An Economic Approach to Consumer Product Ratings

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Abstract

In three essays we examine user-generated product ratings with aggregation. While recommendation systems have been studied extensively, this simple type of recommendation system has been neglected, despite its prevalence in the field. We develop a novel theoretical model of user-generated ratings. This model improves upon previous work in three ways: it considers rational agents and allows them to abstain from rating when rating is costly; it incorporates rating aggregation (such as averaging ratings); and it considers the effect on rating strategies of multiple simultaneous raters. In the first essay we provide a partial characterization of equilibrium behavior. In the second essay we test this theoretical model in laboratory, and in the third we apply established behavioral models to the data generated in the lab. This study provides clues to the prevalence of extreme-valued ratings in field implementations. We show theoretically that in equilibrium, ratings distributions do not represent the value distributions of sincere ratings. Indeed, we show that if rating strategies follow a set of regularity conditions, then in equilibrium the rate at which players participate is increasing in the extremity of agents' valuations of the product. This theoretical prediction is realized in the lab. We also find that human subjects show a disproportionate predilection for sincere rating, and that when they do send insincere ratings, they are almost always in the direction of exaggeration. Both sincere and exaggerated ratings occur with great frequency despite the fact that such rating strategies are not in subjects' best interest. We therefore apply the behavioral concepts of quantal response equilibrium (QRE) and cursed equilibrium (CE) to the experimental data. Together, these theories explain the data significantly better than does a theory of rational, Bayesian behavior — accurately predicting key comparative statics. However, the theories fail to predict the high rates of sincerity, and it is clear that a better theory is needed.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

Until the fall of 2009, YouTube.com (hereafter, YouTube), the third most trafficked website in the world[1], utilized a system of user-generated ratings. At that time, whenever a user viewed a video on YouTube, she had the option to submit an integer rating from 1 to 5 corresponding to the number of "stars" a user wished to rate a video. For each video, YouTube computed the average of these ratings and displayed the average next to the video wherever the video was found: next to the video itself, in search results, etc. Every user was therefore informed of a video's average rating before taking any action, and could use that information to inform his consumption decision.

The size of a visitor to YouTube's choice set is enormous — roughly 48 hours of content are uploaded to YouTube every *minute*.[2] Deciding which video to watch is therefore not a simple choice. Moreover, while the type of product may vary, difficult consumption choices are not unique to YouTube. Indeed, large and varied choice sets are a hallmark of Internet marketplaces. Amazon.com, for example, claims to offer more than 14 million books.[3]

Product ratings and reviews are intended to help consumers choose between products. They are a part of a mechanism designed to organize products, alleviating the need for a consumer to experience every available product for themselves. User-generated ratings in particular allow consumers who *have* consumed a product to send a message to future potential consumers of that

[1] All statistics and information regarding `www.youtube.com` were found at one of the following urls, all of which were accessed first on 05/05/2009 and again on 01/28/11. `http://www.alexa.com/topsites,http://www.youtube.com/t/advertising\_targeting`, `http://www.youtube.com/t/about\_youtube`, `http://help.youtube.com/support/youtube/bin/answer.py?hl=en\&answer=95564`.

[2] `http://www.youtube.com/t/faq`. Accessed 08/20/2012.

[3] `http://www.amazon.com/gp/help/customer/display.html?ie=UTF8&nodeId=14101911`. Visited 08/20/2012.

product, informing them of the product's quality. By many accounts, user-generated ratings have been successful towards this end (Li and Hitt 2008) (Chevalier and Mayzlin 2006) (Dellarocas, Zhang, and Awad 2007).

Nonetheless, on its official blog, in September 2009, YouTube lamented the state of its user-generated rating system, "...it looks like some of you are moved to rate videos when you don't like them, but the overwhelming majority of videos on YouTube have a stellar five-star rating." They concluded, "Seems like when it comes to ratings it's pretty much all or nothing." Indeed, at that time there were more than twice as many one-star ratings on YouTube than any other ratings besides five-star ratings, and there were at least ten times more five-star ratings than one-star ratings.[4] As a result, on January 21st, 2010, YouTube removed its user-generated ratings system entirely.[5] While YouTube's experience highlights the prevalence of extreme-valued ratings, the phenomenon is not unique to YouTube, and indeed appears to be widespread in online communities (Lafky 2010).

In this thesis we study user-generated ratings in three essays. Our goal is to shed light on rating behavior as it exists in the field today. In the first essay, we develop a unique theoretical framework. In the second essay, we test the model using laboratory experiments; and in the third essay, we apply established behavioral models to the data generated in the laboratory experiments. Throughout the thesis we consider three conjectures as the cause of the prevalence of extreme-valued ratings:

Sincere: *Raters report their true valuations and rating distributions are equivalent to valuation distributions.*

Exaggeration: *Raters do not report their true valuations, but instead skew their ratings towards the extremes of the rating space.*

Extreme Participation: *Raters are more likely to rate a product if they have an extreme valuation for that product.*

There has been little work done on this question. The study of user-generated ratings has historically fallen under the heading of "recommendation systems". Recommendation systems at-

[4] This estimate assumes that the scale of the figure accompanying this sentiment is linear

[5] The system was replaced with a binary system of "thumbs up" or "thumbs down." However, unlike typical systems of user-generated ratings, this new system does not display results to future users, removing the system's utility as a one which assigns value or removes uncertainty.

tempt to, "...estimat[e] ratings for the items that have not been seen by a user" (Adomavicius and Tuzhilin 2005). Systems such as the ones we study here which present past product ratings through the use of aggregation functions are referred to as memory- (or heuristic-) based collaborative recommendation systems (Adomavicius and Tuzhilin 2005), and are among the simplest used or studied. Perhaps because of their simplicity, literature in this field has paid these types of recommendation systems little attention, focusing instead, primarily, on developing improved mechanisms. See, for example, Sami and Resnick (2007) or Miller, Resnick, and Zeckhauser (2005); for a survey of recommendation systems, see Adomavicius and Gediminas (2005).

Treatments of rating behavior have been unable to differentiate between our posed conjectures. For example, Chen et al. (2005) consider the determinants of rating frequency of users at MovieLens. They confirm that extreme-valued rating behavior is positively correlated with rating frequency. However, the authors do not attempt to differentiate between a rater's true feelings and his rating. Lafky (2010) studied the causes of the prevalence of extreme ratings, developing a theory of rating behavior and testing it in the lab. While this paper is the best treatment of this topic to date, Lafky considers the rating behavior of only a single rater. As we show in the next chapter, this assumption tremendously simplifies analysis of the game, but fails to account for changes in rater behavior resulting both from the induced simultaneous rating game, and from the loss of information involved in rating aggregation. Importantly, as we prove below, with only one rater, every equilibrium is equivalent to an equilibrium in which there exist only three feasible ratings. The assumption of a single rater therefore precludes analysis of the exaggeration conjecture.

While implementations of user-generated ratings mechanisms vary widely in the field, in the theoretical treatment we develop here we attempt to incorporate the most salient, commonly shared, attributes of these implementations. Following Lafky, among these are the ability of rater-agents to abstain from costly rating. In addition, our model generalizes Lafky's by allowing the number of raters to vary, and by aggregating user ratings into simple statistics viewable by future actors. We find that for a class of informative equilibria, the extreme participation conjecture holds regardless of the (ex ante) distribution of raters' valuations and costs. As a corollary, we can rule out the sincere conjecture: if raters report their true valuations, then in equilibrium selective participation skews rating distributions away from value distributions. We also identify conditions under which equilibrium ratings are sincere, and find equilibria under which messaging strategies involve both exaggeration and dampening relative to raters' valuations (which it is depends on the rater's valuation).

The second essay tests the theoretical predictions in a laboratory environment. The laboratory is well suited to studying user-generated ratings as well as the conjectures posed above. This is because in a laboratory we are able to control aspects of the rating environment that are unobservable in the field. Vitally, we observe each subject's valuation for a product, her cost of rating a product, and her rating for that product. Lafky's is the only paper we are aware of that has tested rating behavior in the lab, and while it is an important first step, it suffers from the same deficiencies as that paper's theoretical treatment. We conduct experiments on three treatments in which we vary the number of senders (players who submit ratings, but do not see others' ratings themselves) between one and five. We find that extreme participation conjecture holds in all treatments, as expected. The essay contains two unexpected results. First, we find that subjects' rating strategies are strongly anchored on sincerity; senders overwhelmingly send their valuations as their rating. Second, we find that when subjects' ratings *are* insincere, subjects almost exclusively exaggerate their rating (as compared to their valuation; with respect to a central, information-less valuation), and the rate of exaggeration increases with the number of senders. These rating trends exist despite the fact that such ratings were often far from subjects' best responses given empirical play.

At the conclusion of the second essay we present two hypotheses for subjects' deviations from equilibrium or rational best response. First, we suggest that subjects may have made erroneous choices as a result of the fact that subjects' expected payouts were relatively flat as a function of their feasible actions. Second, we hypothesize that subjects may not have properly updated their beliefs contingent on the pivotal events of the game. In the final essay, we apply the behavioral concepts of quantal response equilibrium (QRE) and cursed equilibrium (CE) to the data generated in the lab. These concepts address our hypotheses in order. Devised by McKelvey and Palfrey

(1995), QRE generalizes Nash equilibrium by allowing for errors in players' strategies. In particular, regular QRE (Goeree, Holt, and Palfrey 2005), which we utilize here, specifies that subjects play actions in proportion to their expected utility of playing that action. QRE therefore provides a precise theory for how our flat payoffs structure may have affected subjects' actions. Eyster and Rabin's concepts of cursed equilibrium and χ-cursed equilibrium say that players fail to properly update their beliefs given other players' actions. In the context of our game, a cursed player believes that the likelihood of a pivotal event is equal to the average likelihood of that event, averaged over all feasible levels of product quality); in a fully cursed equilibrium, pivotal events are independent to the product's quality (and therefore other players' signals). Combining QRE and CE therefore provides a behavioral model in which players make errors in actions and in formulating their beliefs. This approach has been successful, for example, in explaining subject behavior in common-value auctions (Palfrey, Camerer, and Nunnari 2011). We find that estimating quantal responsiveness and cursedness simultaneously, with separate estimates for each treatment and for each player role, best explains behavior. In every treatment, subjects acting as receivers (that is, players who viewed the averaged ratings of other players but were not asked to submit ratings themselves) behaved as if uncursed, evidently updating their beliefs properly and acting with some error. This is also true for players acting in the sender role in the one-sender treatment. However, we find evidence that increasing the number of senders above one reduced subjects' ability to update their beliefs, with senders in the five sender treatment acting as if they were almost fully cursed. The CE-QRE model improves upon the predictions of rational best response in many ways, including specifying a greater likelihood of sincere messaging, and correctly predicting the comparative static on the rate of exaggerated messages. Nonetheless, it fails to predict the high rates of sincere rating, and it is clear that a better theory is needed.

Chapter 2

A Theoretical Model of Consumer-Generated Product Ratings

2.1 Introduction

In this chapter, we develop a theoretical model of user ratings and discuss equilibrium rating and participation behavior. The main result of this chapter is to show that the sincere conjecture can be rejected in all of the environments that we consider. We prove that, conditional on continuously differentiable rating strategies, if ratings are aggregated using the mean of sent-ratings, then the extreme participation conjecture necessarily holds in all environments. Moreover, while we cannot categorically reject the exaggeration conjecture, it is possible to identify environments in which only the extreme participation conjecture contributes to the extremity of sent messages.

We proceed as follows: In the next section we review the literature, in the third section we define the model, and in the fourth we present the theory. After an introduction to the basic results, we explore the single-rater environment before moving onto a multiple-rater environment in which ratings are aggregated by the mean aggregation mechanism. Finally, we explore mechanisms in which ratings are aggregated using the median of sent-ratings, and in which raters can give only a "thumbs up" or "thumbs down" rating. In the final section we conclude.

Throughout the paper, unless stated, proofs are included in the appendix.

2.2 Literature Review

A positive relationship between average ratings and product sales is noted by (Li and Hitt 2008), (Chai, Potdar, and Chang 2007) and (Drèze and Hussherr 2003) among others. Most interestingly, Chevalier and Mayzlin (2006) found that discrepancies between books' average ratings on amazon.com and barnesandnoble.com were highly correlated with the sales discrepancies of those books across the two sites. User-generated product reviews have also been found to be highly predictive of product sales. Dellarocas et al. (2004) found that the number of user-generated movie reviews predicted box office sales better than a number of other explanatory variables.

Since Spence (1973), economists have recognized the strategic component of information transmission, and there has been much written about information transmission between informed players and an uninformed decision maker. The seminal reference in this vein is Crawford and Sobel(CS) (1982). The primary difference between ours and the CS model is that sending a message in this model carries a cost; in the language of CS, talk here is not cheap. Furthermore, unlike CS, in our model, players' objectives are aligned. These differences result in a game in which senders wish receivers to be as well informed as possible, but wish only to send a message if they expect the message to be sufficiently valuable to the receiver to overcome the cost to the sender. This is in stark contrast to equilibrium behavior in CS, in which the sender sends a message of limited informativeness as a result of the fact that their incentives are not aligned. More recently, this literature has made forays into message aggregation. Gerardi et al. (2007) show that if a decision maker can commit to an aggregation mechanism, then he can induce informed players to send truthful messages.

The political science literature has shown that when an actor chooses an action he must condition his beliefs on the events in which his actions are "pivotal" (Austen-Smith and Banks 1996) (Feddersen and Pesendorfer 1998) (Feddersen and Pesendorfer 1996) (Coughlan 2000). In other words, if an actor's actions only affect his payoffs under certain conditions, then he should update his beliefs about the state of the world as if those conditions were satisfied. The simplest example of this comes from the "winner's curse" in common-value auctions. In a common-value auction, multiple bidders receive correlated signals about an item's value. If a bidder makes his bid solely on his signal then he has overbid because if he wins then it must have been that every other bidder had a lower signal of the item's value. Therefore, the rational bidder must choose a bid conditional not only on his signal, but also conditional on the pivotal event of winning the auction. In more complex environments such as ours or such as an election, there may multiple pivotal events. In such cases, the logic is essentially the same, but rational agents must also consider the relative likelihood of each pivotal event. For example, voters in (Feddersen and Pesendorfer 1998) are pivotal when their vote changes the outcome of an election; they must therefore consider the likelihood that the candidates will tie without their vote, as well as the likelihoods that each candidate is one vote ahead of the other. Here, this logic is carried through to the product rating framework; a rational rater must update his beliefs about product quality conditional on an event in which he can affect the consumption decision of future actors.

Morgan and Krishna (2008) showed that when voting is both costly and voluntary, there exists an equilibrium in which voters play a cutoff strategy (in voting costs) for participation, and when they do vote, they vote honestly. In contrast to their model, players in our game receive one of many different signals about the quality of a product; players can send one of many messages, as opposed to the binary messages of voting; and the purchasing decisions of players who act in the second period must satisfy incentive compatibility, whereas in a voting model, the election decision is made according to a set rule.

Lafky (2010) considers a model of user-generated reviews in which one first-mover sends a review of one of two products to many second-mover consumers. While the assumption of a single first-mover simplifies analysis of the game, as we prove below, under such an assumption every equilibrium is equivalent to an equilibrium in which there exist only three feasible ratings, and is therefore incompatible with investigating our exaggeration conjecture.

In the context of his model, Lafky shows experimentally that reviewers are motivated to send reviews even in the presence of reviewing-costs. He interprets this result as a confirmation of intrinsic valuation for both helping other consumers and for punishing or rewarding producers. We adopt this result, motivating reviewers by assuming that each player who consumes a product wants future consumers to consume the best product possible. Similar other-regarding preferences have been documented experimentally (Palfrey and Prisbrey 1997) (McKelvey and Palfrey 1992). The tension in the model comes from the fact that players who send messages also incur costs, a facet that has also been explored in literature on voting (Palfrey and Rosenthal 1983) (Ledyard 1984).

2.3 Model, Notation, and the Game

2.3.1 Primitives

We study a class of games in which players consume one of two products, in one of two periods. One of the products is considered "safe" in the sense that it gives all players who consume it a known value, which we normalize to 0. The other product is considered "risky," as the value a player gets from consuming it is a random variable. The distribution of values of the risky product depends on the "state of the world," a random variable that is determined by nature at the outset of the game. We denote the possible states of the world as $\Theta = \{\theta^h, \theta^l\}$ where θ^h corresponds to the state in which the risky product gives values according to a "high" distribution, and θ^l a "low."

In the first period, $N \geq 1$ players called "Senders" consume the risky product[1], observe a private random draw from the distribution of values of the risky product, and then decide whether or not to pay a privately known cost in order to send a message to the receivers. For a given sender, i, we denote the draw from the distribution of values of the risky product as v_i, and the private cost to participation as c_i. The players type is thus denoted $t_i = (v_i, c_i)$. The space of values of the risky product is described by V and the space of costs is described by C where $V = [\underline{v}, \overline{v}] \subset \mathcal{R}$ and $C = [\underline{c}, \overline{c}] \subset \mathcal{R}$. Thus, the space of types for a sender is $T_s = V \times C$.

After observing their private types, senders simultaneously decide whether or not to participate in the mechanism, and which message to send to the mechanism. A sender, i's, participation action is denoted $\rho_i \in \{0, 1\}$. If $\rho_i = 0$, then he abstains, and if $\rho_i = 1$, then he participates. We restrict the message space to the value space and denote the message of a single sender, i, as $m_i \in V$. We

[1]This restriction is without loss of generality as the optimal strategy of senders who consume the safe product would be to abstain.

denote the set of all *sent*-messages (messages from senders who participated) as $\mathbf{m}$, and the set of all sent messages *excluding that of player i* as $\mathbf{m}_{-i}$. Since participation is voluntary, $\mathbf{m}$ may contain any number of messages from 0 to N, and $\mathbf{m}_{-i}$ any number from 0 to $N-1$.

In the second period, all sent messages are aggregated into a single message, which is observed by a single[2] player called the "Receiver". The receiver then chooses which product, the safe or the risky, to consume. We denote the receiver's action as $a_r \in \{0,1\}$. If $a_r = 0$, then he consumes the safe product, and if $a_r = 1$, the risky product. The receiver's consumption value is denoted v_r.

The aggregation mechanism is denoted μ, and we also restrict μ such that feasible space of aggregate messages is equivalent to the value space, V. If no messages are sent, we assume that the aggregation mechanism outputs a unique message, which we denote μ^{Φ}. The receiver's type is denoted t_r and is described by the aggregate message he receives. Therefore, his type space is $T_r = V \cup \mu^{\Phi}$.

We model the game as one of complete but imperfect information following Harsanyi (1967), and players share a common prior over both the state of the world and sender types conditional on the state of the world. Nature chooses the state of the world before the first period begins, and we assume that each state is equally likely. This is not a critical assumption, but it does provide mathematical simplicity.

Assumption 2.3.1. $Pr(\theta^h) = Pr(\theta^l) = \frac{1}{2}$

In the high state, $F_{v|\theta^h}$ describes the common prior distribution of values and in the low state, $F_{v|\theta^l}$. We assume that these have corresponding differentiable densities $f_{v|\theta^h}$ and $f_{v|\theta^l}$, respectively, which each have full support over V. In addition, we assume that a sender's posterior belief regarding the likelihood of the high state of the world is increasing in the value of a player's draw. Assumption 2.3.2 is commonly known as the monotone likelihood property. This assumption also ensures both that $F_{v|\theta^h}$ first order stochastic dominates $F_{v|\theta^l}$ and that $E[v_r|\theta^l] < E[v_r|\theta^h]$.

Assumption 2.3.2. $\frac{\partial}{\partial v}\frac{f_{v|\theta^h}(v)}{f_{v|\theta^l}(v)} \geq 0$

[2]The use of a single receiver follows from the symmetry condition of the equilibrium concept and is without further loss of generality. This is discussed in remark 2.3.8.

In addition to assumption 2.3.2, we require that conditional distributions are such that there exists a value in the interior of V that, if observed by the receiver directly, would cause him to be indifferent between the two products. Let v^I solve $E[v_r|v^I] = 0$. As we show below, v^I is important for characterizing equilibria. Assumption 2.3.3 says that v^I exists in the interior of V.

Assumption 2.3.3. $v^I \in (\underline{v}, \overline{v})$.

We assume that players' common prior distribution over private costs, F_c is independent of the state of the world, and that it has a corresponding differentiable density f_c which has full support over C.

Assumption 2.3.4. *For any sender, i and $c_i \in C$, $F_{c|\theta^h}(c_i) = F_{c|\theta^l}(c_i) = F_c(c_i)$.*

We furthermore assume that 0 is among the possible private costs, and that it is possible for a player to have a cost that exceeds the benefit of participation, no matter that player's posterior beliefs.

Assumption 2.3.5. *If $C = [\underline{c}, \overline{c}]$, then $\underline{c} \leq 0$ and $max\{E[v_r|\theta^h], -E[v_r|\theta^l]\} < \overline{c}$.*

Finally, we assume that players wish to maximize their own consumption utility less their costs, that they are altruistic towards future consumers[3], and that each of these enters the player's utility function linearly.

Assumption 2.3.6. *For any sender, i, $u_i = v_r - c_i 1_{\rho_i=1}$*

Where $1_{\rho_i=1}$ is an indicator function equal to 1 if player i participates and 0 otherwise. Excluding v_i is also without further loss of generality because, since senders do not make a consumption choice and since v_i enters linearly, v_i does not affect player's strategies and can therefore be ignored. For the receiver, this construction reduces simply to $u_r = v_r$.

[3]Because we assume that utilities are linear, it is without further loss of generality that we let the altruism coefficient equal 1 for all senders. This is because, as is clear from lemma 2.4.3, it is the ratio of private cost to private altruism coefficient that determines the sender's best response. Therefore, we can hold the altruism coefficient constant across senders, and let any idiosyncrasy be expressed through private costs.

2.3.2 Strategies and Equilibrium

The receiver's strategy is denoted $A : T_r \to [0,1]$, where $A(t_r)$ denotes the probability the receiver chooses the risky product upon receipt of the aggregate message t_r. A sender, i's, strategy is denoted $S_i : T_s \to [0,1] \times V$. For $t_i \in T_s$, we decompose $S_i(t_i)$ into $S_i(t_i) = (P_i(t_i), M_i(t_i))$ to describe player i's participation strategy and his messaging strategy, respectively[4]. We understand $f_{S_i}(m_i)$ to describe the likelihood that i sends the message m_i, given his strategy S_i, and $f_{S_i|t_i}(m_i)$ to describe the likelihood that i sends the message m_i, given his strategy S_i and his type t_i. These are not equivalent to a transformation of v_i via $M_i(v_i)$ because $P_i(t_i)$ may dictate that player i participate at different rates for different values of v_i[5].

For any S_i, we let σ_i be the composite function that takes first-period player, i's, type into the aggregate message space; $\sigma_i(t_i) = (\mu \circ M_i)(t_i)$. Additionally, for any $T' \subseteq T_s$, we define $\Sigma(T')$ to be the image in T_r of T' under σ_i.

We use $E[u_i]$ to denote the expected value of player i's utility. $E[u_i|t_i, \rho_i = 0]$ is therefore his expected utility given that he abstains and his type is t_i, and $E[u_i|t_i, \rho_i = 1, m_i]$ is his expected utility given that he sends message m_i and his type is t_i.

The equilibrium concept we employ is as an adaptation of perfect Bayesian Nash equilibrium (PBNE). This is the standard definition of perfect Bayesian Nash equilibrium, in which we denote the receiver's posterior beliefs by $g(\theta|t_r)$, and to which we add the following conditions:

Symmetry

$$\text{Let } t_s \in T_s. \text{ For any senders, } i \text{ and } j,\ S_i(t_s) = S_j(t_s).$$

[4]As is evident from this specification, we are restricting our analysis to pure messaging strategies. We do this in order to simplify the analysis.

[5]By construction, $f_{S_i}(m_i) = \frac{\int_{T_s} M_i(t_i)(m_i) P_i(t_i) dF_{T_s}(t_i)}{\int_V \int_{T_s} M_i(t')(m') P_i(t') dF_{T_s}(t') dm'}$ and $f_{S_i|t_i}(m_i) = \frac{M_i(t_i)(m_i) P_i(t_i)}{\int_V M_i(t_i)(m') P_i(t_i) dm'}$

Informativeness

There exist types $\mathbf{t}, \mathbf{t}' \in T_s^N$, and sets of messages $\mathbf{m}$ and $\mathbf{m}'$ such that:

$$f_{S|\mathbf{t}}(\mathbf{m}) > 0, f_{S|\mathbf{t}'}(\mathbf{m}') > 0 \text{ and } A(\mu(\mathbf{m})) \neq A(\mu(\mathbf{m}')).$$

The conditions we impose on perfect Bayesian Nash equilibrium are not without loss of generality. As is common in games of information transmission, the standard definition of PBNE, admits babbling equilibria of our game. These are equilibria in which the strategies of players who receive messages do not depend on the messages they receive. Since we are interested in the interplay of players' strategies, we exclude babbling equilibria with the informativeness condition, which says that there are at least two vectors of senders' types that induce different best responses from the receivers.

The symmetry condition and the condition that messaging strategies are pure strategies allow us to focus our search for equilibria and to focus our discussion. Due to the symmetry condition, if the usage is unambiguous, we drop the player subscript from strategies and posteriors (e.g., $S_i(t_i) = S(t_i)$). Indeed, this simplification can be extended to all beliefs.

Lemma 2.3.7. *In equilibrium, conditional on the state of the world, any two players have the same beliefs over the actions of other (third-party) players (regardless of types).*

Since $Pr(\theta|v_i)$ is common knowledge and $Pr(\theta|t_r)$ is known in equilibrium, we can extend lemma 2.3.7 to say that any players of the same type also have the same beliefs. Further, since costs are assumed to be orthogonal to states of the world, lemma 2.3.7 removes the need for conditioning interim beliefs about the actions of others on private costs (that is, beliefs can be conditioned on values alone).

Remark 2.3.8. *The symmetry condition ensures that if we had more than one receiver, they would have the same posterior beliefs and would therefore take action with the same likelihood. Because of this, it is without loss of generality that we assume a single receiver.*

2.4 Theory

Any equilibrium can be reduced to three parts: the senders' participation strategy, the senders' messaging strategy, and the receiver's consumption strategy. We examine each of these in turn, beginning with the receiver's strategy.

2.4.1 The Receiver

In the second period, the receiver receives an aggregate message. Proposition 1 says that, as a function of this message, the receiver's equilibrium strategy takes the form of a cutoff strategy in his beliefs. This result follows simply from the linearity of the receiver's utility function.

Proposition 2.4.1. *Let* $\nu = \frac{E[v_r|\theta^l]}{E[v_r|\theta^l]-E[v_r|\theta^h]}$ *and* $a^I \in (0,1)$*. In equilibrium,*

$$A(t_r) = \begin{cases} 1, \textit{ if } g(\theta^h|t_r) > \nu \\ 0, \textit{ if } g(\theta^h|t_r) < \nu \\ a^I, \textit{ if } g(\theta^h|t_r) = \nu \end{cases} \tag{2.1}$$

Proof. By definition of a PBNE, for all $t_r \in T_r$, $A(t_r)$ must satisfy

$$A(t_r) \in argmax_{a\in[0,1]}\ a \sum_{\theta\in\Theta} g(\theta|t_r)\ E[v_r|\theta] \tag{2.2}$$

If $g(\theta^h|t_r) > \nu$, then

$$a \sum_{\theta\in\Theta} g(\theta|t_r)\ E[v_r|\theta] > a \left(\frac{E[v_r|\theta^l]E[v_r|\theta^h]}{E[v_r|\theta^l] - E[v_r|\theta^h]} - \frac{E[v_r|\theta^l]E[v_r|\theta^h]}{E[v_r|\theta^l] - E[v_r|\theta^h]} \right) = 0$$

which implies that (2.2) is maximized when $a = 1$. By a symmetric argument, we can show that when $g(\theta^h|t_r) < \nu$, (2.2) is maximized when $a = 0$, and when $g(\theta^h|t_r) = \nu$, the choice of a is irrelevant to maximizing (2.2), so any a will do. □

We exclude the cases of $a^I \in \{0,1\}$ for simplicity. Since both of these cases imply actions consistent with strict preference, allowing a^I to be in $\{0,1\}$ requires that we state each result as a list of contingencies on the value of a^I, a practice which is more confusing than it is informative.

Remark 2.4.2. *From the proof of proposition 2.4.1 we can see that if it is not the case that* $E[v_r|\theta^l] < 0 < E[v_r|\theta^h]$*, then either* $A(t_r) = 1$ *or* $A(t_r) = 0$ *for all* t_r *and therefore the set of informative equilibria is empty. Thus, when considering informative equilibria, we can assume distributions with such expectations without loss of generality.*

2.4.2 Senders

A sender's problem consists of two decisions: first, whether or not to send a message; and second, which message he will send. Both problems require a concept of the likelihoods of outcomes contingent on the sender's actions. The simplest way to achieve this is by partitioning the type space of the *receiver* by outcome equivalence. As described in the statement of the game, the receiver's type space is $T_r = V \cup \mu^\Phi$. Let $L = \{t_r \in V : g(\theta^h|t_r) < \nu\}$, $I = \{t_r \in V : g(\theta^h|t_r) = \nu\}$, and $H = \{t_r \in V : g(\theta^h|t_r) > \nu\}$. By proposition 2.4.1, members of these sets are aggregate messages that when received by the receiver, respectively: induce the receiver to purchase the safe product; cause the receiver to be indifferent between the two products; and induce the receiver to purchase the risky product. Left from this categorization is the null message, μ^Φ. Although μ^Φ can also be grouped by outcome equivalence, it is important to separate the null message in our theoretical treatment and we therefore assign it its own partition, which we call $\Phi = \{\mu^\Phi\}$.

Thus, conditional on the first-period player i's sending message m_i, the likelihood that the receiver purchases the risky product is $Pr(H|m_i, \rho_i = 1)$; and conditional on player i abstaining, the likelihood that the receiver purchases the risky product (upon receipt of a non-null message) is $Pr(H|\rho_i = 0)$. When considering player i's optimal participation strategy, we will be interested in the likelihood that outcomes are different when player i does and does not participate. Thus, for example, given that some message is sent, we are interested in the likelihood that the receiver purchases the risky product when player i sends m_i, but the safe product when player i abstains, or, $Pr(L|\rho_i = 0) * Pr(H|\rho_i = 1, m_i)$. To state probabilities like this more concisely, we adopt the following convention. Let $Y \in \{L, I, H, \Phi\}$ and $Z \in \{L, I, H\}$, then $Pr(YZ|m_i) = Pr(Y|\rho_i =$

$0) * Pr(Z|\rho_i = 1, m_i)$. That is, $Pr(YZ|m_i)$ is the probability that the receiver's type is in set Y if player i abstains, and in set Z if player i sends message m_i.

2.4.2.1 Messaging

Thus, we are equipped to state the senders' messaging problem. Consider an arbitrary sender, i, and an arbitrary set of strategies and beliefs (S, A, g). Player i's best response is to choose a message, m_i, that maximizes his expected utility, *given that he participates.* His expected utility for sending m_i, given that he participates, can be expressed as follows[6]:

$$\begin{aligned} E[u_i|t_i, \rho_i = 1, m_i] = -c_i + \sum_{\theta\in\Theta} E[v_r|\theta] Pr(\theta|v_i)\, [& \\ & 0\ Pr(LL|m_i,\theta) + a^I Pr(LI|m_i,\theta) + Pr(LH|m_i,\theta) + \\ & 0\ Pr(IL|m_i,\theta) + a^I Pr(II|m_i,\theta) + Pr(IH|m_i,\theta) + \\ & 0\ Pr(HL|m_i,\theta) + a^I Pr(HI|m_i,\theta) + Pr(HH|m_i,\theta) + \\ & 0\ Pr(\Phi L|m_i,\theta) + a^I Pr(\Phi I|m_i,\theta) + Pr(\Phi H|m_i,\theta) \quad] \end{aligned} \tag{2.3}$$

Where the form of the conditional event probabilities follows from lemma 2.3.7, and where player i's posterior beliefs follow from assumption 2.3.4. We can condense equation 2.3 substantially with the following notation.

$$w(Q) = \begin{cases} 1, & \text{if } Q = H \\ a^I, & \text{if } Q = I \\ 0, & \text{if } Q = L \\ A(\mu^\Phi), & \text{if } Q = \Phi \end{cases}$$

[6]Although there may be equilibria in which certain types never participate (e.g., $P(v_i, c_i) = 0\ \forall c_i$), and therefore any messaging strategy for this type could be supported as part of a Nash equilibrium, PBNE requires that players form contingent plans at each information set.

Thus, equation 2.3 is succinctly stated:

$$-c_i + \sum_{\theta \in \Theta} \sum_{Y \in \{L,I,H,\Phi\}} \sum_{Z \in \{L,I,H\}} w(Z)E[v_r|\theta]Pr(\theta|v_i)P(YZ|m_i, \theta) \tag{2.4}$$

By definition of PBNE, the symmetric equilibrium messaging strategy, M, must satisfy, for all senders i and all types t_i,

$$M(t_i) \in argmax_{m_i \in V} \; - c_i + \sum_{\theta \in \Theta} \sum_{Y \in \{L,I,H,\Phi\}} \sum_{Z \in \{L,I,H\}} w(Z)E[v_r|\theta]Pr(\theta|v_i)P(YZ|m_i, \theta)$$

From this incentive compatibility condition, it is clear that the maximal choice of m_i is independent of c_i, and therefore in equilibrium we substitute $M(t_i)$ for $M(v_i)$.

In equilibrium, it is unlikely that the receiver's action will be deterministic in player i's action. For a given message, m_i, depending on the actions of others, it is possible that the aggregate message may take a value that induces the purchase of the safe product, and it is possible that it may take a value that induces the purchase of the risky product. Moreover, these likelihoods can depend on the state of the world. Thus, when player i chooses his message, he must consider the effect that message will have likelihood of each outcome in each state of the world.

Indeed, optimal messaging strategies must balance a player's certainty regarding the state of the world with his marginal expected utility in each state of the world. To be more clear, the basic incentive compatibility problem can be expanded into parts conditional on the state of the world:

$$max_{m_i} Pr(\theta^h|v_i)E[u_i|\theta^h, t_i, \rho_i = 1, m_i] + Pr(\theta^l|v_i)E[u_i|\theta^l, t_i, \rho_i = 1, m_i]$$

The first order condition (assuming differentiability, briefly) therefore tells us that the optimal messaging strategy requires that relative posterior beliefs be proportional to the inverse of relative marginal utilities.

$$\frac{Pr(\theta^h|v_i)}{Pr(\theta^l|v_i)} = -\left.\frac{1}{\frac{\frac{d}{dm_i}E[u_i|\theta^h, t_i, \rho_i = 1, m_i]}{\frac{d}{dm_i}E[u_i|\theta^l, t_i, \rho_i = 1, m_i]}}\right|_{m_i = M(v_i)} \tag{IC}$$

2.4.2.2 Participation

Player i's participation problem can be formulated in a similar fashion to his messaging strategy. First, given $M(v_i)$, if player i abstains, then his expected utility is

$$\begin{aligned} E[u_i|t_i, \rho_i = 0] = \sum_{\theta\in\Theta} E[v_r|\theta]Pr(\theta|v_i)\,[\\ 0\,\{Pr(LL|\theta, M(v_i)) + Pr(LI|\theta, M(v_i)) + Pr(LH|\theta, M(v_i))\,\} + \\ a^I\,\{Pr(IL|\theta, M(v_i)) + Pr(II|\theta, M(v_i)) + Pr(IH|\theta, M(v_i))\,\} + \\ 1\,\{Pr(HL|\theta, M(v_i)) + Pr(HI|\theta, M(v_i)) + Pr(HH|\theta, M(v_i))\} + \\ A(\mu^\Phi)\,\{Pr(\Phi L|\theta, M(v_i)) + Pr(\Phi I|\theta, M(v_i)) + Pr(\Phi H|\theta, M(v_i))\}\quad] \end{aligned} \tag{2.5}$$

Rewritten, using the notation developed above,

$$E[u_i|t_i, \rho_i = 0, M(v_i)] = \sum_{\theta\in\Theta}\ \sum_{Y\in\{L,I,H,\Phi\}}\ \sum_{Z\in\{L,I,H\}} w(Y)E[v_r|\theta]Pr(\theta|v_i)P(YZ|\theta, M(v_i)) \tag{2.6}$$

Thus, conditional on a messaging strategy $M(v_i)$, the symmetric equilibrium participation strategy, P, must satisfy, for all senders i and all types t_i,

$$P(t_i) \in argmax_{\rho\in[0,1]}\ \rho E[u_i|t_i, \rho_i = 1] + (1-\rho)E[u_i|t_i, \rho_i = 0] \tag{IR}$$

The optimal participation strategy for a sender thus depends on the net expected utility of participation. Lemma 2.4.3 tells us that in equilibrium, participation takes the form of a simple cutoff rule. For concision, for any sender, i, let $X_i(v_i)$ be that sender's net expected utility of participation, given his type and independent of costs. That is,

$$
\begin{aligned}
X_i(v_i) &= c_i + E[u_i|t_i, \rho_i = 1] - E[u_i|t_i, \rho_i = 0] \\
&= \sum_{\theta \in \Theta} E[v_r|\theta] Pr(\theta|v_1) \sum_{Y \in \{L,I,H,\Phi\}} \sum_{Z \in \{L,I,H\}} [w(Z) - w(Y)]\, P(YZ|\theta, M(v_i))
\end{aligned}
$$

Lemma 2.4.3. *In equilibrium,*

$$
P(t_i) = \begin{cases} 0, \text{ if } c_i > X(v_i) \\ 1, \text{ if } c_i \leq X(v_i) \end{cases} \tag{2.7}
$$

Proof. The (IR) condition can be rewritten as

$$
P_i(t_i) \in argmax_{\rho \in [0,1]}\ \rho \{E[u_i|t_i, \rho_i = 1] - E[u_i|t_i, \rho_i = 0]\} + E[u_i|t_i, \rho_i = 0]
$$

Substituting the definition of $X_i(v_i)$, this holds if and only if, for all t_i, $P_i(t_i)$ maximizes $P_i(t_i)\,[X_i(v_i) - c_i]$. Clearly, if $X_i(v_i) - c_i > 0$, then the maximand is maximized iff $P_i(t_i) = 1$, and if $X_i(v_i) - c_i < 0$, then the quantity is maximized iff $P_i(t_i) = 0$. Since we are considering symmetric equilibria, we drop the player subscripts on P and X. Rearranging terms yields the statement of the lemma. □

By lemma 2.4.3, $F_c(X(v_i))$ is equivalent to the probability that player i will participate given the value v_i. Along with our assumptions regarding F_c, lemma 2.4.3 ensures that player i's participation is responsive to his net expected value, given v_i. That is, the higher player i's net expected value of sending some message, the more likely that he is to send it. Moreover, assumption 2.3.5 ensures that, in equilibrium, for any v_i, there exists a set of types (of positive measure) such that participation is not a best response, and therefore the probability that no message is sent is strictly positive. This final point is summarized in lemma 2.4.4, which tells us that if a sender participates, he has a positive probability of being the only participant.

Lemma 2.4.4. *In equilibrium, for any v_i, $Pr(\rho_i = 1|v_i) < 1$, and $\forall\ \theta$, $Pr(\mu^{\Phi}|\theta) > 0$.*

2.4.3 Equilibrium

In the remainder of this section, we build to a characterization of equilibrium participation comparative statics. The result of this will be to identify conditions under which the extreme participation conjecture holds. Proposition 2.4.9 and its corollary contain this result, and build from lemmas 2.4.6 and 2.4.8 which characterize the receiver's equilibrium type partition under certain conditions. First, we require a pair of definitions and another piece of notation.

Let v^Φ be the value of the risky product that makes a sender indifferent between the two products, conditional on no other players sending a message. That is,

$$E[v_r|\mu(\mathbf{m}_{-i}) = \mu^\Phi, v_i = v^\Phi] = 0 \tag{2.8}$$

By assumption 2.3.2, for a given set of strategies v^Φ, like v^I, is unique if it exists. Unlike v^I, v^Φ is determined by equilibrium strategies as well as by the environment (whereas v^I is defined solely by the environment).

Definition *A message aggregator, μ, is **regular** if: for any sender, i; all $m_i, m_i' \in V$; and any $\boldsymbol{m}_{-i}$;*

$$\begin{aligned} &\text{If } m_i \neq m_i', \text{ then } \mu(m_i) \neq \mu(m_i') \\ &\text{If } m_i \leq \mu(\mathbf{m}_{-i}), \text{ then } m_i \leq \mu(\mathbf{m}) \leq \mu(\mathbf{m}_{-i}) \\ &\text{If } m_i \geq \mu(\mathbf{m}_{-i}), \text{ then } m_i \geq \mu(\mathbf{m}) \geq \mu(\mathbf{m}_{-i}) \end{aligned} \tag{2.9}$$

We say that a message aggregator is **strictly regular** if the inequalities of the definition are strict. The mean aggregator (see section 2.6.2) is an example of a strictly regular message aggregator, and the median aggregator (see section 2.6.3) is an example of a regular message aggregator that is not strictly regular. All strictly regular message aggregators are regular.

