	Method 2	NWCM	LCM	VAM	MODI
3.6.2.1	**555**	595	730	555	555	555
3.6.2.2	**5920**	**5920**	6600	6460	5920	5920
3.6.2.3	4000	4100	4200	4100	3900	3900
3.6.2.4	115	150	128	156	114	114
3.6.2.5	**149**	**149**	176	150	149	149

Remark From Table 3.44, it is clear that method 1 gives optimal solution for almost all examples while method 2 gives optimal solution for certain examples of TP.

3.6 Conclusion

This chapter deals two alternate algorithms for TP as very few alternate algorithms for obtaining an optimal solution are available in the textbook and in other literature. These methods are so simple and easy that makes understandable to a wider spectrum of readers. The methods discussed in this chapter give a near optimal solution for certain TP while it gives optimal solution for other certain TP.

CHAPTER 4
A NEW ALTERNATE METHOD OF TRANS-SHIPMENT PROBLEM

4.1 Introduction

In a transportation problem shipment of commodity takes place among sources and destinations. But instead of direct shipments to destinations, the commodity can be transported to a particular destination through one or more intermediate or trans-shipment points. Each of these points in turn supply to other points. Thus, when the shipments pass from destination to destination and from source to source, we have a trans-shipment problem. Since transshipment problem is a particular case of transportation problem hence to solve transshipment problem, we first convert transshipment problem into equivalent transportation problem and then solve it to obtain optimal solution using MODI method of transportation problem. In a transportation problem, shipments are allowed only between source-sink pairs. In many applications, this assumption is too strong. For example, it is often the case that shipments may be allowed between sources and between sinks. Moreover, some points may exist through which units of a product can be transshipped from a source to a sink. Models with these additional features are called transshipment problems. Interestingly, it turns out that any given transshipment problem can be converted easily into an equivalent transportation problem. The availability of such a conversion procedure significantly broadens the applicability of our algorithm for solving transportation problems.

A transportation problem allows only shipments that go directly from supply points to demand points. In many situations, shipments are allowed between supply points or between demand points. Sometimes there may also be points (called transshipment points) through which goods can be transshipped on their journey from a supply point to a demand point. Fortunately, the optimal solution to a transshipment problem can be found by solving a transportation problem.

Some Definition:

Supply point: It can send goods to another point but cannot receive goods from any other point.
Demand point: It can receive goods from other points but cannot send goods to any other point.
Transshipment point: It can both receive goods from other points send goods to other points.

Orden (1956) introduced the concept of Transshipment Problem. He extended the concept of original transportation problem so as to include the possibility of transshipment. In other words, we can say that any shipping or receiving point is also permitted to act as an intermediate point. In fact, the transshipment technique is used to find the shortest route from one point to another point representing the network diagram. Rhody (1963) considered Transshipment model as reduced matrix model. King and Logan (1964) argued that the problem of determining simultaneously the flows of primary products through processor to the market as final product has been formulated alternatively as a transshipment model. However Judge et al. (1965) formulated the Transshipment model into general linear programming model. Garg and Prakash (1985) studied time minimizing Transshipment model. However Herer and Tzur (2001) discussed dynamic Transshipment Problem. In the standard form, the Transshipment problem is basically a linear minimum cost network. For such types of optimization problems, a number of effective solutions are available in the literature since many years. Recently Khurana and Arora (2011) discussed transshipment problem with mixed constraints. In this chapter, we have discussed a simple and alternate method for solving Transshipment Problem which gives either an optimal solution or near to optimal solution.

4.2 Mathematical Statement of the Problem

If we let the sources and destinations in a transshipment problem as T, then x_{ij} would represent the amount of goods shipped from the i^{th} terminal (T_i) to the j^{th} terminal (T_j) and c_{ij} would represent the unit cost of such shipment. Naturally, x_{ij} would equal to zero because no units would be shipped from a terminal to itself. Now, assume that at m terminals ($T_1, T_2, \dots, T_m$), the total out shipment exceeds the total in shipment by amounts equal to $a_1, a_2, \dots, a_m$ respectively and at the remaining n terminals ($T_{m+1}, T_{m+2}, \dots, T_{m+n}$), the total in shipment exceeds the total out shipment by amounts $b_{m+1}, b_{m+2}, \dots, b_{m+n}$ respectively. If the total in shipment at terminals $T_1, T_2, \dots, T_m$ be $t_1, t_2, \dots, t_m$ respectively and the total out shipment at the terminals $T_{m+1}, T_{m+2}, \dots, T_{m+n}$ be $t_{m+1}, t_{m+2}, \dots, t_{m+n}$ respectively, we can write the transshipment problem as:

$$Minimize\ Z = \sum_{i=1}^{m+n} \sum_{j=1}^{m+n} c_{ij}\, x_{ij}$$

Subject to

$$x_{i1} + x_{i2} + \cdots + x_{i,i-1} + x_{i,i+1} + \cdots + x_{i,m+n} - t_i = a_i \qquad (i = 1,2, \dots, m) \qquad (1)$$

$$x_{1i} + x_{2i} + \cdots + x_{i-1,i} + x_{i+1,i} + \cdots + x_{m+n,i} = t_i \qquad (i = 1,2, \dots, m) \qquad (2)$$

$$\begin{aligned} x_{1j} + x_{2j} + \cdots + x_{j-1,j} + x_{j+1,j} + \cdots + x_{m+n,j} - t_j \\ = b_j \quad (j = m+1, m+2, \dots, m+n) \qquad (3) \end{aligned}$$

$$\begin{aligned} x_{j1} + x_{j2} + \cdots + x_{j,j-1} + x_{j,j+1} + \cdots + x_{j,m+n} \\ = t_j \qquad (j = m+1, m+2, \dots, m+n) \quad (4) \end{aligned}$$

$$\sum_{i=1}^{m} a_i = \sum_{j=m+1}^{m+n} b_j \qquad (5)$$

As can be easily observed, these constraints are similar to the constraints of a transportation problem with m+n sources and m+n destinations, with the differences that here there are no x_{ii} and x_{jj} terms, and that $b_j=0$ for $j=1,2,\dots,m$

and a_i = 0 for i=m+1,m+2,…,m+n. The terms t_i and t_j in these constraints may be seen as the algebraic equivalents of x_{ii} and x_{jj}. Now, we can view this problem as an enlarged problem and solve it by using the transportation method.

The transshipment problem can be depicted form as shown in Table 4.1.

Table 4.1 Transshipment Problem

Terminal $T_i\downarrow$ $T_j\rightarrow$	1	2	…	m	m+1	…	m+n	Supply a_i
1	$-t_1$	x_{12}	…	x_{1m}	$x_1,m+1$	…	$x_1,m+n$	a_1
2	x_{21}	$-t_1$	…	x_{2m}	$x_2,m+1$	…	$x_2,m+n$	a_2
⋮			⋮			⋮		
m	x_{m1}	x_{m2}		$-t_m$	$x_{m,m+1}$		$x_{m,m+1}$	a_m
m+1	$x_{m+1,1}$	$x_{m+1,2}$		$x_{m+1,m}$	$-t_{m+1}$		$x_{m+1,m+n}$	0
⋮			⋮			⋮		
m+n	$x_{m+n,1}$	$x_{m+n,2}$		$x_{m+n,m}$	$x_{m+n,m+1}$		$-t_{m+n}$	0
Demand b_j	0	0	…	0	b_{m+1}	…	b_{m+n}	$\sum a_i=\sum b_j$

The first m rows represent the m constraints given in (1) while the remaining n rows show the constraints given in (4). The constraints in (2) and (3) are represented by the first m columns and the remaining n columns respectively. All the t values are placed on the diagonal from left top to right bottom. Each of them bears negative sign which must be considered carefully when a t is involved in the readjustment (during the solution process).

4.3 Solution of the Transshipment Problem

The following are steps for solving a Transshipment problem so far available in the literature.

Step1 If necessary, add a dummy demand point (with a supply of 0 and a demand equal to the problem's excess supply) to balance the problem. Shipments to the dummy and from a point to itself will be zero. Let s= total available supply.

Step2 Construct a transportation table as follows: A row in the table will be needed for each supply point and transshipment point, while a column will be needed for each demand point and transshipment point.

In transshipment problem we consider the following concept. Let each supply point will have a supply equal to its original supply, and each demand point will have a demand to its original demand. Let s= total available supply. Then each transshipment point will have a supply which is equal to point's original supply + s and a demand which is equal to point's original demand + s. This ensures that any transshipment point that is, a net supplier will have a net outflow equal to point's original supply and a net demander will have a net inflow equal to point's original demand. Although we don't know how much will be shipped through each transshipment point. However, we can be sure that the total amount will not exceed s.

4.3.1 Illustrated Example In this section first we solve numerical example related with transshipment problem using the method available so far. Next we will solve the same problem using the new alternate method developed by us.

Example 4.3.1.1 Consider a firm having two factories to ship its products from the factories to three-retail stores. The number of units available at factories X and Y are 200 and 300 respectively, while those demanded at retail stores A, B and C are 100,150 and 250 respectively. In stead of shipping directly from factories to retail stores, it is asked to investigate the possibility of transshipment. The transportation cost (in rupees) per unit is given in the table 4.2

Table 4.2

		Factory		Retail store		
		X	**Y**	**A**	**B**	**C**
Factory	**X**	0	8	7	8	9
	Y	6	0	5	4	3
Retail store	**A**	7	2	0	5	1
	B	1	5	1	0	4
	C	8	9	7	8	0

Find the optimal shipping schedule.

Solution We solve this problem using VAM to find the initial solution and then use this initial solution to obtain optimal solution using MODI method in the following way.