Definition *A partition of* T_r *is* ***simple*** *if there exists a* $v' \in V$ *such that for* $Y, Z \in \{L, I, H\}$*:* $\Sigma\left([\underline{v}, v')\right) \subseteq Y$ *and* $\Sigma\left((v', \overline{v}]\right) \subseteq Z$*. We say this partition is described by* (Y, Z)*, and is formed around* v'*.*

Working with simple partitions simplifies the problem immensely, and therefore we work to find conditions under which we can be certain that equilibrium is characterized by a simple partition. This is the concern of lemma 2.4.6. Lemma 2.4.6 relies on a continuity argument, which follows from lemma 2.4.5.

Lemma 2.4.5. *For any sender, i, in equilibrium,* $E[u_i|t_i, \rho_i = 1]$ *and* $E[u_i|t_i, \rho_i = 0]$ *are continuous in* v_i*.*

Lemma 2.4.5 says that since $Pr(\theta^h|v_i)$ changes continuously in v_i, in any PBNE, so too must the quantities that dictate i's best response. It is also easy to see that lemma 2.4.5 implies that $X(v_i)$ and $F_c(X(v_i))$ must be continuous as well. Lemma 2.4.5 also leads directly to lemma 2.4.6 which describes how the receiver's type partition in equilibrium.

Lemma 2.4.6. *If, in equilibrium,* $E[v_r|\mu(\boldsymbol{m}_{-i}) \neq \mu^{\Phi}, v_i]$ *is continuous in* v_i*, then the receiver's type space is described by a simple partition, formed around* v^{Φ}*.*

Therefore, the only way that the necessary condition of lemma 2.4.5 could fail to hold is if the conditions of lemma 2.4.6 fail to hold; that is, if $E[v_r|\mu(\mathbf{m}_{-i}) = \mu^{\Phi}, v_i]$ is discontinuous. What the proof of lemma 2.4.6 shows is that if $\sigma(v')$ is a point of transition between any partition members of T_r, then $E[v_r|\mu(\mathbf{m}_{-i}) = \mu^{\Phi}, v_i]$ is continuous if and only if $v' = v^{\Phi}$ (and the necessary condition of lemma 2.4.5 is thus satisfied if and only if $v' = v^{\Phi}$).

Another way to interpret lemma 2.4.6 is to fix some v' around which the receiver's type-space partition is formed. In this interpretation, lemma 2.4.6 constrains participation probabilities, requiring that the ratio of participation probabilities conditional on the state of the world must be such that $v' = v^{\Phi}$.

Remark 2.4.7. *Lemma 2.4.6 also implies that if its conditions are met, then in equilibrium v^{Φ} must exist in V. Otherwise, it would be the case that for every possible value of the aggregate message, the receiver would take the same action, violating the informativeness condition of our equilibrium concept.*

Next, lemma 2.4.8 provides a refinement of the set of equilibria to which we are attending.

Lemma 2.4.8. *For any game with a strictly regular message aggregator, in equilibrium, if M is strictly monotonic, then at least one of: $\sigma(\underline{v}) \in L$; $\sigma(\overline{v}) \in H$.*

The proof uses the fact that because M is strictly monotonic and μ is strictly regular, the only way for $\mu(\mathbf{m})$ to equal an extreme message (for example, $M(\underline{v})$), is for all participating senders to have obtained the corresponding extreme value (e.g., $\underline{v}$). By assumption 2.3.2, it is clear which state of the world is more likely, *conditional on a certain level of participation.* However, it may actually be the opposite state of the world that makes that level of participation more likely. Strict monotonicity of the messaging strategy does not tell us, therefore, that *both* $\sigma(\underline{v}) \in L$ and $\sigma(\overline{v}) \in H$ because it is not necessarily the case that sender participation is equally likely in both states of the world (as was identified in the discussion of lemma 2.4.6).

However, corollary 2.4.8 says that if the conditions of both lemma 2.4.6 and lemma 2.4.8 hold, then in equilibrium both $\sigma(\underline{v}) \in L$ and $\sigma(\overline{v}) \in H$. Corollary 2.4.8 utilizes the fact that if there is a simple partition formed around v^{Φ}, then it must be that $\frac{Pr(\rho=0|\theta^l)}{Pr(\rho=0|\theta^h)}$ is sufficiently bounded such that both conditions of lemma 2.4.8 always hold.

Corollary 2.4.8 *If the conditions of lemmas 2.4.6 and 2.4.8 hold, then the receiver's type space is described by the simple partition* (L, H).

Finally, proposition 2.4.9 and its corollary describe the comparative statics of equilibrium participation under a certain set of conditions. In particular, the results of proposition 2.4.9 require the receiver's type space be described by a simple partition.

Proposition 2.4.9. *For any game with a regular message aggregator, if in equilibrium: the receiver's type space is described by a simple partition around* v'*;* $E[u_i|t_i, \rho_i = 1]$ *is differentiable w.r.t.* v_i *everywhere but at a finite set of points; and both the distribution of aggregate messages conditional on* θ *and the distribution of aggregate messages conditional on* θ *and* m_i *are massless on* ($M(\underline{v})$ *or* $M(\overline{v})$)*, then, letting* $Q, K \in \{[\underline{v}, v'), (v', \overline{v}]\}$:

A) $X(v_i)$ *is increasing on* Q *if either:*

i) $\Sigma(Q) \subseteq H$

ii) $\Sigma(Q) \subseteq I$, $\Sigma(K) \subseteq L$, *and* $A(\mu^{\Phi}) \leq a^I$

B) $X(v_i)$ *is decreasing in* Q *if either:*

i) $\Sigma(Q) \subseteq L$

ii) $\Sigma(Q) \subseteq I$, $\Sigma(K) \subseteq H$, *and* $A(\mu^{\Phi}) \geq a^I$

If its conditions are met, proposition 2.4.9 is powerful. To see this, imagine an equilibrium that satisfies the conditions of the proposition such that $\sigma(\underline{v}) \in L$ and $\sigma(\overline{v}) \in H$. By lemma 2.4.6, this implies that if a player of any type with $v_i < v^{\Phi}$ participates, he sends a message that (by itself) induces purchase of the safe product and any type with $v_i > v^{\Phi}$, sends a message that induces purchase of the risky product. Moreover, by proposition 2.4.9, the likelihood that i participates is increasing as his type moves away from v^{Φ}. Under these conditions, we would accept the extreme participation conjecture: the prevalence of extreme-valued ratings in the field is at least in part due to the fact that participation rates increase as values become more extreme. This particular refinement is the topic of corollary 2.4.9.

Corollary 2.4.9 *For any game with a regular message aggregator, if in equilibrium, the receiver's type space is described by a simple partition (L, H) formed around v', and $E[u_i|t_i, \rho_i = 1]$ is differentiable w.r.t. v_i everywhere but at a finite set of points, then $X(v_i)$ is decreasing in $[\underline{v}, v')$ and increasing in $(v', \overline{v}]$.*

Thus, since the result of corollary 2.4.8 is the simple partition (L, H), then if the conditions of both lemmas 2.4.6 and 2.4.8 are met, then participation comparative statics are described by the substantially simpler corollary 2.4.9.

To demonstrate a simple equilibrium of the game, consider the strategies in which the receiver chooses the risky product unless he receives the null message; in which $M(\cdot)$ is arbitrary; and in which senders participate at an increasing rate for $v_i > v^{\Phi}$ and abstain when $v_i \leq v^{\Phi}$. These strategies are sustained as an equilibrium since, because senders with negative signals do not participate, every non-null message is more likely to be received in the high state and therefore the receiver chooses the risky product when he receives a message. Similarly, abstention is more likely in the low state of the world and therefore the null message induces the receiver to choose the safe product. There is therefore only one case in which a sender's action is pivotal, the case in which the sender is the only participant. Conditioning on this case is a strong signal that the state is the low state. For a sender with a value less than v^{Φ}, this effect works in the same direction as her posterior, and the best response is therefore to abstain (since messaging results in the selection of the risky product). For a player with a weak posterior and value greater than v^{Φ}, the effect of conditioning on the pivotal event will overwhelm her posterior and her best response will therefore also be to abstain. As a sender's value increase, eventually the strength of her posterior will overwhelm the effect of conditioning on the pivotal event, and participation will be associated with a positive expected utility. Finally, since we are considering PBNE, we are free to specify the receiver's beliefs in zero probability events. Thus, if there exists a message, m, not sent in equilibrium, we let $g(m) > \nu$, and therefore $A(m) = 1$.

In the following section, we build intuition by examining the simple case in which there is only one sender. In that case many of the lemmas developed in this section hold trivially, and we can therefore build to conclusions easily. In the section that follows that, we will discuss the much more difficult case of many senders, in which the type of aggregation mechanism defines which of the lemmas developed in this section we may apply.

2.5 One Sender

The two important differences between our one-sender game and the canonical Crawford and Sobel model are, first, that in this game the sender's and receiver's goals are aligned, instead of opposed, and second, that here the sender's message is costly and he may choose to abstain. This subsection uses the tools that we developed in the previous sections to characterize equilibria in our game when there is only one sender. This serves to both highlight the differences between ours and the Crawford and Sobel model as well as to preview the main results of this paper.

In the one-sender case, the game takes a peculiar form, as messages are directly received by the receiver without the loss of information associated with aggregation. This also means that abstention, or, the lack of a message, can be used as a direct, costless message. Indeed, as we see in lemma 2.5.3, equilibria in these games are shaped by action the receiver takes when he receives no message. We proceed by examining the sender's messaging decision, the sender's participation decision, and finally the consistency of the receiver's beliefs. This culminates in proposition 2.5.4.

Throughout this subsection we assume that μ is regular, which in this case implies simply that for any two distinct sent messages, the aggregate messages are also distinct. A simple form of this is the identity function; $\mu(m_1) = m_1$.

Remark 2.5.1. *In this subsection we strengthen the informativeness criterion of our equilibrium concept to say that in equilibrium there must exist a sender-type that induces purchase of the risky product with certainty and there must exist a sender-type that induces purchase of the safe product* *with certainty. This is not a necessary simplification, but it allows us to condense our exposition significantly.*

From the results from the previous section a great deal is already known about the class of game in which there is only one sender. We know that $Pr(\Phi|\theta) = 1$ and therefore lemma 2.4.6 holds trivially. This implies that in all equilibria, the receiver's type space is described by a simple partition around v^{Φ}. Moreover, $E[u_i|t_i, \rho_i = 1, m_i]$ simplifies such that if $M(v_1)$ is differentiable, then the conditions of proposition 2.4.9 are met as well. Finally, since there are never any other messages sent, v^{Φ} is equivalent to v^I.

2.5.1 Messaging

Equilibrium messaging strategies are constrained very little in the one-sender environment. To see this, note that $E[u_1|t_1, \rho_1 = 1, m_1]$ depends only on the partition member in which $\mu(m_1)$ resides:

$$E[u_1|t_1, \rho_1 = 1, m_1] + c_1 = \begin{cases} E[v_r|v_1], & \text{if } \mu(m_1) \in H \\ a^I E[v_r|v_1], & \text{if } \mu(m_1) \in I \\ 0, & \text{if } \mu(m_1) \in L \end{cases}$$

Thus, depending on his posterior belief about the state of the world, there is a clear ranking of outcome-equivalent messages, (e.g., if $E[v_r|v_1] < 0$, then he prefers to send a message such that $\mu(m_1) \in L$), but he is indifferent between messages that induce the same outcome (e.g., if $\mu(m_1), \mu(m_1') \in L$, then $E[u_1|t_1, \rho_1 = 1, m_1] = E[u_1|t_1, \rho_1 = 1, m_1'])$.

This simple equilibrium messaging rule leads to proposition 2.5.2 which tells us that although the set of messaging strategies supported in equilibrium may be large, the form of the messaging strategy is unimportant so long as there are at least 3 unique messages. First, a definition.

Definition The tuples (S, A, g) and (S', A', g') are **outcome equivalent** if for every vector of sender types $\mathbf{t}$, $Pr(a_r = 1|\mathbf{t}, (S, A, g)) = Pr(a_r = 1|\mathbf{t}, (S', A', g'))$.

In other words, two sets of strategies and beliefs are outcome equivalent if no matter the players' types, the receiver chooses the risky good with the same likelihood.

Proposition 2.5.2. *For any equilibrium of any game in which $N = 1$, there exists an outcome-equivalent equilibrium of the same game in which each type of sender sends one of only three (or fewer) messages.*

2.5.2 Participation

Because there is only one sender, the receiver can interpret abstention by the sender as a direct message. Therefore, the sender's expected value to abstention depends on the way in which the receiver interprets his abstention.

$$E[u_1|t_1, \rho_1 = 0, m_1] = \begin{cases} E[v_r|v_1], & \text{if } g(\theta^h|\mu^\Phi) > \nu \\ a^I E[v_r|v_1], & \text{if } g(\theta^h|\mu^\Phi) = \nu \\ 0, & \text{if } g(\theta^h|\mu^\Phi) < \nu \end{cases}$$

Lemma 2.5.3 describes necessary conditions for the sender's equilibrium participation strategy. Let $q(\cdot)$ be the inverse of $E[v_r|v_1]$ (by assumption 2.3.2, $q(\cdot)$ exists for all v_i). Then,

Lemma 2.5.3. *For any game in which $N = 1$ with $c_1 > 0$, the following are necessary conditions for equilibrium, conditional on the receiver's interpretation of μ^Φ:*

A) If $g(\theta^h|\mu^\Phi) < \nu$, then

$$P(t_1) = \begin{cases} 0, & \textit{if } v_1 < q(c_1) \\ 1, & \textit{otherwise} \end{cases}$$

B) If $g(\theta^h|\mu^\Phi) > \nu$*, then*

$$P(t_1) = \begin{cases} 0, & \textit{if } v_1 > q(-c_1) \\ 1, & \textit{otherwise} \end{cases}$$

C) If $g(\theta^h|\mu^\Phi) = \nu$*, then*

$$P(t_1) = \begin{cases} 0, & \textit{if } v_1 \in \left(q(\frac{-c_1}{a^I}), q(\frac{c_1}{1-a^I})\right) \\ 1, & \textit{otherwise} \end{cases}$$

Proof. A) By assumption $g(\theta^h|\mu^\Phi) < \nu$, and therefore $A(\mu^\Phi) = 0$. Since for all $v_1 \in [\underline{v}, v^I)$, $E[v_r|v_1] < 0$, we know that in equilibrium $\sigma(v_1) \in L$. This implies that in equilibrium participation yields the same outcome as abstention and therefore $X(v_1) = 0$. Therefore, since we assumed $c_1 > 0$, $P(t_1) = 0$ by lemma 2.4.3.

Because for all $v_1 \in (v^I, \overline{v}]$, $E[v_r|v_1] > 0$, we know that in equilibrium $\sigma(v_1) \in H$. This implies that in equilibrium participation yields a positive net outcome in expectation; $X(v_1) = E[v_r|v_1] > 0$. Therefore, by lemma 2.4.3, $P(t_1) = 1$ if and only if $E[v_r|v_1] > c_1$, or, rewriting, $v_1 > q(c_1)$.
Cases B and C are proved by the same process.

□

The intuition is simple for all the cases of lemma 2.5.3: the sender will send a message if and only if the net benefit of sending his best available message outweighs the cost of doing so. The existence of equilibrium cutoffs is guaranteed by our informativeness criterion in all of the cases.

2.5.3 Equilibrium

Lemma 2.5.3 was stated without the constraint of consistency of beliefs. By definition of a PBNE, in order for the participation strategies in lemma 2.5.3 to constitute a part of an equilibrium, players' beliefs about the likelihoods of others' actions must match the actually likelihoods of those actions. Proposition 2.5.4 checks for consistency in each case described by lemma 2.5.3 and finds that at least one of the one-sided cutoff strategies will always be rationalizable as part of an equilibrium, and that while case C may not describe equilibrium so generally, there exist equilibria in which case C describes equilibrium participation.

Proposition 2.5.4. *In the class of games in which there is only one sender,*

1) For any realization of the game, there exist strategies such that at least one of A or B of lemma 2.5.3 describes equilibrium participation.

2) There exist conditions on the primitives of the game under which each case of lemma 2.5.3 describes equilibrium participation.

Thus, as in the Crawford and Sobel model, in equilibria of the one sender game, the sender's value space is partitioned into convex subsets within which the sender sends outcome-equivalent messages. Unlike in that model, players here wish to abstain if either abstention induces the same outcome as does the sender's optimal message, or if their expected value to participation does not justify the cost.

Figure 2.1.a presents example conditional densities of a game in which $V = [-1, 1]$, and the conditional densities of values are the simple triangle densities $f_{v|\theta^h}(v) = 0.5 + .45v = f_{v|\theta^l}(-v)$. Figure 2.2 displays example equilibrium participation strategies of a this example game when there is only one-sender game. Figure 2.2.a describes an equilibrium participation strategy of the type described by lemma 2.5.3 part B, while Figure 2.2.b describes an equilibrium participation strategy of the type described by lemma 2.5.3 part C.

We know from lemma 2.4.3 that participation is a cutoff in net expected value. Proposition 2.5.4 tells us that when there is only one sender, participation is also a cutoff in v_1. In each case, $X(v_1)$ is decreasing in v_1 for $v_1 < v^I$ and increasing in v_1 for $v_1 > v^I$. The net expected value of participation, as a function of v_1, therefore exceeds a player's cost either once or twice, resulting in either a one-sided cutoff strategy, as in cases A and B, or two-sided cutoff strategy, as in case C. Thus, no matter which of these describes equilibrium, the likelihood of participation increases weakly as the sender's value moves away from v^I. In terms of our conjectures, this implies that in the one-sender case we reject both the sincere and exaggeration conjectures (the latter because of proposition 2.5.2) and accept the extreme participation conjecture.

While this conclusion previews the result of the multiple senders case, that case will be much more difficult to prove. This is in part because in the one-sender case comparative statics regarding participation are due entirely to changes in the strength of a sender's posterior: by proposition 2.5.2, in the one-sender case players with values less than (greater than) v^I send outcome-equivalent messages; therefore as a player's value becomes more extreme and his posterior becomes stronger, the effect of his message remains constant. This intuition remains important in the multiple sender case. However, in the multiple sender case, a sender's message is aggregated with other messages, and senders of different types will therefore not necessarily choose outcome-equivalent messages. Thus, differences in a player's net expected value will be generated by both differences in his posterior as well as differences in the expected effect of his equilibrium message.

2.6 Multiple Senders

The previous subsection highlighted some of the ways in which the game studied here differs from the canonical Crawford and Sobel model. The distinction becomes more stark when this game includes more than one sender. This is because in the game with multiple senders, no matter how many messages are sent in total, the aggregation mechanism sends only one message to the receiver. Because of this, information may be lost between the first and second periods, potentially impeding coordination. Moreover, because messaging is costly in this model, even if senders are of the same type, there is conflict between them as each would prefer the other players to pay the cost of participation to obtain their desired result.

2.6.1 An Extended Example

Before continuing with our theoretical results, we explore an example in depth. This example is meant both to provide intuition for the concepts we have introduced up to this point as well as to serve as a focal point to which we may return throughout the paper as new results are introduced.

We split this example into two parts. First, we explore messaging in the absence of the participation complication; that is, we assume full participation. After this, we move to the full model, in which players may abstain instead of sending messages for a cost.

The details of this example coincide with the details the example game described in section 2.5 but with three senders instead of one[7], and where the receiver receives the mean of sent messages. Unless otherwise noted, assume that $M_2(v_2) = v_2$ and $M_3(v_3) = v_3$, that the receiver plays a cutoff around $t_r = 0$ ($A(t_r) = 0$ if $t_r < 0$, and that $A(t_r) = 1$ if $t_r > 0$) and that the receiver mixes evenly when he receives the null message[8].

2.6.1.1 Full Participation

In this full participation version of the game, a first-period player's only problem is to choose an optimal message; that is, to solve the incentive compatibility problem (labeled IC in section 2.4). From our discussion of the incentive compatibility constraint in section 2.4, it is evident that a sender must choose a message that balances his marginal expected utility in each state of the world with his relative posterior about the state of the world. For our example, the relative posterior belief about the state of the world is displayed in Figure 2.1.b as a function of v_1. As is evident, as sender's value increases, he becomes more certain that the state of the world is the high state.

To understand a sender's marginal expected utility, consider some message $m_1 > 0$, and recall our construction of the sender's expected utility of sending message m_1 (equation 2.3). In this example, by the assumptions we made on players' strategies, the only way the sender can change the outcome

[7] Recall that in that example $V = [-1, 1]$ and the conditional densities of values are the simple triangle densities $f_{v|\theta^h}(v) = 0.5 + .45v = f_{v|\theta^l}(-v)$. Figure 2.1.a presents the conditional value densities.

[8] In other words, the receiver's type space is described by a simple partition around 0. It should be noted that this assumption allows this exposition to bypass a significant complicating factor which is determining the composition of the receiver's type space. Nonetheless, even under this simplification, this example shows the complexity of the senders' messaging decision.

is by changing the receiver's choice from the safe product to the risky product ($Pr(LH|\theta, m_1) > 0$).

We assumed that the message aggregator in this example is the mean. Solving for $Pr(LH|\theta, m_1)$ is therefore very simple; for a given m_1, we are looking for the value of $\mu(m_2, m_3)$ that is less than 0 and which gives $\mu(m_1, m_2, m_3) = 0$. We call this value of $\mu(m_2, m_3)$, $\hat{\mu}(m_1)$. For any value of $\mu(m_2, m_3)$ between $\hat{\mu}(m_1)$ and 0, $\mu(m_1, m_2, m_3) > 0$. In other words, for values of $\mu(m_2, m_3)$ between $\hat{\mu}(m_1)$ and 0, the message m_1 changes the receiver's choice from safe to risky. And for values of $\mu(m_2, m_3)$ outside this range, the receiver makes the same choice irrespective of m_1. Therefore the likelihood that $\mu(m_2, m_3)$ is between $\hat{\mu}(m_1)$ and 0 is equivalent to $Pr(LH|\theta, m_1)$, and the likelihood that $\mu(m_2, m_3) = \hat{\mu}(m_1)$ therefore represents the sender's *marginal* expected utility.

Applying this intuition, the ratio of sender 1's marginal expected utility conditional on sending m_1 and on the states of the world (the right-hand side of IC), is therefore $\frac{f_{\mu(\mathbf{m}_{-1})|\theta^l}(\hat{\mu}(m_1))}{f_{\mu(\mathbf{m}_{-1})|\theta^h}(\hat{\mu}(m_1))}$.

Figure 2.3a presents the density of $\mu(m_2, m_3)$ conditional on each state of the world, on the domain[9] $[-1, 0]$, while Figure 2.3b both displays the fraction $\frac{f_{\mu(\mathbf{m}_{-1})|\theta^l}(\hat{\mu}(m_1))}{f_{\mu(\mathbf{m}_{-1})|\theta^h}(\hat{\mu}(m_1)}$ as a function of m_1, as well as reproduces the relative posterior as a function of v_1. By (IC), in order for sincerity ($M_1(v_1) = v_1$) to be a best response, these two functions must correspond at each value of v_1 (letting $m_1 = v_1$). Clearly, this is not the case. Figure 2.3b traces out the best response, for example, points v_1^A and v_1^B. At v_1^A, his best response is to dampen his value, while at v_1^B, his best response is to exaggerate his value. These best response messages are represented in the figure as $M_1^*(v_1^A)$ and $M_1^*(v_1^B)$, respectively. Figure 2.3a also projects a vertical line up from the marginal values of $\mu(m_2, m_3)$ implied by $m_1 = v_1^A$, $m_1 = v_1^B$, $m_1 = M_1^*(v_1^A)$, and $m_1 = M_1^*(v_1^B)$. By tracing these vertical lines, you can see how the ratio of the likelihoods of the marginal aggregate values differs for different messages.

Figure 2.4a plots the entire best response correspondence. As can be seen in the figure, best responses dampen values close to 0 (as at v_1^A), and best responses exaggerate more extreme values (as at v_1^B). Both effects are results of the fact that the density of the mean of $\mathbf{m}_{-1}$ does not converge at the precise rate that would be required to match player 1's marginal effect of messaging with his

[9]On the domain $[0, 1]$, the graph is the mirror image, with $f_{\mu(\mathbf{m}_{-1})|\theta^h}(t_r)$ taking the form of a bell-curve and with $f_{\mu(\mathbf{m}_{-1})|\theta^l}(t_r)$ declining to 0.

posterior. For example, v_1^A does not convey much certainty about the state of the world and so, as conveyed by Figure 2.3, his best response is to dampen his message in order to reduce the chance that his message is pivotal. In contrast, a value of v_1^B tells the sender that it is highly likely that the state of the world is the high state. However, as can be seen in Figure 2.3, sending message $m_1 = v_1^B$ does not achieve a ratio of aggregate densities to match his posterior at $\frac{v_1^B}{2}$. He therefore chooses a message that exaggerates his value.

Finally, Figure 2.4b displays a symmetric equilibrium of the full participation game. As is evident, this equilibrium exaggerates the logic of the above best response; almost all values are dampened by the equilibrium messaging strategy, while only at the extremes of the value space are messages exaggerated.

2.6.1.2 Voluntary Participation

The first challenge to a sender transitioning to the voluntary participation case is that that he no longer knows the number of other senders' messages to which he is responding. He may be responding to any number of participants from 0 to $N-1$. This affects his best response for two reasons. First, for a given value of $\mu(\mathbf{m}_{-i})$, it alters the expected effect of m_1. For example, if there is one other participant then $\hat{\mu}(m_1) = -m_1$, while if there are two other participants, $\hat{\mu}(m_1) = -\frac{m_1}{2}$. In other words, the marginal effect of m_1 on the aggregate message decreases as the number of other participants increases. This can be seen in the different placements of $\hat{\mu}(m_1)$ between Figure 2.6a and Figure 2.6b.

Second, because senders participate at different rates for different values, the densities of messages and, consequently, the densities of aggregate messages are severely distorted. Figure 2.5b displays the density of messages when players send their values under both full participation and voluntary participation (assuming participation as displayed in Figure 2.5a). As can be surmised from the figure, sent messages near 0 become rare, while messages at the extremes of the message space become comparatively more likely.

The effect of this change in sent message likelihood on aggregate message likelihood can be seen in Figure 2.6. Figure 2.6 is the voluntary participation analog to Figure 2.3. Figure 2.6 is divided into 4 sub-figures, differentiated by level of participation. Figures 2.6a and Figure 2.6b correspond to 2 participants (counting sender 1) while Figures 2.6c and Figure 2.6d correspond to 3 participants. Like Figure 2.3a, Figures 2.6a and 2.6c present the density of $\mu(\mathbf{m}_{-1})$ conditional on each state of the world, while, like Figure 2.3b, Figures 2.6b and 2.6d display both the relative marginal expected utilities as a function of m_1, as well as reproduce the sender's relative posterior as a function of v_1. For ease of comparison, we have reproduced the values v_1^A and v_1^B here, and traced the best responses as we did in the full participation case. As can be seen in the figure, when there are exactly two participants, $m_1 = v_1$ is a best response[10].

Unlike the density of aggregate messages under full participation, the density of messages under voluntary participation when there are 3 participants does not take the form of a bell curve but that of a confluence of three bell curves. This results in an exaggeration of the best responses from the full participation case. That is, compared to the full participation case, the sender's best response with value v_1^A is to dampen this value further, and the sender's best response with value v_1^B is to exaggerate this value further.

As was the case under full participation, in the symmetric equilibrium it is the dampening of messages that characterizes the messaging strategy. Figure 2.7 displays an approximate symmetric messaging equilibrium[11]. Compared to the full participation symmetric equilibrium, messages here are dampened far more. This is because of the shape of participation. As we can see from Figure 2.5a, participation is a positive function of certainty of the state of the world. What this implies is that under voluntary participation, players who are *un*certain about the state of the world know that they are best responding to players who are *certain* about the state of the world, and it is therefore in their best interest to dampen their own message (that is, lower the likelihood that they are pivotal) and to therefore not overwhelm the message of the those more-informed players.

In this example we demonstrated that optimal messaging is a careful balance of matching the expected effects in the low state versus the expected effects in the high state with posterior beliefs,

[10]Indeed, that sincerity is an equilibrium when two or fewer senders participate holds with some generality and is explored further in Section 2.6.

[11]This equilibrium was found using numerical methods.

and that this can result in best response messages that are an exaggeration of types, as well as best response messages that are a dampening of types. Moreover, we showed that voluntary participation is a significant complicating factor in senders' optimization problems. Not only does it require senders to weigh the effect of their message under all levels of participation, it seriously distorts the distributions of others' actions upon which senders must base their decisions. Finally, we have presented a very simple case in which the symmetric equilibrium messaging strategies in both the voluntary participation and in the full participation cases are to send messages that dampen values. Furthermore, in the voluntary participation case, this pattern of messaging is accompanied by a pattern of participation that is increasing in the extremity of players' values. Therefore, this provides an example in which can reject the sincere conjecture and the exaggeration conjecture (at least for some equilibria) and accept the extreme participation conjecture.

2.6.2 The Mean

In this section we present a partial characterization of equilibrium behavior when there are multiple senders and when the receiver is sent the mean of sent messages. The mean aggregation mechanism is important because of its prevalence. It is widely used on many highly trafficked websites such as YouTube (previously; as discussed in the introduction), amazon.com, alexa.com, google.com/local, imdb.com, and netflix.com[12]. In this section we show that in such environments we can reject the sincere conjecture and conditionally accept the extreme participation conjecture. This result is summarized in proposition 2.6.5. First, lemma 2.6.1 establishes conditions under which a number of important densities are continuous. While lemma 2.6.2 establishes that the likelihood that $\mu(\mathbf{m}_{-i})$ and $\mu(\mathbf{m})$ belong to any two disjoint, open, convex subsets of V, can be expressed as a function of $F_{\mu(\mathbf{m}_{-i})|\theta}(\cdot)$. These lead to lemmas 2.6.3 and 2.6.4 which establish conditions under which lemma 2.4.6 and proposition 2.4.9 hold. We remain agnostic about the exaggeration conjecture in such environments, but we do identify environments in which we reject the exaggeration conjecture.

The results of this section rely on a number of constructions. In particular, we construct the

[12] Usage varies by website. Websites visited 03/01/11.

distribution of sent messages, $F_{S|\theta}(m_i)$, and the density of the aggregate message, $f_{\mu(\mathbf{m})|\theta}(t_r)$. These constructions can be found in Appendix B.

Lemma 2.6.1. *For any game with a mean message aggregator, if the equilibrium messaging strategy is strictly monotonic and continuously differentiable, then the conditional densities of sent messages are continuous. Moreover, the conditional densities of aggregate messages and the receiver's posterior beliefs are continuous.*

Lemma 2.6.2. *Choose any open, convex and disjoint sets $Q, K \subseteq V$. For any game with a mean message aggregator, for any sender, i, and any message, m_i, in equilibrium, the probability that $\mu(\boldsymbol{m}_{-i}) \in Q$ and $\mu(\boldsymbol{m}) \in K$ may be expressed as a function of the distributions of aggregate messages that exclude i's message, $F_{\mu(\boldsymbol{m}_{-i})}(\cdot)$.*

The exact form of this function is derived in the proof of the lemma.

Lemma 2.6.3 uses lemmas 2.6.1 and 2.6.2 to show, first, that the probability that a sender changes the outcome with his message is continuous in that message, and second that this implies that the conditions of lemma 2.4.6 are satisfied.

Lemma 2.6.3. *For any game with a mean message aggregator, in equilibrium, if the equilibrium messaging strategy is strictly monotonic and continuously differentiable, then the conditions of lemma 2.4.6 are satisfied.*

Similarly, lemma 2.6.4 uses lemmas 2.6.1 and 2.6.2 to show that the conditions of proposition 2.4.9 are satisfied.

Lemma 2.6.4. *For any game with a mean message aggregator, in equilibrium, if the equilibrium messaging strategy is strictly monotonic and continuously differentiable, then the conditions of proposition 2.4.9 are satisfied.*

Therefore, if the conditions of lemma 2.6.3 are met, then the receiver's type space is described by a simple partition, formed around v^Φ, and if the conditions of lemma 2.6.4 are met, we know generally the comparative statics of participation. Moreover, since the additional conditions of lemmas 2.6.3 and 2.6.4 imply that the conditions of corollary 2.4.8 are met, we can pin down the receiver's type space to the simple partition described by (L, H). Therefore, finally, by corollary 2.4.9, we know that participation is increasing in the extremism of a sender's value. This intuition is reflected in proposition 2.6.5.

Proposition 2.6.5. *Choose an arbitrary sender, i, and let v^Φ solve $E[v_r|\mu(\boldsymbol{m}_{-i}) = \mu^\Phi, v_i = v^\Phi] = 0$. In any game with a mean message aggregator, if the equilibrium messaging strategy is strictly monotonic and continuously differentiable, then for $v_i < v^\Phi$, the likelihood that sender i participates is strictly decreasing in v_i and for $v_i > v^\Phi$, the likelihood that sender i participates is strictly increasing in v_i.*

Proposition 2.6.5 says that if the mean of sent messages is sent to the receiver, then we can reject the sincere conjecture and conditionally accept the extreme participation conjecture[13]. That is, we know that in these environments, the data is not generated by a sincere, representative data generating process, and we know that so long as messaging is monotonic and continuously differentiable, then participation is increasing in the extremity of senders' values.

[13]Under the sincere conjecture, $M(v_i) = v_i$ is continuously differentiable so expression B.1 is valid. Moreover, in order for $F_{S|\theta}(v_i) = F_{v|\theta}(v_i)$, expression B.1 requires that $F_c(X(v_i)) = Pr(\rho_1 = 1|\theta)$ for all v_1. The right side of this expression is constant in v_i. However, by proposition 2.6.5, $X(v_i)$ is not constant in v_i, which by assumption 2.3.5, implies that $F_c(X(v_i))$ is not constant in v_i.

Without a complete characterization of equilibrium, it is difficult to rule out the hypothesis that raters also skew their ratings towards the extreme (the exaggeration conjecture). However, proposition 2.6.6 provides a class of games in which only participation contributes to extreme-valued ratings.

Proposition 2.6.6. *For any game of the class described by example 1, but in which $N = 2$, there exists an equilibrium in which $M(v_i) = v_i$.*

Proposition 2.6.6 is also important because although we have not fully characterized equilibria in the multiple senders case, proposition 2.6.6 says that the types of strategies we have supposed (strictly monotonic, differentiable) are not unreasonable.

2.6.3 The Median

It is not known to this author if the median is utilized in the field. However the median is known to possess nice properties, such as strategy proof-ness when used as a social choice function when individuals have single-peaked preferences (Moulin 1980). The median has also been suggested as a desirable replacement to the mean aggregation mechanism by some related work (Garcin, Faltings, and Jurca 2009).

For any set of messages, let $y_l(\mathbf{m})$ be the l-smallest message in $\mathbf{m}$, let $n = |\mathbf{m}|$, and define the "median" aggregation mechanism as $\mu^d(\mathbf{m}) = y_{k(n)}(\mathbf{m})$ where $k(n)$ is a random variable defined as follows:

$$Pr\left(k(n) = \hat{k}\right) = \begin{cases} \frac{1}{2}, \text{ if } \hat{k} \in \left\{floor(\frac{n+1}{2}), ceiling(\frac{n+1}{2})\right\} \\ 0, \text{ otherwise} \end{cases} \tag{2.10}$$

If n is odd, this "median" is a degenerate random variable at the order statistic that is usually defined as the median. We deviate from the traditional definition of the median when n is even. Instead of taking the average of the two most central order statistics, our random variable puts equal weight on each of these (and only on these) order statistics. To better understand this definition of the median aggregation mechanism, we consider the following example.

Example 2 Suppose $L = [\underline{v}, 0)$, $I = [0]$, and $H = (0, \overline{v}]$, and $\mathbf{m}_{-i} = (-0.5, -0.25, 0.5)$. This implies that $\mu^d(\mathbf{m}_{-i}) = -0.25$. However, since $\mathbf{m}$ is even, $\mu^d(\mathbf{m})$ is uncertain. If, for example, $m_i = 1$, there is an equal likelihood that $\mu^d(\mathbf{m})$ will equal -0.25 as there is that $\mu^d(\mathbf{m})$ will equal 0.5. Under the more common definition of the median, the median of $\mathbf{m}$ would equal $\frac{3}{16}$ with certainty.

Suppose instead that $\mathbf{m}_{-i} = (-0.5, -0.25, 0.25, 0.5)$ and $m_i = 1$. In this case $\mu^d(\mathbf{m}) = 0.25$ with certainty, but $\mu^d(\mathbf{m}_{-i})$ equals -0.25 and 0.25 with equal probability.

Notice that in both cases in our example, despite the fact that m_i adds a new, largest element to the set of sent messages, there is only a 50% chance that m_i increases the aggregate message and this likelihood is independent of the value of m_i so long as m_i is the largest element.

This independence characterizes the likelihoods that a sender affects the outcome of the game; in equilibrium, any two sent messages within a convex subset of outcome-equivalent aggregate-messages result in equivalent likelihoods of affecting the outcome. To see this, take any two disjoint subsets of V called Y, Z, and assume, for all $y \in Y$ and $z \in Z$, $y < z$. By definition of the median, the probability that the aggregate message transitions from Y to Z via message m_i, given that there are n total messages, is the probability that $y_{k(n-1)}(\mathbf{m}_{-i}) \in Y$ and $y_{k(n)}(\mathbf{m}) \in Z$. Moreover, this probability can be reduced to a function of just $\mathbf{m}_{-i}$. Assume that $\mu(\mathbf{m}_{-i}) \in Y$ and consider three cases. First, let $m_i < minZ$. In this case the probability of a transition from Y to Z is 0 because the highest value the new median can attain is $max\{m_i, \mu^d(\mathbf{m}_{-i})\} < minZ$. Next, let $m_i > maxZ$. Since m_i will raise the aggregate message from $y_{k(n-1)}(\mathbf{m}_{-i})$ to $min\{m_i, y_{k(n-1)+1}(\mathbf{m}_{-i})\}$, the only way for the new value to be in Z is for $y_{k(n-1)+1}(\mathbf{m}_{-i}) \in Z$. Finally, let $m_i \in Z$. Similarly to the previous case, m_i raises the value of the median to $min\{m_i, y_{k(n-1)+1}(\mathbf{m}_{-i})\}$, but the highest value the median can attain is m_i, which is in Z. Therefore, the likelihood that $\mu(\mathbf{m}) \in Z$ is the likelihood that $y_{k(n-1)+1}(\mathbf{m}_{-i}) > infZ$.

In each of these cases, the likelihood of transition is independent of the choice of m_i. That is, if messages m_i, m_i' each satisfy the same of the above cases, the probability of transition from Y to Z

is independent of which of m_i or m_i' is sent.

We now derive the likelihood that an arbitrary sender, i, changes the receiver's choice from the safe product to the risky product. We do this step-by-step, in terms of order statistics. The likelihoods of all pairwise transitions are then summarized in Table 2.1.