Table 4.3 showing Initial solution using VAM

	X	Y	A	B	C	Supply
X	$\boxed{0}$ 0	8	$\boxed{100}$ 7	$\boxed{100}$ 8	9	200
Y	6	$\boxed{0}$ 0	5	$\boxed{50}$ 4	$\boxed{250}$ 3	300
A	7	2	$\boxed{0}$ 0	5	1	0
B	1	5	1	$\boxed{0}$ 0	4	0
C	8	9	7	8	$\boxed{0}$ 0	0
Demand	0	0	100	150	250	500

Table 4.4 showing optimal solution using MODI

	X	Y	A	B	C	Supply	u_i
X	$\boxed{0}$ 0	8	$\boxed{100}$ 7	$\boxed{100}$ 8	9	200	u_1=4
Y	6	$\boxed{0}$ 0	5	$\boxed{50}$ 4	$\boxed{250}$ 3	300	u_2=0
A	7	2	$\boxed{0}$ 0	5	1	0	u_3=-3
B	1	5	1	$\boxed{0}$ 0	4	0	u_4=-4
C	8	9	7	8	$\boxed{0}$ 0	0	u_5=-3
Demand	0	0	100	150	250	500	
v_j	v_1=-4	v_2=0	v_3=3	v_4=4	v_5=3		

$d_{12}= c_{12}-(u_1+v_2) = 8-4 = 4$

$d_{15}= c_{15}-(u_1+v_5) = 9-7 = 2$

$d_{21}= c_{21}-(u_2+v_1) = 6+4 = 10$

$d_{23}= c_{23}-(u_2+v_3) = 5-3 = 2$

$d_{31}= c_{31}-(u_3+v_1) = 7+7 = 14$

$d_{32}= c_{32}-(u_3+v_2) = 2+3 = 5$

$d_{34}= c_{34}-(u_3+v_4) = 5-1 = 4$

$d_{35}= c_{35}-(u_3+v_5) = 1-0 = 1$

$d_{41}= c_{41}-(u_4+v_1) = 1+8 = 9$

$d_{42}= c_{42}-(u_4+v_2) = 5+4 = 9$

$d_{43}= c_{43}-(u_4+v_3) = 1+1 = 2$

$d_{45}= c_{45}-(u_4+v_5) = 4+1 = 5$

$d_{51}= c_{51}-(u_5+v_1) = 8+7 = 15$

$d_{52}= c_{52}-(u_5+v_2) = 9+3 = 12$

$d_{53}= c_{53}-(u_5+v_3) = 7-0 = 7$

$d_{54}= c_{54}-(u_5+v_4) = 8-1 = 7$

Since opportunity cost corresponding to each unoccupied cell is positive, therefore, the solution given in Table 4.4 is optimal.

Total cost = (100*7) + (100*8) + (50*4) + (250*3) = 700+800+200+750 = **2450**. This solution is optimal solution.

4.4 A New Alternate Method for Solving Transshipment Problem

The alternate method developed by us in this investigation seems to be easiest as compare to available methods of Transshipment problem.

4.4.1 Algorithm of New Alternate Method for Solving Transshipment Problem

Step 1 Prepare a transshipment table which will be in the form of square matrix always.

Step 2 In transshipment table, write 0 for that demand/supply for which demand/supply is unknown.

Step 3 Find the minimum value for each row. Certainly this value will be zero for each row.

Step 4 Find minimum demand/supply. Certainly it will be zero for unknown demand and supply. So allocate zero to each of the row where minimum cost is zero. In this method, we put minimum cost as zero in the diagonal of the matrix.

Step 5 Delete those rows and columns which are already allocated.

Step 6 Find minimum unit cost for remaining rows. If minimum unit cost is distinct then allocate min (supply, demand) to that cell. For example, let for row x minimum unit cost is 3 in column B and for row y minimum unit cost is 2 in column A. So allocate min (supply, demand) to cell (x, B) and cell (y, A). If it is not distinct then go to step 7.

Step 7 Delete those rows and columns where supply and demand exhausted. Next adjust supply/demand for undeleted row/column. If they are not distinct then find difference between minimum and next minimum unit cost for those rows where they are identical. Find the maximum difference and allocate those cells where demand/supply is minimum. For example, column B contains the minimum unit cost for row x and y. For row x, difference between minimum and next minimum unit cost is 2 while for row y, difference between minimum and next minimum unit cost is 1, so 2 is maximum and hence allocate a min(supply, demand) to cell (x, B).

Step 8 Continue steps 6 and 7 until (m+n-1) cells are allocated. This gives initial feasible solution.

Step 9 Find total cost as sum of the product of allocated demand/supply and cost value for the respective cells. That is,

$$Total\ cost = \sum_{i=1}^{m+n} \sum_{j=1}^{m+n} c_{ij}\, x_{ij}\,.$$

This total cost gives optimal or near to optimal solution.

4.4.2 Numeric Examples

Example 4.4.2.1 Consider the example 4.3.1.1.

Solution We solve this Transshipment problem using new alternate method in following steps.

Step 1 Table 4.5 shows the Transshipment matrix

Step 2 In table 4.5 write zero for unknown demand/supply for respective column of the row.

Step 3 The minimum cost value in each row is zero.

Step 4 We find that zero is minimum unit cost in each row and hence allocate zero in the diagonal cell of the Transshipment matrix .Which are shown in Table 4.5.

Table 4.5 Transshipment Matrix

	X	**Y**	**A**	**B**	**C**	Supply
X	$\boxed{0}$ 0	8	7	8	9	200
Y	6	$\boxed{0}$ 0	5	4	3	300
A	7	2	$\boxed{0}$ 0	5	1	0
B	1	5	1	$\boxed{0}$ 0	4	0
C	8	9	7	8	$\boxed{0}$ 0	0
Demand	0	0	100	150	250	500

Step 5 Delete columns X, Y and rows A, B, C. The remaining is shown in Table 4.6.

Table 4.6

	A	**B**	**C**	Supply
X	7	8	9	200
Y	5	4	3	300
Demand	100	150	250	500

Step 6 In row X, the minimum unit cost is 7 which represents column A. Similarly in row Y, the minimum unit cost is 3 which represents column C. So allocate min (100, 200) = 100 to cell (X, A) and a min (250, 300) = 250 to cell (Y, C). Next adjust the supply for rows X and Y which is shown in Table 4.7.

Table 4.7

	A	**B**	**C**	Supply
X	$\boxed{100}$ 7	8	9	200-100=100
Y	5	4	$\boxed{250}$ 3	300-250=50
Demand	100-100=0	150	250-250=0	500

Delete column A and C. Next, the minimum unit cost in rows X and Y are 8 and 4 respectively which represent column B. So allocate min (100, 150) =100 to cell

(X, B). Remaining demand for column B is 150-100=50 which is to be allocated to cell (Y, B). Finally in Table 4.8 we showed that (5+5-1) =9 cells are occupied and hence we get initial feasible solution.

Table 4.8 Basic feasible solution using new method

	X	Y	A	B	C	Supply
X	$\boxed{0}$ 0	8	$\boxed{100}$ 7	$\boxed{100}$ 8	9	200
Y	6	$\boxed{0}$ 0	5	$\boxed{50}$ 4	$\boxed{250}$ 3	300
A	7	2	$\boxed{0}$ 0	5	1	0
B	1	5	1	$\boxed{0}$ 0	4	0
C	8	9	7	8	$\boxed{0}$ 0	0
Demand	0	0	100	150	250	500

Total cost = (100*7) + (100*8) + (50*4) + (250*3) = 700+800+200+750 = **2450.**

Result This gives an **optimal solution**. This is similar to that of MODI method.

Example 4.4.2.2 A firm owns facilities at six places. It has manufacturing plants at places A, b, and C with daily production of 50, 40 and 60 units respectively. At point D, E and F, it has three warehouses with daily demands of 20, 95 and 35 units respectively. The firm instead of shipping from plant to warehouse decides to investigate the possibility of trans-shipment. The unit transportation cost (in Rs) is given in the table 4.9.

Table 4.9

		Plant			Warehouse			Supply
		A	**B**	**C**	**D**	**E**	**F**	
Plant	**A**	0	3	2	6	4	1	50
	B	3	0	4	3	8	7	40
	C	2	4	0	4	4	2	60
Warehouse	**D**	6	3	4	0	2	5	0
	E	4	8	4	2	0	1	0
	F	1	7	2	5	1	0	0
Demand		0	0	0	50	40	60	150

Solution Now following algorithm 4.4.1, we solve example 4.4.2.2 using new alternate method and obtained the Basic feasible solution which is shown in Table 4.10.

Table 4.10 Basic feasible solution using new method

	A	B	C	D	E	F	Supply
A	$\boxed{0}$ 0	3	2	6	4	$\boxed{50}$ 1	50
B	3	$\boxed{0}$ 0	4	$\boxed{40}$ 3	8	7	40
C	2	4	$\boxed{0}$ 0	$\boxed{10}$ 4	$\boxed{40}$ 4	$\boxed{10}$ 2	60
D	6	3	4	$\boxed{0}$ 0	2	5	0
E	4	8	4	2	$\boxed{0}$ 0	1	0
F	1	7	2	5	1	$\boxed{0}$ 0	0
Demand	0	0	0	50	40	60	150

Total cost: (50*1) + (40*3) + (10*4) + (40*4) + (10*2) = 50+120+40+160+20 = **390**.

Result This gives an **optimal solution**. This is similar to that of MODI method.

Example 4.4.2.3 A firm having two sources, S1 and S2 wishes to ship its product to two destinations, D1 and D2. The number of units available at S1 and S2 are 10 and 30 and the product demanded at D1 and D2 are 25 and 15 units respectively. The firm instead of shipping from sources to destinations decides to investigate the possibility of trans-shipment. The unit transportation cost (in Rs) is given in the table 4.11.

Table 4.11

		Source		Destination		Supply
		S_1	S_2	D_1	D_2	
Source	S_1	0	3	4	5	10
	S_2	3	0	3	5	30
Destination	D_1	4	3	0	2	0
	D_2	5	5	2	0	0
Demand		0	0	25	15	40

Determine the shipping schedule.

Solution Now following algorithm 4.4.1, we solve example 4.4.2.2 using new alternate method and obtained the Basic feasible solution which is shown in Table 4.10.

Table 4.12 Basic feasible solution using new method

		Source		Destination		Supply
		S_1	S_2	D_1	D_2	
Source	S_1	$\boxed{0}$ 0	3	4	$\boxed{10}$ 5	10
	S_2	3	$\boxed{0}$ 0	$\boxed{25}$ 3	$\boxed{5}$ 5	30
Destination	D_1	4	3	$\boxed{0}$ 0	2	0
	D_2	5	5	2	$\boxed{0}$ 0	0
Demand		0	0	25	15	40

Total minimum cost = (10*5) + (25*3) + (5*5) = 50+75+25=**150**.

Result This gives an **optimal solution**. This is similar to that of MODI method.

4.5 Conclusion

In this chapter we have developed a simple algorithm for solving a Transshipment Problem. The proposed algorithm is easy to understand and apply. The optimal solution obtained in this investigation is same as that of MODI method.

CHAPTER 5

A NEW ALTERNATE METHOD OF ASSIGNMENT PROBLEM

5.1 Introduction

An assignment problem is a particular case of transportation problem where the objective is to assign a number of resources to an equal number of activities so as to minimize total cost or maximize total profit of allocation.