First, assume that the receiver's type space is described by the simple partition (L, H) formed around v^d, that $M(v^d) \in I$, that $M(v_i)$ is continuous, and that $m^h \in H$. Then,

$$Pr(LH|m^h, n) = Pr\left(y_{k(n-1)}(\mathbf{m}_{-i}) < M(v^d), y_{k(n)}(\mathbf{m}) > M(v^d)|n, m^h\right)$$

How this translates into precise order statistics, depends on if n is odd or even.

$$\begin{aligned} Pr(LH|m^h, n) =& 1_{n\ even}\left[\frac{1}{2}Pr\left(y_{\frac{n}{2}}(\mathbf{m}_{-i}) < M(v^d), y_{\frac{n}{2}}(\mathbf{m}) > M(v^d)|n, m^h\right) + \right. \\ & \left. \frac{1}{2}Pr\left(y_{\frac{n}{2}}(\mathbf{m}_{-i}) < M(v^d), y_{\frac{n+1}{2}}(\mathbf{m}) > M(v^d)|n, m^h\right)\right] + \\ & 1_{n\ odd}\left[\frac{1}{2}Pr\left(y_{\frac{n-1}{2}}(\mathbf{m}_{-i}) < M(v^d), y_{\frac{n+1}{2}}(\mathbf{m}) > M(v^d)|n, m^h\right) + \right. \\ & \left. \frac{1}{2}Pr\left(y_{\frac{n+1}{2}}(\mathbf{m}_{-i}) < M(v^d), y_{\frac{n+1}{2}}(\mathbf{m}) > M(v^d)|n, m^h\right)\right] \end{aligned}$$

As was argued above, given a value of sender i's message, conditions on the order statistics of $\mathbf{m}$ are easily translated into conditions on the order statistics of $\mathbf{m}_{-i}$.

$$\begin{aligned} Pr(LH|m^h, n) =& 1_{n\ even}\left[\frac{1}{2}Pr\left(y_{\frac{n}{2}}(\mathbf{m}_{-i}) < M(v^d), y_{\frac{n}{2}}(\mathbf{m}_{-i}) > M(v^d)|n\right) + \right. \\ & \left. \frac{1}{2}Pr\left(y_{\frac{n}{2}}(\mathbf{m}_{-i}) < M(v^d), y_{\frac{n}{2}+1}(\mathbf{m}_{-i}) > M(v^d)|n\right)\right] + \\ & 1_{n\ odd}\left[\frac{1}{2}Pr\left(y_{\frac{n-1}{2}}(\mathbf{m}_{-i}) < M(v^d), y_{\frac{n+1}{2}}(\mathbf{m}_{-i}) > M(v^d)|n\right) + \right. \\ & \left. \frac{1}{2}Pr\left(y_{\frac{n+1}{2}}(\mathbf{m}_{-i}) < M(v^d), y_{\frac{n+1}{2}}(\mathbf{m}_{-i}) > M(v^d)|n\right)\right] \end{aligned}$$

This equivalence follows from the logic presented in example 2, and relies on the fact that $m^h \in H$. Presented like this, it is clear that the first and fourth terms are equal to 0, and thus, we create our final representation of $Pr\left(LH|m^h,n\right)$:

$$Pr(LH|m^h,n) = \frac{1}{2}\left[1_{n\ even}Pr\left(y_{\frac{n}{2}}(\mathbf{m}_{-i}) < M(v^d), y_{\frac{n}{2}+1}(\mathbf{m}_{-i}) > M(v^d)|n\right) + \right.$$
$$\left. 1_{n\ odd}Pr\left(y_{\frac{n-1}{2}}(\mathbf{m}_{-i}) < M(v^d), y_{\frac{n+1}{2}}(\mathbf{m}_{-i}) > M(v^d)|n\right)\right]$$

By this same process, we create Table 2.1, a list of non-zero probabilities of outcome transitions as a function of a sender's message, in terms of order statistics.

Table 2.1 makes clear that many event probabilities are equivalent. Specifically, for a given θ,

$$Pr(LL|m^h,\theta,n) = Pr(LL|M(v^d),\theta,n)$$
$$Pr(HH|m^l,\theta,n) = Pr(HH|M(v^d),\theta,n)$$

Moreover, if $M(\cdot)$ is continuously differentiable (and therefore $F_{S|\theta}(\cdot)$ is continuous and for all l and t_r $Pr(y_l(\mathbf{m}_{-i}) = t_r) = 0$), then,

$$Pr(LH|m^h,\theta,n) = Pr(LI|M(v^d),\theta,n)$$
$$Pr(HL|m^l,\theta,n) = Pr(HI|M(v^d),\theta,n)$$

Unlike with the mean, we cannot prove that in equilibrium L, I and H must be convex and ordered in a natural way. Nevertheless, there are environments in which we can prove that such equilibria exist. Thus, we restrict our examination of equilibria to those in which the receiver's type space is described by a simple partition. Moreover, as we did in proposition 2.6.6, we focus on symmetric environments ($\underline{v} = -\overline{v}$ and $f_{v|\theta^h}(v) = f_{v|\theta^l}(-v)$). Proposition 2.6.9 characterizes a class of equilibria. Before we state proposition 2.6.9, however, we need two lemmas.

In our treatment of the mean message aggregator, it was lemma 2.4.6 that ensured that receiver's type space was described by a simple partition. However, unlike in the case of the mean message aggregator, in the case of the median message aggregator, $E[v_r|\mu(\mathbf{m}_{-i}) \neq \mu^{\Phi}, v_i]$ is not necessarily continuous. In particular, it is *likely* to be *dis*continuous at points of transition between

Outcome	**Likelihood**
$Pr(\Phi H\|m^h,\theta)$	$Pr(\Phi\|\theta)$
$Pr(LL\|m^h,\theta,n)$	$1_{n\ odd}\left(Pr(y_{\frac{n+1}{2}}(\mathbf{m}_{-i})<M(v^d)\|\theta,n)\right) +$ $\frac{1}{2}1_{n\ even}\left(Pr(y_{\frac{n}{2}}(\mathbf{m}_{-i})<M(v^d)\|\theta,n)+Pr(y_{\frac{n+2}{2}}(\mathbf{m}_{-i})<M(v^d)\|\theta,n)\right)$
$Pr(LH\|m^h,\theta,n)$	$\frac{1}{2}\Big[1_{n\ odd}\left(Pr(y_{\frac{n-1}{2}}(\mathbf{m}_{-i})<M(v^d),y_{\frac{n+1}{2}}(\mathbf{m}_{-i})>M(v^d)\|\theta,n)\right) +$ $1_{n\ even}\left(Pr(y_{\frac{n}{2}}(\mathbf{m}_{-i})<M(v^d),y_{\frac{n+2}{2}}(\mathbf{m}_{-i})>M(v^d)\|\theta,n)\right)\Big]$
$Pr(HH\|m^h,\theta,n)$	$\frac{1}{2}1_{n\ odd}\left(Pr(y_{\frac{n-1}{2}}(\mathbf{m}_{-i})>M(v^d)\|\theta,n)+Pr(y_{\frac{n+1}{2}}(\mathbf{m}_{-i})>M(v^d)\|\theta,n)\right) +$ $1_{n\ even}\left(Pr(y_{\frac{n}{2}}(\mathbf{m}_{-i})>M(v^d)\|\theta,n)\right)$
$Pr(\Phi L\|m^l,\theta)$	$Pr(\Phi\|\theta)$
$Pr(LL\|m^l,\theta,n)$	$\frac{1}{2}1_{n\ odd}\left(Pr(y_{\frac{n-1}{2}}(\mathbf{m}_{-i})<M(v^d)\|\theta,n)+Pr(y_{\frac{n+1}{2}}(\mathbf{m}_{-i})<M(v^d)\|\theta,n)\right) +$ $1_{n\ even}\left(Pr(y_{\frac{n}{2}}(\mathbf{m}_{-i})<M(v^d)\|\theta,n)\right)$
$Pr(HL\|m^l,\theta,n)$	$\frac{1}{2}\Big[1_{n\ odd}\left(Pr(y_{\frac{n-1}{2}}(\mathbf{m}_{-i})<M(v^d),y_{\frac{n+1}{2}}(\mathbf{m}_{-i})>M(v^d)\|\theta,n)\right) +$ $1_{n\ even}\left(Pr(y_{\frac{n-2}{2}}(\mathbf{m}_{-i})<M(v^d),y_{\frac{n}{2}}(\mathbf{m}_{-i})>M(v^d)\|\theta,n)\right)\Big]$
$Pr(HH\|m^l,\theta,n)$	$1_{n\ odd}\left(Pr(y_{\frac{n-1}{2}}(\mathbf{m}_{-i})>M(v^d)\|\theta,n)\right) +$ $\frac{1}{2}1_{n\ even}\left(Pr(y_{\frac{n}{2}}(\mathbf{m}_{-i})>M(v^d)\|\theta,n)+Pr(y_{\frac{n-2}{2}}(\mathbf{m}_{-i})>M(v^d)\|\theta,n)\right)$
$Pr(\Phi I\|M(v^d),\theta)$	$Pr(\Phi\|\theta)$
$Pr(LL\|M(v^d),\theta,n)$	$1_{n\ odd}\left(Pr(y_{\frac{n+1}{2}}(\mathbf{m}_{-i})<M(v^d)\|\theta,n)\right) +$ $\frac{1}{2}1_{n\ even}\left(Pr(y_{\frac{n}{2}}(\mathbf{m}_{-i})<M(v^d)\|\theta,n)+Pr(y_{\frac{n+2}{2}}(\mathbf{m}_{-i})<M(v^d)\|\theta,n)\right)$
$Pr(LI\|M(v^d),\theta,n)$	$\frac{1}{2}\Big[1_{n\ odd}\left(Pr(y_{\frac{n-1}{2}}(\mathbf{m}_{-i})<M(v^d),y_{\frac{n+1}{2}}(\mathbf{m}_{-i})\geq M(v^d)\|\theta,n)\right) +$ $1_{n\ even}\left(Pr(y_{\frac{n}{2}}(\mathbf{m}_{-i})<M(v^d),y_{\frac{n+2}{2}}(\mathbf{m}_{-i})\geq M(v^d)\|\theta,n)\right)\Big]$
$Pr(HI\|M(v^d),\theta,n)$	$\frac{1}{2}\Big[1_{n\ odd}\left(Pr(y_{\frac{n-1}{2}}(\mathbf{m}_{-i})<M(v^d),y_{\frac{n+1}{2}}(\mathbf{m}_{-i})\geq M(v^d)\|\theta,n)\right) +$ $1_{n\ even}\left(Pr(y_{\frac{n-2}{2}}(\mathbf{m}_{-i})\leq M(v^d),y_{\frac{n}{2}}(\mathbf{m}_{-i})>M(v^d)\|\theta,n)\right)\Big]$
$Pr(HH\|M(v^d),\theta,n)$	$1_{n\ odd}\left(Pr(y_{\frac{n-1}{2}}(\mathbf{m}_{-i})>M(v^d)\|\theta,n)\right) +$ $\frac{1}{2}1_{n\ even}\left(Pr(y_{\frac{n}{2}}(\mathbf{m}_{-i})>M(v^d)\|\theta,n)+Pr(y_{\frac{n-2}{2}}(\mathbf{m}_{-i})>M(v^d)\|\theta,n)\right)$

Table 2.1: Non-zero equilibrium likelihoods of outcome transitions as a function of a sender's message, when the receiver's type space is described by the simple partition (L,H) formed around v^d, and where $m^h\in H$, $m^l\in L$, $M(v^d)\in I$

outcome-equivalent sets. Therefore, while the necessary conditions of lemma 2.4.5 are met trivially intra-outcome-equivalent sets, they are not necessarily met at points of transition between outcome equivalent sets. Lemma 2.6.7 follows the implications of this logic, finding conditions that ensure that the necessary conditions of lemma 2.4.5 are met everywhere.

Lemma 2.6.7. *If, in equilibrium, the receiver's type space is defined by the simple partition* (L, H) *formed around* v^d*, then* v^d *must solve:*

$$0 = \sum_{\theta \in \Theta} Pr(\theta|v^d)E[v_r|\theta] \left(1 - \left(Pr(LL|M(v^d)), \theta) + Pr(HH|M(v^d), \theta)\right)\right) \tag{2.11}$$

Equation 2.11 depends on v^d not only through $Pr(\theta|v^d))$ but also through the relevant event probabilities. Therefore, we cannot prove that v^d is unique, or even that outcome equivalent sets must be convex (although we have restricted our search to such equilibria here).

Lemma 2.6.8 tells us that in this class of symmetric environments, if the equilibrium is of the form described by proposition 2.6.6, then there is additional equivalence of outcome-transition likelihoods.

Lemma 2.6.8. *Let* $Y, Z \in \{L, H\}$*,* $Y' \in \{L, H\} \setminus Y$*,* $Z' \in \{L, H\} \setminus Z$*,* $\theta \in \Theta$*,* $\theta' \in \Theta \setminus \theta$*,* $v^l \in L$ *and* $v^h \in H$*. For any game with a symmetric environment; in equilibrium, if* $M(v_i) = v_i$*,* $X(v_i)$ *is symmetric around* 0*, the receiver's type space is described by the simple partition* (L, H) *formed around* 0*, and* $M(0) \in I$*, then* $Pr(YZ|M(v^l), \theta) = Pr(Y'Z'|M(v^h), \theta')$*.*

Specifically, lemma 2.6.8 proves that in these circumstances, for disjoint θ, θ':

$$Pr(LH|M(v^l),\theta) = Pr(HL|M(v^h),\theta')$$

$$Pr(LL|M(v^l),\theta) = Pr(HH|M(v^h),\theta')$$

$$Pr(HH|M(v^l),\theta) = Pr(LL|M(v^h),\theta')$$

$$Pr(HL|M(v^l),\theta) = Pr(LH|M(v^h),\theta')$$

These equalities are necessary for the proof of proposition 2.6.9.

Proposition 2.6.9. *For any game with an environment such that* $\underline{v} = -\overline{v}$ *and* $f_{v|^h}(v) = f_{v|^l}(-v)$*, there exists an equilibrium in which the receiver's type space is described by the simple partition* (L,H) *formed around* 0*,* $M(v_i) = v_i$*, and in which* $X(v_i)$ *is strictly decreasing for* $v_i < 0$ *and strictly increasing for* $v_i > 0$*.*

Thus, the equilibrium constructed for the proof of proposition 2.6.6 is also supported as an equilibrium when the median sent message is sent to the receiver. Unlike proposition 2.6.6, proposition 2.6.9 holds for any number of senders. Moreover, as direct revelation, or "sincerity," is a focal point of both research and popular debate, proposition 2.6.9 presents the median aggregation mechanism in a positive light: In these simple environments, the median aggregation mechanism supports sincere messaging in equilibrium. It should be mentioned, of course, that direct revelation is one of an infinite number of sustainable equilibrium strategies. Indeed, proposition 2.6.9 is proved assuming $M(v_i)$ is any increasing function that is a reflection through $M(0)$ (of which $M(v_i) = v_i$ is a special case). Because of this, in terms of our conjectures, there are equilibria in which we accept the exaggeration conjecture and equilibria in which we reject it.

Moreover, the median does not escape the main result of the previous section. That is, in the equilibria described here, the median message aggregator satisfies the conditions of corollary 2.4.9, and thus participation remains an increasing function of the extremism of players' values.

2.6.4 Thumbs Up or Thumbs Down

When YouTube abandoned its user generated rating system, it replaced it with a system that prompted users to rate a video as "thumbs up" or as "thumbs down". Binary-message mechanisms, such as this, can be found in many implementations, and are therefore examined here.

To examine the "thumbs up, thumbs down" (TUTD) environment, we must first define a binary message space. Thus, let each sender's message space be $\{\underline{m}, \overline{m}\}$, where $\underline{m}$ and $\overline{m}$ are arbitrary but unique messages. Next, we define the TUTD message aggregator, μ^t, to display the number of senders who send $\underline{m}$ and the number of senders who send $\overline{m}$; that is, the receiver's type space becomes $\{0, 1, ..., N\} \times \{0, 1, ..., N\}$. The receiver's inference problem is thus vastly different than in the previous treatments. Most significantly, the receiver can now see precisely which messages are sent, and precisely the level of participation. This will result in the receiver playing a cutoff rule for each level of participation. For the senders, despite best responding to a potentially more complex strategy on the part of the receiver, the best response strategy is simple.

Lemma 2.6.10. *In any TUTD game, the equilibrium messaging strategy is a cutoff strategy.*

Lemma 2.6.10 simply says that there exists some value below which senders send one message, and above which, they send the other. We call this switch-point v^t, and for ease of exposition (and without loss of generality) we assume that players send $\underline{m}$ for values below v^t.

$$M(v_i) = \begin{cases} \underline{m}, \text{ if } v_i < v^t \\ \overline{m}, \text{ if } v_i > v^t \end{cases}$$

This simple message strategy results in a simple density of sent messages.

$$Pr(m|\theta) = \begin{cases} \int_{\underline{v}}^{v^t} f_{v|\theta}(v) F_c(X(v)) dv, \text{ if } m = \underline{m} \\ \int_{v^t}^{\overline{v}} f_{v|\theta}(v) F_c(X(v)) dv, \text{ if } m = \overline{m} \end{cases}$$

By our assumptions on the conditional distributions of values, it is quick to see that $Pr(\underline{m}|\theta^l) > Pr(\underline{m}|\theta^h)$ and $Pr(\overline{m}|\theta^l) < Pr(\overline{m}|\theta^h)$. That is, players are more likely to send the low message in the low state than they are in the high state, and players are more likely to send the high message in the high state than they are in the low state. As mentioned above, this results in the receiver playing a set of contingent cutoff strategies.

Lemma 2.6.11. *In any TUTD game, in equilibrium, conditional on total participation levels, the receiver plays a cutoff strategy in the difference between the number of low messages received and the number of high messages received.*

Lemma 2.6.11 says, quite intuitively, that for a given number of total messages, as the number of high messages becomes greater, and the number of low messages becomes fewer, the receiver's posterior belief that the state of the world is the high state, increases.

Proposition 2.6.12. *In any TUTD game, in equilibrium, at least one of the following is true: $X(v_i)$ is decreasing for $v_i < v^t$; and $X(v_i)$ is increasing for $v_i > v^t$.*

Thus, in all equilibria of the class of TUTD game, participation patterns resemble the extreme participation conjecture on at least part of the domain. Corollary 2.6.12 gives a condition under which the conjecture holds on the entire domain.

Corollary 2.6.12: *There exists an open neighborhood around v^I, V', such that in equilibrium, if $v^t \in V'$, then $X(v_i)$ is decreasing for $v_i < v^t$ and increasing for $v_i > v^t$.*

The issue here is that the messages $\underline{m}$ and $\overline{m}$ are both generated by a wide range of values. Corollary 2.6.12 uses the fact that if the cutoff, v^t, is sufficiently close to v^I, then the set of values that generate each message are similar to one another.

Restricting the set of environments we are considering to the simple symmetric environment that we established with example 1 and proposition 2.6.6, we see that the requirement of corollary 2.6.12 for v^t to be near v^I is not unreasonable.

Proposition 2.6.13. *For any game with an environment such that* $\underline{v} = -\overline{v}$ *and* $f_{v|^h}(v) = f_{v|^l}(-v)$*, there exists an equilibrium in which* $v^t = v^I = 0$*, and therefore* $X(v_i)$ *is strictly decreasing for* $v_i < 0$ *and strictly increasing for* $v_i > 0$*.*

Focusing on YouTube's newest implementation, users cannot see ratings before they begin to view a video. This puts YouTube's implementation outside of the scope of the TUTD analysis, but not outside the scope of our model. Our predicted comparative statics depend on the environment, but if we assume the symmetric environment and an equilibrium of the type described by proposition 2.6.13, we find that $X(v_i)$ is positive everywhere but at v^Φ. In YouTube's implementation, since there is no interaction between senders and receivers, $X(v_i)$ would be 0 for all v_i. That is, our model predicts that under their new implementation only those consumers who enjoy the act of rating (negative c_i) will submit ratings. This implies that we would predict that YouTube's change of mechanism decreased ratings for all values, excluding some central value. Of course, since there are but two messages in the TUTD system, the only testable prediction is that participation ought to have decreased in aggregate.

2.7 Discussion and Conclusion

For games utilizing each of the aggregation mechanisms we considered, we showed that equilibrium messaging distributions are not representative of the underlying value distributions (rejection of the sincere conjecture). Indeed, for each type of message aggregation, we found classes of equilibria in which it is always the case that participation increases in the extremity of how an individual feels about a product (confirmation of the extreme participation conjecture). We prove each of these with a unique model that includes rating aggregation; costly, optional participation; diffuse but correlated valuations; and intrinsic rating motivation. No other work has examined this topic while considering each of these facets simultaneously.

In the case of mean message aggregation, we showed that so long as messaging strategies are continuously differentiable, this pattern of participation must occur in *all* environments. We remain agnostic about the exaggeration conjecture, that raters skew their ratings towards the extreme. However example 1 and proposition 2.6.6 provided classes of games in which the exaggeration conjecture is rejected.

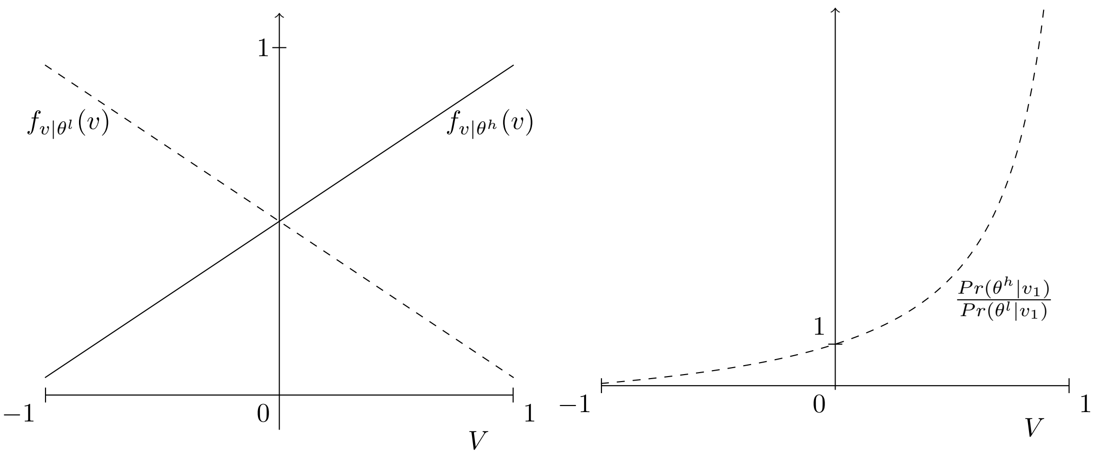
Similarly, in the case of median message aggregation we examined a class of environments and equilibria in which we can reject the sincere conjecture and accept the extreme participation conjecture. Unique to this message aggregator, we show that there are each of: equilibria in which the symmetric messaging strategy is to dampen players' values; equilibria in which the symmetric messaging strategy is to exaggerate players' values; and equilibria in which the symmetric messaging strategy is the sincere messaging strategy. This is because with the median aggregation mechanism, the marginal effect of a player's message is either 1 or 0 (a message is either the median or it is not; if it is not, local changes do not affect the median value) and this implies that there exist many messages over which each player is indifferent.

In the case of a binary "thumbs up" or "thumbs down" mechanism, we showed that both senders and receivers utilize a cutoff strategy in equilibrium. We showed that in all equilibria, participation is increasing in the extremism of a sender's value on at least part of the value domain, and we identify conditions under which the extreme participation conjecture holds over the entire domain. Similarly to the other aggregation mechanisms, we showed that such equilibria exist in symmetric environments.

The selective participation result is highly reminiscent of the "Swing Voter's Curse" (Feddersen and Pesendorfer 1996) in which voters who are uninformed about which candidate is best prefer to abstain in some equilibria. In contrast to that model, actors in our environment receive signals that result in a continuum of posterior beliefs, and therefore there exists the possibility that in equilibrium players with different values participate at different (interior) rates. The participation result is therefore also partially due to the assumption that rating is idiosyncratically costly, and that senders' costs are not related to their values. Restated, in equilibrium, players with stronger posteriors are more willing to pay to rate a product; therefore, since there exist a continuum of costs, we can say that players with extreme values are more likely to rate the product. If instead rating was costless or beneficial to all consumers, then we would not expect the hypothesized participation behavior to persist. Instead, we would observe full participation. However, full participation is plainly not observed.

The other factor behind the extreme participation result is that we were able to identify equilibria in which the marginal effect of equilibrium messages, as a function of senders' values, exhibited the same comparative static as did posterior strength. We achieved this by narrowing the focus to equilibria in which the receiver played a cutoff strategy. Such a consideration was not a factor in the swing voter literature since both signals and actions in that model were binary.

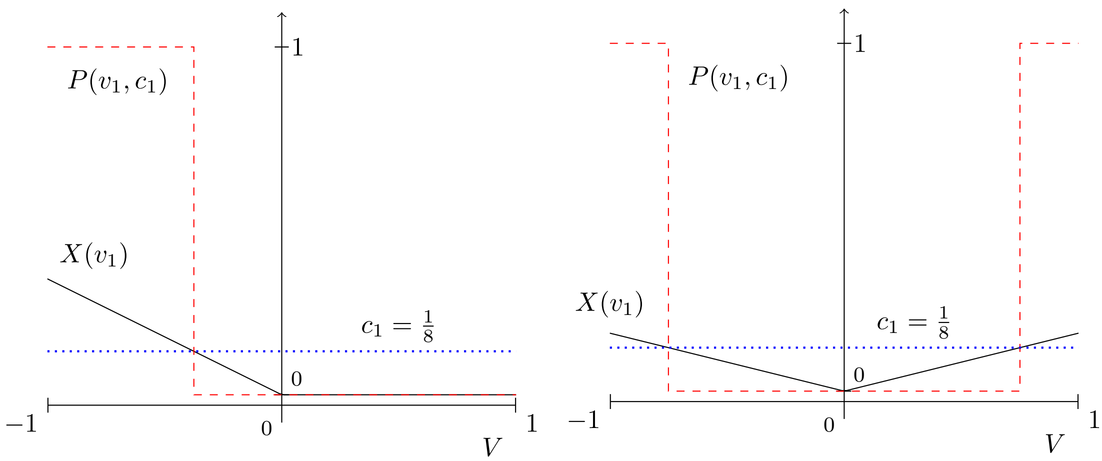
It is through this vector that the choice of aggregation mechanism affects outcomes. Clearly, aggregation does not affect the informativeness of signals. It does affect the way messages are incorporated with each other, and therefore it does effect the marginal effect of any given message. It should be noted that with rational agents, message aggregation is weakly welfare reducing. An aggregation mechanism cannot do better than allowing the receiver to view each individual rating because the receiver can best infer the state of the world by observing senders' signals directly. A proper treatment of the welfare effects of message aggregation is therefore likely to include agents with limited capabilities to consume or update properly given a multitude of signals. This is a potential direction for future work.



a) Densities of values, conditional on the state of the world

b) Relative Bayesian posterior beliefs

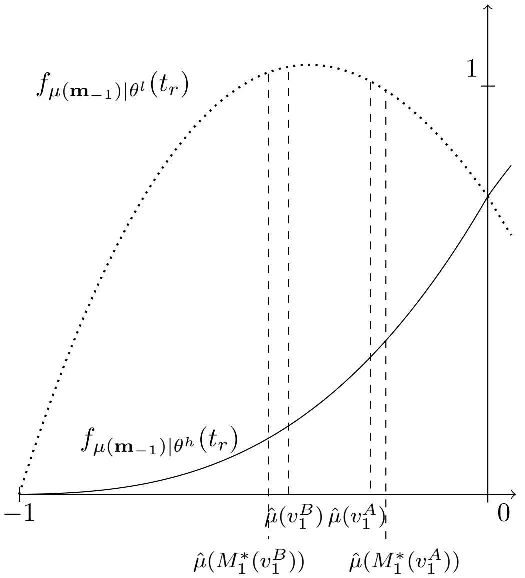
Figure 2.1: Example conditional value densities and posterior beliefs



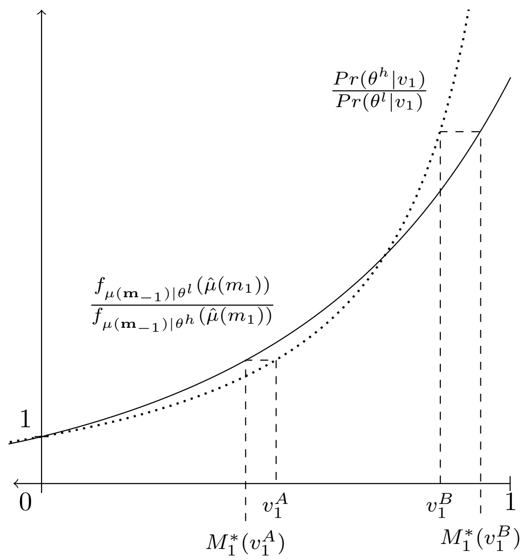
a) No messages sent implying preference for the risky product ($\mu^\Phi \in H$) results in a one-sided cutoff participation strategy

b) No messages sent implying indifference ($\mu^\Phi \in I$) results in a two-sided cutoff participation strategy

Figure 2.2: Example equilibrium participation strategies with one sender when $c_1 = 0.2$

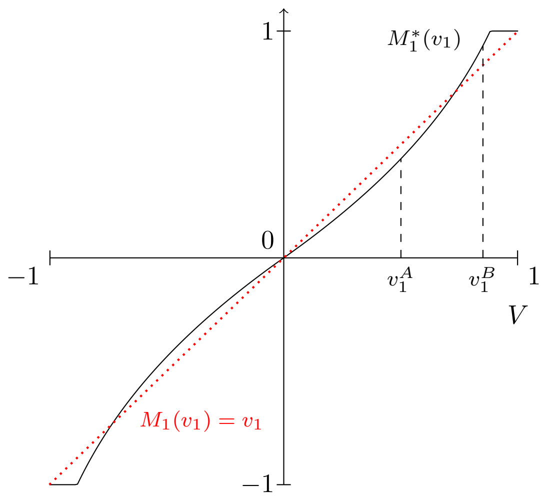


a) Full participation, $N = 3$, densities of aggregate messages when excluding m_1 and conditional on the state of the world

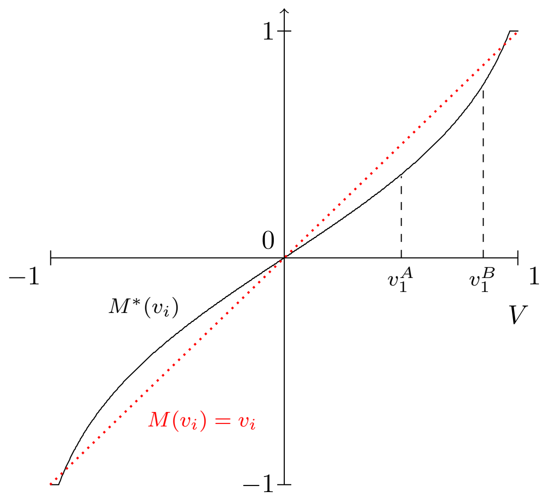


b) Full participation, $N = 3$, relative marginal utility of m_1, compared to the sender's relative posteriors as a function of v_1

Figure 2.3: Example best response messages for example values v_1^A and v_1^B, given full participation

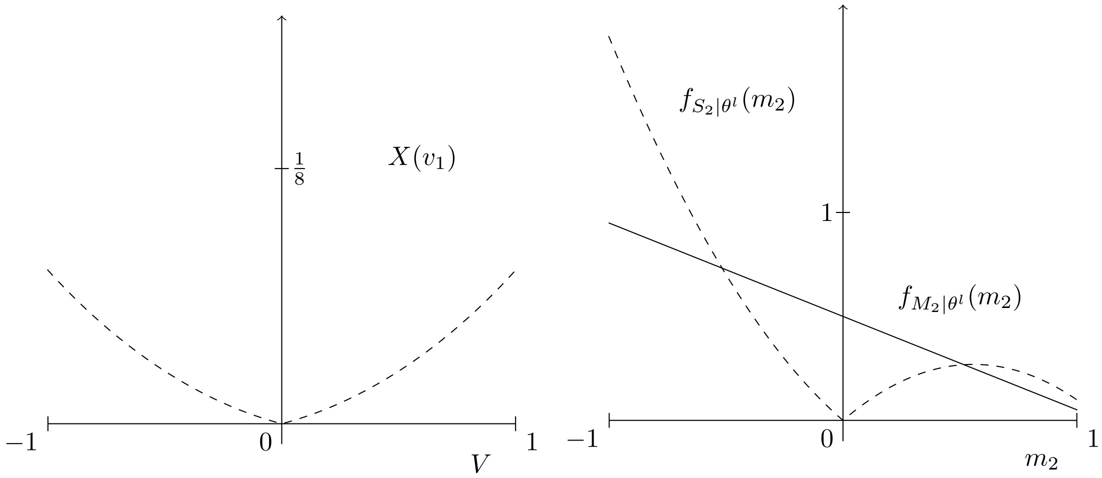


a) Sender 1's best response, given the assumed strategies



b) A symmetric equilibrium messaging strategy

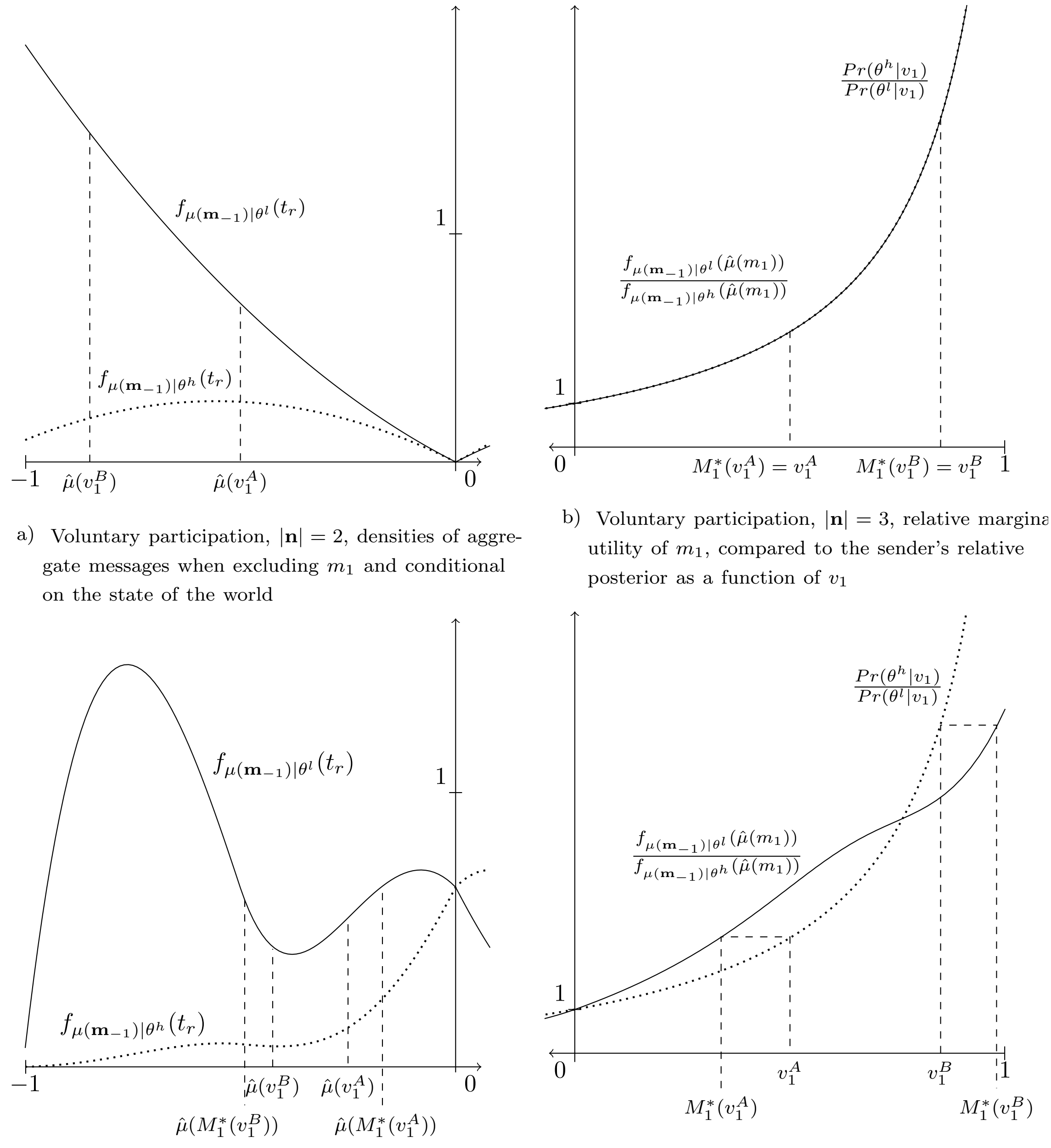
Figure 2.4: Example Messaging Under Full Participation



a) Example equilibrium net expected utility to participation as a function of a player's value

b) Densities of sent messages under full participation and voluntary participation when $M_2(v_2) = v_2$

Figure 2.5: Example voluntary participation and its effect on message likelihood



a) Voluntary participation, $|\mathbf{n}| = 2$, densities of aggregate messages when excluding m_1 and conditional on the state of the world

b) Voluntary participation, $|\mathbf{n}| = 3$, relative marginal utility of m_1, compared to the sender's relative posterior as a function of v_1

c) Voluntary participation, $|\mathbf{n}| = 3$, densities of aggregate messages when excluding m_1 and conditional on the state of the world

d) Voluntary participation, $|\mathbf{n}| = 2$, relative marginal utility of m_1, compared to the sender's relative posterior as a function of v_1

Figure 2.6: Example best responses for example values v_1^A and v_1^B as a function of level of participation, given the assumed strategies of other players

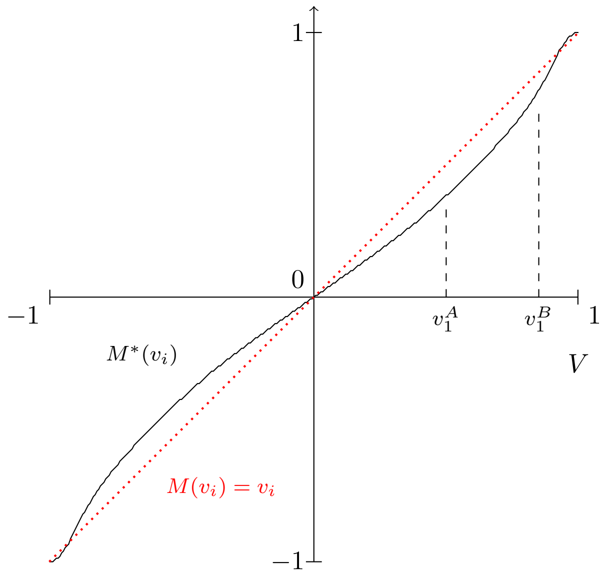


Figure 2.7: Example symmetric equilibrium messaging strategy, given voluntary participation

Chapter 3

Rating Behavior in the Lab

3.1 Introduction

The previous chapter developed a theoretical framework for studying rating behavior in an user-generated product rating environment. In this chapter we study a simplified version of the rating model and examine behavior in this game in a laboratory setting. Unlike the previous chapter, we assume a discrete type space, which makes the theoretical analysis more tractable and the experimental game simpler. The altered model is described in section 3.3.

We find that subjects exhibit participation behavior consistent with the extreme participation conjecture, as expected by the theory. Unexpectedly, we find both that subjects anchor strongly to the sincere messaging strategy, and that when they are insincere they almost always send messages that are exaggerations of their valuation for the risky product. We show that these behaviors persist despite being neither in line with equilibrium behavior nor best response to empirical rates of play.

In the following section we discuss related literature. In section 3.4 we reestablish the theoretical results of the previous chapter, and establish new results, made possible by the simpler model. In the following section we describe the experimental design. In section 3.6 we present the experimental results, and finally we discuss the results.

3.2 Related Experiments

Lafky's (2010) is the only paper we know of that studies product rating in the lab. He showed that when messaging is costly, subjects participate at much lower rates compared to when it is free, and that they do so, on average, in the u-shaped pattern commonly seen in real world collaborative recommendation systems. Lafky considered the problem in three stages. Unlike our model, product quality in Lafky's game was endogenous, with two subjects ("sellers") simultaneously choosing the quality of a product. Like our model, Lafky modeled "buyers" in two stages. First, a single sender chose one of the two available products, observed the quality of the selected product, and then chose between sending a costly message and abstaining. Finally, two receivers saw the message sent by the sender, and then selected one of the two products.[1] Lafky studied two treatments, a "Free" treatment in which the cost to the sender of sending a message was 0, and a "Costly" treatment, in which the cost of sending a message was $0.25 (about 6% of the average sender single-round payout when excluding the cost of participation). He separates motivation for senders' participation by devising "seller-fixed" and "buyer-fixed" rounds, in which sellers' and receivers' outcomes were fixed to be independent of all players' actions, respectively. All players were informed of the type of round after the round completed, but only the sender knew the type of round during the round itself.

Lafky's seller-fixed rounds are similar to the one-sender model discussed in the previous chapter, and it is therefore through the multiple senders treatments that this chapter makes its contribution.[2] As we showed in the preceding chapter, any equilibrium in the one sender setting is equivalent to an equilibrium in which there are at most three sent messages, each essentially acting as a selection recommendation for the receiver ("choose one product", "choose the other product", "be indifferent"). In contrast, in our multiple senders models senders consider how their message will

[1] If the sender abstained, the receivers were not informed which product the sender selected.

[2] The most notable caveat to this comparison is that in our model abstention by the sender is itself an informative message. This results in the three types of equilibria discussed in the previous chapter; two in which the sender plays a one-sided cutoff participation strategy, and one in which the sender plays a two-sided cutoff participation strategy. We furthermore showed that the type of equilibrium that obtains is driven by the receiver's interpretation of abstention by the sender. In Lafky's model, after abstention by the sender, receivers do not know which product was selected and therefore cannot update their beliefs about the quality of that product. This restriction therefore essentially restricts the type of equilibrium to the one that obtains when the abstention message implies indifference: the two-sided cutoff participation strategy. This helps to explain why Lafky's subjects participated at similar rates at both extremes of the type space.

interact with other senders' messages, in which circumstances those interactions are payoff relevant, and the informational implication of those circumstances.[3]

The multiple sender model therefore has much in common with models of other domains, such as common value auctions or elections, that assume common values but private information. The hallmark of these models is that a rational actor conditions his beliefs on his actions being pivotal, and that updating his beliefs in this way provides him with more information than is contained in his private signal alone. The "winner's curse" describes a phenomenon in which a winner of a common value auction discovers that the value of the item he won is less than he expected it to be (possibly because he did not fully appreciate the information contained in the fact that if he won, he had the highest private valuation of the item). The "swing voter's curse" similarly describes a phenomenon in which conditioning on the fact that his vote swings an election, a rational voter may prefer to abstain, even if he prefers one candidate to another.

The winner's curse has been demonstrated many times in the lab. Bazerman and Samuelson (1983, 1985) showed in a series of experiments that the winner's curse is prevalent in a laboratory setting, and Ball et al. (1991) showed in a similar but repeated experiment that only a small fraction of subjects learn to avoid the winner's curse. Kagel and Levin (1986) and Dyer, Kagel, and Levin (1989) demonstrated the prevalence of the winner's curse, and showed that its effects increased with the the number of bidders and were not mitigated with professional bidders.

In contrast, experimental work examining the swing voter's curse has found that subjects act similarly to theoretical predictions of rational actors. Battaglini et al. (2010) examine the swing voter's curse in the laboratory and find that subjects generally conform to expected equilibrium behavior, with, generally, uninformed voters abstaining except to nullify the vote of a partisan voter, in expectation. Similarly, Guarnaschelli et al. (2000) find that subjects choose actions near expected equilibrium rates in a laboratory test of jury decision rules.

At its core, the game we describe here is a game of information transmission. Strategic information transmission and cheap talk in the vein of Crawford and Sobel (1982) was tested in the

[3]This point was expounded in detail in the previous chapter. In particular, see section 2.4.2.

laboratory by Dickhaut et al. (1995) and more recently by Cai and Wang (2006). These papers confirm the comparative statics of their theoretical inspiration — that a greater divergence in the interests of senders and receivers results in less information transmission, but also show significant over-transmission of information on the part of senders and significant over-reliance on senders' messages on the part of receivers. Vespa and Wilson (2012) reinterpret the information transmission game as a multi-dimensional game of trustworthiness and show that receivers are generally unable to extrapolate trustworthiness of a sender from one dimension into another, a phenomenon they suggest may be similar to other failures of inference such as the winner's curse.

3.3 The Model

The game we study in this chapter is the same as the game we studied in Chapter 2, with a few key simplifications. In the previous chapter we normalized the value of the safe product to 0. This is undesirable in an experimental setting, both because we do not wish subjects to have a chance of receiving negative payouts and because we do not wish players to conflate their incentives with loss aversion. Therefore, in this chapter we consider a strictly positive set of values, $\mathcal{V}$, and we describe the value of the risky product as $\hat{v}$. Moreover, in order to simplify the experimental environment, we move to a discrete type space, for both values and costs. In addition, we restrict the conditional value densities to be reflections of one other across $\hat{v}$. That is, for any value of the risky product, v:

$$Pr(v|\theta^h) = Pr(2\hat{v} - v|\theta^l)$$

Figure 2.1.a demonstrates this for $\hat{v} = 0$. In addition, in order to allow senders the ability to send obviously exaggerated messages, messages are chosen from $\mathcal{M}$, a discrete, compact subset of $\mathcal{R}^1$ such that $\mathcal{V} \subseteq \mathcal{M}$. As a final simplification, we consider only the message aggregation mechanism that returns the average of all sent messages.

Finally, a receiver is said to play a cutoff strategy if there exists some aggregate message, t_r, such that for all $t'_r < t_r$, $A(t'_r) = 0$ and for all $t'_r > t_r$, $A(t'_r) = 1$. We analyze symmetric, informative equilibria in which senders play monotone increasing (in values) messaging strategies, $M(\cdot)$, and in which receivers play a cutoff strategy around $M(\hat{v})$, and we refer to them simply as equilibria.

3.4 Theory

In this section we characterize the equilibria of the simplified, discrete version of the game. Generically, the game has many equilibria, and the experimental parameters admit multiple equilibria. In this section, without loss of generality, we assume that the safe product gives players no value; that is, $\hat{v} = 0$. We utilize definitions and notation developed in the previous chapter, and formal proofs appear in the appendix.

3.4.1 Messaging

Following the intuition of incentive compatibility developed in the previous chapter, optimal messaging strategies balance a player's certainty regarding the state of the world with his marginal expected utility in each state of the world. The analog of that incentive compatibility condition in this discrete setting is stated in the following lemma.

Lemma 3.4.1. *For any sender, i, and types $t_i = (v_i, c_i)$ and $t'_i = (v'_i, c_i)$; in equilibrium, for all $v'_i > v_i > \hat{v}$ or $v'_i < v_i < \hat{v}$:*

$$\frac{\sum\limits_{Y\in\{L,I,H,\Phi\}}\sum\limits_{Z\in\{L,I,H\}} w(Z)\left[P(YZ|M(t'_i),\theta^l) - P(YZ|M(t_i),\theta^l)\right]}{\sum\limits_{Y\in\{L,I,H,\Phi\}}\sum\limits_{Z\in\{L,I,H\}} w(Z)\left[P(YZ|M(t'_i),\theta^h) - P(YZ|M(t_i),\theta^h)\right]} \leq \frac{Pr(\theta^h|t'_i)}{Pr(\theta^l|t'_i)} \tag{IC.1}$$

$$\frac{Pr(\theta^h|t_i)}{Pr(\theta^l|t_i)} \leq \frac{\sum\limits_{Y\in\{L,I,H,\Phi\}}\sum\limits_{Z\in\{L,I,H\}} w(Z)\left[P(YZ|M(t'_i),\theta^l) - P(YZ|M(t_i),\theta^l)\right]}{\sum\limits_{Y\in\{L,I,H,\Phi\}}\sum\limits_{Z\in\{L,I,H\}} w(Z)\left[P(YZ|M(t'_i),\theta^h) - P(YZ|M(t_i),\theta^h)\right]} \tag{IC.2}$$

The analog of the left hand side of inequality IC.1 and the right-hand side of inequality IC.2 to the ratio of marginal effects of the optimal message in each state of the world should be clear (one could divide both the numerator and the denominators by $M(t_i') - M(t_i)$). Considering the case in which $v_i' > v_i > \hat{v}$, the intuition here is that for any type, a sender increases his message until the ratio of the marginal effect of his message matches the ratio of his posterior as closely as possible, given the discreteness of the message space; any greater (smaller) and the effect of his message in the low (high) state would exceed the effect of his message in the high (low) state to an extent that outweighs his certainty that the state is high (low).

3.4.2 Equilibrium

Because we moved to the simpler discrete environment, and because we are constraining the receiver to playing a cutoff strategy, Kakutani's fixed point theorem tells us that there exists an (possibly mixed strategy or asymmetric) equilibrium to the game. In the remainder of this section, we characterize equilibrium comparative statics of expected utility and participation. These show that in the type of equilibria we consider, the extreme participation conjecture holds. Proposition 3.4.3 contains the result, and builds from lemma 3.4.2.

Lemma 3.4.2. *In equilibrium,* $E[u|(v_i, c_i), \rho_i = 1]$ *is decreasing in* v_i *when* $v_i < \hat{v}$*, and increasing in* v_i *when* $v_i > \hat{v}$*.*

Although a sender becomes more certain of the state of the world as his signal moves towards the extremities of the value space, it is also the case that more extreme messages are more likely to be pivotal in the state that is less likely according to the sender's posterior. Conditioning on pivotal events is therefore likely to move a sender's posterior in the opposite direction than did his signal. Lemma 3.4.2 therefore tells us that the latter effect is not strong enough to overpower the former, in equilibrium.

Proposition 3.4.3 uses this result to prove that sender participation follows a similar comparative static. This result is non-trivial because $E[u|(v_i, c_i), \rho_i = 0]$ has the same comparative static as $E[u|(v_i, c_i), \rho_i = 1]$. That is, in equilibrium, $E[u|(v_i, c_i), \rho_i = 0]$ is decreasing when $v_i < \hat{v}$, increasing when $v_i > \hat{v}$ (since $E[u|(v_i, c_i), \rho_i = 0]$ is not affected by sender i's actions, this follows immediately from the fact that $Pr(\theta^h|v_i)$ is increasing in v_i).

Proposition 3.4.3. *In equilibrium, $X(v_i)$ is decreasing when $v_i < \hat{v}$, and increasing when $v_i > \hat{v}$.*

The proof of proposition 3.4.3 shows that the incentive compatibility inequalities imply that the components that drive the comparative static of $E[u|(v_i, c_i), \rho_i = 1]$ are shared by $X(v_i)$.

Proposition 3.4.3 tells us that in the environment and equilibria we are considering here, that we accept the extreme participation conjecture: participation increases as types move away from the neutral signal $\hat{v}$. The incentive compatibility conditions of lemma 3.4.1 constrain the messaging strategies we can expect to see in equilibrium, however not sufficiently to speak to the exaggeration conjecture. We therefore remain agnostic about the precise shape of equilibrium messaging strategies.

3.4.3 One Sender

We remain particularly interested in the one-sender game. In addition to the reasons stated in the previous chapter, we are interested in the one-sender game here because it is simpler than the multiple-senders game. Unlike the multiple-senders case, players in the one-sender case do not need to infer the types or strategies of other senders, or to update their beliefs on more than one pivotal event. Like the multiple-senders case, players in the single sender game receive a signal about the state of the world, upon which they update beliefs and choose an optimal action. When we study the experimental data, this one-sender case therefore provides a useful baseline for how subjects behave when they are faced with the same basic task and environment, but a substantially simplified inference problem.

The one-sender game is substantially more tractable from a theoretical standpoint as well. Like the multiple-sender games, there are multiple possible equilibria of a one-sender game. However, in this restricted environment, the number of potential equilibria is significantly reduced.

Proposition 3.4.4. *For every game, there exists a* $\delta < 0.5$ *such that in every equilibrium of the game, one of the following must hold:* $a^\Phi = 0$*;* $a^\Phi = 1$*; or* $a^\Phi \in (0.5 - \delta, 0.5 + \delta)$*. The final case may constitute part of an equilibrium if and only if the strategies of that equilibrium also form an equilibrium when* $a^\Phi = 0.5$*.*

Proposition 3.4.4 says that there can only be an equilibrium in which the receiver mixes upon receipt of the null message if the strategies that make up that equilibrium are also an equilibrium when the receiver mixes evenly between the two products.

This result is particularly useful because, as discussed in Chapter 2, a^Φ has a large effect on the form equilibrium strategies. Proposition 3.4.4 says that we may, without significant loss of generality, limit our search for equilibria to the three cases in which $a^\Phi \in \{0, 0.5, 1\}$.

3.5 Experimental Design

We used a controlled laboratory experiment to evaluate the theoretical results. The value of the safe product in all of the laboratory sessions was $\hat{v} = 100$. Each session used the value space $\mathcal{V} = \{25, 50, 75, 100, 125, 150, 175\}$, the message space $\mathcal{M} = \{0, 25, 50, 75, 100, 125, 150, 175, 200\}$, the cost space $\mathcal{C} = \{1, 2.5, 5, 7.5, 10, 12.5, 15, 20, 25\}$, and the type densities depicted in Figure 3.1. We studied three treatments in which we varied only the number of senders, N, between 1, 2, and 5.

The experiment was conducted at the Social Science Experimental Laboratory at The California Institute of Technology (Caltech). We utilized a within-subjects design in order to reduce variance across treatments stemming from individual subject differences. Each session was therefore divided by treatment into three parts. We also varied the treatment order across sessions in order to control for any learning effects. Each of the possible sequences was implemented one time, yielding six total sessions. Each treatment of each session lasted 30 matches, and as a result we conducted 180 total matches for each treatment.

Each session had 12 participants, each of whom were Caltech students. Subjects took part in one session each, and we therefore had 72 total, unique participants.

In each match subjects were randomly grouped into groups of size $N + 1$, where N was the number of senders in that treatment. N of the subjects in each group were randomly selected and assigned the role of sender. The remaining subject in each group was assigned the role of receiver. The number of receivers in each group in each match did not change. After each match, groups and roles were re-assigned at random.

Instructions were read aloud to subjects while slides depicting important points in the instructions were displayed at the front of the room.[4] Experiments were conducted via networked, private computer terminals, each separated by a physical divider. Subjects' computers were coordinated by a central server computer, controlled by the experimenter, and which also was responsible for realizing values of random variables in each match.

Subjects were told that they were participating in an experiment evaluating product rating behavior, and that they would need to discern between an old product of known quality, and a new product of unknown quality. This change in diction from "safe" to "old" and "risky" to "new" was done to avoid activating subjects' risk preferences. The instructions and slides took the subjects through an example match step-by-step. Subjects were fully informed of the parameters and distributions of the game; they knew the distribution of costs as well as the conditional densities of values. It was furthermore explained to them that the larger the value they received, the more likely it was that the state was the high state. Whenever an example random variable was realized in this step-by-step instructional process, an additional example draw was shown in which the example subject got a different draw (in the opposite direction, if applicable). After the instructions concluded, subjects participated in 5 practice matches and were given an opportunity to ask questions before

[4]A copy of the instructions and slides for the session in which the treatment order was (N=2, N=5, N=1) can be found in Appendix D. The final three slides were revealed one bullet point at a time.

and after the practice matches. A summary slide of the important points of the game was left on display at the front of the room throughout the session.

For a given group, matches proceeded in the following way (all actions and random variables were independent across groups). First, the state of the world was determined by the server. Next, each sender received an i.i.d. draw from the appropriate distribution of the new/risky product (their signal). This draw was described to participants as a “clue about the quality of the new product”. Each sender was asked which message from $\mathcal{M}$ they would prefer to send *if they were to send a message*. Senders were then each shown their private cost of sending their chosen message, and asked whether or not they wished to participate.[5] The group's receiver then viewed the average of all the messages that were sent. If no messages were sent, the receiver saw the message “No Messages Were Sent”. Finally, the receiver chose either the new or the old product. If he chose the old product, then he received 100 points. If he chose the new product, then he received a draw from the same conditional distribution of values from which the senders previously drew. Senders' payouts for the match were equal to the receiver's payouts for that match less whatever costs they may have incurred through messaging. Therefore, it was in each sender's best interest to guide the receiver to the best choice.

During the instructions, the game was described in terms of the first treatment, and subjects were told that these conditions would last for 30 matches. Subjects were told that there would be two additional parts of the experiment after the initial 30 matches were completed, but were not immediately informed what those parts would entail. At the conclusion of the first treatment, the experimenter described the change in the number of senders for the second part of the experiment. The summary slide at the front of the room was updated, subjects were allowed to ask questions, and then the second treatment proceeded as did the first. At the conclusion of the second part, this process was repeated. At the conclusion of the session each subject was paid in cash, in private, the sum of his or her earnings over all 90 matches. Subjects earned approximately $22, on average plus a $5 show-up payment, and each session lasted approximately one hour.

[5] The order of these events was meant to enforce the theoretical independence of message selection and costs.

Figures 3.2 and 3.3 display the unique equilibrium messaging and participation strategies, as a function of values, for senders in each treatment when $a^{\Phi} = 0$ and when $a^{\Phi} = 0.5$.[6] For the $N = 1$ case, the displayed strategies are exact. For $N > 1$, the equilibria were computed numerically, and this reflects the fact that the game is more complex with multiple senders. In each of the figures, the black dotted line corresponds to the $N = 5$ case, the red dashed line corresponds to the $N = 2$ case, and the blue solid line corresponds to the $N = 1$ case. In Figures 3.2b and 3.3b, for a given signal, multiple values of the same color indicate indifference between those values; when $N = 1$, senders are indifferent between which message they send, so long as it induces their desired action from the receiver. The $a^{\Phi} = 0$ equilibria specify values for which there exist no cost low enough such that the sender's best response is participation. For these types, the message specified in Figure 3.3b still represents the optimal message, conditional on sending a message. However, that information set will never be reached, in equilibrium.

3.6 Experimental Results

In all three of the treatments, subjects' participation behavior was in line with the theoretically predicted comparative statics, with increasing participation, on average, as sender types moved away from $\hat{v} = 100$. Subjects' messaging strategies were anchored quite strongly to the sincere messaging strategy of sending their signal as their message, even when this was neither an equilibrium strategy or a best response to empirical play. In each of the treatments, receivers' play closely matched that of a cutoff strategy around $t_r = 100$, mixing when receiving $t_r = 100$, and choosing the safe product after receiving the null message.

Tables 3.1 and 3.2 display summary statistics for each treatment. Of the equilibria we computed, empirical play most closely matched the $a^{\Phi} = 0$ equilibria in all treatments. These equilibria are included in the tables for reference.[7] In Table 3.1, "Expected" numbers are the numbers expected

[6]For each treatment, there is also an equilibrium when $a^{\Phi} = 1$. The strategies of this equilibrium are a reflection around $\hat{v}$ of those in $a^{\Phi} = 0$ equilibrium.

[7]In the $N = 1$ and $N = 2$ cases, aggregate messages less than or equal to 100 are not received in equilibrium. $A(t_r) = 0$ was assumed for $t_r \leq 100$ when computing the $a^{\Phi} = 0$ equilibria for all cases. In the $N = 5$ case, aggregate

		% μ^Φ	% participate	avg sender util	avg receiver util	% "correct" selection
$N = 1$	Empirical	49.4	50.6	114.6	119.2	79.8
	Expected	47.6	52.4	108.0	112.3	78.6
	Eq: $a^\Phi = 0$	56.4	43.6	108.9	113.2	80.8
$N = 2$	Empirical	28.2	50.6	120.7	124.6	77.5
	Expected	27.6	50.2	111.4	113.7	81.9
	Eq: $a^\Phi = 0$	57.3	27.4	111.4	113.3	80.9
$N = 5$	Empirical	8.6	45.3	126.7	129.9	85.5
	Expected	7.0	44.2	115.7	116.3	87.9
	Eq: $a^\Phi = 0$	40.7	18.5	113.6	114.2	83.0

Table 3.1: Summary statistics by treatment and compared to equilibrium outcomes

	$t_r < 100$	$t_r = 100$	$t_r > 100$	$t_r = a^\Phi$
$N = 1$	2.2	37.5	99.3	6.7
$N = 2$	1.4	33.3	97.2	8.9
$N = 5$	0.9	14.3	99.5	6.5
Eq: $a^\Phi = 0$	0.0	0.0	100.0	0.0

Table 3.2: Percentage receivers chose the risky action, conditional on binned average message

given the empirical play of the subjects. These numbers diverge slightly from the raw empirical numbers due to an error in the experimental software in which risky values were too high, on average, in the low state. The details of this error are discussed in Appendix E.[8] The final column of Table 3.1 displays the percentage of matches (or the expected percentage of matches, where appropriate) in which the receiver choose the "correct" product, which we define to be the risky product in the high state and the safe product in the low state.

The summary statistics reveal moderate trends across treatments. Expected rates of participation decrease by 8 percentage points as the number of senders increases from one to five; the percentage of "correct" choices increases by 9 percentage points; and expected payouts increase slightly for both senders and receivers. The participation trend is in stark contrast to the trend indicated by our computed equilibria in which participation decreases dramatically as the number of senders increases. We discuss empirical play and its deviation from both equilibrium and best responses to empirical play in the following sub-sections. It is worth noting here that despite the fact that

messages both less than and equal to 100 are received in equilibrium and Table 3.2 accurately reflects the equilibrium rates of play conditional on these aggregate messages.

[8]In the Appendix we conclude that this bug was difficult for subjects to notice, and that there is no evidence that the bug had behavioral consequences.

play is out of equilibrium, average payouts are actually *greater* than average equilibrium payouts in the multiple sender treatments. This is because the higher rates of participation resulted in the receiver choosing the correct product more frequently, which in turn yielded larger receiver payoffs, on average. In the multiple sender treatments these gains in payoffs more than offset the average costs of increased participation. As we discuss in detail below, this does not describe an equilibrium as an individual sender's best response to empirical play is to reduce participation rates for almost every type.

In subsection 3.6.1 we discuss the results from the one sender treatment and in subsection 3.6.2, the results from the multiple senders treatments. In addition to the aggregate analysis discussed below, we analyzed the experimental data for each of the individual sessions, as well as for the subset of the data generated by the final 15 matches of each treatment. These analyses yielded no substantial deviations from the aggregate results discussed below. Thus, there does not appear to be any significant convergence towards equilibrium over time, and the order of treatments (which varied across sessions) do not appear to have affected behavior significantly.

3.6.1 One Sender

There are four main results from this treatment. The first is that empirical participation rates confirm the theoretical expectation expressed in proposition 3.4.3: as values increased beyond $\hat{v} = 100$, players participated more, on average; and as values decreased below $\hat{v} = 100$, players participated more, on average[9]. This can be seen in the black solid line of Figure 3.5b which represents average rates of participation. Figure 3.6 breaks this down into average rates of participation by type (cost and value). While the figure does not reveal strict monotonicity, there is a clear upward trend in willingness to pay as subjects' values move away from $\hat{v} = 100$.

The second striking aspect of the data is subjects' apparent adherence to sincerity as a messaging strategy. Of the 546 total sent messages 500 (92%) were equal to the sender's signal. Average messages as a function of senders' signals are contrasted with the best response correspondence in

[9] The only exception was that players participated less at $v_i = 50$ than at $v_i = 75$, however, these rates are not statistically significantly different.

Figure 3.5a. In addition, Figure 3.6 displays the frequency with which each message was sent as a function of the sender's signal. Recall that in this treatment, the optimal messaging strategy is directional; any message that results in the sender's desired outcome is equally good in expectation. This is true even when empirical play is out of equilibrium, and means that there is no theoretical reason we should expect an adherence to sincerity here. Another trend apparent from Figure 3.6 is that deviations from sincerity appear to be predominantly towards more extreme numbers in the same direction from $\hat{v}$ as the sender's value (e.g., a value of 150 and a message of 175). We refer to this behavior as exaggeration of the sender's signal. Of the 46 insincere messages, 42 (91%) were exaggerations.

Third, receiver behavior is almost exactly as if subjects played a cutoff strategy around $\hat{v} = 100$. Figure 3.4 displays receivers' average actions in this treatment. The black, solid line is the frequency with which receivers choose the risky product, conditional on the aggregate message (error bars are standard errors). The orange, dashed line is the Bayesian posterior likelihood of the high state, conditional on aggregate message and given empirical play. As is clear from the figure, the posterior dictates that subjects choose the safe product upon receipt of a message below 100 and the risky product if they receive a message above 100, and this is almost precisely how subjects played. Of note is that receivers chose the risky product only 6.7% of the time after receiving the null message. While not an equilibrium response, this rate is near 0, and therefore dictates that sender's best responses will be similar to the equilibrium strategies specified by the $a^{\Phi} = 0$ case.

Thus, the final main result concerns equilibrium selection. Summary statistics from the one-sender treatments are listed in Table 3.1. From this table, it appears that subjects' behavior had results similar to what we'd expect from the $a^{\Phi} = 0$ equilibrium. This appearance is made more dramatic by Figure 3.5b, which displays senders' average rates of participation as a function of their signal. In the figure, the equilibrium participation rate is the dashed orange line; the best response to empirical play participation rate is represented by the blue dotted line; and the empirical rates of play are represented by the black solid line with standard errors shown as error bars. Because the $a^{\Phi} = 0$ equilibrium specifies that players with values below 125 never send a signal, we can say conclusively that players are not playing the $a^\Phi = 0$ equilibrium. However, requiring adherence to strict abstention may be overly stringent considering the non-zero likelihood of human error. When we consider only types that, in the $a^\Phi = 0$ equilibrium, participate at a strictly positive rate, we cannot reject the hypothesis that empirical rates of participation follow the $a^\Phi = 0$ equilibrium distribution (Pearson's cumulative test statistic of 1.39 with 2 degrees of freedom yields a p-value of 0.50). Figure 3.18a shows the empirical distribution function of subjects' participation errors when compared to the best response to empirical play. It shows that most subjects do very well, with 75% of subjects making participation errors no more than 15% of the time, and 20% of subjects responding to empirical play perfectly.

Thus, although subjects did not follow equilibrium strategies on average, they were, in a sense, close. This closeness is perhaps more striking when we consider that there are two other equilibria of this game, one of which, the equilibrium when $a^\Phi = 1$, is equivalent to the $a^\Phi = 0$ equilibrium in terms of the distribution of equilibrium payouts. There is no clear reason, therefore, that subjects would select the $a^\Phi = 0$ equilibrium.

3.6.2 Two and Five Senders

If the most impressive feature of the one-sender case was subjects' coordination near one of the game's equilibria, the most impressive feature of the multiple-senders case may be their complete failure to replicate the feat. While we are not certain, as we were with only one sender, that there are only three equilibria of these games, subjects' best response to empirical play was significantly different from both empirical and theoretical play, sometimes drastically so.

3.6.2.1 Receivers

Receivers, as in the single-sender treatment, acted largely as though they were playing a cutoff strategy around $t_r = 100$.[10] The rates at which receivers chose the risky product as a function of the average message are displayed as a black solid line in Figure 3.7. The orange dashed line

[10]The notable exception occurred in the two-sender treatment in which players chose the risky product only 4 of the 7 times 112.5 was the average message.

	% Sincere	% Exaggeration	% Insincere and Exaggeration
$N = 1$	91.6	7.7	93.3
$N = 2$	84.4	15.0	95.6
$N = 5$	77.1	21.3	95.1

Table 3.3: Experimental Messaging Statistics

represents the likelihood that the state was the high state, conditional on the average message and given empirical play. The rate at which receiver's chose the risky product conditional on the null message remained relatively constant, and low, across treatments. These rates across treatments are displayed in Table 3.2.

3.6.2.2 Messaging

Senders in the two- and five-sender treatments continued to anchor strongly to sincere messaging. Figure 3.8 displays the frequency of each message as a function of player's types in each treatment. In the two-sender treatment, 615 of the 729 total sent messages (84%) were equivalent to the subject's signal (they were sincere), and in the five-sender treatment, 629 of the 816 total sent messages (77%) were sincere. Despite both treatments maintaining high levels of sincerity, there is a clear downward trend (recall that 92% of messages were sincere in the one-sender treatment). Table 3.3 organizes this data, and presents a second notable trend: over 90% of all insincere messages that are sent are exaggerations of the sender's signal; this translates to 15 and 21 percent of all sent messages in the two- and five-sender treatments, respectively; up from 8% in the one-sender case.

Subjects' proclivity towards sincere and exaggerated messages does not appear to be related to their best response in any systematic way. Figure 3.9 displays both senders' average message as a function of their value (in black), as well as their best response as a function of their value (in blue). In the two-sender case, the optimal message given empirical play ranges from exaggeration for $v_i = 25$ or $v_i = 175$ to sincerity everywhere else (excluding at the neutral point, as is discussed further below). However, subjects played a mix of sincere and exaggerated messages for *every* value in the type space; this is made clear in Figure 3.8. The best response to empirical play in the five-sender treatment is more confounding still. The best response correspondence specifies that the sender exaggerate his signal for values below 100, but *dampen* (weakly) his messages for values above 100. Therefore, for this latter class of types, the best response to the data is to send signals in the opposite direction from sincerity than those exhibited in the patterns we see in the data.

Figure 3.10 breaks down the expected utility of participating as a function of sent message, conditional on the sender's type. This figure indicates that in both treatments, for a given type, messages with the same sign (where we are using sign to describe if the message is greater than or less than 100), produce very similar expected utilities. A theory of subject errors across decisions with similar outcomes could therefore help to explain a uniform distribution of subjects' messages across messages with the same sign. However, the data show a clear bias towards both sincere and extreme messages, and almost no deviations away from sincerity towards $\hat{v}$, even when such a deviation was associated with a higher expected utility.

At a glance, the best response messaging correspondences may appear counter-intuitive. To help with intuition, we decompose the expected utility of sending each message, conditional on the high state (top panels) and conditional on the low state (bottom panels), into their weighted event probability parts in Figures 3.11 and 3.12.[11] For example, the top panel of Figure 3.11 reveals that, conditional on the high state, a sender's expected utility is composed predominantly of two weighted likelihoods: the likelihood that the receiver chooses the risky product regardless of the sender's action (HH), and the likelihood that the sender sends the sole message which induces the receiver to choose the risky product (ΦH). Decomposing the utility conditional on the state is useful because the expected utility of every type of sender (the distinct lines in Figure 3.10) are only distinguished by the amount that they weight each of the state-conditional expected utilities. For example, a sender of type $v_i = 100$ has no information about the state of the world, and so weights the expected utilities (as well as the event probabilities) in the top and bottom panels equally.

Because of their uniform posterior, considering a sender of type $v_i = 100$ is particularly informative. Contrast, in the two-sender treatment, a sender of type $v_i = 100$'s expected utility when changing his message from $m_i = 75$ to $m_i = 125$. The sender nets a gain of 0.76 expected utility in

[11] Note that for the economy of space in these figures, the weighting function has been adapted to also include the expected value of the risky product. Thus, $W(H) = 1 * E[v_r|\theta]$.

the HH event (magenta dashed line in Figure 3.11). This means that by increasing his message to 125 the sender increases the likelihood that the message he sends will not overturn a high message sent by another sender, and that this is more likely to happen in the high state. However, the change from $m_i = 75$ to $m_i = 125$ also nets the sender a loss of 15.93 expected utility in the ΦH event (cyan dashed line in Figure 3.11). This means that by increasing his message to 125, the sender increases the likelihood that his message will induce the receiver to choose the risky product when he would have otherwise received the null message, and that this is far more likely to happen in the low state (from which we can infer that abstention is far more likely in the low state). Taking both of these changes to expected utility under consideration, it is clear that the sender would be better off by choosing $m_i = 75$ than she would by choosing $m_i = 125$. There are, of course, other weighted event likelihoods that enter into the senders' calculus, but the best response message-selection problem is essentially the same for all types. The only difference between types of sender is the amount they weight the high and low states (and this amount is dictated by their posterior). For example, as is evident in the figure, when $v_i = 175$ the sender's posterior is so strong towards the high state that the small gains of choosing a large message (e.g., $m_i = 200$) associated with the high state outweigh the larger losses associated with the low state.

The result of these calculations has an even more surprising result in the five-sender treatment: the best response message of a sender of type $v_i = 125$ is $m_i = 100$. That is, one type of sender with a value indicating that the high state is more likely, chooses the neutral message. The reasoning is the same as before, and is clear to see in Figure 3.12. Choosing $m_1 = 125$ instead of $m_i = 100$ is associated with a very small gain in expected utility in the high state and a much higher reduction in expected utility in the low state. Since players of this type have a relatively weak posterior, the low state is weighted only slightly less than the high state in his expected utility calculation. As a result, the neutral message is more attractive to the sender than is a message with the same sign as his type.

Another way to think of this calculations is the following: the state of the world generates actions of other players which generate events. Since these events occur at different rates in different

	Ind. Var.	Coefficient	Robust Std. Error	z	$Pr > \|z\|$
$N = 1$	α	1.16	0.62	1.88	0.061
	$v_i \leq 100$	-0.24	0.11	-2.25	0.024
	Log Pseudolikelihood= -200.7, Pseudo $R^2 = 0.26$, Obs= 596				
$N = 2$	α	0.20	0.51	0.38	0.700
	$v_i \leq 100$	-0.29	0.09	-3.10	0.002
	Log Pseudolikelihood= -270.4, Pseudo $R^2 = 0.34$, Obs= 716				
$N = 5$	α	1.14	0.44	2.57	0.010
	$v_i \leq 100$	-0.21	0.10	-2.13	0.033
	Log Pseudolikelihood= -330.8, Pseudo $R^2 = 0.37$, Obs= 716				

Table 3.4: Probit Estimations of Participation as Functions of Signals Less Than 100. Cost dummy variables and sequencing explanatory variables are suppressed. Standard errors clustered by subject. Values coded as number of steps away from 100 (e.g., $v_i = 75$ is coded as 1 and $v_i = 25$ as 3.

states of the world, when an event occurs it carries information about the state of the world. For example, conditional on all other senders abstaining, the low state is more likely than is the high state ($Pr(\theta^l|\Phi) > Pr(\theta^h|\Phi)$). Thus, since a player of type $v_i = 125$ finds the neutral message ($m_i = 100$) to be a better choice than a message with the same sign as his type (e.g., $m_i = 125$), we can say that the information in the event likelihoods was sufficient to overturn the player's posterior. In this case, the sender was so much more likely to affect the outcome with a message greater than 100 in the low state than he was in the high state, that contingent on both Φ and his type, his posterior was approximately 0.5 for each state.

3.6.2.3 Participation

Each of the two- and five-sender treatments support the extreme participation conjecture. Figure 3.13 shows participation rates for each type while Figure 3.14 presents the participation rates for each value, aggregated across costs. As was the case in the one-sender treatment, Figure 3.13 does not show strict monotonicity across values, but there are clear positive trends in participation as sender's values move away from 100. For the two-sender treatment, Figure 3.14 reveals that subject average participation was strictly increasing in the extremism of player's values. This is also true of the five-sender treatment excluding the fact that players participated slightly less when they were of type $v_i = 25$ than they did $v_i = 50$. The difference in participation rates is not significantly different however. Finally, Tables 3.4 and 3.5 presents data from a probit regressions, all of which

	Ind. Var.	Coefficient	Robust Std. Error	z	$Pr > \|z\|$
$N = 1$	α	1.33	0.70	1.90	0.058
	$v_i \geq 100$	1.38	0.24	5.81	0.000
	Log Pseudolikelihood= -127.1, Pseudo $R^2 = 0.50$, Obs= 549				
$N = 2$	α	-0.12	0.44	-0.27	0.785
	$v_i \geq 100$	1.06	0.13	8.12	0.000
	Log Pseudolikelihood= -322.6, Pseudo $R^2 = 0.37$, Obs= 817				
$N = 5$	α	0.94	0.50	1.87	0.062
	$v_i \geq 100$	0.48	0.08	5.68	0.000
	Log Pseudolikelihood= -536.7, Pseudo $R^2 = 0.26$, Obs= 1061				

Table 3.5: Probit Estimations of Participation as Functions of Signals Greater Than 100. Cost dummy variables and sequencing explanatory variables are suppressed. Standard errors clustered by subject. Values coded as number of steps away from 100 (e.g., $v_i = 125$ is coded as 1 and $v_i = 175$ as 3)

confirm that participation decreases in senders' values when values are less than 100, and increase in sender's values when values are greater than 100.

Table 3.5 and Figures 3.5 and 3.14 also suggest another trend: the magnitude of the extreme participation effect appears to decrease as the number of senders increases. The regression coefficient on a sender's value decreases from 1.38 in the one-sender treatment to 1.06 in the two-sender treatment and 0.48 in the five-sender treatment.

Figure 3.14 also displays subject's best response to empirical play (blue, dotted line), and the $a^\Phi = 0$ equilibrium participation rate (orange, dashed line). While the $a^\Phi = 0$ equilibrium is displayed, we are not certain, as we were in the one-sender case, that it is the nearest possible equilibrium. However, we can, as before, compare players' best response functions, given empirical play, to empirical rates of participation. If subjects were in, or near, equilibrium, then the best response should match or approximate the empirical rates of participation. However, as is clear from the figure, neither the best response nor the $a^\Phi = 0$ equilibrium participation rates come close to approximating empirical rates of play.[12]

The best response participation functions of both the two- and five-sender treatments exhibit a striking attribute: there exist types with non-uniform posteriors whose best response is to abstain,

[12]In the two-sender treatments, for values less than 100, the best response participation function tracks empirical participation rates well, but approximately 10 points too high at each value. This data alone suggests again that deviations from equilibrium may in part be due to subjects' errors since at low levels of participation, errors are more likely to occur in the direction of over-participation. However, this close tracking and constant error rate does not persist for values greater than 100 or on either half of the domain for the five-sender treatment.

no matter their cost. In the five-sender treatment this is so pronounced that for any type with a value greater than 100 other than $(v_i = 175, c_i = 1)$, the best response is to abstain. This abstention is different in kind than the abstention we observe in the $a^{\Phi} = 0$ equilibria. In those equilibria, abstention is the best response for certain type because the player's most preferred outcome was chosen by the receiver when the receiver obtained the null message. In these best response correspondences abstention is the best response for two reasons. The first is that the expected utility of abstention is high. Since other players are participating at such high rates, and since all players are equally well informed, a sender can expect with high probability that other senders will send sufficient information for the receiver to make the correct choice. This intuition is reflected in Figure 3.15 which displays sender's expected utility of abstention conditional on each value-type. In order to match the domain of Figure 3.10, these expected utilities are presented as functions of the sent message, and are therefore constant. This is a useful construction when we discuss net expected utility below.

There are a number of points of interest in Figure 3.10. The first is that in the worst case scenario, the expected utility of abstention is far above the expected value of the risky product in the low state. This reflects the fact the in the low state it is likely that another player will send a message, inducing the receiver to choose the safe product and therefore receive 100 points. The second is that the expected value of abstention is higher for every type in the five-sender treatment than it is in the two-sender treatment. This reflects the fact that the receiver makes the correct choice more often (in terms of expected value) in the five-sender treatment. Third, players with type $v_i = 175$ in the five-sender treatment have an expected utility of abstention near the theoretical upper bound. This means that *by abstaining* a sender of this type can expect to attain a level of utility near what he would obtain if he knew with certainty that the receiver would draw from the risky product in the high state.

The second reason abstention is a best response for these types is that participation does not yield senders high returns. As we discussed in the previous subsection, the events in which these types' messages affect their expected utility are far more likely in the low state of the world, which is the opposite of the state indicated by their posterior given just their type. Thus, although players of these types believe that the high state is more likely conditional on their type, they also know that their message is much more likely to be effective in the low state, and therefore sending their preferred message is not highly valuable to them.

The net expected value of participation given empirical play is found by subtracting the expected utility given abstention functions found in Figure 3.15 from the expected utility given participation functions found in Figure 3.10. The result for the two-sender treatment is displayed in Figure 3.16, and the result for the five-sender treatment is displayed in Figure 3.17. In both figures the feasible costs of the experiment are displayed as dotted horizontal lines.

We search for heterogeneity in subjects' participation strategies by examining the rates at which each subject committed participation errors (either participating when they should have abstained or abstaining when they should have participated) compared to the best response to empirical play participation strategy. Figure 3.18 presents these rates as empirical distribution functions (EDFs) for each treatment. Error rates were highest (in terms of first order stochastic domination) in the five sender treatment and lowest in the one sender treatment. While the figures do reveal substantial heterogeneity in subjects' abilities to respond to empirical play, the large average diversion from best response is not generated by a small set of very bad actors. Only in the one sender case did a substantial portion of subjects act while making few errors (75% of subjects made errors fewer than 15% of the time; this is compared to 20% in the two sender treatment and 13% in the five sender case). In contrast, many subjects in the multiple-senders treatments made frequent errors. In the five sender treatment, more than 20% of subjects did worse than chance, committing participation errors more than half of the time.

3.7 Discussion and Conclusion

Significant evidence suggests that consumer generated product ratings produce an abundance of ratings at the extremes of the rating space. Theoretical analysis suggests that this pattern must at least in part be due to selective participation; in equilibrium consumers who feel more strongly about a product, good or bad, are more likely to wish to submit a rating. Like most signaling games, the theoretical environment admits the possibility of many equilibria, and the theoretical literature has therefore been inconclusive thus far about the equilibrium rating strategies or comparative statics. In this paper we provide an experimental test of this theory in a simplified consumer product rating environment. The use of laboratory experiments allows us to control for key parameters in the raters' decision calculations which would not be possible to measure or control in a natural environment, including the number of product raters, subjects' signals of product quality, and their costs of rating generation. We find strong support for the theoretically predicted participation comparative statics, as well as for the exaggeration conjecture that ratings tend to be exaggerations of true evaluations. In all treatments, the more extreme (beyond a central, information-less type) their signal of product quality, subjects in a sender role were more likely to send a costly message to subjects in a receiver role. This result is clear and significant in all treatments. Moreover, subjects in the sender role appear to be strongly anchored to sincere messaging, sending their signal as a message. However, almost all deviations from sincere messaging, over 90% in all treatments, are in the direction of exaggerating their signal away from the central, neutral message.