The problem of assignment arises because available resources such as men, machines, etc. have varying degrees of efficiency for performing different activities. Therefore, cost, profit or time of performing the different activities is different. Thus, the problem is how the assignments should be made so as to optimize the given objective. The assignment problem is one of the fundamental combinatorial optimization problems in the branch of optimization or operations research in Mathematics. It consists of finding a maximum weight matching in a weighted bipartite graph.

In general, the assignment problem is of following type:

There are a number of *agents* and a number of *tasks*. Any agent can be assigned to perform any task, incurring some *cost* that may vary depending on the agent-task assignment. In such problem, it is required to perform all tasks by assigning exactly one agent to each task in such a way that the *total cost* of the assignment is minimized. If the numbers of agents and tasks are equal and the total cost of the assignment for all tasks is equal to the sum of the costs for each agent (or the sum of the costs for each task, which is the same thing in this case), then the problem is called the *linear assignment problem*. Commonly, when speaking of the *assignment problem* without any additional qualification, then the *linear assignment problem* is meant.

Suppose that a taxi firm has three taxis (the agents) available, and three customers (the tasks) wishing to be picked up as soon as possible. The firm prides itself on speedy pickups, so for each taxi the "cost" of picking up a particular customer will depend on the time taken for the taxi to reach the pickup point. The solution to the assignment problem will be whichever combination of taxis and customers results in the least total cost. However, the assignment problem can be made rather more flexible than it first appears. In the above example, suppose that there are four taxis available, but still only three customers. Then a fourth dummy task can be invented, perhaps called "sitting still doing nothing", with a cost of 0 for the taxi assigned to it. The assignment problem can then be solved in the usual way and still give the best solution to the problem. Similar tricks can be played in order to allow more tasks than agents, tasks to which multiple agents must be assigned (for instance, a group of more customers than will fit in one taxi), or maximizing profit rather than minimizing cost.

So far in the literature, there are mainly four methods so called Enumeration Method, Simplex Method, Transportation Method and Hungarian Method for solving Assignment Problem. Out of which Hungarian method is one of the best available for solving an assignment problem.

Hungarian mathematician Konig (1931) developed the Hungarian method of assignment which provides us an efficient method of finding the optimal solution without having to make a direct comparison of every solution. It works on the principle of reducing the given cost matrix to a matrix of opportunity costs. Opportunity costs show the relative penalties associated with assigning resource to an activity as opposed to making the best or least cost assignment. If we can reduce the cost matrix to the extent of having at least one zero in each row and column, it will be possible to make optimal assignments (opportunity costs are all zero). The Hungarian method is a combinatorial optimization algorithm which solves the assignment problem in polynomial time and which anticipated later primal-dual methods. Kuhn (1955) further developed the assignment problem which has been as "Hungarian method" because the algorithm was largely based on the earlier works of two Hungarian mathematicians: Dénes Kőnig and Jenő Egerváry. Ford and Fulkerson (1956) extended the method to general transportation problems. Munkres (1957) reviewed the algorithm and observed that it is strongly polynomial. Since then the algorithm has been known also as Kuhn–Munkres algorithm or Munkres assignment algorithm. The time complexity of the original algorithm was $O\,(n^4)$, however Edmonds and Karp (1972) studied the theoretical improvements in algorithmic efficiency for network flow problems while Tomizawa (1990b) studied polynomial diagonal-parameter symmetry model for a square contingency table however both of them independently noticed that it can be modified to achieve an $O(n^3)$ running time. In 2006, it was discovered that Carl Gustav Jacobi (1890) had solved the assignment problem in the 19th century but it was not published during his tenure however it was published posthumously in 1890 in Latin. Jacobi (1890) developed the concept of assignment algorithm.

Thompson (1981) discussed a recursive method for solving assignment problem which is a polynomially bounded non simplex method for solving assignment problem. Li and Smith (1995) discuss an algorithm for Quadratic assignment problem. Ji et. al. (1997) discussed a new algorithm for the assignment problem which they also called an alternative to the Hungarian Method. There assignment algorithm is based on a 2n*2n matrix where operations are performed on the matrix until an optimal solution is found.

In this chapter, we have developed an alternative method for solving an assignment problem to achieve the optimal solution. It has been found that the optimal solution obtained in this method is same as that of Hungarian method.

5.2 Mathematical Statement of the Problem

Given n resources (or facilities) and n activities (or jobs), and effectiveness (in terms of cost, profit, time, etc.) of each resource (facility) for each activity (job), the problem lies in assigning each resource to one and only one activity (job) so that the given measure of effectiveness is optimized. The data matrix for this problem is shown in Table 5.1.

Table 5.1 Data Matrix

Resources (workers)	Activities(jobs)				Supply
	J_1	J_2	…	J_n	
W_1	c_{11}	c_{12}	…	c_{1n}	1
W_2	c_{21}	c_{22}	…	c_{2n}	1
⋮	⋮	⋮		⋮	⋮
W_n	c_{n1}	c_{n2}	…	c_{nn}	1
Demand	1	1	…	1	n

From Table 5.1, it may be noted that the data matrix is the same as the transportation cost matrix except that supply (or availability) of each of the resources and the demand at each of the destinations is taken to be one. It is due to this fact that assignments are made on a one-to-one basis.

Let x_{ij} denote the assignment of facility i to job j such that

$$x_{ij} = \begin{cases} 1 & \text{if facility } i \text{ is assigned to job } j \\ 0 & \text{otherwise} \end{cases}$$

Then, the mathematical model of the assignment problem can be stated as:
The objective function is to,

$$\text{Minimise } Z = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij}$$

Subject to the constraints

$$\sum_{j=1}^{n} x_{ij} = 1, \textit{ for all } i \,(\textit{resource availability})$$

$$\sum_{i=1}^{n} x_{ij} = 1, \textit{ for all } j \,(\textit{activity requirement})$$

Where, x_{ij}=0 or 1 and c_{ij} represents the cost of assignment of resource i to activity j.

From the above discussion, it is clear that the assignment problem is a variation of the transportation problem with two characteristics: (i) the cost matrix is a square matrix, and (ii) the optimal solution for the problem would always be such that there would be only one assignment in a given row or column of the cost matrix.

5.3 Solution of the Assignment Problem

So far, in the literature it is available that an assignment problem can be solved by the following four methods.

I. Enumeration method
II. Simplex method
III. Transportation method
IV. Hungarian method

Here, we discuss each of four, one by one.

I. Enumeration method

In this method, a list of all possible assignments among the given resources (like men, machines, etc.) and activities (like jobs, sales areas, etc.) is prepared. Then an assignment involving the minimum cost (or maximum profit), time or distance is selected. If two or more assignments have the same minimum cost (or maximum profit), time or distance, the problem has multiple optimal solutions.

In general, if an assignment problem involves n workers/jobs, then there are in total n! Possible assignments. As an example, for an n=3 workers/jobs problem, we have to evaluate a total of 3! or 6 assignments. However, when n is large, the method is unsuitable for manual calculations. Hence, this method is suitable only for small n.

II. Simplex Method

Since each assignment problem can be formulated as a 0 or 1 which becomes integer linear programming problem. Such a problem can be solved by the simplex method also. As can be seen in the general mathematical formulation of the assignment problem, there are $n \times n$ decision variables and n+n or 2n equalities. In particular, for a problem involving 5 workers/jobs, there will be 25 decision variables and 10 equalities. It is, again, difficult to solve manually.

III. Transportation Method

Since an assignment problem is a special case of the transportation problem, it can also be solved by transportation methods. However, every basic feasible solution of a general assignment problem having a square payoff matrix of order n should have more m+n-1= n+n-1= 2n-1 assignments. But due to the special structure of this problem, any solution cannot have more than n assignments. Thus, the assignment problem is inherently degenerate. In order to remove degeneracy, (n-1) number of dummy allocations will be required in order to proceed with the transportation method. Thus, the problem of degeneracy at each solution makes the transportation method computationally inefficient for solving an assignment problem.

IV. Hungarian Method

Assignment problems can be formulated with techniques of linear programming and transportation problems. As it has a special structure, it is solved by the special method called Hungarian method. This method was developed by D. Konig, a Hungarian mathematician and is therefore known as the Hungarian method of assignment problem. In order to use this method, one needs to know only the cost of making all the possible assignments. Each assignment problem has a matrix (table) associated with it. Normally, the objects (or people) one wishes to assign are expressed in rows, whereas the columns represent the tasks (or things) assigned to them. The number in the table would then be the costs associated with each particular assignment. It may be noted that the assignment problem is a variation of transportation problem with two characteristics. (i) The cost matrix is a square matrix, and (ii) The optimum solution for the problem would be such that there would be only one assignment in a row or column of the cost matrix**.** Hungarian method is based on the principle that if a constant is added to the elements of cost matrix, the optimum solution of the assignment problem is the same as the original problem. Original cost matrix is reduced to another cost matrix by adding a constant value to the elements of rows and columns of cost matrix where the total completion time or total cost of an assignment is zero. This assignment is also referred as the optimum solution since the optimum solution remains unchanged after the reduction.

Hungarian Method (minimization case) can be summarized in the following steps:

Step 1 Develop the cost table from the given problem

If the number of rows is not equal to the number of columns and vice versa, a dummy row or dummy column must be added. The assignment costs for dummy cells are always zero.

Step 2 Find the opportunity cost table

(a) Locate the smallest element in each row of the given cost table and then subtract that from each element of that row, and

(b) In the reduced matrix obtained from 2(a), locate the smallest element in each column and then subtract that from each element of that column. Each row and column now have at least one zero value.

Step 3 Make assignments in the opportunity cost matrix

The procedure of making assignments is as follows:

(a) Examine rows successively until a row with exactly one unmarked zero is obtained. Make an assignment to this single zero by making a square ☐ around it.

(b) For each zero value that become assigned, eliminate (strike off) all other zeros in the same row and/or column.

(c) Repeat Steps 3(a) and 3(b) for each column also with exactly single zero value cells that have not been assigned.

(d) If a row and/or column have two or more unmarked zeros and one cannot be chosen by inspection, then choose the assigned zero cell arbitrarily.

(e) Continue this process until all zeros in rows/columns are either enclosed(assigned) or struck off($\times$)

Step 4 Optimality criterion

If the number of assigned cells is equal to the number of rows/columns, then it is an optimal solution. The total cost associated with this solution is obtained by adding original cost figures in the occupied cells.

If a zero cell was chosen arbitrarily in Step 3, there exists an alternative optimal solution. But if no optimal solution is found, then go to Step 5.

Step 5 Revise the opportunity cost table

Draw a set of horizontal and vertical lines to cover all the zeros in the revised cost table obtained from Step 3, by using the following procedure:

(a) For each row in which no assignment was made, mark a tick($\surd$)

(b) Examine the marked rows. If any zero cells occur in those rows, mark to the respective columns that contain those zeros.

(c) Examine marked columns. If any assigned zero occurs in those columns, tick the respective rows that contain those assigned zeros.

(d) Repeat this process until no more rows or columns can be marked.

(e) Draw a straight line through each marked column and each unmarked row.

If the number of lines drawn (or total assignments) is equal to the number of rows (or columns), the current solution is the optimal solution, otherwise go to Step 6.

Step 6 Develop the new revised opportunity cost table

(a) From among the cells not covered by any line, choose the smallest element. Call this value k.

(b) Subtract k from every element in the cell not covered by a line.

(c) Add k to every element in the cell covered by the two lines, i.e. intersection of two lines.

(d) Elements in cells covered by one line remain unchanged.

Step 7 Repeat Steps 3 to 6 until an optimal solution is obtained.

5.3.1 Solution of Assignment Problem using Hungarian Method

Example 5.3.1.1 A department has five employees with five jobs to be performed. The time (in hours) each men will take to perform each job is given in the effectiveness matrix.

Table 5.2

	Employees					
		I	II	III	IV	V
Jobs	A	10	5	13	15	16
	B	3	9	18	13	6
	C	10	7	2	2	2
	D	7	11	9	7	12
	E	7	9	10	4	12

How should the jobs be allocated, one per employee, so as to minimize the total man-hours? (Refer Sharma, 2007)

Solution Here I solve this example using only Hungarian Method because other three methods are not mostly used for solving an assignment problem. And then I show that how our method is better than Hungarian Method.

Applying step 2 of the algorithm, we get the reduced opportunity time matrix as shown in Table 5.3.

Table 5.3

	I	II	III	IV	V
A	5	0	8	10	11
B	0	6	15	10	3
C	8	5	0	0	0
D	0	4	2	0	5
E	3	5	6	0	8

Steps 3 and 4

(a) We examine all the rows starting from A one-by-one until a row containing only single zero element is located. Here rows A, B and E have only one zero element in the cells (A, II), (B,I) and (E,IV). Assignment is made in these cells. All zeros in the assigned columns are now crossed off as shown in Table 5.4.

(b) We now examine each column starting from column 1. There is one zero in column III, cell (C, III). Assignment is made in this cell. Thus cell (C, V) is crossed off. All zeros in the table now are either assigned or crossed off as shown in Table 5.4.

Table 5.4

	I	II	III	IV	V
A	5	**0**	8	10	11
B	**0**	6	15	10	3
C	8	5	**0**	0 ×	0 ×
D	0 ×	4	2	0 ×	5
E	3	5	6	**0**	8

The solution is not optimal because only four assignments are made.

Step 5

Cover the zeros with minimum number of lines (=4) as explained below:

(a) Mark (√) row D since it has no assignment.

(b) Mark (√) columns I and IV since row D has zero element in these columns.

(c) Mark (√) rows B and E since columns I and IV have an assignment in rows B and E, respectively.

(d) Since no other rows or columns can be marked, draw straight lines through the unmarked rows A and C and the marked columns I and IV, as shown in Table 5.5.

Table 5.5

	I	II	III	IV	V	
A	5	**0**	8	10	11	
B	**0**	6	15	10	3	√
C	8	5	**0**	0 ×	0 ×	
D	0 ×	4	2	0 ×	5	√
E	3	5	6	**0**	8	√
	√			√		

Step 6

Develop the new revised table by selecting the smallest element among all uncovered elements by the lines in Table 3; viz. 2. Subtract k=2 from uncovered elements including itself and add it to elements 5,10,8 and 0 in cells (A, I), (A, IV), (C, I) and (C, IV), respectively which lie at the intersection of two lines. Another revised table so obtained is shown in Table 5.6.

Table 5.6

	I	II	III	IV	V
A	7	**0**	8	12	11
B	**0**	4	13	10	1
C	10	5	**0**	2	0
D	0	2	0	0	3
E	3	3	4	**0**	6

Step 7

Repeat Steps 3 to 6 to find a new solution. The new assignment is shown in Table 5.7.

Table 5.7

	I	II	III	IV	V
A	7	$\boxed{0}$	8	12	11
B	$\boxed{0}$	4	13	10	1
C	10	5	0 ×	2	$\boxed{0}$
D	0 ×	2	$\boxed{0}$	0 ×	3
E	3	3	4	$\boxed{0}$	6

Since the number of assignments (=5) equals the number of rows (or columns), the solution is optimal.

The pattern of assignments among jobs and employees with their respective time (in hours) is given below.

Job	Employee	Time (in hours)
A	II	5
B	I	3
C	V	2
D	III	9
E	IV	4
	Total	**23 hours**

The Hungarian method has many steps to solve the problem so many times it will be very complicated. So we illustrated a very easy method to solve the assignment problem.

5.4 A NEW ALTERNATE METHOD OF ASSIGNMENT PROBLEM

The new alternate method of assignment problem discussed here gives optimal solution directly within few steps. It is very easy to calculate and understand. The alternate method developed by us in this investigation seems to be easiest as compare to available methods of assignment problem. Here we explain algorithm for alternate method of solving assignment problem for minimization and maximization cases.

5.4.1 Algorithm for Minimization Case

Let A, B, C … Z denote resources and I, II, III, IV… denote the activities. Now we discussed various steps for solving assignment problem which are following.

Step 1 Construct the data matrix of the assignment problem. Consider row as a worker (resource) and column as a job (activity).

Step 2 Write two columns, where column 1 represents resource and column 2 represents an activity. Under column 1, write the resource, say, A, B, C … Z.

Next find minimum unit cost for each row, whichever minimum value is available in the respecting column, select it and write it in term of activities under column 2. Continue this process for all the Z rows and write in term of I, II …

Step 3 Let for each resource; if there is unique activity then assigned that activity for the corresponding resource, hence we achieved our optimal solution. For example, let we have 5 resources A, B, C, D, E and 5 activities I, II, III, IV, V. This is shown in Table 5.8.

Table 5.8

Column 1 Resource	Column 2 Activity
A	V
B	III
C	I
D	IV
E	II

If there is no unique activity for corresponding resources (which is shown in Example 5.4.2.1 and Example 5.4.2.2) then the assignment can be made using following given steps:

Step 4 Look at which of any one resource has unique activity and then assign that activity for the corresponding resource. Next delete that row and its corresponding column for which resource has already been assigned.

Step 5 Again find the minimum unit cost for the remaining rows. Check if it satisfy step 4 then perform it. Otherwise, check which rows have only one same activity. Next find difference between minimum and next minimum unit cost for all those rows which have same activity. Assign that activity which has maximum difference. Delete those rows and corresponding columns for which those resources have been assigned.

Remarks 1 However if there is tie in difference for two and more than two activity then further take the difference between minimum and next to next minimum unit cost. Next check which activity has maximum difference, assign that activity.

Step 6 Repeat steps 4 to 5 till all jobs are assigned uniquely to the corresponding activity.

Step 7 Once all the jobs are assigned then calculate the total cost by using the expression, **Total cost =** $\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} x_{ij}$

5.4.2 Numerical Examples In this chapter, we illustrate some numerical examples to solve the assignment problem using New Alternate Method.

Example 5.4.2.1 A computer centre has five expert programmers. The centre needs five application programs to be developed. The head of the computer centre, after studying carefully the programs to be developed. Estimate the computer time in minutes required by the experts for the application programs. The observations are shown in Table 5.9.

Table 5.9 Data matrix

Programmers	Programs				
	I	II	III	IV	V
A	100	90	60	80	120
B	70	90	110	85	105
C	130	110	120	125	140
D	100	120	90	105	85
E	120	140	135	100	115

Assign the programmers to the programs in such a way that the total computer time is minimum.

Solution Consider Table 5.9, Select row A, where the minimum value is 60 representing program **III**. Similarly, the minimum value for row-B to row-E are 70, 110, 85 and 100 representing programs I, II, V and IV respectively. This is shown in Table 5.10.

Table 5.10

Programmers	Programs
A	**III**
B	**I**
C	**II**
D	**V**
E	**IV**

Table 5.10a

Programmers	Programs				
	I	II	III	IV	V
A	100	90	$\boxed{60}$	80	120
B	$\boxed{70}$	90	110	85	105
C	130	$\boxed{110}$	120	125	140
D	100	120	90	105	$\boxed{85}$
E	120	140	135	$\boxed{100}$	115

From Table 5.10, we can easily see that different programs are meant for different programmers. That is, we can assign programs uniquely to the programmers, which is shown in Table 5.10a and hence we achieved our optimal solution, which is shown in Table 5.11.

Table 5.11

Programs	Programmers	Time
I	B	70
II	C	110
III	A	60
IV	E	100
V	D	85
	Total	**425**

Result This answer is happened to be same as that of Hungarian method. Hence we can say that the minimum time is still 425 in both the methods. So our method also gives **Optimal Solution**. However our method seems to be very simple, easy and takes very few steps in solving the method.

Now we consider another example where (i) different resources do not have unique activity and (ii) resource has more than one minimum cost. This is discussed in Example 5.4.2.2.

Example 5.4.2.2 The department has five employees with five jobs to be performed. The time (in hours) each men will take to perform each job is given in the effectiveness matrix.

Table 5.12 Data matrix

	Activities(jobs)					
Resources (Employees)		I	II	III	IV	V
	A	10	5	13	15	16
	B	3	9	18	13	6
	C	10	7	2	2	2
	D	7	11	9	7	12
	E	7	9	10	4	12

Solution Consider the data matrix 5.12. Now, we select row A and select that column (activity) for which row A has minimum unit cost. In this example, for row A, column II (activity) has the minimum unit cast. So we write resource A under column I and activity II under column II. In the similar way, we select all the rows (resources) and find the minimum unit cost for the respective columns, which are shown in Table 5.13.

Table 5.13

Resource	Activity
A	**II**
B	I
C	III, IV, V
D	I, IV
E	IV

Table 5.13a

	I	II	III	IV	V
A	10	$\boxed{5}$	13	15	16
B	3	9	18	13	6
C	10	7	2	2	2
D	7	11	9	7	12
E	7	9	10	4	12

Here, Activity II is unique as it doesn't occur again and hence assigned resource A to activity II and is shown in Table 5.13a. Next, delete Row A and Column II.

Again select minimum cost value for the remaining resources, B, C, D and E. which is shown below Table 5.14.