Subjects were remarkable both for their coordination on equilibrium, and for their failure to coordinate on equilibrium. In all treatments subject-receivers' aggregate rates of play tracked very closely to that which would be generated by a cutoff strategy around a central aggregate message, and in which, $a^{\Phi} \in [6.5\%, 9.0\%)$. In the one-sender treatment, we showed theoretically that there are essentially three equilibria, each described sufficiently by a^{Φ}. The three possible equilibrium rates are 0, 1, and 0.5, one each for whether the receiver chooses the safe product, the risky product, or mixes evenly between the two upon receipt of the null message. In the experiment, subject-senders' rates of participation and messaging strategies were indistinguishable from those of the $a^{\Phi} = 0$ equilibrium, when comparing types for whom equilibrium rates of play were positive. Thus, while subjects were not precisely at equilibrium, they were close. This coordination is all the more striking considering there were two other equilibria of this game, one of which, the equilibrium when $a^{\Phi} = 1$, was equivalent to the $a^{\Phi} = 0$ equilibrium in terms of the distribution of equilibrium payouts.

In contrast, in the multiple-senders treatments, senders did not coordinate on an equilibrium. Unlike the one-sender case, we were not certain of a sparse set of equilibria, or which equilibria existed outside of $a^{\Phi} \in \{0, 1, 0.5\}$. Despite this, we were able to compare best response strategies to empirical rates of play, and found that these were statistically divergent for almost every type of sender. Subjects' messaging behavior was particularly difficult to explain. Subjects anchored strongly to sincere messaging, even when, as was the case in the five-sender treatment, sincerity was not a best response for almost any types. In fact, in the five-sender treatment, for two types the best response was to *dampen* (towards the central message, away from sincerity) their signal. Subjects of those two types played their optimal, dampened, messages exactly 0 times (given 354 opportunities, resulting in 193 suboptimal sent messages for these types alone). Moreover, the proportion of exaggerated messages trends clearly upwards as the number of senders increases; from 8% of all messages in the one-sender treatment to 21% of all messages in the five-sender treatment. Therefore, unexpectedly, and although it may be an out of equilibrium response, we find that the experimental data appear to confirm the exaggeration conjecture, and to provide it with an unexpected addendum: senders appear to be more likely to send an exaggerated message the more senders there are. It is unclear how this trend would develop with the addition of more senders, and further research is needed if we are to answer this question.

The reasons for subjects' deviations from their optimal messages are not clear from the data, but we suspect at least two contributing factors. First, expected utilities as a function of sent messages are relatively flat. This does not explain subjects' coordination on only sincere or exaggerated messages, but it does say that they are not sacrificing much in terms of expected utility by choosing those particular suboptimal messages. Second, the data is consistent with subjects improperly estimating the relative likelihoods of events in the respective states. Deviations in the relative likelihoods of events are the primary factor in shaping best response correspondences. For example, as discussed in the text, in the five-sender treatment, the best response message for a sender of type $v_i = 125$ (a type that indicates weakly that the state is the high state) was the neutral message, 100, because the likelihood that all other senders abstained was so much higher in the low state than it was in the high state. This means that although the sender's posterior belief, conditional on his type, was that the high state was more likely, his message was much more likely to be effective in the low state. These effects worked in opposite directions and with approximately equal magnitude, resulting in the best response for senders of this type being to abstain, and, if they found themselves at the participation information set, their best response was to send a message that would not affect the outcome. That this inference problem was a source of subject errors is corroborated by subjects' stellar performance and coordination on equilibrium in the one-sender treatment. In that treatment confusion about the relative likelihood of events is unlikely since there was only one feasible event (the event in which the sender was the only sender) and that event occurred with certainty in both states of the world.

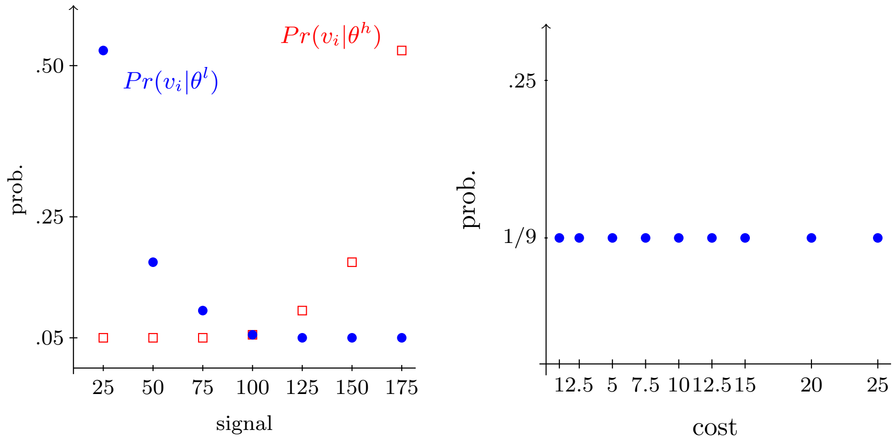
Choosing a message therefore represents a complex inference problem, and we would not be surprised if some fraction of the subjects failed to comprehend it. Such a failure is the essential intuition behind Eyster and Rabin's "cursed equilibrium" (2005), which says that people do not properly update their beliefs about the state of the world conditional on the actions of other players. We believe, therefore, that the cursed equilibrium concept could prove fruitful in explaining subject behavior in our experiments, and exploring the implications of cursed equilibrium in our model and how it might help to explain our data is a clear priority for future work.

In terms of participation, we found support for the extreme participation conjecture in both the two- and five-sender treatments. Although this *comparative static* conforms to equilibrium comparative statics, as with messaging behavior, subjects failed to best respond to the actions of other subjects. Conditioning on their signal, subjects over-participated compared to their best response for 13 of 14 total possible value-types in the two- and five-sender treatments. Moreover, while deviations from optimal play with respect to message selection could perhaps be rationalized by small differences in expected utility, deviations from optimal play with respect to participation resulted in large deviations in subjects' expected payoffs. In the five-sender treatment, over-participation was so pronounced that for types whose posterior indicated the high state, *the best response for every type*, excluding the most informative signal paired with the lowest possible cost, *was to abstain.*

Because an optimal participation decision requires a comprehension of the messaging decision, participation too represents a complex inference problem. We therefore suspect that the cursed equilibrium concept may help explain subject participation behavior in this experiment. In addition, over-participation could be, at least in part, due to subjects' errors. The one-sender data tells this story fairly clearly; although subjects very nearly play the $a^{\Phi} = 0$ equilibrium, since equilibrium rates of participation are 0 for many types, subjects can only err in the positive direction, resulting in over-participation. A theory that incorporates subject errors could therefore help to explain subject behavior. We find it unlikely that over-participation could be explained by a simple preference for the act of messaging. This is because the rate of participation varies considerably across the value-type domain. Indeed, in each of our treatments, probit estimates of the coefficient on senders' values were significantly different than zero.

The normative implications of this study as it applies to rating environments depends on how one reads the validity of our assumptions. We motivated subjects by paying them based on the performance of future consumers. We did this first to parallel the previous theoretical literature which hypothesizes altruism as the primary motivator in rating environments. Moreover, it was not the goal of this paper to prove or disprove the existence of altruism in the lab, especially since, without generating a new motivation for action, actions that could be interpreted as altruism could be explained by many other factors such as subject-boredom or as an experimenter demand effect. By enforcing a high level of "altruism" in our experiment, we were able to focus on the many other aspects of subjects' behavior in an environment in which altruism is assumed to exist. If we accept that this assumption approximates the *form* of raters' motivations in the field (and not necessarily the correct magnitude or the correct distribution of types), then we may attempt to generalize our results to field product rating environments.

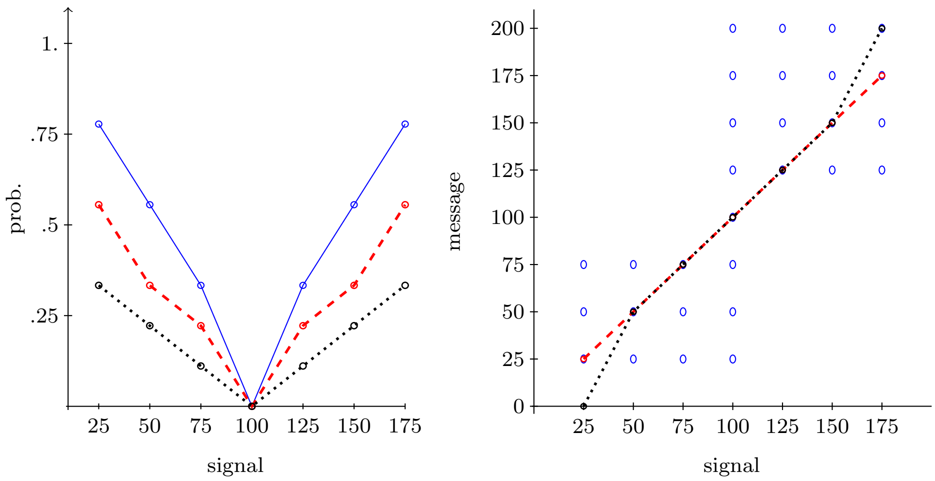
In that case, the most basic lesson we have learned, is perhaps the most important: on average, rating environments are utility improving mechanisms. Despite subjects taking out of equilibrium actions, expected utilities increased in the presence of the product rating mechanism for both senders and receiver in all treatments. Moreover, subjects' expected utility and the expected rate at which receivers choose the correct product (where correct is judged to be the risky product in the high state and the safe product in the low state) are both increasing in the number of senders, while the rate of individual participation is decreasing. This means that the more senders there are, the more information that is conveyed, despite the fact that individuals send ratings less often, on average. This could be very good news if we extrapolate to the multitude of potential raters found in the field, but whether or not these trends continue as the number of senders increases beyond five is a question for future research.



a) Experimental densities of values, conditional on the state

b) The density of costs used in the laboratory experiments

Figure 3.1: Experimental type densities



a) The probability of participation, conditional on a sender's signal

b) Symmetric messaging strategy. Polygons represent indifference.

Figure 3.2: Equilibrium sender strategies, by treatment, when $a^{\Phi} = 0.5$. For a given signal, multiple values of the same color indicate indifference between those values.

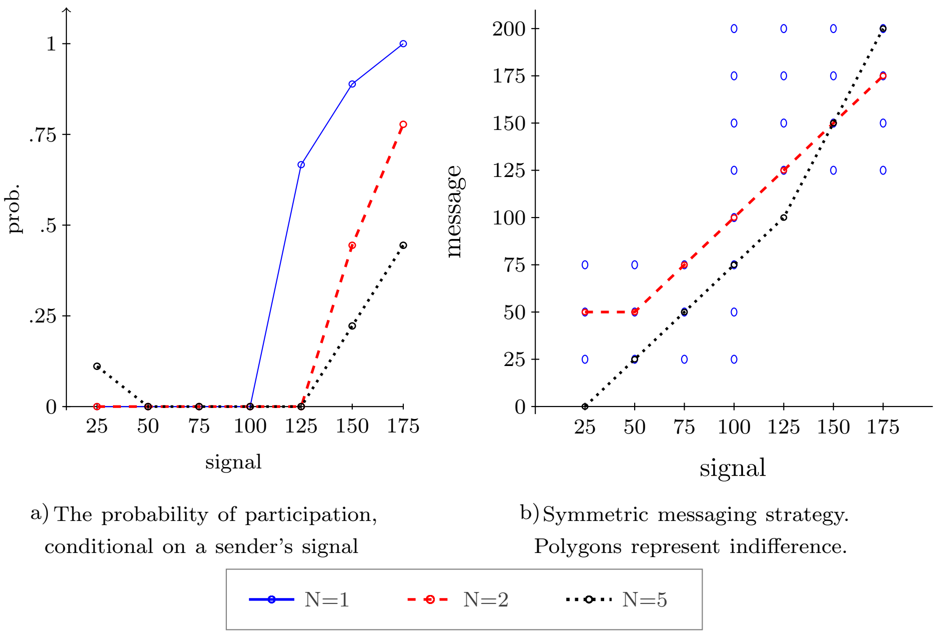


Figure 3.3: Equilibrium sender strategies, by treatment, when $a^{\Phi} = 0$. For a given signal, multiple values of the same color indicate indifference between those values.

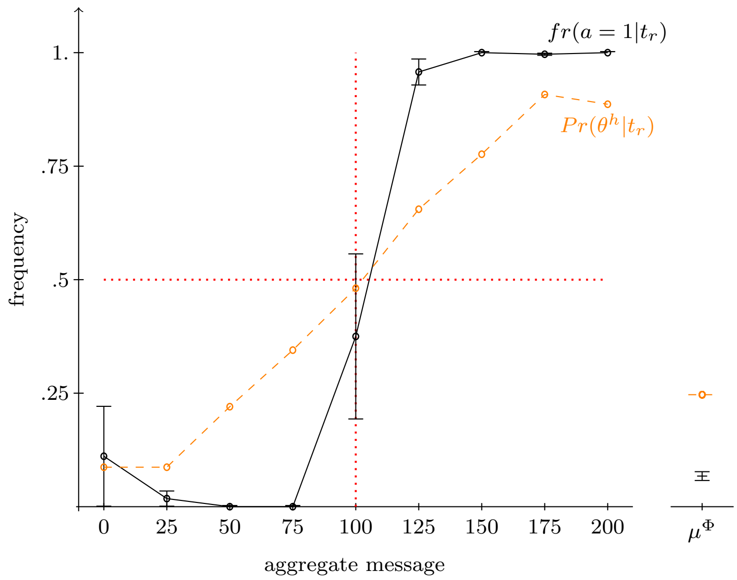


Figure 3.4: $N = 1$ experimental data. Frequency receivers choose the risky product conditional on aggregate message (error bars are SE's), versus the empirical likelihood of the high state, conditional on aggregate message and given empirical play.

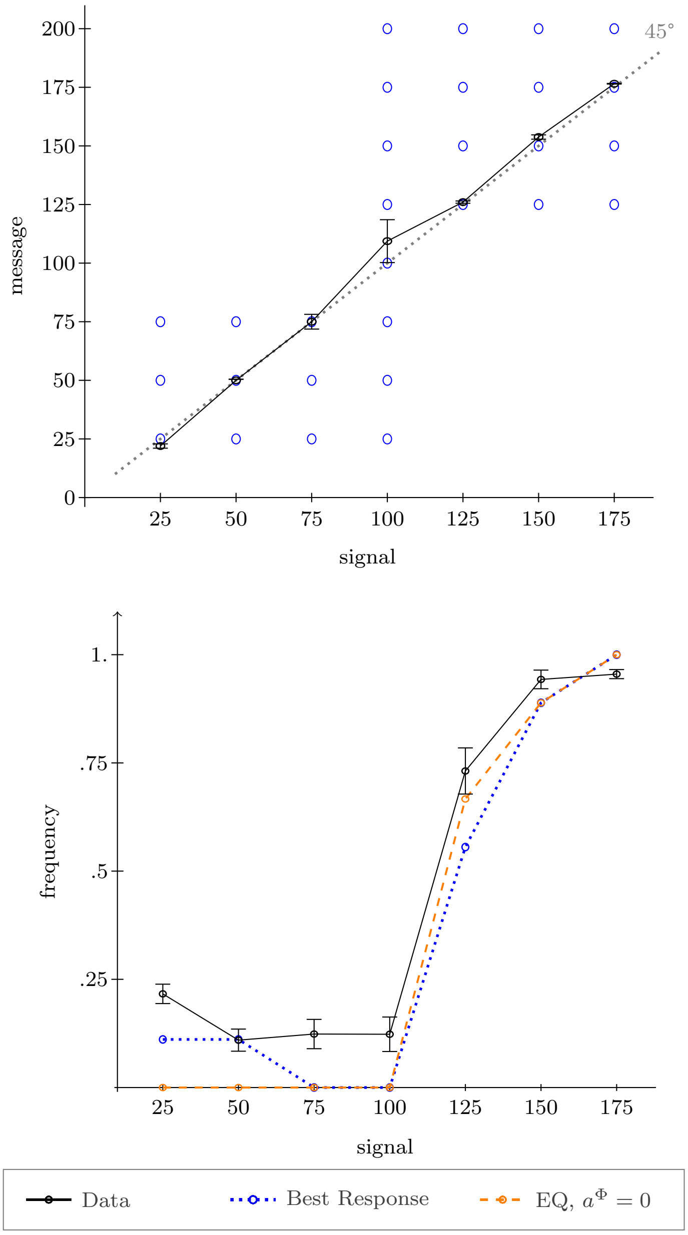


Figure 3.5: $N = 1$, average message by type (above) and average rate of participation by signal (below). For a given signal, multiple values of the same color indicate indifference between those values.

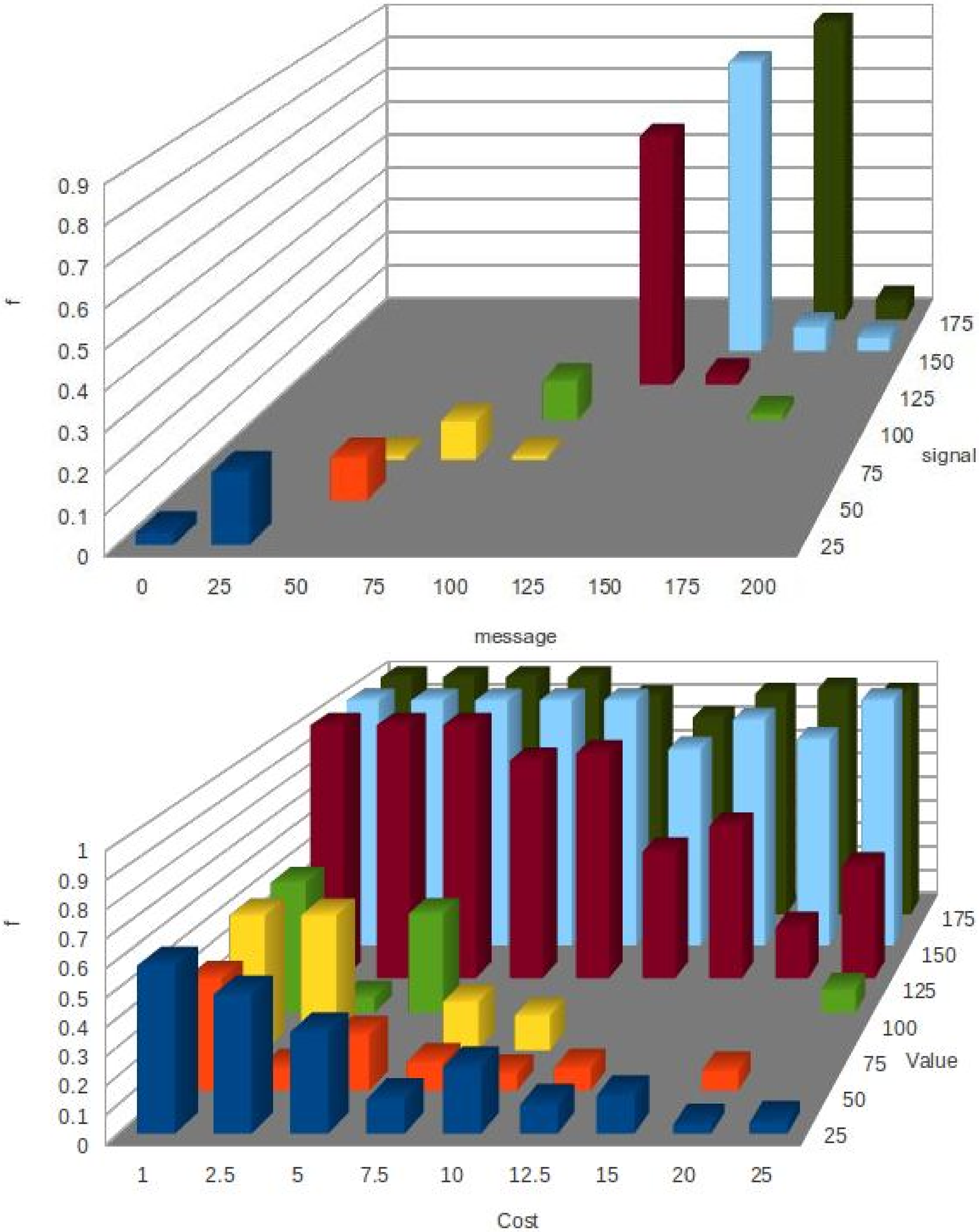


Figure 3.6: $N = 1$ Message frequency by signal (above), and participation rates by signal and cost (below)

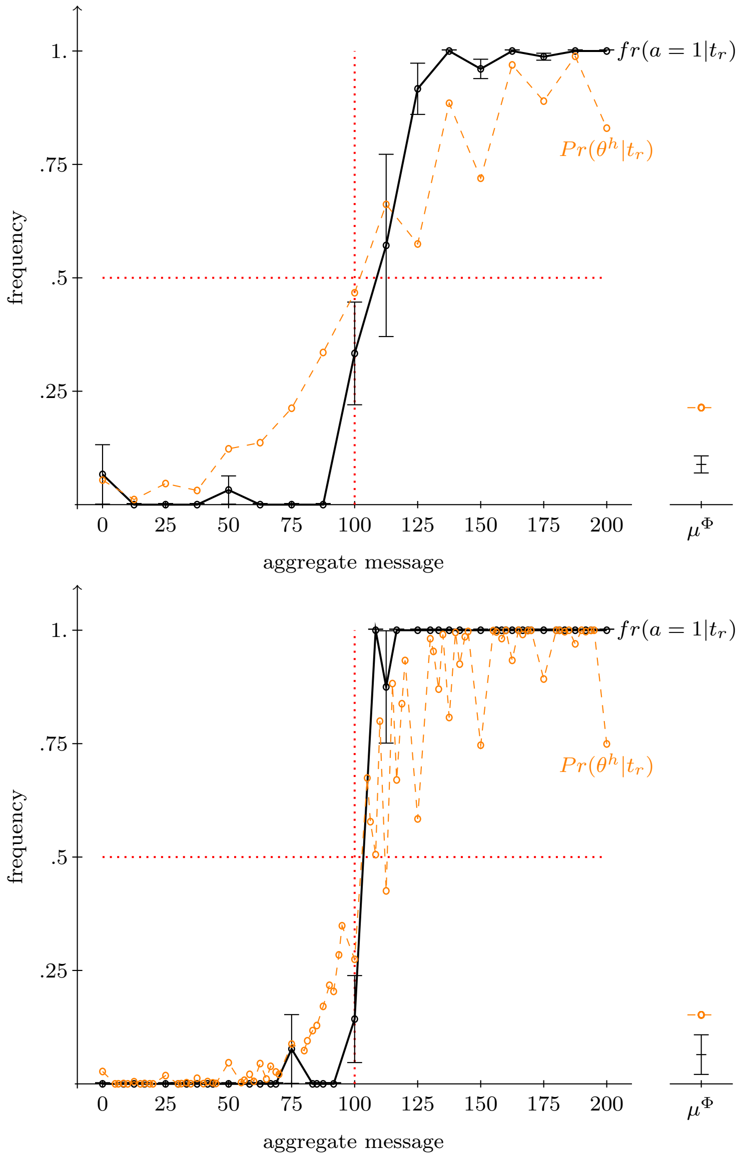


Figure 3.7: $N = 2$ (Above), $N = 5$ (Below) experimental data. Frequency receivers choose the risky product conditional on aggregate message (error bars are SE's), versus the Bayesian posterior likelihood of the high state, conditional on aggregate message and given empirical play.

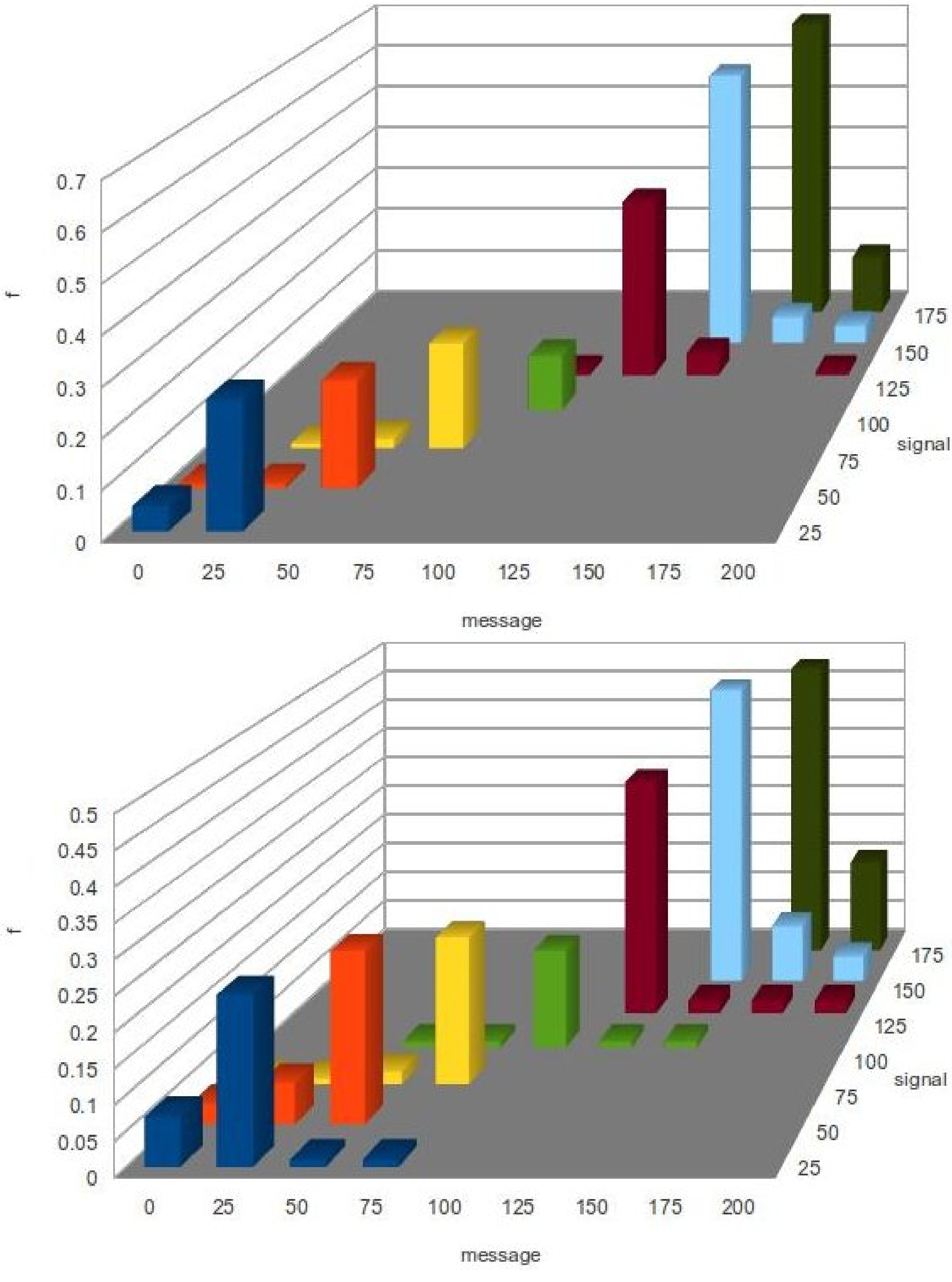


Figure 3.8: $N = 2$ (Above), $N = 5$ (Below) message frequency by signal

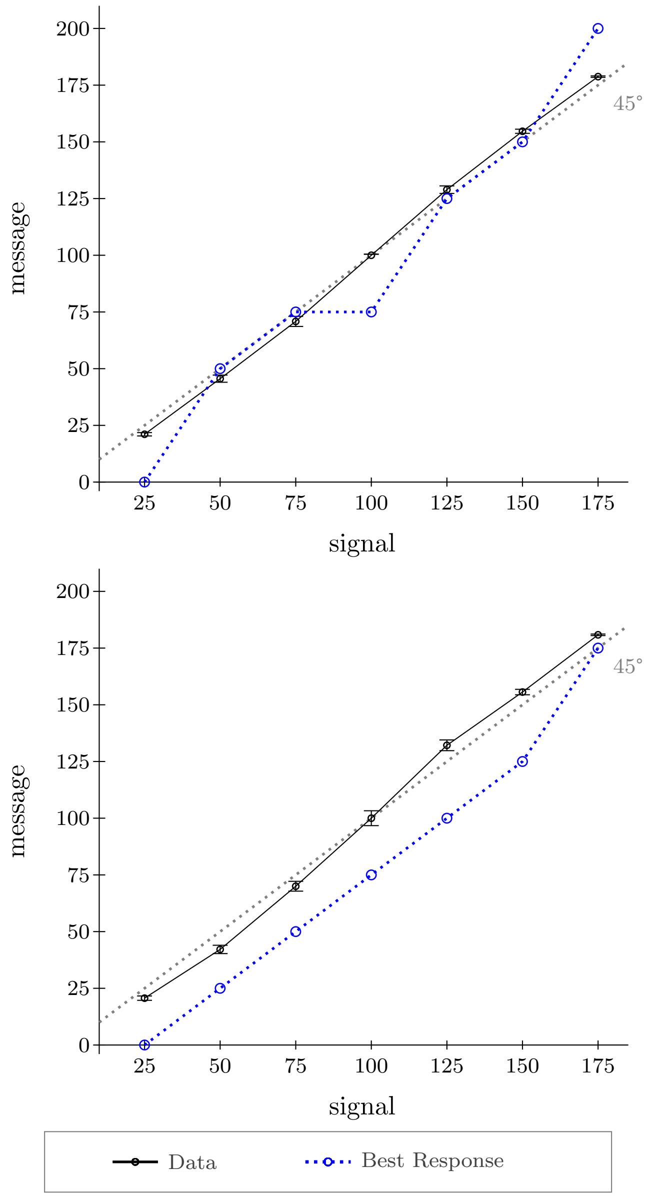


Figure 3.9: $N = 2$ (Above), $N = 5$ (Below) messaging behavior and best response to empirical play messages

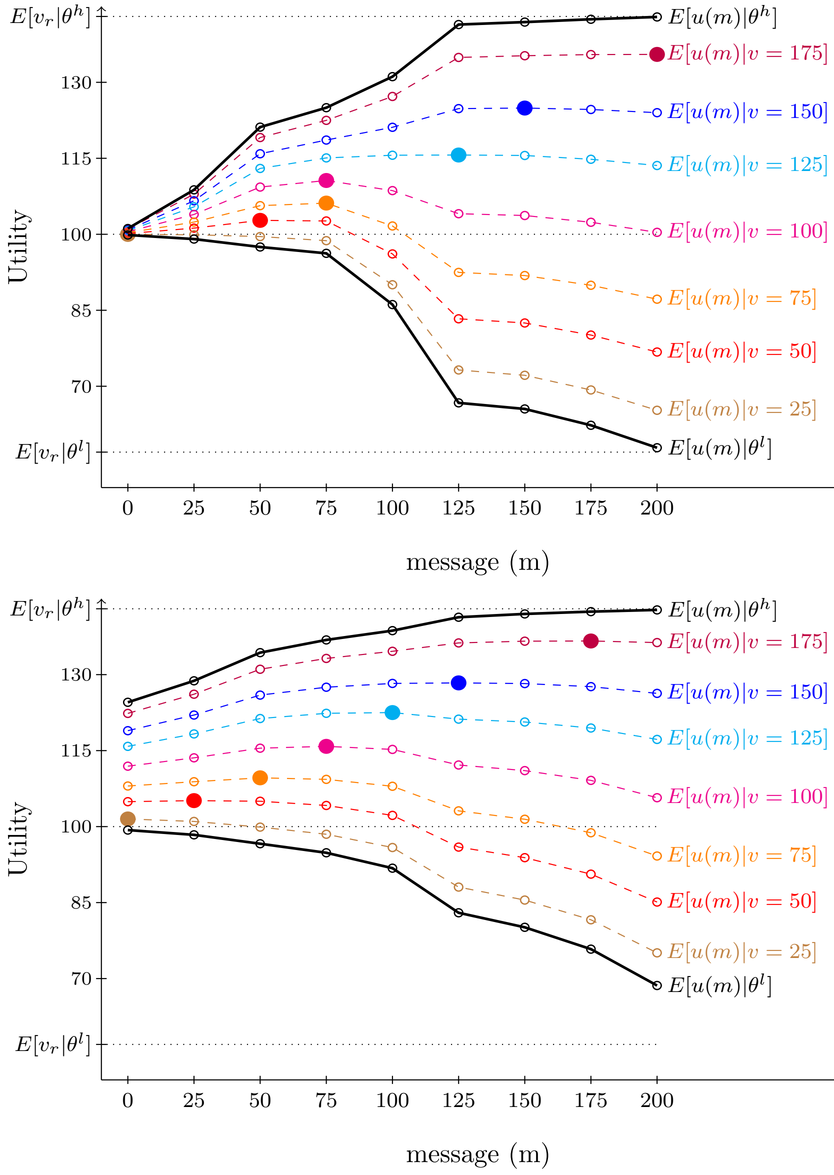


Figure 3.10: $N = 2$ (Above), $N = 5$ (Below). Expected utility as a function of sending each message, conditional on a sender's value, and given empirical play. Solid black lines represent expected utility of sending a message given certainty of the respective states. Best response messages are marked by filled circles.

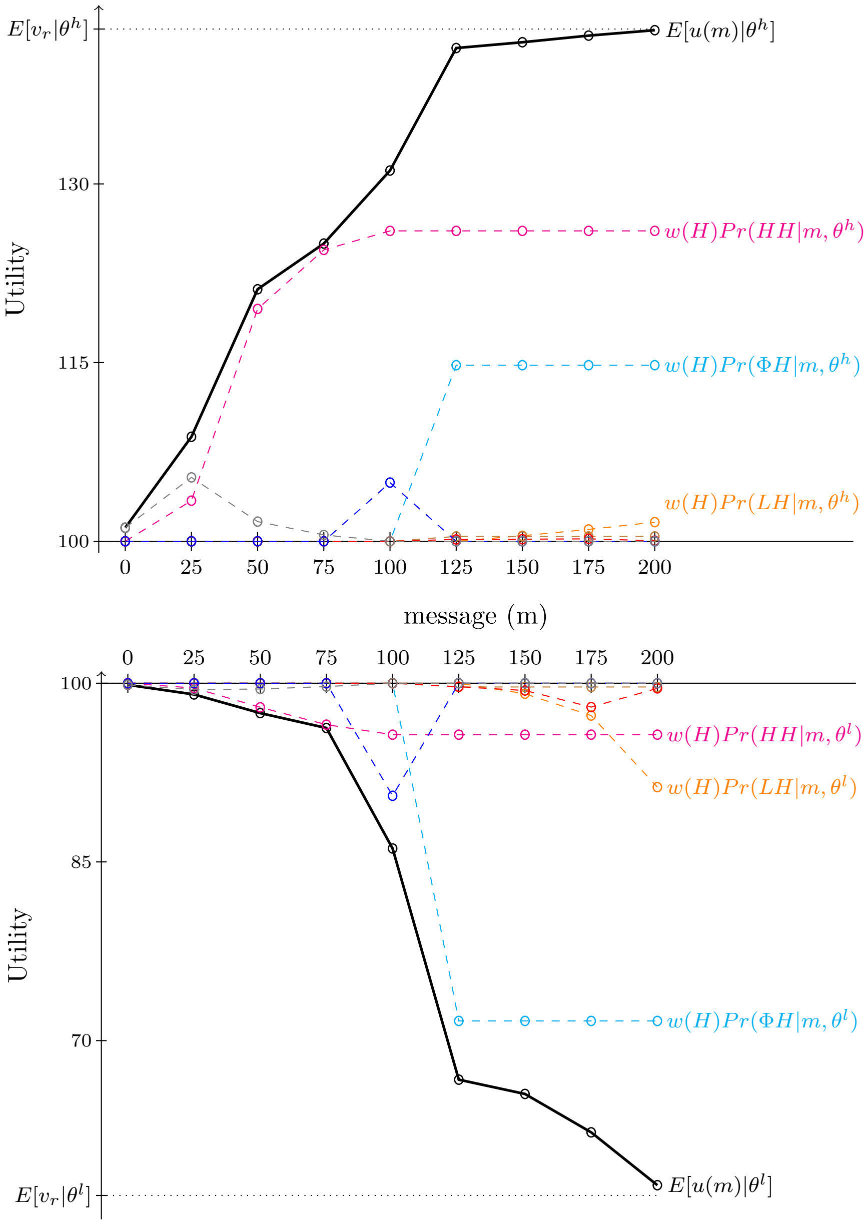


Figure 3.11: $N = 2$. Anatomy of expected utility as a function of sending a message, conditional on the high state (above), and conditional on the low state (below). Dark lines represent the sum of all parts. Some labels suppressed for clarity.

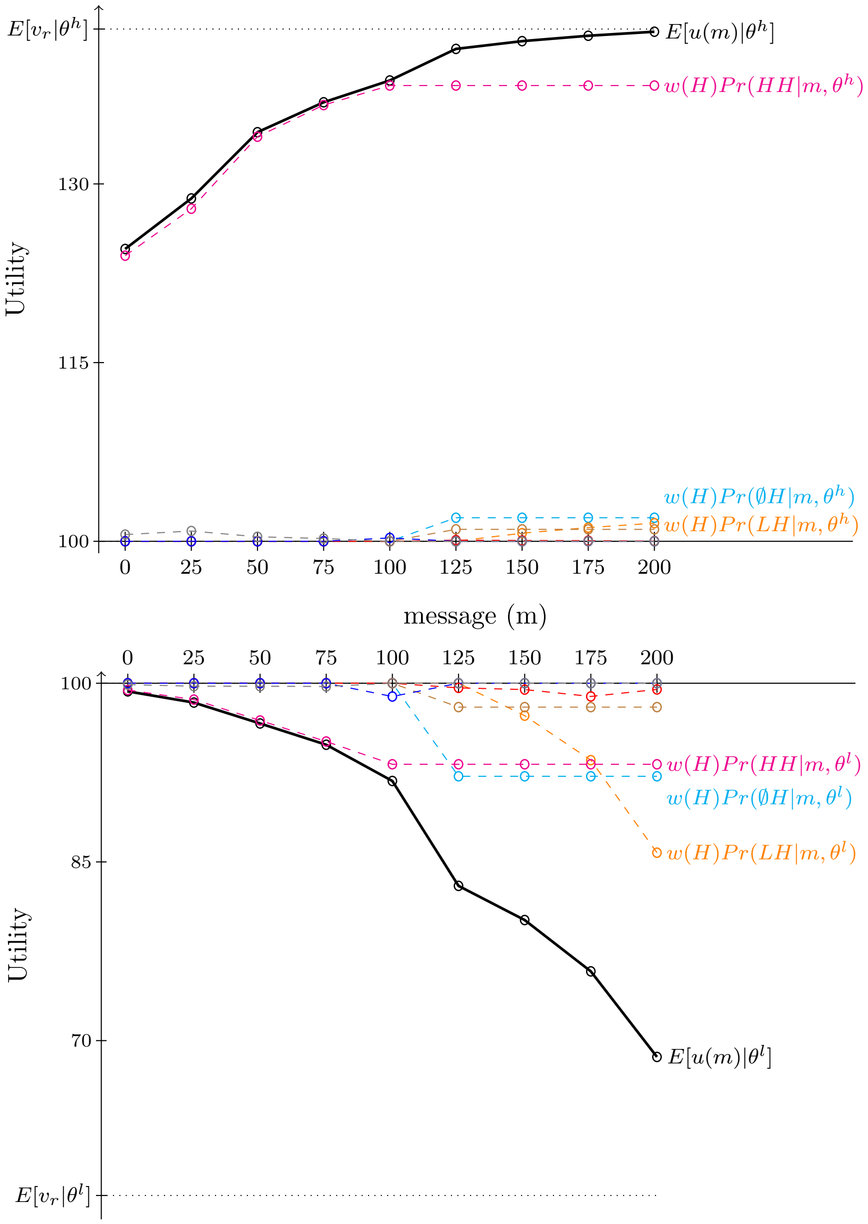


Figure 3.12: $N = 5$. Anatomy of expected utility as a function of sending a message, conditional on the high state (above), and conditional on the low state (below). Dark lines represent the sum of all parts. Some labels suppressed for clarity.