Table 5.14

Resource	Activity
B	**I**
C	III, IV, V
D	I, IV
E	IV

Table 5.14a

	I	III	IV	V
B	3	18	13	6
C	10	2	2	2
D	7	9	7	12
E	7	10	4	12

Since resource B has single activity I. Next we see that resource D has also activity I and hence we take the minimum unit cost difference for resource B and D. Here minimum cost difference for resource B is 3(6-3) while minimum cost difference for resource D is 0(7-7). Since 3 is the maximum difference which represents resource B and hence assign resource B to activity I and is shown in Table 5.14a. Further delete row B and Column I.

Again select minimum unit cost for the remaining resources, C, D and E. which is shown in Table 5.15.

Table 5.15

Resource	Activity
C	III, IV, V
D	IV
E	**IV**

Table 5.15a

	III	IV	V
C	2	2	2
D	9	7	12
E	10	4	12

Since resource D and E have single activity IV. Next we see that resource C has also same activity IV and hence we take the minimum cost difference for resources C, D and E. Here minimum cost difference for resource C is 0, minimum cost difference for resource D is 2 while minimum cost difference for resource E is 6. Since 6 is a maximum difference which represents resource E and hence assign resource E to activity IV and is shown in Table 4.15a. Further delete row E and Column IV.

Again select minimum cost value for the remaining resources, C and D. which is shown in Table 5.16.

Table 5.16

Resource	Activity
C	III, V
D	**III**

Table 5.16a

	III	V
C	2	$\boxed{2}$
D	$\boxed{9}$	12

Since resource D has single activity **III**. Next we see that resource C has also same activity **III** and hence we take the minimum cost difference for resources C and D. Here minimum cost difference for C is 0, while minimum cost difference for D is 3. Since 3 is the maximum difference which represents resource D and hence assign resource D to activity **III**. Finally only row C and column V remains and hence assign resource C to activity V and is shown in Table 4.16a.

Finally, different employees have assigned jobs uniquely, which is shown in Table 5.17.

Table 5.17

Employees	Jobs	Time (in hour)
A	II	5
B	I	3
C	V	2
D	III	9
E	IV	4
	Total	**23**

Result This answer is happened to be same as that of Hungarian method. Hence we can say that the minimum value is still 23 in both the methods. So our method also gives **Optimal Solution**. However our method seems to be very simple, easy and takes very few steps in solving the problem.

Next we consider another example where different resources do not have unique activity but each resource has only one minimum unit cost. This is discussing in Example 5.4.2.3.

Example 5.4.2.3 A factory has six machines and employed six workers to work on the given six machines. Based on the workers experience and their personal efficiency, the workers have assigned to work on six machines. Following is the times (minutes) taken by workers in completion of that work in the respective machines. The times are as follow.

Table 5.18 Data matrix

		Machine					
		I	II	III	IV	V	VI
Workers	A	9	8	7	5	3	10
	B	6	4	2	8	7	9
	C	8	10	9	6	4	12
	D	9	6	5	4	1	11
	E	3	5	6	7	11	8
	F	2	4	3	5	8	9

Obtain optimal assignment of the workers.

Solution Consider the data matrix 5.18. Now, we select row A and select that column (activity) for which row A has minimum value. In this example, for row A, column V (activity) has the minimum value. So we write resource A under column I and activity V under column II. In the similar way, we select all the rows (resources) and find the minimum value for the respective columns. This is shown in Table 5.19.

Table 5.19

Column 1	Column 2
A	V
B	**III**
C	V
D	V
E	I
F	I

Table 5.19a

	I	II	III	IV	V	VI
A	9	8	7	5	3	10
B	6	4	$\boxed{2}$	8	7	9
C	8	10	9	6	4	12
D	9	6	5	4	1	11
E	3	5	6	7	11	8
F	2	4	3	5	8	9

Here, Activity III is unique as it doesn't occur again and hence assigned resource B to activity III and is shown in Table 5.19a. Next, delete Row B and Column III.

Again select minimum cost value for the remaining resources, A, C, D, E and F. which is shown below Table 5.20.

Table 5.20

Column 1	Column 2
A	V
C	V
D	**V**
E	**I**
F	I

Table 5.20a

	I	II	IV	V	VI
A	9	8	5	3	10
C	8	10	6	4	12
D	9	6	4	$\boxed{1}$	11
E	$\boxed{3}$	5	7	11	8
F	2	4	5	8	9

Since resource A has single activity V. Next we see that resource C and D have also activity V and hence take the minimum cost difference for resource A, C and D. Here minimum cost difference for resource A is 2, for resource C is 2, while minimum cost difference for resource D is 3. Since 3 is the maximum difference which represents resource D and hence assign resource D to activity V and is shown in Table 5.20a. Again resource E has single activity I. Next we see that resource F has also same activity I and hence we take the minimum cost difference for resource E and F. Here minimum cost difference for resource E is 2 while minimum cost difference for resource F is also 2. So we will find next minimum difference for resource E and F which comes out as 2 and 1 for E and F respectively. Since 2 is the maximum difference which represents resource E and hence assign resource E to activity I and is shown in Table 3a. Further delete (i) row D and Column V and (ii) row E and Column I.

Again select minimum unit cost for the remaining resources, A, C and F. which is shown below Table 5.21.

Table 5.21

Column 1	Column 2
A	IV
C	IV
F	**II**

Table 5.21a

	II	IV	VI
A	8	5	10
C	10	6	12
F	$\boxed{4}$	2	9

Since resource F has unique single activity II and hence assigned resource F to activity II shown in Table 5.21a. Further delete row F and Column II. Again select minimum unit cost for the remaining resources, A and C. which is shown in Table 5.22.

Table **5.22**

Column 1	Column 2
A	IV
C	**IV**

Table 5.22a

	IV	VI
A	5	$\boxed{10}$
C	$\boxed{6}$	12

Since resource A has single activity IV. Next we see that resource C has also same activity IV and hence take the minimum cost difference for resource A and C. Here minimum cost difference for resource A is 5 while for resource C is 6. Since 6 is the maximum difference which represents resource C and hence assign resource C to activity IV. Finally, the remaining resource is A with single activity VI and hence assigned resource A to activity VI and is shown in Table 5.22a.

Since it gives unique assignment and hence we achieved our optimal solution, which is shown in Table 5.23.

Table 5.23

Workers	Machines	Time (in minute)
A	VI	10
B	III	2
C	IV	6
D	V	1
E	I	3
F	II	4
	Total	**26**

Result This answer is happened to be same as that of Hungarian method. Hence we can say that the minimum value is still 26 in both the methods. So our method also gives **Optimal Solution**. However our method seems to be very simple, easy and takes very few steps in solving the problem.

5.4.3 Algorithm for Maximization Case

Let A, B, C … Z denote resources and I, II, III, IV… denote the activities. Now we discussed various steps for solving assignment problem which are following.

Step 1 Construct the data matrix of the assignment problem. Consider row as a worker (resource) and column as a job (activity).

Step 2 Write two columns, where column 1 represents resource and column 2 represents an activity. Under column 1, write the resource, say, A, B, C … Z. Next find maximum unit cost for each row, whichever maximum unit cost is available in the respecting column, select it and write it in term of activities under column 2. Continue this process for all the Z rows and write in term of I, II …

Step 3 Let for each resource; if there is unique activity then assigned that activity for the corresponding resource, hence we achieved our optimal solution. For example, let we have 5 resources A, B, C, D, E and 5 activities I, II, III, IV, V. This is shown in Table 5.24.

Table 5.24

Column 1 Resource	Column 2 Activity
A	V
B	III
C	I
D	IV
E	II

If there is no unique activity for corresponding resources then the assignment can be made using following given steps:

Step 4 Look at which of any one resource has unique activity and then assign that activity for the corresponding resource. Next delete that row and its corresponding column for which resource has already been assigned.

Step 5 Again find the maximum unit cost for the remaining rows. Check if it satisfy step 4 then perform it. Otherwise, check which rows have only one same activity. Next find difference between maximum and next maximum unit cost for all those rows which have same activity. Assign that activity which has maximum difference. Delete those rows and corresponding columns for which those resources have been assigned.

Remarks 1 However if there is tie in difference for two and more than two activity then further take the difference between maximum and next to next maximum unit cost. Next check which activity has maximum difference, assign that activity.

Step 6 Repeat steps 4 to 5 till all jobs are assigned uniquely to the corresponding activity.

Step 7 Once all the jobs are assigned then calculate the total cost by using the expression, **Total cost =** $\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij}$

Example 5.4.3.1 A marketing manager has five salesmen and five sales districts. Considering the capabilities of the salesman and the nature of districts, the marketing manager estimates that sales per month (in hundred rupees) for each district would be as follows:

Table 5.25

		Districts				
		A	B	C	D	E
Salesman	1	32	38	40	28	40
	2	40	24	28	21	36
	3	41	27	33	30	37
	4	22	38	41	36	36
	5	29	33	40	35	39

Find the assignment of salesmen to districts that will result in maximum sales.

Solution Consider the effective matrix. Now, we select row (salesman) 1 and select that column (district) for which it has maximum sale value. In this example, for row 1, column C and E have the maximum value. So we write salesman 1 under column I and district C and E under column II. In the similar way, we select all the salesmen and find the maximum value for the respective districts, which are shown in Table 5.26.

Table 5.26

Resource	Activity
1	C, E
2	**A**
3	A
4	C
5	C

Table 5.26a

	A	B	C	D	E
1	32	38	40	28	40
2	$\boxed{40}$	24	28	21	36
3	41	27	33	30	37
4	22	38	41	36	36
5	29	33	40	35	39

In this example, there is no unique activity for any resource and hence proceed step 4. The maximum difference between maximum and next maximum profit for salesman 2 and 3 are 12 and 8 respectively (as for this two resources activity is same (A)). Here 12 is maximum so assign salesman 2 to district A. This is shown in Table 5.26a.

Delete salesman 2 and district A. For remaining resources, maximum profit of the corresponding activity is shown in table 4.27. The maximum difference between maximum and next maximum profit for salesman 1 and 3 are 0 and 4 respectively (as for this two resources activity is same (E)). Here 4 is maximum so assign salesman 3 to district E. This is shown in Table 5.27a.

Table 5.27

Column 1	Column 2
1	C, E
3	**E**
4	C
5	C

Table 5.27a

	B	C	D	E
1	38	40	28	40
3	27	33	30	$\boxed{37}$
4	38	41	36	36
5	33	40	35	39

Delete salesman 3 and district E. For remaining resources, maximum profit of the corresponding activity is shown in table 5.28. The maximum difference between maximum and next maximum profit for salesman 1, 4 and 5 are 4, 3 and 5 respectively (as for this two resources activity is same (C)). Here 5 is maximum so assign salesman 5 to district C. This is shown in Table 5.28a.