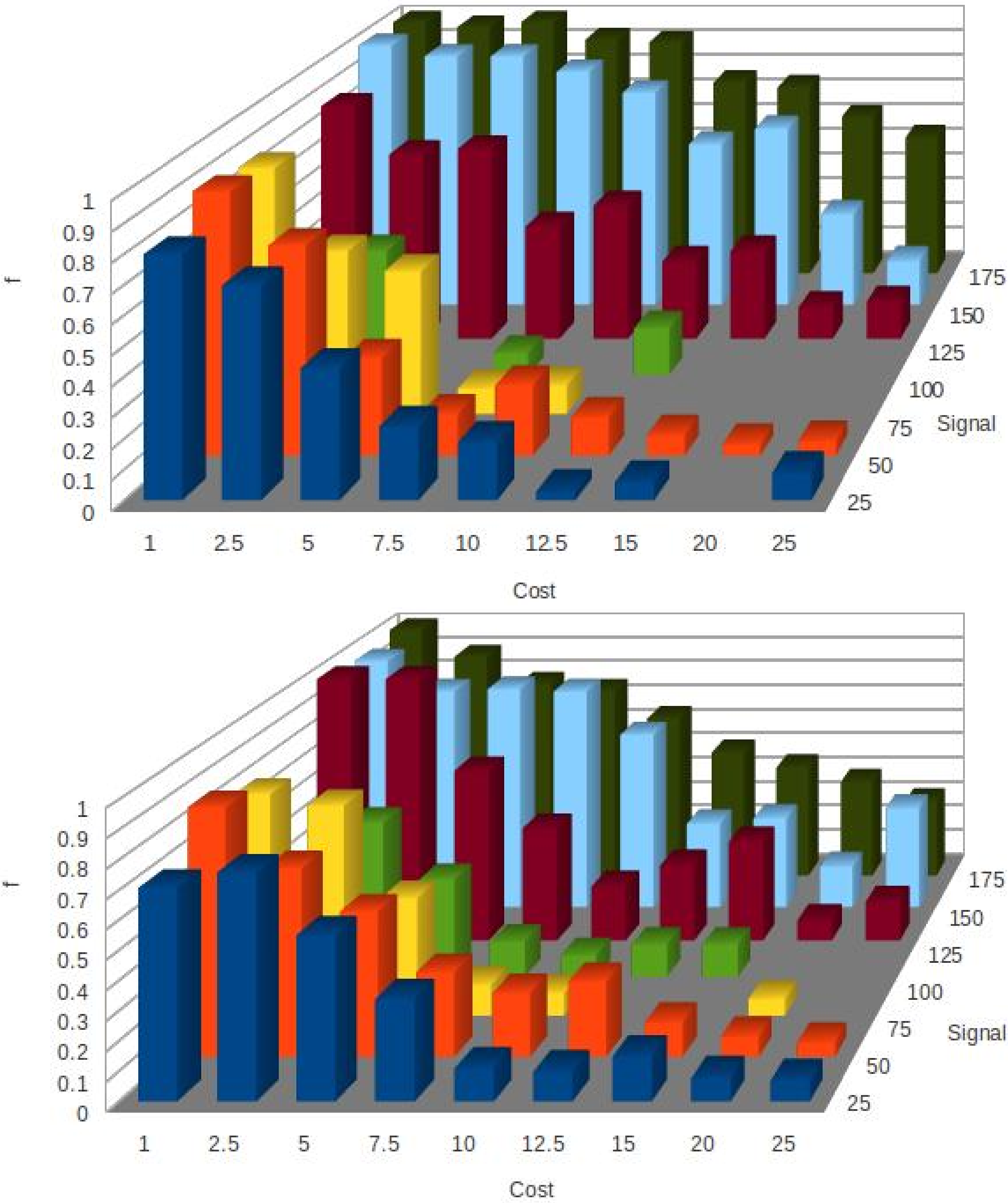


Figure 3.13: $N = 2$ (Above), $N = 5$ (Below) participation rates by type

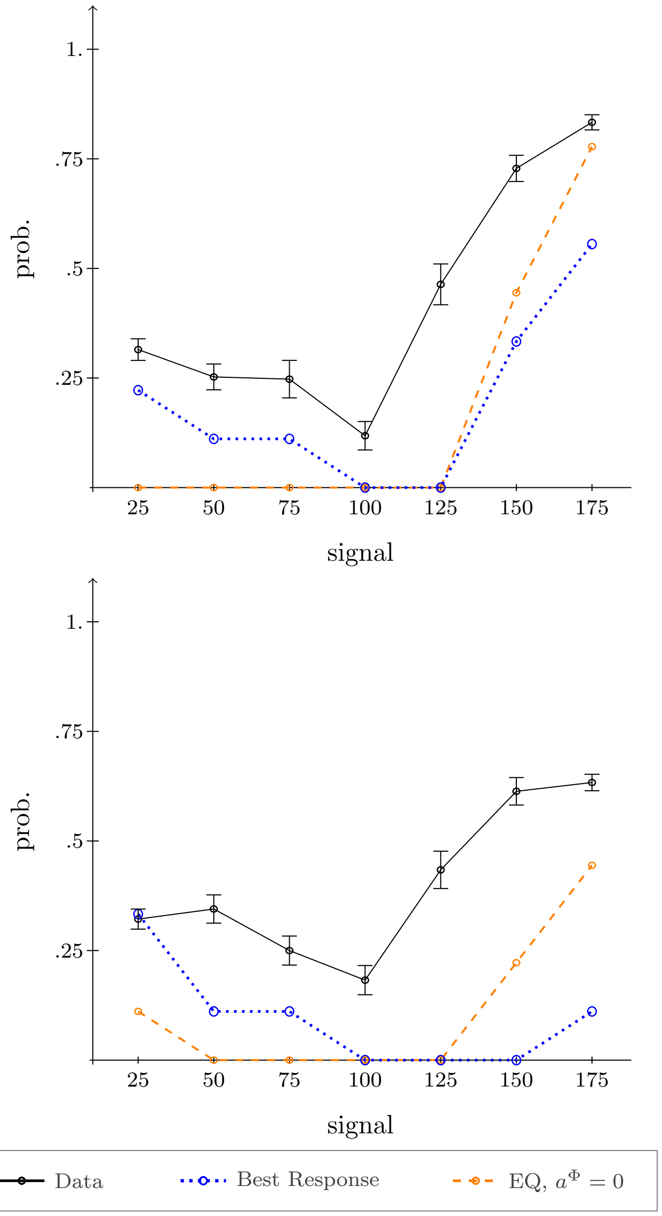


Figure 3.14: $N = 2$ (Above), $N = 5$ (Below) participation behavior and best response to empirical play participation

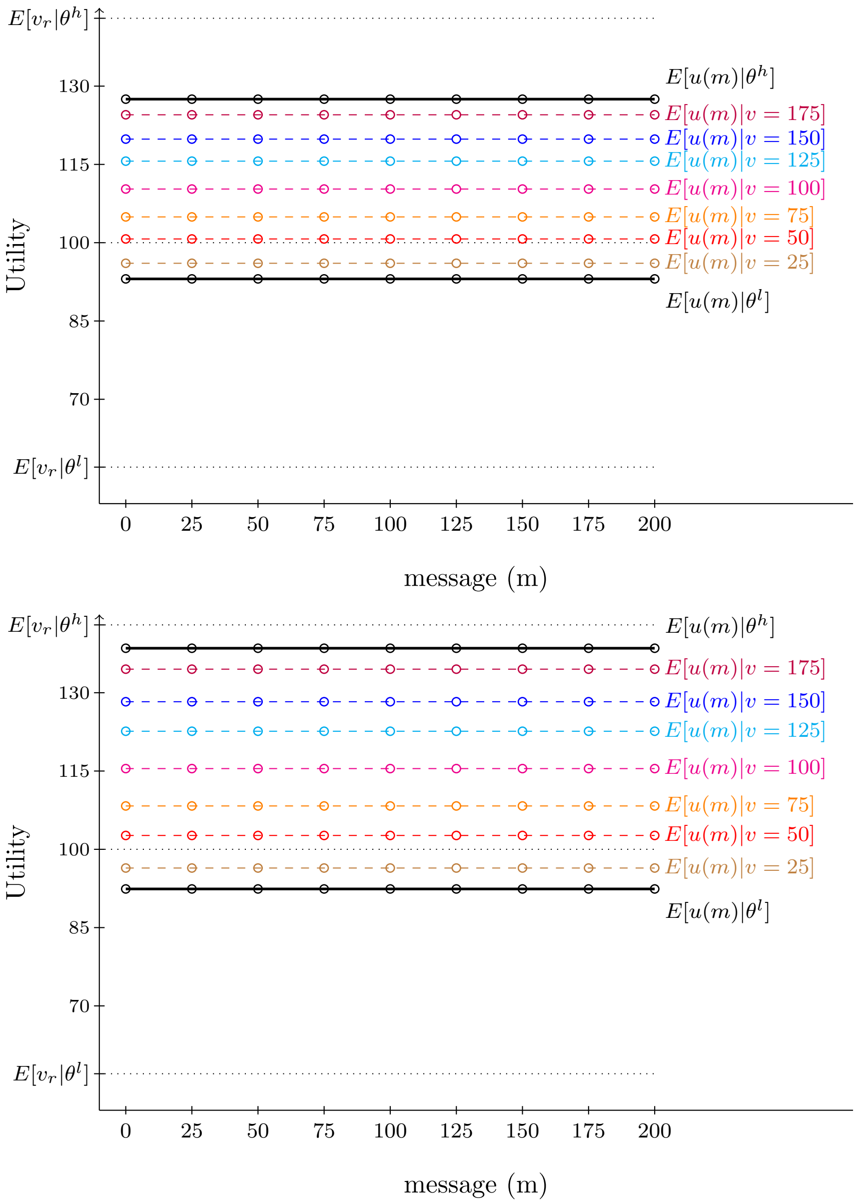


Figure 3.15: $N = 2$ (Above), $N = 5$ (Below). Expected utility of abstention as a function of sent message, conditional on a sender's value, and given empirical play. Solid black lines represent expected utility given certainty of the respective states.

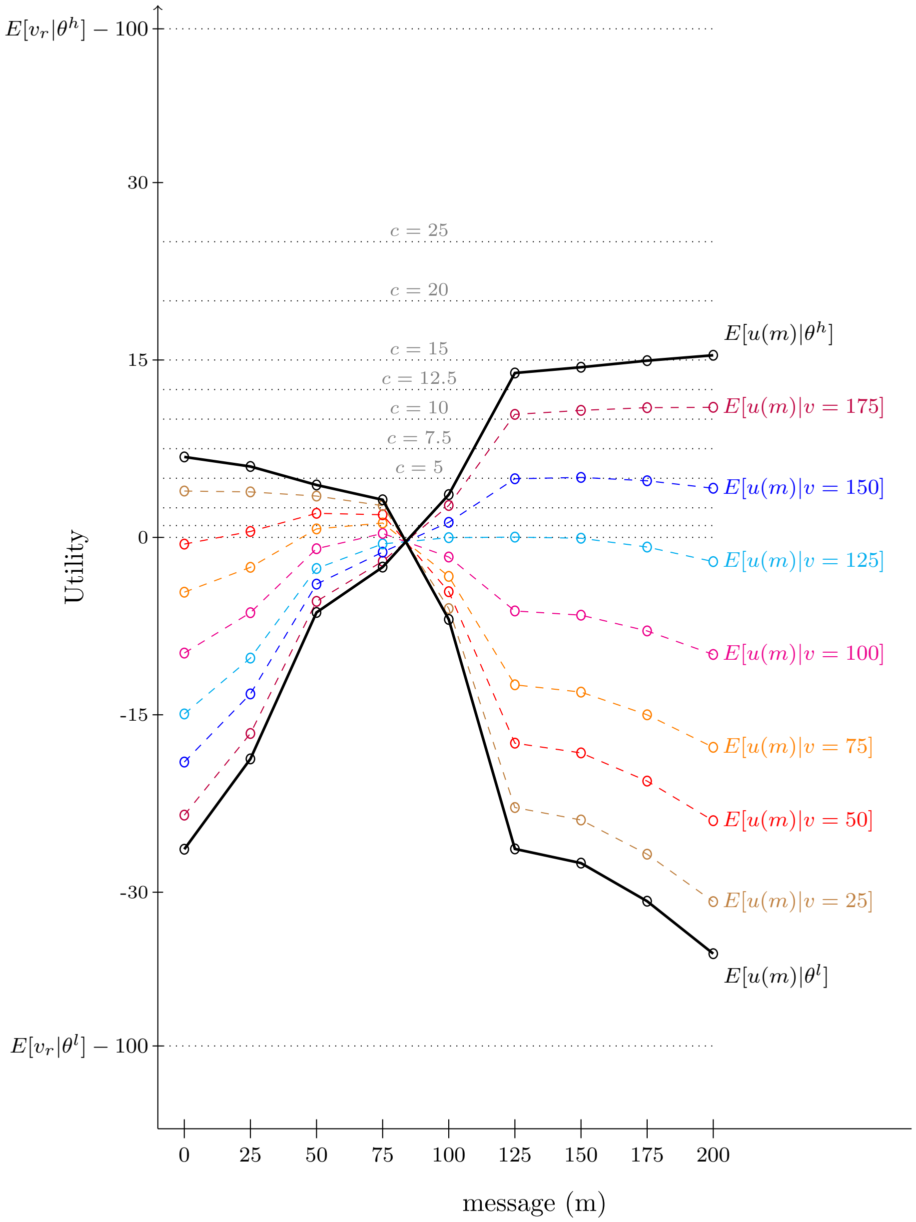


Figure 3.16: $N = 2$ Net expected utility as a function of sending a message, conditional on a sender's value, and given empirical play. Solid black lines represent expected utility given certainty of the respective states. Dotted horizontal lines are feasible costs to participation.

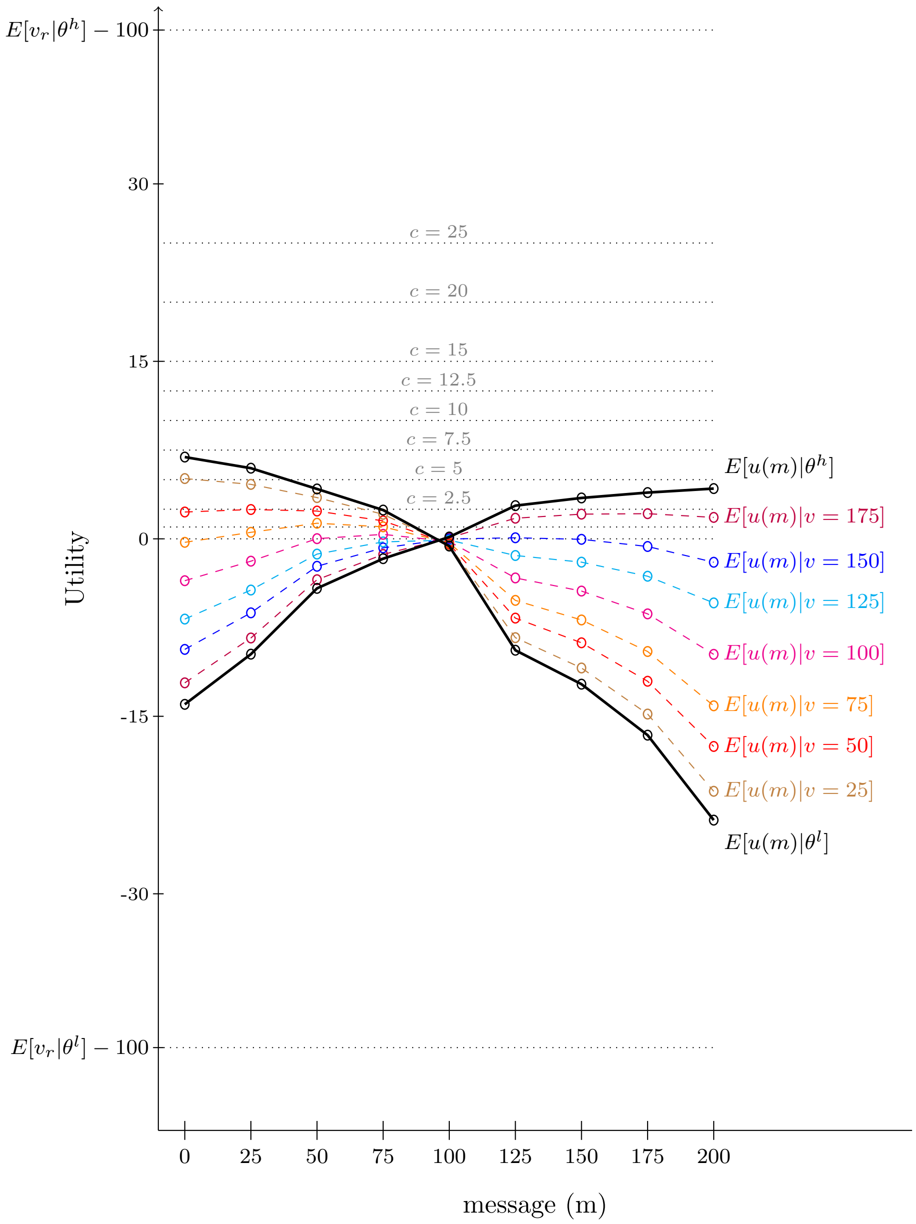


Figure 3.17: $N = 5$ Net expected utility as a function of sending a message, conditional on a sender's value, and given empirical play. Solid black lines represent expected utility given certainty of the respective states. Dotted horizontal lines are feasible costs to participation.

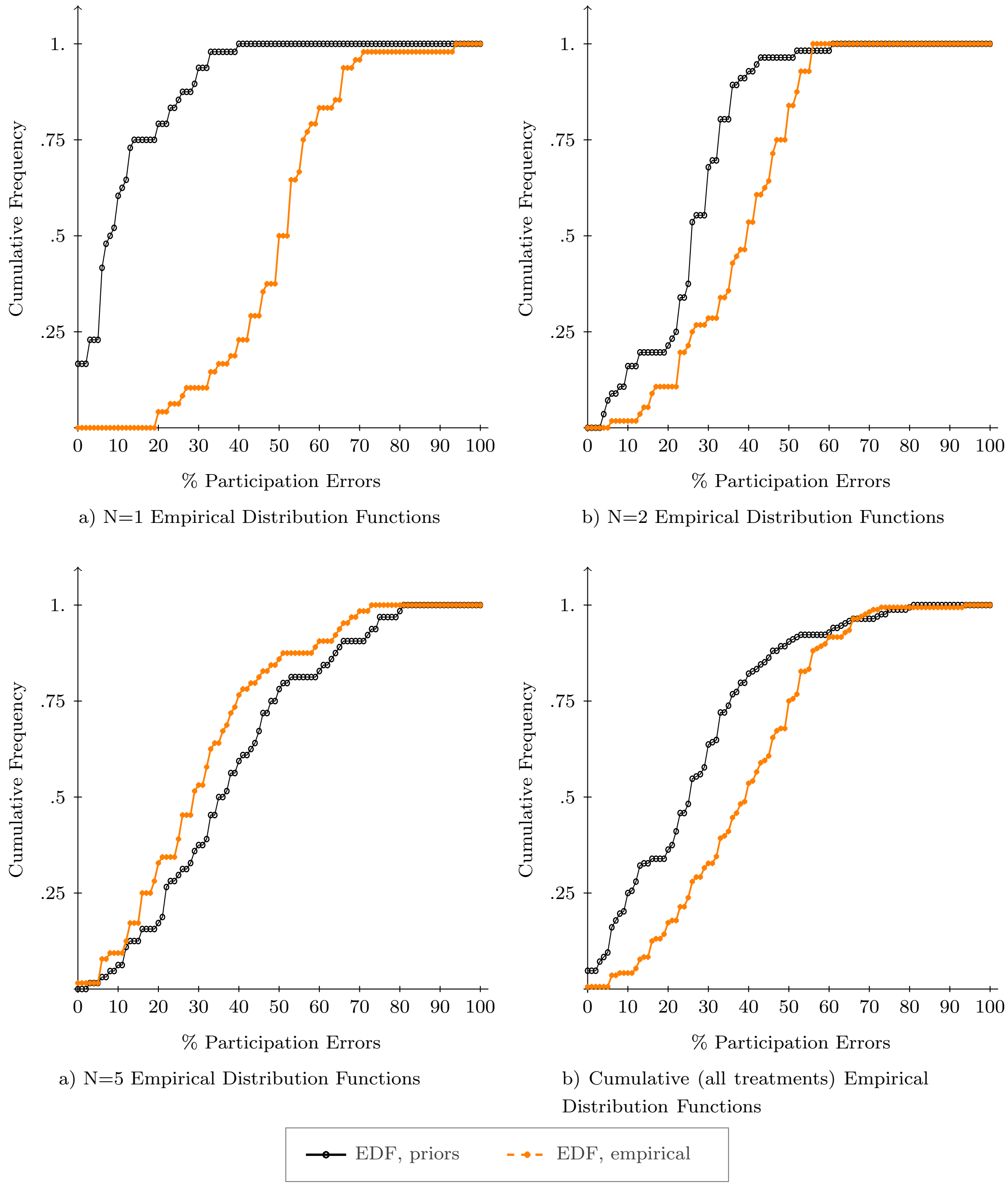


Figure 3.18: Empirical distribution functions (EDF) of the percentage of participation errors by each subject. Black lines represent the EDF of errors if subjects' beliefs regarding risky product draws were equivalent to the stated priors. Oranges lines represent the EDF of errors if subjects' beliefs regarding risky product draws were equivalent to the empirical density functions.

Chapter 4

Using Behavioral Models to Explain Rating Behavior in the Lab

4.1 Introduction

This paper discusses experimental evidence of bidding and participation behavior in the product rating game introduced in the previous chapter. One of the striking results of that experiment was that subjects closely adhered to equilibrium in the one-sender treatment but failed to coordinate on an equilibrium in the multiple-sender cases. At the conclusion of Chapter 3, we presented two hypotheses for this result. The first was that subjects in the multiple senders treatments may have played a wider array of actions as a result of the fact that expected payoffs as functions of actions were relatively flat; that is, subjects may have found their optimal actions to have been non-obvious or found calculating their best response to be unworthy of the effort. Second, we hypothesized that subjects failed to properly incorporate into their posterior beliefs the fact that the events in which subjects' messages were pivotal occurred at different rates depending on the quality of the risky product, and this resulted in subjects taking actions based upon incorrect beliefs.

These hypotheses correspond closely to existing theories of behavior in games, and in this chapter we apply these theories to the experimental data. We find that cursed equilibrium with quantal response equilibrium (CE-QRE) represents a substantial improvement in predictive power over the standard notion of equilibrium (PBNE). We estimate eight nested models in which we produce maximum likelihood estimates for the appropriate behavioral models. We find that each treatment and each player role represents a significantly different task, and that the model in which cursedness and quantal responsiveness are allowed to vary with these tasks explains behavior to an extent that we can reject that the data was generated by all other estimated models. We find that players in the one-sender treatment, as well as receivers in all treatments are essentially uncursed, while cursedness in the multiple senders treatments increases in the number of senders, with senders in the five-senders treatment being nearly fully cursed. CE-QRE approximates well the observed messaging comparative statics across treatments, however we find that the model substantially under-predicts the extent of sincere messaging that occurs in the data.

In the following section, we discuss quantal response equilibria (QRE), CE, and CE-QRE and how the concepts relate to the game presented in chapter 2. We then discuss methods and results of our estimation, and offer concluding remarks. In the appendices we first derive expected utility in a fully cursed equilibrium, and then present the code used to generate estimates for our models.

4.2 Theory

4.2.1 QRE

First proposed by McKelvey and Palfrey (1995), QRE has been used to great success in explaining experimental outcomes (Guarnaschelli, McKelvey, and Palfrey (2000) and Goeree and Holt (2005) are examples among many). QRE generalizes Nash equilibrium by allowing for errors in players' decisions that are sensitive to deviation in expected utility. A "responsiveness" parameter (denoted by λ, alternatively called a "rationality" parameter) determines the sensitivity of choice probabilities to differences in expected utility for each action.

In regular quantal response equilibria (Goeree, Holt, and Palfrey (2005)), the rate at which players take a given action is proportional to the expected utility of taking that action.[1] In the

[1] Regular QRE is a refinement of the notion of QRE proposed by McKelvey and Palfrey in which quantal response functions are restricted to satisfy four axioms: continuity, interiority, responsiveness, and monotonicity. The logit version of QRE, which we consider here, satisfies these axioms and is therefore a regular QRE. We henceforth refer to regular QRE simply as QRE.

logit version of QRE, the log probability that a player takes a certain action is proportional to the expected utility of that action. In particular, for a player acting at the information set h, the likelihood of playing action a is:

$$Pr(a|h) = \frac{e^{\lambda E[u|a,h]}}{\sum_{a' \in A} e^{\lambda E[u|a',h]}} \tag{4.1}$$

where A is the set of feasible actions. The value of λ determines how responsive players' behavioral strategies are to differences in expected utility. At one extreme, as $\lambda \to \infty$, QRE corresponds to Nash equilibrium; players choose the action with the highest expected utility with certainty. In the other extreme, when $\lambda = 0$, players randomize over actions, irrespective of the actions' expected utilities.

A quantal response equilibrium is a set of strategies that are a fixed point of (4.1). That is, in a quantal response equilibrium, the responsiveness parameter, (behavioral) strategies, and expected utilities are consistent in the sense that the responsiveness parameter and the expected utilities generate strategies that in turn generate the original expected utilities. While computing such equilibria can be computationally demanding, the estimation procedure of Bajari and Hortacsu (2005) enables us to calculate QRE consistently and with far less computational demands. Bajari and Hortacsu observe that if the model is correct, subjects can be assumed to have played a quantal response equilibrium, and the experimental data provide a consistent estimator of the equilibrium joint distribution of types and actions.

Thus, we assumed subjects played a QRE, which implied that empirical rates of play were consistent estimates of the behavioral strategies subjects played. Since QRE is an equilibrium concept, this implies that we were able to use the experimental data to generate expected utilities for each action, for each type. This approach assumes that all players played the same QRE, but given that there were not large differences in behavior across sessions, it is unlikely that there were multiple. For both sender and receiver estimates we used the aggregate experimental data pooled across all experimental sessions.

For receivers, this entailed generating probability density functions for the likelihood of each aggregate message (including the null message) in each state. This in turn yielded Bayesian posterior beliefs and the expected value of playing each action conditional on receiving each aggregate message. Generating sender's expected utilities was similar, but instead of aggregate messages, we generated the likelihood in each state, conditional on the player's type, of each aggregate message when one sender's message was left out. Using this, we were able to generate the expected outcome of each feasible action, and thereby its expected utility. Having generated expected utilities for each type-action pair, it was straightforward to estimate the $\hat{\lambda}$ that best fits the data using maximum likelihood estimation.

As noted in the previous chapter, conditional on empirical rates of play, expected utilities in the lab as functions of available actions were relatively flat (see Figures 3.10 and 3.16). QRE will therefore typically entail similar rates of selection of each action, and ex-ante can be expected to help to explain the subjects' observed over-participation relative to their best response for almost every type and in all treatments. However, because subjects anchored strongly on sincere messaging (see Table 4.5), and because sincerity was often a suboptimal response, the flatness of the expected utility curves is also a virtual guarantee that QRE alone will not be able to explain subject's behavior.

4.2.2 CE

Cursed Equilibrium was introduced by Eyster and Rabin (Eyster and Rabin 2005) to explain the "winner's curse" in experiments of common value auctions. The intuition behind CE is that "players in a Bayesian game underestimate the extent to which other players' actions are correlated with their information." In a fully cursed equilibrium, players correctly update their beliefs given their types, but fail to properly account for the correlation between other players' actions and their types. Instead, for a given action profile, players attribute the average likelihood of that profile to all other players' types. For a rational, uncursed player, k, the expected utility of playing action a_k, given that his type is t_k is:

$$E[u|t_k, a_k] = \sum_{t_{-k} \in T_{-k}} p(t_{-k}|t_k) \sum_{a_{-k} \in A_{-k}} \sigma_{-k}(a_{-k}|t_k) u_k(a_k, a_{-k}; t_k, t_{-k})$$

where T_{-k} and A_{-k} are the feasible sets of type and action profiles of the other players, respectively, and where $\sigma_{-k}(\cdot)$ is a joint strategy profile of the other players. In contrast, a cursed player has the following expected utility:

$$E_{CE}[u|t_k, a_k] = \sum_{t_{-k} \in T_{-k}} p_k(t_{-k}|t_k) \sum_{a_{-k} \in A_{-k}} \overline{\sigma}_{-k}(a_{-k}|t_k) u_k(a_k, a_{-k}; t_k, t_{-k})$$

where $\overline{\sigma}_{-k}(a_{-k}|t_k)$ is "the average strategy of other players, averaged over the other players types," or, equivalently, "the probability that players $j \neq k$ play action profile a_{-k} when they follow strategy σ_{-k}."

$$\overline{\sigma}_{-k}(a_{-k}|t_k) \equiv \sum_{t'_{-k} \in T_{-k}} p_k(t'_{-k}|t_k) \cdot \sigma_{-k}(a_{-k}|t'_{-k})$$

In a common-value auction game for which this concept was first developed, the fully cursed expected utility essentially translates simply to the player updating his beliefs about the value of the product given his type, and only given his type, while correctly estimating his *average* likelihood of winning for each message.

Eyster and Rabin also specify partially cursed equilibria. A partially cursed equilibrium is parametrized by the cursedness parameter $\chi \in [0, 1]$, which acts as a coefficient for a convex combination of cursed and uncursed expected utilities. A χ of 1 corresponds to a fully-cursed equilibrium and a χ of 0 corresponds to an uncursed, Nash equilibrium. All equilibria are called χ-cursed equilibria, and for each player k of type t_k, the equilibrium action, a_k^*, must satisfy:

$$a_k^* \in argmax_{a_k \in A_k} \; E_{CE}^{\chi}[u|t_k, a_k] = \chi E_{CE}[u|t_k, a_k] + (1 - \chi) E[u|t_k, a_k]$$

A formal derivation of expected utility in a χ-equilibrium in our game is presented in the appendix. The result of this derivation is that fully cursed senders do not differentiate between the likelihoods of an event occurring in the respective states of the world. Instead, a fully cursed sender believes that the likelihood of each event occurring in each state of the world is the average likelihood of that event, averaged across states of the world (using the correct posterior belief about the states of the world).

Example 4.2.1. *Suppose that there was only one possible event in the our game: the event in which only one sender sends a message (all others abstained; represented by* Φ*). In the five sender treatment, the empirical frequencies of this event were approximately* 12% *in the low state and* 2% *in high state. For a sender with the signal of* $v_k = 175$*, the Bayesian posterior that the state was the high state was approximately* 95%*. Therefore, for an uncursed sender, the expected utility of playing (arbitrary) action,* a_k*, was*

$$\begin{aligned}E[u|v_k = 175, a_k] =& \sum_{\theta} Pr(\theta|v_k = 175)Pr(\Phi|\theta)E[u_k|\theta, \Phi, a_k]\\ =&(0.95 * 0.02)E[u_k|\theta^h, \Phi, a_k] + (0.05 * 0.12)E[u_k|\theta^l, \Phi, a_k]\\ =&0.019E[u_k|\theta^h, \Phi, a_k] + 0.006E[u_k|\theta^l, \Phi, a_k] \qquad (4.2)\end{aligned}$$

Fully cursed senders incorrectly estimate the likelihoods of the event; for each state, they used the posterior-weighted average of the two event likelihoods: $(0.02*0.95+0.12*0.05) = 0.025$*. Therefore, the expected utility of the same action for a fully cursed sender was:*

$$\begin{aligned}E_{CE}[u|v_k = 175, a_k] =& \sum_{\theta} 0.025Pr(\theta|v_k = 175)E[u_k|\theta, \Phi, a_k]\\ =&(0.95 * 0.025)E[u_k|\theta^h, \Phi, a_k] + (0.05 * 0.025)E[u_k|\theta^l, \Phi, a_k]\\ \approx&0.024E[u_k|\theta^h, \Phi, a_k] + 0.001E[u_k|\theta^l, \Phi, a_k] \qquad (4.3)\end{aligned}$$

Comparing the first term of 4.2 with the first term of 4.3 we see that fully cursed senders over-

weighted their high-state expected utility by about 25%*, and comparing the second terms, we see that fully cursed senders under-weighted their low-state expected utility by approximately* 80%*. Thus, fully cursed senders were more likely to play actions that traded utility in the low state for utility in the high state (such as sending a high message).*

Figures 4.1 and 4.2 display fully cursed best response participation and messaging strategies, respectively, to empirical rates of play. These figures display a remarkably strong fit for participation behavior, especially when compared to Bayesian best response functions, but show a remarkably poor fit for messaging behavior. The fully cursed best response messaging strategies take the form they do because fully cursed senders update their beliefs solely on their types. Given this, fully cursed senders simply attempt to maximize the likelihood that the receiver makes his choice in accord with the sender's posterior. Since messages are averaged together, the sender has her largest effect by choosing a message at the extreme of the messaging space.

As we show in the appendix, no matter what average message the fully cursed receiver observes, his expectation of the value of the risky product is equivalent to his ex-ante expectation of the value of the risky product.

4.2.3 CE-QRE

CE-QRE combines the concepts of quantal response equilibrium and cursed equilibrium. In particular, we assume that χ-cursed players respond imperfectly, according to a responsiveness parameter, λ, as in a QRE:

$$Pr(a|h) = \frac{e^{\lambda E^{\chi}_{CE}[u|a,h]}}{\sum\limits_{a' \in A} e^{\lambda E^{\chi}_{CE}[u|a',h]}}$$

As with QRE, we used the empirical joint distributions of types and actions to compute expected utilities. In this case we computed the fully cursed expected utility of each action as well as the Bayesian expected utility. We then estimated values for λ and χ using maximum likelihood estima-

	Treatment	Role	$(\hat{\lambda}^{sender}, \hat{\lambda}^{receiver})$	Log Likelihood
(1)	Pooled	Pooled	2.2	-5860.8
(2)	Pooled	Separate	$(2.4, 1.3)$	-5753.1
(3)	Separate	Pooled	N=1: 2.1 N=2: 2.3 N=5: 2.2	-5858.8
(4)	Separate	Separate	N=1: $(3.0, 1.3)$ N=2: $(2.7, 1.2)$ N=5: $(2.2, 1.4)$	-5728.9

Table 4.1: QRE Estimates of Three Embedded Models

tion.

4.3 Experimental Results

We estimated eight models. Each model contained different assumptions on the extent to which estimates for λ and χ applied across treatments and player-roles. Four of these models assumed that players were uncursed ($\chi = 0$). The results of these estimations, as well as the log likelihood of each model, can be found in Table 4.1. The rows of the table represent the separate models. In the model represented by the first row, we assumed that behavior in all treatments and by both senders and receivers were generated by the same responsiveness parameter. In the second, we split the data by player-roles, estimating separate responsiveness parameters for the sender and receiver roles. Next, we split the data by treatment, estimating $\hat{\lambda}$ for each treatment. Finally, we split the data by both player-roles and treatment, and these estimates are presented in the final row of the table. The first column numbers the models. Columns 2 and 3 present each row's respective modeling assumptions. Column 4 presents the respective $\hat{\lambda}$ estimates, while column 5 presents the log likelihood of the data being produced by each model.

To compare the fit of these models we utilized likelihood ratio tests on each of the embedded models. The p-values resulting from tests can be found in Table 4.2.[2] As the table shows, model 3 cannot reject that the data were produced by the fully pooled model. However, models 2 and 4 can reject this null hypothesis, and model 4, the fully separate model, in which estimates were made for

[2]Since models 2 and 3 are not embedded models, they were not directly compared.

		Null Model		
		(1)	(2)	(3)
Alternative Model	(2)	0.00	–	–
	(3)	0.13	–	–
	(4)	0.00	0.00	0.00

Table 4.2: p-Values of Likelihood Ratio Tests for Embedded QRE Models. Model numbers correspond to the numbered rows of Table 4.1

	Treat.	Role	$(\hat{\lambda}^{send.}, \hat{\lambda}^{rec.})$	$(\hat{\chi}^{send.}, \hat{\chi}^{rec.})$	LL
(5)	Pooled	Pooled	2.5	0.57	-5685.2
(6)	Pooled	Separate	(2.9, 1.3)	(0.61, 0.04)	-5518.1
(7)	Separate	Pooled	N=1: 2.1 N=2: 2.6 N=5: 2.8	N=1: 0.06 N=2: 0.36 N=5: 0.96	-5613.4
(8)	Separate	Separate	N=1: $(3.0, 1.3)$ N=2: $(3.3, 1.2)$ N=5: $(2.9, 1.4)$	N=1: $(0.00, 0.04)$ N=2: $(0.41, 0.07)$ N=5: $(0.97, 0.00)$	-5446.3

Table 4.3: Cursed-QRE Estimates of Four Embedded Models

$\hat{\lambda}^{sender}$ and $\hat{\lambda}^{receiver}$ in all three treatments, can reject that the data was generated by each of the other three models, at all standard levels of significance.

The final four models allow χ and λ to vary along parallel dimensions to the previous four models (the first model assumes the data is pooled, the second splits the data by player-roles, etc.). The results of these estimations can be found in Table 4.3, which mirrors Table 4.1 in its presentation. As with the uncursed models, we tested the fit of the models using likelihood ratio tests. Each of the embedded models reject the hypothesis that its parent model(s) generated the empirical data at all standard levels of significance. The results of the two sets of tests therefore imply that the best model is model 8, the model that specifies separate levels of cursedness and separate level of quantal responsiveness for both roles, for each treatment.

The estimated behavior of models 4 (fully separate QRE) and 8 (fully separate CE-QRE) are graphed in Figures 4.3 through 4.5. In addition, Figures 4.3 and 4.4 display the empirical rates of play and the rational best response. Figure 4.5 displays experimental messaging behavior as box plots that are a function of the sender's signal. For almost every signal in every treatment, all quartiles were equal to the sincere message. In order to visualize the variance in the data, the box

	Average Sent-Message							
	< 100		100		> 100		a^{Φ}	
	Data	CE-QRE	Data	CE-QRE	Data	CE-QRE	Data	CE-QRE
$N = 1$	2.2	2.7	37.5	44.6	99.3	92.0	6.7	5.1
$N = 2$	1.4	1.7	33.3	32.1	97.2	80.5	8.9	5.1
$N = 5$	0.9	0.4	14.3	5.9	99.5	88.2	6.5	1.5

Table 4.4: Percentage Receivers Chose the Risky Action, Conditional on Binned Average Message

plots of Figure 4.5 therefore display sextiles (6-quantiles). Consistent with this, we present the QRE and CE-QRE estimates for first and fifth sextile messages as well as for the median message, for each signal.

One of the most striking aspects of the figures and of Table 4.3 is the variance in the maximum likelihood estimate of $\hat{\chi}$ across roles and treatments. Estimates of $\hat{\chi}$ for receivers are all nearly zero. Recall, that a fully cursed receiver plays based on his prior. Therefore, for receivers in every treatment, χ-cursedness amounts simply to a dampening of beliefs towards the receiver's prior. Every level of cursedness less than fully cursed will therefore result in the same best response. When combined with QRE, for any given positive value of λ, increasing χ results in the receiver mixing across all feasible actions more evenly at each information set. That is, for receivers, χ and λ have the same effect (but in opposite directions). MLE estimates of $\hat{\chi}$ greater than 0 for receivers therefore represent extremely small gains in likelihood. Note that in Figure 4.3, which displays predicted receiver behavior, we've plotted estimates from both fully separate models, and there is no discernible difference in predicted rates of play. Table 4.4 displays the predicted likelihood that receivers chose the risky product as a function of which side of 100 was the average message and as a function of the null message, and shows that the models predict receiver behavior well, on average.

The predicted level of cursedness increases in the number of senders. Senders in the single-sender treatment are completely uncursed. Senders in the multiple-sender treatments, however, appear to be strongly cursed, with $\hat{\chi} = 0.41$ in the two-sender case, and with senders almost fully cursed in the five-sender case; $\hat{\chi} = 0.97$. There are no clear trends in the estimations of λ, however each $\hat{\lambda}$ must be taken in the context of the estimation of χ. We estimate $\hat{\lambda}$ to be lower in the one-sender treatment than in the five-senders treatments, however because of the respective estimates of χ,

	% Sincere				% Exaggeration			
	Data	QRE	CE-QRE	Random	Data	QRE	CE-QRE	Random
N=1	91.6	23.9	23.9	11.1	7.7	35.6	35.6	16.1
N=2	84.4	25.7	25.8	11.1	15.0	30.5	38.8	16.7
N=5	77.1	19.7	21.3	11.1	21.3	26.1	41.0	16.5

Table 4.5: Select Message Frequencies/Likelihoods. QRE and CE-QRE data generated by maximum likelihood parameters estimated separately for each role and each treatment.

the appropriate interpretation is that the single senders are playing slightly more noisily around a rational best response, than multiple senders are playing around a highly cursed, suboptimal response.

Both the QRE and CE-QRE models pick up the theoretically expected and empirically demonstrated comparative static of increasing rates of participation as sender's signals move away from 100. The cursed model improves upon the fit of the QRE predictions substantially by predicting higher rates of participation at higher values. QRE estimates for the five sender treatment predict the average rate of participation well, but provides a poor fit as a function of players' types, predicting that players participate more frequently given signals below 100 than they do with signals above.

Recall from the previous chapter that subject behavior was strongly anchored to the sincere messaging strategy, with increasing rates of exaggerated messaging as a function of the number of senders. This trend is reproduced in Table 4.5 along with parallel estimates generated by the QRE and CE-QRE models. As with participation, QRE alone does poorly in predicting messaging behavior. As is evident in Figure 4.5, QRE predicts too high a level of variance in messaging behavior. In particular, it predicts a large amount of dampened messages (in the direction of 100 from their signal), and even predicts messaging on the side of 100 opposite to a sender's signal. Both of these predictions are rarely observed in the data. Moreover, as is evident from Table 4.5, QRE significantly under-predicts sincerity, significantly over predicts exaggeration, and predicts a comparative statics in exaggeration across treatments opposite to what is observed in the data. CE-QRE is not immune to these shortcomings. However it does improve upon them in many ways. The CE-QRE messaging predictions are in narrower bands around the observed data, and predict far fewer messages on the side of 100 opposite to a sender's signal. It predicts slightly higher rates of sincerity and exaggeration, and correctly estimates the comparative static on exaggeration across treatments.

4.4 Discussion and Conclusion

In this chapter, we analyzed subject behavior in the product rating game introduced in the previous chapters using the behavioral concepts of quantal response equilibrium and cursed equilibrium. We estimated eight models. Four of these models estimated the maximum likelihood quantal response equilibrium and differed in their assumptions about the extent to which responsiveness estimates applied across treatments and player-roles. The remaining four models combined the concepts of quantal response equilibrium and cursed equilibrium. These four models also differed in their assumptions about the extent to which the behavioral parameters applied across treatments and player-roles. For each of these models we simultaneously estimated the maximum likelihood quantal responsiveness parameter and the level of χ-cursedness. Of the eight models, we found that the model in which we estimated the level of cursedness and the quantal responsiveness parameter separately for senders and for receivers in each treatment explained the data best, and to an extent that we could reject that the data was generated by all of the other models, at all standard levels of significance. The predictions of this model represent a substantial improvement over rational, Bayesian best response in predicting subjects' behavior.

Our quantal response equilibrium (QRE) estimates provide a counter-example to the occasional refrain that "QRE can explain anything". QRE alone does a poor job of explaining the data. It does not predict the empirical messaging pattern of anchoring on sincere messaging, and in the five sender treatment it predicts that senders with negative signals actually participate at higher rates than do senders with positive signals, a clear inconsistency with the data. The range of behavior that regular QRE can explain begins with perfect best response and ends with random play. QRE cannot explain, however, consistent suboptimal, low-noise behavior like that which we observe in subjects' messaging behavior. Subjects played the sincere message with overwhelming frequency despite the fact that sincerity was often a suboptimal response. In order for QRE to fit this data, we were forced to decrease our estimate of the rationality parameter, λ. However, as λ decreased, the predicted likelihoods that *all* suboptimal actions were taken increased, not just that of the sincere message. As a result, the best QRE fit of the data predicts a wide array of sender message selection, a trend we do not observe in the data.

Our cursed quantal response equilibrium (CE-QRE) estimates explained behavior significantly better than QRE alone. Compared to QRE, CE-QRE, predicted participation better, anticipating greater participation at higher signals than at lower signals, and predicted tighter bands around sincere messaging. Moreover, on message exaggeration CE-QRE correctly estimated the comparative static that the rate of exaggeration increases with the number of senders. In the one-sender treatment, we found that senders are uncursed; in the two sender treatment, we found that sender-subjects were approximately half-cursed, and in the five sender treatment we found that sender-subjects were nearly fully cursed. We believe that the comparative static of increasing cursedness in the number of senders is consistent with the spirit of the cursed equilibrium: since cursedness is derived from players' failure to properly infer information about other players' types from their actions, it is appropriate that this failure becomes greater as the inference problem becomes more complex, as it does with a greater number of senders. One clear path for future research is to replicate this experiment with a greater number of senders. We identified clear comparative statics in the data, but among them is that senders are already fully cursed with only five senders. It is therefore not clear which of the trends we identified may continue, and which have already run their course.

As for subjects acting in the receiver role, we found that in all treatments receivers were essentially uncursed. This is true despite the fact that the same subjects were highly cursed when playing in the sender role in the same treatment. Along with our finding that senders in the one-sender treatment were uncursed, we believe this result points towards a potential simple improvement in the cursed equilibrium concept when applied to sequential games. Cursed Equilibrium is a theory of a precise way in which players fail to properly update their beliefs given the expected *simultaneous* play of others. It is therefore not obvious that it is appropriate in a sequential game in which there does not exist the possibility of an inference failure on simultaneous action. As discussed, this is borne out by the data presented here. A concept of cursed equilibrium that specified that a player update her beliefs given her *information set*, as opposed to her type, would resolve our complaint elegantly, and alleviate the need to estimate multiple χ for different player-roles. For example, if receivers in a cursed equilibrium were allowed to update given their information, then their problem would naturally coincide with the uncursed, Bayesian receiver's problem.

One question posed in the cursed equilibrium literature is whether the observed failures by subjects represent cursedness, as it was postulated by Eyster and Rabin, or if it simply represents a failure by subjects to compute a conditional expected value. We believe that our results provide evidence for the former. In our one-sender treatment cursedness is not a meaningful concept for senders since there is no inference to be made on the simultaneous actions of other players. All that was required of senders in that treatment was to compute a conditional expected value given their type. Subjects perform remarkably well in this treatment. However, despite the fact that the same subjects participated in each treatment, subject-senders fared poorly in the multiple-sender treatments. The problem of computing their conditional expected value given their type was unchanged between treatments; the only difference was the saliency of the inference on simultaneous sender actions problem. Moreover, a receiver's conditional expected utility problem was at least as computationally hard as the senders'. Bayesian receivers were required to calculate the likelihood of each aggregate message in each state of the world in order to best respond to their information. Despite subjects playing both the role of sender and receiver, when in the role of receiver, subjects rarely failed to take the expected value-maximizing action. We believe these facts are strong evidence that subjects' failures when in the subject role are due to phenomena much more akin to cursedness than to a lack of computational prowess.

Despite the relative success of CE-QRE in explaining our experimental data, the models had significant shortcomings. Most striking is the failure to predict subjects' proclivity for sending their signal as their message. Subjects played an overwhelming number of their actions in this way, but each of the models explored here predict a substantially greater amount of variance in messaging behavior. Although not explained by our models, it is not hard to imagine subjects settling on such a strategy. Sincerity may simply have appealed to subjects as a rule of thumb that was near enough their optimal best response. Indeed, expected utilities as a function of actions were relatively flat given empirical play and this may have made the calculation of best response comparatively too computationally taxing. Alternatively, since sincerity is a weak best response in the one-sender case, it may also be that subjects employed simple (albeit incorrect) inductive logic to the problem. Whatever it is, deciphering subjects' motivation for sincere messaging will be at the heart of any theory that is to fully explain behavior in this environment. It appears likely to us that quantal response and cursedness, or concepts like them, are behind subjects' behavior, but it is clear that there is more remaining to be explained.

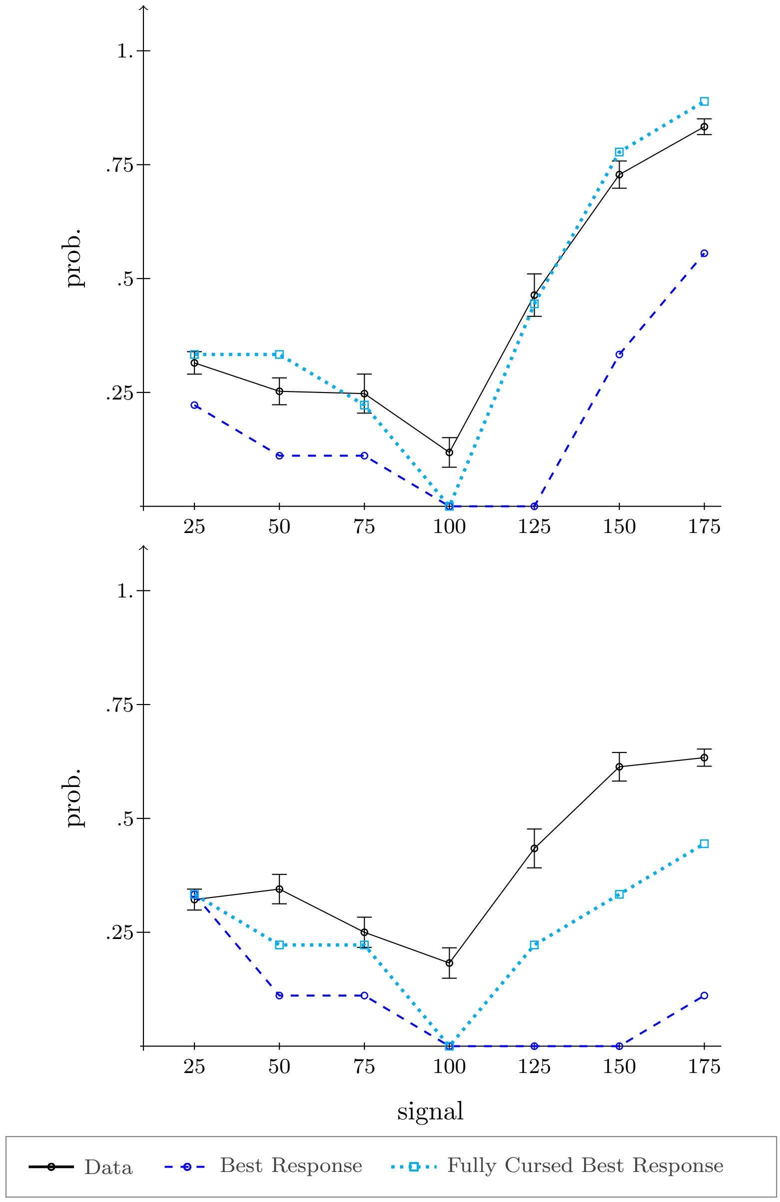


Figure 4.1: Fully cursed participation as a function of signal. $N = 2$ (Above), $N = 5$ (Below)

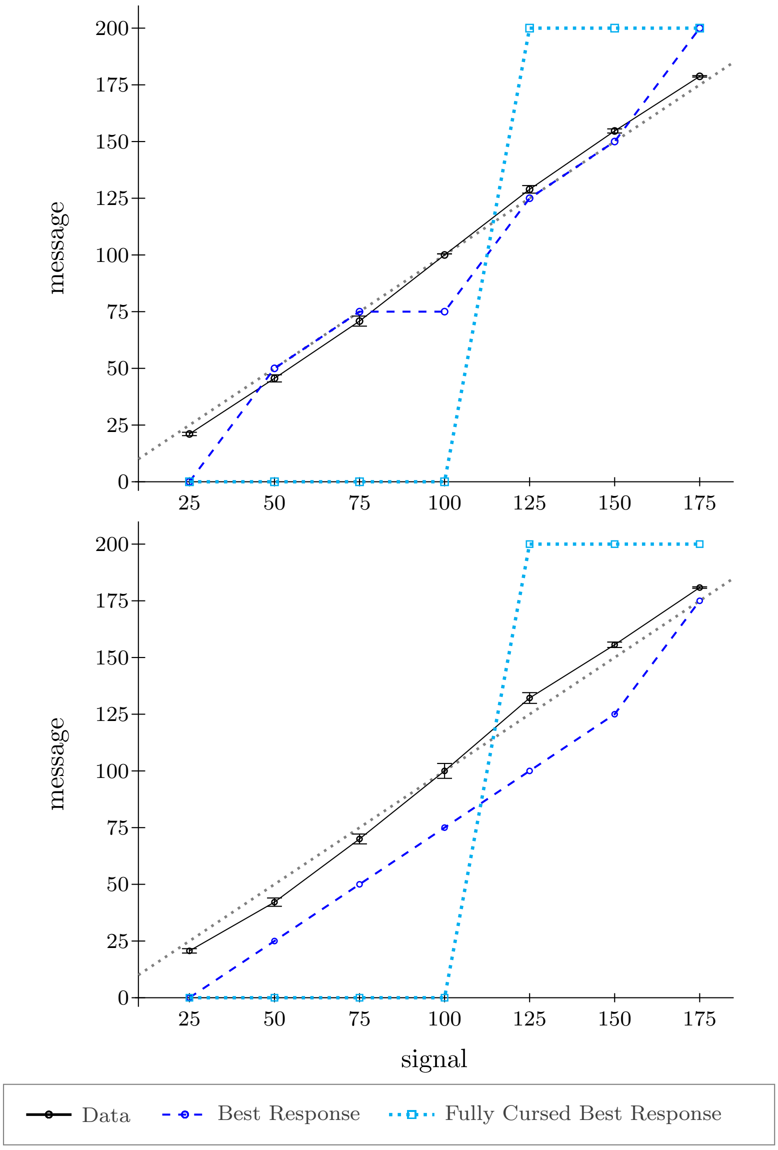


Figure 4.2: Fully cursed messaging strategy as a function of signal, $N = 2$ (Above), $N = 5$ (Below)

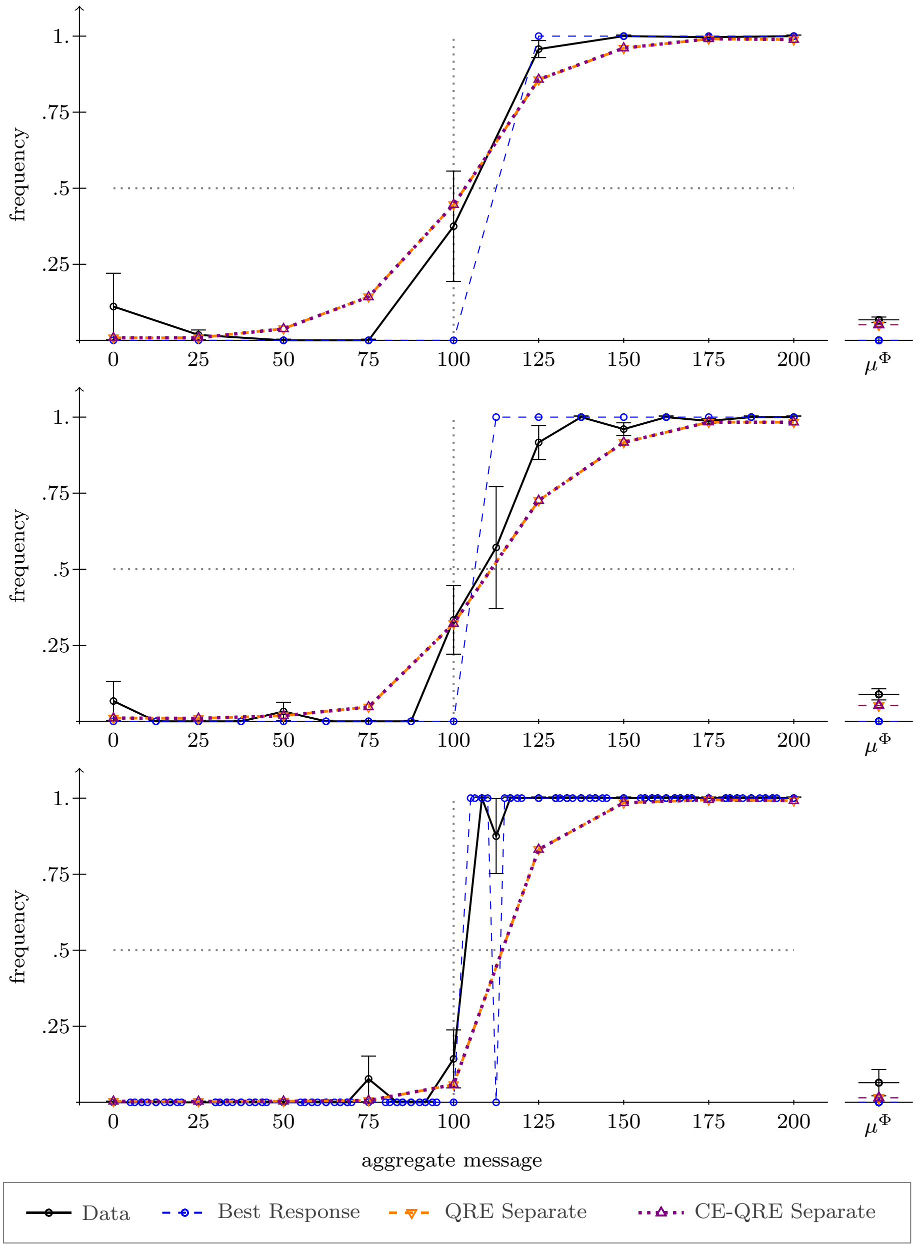


Figure 4.3: CE-QRE, rate receiver makes risky selection. N=1 (Above), N=2 (Middle), N=5 (Below). “Pooled” data pooled across treatments and roles. “Separate” estimates for each role and each treatment

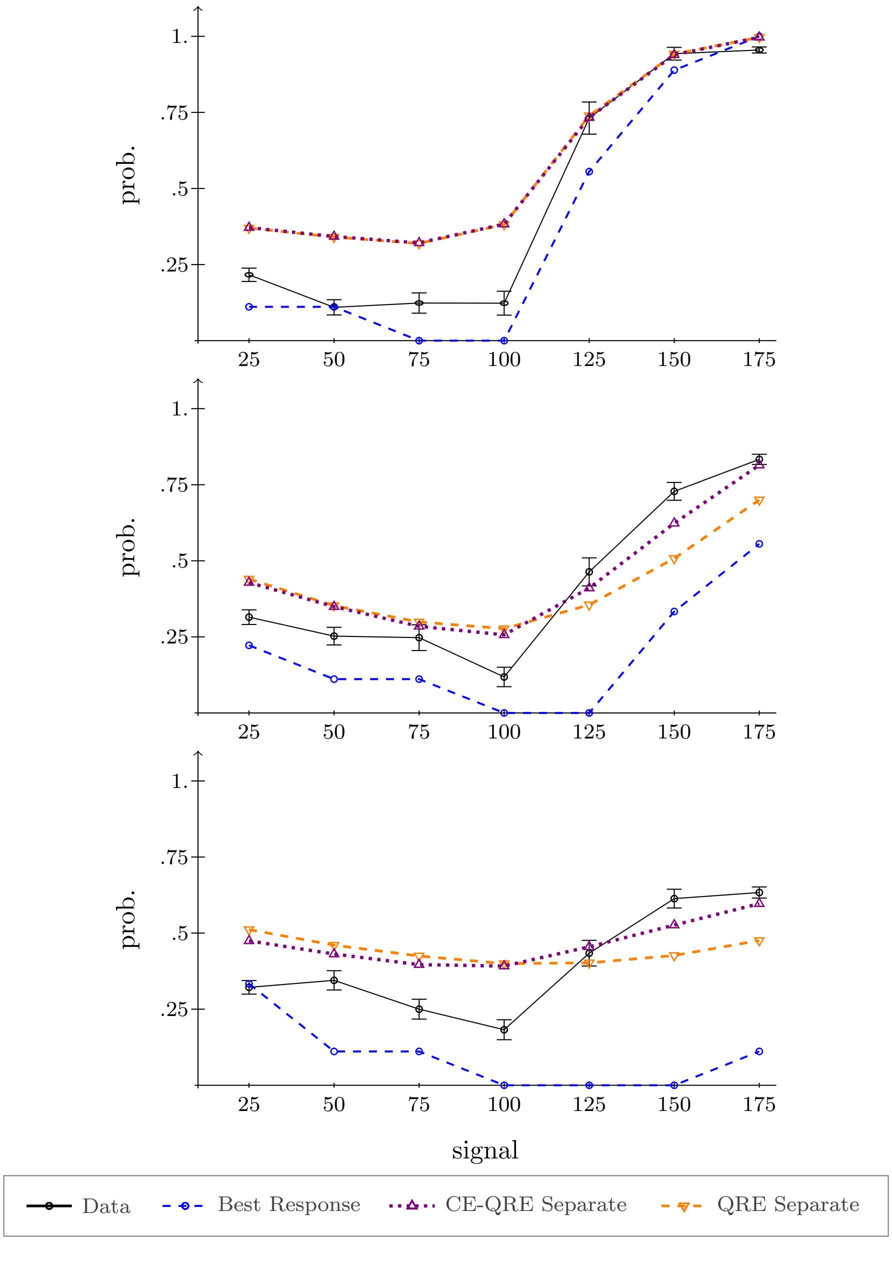


Figure 4.4: CE-QRE, sender participation, N=1 (Above), N=2 (Middle), N=5 (Below). “Separate” estimates for each role and each treatment

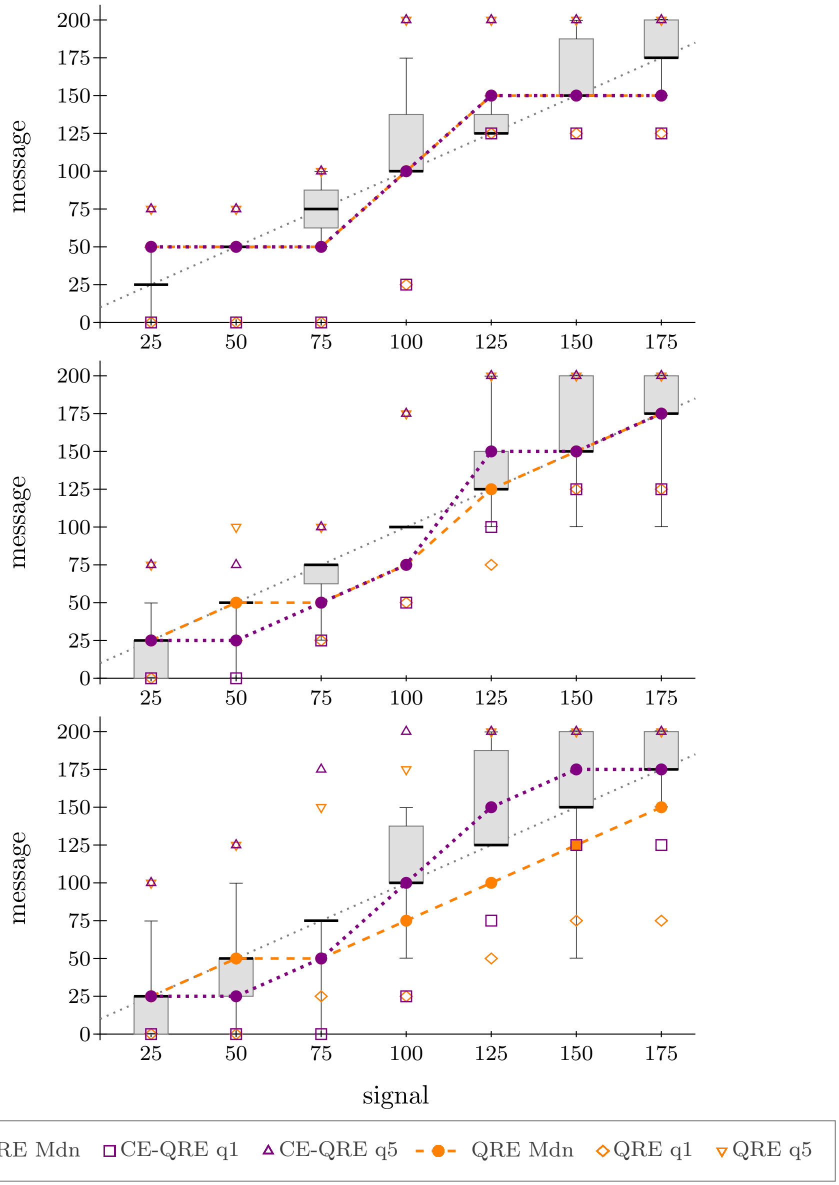


Figure 4.5: CE-QRE, sender messaging, N=1 (Above), N=2 (Middle), N=5 (Below). Boxplots display behavior in sextiles (6-quantiles). CE-QRE estimates for median, first, and fifth sextile predictions estimated separately for each role and each treatment

Appendices

Appendix A

Proofs for Chapter 2

Remark A.0.1. *Because we are considering a simultaneous-move, Bayesian game, there are many relevant beliefs. Because of this, we consolidate notation in this appendix by adopting for all beliefs the notational convention in the main text. To describe a distribution or a density of a continuous variable (or a variable the properties of which we are uncertain), we use a generic* F *and* f*, respectively, followed by a subscript containing: the player who holds the belief*[1]*; a semi-colon; the random variable about which the belief is formed; a vertical bar; and then any conditional variables. For example, player* i*'s belief that the outcome of the aggregate message is* t_r*, given the state* θ^h*, is described by* $f_{i;\mu(\boldsymbol{m})|\theta^h}(t_r)$*. For discrete variables we use standard probability notation. For example, we describe* i*'s belief that* $\boldsymbol{n} = \{1,2\}$*, given that the state of the world is* θ^h*, by* $Pr_i(\boldsymbol{n} = \{1,2\}|\theta^h)$*.*

Proof of lemma 2.3.7.

Given any $\theta \in \Theta$, consider the beliefs of an arbitrary player $j \in \mathcal{I}_1$ over another arbitrary first-period player, $i \in \mathcal{I}_1$,'s actions, $Pr_j(s_i \in Y|t_j, \theta)$. Where Y is any set of actions. If player i plays the mixed

[1]This is only included if it is relevant. For commonly held beliefs, the name of the player who holds the belief is excluded. Moreover, for the most part, the complication of individual sets of beliefs is avoided through a symmetry refinement of the equilibrium concept, which is made in the next section.

strategy $\pi_i(s|t_i)$, and $Q \subseteq T_s$ is the pre-image of π_i under Y, then

$$\begin{aligned} Pr_j(s_i \in Y|t_j,\theta) &= \sum_{t_i \in Q} \pi_i(s_i;t_i) Pr_j(s_i|\theta,t_j) \\ &= \sum_{t_i \in Q} \pi_i(s_i;t_i) Pr_j(t_i|\theta) \\ &= Pr_j(s_i \in Y|\theta) = Pr(s_i \in Y|\theta) \end{aligned} \tag{A.1}$$

The second equality follows from the fact that types are independently distributed conditional on θ, and the third from the fact that the quantity no longer relies on t_j. After the final equality we drop the subscript j to denote the fact that since the quantity does not depend on j or t_j in any way, and all other quantities are known in equilibrium, all first-period agents will have the same beliefs. By a similar process it is easy to show that for any set of agents, $-j$, and any set of sets of actions $\mathcal{Y}$, $Pr_j(s_{-j} \in \mathcal{Y}|\theta,t_j) = Pr(s_{-j} \in \mathcal{Y}|\theta)$.

Let $Z \subseteq T_r$, and let $\mathcal{Y} \subseteq \mathcal{M}$ be the pre-image of μ under Z. Since μ is a known function of sent messages, $\mu(\mathbf{m}_{-i}) \in Z$ if and only if $s_{-j} \in \mathcal{Y}$. Therefore,

$$\begin{aligned} Pr_j(\mu(\mathbf{m}_{-i}) \in Z|\theta,t_j) &= Pr_j(s_{-j} \in \mathcal{Y}|\theta,t_j) \\ &= Pr(s_{-j} \in \mathcal{Y}|\theta) \\ &= Pr(\mu(\mathbf{m}_{-i}) \in Z|\theta) \end{aligned}$$

We can construct beliefs over $\mathbf{n}_{-i} = \{j \in \mathcal{I}_1 : \rho_j = 1\}$ in similar terms to (A.1). Let s_i^1 be the set of actions for i in which i participates. Then if i plays the mixed strategy $\pi_i(s_i;t_i)$,

$$\begin{aligned} Pr_j(\mathbf{n}_{-i}|\theta,t_j) &= \prod_{i \in \mathbf{n}_{-i}} \sum_{t_i} \sum_{s_i \in s_i^1} \pi_i(s_i;t_i) Pr(t_i|\theta,t_j) \prod_{k \notin \mathbf{n}_{-i}} \sum_{t_k} \sum_{s_k \notin s_k^1} \pi_k(s_k;t_k) Pr(t_k|\theta,t_j) \\ &= \prod_{i \in \mathbf{n}_{-i}} \sum_{t_i} \sum_{s_i \in s_i^1} \pi_i(s_i;t_i) Pr(t_i|\theta) \prod_{k \notin \mathbf{n}_{-i}} \sum_{t_k} \sum_{s_k \notin s_k^1} \pi_k(s_k;t_k) Pr(t_k|\theta) \\ &= Pr(\mathbf{n}_{-i}|\theta) \end{aligned}$$

□

Lemma 2.4.4 In equilibrium, for any v_i, $Pr(\rho_i = 1|v_i) < 1$, and $\forall\ \theta$, $Pr(\mu^\Phi|\theta) > 0$.

Proof of lemma 2.4.4. Let i be any sender. By lemma 2.4.3, $Pr(\rho_i = 1|v_i) = F_c(X(v_i))$, so $Pr(\rho_i = 1|v_i) < 1$ iff $F_c(X(v_i)) < 1$. By assumption 2.3.5, for any $c_i \leq max\{E[v_r|\theta^h], -E[v_r|\theta^l]\}$, $F_c(c_i) < 1$. Thus, we must only show that $X(v_i)$ is bounded above by $max\{E[v_r|\theta^h], -E[v_r|\theta^l]\}$. By definition, for all v_i and θ,

$$X(v_i) = (1-Pr(\theta^l|v_i))E[v_r|\theta^h] \sum_{Y\in\{L,I,H,\Phi\}} \sum_{Z\in\{L,I,H\}} [w(Z) - w(Y)]\, P(YZ|\theta^h, M(v_i))+ \qquad (A.2)$$

$$Pr(\theta^l|v_i)\ E[v_r|\theta^l] \sum_{Y\in\{L,I,H,\Phi\}} \sum_{Z\in\{L,I,H\}} [w(Z) - w(Y)]\, P(YZ|\theta^l, M(v_i))$$

where

$$\sum_{Y\in\{L,I,H,\Phi\}} \sum_{Z\in\{L,I,H\}} [w(Z) - w(Y)]\, P(YZ|m_i, \theta) = \{ \qquad (A.3)$$

$$a^I Pr(LI|m_i,\theta) + \qquad Pr(LH|m_i,\theta)$$

$$-\quad a^I Pr(IL|m_i,\theta) \qquad + \quad (1-a^I)Pr(IH|m_i,\theta)$$

$$-\quad Pr(HL|m_i,\theta) - \quad (1-a^I)Pr(HI|m_i,\theta)$$

$$-A(\mu^\Phi)Pr(\Phi L|m_i,\theta)\ + (a^I - A(\mu^\Phi))Pr(\Phi I|m_i,\theta)\ + (1-A(\mu^\Phi))Pr(\Phi H|m_i,\theta)\}$$

From (A.3) we know that the summation weights the probabilities of members of a partition with weights that are each in $[-1, 1]$, and this implies that (A.3) must also be in $[-1, 1]$. It is therefore clear from (A.2), that $X(v_i) \leq max\{E[v_r|\theta^h], -E[v_r|\theta^l]\}$.

Finally, since we are considering symmetric strategies,

$$Pr(\mu^\Phi|\theta) = Pr(\rho_i = 0|\theta)^N = \left(1 - \int_V F_c(X(v_i)) f_{v|\theta}(v_i) dv_i\right)^N > 0 \qquad (A.4)$$

where the final inequality follows from the fact that $F_c(X(v_i)) < 1\ \forall v_i$. □

Proof of lemma 2.4.5. First, we show that $E[u_i|t_i, \rho_i = 1]$ must be continuous in v_i. Consider an arbitrary first period agent, i. Since we are considering PBNE, equilibrium strategies must be utility maximizing at each information set. This implies that M must satisfy incentive compatibility regardless of P.

Choose an arbitrary set of strategies $((P, M), A, g) \in \beta(G)$, $i \in \mathcal{I}_1$, $\delta > 0$, and $v_i \in V$. We want to show that there exists an ϵ such that $\forall\ v_i' \in (v_i - \epsilon, v_i + \epsilon)$, $|E[u_i|t_i, \rho_i = 1] - E[u_i(M(v_i'), \rho_i = 1, t_i')|t_i']| < \delta$.

Since $Pr(\theta^h|v_i)$ is continuous in v_i by assumption, $\forall\ \delta' > 0$, $\exists\ \epsilon'$ s.t. $\forall\ v_i' \in (v_i - \epsilon', v_i + \epsilon')$, $|Pr(\theta^h|v_i) - Pr(\theta^h|v_i')| < \delta'$. Choose any $0 < \delta' < \frac{\delta}{E[v_r|\theta^h] - E[v]\theta^l]}$ and any $v_i' \in (v_i - \epsilon', v_i + \epsilon')$. Then there exists $\delta'' \geq 0$ such that $\delta'' < \delta'$ and $|Pr(\theta^h|v_i) - Pr(\theta^h|v_i')| = \delta''$. Without loss of generality, let $Pr(\theta^h|v_i) = Pr(\theta^h|v_i') - \delta''$ (the proof in the other direction follows the same steps). The IC conditions insist that,

$$E[u_i|t_i, \rho_i = 1] \geq E[u_i(M(v_i'), \rho_i = 1, t_i)|t_i] \tag{A.5}$$

$$E[u_i(M(v_i'), \rho_i = 1, t_i')|t_i'] \geq E[u_i(M(v_i'), \rho_i = 1, t_i')|t_i'] \tag{A.6}$$

Subtracting $E[u_i(M(v_i'), \rho_i = 1, t_i')|t_i']$ from both sides of (A.5), we get

$$\begin{aligned} & E[u_i|t_i, \rho_i = 1] - E[u_i(M(v_i'), \rho_i = 1, t_i')|t_i'] && \text{(A.7)} \\ & \geq -\delta'' \sum_{Y \in \{L,I,H,\Phi\}} \sum_{Z \in \{L,I,H\}} w(Z) \left[E[v_r|\theta^h] P(YZ|M(v_i'), \theta^h) - E[v_r|\theta^l] P(YZ|M(v_i'), \theta^l) \right] \end{aligned}$$

Switching the sign and substituting δ',

$$
\begin{aligned}
&E[u_i(M(v_i'), \rho_i = 1, t_i')|t_i'] - E[u_i|t_i, \rho_i = 1] \\
&\leq \delta' \sum_{Y \in \{L,I,H,\Phi\}} \sum_{Z \in \{L,I,H\}} w(Z) \left[E[v_r|\theta^h] P(YZ|M(v_i'), \theta^h) - E[v_r|\theta^l] P(YZ|M(v_i'), \theta^l) \right] \\
&< \delta \frac{\sum_{Y \in \{L,I,H,\Phi\}} \sum_{Z \in \{L,I,H\}} w(Z) \left[E[v_r|\theta^h] P(YZ|M(v_i'), \theta^h) - E[v_r|\theta^l] P(YZ|M(v_i'), \theta^l) \right]}{E[v_r|\theta^h] - E[v_r|\theta^l]} \\
&\leq \delta
\end{aligned} \tag{A.8}
$$

where the last inequality follows from the fact that $\sum_{Y \in \{L,I,H,\Phi\}} \sum_{Z \in \{L,I,H\}} w(Z) P(YZ|m_i, \theta) \in [0, 1]$. Subtracting $E[u_i|t_i, \rho_i = 1]$ from both sides of (A.6) yields

$$
\begin{aligned}
&E[u_i(M(v_i'), \rho_i = 1, t_i')|t_i'] - E[u_i|t_i, \rho_i = 1] \\
&\geq \delta'' \sum_{Y \in \{L,I,H,\Phi\}} \sum_{Z \in \{L,I,H\}} w(Z) \left[E[v_r|\theta^h] P(YZ|\theta^h, M(v_i)) - E[v_r|\theta^l] P(YZ|\theta^l, M(v_i)) \right] \\
&\geq 0 > -\delta
\end{aligned} \tag{A.9}
$$

Together, (A.8) and (A.9) imply for all $v_i' \in (v_i - \epsilon', v_i + \epsilon')$, $|E[u_i|t_i, \rho_i = 1] - E[u_i(M(v_i'), \rho_i = 1, t_i')|t_i']| < \delta$, which completes the proof.

To see that $E[u_i|t_i, \rho_i = 0]$ is continuous in v_i, we need only observe equation 2.6; $E[u_i|t_i, \rho_i = 0]$ is a function of v_i only through a linear transformation of $Pr(\theta|v_i)$, which is continuous by assumption. These results immediately implies that $W(\cdot)$ is continuous as well. □

Lemma 2.4.6 If, in equilibrium, $E[v_r|\mu(\mathbf{m}_{-i}) \neq \mu^\Phi, v_i]$ is continuous in v_i, then the receiver's type space is described by a simple partition, formed around v^Φ.

Proof of lemma 2.4.6. Given the assumptions, we wish to show that for any v_i, $Q \in \{L, I, H\}$, and $K \in \{[\underline{v}, v^\Phi), (v^\Phi, \overline{v}]\}$, such that $\sigma(v_i) \in Q$ and $v_i \in K$, $\Sigma(K) \subseteq Q$.

We know from lemma 2.4.5 that, in equilibrium, $E[u_i|t_i, \rho_i = 1]$ must be continuous in v_i.

Expanding,

$$E[u_i|t_i, \rho_i = 1] = \sum_{\circledast \in \{=, \neq\}} Pr(\mu(\mathbf{m}_{-i}) \circledast \mu^{\Phi}) E[v_r|\mu(\mathbf{m}_{-i}) \circledast \mu^{\Phi}, v_i, \rho_i = 1]$$

We assumed that $E[v_r|\mu(\mathbf{m}_{-i}) \neq \mu^{\Phi}, v_i]$ is continuous is v_i, which implies that $E[v_r|\mu(\mathbf{m}_{-i}) \neq \mu^{\Phi}, v_i, \rho_i = 1]$ is continuous in v_i. Moreover, $Pr(\mu(\mathbf{m}_{-i}) = \mu^{\Phi}) > 0$ by lemma 2.4.4. Therefore, in equilibrium, $E[v_r|\mu(\mathbf{m}_{-i}) = \mu^{\Phi}, v_i, \rho_i = 1]$ must also be continuous in v_i. $E[v_r|\mu(\mathbf{m}_{-i}) = \mu^{\Phi}, v_i, \rho_i = 1]$ is continuous if and only if for any $\delta > 0$, there exists an $\epsilon > 0$ such that, if $v_i' \in (v_i - \epsilon, v_i + \epsilon)$, then

$$\left| \sum_{\theta \in \Theta} E[v_r|\theta] Pr(\theta|v_i) \left(\sum_{Z \in \{L,I,H\}} w(Z) P(\Phi Z|M(v_i), \theta) - \sum_{Z' \in \{L,I,H\}} w(Z') P(\Phi Z'|M(v_i'), \theta) \right) \right| < \delta \tag{A.10}$$

where the first term follows from the fact that $Pr(\theta|v_i)$ is continuous by definition. Assume $\sigma(v_i) \in H$. Then $Pr(\Phi H|M_i(v_i), \theta) = Pr(\Phi|\theta)$. Furthermore, this implies that if $\sigma(v_i') \in H$, then the left-hand side of (A.10) is equal to 0, and (A.10) therefore holds for all δ. However, if $\sigma(v_i') \in I$, then (A.10) simplifies to

$$\left| (1 - a^I) \sum_{\theta \in \Theta} E[v_r|\theta] Pr(\theta|v_i) Pr(\Phi|\theta) \right| < \delta \tag{A.11}$$

and if $\sigma(v_i') \in L$, then (A.10) simplifies to

$$\left| \sum_{\theta \in \Theta} E[v_r|\theta] Pr(\theta|v_i) Pr(\Phi|\theta) \right| < \delta \tag{A.12}$$

By lemma 2.4.4, $Pr(\Phi|\theta) > 0$. Therefore, since we assumed that $a^I \in (0, 1)$, the left-hand sides of (A.11) and (A.12) are strictly positive for all $v_i \neq v^{\Phi}$.

Choose δ' such that $0 < \delta' < (1 - a^I) \left| \sum_{\theta \in \Theta} E[v_r|\theta] Pr(\theta|v_i) Pr(\Phi|\theta) \right|$. By expression A.12 (A.11), to achieve continuity, for all v_i such that $\sigma(v_i) \in H \setminus \sigma(v^{\Phi})$, there must exist a neighborhood around

v_i that for all v' in that neighborhood, $\sigma(v_i') \in H$. That is, for all v_i, there exists $\epsilon > 0$ such that if $v_i' \in (v_i - \epsilon, v_i + \epsilon)$ then $\sigma(v_i') \in H$. If not; if for all $\epsilon > 0$ there exists a $v_i' \in (v_i - \epsilon, v_i + \epsilon)$ such that $\sigma(v') \in L$ (or $\sigma(v') \in I$); then there does not exist an $\epsilon > 0$ such that expression (A.12) (A.11) holds for δ', contradicting lemma 2.4.5.

The proof to this point follows the same steps if we assume $\sigma(v_i) \in L$ or $\sigma(v_i) \in I$.