Table 5.28

Column 1	Column 2
1	C
4	C
5	**C**

Table 5.28a

	B	C	D
1	38	40	28
4	38	41	36
5	33	$\boxed{40}$	35

Delete salesman 5 and district C. For remaining resources, maximum profit of the corresponding activity is shown in table 5.29. The maximum difference between maximum and next maximum profit for salesman 1 and 4 are 10 and 2 respectively (as for this two resources activity is same (B)). Here 10 is maximum so assign salesman 1 to district B. This is shown in Table 5.29a.

Table 5.29

Column 1	Column 2
1	**B**
4	B

Table 5.29a

	B	D
1	$\boxed{38}$	28
4	38	36

At the last the salesman 4 and district D remains and hence we assign salesman 4 to district D.

Finally, different salesman have assigned district uniquely, which is shown in Table 5.30.

Table 5.30

Salesman	District	Sales (in hundred)
1	B	38
2	A	40
3	E	37
4	D	36
5	C	40
	Total	**191**

Result This answer is happened to be same as that of Hungarian method because the maximum sale value is 191 in both the methods. So our method also gives **Optimal Solution**. However our method seems to be very simple, easy and takes very few steps in solving the problem.

5.4.4 Unbalanced Assignment Problem

The method of assignment discussed above requires that the number of columns and rows in the assignment matrix be equal. However, when the given cost matrix is not a square matrix, the assignment problem is called an unbalanced problem. In such cases a dummy row(s) or column(s) are added in the matrix (with zeros as the cost elements) to make it a square matrix. For example, when the given cost matrix is of order 4×3, a dummy column would be added with zero cost element in that column. After making the given cost matrix a square matrix, the new alternate method will be used to solve the problem. Once the unbalanced assignment problem is converted into balanced assignment problem then we can follow usual algorithm to solve the assignment problem.

Remark Dummy row/column will not be considered for selecting minimum value in our method for unbalanced assignment problem.

Example 5.4.4.1

A city corporation has decided to carry out road repairs on main four arteries of the city. The government has agreed to make a special grant of Rs 50 lakh towards the cost with a condition that the repairs are done at the lowest cost and quickest time. If the conditions warrant, a supplementary token grant will also be considered favorably. The corporation has floated tenders and five contractors have sent in their bids. In order to expedite work, one road will be awarded to only one contractor.

Table 5.31

		Cost of repairs (Rs lakh)			
		R_1	R_2	R_3	R_4
Contractors	C_1	9	14	19	15
	C_2	7	17	20	19
	C_3	9	18	21	18
	C_4	10	12	18	19
	C_5	10	15	21	16

Find the best way of assigning the repair work to the contractors and the costs.

Solution The given cost matrix is not balanced; hence we add one dummy column (road, R_5) with a zero cost in that column. The cost matrix so obtained is given in following Table 5.32.

Table 5.32

		Cost of repairs (Rs lakh)				
		R_1	R_2	R_3	R_4	R_5
Contractors	C_1	9	14	19	15	0
	C_2	7	17	20	19	0
	C_3	9	18	21	18	0
	C_4	10	12	18	19	0
	C_5	10	15	21	16	0

We apply the new alternate method to solve this problem using algorithm discussed in 5.4.2.

Table 5.33

Column 1	Column 2	Difference
C_1	R_1	5
$\mathbf{C_2}$	$\mathbf{R_1}$	**10**
C_3	R_1	9
C_4	R_1	2
C_5	R_1	5

Assign contractor C_2 to repair road R_1. Next delete Row C_2 and Column R_1 and apply step 4 to 5 which is shown in Table 5.34.

Table 5.34

Column 1	Column 2	Difference
C_1	R_2	1
C_3	R_2, R_4	0
$\mathbf{C_4}$	$\mathbf{R_2}$	**6**
C_5	R_2	1

Assign contractor C_4 to repair road R_2. Next delete Row C_4 and Column R_2 and apply step 4 to 5 which is shown in Table 5.35.

Table 5.35

Column 1	Column 2	Difference
C_1	R_4	4
C_3	R_4	3
C_5	**R_4**	**5**

Assign contractor C_5 to repair road R_4. Next delete Row C_5 and Column R_4 and apply step 4 to 5 which is shown in Table 5.36.

Table 5.36

Column 1	Column 2	Difference
C_1	R_3	19
C_3	**R_3**	**21**

In this step, only one column (except dummy column) remains along with two rows C_1 and C_3. So assign C_1 to R_3 (as value is minimum). Next only row C_3 remains to assign which will be assign to dummy column R_5. Finally each contractor is assigning to repaired road uniquely which is shown in Table 4.37 along with its corresponding cost.

Table 5.37

Column 1	Column 2	Cost
C_1	R_3	19
C_2	R_1	7
C_3	R_5	0
C_4	R_2	12
C_5	R_4	16
	Total	**54**

Result This answer is happened to be same as that of Hungarian method because the maximum sale value is 54 in both the methods. So our method also gives **Optimal Solution**. However our method seems to be very simple, easy and takes very few steps in solving the problem.

Applications Some of the problems where the assignment technique may be useful are Assignment of workers to machines, salesmen to different sales areas, clerks to various checkout counters, classes to rooms, etc.

5.5 Conclusion

Hungarian method is used to obtained optimal solution for an assignment problem. In this chapter, we have developed a new alternate method for solving an assignment problem where it is shown that this method also gives optimal solution. Moreover the optimal solution obtained using this method is same as that of optimal solution obtained by Hungarian method. So we conclude that the Hungarian method and our method give same optimal solution. However the technique for solving an assignment problem using our method is more simple and easy as it takes few steps for the optimal solution.

Chapter 6
TRAVELLING SALESMAN PROBLEM

6.1 Introduction

The Traveling Salesman Problem (TSP) is a problem in combinatorial optimization studied in operations research and theoretical computer science. Given a list of cities and their pair wise distances, the task is to find a shortest possible tour that visits each city exactly once.

The problem was first formulated as a mathematical problem by Menger (1930) and is one of the most intensively studied problems in optimization. However it was unnoticed till Menger (1994) published a book where he narrated the foundation of mathematical problem for the travelling salesman problem. It is used as a benchmark for many optimization methods. Even though the problem is computationally difficult, a large number of heuristics and exact methods are known, so that some instances with tens of thousands of cities can be solved. The TSP has several applications even in its purest formulation, such as planning, logistics, and the manufacture of microchips. If a slight modification is made in the problem, it appears as a sub-problem in many areas, such as genome sequencing. In these applications, the concept *city* represents, for example, customers, soldering points, or DNA fragments, and the concept *distance* represents traveling times or cost, or a similarity measure between DNA fragments. In many applications, additional constraints such as limited resources or time windows make the problem considerably harder. In the theory of computational complexity, the decision version of TSP belongs to the class of NP-complete problems. Thus, it is assumed that there is no efficient algorithm for solving TSP problems. In other words, it is likely that the worst case running time for any algorithm for TSP increases exponentially with the number of cities, so even some instances with only hundreds of cities will take many CPU years to solve exactly.

The origins of the traveling salesman problem are unclear. A handbook for traveling salesmen from 1832 mentions the problem and includes example tours through Germany and Switzerland, but contains no mathematical treatment. Hamilton (1800) and Kirkman (1800) expressed the concept of Mathematical problems related to the travelling salesman problem. The general form of the TSP appears to have been first studied by mathematicians notably by Menger (1930). Further Menger (1930) also defines the problem related with salesman ship based on brute-force algorithm, and observes the non-optimality of the nearest neighbor heuristic. However Whitney (1930) introduced the name *travelling salesman problem*.

During the period 1950 to 1960, the travelling salesman problem started getting popularity in scientific circle is especially in Europe and the USA. Many researchers like Dantzig, Fulkerson and Johnson (1954) at the RAND Corporation in Santa Monica expressed the problem as an integer linear program and developed the cutting plane method for its solution. With these new methods they solved an instance with 49 cities to optimality by constructing a tour and proving that no other tour could be shorter. In the following decades, the problem was studied by many researchers from mathematics, science, chemistry, physics, and other sciences. Karp (1972) showed that the Hamiltonian cycle problem was NP-complete, which implies the NP-hardness of TSP. This supplied a scientific explanation for the apparent computational difficulty of finding optimal tours.

6.2 Application of New Alternate Method of Assignment Problem in TSP

We have so far already discussed the algorithm and examples for solving an assignment problem using a new alternate method in Chapter 5. Now in this chapter we discuss how the new alternate method for solving an assignment problem can be applied for Travelling Salesman Problem. For this we have considered an example related with travelling salesman problem and explain in detail how to find optimal solution using new alternate method of assignment problem.

Example 6.2.1 A salesman has to visit five cities A.B, C, D and E. The distances (in hundred kilometers) between the five cities are shown in Table 6.1.

Table 6.1

		To city				
		A	B	C	D	E
From City	A	-	1	6	8	4
	B	7	-	8	5	6
	C	6	8	-	9	7
	D	8	5	9	-	8
	E	4	6	7	8	-

If the salesman starts from city A and has to come back to city A, which route should he select so that total distance traveled become minimum?

Solution

Consider the effective matrix. This is shown in Table 6.2.

Table 6.2

		To city				
		A	B	C	D	E
From City	A	-	1	6	8	4
	B	7	-	8	5	6
	C	6	8	-	9	7
	D	8	5	9	-	8
	E	4	6	7	8	-

In this matrix first, we will take first row which is referred a city. We select that column (assignment) for which it contains minimum distance. For this example, incase of first row, column B (assignment) has the minimum value. In the similar way, we select all the rows and find the minimum value for the respective columns. These are given in Table 6.3.

Table 6.3

Column 1(City)	Colum 2(Assignment)
A	B
B	**D**
C	A
D	B
E	A

In this table, we observed that assignment D occur only once with city B. That is city B is unique for city D and hence we assign city B to D. This is shown in Table 6.4.

Table 6.4

	A	B	C	D	E
A	-	1	6	8	4
B	7	-	8	$\boxed{5}$	6
C	6	8	-	9	7
D	8	5	9	-	8
E	4	6	7	8	-

However, for other job assignment occur more than once. Hence they are not unique. So how other job will be assigned further we discuss below.

Next delete row B and column D. Again select minimum cost value for the remaining cities which is shown below Table 6.5.

Table 6.5

Column 1(City)	Colum 2(Assignment)
A	**B**
C	A
D	B
E	A

Since assignment B occur with city A and D. Hence first we take the difference between the value of B and next minimum value (here tie is happens). Here maximum difference is 2 for A and hence we assign B to city A. This is shown in Table 6.6.

Table 6.6

	A	B	C	E
A	-	$\boxed{1}$	6	4
C	6	8	-	7
D	8	5	9	8
E	4	6	7	-

Next delete row A and column B. Again select minimum cost value for the remaining cities which is shown below Table 6.7.

Table 6.7

Column 1(City)	Colum 2(Assignment)
C	A
D	A, E
E	**A**

Since assignment A occur with city C, D and E. Hence we take the difference between the value of A and next minimum value, here the maximum difference is 3 for Job E. And hence we assign A to city E. This is shown in Table 6.8.

Table 6.8

	A	C	E
C	6	-	7
D	8	9	8
E	$\boxed{4}$	7	-

Next delete row E and column A. Again select minimum cost value for the remaining cities which is shown below Table 6.9.

Table 6.9

Column 1(City)	Column 2(Assignment)
C	**E**
D	E

Here, we cannot assign C to city C. Therefore we only assign E to city C. Then obviously, we have no other choice rather to assign C for City D.

Finally, we can assign all the cities along with distance which is shown in Table 6.10.

Table 6.10

Column 1(City)	Colum 2(Assignment)	Distance
A	B	1
B	D	5
C	E	7
D	C	9
E	A	4
	Total	**26**

This solution is happened to be same as that of Hungarian method. Hence we can say that the minimum value is still 26 in both the methods. So this solution is **optimal**. However our method seems to be very simple, easy and takes very few steps in solving the method.

6.3 Conclusion

In this chapter, we have applied new alternate method of an assignment problem for solving Travelling salesman problem where it is shown that this method also gives optimal solution. Moreover the optimal solution obtained using this method is same as that of optimal solution obtained by Hungarian method. So we conclude that the Hungarian method and our method give same optimal solution. However the technique for solving Travelling Salesman problem using our method is more simple and easy as it takes few steps for the optimal solution.

CHAPTER 7
SUPPLY CHAIN MANAGEMENT

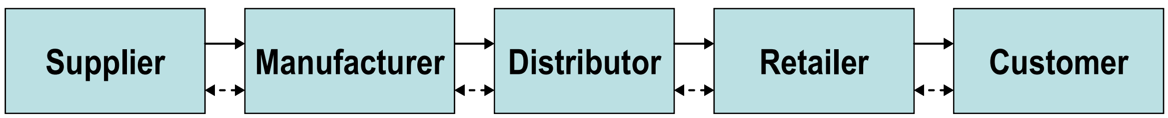
7.1 Introduction

The phrase 'supply chain management' appears to have originated in the early 1980s. Oliver and Webber (1982) discussed the potential benefits of integrating the internal business functions of purchasing, manufacturing, sales and distribution. In the modern era, it is a phrase that appears in many company's strategies and reports, practitioner and academic journals and texts. However, there is little consistency in the use of the term and little evidence of clarity of meaning expressed by Harland (1995a). Rather it appears to be a term used in several emerging bodies of knowledge which have remained largely unconnected till to date.

Oliver and Webber (1982), Houlihan (1984), Stevens (1989), Saunders (1994), Jones and Riley (1985) etc. developed the concept and the meaning of supply chain management in detail. Ammer (1968) and Lee and Dobler (1965) explained the pre-existing concepts of Supply Chain Management and found that It relates closely to materials management. Porter (1985), Johnston and Lawrence (1988) and Kogut (1985) explained the concepts of Supply Chain Management in terms of the value of Materials. However, Harland argued that the supply chain management is concerned with inter business, not intra-business integration.

There seems to be a universal agreement on what a supply chain is? A supply chain is a network of autonomous or semi-autonomous business entities collectively responsible for procurement, manufacturing, and distribution activities associated with one or more families of related products. A supply chain is a network of facilities that procure raw materials, transform them into intermediate goods and then finished products, and then finally deliver the products to customers through a distribution system or a chain. Moreover we can also express that a supply chain is a network of facilities and distribution options that performs the functions of procurement of materials. This also transforms these materials into intermediate and finished products, and finally the distribution of these finished products to customers.

Next we discuss the supply chain management system using networking in the following way.



This network is consists of all parties involved directly or indirectly in fulfilling a customer request.

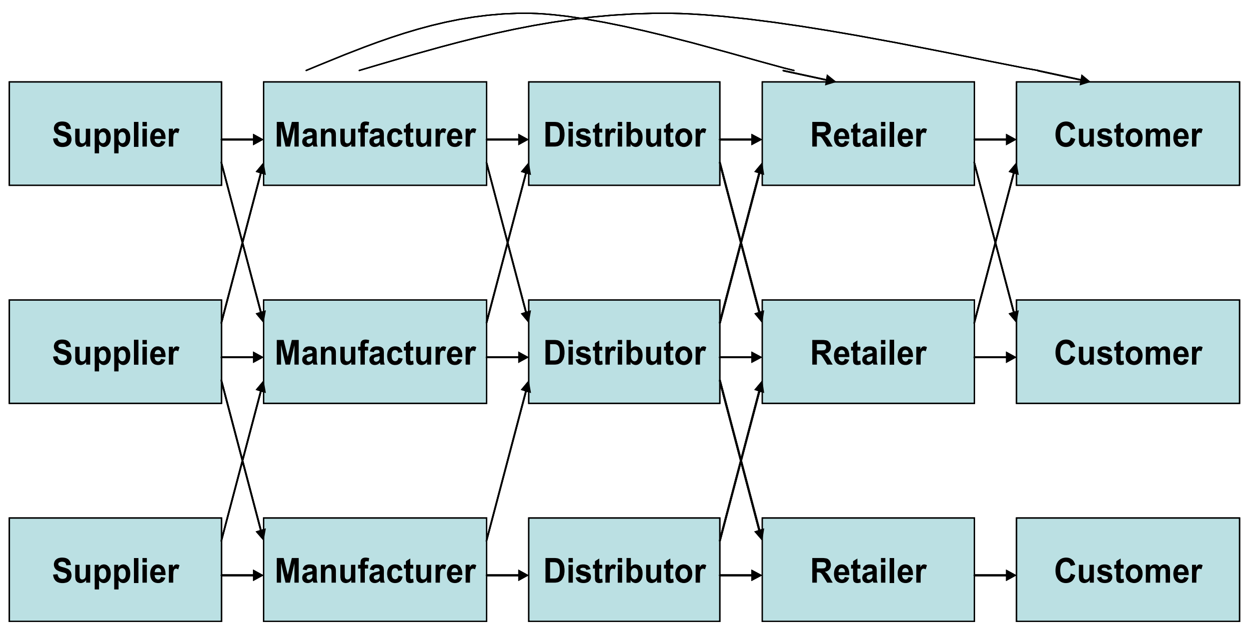


Figure 7.1

Upstream **Downstream**

Supply chain can explain with the following example.

A manufacturer produced a product which cost is Rs. 1 per product. Now he sells to the Retailer at a cost of Rs. 6 per product and finally retailer sells to the customer same product at a cost of Rs. 11 per product.

For this example, we will see the exact profit received by manufacturer and retailer.

In Supply Chain Management System, any product which is manufactured in a company, first reaches directly from manufacturer to distributors where manufacturer sold the product to the distributor with some profit of margin. Distributors supply that product to retailer with his profit and then finally customers received that product from retailer. That is called supply chain management system which implies that a product reaches from manufacturer to customer through supply. Let manufacturer cost of one product is Rs. 1 and have produce 1000 such products. Therefore manufacturer cost of 1000 products is Rs. 1000. Manufacturer supply these products to the distributor at a cost of Rs. 2 per product. Which means that distributor received those products at a cost of Rs. 2000. So manufacturer's profit is Rs. 1000. Then retailer received those products at a cost of Rs. 4 per product from the distributor. That is the retailer received those product at a cost of Rs. 4000. So distributor's profit is Rs. 2000. Finally the customer received these products at a cost of Rs. 7 per product from the retailer. That is the customer received those products at the cost of Rs. 7000. So retailer's profit comes out to be Rs. 3000 only. This shows that customer got 1000 product with a cost of Rs. 5000 more provided customer purchase those products directly from the manufacturer. To understand this chain we have developed a mathematical model of Supply of a product from manufacturer-distributor-retailer-customer. This we discuss in section 7.2.

What is supply chain management?

Supply chain management (SCM) is the combinations of art and science that goes into improving the way your company finds the raw components it needs to make a product or service and deliver it to customers. The following are five basic components of SCM.

1. **Plan.** This is the strategic portion of SCM. Companies need a strategy for managing all the resources that go toward meeting customer demand for their product or service. A big piece of SCM planning is developing a set of metrics to monitor the supply chain so that it is efficient, costs less, and delivers high quality as well as value to customers.

2. **Source**. Next, companies must choose suppliers to deliver the goods and services they need to create their product. Therefore, supply chain managers must develop a set of pricing, delivery and payment processes with suppliers and create metrics for monitoring and improving the relationships. And then, SCM managers can put together processes for managing their goods and services inventory, including receiving and verifying shipments, transferring them to the manufacturing facilities and authorizing supplier payments.

3. **Make**. This is the manufacturing step. Supply chain managers schedule the activities necessary for production, testing, packaging and preparation for delivery. This is the most metric-intensive portion of the supply chain where companies are able to measure quality levels, production output and worker productivity.

4. **Deliver**. This is the part that many SCM insiders refer to as logistics, where company manager coordinate the receipt of orders from customers, develop a network of warehouses, pick carriers to get products to customers and set up an invoicing system to receive payments.

5. **Return**. This can be a problematic part of the supply chain for many companies. Supply chain planners have to create a responsive and flexible network for receiving defective and excess products back from their customers and supporting customers who have problems with delivered products.

Definition

A supply chain is the stream of processes of moving goods from the customer order through the raw materials stage, supply, production, and distribution of products to the customer. All organizations have supply chains of varying degrees, depending upon the size of the organization and the type of product manufactured. These networks obtain supplies and components, change these materials into finished products and then distribute them to the customer.

Concept of Supply Chain System

Managing the chain of events in this process is known as supply chain management. Effective management must take into account coordinating all the different pieces of this chain as quickly as possible without losing any of the quality or customer satisfaction, while still keeping costs down.

The first step is obtaining a customer order, followed by production, storage and distribution of products and supplies to the customer site. Customer satisfaction is paramount. In supply chain process, we are including the customer orders, order processing, inventory, scheduling, transportation, storage, and customer service. A necessity in coordinating all these activities is the information service network.

In addition, key to the success of a supply chain is the speed in which these activities can be accomplished and the realization that customer needs where customer satisfaction are the very reasons for the network. Reduced inventories, lower operating costs, product availability and customer satisfaction are all benefits which grow out of effective supply chain management.

The decisions associated with supply chain management cover both the long-term and short-term period. Strategic decisions deal with corporate policies, and look at overall design and supply chain structure. Operational decisions are those dealing with every day activities and problems of an organization. These decisions must take into account the strategic decisions already in place. Therefore, an organization must structure the supply chain through long-term analysis and at the same time focus on the day-to-day activities. Manufacturer to customer will be benefited in real sense if day to day activities are performed.

Furthermore, market demands, customer service, transport considerations, and pricing constraints all must be understood in order to structure the supply chain effectively. These are all factors, which change constantly and sometimes unexpectedly. Moreover an organization must realize this fact and be prepared to structure the supply chain accordingly.

Structure of the supply chain requires an understanding of the demand patterns, service level requirements, distance considerations, cost elements and other related factors. It is easy to see that these factors are highly variable in nature and this variability needs to be considered during the supply chain analysis process. Moreover, the interplay of these complex considerations could have a significant bearing on the outcome of the supply chain analysis process.

There are six key elements to a supply chain:

- Production
- Supply
- Inventory
- Location
- Transportation, and
- Information

Application of Supply Chain Management

There are four main uses of the term 'supply chain management':
First, the internal supply chain that integrates business functions involved in the flow of materials and information from inbound to outbound ends of the business. Secondly, the management of dike or two parties are related with immediate suppliers. Thirdly, the management of a chain of businesses includes a supplier and a customer and so on. Fourthly, the management of a network of interconnected businesses involved in the ultimate provision of product and service packages required by end customers.

Now we develop a mathematical model for Supply Chain Management. In this model we want to explain how one can find out manufacturer's price once the customer price is known. In visa- versa we are also interested to know the customer price once the manufacturer's price are known. Such phenomena are discussed in mathematical model of supply Chain Management developed by us in section 7.2.

7.2 Mathematical Model of SCM

Let denote price of manufacturer, distributor, retailer and customer by M, D, R and C respectively. Suppose manufacturer sells his product with more than d_1% of the manufacturer price to the distributor, distributor sells that product with more than d_2% of the distributor price to the retailer. Finally, retailer sells that product with more than d_3% of the retailer price to the customer. We can explain this in equation form as:

$$D = M+d_1\%M \tag{7.1}$$

$$R = D+d_2\%D \tag{7.2}$$

$$C = R+d_3\%R \tag{7.3}$$

On the basis of the price and the equation provided in (7.1) to (7.3), we developed a mathematical model to know the price of manufacturer provided price of customer is known in the following way.

Using (7.3) we can find retailer's price provided final customer's price is known in the following way.

$$C = R\ (1+d_3)$$

$$Hence\ R = \frac{C}{(1+d_3)} \tag{7.4}$$

We get price relation between retailer and customer from (7.4).

Again using (7.2), we find distributor's price provided retailer price is known from (7.4) in the following way.

$R = D\ (1+d_2)$

$$D = \frac{R}{(1+d_2)}$$

On putting the value of R from (7.4), we get D as,

$$D = \frac{\frac{C}{(1+d_3)}}{(1+d_2)}$$

$$\text{Hence } D = \frac{C}{(1+d_2)*(1+d_3)} \qquad (7.5)$$

Now we get price relation between distributor and customer from (7.5).

Finally using (7.1), we can find out manufacturer price provided distributor price is available from (7.5) in the following way.

$D = M\ (1+d_1)$

$$\text{Therefore, } M = \frac{D}{(1+d_1)}$$

On putting the value of D from (7.5), we get price of Manufacturer as,

$$M = \frac{\frac{C}{(1+d_2)*(1+d_3)}}{(1+d_1)}$$

$$\text{Hence } M = \frac{C}{(1+d_1)*(1+d_2)*(1+d_3)} \qquad (7.6)$$

Now we get price relation between manufacturer and customer from (7.6).

In visa-versa, we can also find out customer price provided retailer, distributor and manufacturer price is given along with d_1, d_2 and d_3. This is explained by the following model.

$$C = M\ (1+d_1)*(1+d_2)*(1+d_3) \qquad (7.7)$$

From this we found the relation between manufacturer and distributor, manufacturer and retailer, manufacturer and customer.

Using from (7.7), we have C as,

$C=M\,(1+d_1)*(1+d_2)*(1+d_3)$

This relation is between customer and manufacturer.

Putting the value of C in eq. (7.5), we get D as,

$$D=\frac{M\,(1+d_1)*(1+d_2)*(1+d_3)}{(1+d_2)*(1+d_3)}$$

Hence we have $D=M\,(1+d_1)$ (7.8)

This relation is between distributor and manufacturer.

Again the putting the value of C in eq. (7.4), we get R as,

$$R=\frac{M\,(1+d_1)*(1+d_2)*(1+d_3)}{(1+d_3)}$$

Finally we have the Price of Retailer as, $R= M\,(1+d_1)*(1+d_2)$ (7.9)

This relation is between retailer and manufacturer.

Remark

Using (7.1) to (7.9), one can find price relation of product between customer and manufacturer, customer and retailer, customer and distributor and visa-versa.

7.2.1 Numeric example Here we demonstrate the price from manufacturer to customer and vice-versa by a hypothecated numerical example in 7.2.1.1.

Example 7.2.1.1

Manufacturer sold a product with 10% more than the manufacturing price to distributor, Distributor sold that product with 10% more than the distributor price to retailer and Retailer sold that product with 35% more than the retailer price to customer. For this example we have calculated distributor, retailers and customer price for given manufacturer price.

Table 7.1

Manufacturer price	100	200	300	400	500	600	700	800	900	1000

Solution

Here given is d_1=0.10, d_2=0.10 and d_3=0.35. We have to find distributor, retailers and customer price for given manufacturer price using (7.8), (7.9) and (7.7).

Table 7.2

M Manufacturer price	D=M (1+d_1) Distributor price	R=M (1+d_1)*(1+d_2) Retailer price	C=M (1+d_1)*(1+d_2)*(1+d_3) Customer price	Round figure Customer price
100	110	121	163.35	**163**
200	220	242	326.7	**327**
300	330	363	490.05	**490**
400	440	484	653.4	**653**
500	550	605	816.75	**817**
600	660	726	980.1	**980**
700	770	847	1143.45	**1143**
800	880	968	1306.8	**1307**
900	990	1089	1470.15	**1470**
1000	1100	1210	1633.5	**1634**

Table 7.3

C Customer price	C/(1+d_3) Retailer price	C/(1+d_2)*(1+d_3) Distributor price	C/(1+d_1)*(1+d_2)*(1+d_3) Manufacturer price	Round figure (Manufacturer Price)
163	120.740741	109.7643098	99.78573615	**100**
327	242.222222	220.2020202	200.1836547	**200**
490	362.962963	329.96633	299.9693909	**300**
653	483.703704	439.7306397	399.755127	**400**
817	605.185185	550.1683502	500.1530456	**500**
980	725.925926	659.9326599	599.9387818	**600**
1143	846.666667	769.6969697	699.7245179	**700**
1307	968.148148	880.1346801	800.1224365	**800**
1470	1088.88889	989.8989899	899.9081726	**900**
1634	1210.37037	1100.3367	1000.306091	**1000**

Remark We also calculated retailer, distributor and manufacturer price by taking the price of customer obtained in (7.2) in order to check the price of manufacturer as given in Table (7.2). From Table (7.2) and (7.3), it is clear that the known manufacturer price in Table (7.2) is same as calculated manufacturer price in Table (7.3). Similarly known customer price in Table (7.3) is same as calculated customer price Table (7.2).

7.3 Conclusion

Using this model, if we know customer price and d_3 we can find retailer price, if we know customer price and d_2, d_3 we can find distributor price, if we know customer price and d_1, d_2, d_3 we can find manufacturer price and also vice-vers~~a.~~

CHAPTER 8

CONCLUSIONS AND SCOPE OF FUTURE WORK

8.1 CONCLUSION

In this chapter we discuss the final findings obtained in chapters 3, 4, 5, 6 and 7.

8.1.1 Transportation problem

Chapter 3 deals both the alternate algorithms for a TP as very few alternate algorithms for obtaining an optimal solution are available in the textbook and in other literature. These methods are so simple and easy that makes understandable to a wider spectrum of readers. The methods discussed in chapter 3, either gives a near optimal solution for certain TP while it gives optimal solution for other certain TP.

8.1.2 Transshipment Problem

In Chapter 4, we have developed a simple algorithm for solving a Transshipment Problem. The proposed algorithm is easy to understand and apply. The optimal solution obtained in this investigation is same as that of MODI method. It will be possible that basic feasible solution obtained using new alternate method developed in chapter 4 may yield near to the optimal solution for certain TP compared to MODI method.

8.1.3 Assignment Problem

So far in the literature Hungarian method is used to obtained optimal solution for an assignment problem. In chapter 5, we have developed a new alternate method for solving an assignment problem where it is shown that this method always gives optimal solution. Moreover the optimal solution obtained using new alternate method is same as that of optimal solution obtained by Hungarian method. So we conclude that the Hungarian method and our method give same optimal solution. However the technique for solving an assignment problem using new alternate method is more simple and easy as it takes few steps for obtaining the optimal solution.

8.1.4 Travelling Salesman Problem

In chapter 6, we have applied a new alternate method of an assignment problem for solving Travelling salesman problem where it is shown that this method also gives optimal solution. Moreover the optimal solution obtained using new alternate method is same as that of optimal solution obtained by Hungarian method. So we conclude that the Hungarian method and new alternate method gives same optimal solution. However the technique for solving travelling salesman problem based on a new alternate method of assignment problem is more simple and easy as it takes few steps for the optimal solution.

8.1.5 Supply Chain Management

Using our model discussed in Chapter 7, one can find customer's price provided manufacturer's price is known, similarly manufacturer's price can be calculated provided customer's price is known when value of d_1, d_2 and d_3 are supplied.

8.2 SCOPE OF FUTURE WORK

From chapter 3 to chapter 6, we have developed a new alternate method for solving Transportation Problem, Transshipment Problem, Assignment Problem and Travelling Salesmen Problem which gives either near to optimal solution or optimal solution.

The new alternate methods developed from chapter 3 to 6 are used only for balanced cases except assignment problem. However we could not made any attempt to solve unbalanced problem related with transportation and transshipment problem. Hence one can try to solve unbalanced problem using new alternate methods developed in this thesis as scope of future work.

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