We conclude that for any $Q \in \{L, I, H\}$ and any $K \in \{[\underline{v}, v^\Phi), (v^\Phi, \overline{v}]\}$, if $\sigma(v_i) \in Q$ and $v_i \in K$, then $\Sigma(K) \subseteq Q$. Assume to the contrary, and choose disjoint $Z, Q, Y \in \{L, I, H\}$. By contradiction, $\Sigma(K) \nsubseteq Q$. By the above arguments, there exists an open ball around v_i whose image under σ is also in Q. Let Q' be the largest possible such open ball[2]. By the above arguments and by the definition of Q', there exists a v' in the closure of Q' such that $\sigma(v') \notin Q$, and since $\Sigma(K) \nsubseteq Q$, $v' \in K$. This implies that $\sigma(v') \in Y \cup Z$, which implies, by the above arguments, that there is an open ball around v' whose image under σ is a subset of $Y \cup Z$. However, this contradicts the definition of Q'. Therefore, it must be that $\Sigma(K) \subseteq Q$. □

Lemma 2.4.9 For any game with a regular message aggregator, if in equilibrium: the receiver's type space is described by a simple partition around v'; $E[u_i|t_i, \rho_i = 1]$ is differentiable w.r.t. v_i everywhere but at a finite set of points; and both the distribution of aggregate messages conditional on θ and the distribution of aggregate messages conditional on θ and m_i are massless on ($M(\underline{v})$ or $M(\overline{v})$), then letting $Q, K \in \{[\underline{v}, v'), (v', \overline{v}]\}$ be disjoint:

A) $X(v_i)$ is increasing on Q if either:

i) $\Sigma(Q) \subseteq H$

ii) $\Sigma(Q) \subseteq I$, $\Sigma(K) \subseteq L$, and $A(\mu^\Phi) \leq a^I$

B) $X(v_i)$ is decreasing in Q if either:

i) $\Sigma(Q) \subseteq L$

ii) $\Sigma(Q) \subseteq I$, $\Sigma(K) \subseteq H$, and $A(\mu^\Phi) \geq a^I$

[2]Because of the boundaries at $\underline{v}$ and $\overline{v}$, Q' may technically be half open. This does not affect the proof. If Q' does not exist, then let Q' be any point in Q.

Proof of proposition 2.4.9. By assumption $E[u_i|t_i, \rho_i = 1]$ is differentiable everywhere but at a finite set of points in Q. Let $Q' \subset Q$ be any open, convex subset in which $E[u_i|t_i, \rho_i = 1]$ is differentiable. Then $X(v_i)$ is strictly increasing on Q' if and only if $\forall\ v_i \in Q'$, $\frac{d}{dv_i}(E[u_i|t_i, \rho_i = 1] - E[u_i|t_i, \rho_i = 0]) > 0$ and strictly decreasing on Q' if and only if $\forall\ v_i \in Q'$, $\frac{d}{dv_i}(E[u_i|t_i, \rho_i = 1] - E[u_i|t_i, \rho_i = 0]) < 0$. We know $\frac{d}{dv_i}(E[u_i|t_i, \rho_i = 1] - E[u_i|t_i, \rho_i = 0]) = \frac{\partial E[u_i|t_i,\rho_i=1]}{\partial m_i}\frac{dm_i}{dv_i} + \frac{\partial E[u_i|t_i,\rho_i=1]}{\partial v_i} - \frac{\partial E[u_i|t_i,\rho_i=0]}{\partial v_i}|_{m_i=M(v_i)}$. Moreover, by IC, $\frac{dE[u_i|t_i,\rho_i=1]}{dm_i} = 0|_{m_i=M(v_i)}$. This implies that, in equilibrium, $X(v_i)$ is increasing on Q' if and only if, $\forall\ v_i \in Q'$, $\frac{\partial E[u_i|t_i,\rho_i=1]}{\partial v_i} - \frac{\partial E[u_i|t_i,\rho_i=0]}{\partial v_i} > 0$. By (A.2), this is true if and only if

$$\frac{dPr(\theta^h|v_i)}{dv_i}\Bigg[E[v_r|\theta^h]\sum_{Y\in\{L,I,H,\Phi\}}\sum_{Z\in\{L,I,H\}}[w(Z)-w(Y)]\,P(YZ|\theta^h, M(v_i)) - E[v_r|\theta^l]\sum_{Y\in\{L,I,H,\Phi\}}\sum_{Z\in\{L,I,H\}}[w(Z)-w(Y)]\,P(YZ|\theta^l, M(v_i))\Bigg] \geq 0 \qquad \text{(A.13)}$$

Proof of B.ii: Assume $\Sigma\left([\underline{v}, \sigma(v')\right) = I$, $\{\sigma\left(v'\right)\} = L$, $\Sigma\left((v', \overline{v}]\right) = H$, and $\sigma(v_i) \in I$. Since μ is regular, if $\mu(\mathbf{m}_{-i}) \in I$, then $Pr(\mu(\mathbf{m}) \in I) = 1$, or, equivalently, $Pr(IH|M(v_i)) = Pr(IL|M(v_i)) = 0$. By the same argument, $Pr(LH|M(v_i)) = 0$. Moreover, since $F_{\mu(\mathbf{m}_{-i})|\theta}(t_r)$ are massless on $(\underline{v}, \overline{v})$ and $|L| = 1$, $Pr(LI|M(v_i)) = 0$. Therefore, by (A.3), (A.13) reduces to

$$\frac{dPr(\theta^h|v_i)}{dv_i}\left[E[v_r|\theta^h]\left((a^I - A(\mu^\Phi))Pr(\Phi|\theta^h) - Pr(HL|M_i(v_i), \theta^h) - (1-a^I)Pr(HI|M_i(v_i), \theta^h)\right) + \right. \qquad \text{(A.14)}$$

$$\left. -E[v_r|\theta^l]\left((a^I - A(\mu^\Phi))Pr(\Phi|\theta^l) - Pr(HL|M_i(v_i), \theta^l) - (1-a^I)Pr(HI|M_i(v_i), \theta^l)\right)\right]$$

since $a^I \leq A(\mu^\Phi)$ by assumption, all of the weighted event probabilities are non-positive. The remaining terms are positive by inspection and so (A.14) must be non-positive. Of course, if any of the event probabilities are positive, then (A.14) is negative, and participation is *strictly* decreasing.

We showed that participation is decreasing on any open, convex set $Q' \subset Q$, where $E[u_i|t_i, \rho_i = 1]$ is differentiable with respect to v_i for all $v_i \in Q'$. By assumption $E[u_i|t_i, \rho_i = 1]$ is differentiable with respect to v_i at all but a set of points of measure-0 in Q. Call this set $O \subset Q$. By definition, for any $v_i \in O \setminus \{\underline{v}, \overline{v}\}$, there exist open, convex subsets $Q', Q'' \subset Q$ such that $supQ' = v_i = infQ''$. By lemma 2.4.5, in equilibrium $X(v_i)$ is continuous in v_i. This implies that $\lim_{v \to v_i^-} W(v) = X(v_i) = \lim_{v \to v_i^+} W(v)$. Since $W(\cdot)$ is decreasing over Q' and Q'', this implies that $W(\cdot)$ is decreasing over $Q' \cup v_i \cup Q''$. We can not repeat this process for all $v_i \in O$ (adapting it in the obvious way for the endpoints $\underline{v}$ and $\overline{v}$), to show that $W(\cdot)$ is decreasing over all of Q.

Proof of A.i: Assume the same value sets as above, but let $\sigma(v_i) \in H$. By the same arguments found in the proof of B.ii, $Pr(HI|M(v_i)) = Pr(HL|M(v_i)) = Pr(LI|M(v_i)) = 0$. Moreover, since $F_{\mu(\mathbf{m})|\theta,m_i}(t_r)$ are massless on $(\underline{v}, \overline{v})$ and $|L| = 1$, $Pr(IL|M(v_i)) = 0$. Therefore, by (A.3), (A.13) reduces to

$$\frac{dPr(\theta^h|v_i)}{dv_i}\big[E[v_r|\theta^h]\left((1 - A(\mu^\Phi))Pr(\Phi|\theta^h) + Pr(LH|M_i(v_i), \theta^h) + (1 - a^I)Pr(IH|M_i(v_i), \theta^h)\right) + \\ -E[v_r|\theta^l]\left((1 - A(\mu^\Phi))Pr(\Phi|\theta^l) + Pr(LH|M_i(v_i), \theta^l) + (1 - a^I)Pr(IH|M_i(v_i), \theta^l)\right)\big]$$

which is positive by inspection. By the same arguments as above, this implies that $W(\cdot)$ is increasing over all of Q. The proofs of A.ii and B.i follow a similar set of steps. □

Corollary 2.4.9 *For any game with a regular message aggregator, if in equilibrium, the receiver's type space is described by a simple partition (L, H) formed around v', and $E[u_i|t_i, \rho_i = 1]$ is differentiable w.r.t. v_i everywhere but at a finite set of points, then $X(v_i)$ is decreasing in $[\underline{v}, v')$ and increasing in $(v', \overline{v}]$.*

Proof of corollary 2.4.9. Consider the case in which $[\underline{v}, v') \subseteq L$ and $(v', \overline{v}] \subseteq H$ and let $\sigma(v_i) \in H$. Since μ is regular $Pr(HL|M_i(v_i), \theta^h) = Pr(HI|M_i(v_i), \theta^h) = Pr(IL|M_i(v_i), \theta^h) = 0$. Therefore, by (A.3), (A.13) reduces to

$$\frac{dPr(\theta^h|v_i)}{dv_i}\big[E[v_r|\theta^h]\left((1 - A(\mu^\Phi))Pr(\Phi|\theta^h) + a^I Pr(LI|M_i(v_i), \theta^h) + Pr(LH|M_i(v_i), \theta^h) + (1 - a^I)Pr(IH|M_i(v_i), \theta^h)\right) + \\ -E[v_r|\theta^l]\left((1 - A(\mu^\Phi))Pr(\Phi|\theta^l) + a^I Pr(LI|M_i(v_i), \theta^l) + Pr(LH|M_i(v_i), \theta^l) + (1 - a^I)Pr(IH|M_i(v_i), \theta^l)\right)\big]$$

which is non-negative by inspection. The proofs for when $\sigma(v_i) \in L$ or when $[\underline{v}, v') \subseteq H$ and

$(v', \overline{v}] \subseteq L$ follows the same steps. □

Lemma 2.4.8 For any game with strictly regular message aggregator, in equilibrium, if M is strictly monotonic, then at least one of: $\sigma(\underline{v}) \in L$; $\sigma(\overline{v}) \in H$.

Proof of lemma 2.4.8. Assume M is strictly increasing. Then $M(\underline{v}) < M(v)$ for all v. We want to show that $g(\theta^h | t_r = M(\underline{v})) < \nu$. By Bayes' rule, this is true if and only if

$$f_{\mu(\mathbf{m})|\theta^h}(M(\underline{v}))E[v_r|\theta^h] < -f_{\mu(\mathbf{m})|\theta^l}(M(\underline{v}))E[v_r|\theta^l] \tag{A.15}$$

Since μ is strictly regular, $\mu(\mathbf{m}) = \mu(M(\underline{v}))$ if and only if, for all $j \in \mathbf{n}$, $v_j = \underline{v}$. Since $Pr(v_j = \underline{v}, \rho_j = 1|\theta) = Pr(\rho_j = 1|\theta, \underline{v}) f_{v|\theta}(\underline{v}) = F_c(W(\underline{v})) f_{v|\theta}(\underline{v})$, (A.15) is true if and only if

$$\sum_{n=1}^{N} \binom{N}{n} \left(f_{v|\theta^h}(\underline{v}) F_c(W(\underline{v}))\right)^n Pr(\rho = 0|\theta^h)^{N-n} E[v_r|\theta^h] <$$
$$-\sum_{n=1}^{N} \binom{N}{n} \left(f_{v|\theta^l}(\underline{v}) F_c(W(\underline{v}))\right)^n Pr(\rho = 0|\theta^l)^{N-n} E[v_r|\theta^l]$$

or, rearranging, if and only if

$$\sum_{n=1}^{N} \binom{N}{n} F_c\left(W(\underline{v})\right)^n \Big[f_{v|\theta^h}(\underline{v})^{n-1} Pr(\rho = 0|\theta^h)^{N-n} \left(E[v_r|\theta^h] f_{v|\theta^h}(\underline{v})\right) - f_{v|\theta^l}(\underline{v})^{n-1} Pr(\rho = 0|\theta^l)^{N-n} \left(-E[v_r|\theta^l] f_{v|\theta^l}(\underline{v})\right)\Big] < 0 \tag{A.16}$$

Inside the summation, the first two terms are strictly positive. Therefore, we want to show that, for all n, the bracketed part of the summand is negative. That is, we want to show that in any informative equilibrium, for any n,

$$\left(\frac{Pr(\rho = 0|\theta^h)}{Pr(\rho = 0|\theta^l)}\right)^{N-n} < \left(\frac{f_{v|\theta^l}(\underline{v})}{f_{v|\theta^h}(\underline{v})}\right)^{n-1} \left(\frac{-E[v_r|\theta^l] f_{v|\theta^l}(\underline{v})}{E[v_r|\theta^h] f_{v|\theta^h}(\underline{v})}\right) \tag{A.17}$$

By definition, $\frac{f_{v|\theta^l}(\underline{v})}{f_{v|\theta^h}(\underline{v})} > 1$. Moreover, by assumption 2.3.3, $\frac{-E[v_r|\theta^l] f_{v|\theta^l}(\underline{v})}{E[v_r|\theta^h] f_{v|\theta^h}(\underline{v})} > 1$. Therefore, if $\frac{Pr(\rho=0|\theta^h)}{Pr(\rho=0|\theta^l)} \leq 1$ then (A.17) holds for all n. This implies that every term of the summation in (A.16) is negative, and therefore that $\mu((\underline{v})) \in L$.

If $Pr(\rho = 0|\theta^h) \geq Pr(\rho = 0|\theta^l)$ then the same steps will show that $g(\theta^h|t_r = M(\overline{v})) > \nu$ and therefore $\mu(M(\overline{v})) \in H$. These things together imply that if $Pr(\rho = 0|\theta^h) = Pr(\rho = 0|\theta^l)$ then both $\mu(M(\underline{v})) \in L$ and $\mu(M(\overline{v})) \in H$, and since at least one of $Pr(\rho = 0|\theta^h) \geq Pr(\rho = 0|\theta^l)$ and $Pr(\rho = 0|\theta^h) \leq Pr(\rho = 0|\theta^l)$ must be true, the lemma is proved. Finally, if M is decreasing, then $\mu(M(\underline{v})) > \mu(M(v))$ for all v, and the proof proceeds in the same way. □

Corollary 2.4.8 *If the conditions of lemmas 2.4.6 and 2.4.8 hold, then* $\sigma(\underline{v}) \in L$ *and* $\sigma(\overline{v}) \in H$.

Proof of corollary 2.4.8. There are three possibilities: (a) $Pr(\rho = 0|\theta^h) < Pr(\rho = 0|\theta^l)$; (b) $Pr(\rho = 0|\theta^h) > Pr(\rho = 0|\theta^l)$; or (c) $Pr(\rho = 0|\theta^h) = Pr(\rho = 0|\theta^l)$. Lemma 2.4.8 showed that if (b) or (c) then $\mu(M(\underline{v})) \in L$ and if (a) or (c) then $\mu(M(\overline{v})) \in H$. Therefore it only remains to be shown that if the conditions of both lemmas 2.4.6 and 2.4.8 hold, then (b) implies $\mu(M(\underline{v})) \in L$ and (a) implies $\mu(M(\overline{v})) \in H$. Assume (b).

By lemma 2.4.6, $v^\Phi \in V$. Rearranging the definition of v^Φ, this implies

$$\frac{Pr(\Phi|\theta^h)}{Pr(\Phi|\theta^l)} \leq \frac{-E[v_r|\theta^l]f_{v|\theta^l}(\underline{v})}{E[v_r|\theta^h]f_{v|\theta^h}(\underline{v})} \tag{A.18}$$

By definition, $Pr(\Phi|\theta) = Pr(\rho = 0|\theta)^{N-1}$, and multiplying the right-hand side of (A.18) by $\left(\frac{f_{v|\theta^l}(\underline{v})}{f_{v|\theta^h}(\underline{v})}\right)^0$, (A.18) becomes a weak version of (A.17) when $n = 1$:

$$\left(\frac{Pr(\rho = 0|\theta^h)}{Pr(\rho = 0|\theta^l)}\right)^{N-1} \leq \left(\frac{f_{v|\theta^l}(\underline{v})}{f_{v|\theta^h}(\underline{v})}\right)^0 \left(\frac{-E[v_r|\theta^l]f_{v|\theta^l}(\underline{v})}{E[v_r|\theta^h]f_{v|\theta^h}(\underline{v})}\right) \tag{A.19}$$

We now show that (A.19) implies that (A.17) holds for all $n > 1$. Since $\frac{Pr(\rho=0|\theta^h)}{Pr(\rho=0|\theta^l)} > 1$ by assumption, $\left(\frac{Pr(\rho=0|\theta^h)}{Pr(\rho=0|\theta^l)}\right)^{N-n} < \left(\frac{Pr(\rho=0|\theta^h)}{Pr(\rho=0|\theta^l)}\right)^{N-1}$, for all $n > 1$. Furthermore, it remains that $\frac{f_{v|\theta^l}(\underline{v})}{f_{v|\theta^h}(\underline{v})} > 1$ and therefore $\left(\frac{f_{v|\theta^l}(\underline{v})}{f_{v|\theta^h}(\underline{v})}\right)^0 < \left(\frac{f_{v|\theta^l}(\underline{v})}{f_{v|\theta^h}(\underline{v})}\right)^{n-1}$, for all $n > 1$. Bringing these two inequalities together, (A.19) yields, for all $n > 1$,

$$\left(\frac{Pr(\rho = 0|\theta^h)}{Pr(\rho = 0|\theta^l)}\right)^{N-n} < \left(\frac{f_{v|\theta^l}(\underline{v})}{f_{v|\theta^h}(\underline{v})}\right)^{n-1} \left(\frac{-E[v_r|\theta^l]f_{v|\theta^l}(\underline{v})}{E[v_r|\theta^h]f_{v|\theta^h}(\underline{v})}\right)$$

which is a reproduction of condition (A.17) for all n. Therefore the summand of (A.16) is non-positive for all n and negative for all $n > 1$, which implies that (A.16) is negative. By the logic of lemma 2.4.8, this implies that $g(\theta | t_r = M(\underline{v})) < \nu$ which implies $\mu(M(\underline{v})) \in L$. The proof that $Pr(\rho = 0|\theta^h) < Pr(\rho = 0|\theta^l)$ implies $\mu(M(\overline{v})) \in H$ follows a similar set of steps. □

Proof of proposition 2.5.2. Consider any game and any equilibrium $((P, M), A, g)$. We prove the proposition by constructing an outcome-equivalent set of strategies, denoted $((P', M'), A', g')$, and showing that these strategies form an equilibrium of the game. First, let $P'(t_1) = P(t_1)$. Second, let

$$M^3(v_1) = \begin{cases} m^L, \text{ iff } \mu(M(v_1)) \in L \\ m^I, \text{ iff } \mu(M(v_1)) \in I \\ m^H, \text{ iff } \mu(M(v_1)) \in H \end{cases}$$

where m^L, m^I, m^H are arbitrary but distinct messages. Applying Bayes' rule, this construction implies $g'(\cdot)$ must be such that $\mu(m^L) \in L$, $\mu(m^I) \in I$, and $\mu(m^H) \in H$. To see this, take any v_1 such that $\mu(M(v_1)) \in L$. Since $((P, M), A, g)$ is an equilibrium, $\mu(M(v_1)) \in L$ implies that $E[v_r|\theta^h]Pr(\mu(M(v_1))|\theta^h) < -E[v_r|\theta^l]Pr(\mu(M(v_1))|\theta^l)$. Therefore, by construction of $M'(\cdot)$, $E[v_r|\theta^h]Pr(\mu(M'(v_1))|\theta^h) < -E[v_r|\theta^l]Pr(\mu(M'(v_1))|\theta^l)$, which implies that $g'(\mu(M'(v_1))) < \nu$ and therefore $\mu(M'(v_1)) = \mu(m^L) \in L$. By the same logic $\mu(m^I) \in I$ and $\mu(m^H) \in H$.

We showed that for all v_1, the sign of $g(\mu(M(v_1))) - \nu$ is equal to the sign of $g'(\mu(M'(v_1))) - \nu$. By proposition 2.4.1, this implies that $A(\mu(M(v_1))) = A'(\mu(M'(v_1)))$ for all v_1. Since we already assumed that $P'(t_1) = P(t_1)$, the two sets of strategies are outcome equivalent.

Next we show that $((P', M'), A', g')$ is an equilibrium. First we show that $M'(\cdot)$ satisfies incentive compatibility. Since $A(\mu(M(v_1))) = A'(\mu(M'(v_1)))$ for all v_1, the outcome is equivalent under both tuples of strategies/beliefs no matter the sender's type. That is, $E[u_1|t_1, \rho_1 = 1, M(v_1)] = E[u_1|t_1, \rho_1 = 1, M'(v_1)]$. Therefore, since $M(v_1)$ is an equilibrium strategy that satisfies incentive compatibility, so too must $M'(v_1)$.

Next, let $A'(\mu^\Phi) = A\left(\mu^\Phi\right)$. This implies that $E[u_1|t_1, \rho_1 = 0, M(v_1)] = E[u_1|t_1, \rho_1 = 0, M'(v_1)]$.

Along with the above arguments, this implies that the sender's net expected value to participation, $X(v_1)$, is equivalent for all v_1 under both (S, A, g) and (S', A', g'), which implies that under both sets of strategies/beliefs, the sender's optimal participation strategy is $P(t_1) = P'(t_1)$. $P'(t_1)$ therefore satisfies (IR).

Finally, since $P'(t_1) = P(t_1)$ and $A(\mu^\Phi)$ satisfies (2.2) then $A'(\mu^\Phi)$ satisfies (2.2) as well. For $t_r \in V$, $A'(t_r)$ satisfies (2.2) by construction. □

Proof of proposition 2.5.4. By lemma 2.5.3, for each feasible equilibrium, there are 4 quantities about which $g(\cdot)$ must be consistent. If we let Q, Y, and Z denote the distinct members the type-space partition, and let $\mu^\Phi \in Y$, then beliefs over the following likelihoods must match the implied likelihoods, given the strategies of the players: $Pr(t_r = \mu^\Phi)$, $Pr(t_r \in Y \setminus \mu^\Phi)$, $Pr(t_r \in Q)$, and $Pr(t_r \in Z)$. If $\mu^\Phi \in Y$ (for any $Y \in \{L, I, H\}$), let $Y_1 = \{v_i : \sigma(v_i) \in Y\}$, then[3]

$$Pr(t_r = \mu^\Phi|\theta) = (1 - F_c(0))Pr(v \in Y_1|\theta) + \int_{v \in V \setminus Y_1} f_{v|\theta}(v_1)\left(1 - F_c(X(v_1))\right) dv_1$$

$$Pr(t_r \in Y \setminus \mu^\Phi|\theta) = F_c(0)Pr(v \in Y_1) \tag{A.20}$$

$$Pr(t_r \in V \setminus Y|\theta) = \int_{v \in V \setminus Y_1} f_{v|\theta}(v_1)F_c(X(v_1))dv_1$$

Proof of 1, case B: In order for the strategies/beliefs described by part B of lemma 2.5.3 to be consistent, $Pr(t_r = \mu^\Phi|\theta^h) > \nu$, $Pr(t_r \in Y \setminus \mu^\Phi|\theta^h) > \nu$, and $Pr(t_r \in V \setminus Y|\theta^h) < \nu$. Applying Bayes' rule yields, respectively:

$$\begin{aligned}(1 - F_c(0)) \int_{v^I}^{\overline{v}} E[v_r|\theta^h]f_{v|\theta^h}(v) + E[v_r|\theta^l]f_{v|\theta^l}(v)dv + \\ \int_{\underline{v}}^{v^I} (1 - F_c(-E[v_r|v])) \left[E[v_r|\theta^h]f_{v|\theta^h}(v) + E[v_r|\theta^l]f_{v|\theta^l}(v)\right] dv > 0\end{aligned} \tag{A.21}$$

[3]Equation A.20 holds because $W(\cdot)$ is bounded weakly below by 0, as can be inferred from the above statements of expected utility and from lemma 2.5.3. Therefore, whenever $c_1 < 0$, $P(t_1) = 1$.

$$E[v_r|\theta^h]\int_{v^I}^{\overline{v}} f_{v|\theta^h}(v)dv + E[v_r|\theta^l]\int_{v^I}^{\overline{v}} f_{v|\theta^l}(v)dv > 0$$

$$E[v_r|\theta^h]\int_{\underline{v}}^{v^I} f_{v|\theta^h}(v)F_c(-E[v_r|v])dv + E[v_r|\theta^l]\int_{\underline{v}}^{v^I} f_{v|\theta^l}(v)F_c(-E[v_r|v])dv < 0$$

The last two inequalities hold by the definitions of v^I and assumption 2.3.2. We know the second term of inequality A.21 is negative by the definition of v^I. We also know that $-E[v_r|v_1]$ is positive when $v_1 \in [\underline{v}, v^\Phi)$. Therefore, if we replace $1-F_c(-E[v_r|v])$ with $1-F_c(0)$ in the integrand of (A.21), we know that the left-hand side of (A.21) is strictly greater than the resulting quantity. Thus, if the resulting quantity is weakly positive, it implies that the original inequality is strictly positive. That is

$$(A.21) > (1 - F_c(0))\int_{\underline{v}}^{\overline{v}} E[v_r|v]f_v(v)dv = E[v_r|\theta^h] + E[v_r|\theta^l] \tag{A.22}$$

Therefore, if $E[v_r|\theta^h] \geq -E[v_r|\theta^l]$, then (A.21) holds. Therefore, we showed that if $E[v_r|\theta^h] \geq -E[v_r|\theta^l]$ then the participation described by lemma 2.5.3 part B constitutes part of an equilibrium to the game.

By the same steps we can show that if $E[v_r|\theta^h] \leq -E[v_r|\theta^l]$ then part A of lemma 2.5.3 describes equilibrium participation. Since at least one of these conditions must be true for any game, at least one of A or B must describe an equilibrium for every game.

2) The proof for parts A and B is above. It only remains to show that there are environments for which the strategy/beliefs described by part C form part of an equilibrium. To prove this we first choose value distributions as in example 1 such that $f_{v|\theta^h}(v) = f_{v|\theta^l}(-v)$, $E[v|\theta^l] = -E[v|\theta^l]$, $v^I = 0$, $V = [-1, 1]$. Thus, applying the same logic to our consistency constructions as above, for

these to be part of an equilibrium, it must hold that

$$\int_0^{\overline{v}} (1 - F_c(E[v_r|v](1-a^I))) \left[f_{v|\theta^h}(v) - f_{v|\theta^l}(v)\right] dv +$$
$$\int_{\underline{v}}^0 (1 - F_c(-E[v_r|v]a^I)) \left[f_{v|\theta^h}(v) - f_{v|\theta^l}(v)\right] dv = 0 \tag{A.23}$$

$$f_{v|\theta^h}(0) - f_{v|\theta^l}(0) = 0$$

$$\int_{\underline{v}}^0 \left[f_{v|\theta^h}(v) - f_{v|\theta^l}(v)\right] F_c(-E[v_r|v]a^I)dv < 0$$

$$\int_0^{\overline{v}} \left[f_{v|\theta^h}(v) - f_{v|\theta^l}(v)\right] F_c(E[v_r|v](1-a^I))dv > 0$$

The final three consistency conditions clearly hold. Let $a^I = 0.5$. To see that (A.23) holds, note that by the symmetry of the environment $E[v_r|v] = -E[v_r|-v]$. Next, performing a change of variable on the second term of (A.23) yields:

$$\int_0^{\overline{v}} (1 - \frac{1}{2}F_c(E[v_r|v])) \left[f_{v|\theta^h}(v) - f_{v|\theta^l}(v)\right] dv - \int_{-\underline{v}}^0 (1 - \frac{1}{2}F_c(-E[v_r|-v])) \left[f_{v|\theta^h}(-v) - f_{v|\theta^l}(-v)\right] dv$$
$$= \int_0^{\overline{v}} (1 - \frac{1}{2}F_c(E[v_r|v])) \left[f_{v|\theta^h}(v) - f_{v|\theta^l}(v)\right] dv - \int_0^{\overline{v}} (1 - \frac{1}{2}F_c(E[v_r|v])) \left[f_{v|\theta^h}(v) - f_{v|\theta^l}(v)\right] dv$$
$$=0$$

□

Proof of lemma 2.6.1. This proof uses the definitions of Appendix B and densities constructed in Appendix B.

First, we prove the continuity of $f_{S|\theta}(m_i)$ for an arbitrary θ. By assumption, $f_{v|\theta}(v_i)$ are continuous. Since $M(\cdot)$ is monotonic and continuously differentiable by statement of the lemma, so too is $M^{-1}(\cdot)$.

By lemma 2.4.5, in equilibrium, $X(v_i)$ is continuous. Therefore, in equilibrium, each of $\frac{dM^{-1}(m_i)}{dm_i}$, $f_{v|\theta}(M^{-1}(m_i))$ and $W(M^{-1}(m_i))$ are continuous in m_i. Therefore, by our construction of $f_{S|\theta}(m_i)$ (see equation B.1), $f_{S|\theta}(m_i)$ is continuous in m_i, in equilibrium.

For a fixed set of senders, $\mathbf{n}$, we now prove the continuity of $f_{\mu(\mathbf{m})|\theta,\mathbf{n}}(\hat{\mu})$. The proofs that the remaining quantities are continuous either follow directly from this proof, or follow a similar set of steps to this proof.

We wish to show that the integrand and the bounds of integration of the construction of equation B.2 are continuous. First, we show the continuity of the integrand. We know that $f_{S|\theta}(\cdot)$ and $\mu^{-1}(\hat{\mu},\cdot)$ are continuous. Thus, $f_{S|\theta}(\mu^{-1}(\hat{\mu},\cdot))$, and therefore, the integrand, is continuous.

We now establish that the bounds of integration are continuous using the maximum theorem. For this application, the maximum theorem says that if $\eta(\cdot)$ is compact, and both upper and lower semi-continuous, then both $\overline{\eta}(\cdot)$ and $\underline{\eta}(\cdot)$ are continuous as well.

Compactness Let $\hat{\mu}_{<i} = (\hat{\mu}, \hat{\mu}_1, \hat{\mu}_2, ...\hat{\mu}_{i-1})$. Choose an arbitrary sender, i. Since V is bounded, $\eta(\hat{\mu}_{<i})$ is bounded. To show that $\eta(\hat{\mu}_{<i})$ is closed, we show that $\mathcal{R}^1 \setminus \eta(\hat{\mu}_{<i})$ is open. If $m_i \notin \eta(\hat{\mu}_{<i})$, it implies one of 3 cases: (1) $m_i \notin V$; (2) $m_i \in V$ and $\forall\ \mathbf{m}_{-i}$, $\mu(\mathbf{m}_{-i}, m_i) > \hat{\mu}$; or (3) $m_i \in V$ and $\forall\ \mathbf{m}_{-i}$, $\mu(\mathbf{m}_{-i}, m_i) < \hat{\mu}$. The set of m_i belonging to the first case is open because strict upper contour sets are open in $\mathcal{R}$. This, along with the fact that $\mu(\cdot)$ is continuous, imply that the set of m_i belonging to the second (third) case is open to the right (left). The set of m_i belonging to the second (third) case is closed to the left (right) because $m_i \in V$, which is closed. However, this constraint should also make it clear that the union of these three sets is open. $\eta(\hat{\mu}_{<i})$ is therefore closed.

Upper semi-continuity We want to show that for any open ball Y such that $\eta(\hat{\mu}_{<i}) \subseteq Y$, there exists an open ball around $\hat{\mu}_{<i}$, Z, such that for all $z \in Z$, $\eta(z) \subseteq Y$. Choose an arbitrary such Y. By the continuity of u^{-1}, for all $m_i \in \eta(\hat{\mu}_{<i})$, there exists a δ_{m_i} such that for all $\hat{\mu}'_{<i} \in (\hat{\mu}_{<i} - \delta_{m_i}, \hat{\mu}_{<i} + \delta_{m_i})$, $\eta(\hat{\mu}'_{<i}) \in Y$. Let $Z = \cap_{m_i \in \eta(\hat{\mu}_{<i})} (\hat{\mu}_{<i} - \delta_{m_i}, \hat{\mu}_{<i} + \delta_{m_i})$. Z is thus both open and contains $\hat{\mu}_{<i}$, and for all $z \in Z$, $\eta(z) \subseteq Y$.

Lower semi-continuity We want to show that for any $\mu_{<i}$ and any open ball Y such that $\eta(\hat{\mu}_{<i}) \cap Y \neq$

ϕ, there exists an open ball Z around $\hat{\mu}_{<i}$ such that for all $z \in Z$, $\eta(z) \cap Y \neq \phi$. Take an arbitrary $\mu_{<i}$ and assume such a Y. By the openness of Y, $\eta(\hat{\mu}_{<i}) \cap Y$ is not closed. Therefore, we can take an interior $m_i \in \eta(\hat{\mu}_{<i}) \cap Y$. This implies $M(\underline{v}) < m_i < M(\overline{v})$.

Define $\hat{\mu}_{>i}$ to be the set of messages from players $i+1, i+2, ..., n$ such that $\mu^{-1}(\hat{\mu}_{<i}, \hat{\mu}_{>i}) = m_i$. Then, by the continuity of $\mu^{-1}(\cdot)$ there exists an open ball Z around $\hat{\mu}_{<i}$, such that for all $z \in Z$, $\mu^{-1}(z, \hat{\mu}_{>i}) \in (M_i(\underline{v}), M_i(\overline{v}))$ and $\mu^{-1}(z, \hat{\mu}_{>i}) \in Y$. Of course, this implies that $\eta(z) \cap Y \neq \Phi$. □

Proof of lemma 2.6.2. This proof uses the densities constructed in Appendix B.

For any $V' \subseteq V$, $t \in V$, and participation level n, let $\underline{V'} = inf(V')$, $\overline{V'} = sup(V')$, and $\kappa(t, m_i, n) = \frac{t*n-m_i}{n-1}$. Thus, for example, $\kappa(\overline{V'}, m_i, n)$ is the value of the aggregate message (obtained when excluding i's message) such that when m_i is sent, $\mu(\mathbf{m}) = sup(V')$.

We intend to show that

$$Pr\left(\mu(\mathbf{m}_{-i}) \in Q,\ \mu(\mathbf{m}) \in K|\theta, n\right) = \tag{A.24}$$
$$max\left\{0, F_{\mu(\mathbf{m}_{-i})|\theta,n}\left(min\left\{\overline{Q}, \kappa\left(\overline{K}, m_i, n\right)\right\}\right) - F_{\mu(\mathbf{m}_{-i})|\theta,n}\left(max\left\{\underline{Q}, \kappa\left(\underline{K}, m_i, n\right)\right\}\right)\right\}$$

First we note that, by definition of a CDF, the right-hand side of (A.24) is equivalent to

$$Pr(\mu(\mathbf{m}_{-i}) \in (max\{\underline{Q}, \kappa(\underline{K}, m_i, n)\}, min\{\overline{Q}, \kappa(\overline{K}, m_i, n)\})|n, \theta) \tag{A.25}$$

Next, we wish to show equivalence of (A.25) and the left-hand side of (A.24). First we show that when $max\{\underline{Q}, \kappa(\underline{K}, m_i, n)\} \geq min\{\overline{Q}, \kappa(\overline{K}, m_i, n)\}$ both quantities are equal to 0. This is plainly true for (A.25). To see that this is true for the left-hand side of (A.24), note that in this case, one of two things can be true: $\kappa(\overline{K}, m_i, n) \leq \underline{Q}$; or $\kappa(\underline{K}, m_i, n) \geq \overline{Q}$. In the first case, since with the mean aggregation mechanism, $\frac{d}{dm_i}\mu(\mathbf{m})$ is invariant to realizations of $\mathbf{m}_{-i}$ (given n), $\kappa(\overline{K}, m_i, n) \leq \underline{Q}$ implies that $\forall\ \mathbf{m}_{-i} \in \{\mathbf{m}_{-i} : \mu(\mathbf{m}_{-i}) \in Q\}$, $\mu(\mathbf{m}) > \overline{K}$, and therefore $Pr(\mu(\mathbf{m}_{-i}) \in Q, \mu(\mathbf{m}) \in K|\theta, n) = 0$. By the same logic, the second case, $\kappa(\underline{K}, m_i, n) \geq \overline{Q}$, implies that $\forall\ \mathbf{m}_{-i} \in \{\mathbf{m}_{-i} : \mu(\mathbf{m}_{-i}) \in Q\}$, $\mu(\mathbf{m}) < \underline{K}$, and therefore $Pr(\mu(\mathbf{m}_{-i}) \in Q, \mu(\mathbf{m}) \in K|\theta, n) = 0$.

Next, we consider the case that $max\{\underline{Q}, \kappa(\underline{K}, m_i, n)\} < min\{\overline{Q}, \kappa(\overline{K}, m_i, n)\}$. To prove equivalence, we want to show that for any n and θ, $\mu(\mathbf{m}_{-i}) \in Q$ and $\mu(\mathbf{m}) \in K$ if and only if $\mu(\mathbf{m}_{-i}) \in (max\{\underline{Q}, \kappa(\underline{K}, m_i, n)\}, min\{\overline{Q}, \kappa(\overline{K}, m_i, n)\})$. First, assume $\mu(\mathbf{m}_{-i}) \in (max\{\underline{Q}, \kappa(\underline{K}, m_i, n)\}, min\{\overline{Q}, \kappa(\overline{K}, m_i, n$ By definition, this membership can occur if and only if $\mu(\mathbf{m}_{-i}) \in (\underline{Q}, \overline{Q}) \cap (\kappa(\underline{K}, m_i, n), \kappa(\overline{K}, m_i, n))$. Therefore, it is immediate that $\mu(\mathbf{m}_{-i}) \in Q$. Furthermore, since with the mean aggregation mechanism, $\frac{d}{dm_i}\mu(\mathbf{m})$ is invariant to $\mathbf{m}_{-i}$, and since $\mu(\mathbf{m}_{-i}) \in (\kappa(\underline{K}, m_i, n), \kappa(\overline{K}, m_i, n))$, it follows that $\mu(\mathbf{m}) \in (\underline{K}, \overline{K})$.

Next assume to the contrary that $\mu(\mathbf{m}_{-i}) \notin (max\{\underline{Q}, \kappa(\underline{K}, m_i, n)\}, min\{\overline{Q}, \kappa(\overline{K}, m_i, n)\})$. This implies that at least one of $\mu(\mathbf{m}_{-i}) \notin Q$ or $\mu(\mathbf{m}_{-i}) \notin (\kappa(\underline{K}, m_i, n), \kappa(\overline{K}, m_i, n))$. If it is the former, then the contrapositive is proved. If it is the latter, then either $\mu(\mathbf{m}_{-i}) > \kappa(\overline{K}, m_i, n)$ or $\mu(\mathbf{m}_{-i}) < \kappa(\underline{K}, m_i, n)$, which by the above logic imply $\mu(\mathbf{m}) > \overline{K}$ or $\mu(\mathbf{m}) < \underline{K}$, respectively; both implying that $\mu(\mathbf{m}) \notin K$, and therefore proving the contrapositive. □

Lemma 2.6.3 For any G with mean μ, in equilibrium, if M is strictly monotonic and continuously differentiable, then the conditions of lemma 2.4.6 are satisfied.

Proof of lemma 2.6.3. We wish to prove that $\sum_{Y\in\{L,I,H\}} \sum_{Z\in\{L,I,H\}} w(Z)P(YZ|M_i(v_i), \theta)$ is continuous in v_i. By lemma 2.6.1, $g(\theta^h|t_r)$ is continuous in t_r. This implies that each $Z \in \{L, H\}$ is composed of convex open sets. That is, for any $t_r \in Z$, there exists an $\epsilon > 0$ such that for all $t'_r \in (t_r - \epsilon, t_r + \epsilon) \cap V$, $t'_r \in Z$, although it is not necessarily the case that for all $t_r, t'_r \in Z$, if $t''_r \in (t_r, t'_r)$, then $t''_r \in Z$. Define a sub-outcome set of Z to be any convex, open region, $Z' = (\underline{Z}', \overline{Z}')$, such that for all $t_r \in Z'$, $t_r \in Z$, but: either $\underline{Z}' \notin Z$ or $\underline{Z}' = \underline{v}$; and either $\overline{Z}' \notin Z$ or $\overline{Z}' = \overline{v}$. By construction, the set of all such sub-outcome sets form a partition of Z (along with $\underline{v}$ and/or $\overline{v}$, if appropriate).

The continuity of $g(\theta^h|t_r)$ also implies that I is composed of convex, *closed* sets. Similar to above definition, define a sub-outcome set of I to be a closed region $I' = [\underline{I}', \overline{I}']$ such that for all $t_r \in I'$, $t_r \in I$, but for all $\epsilon > 0$, $\exists\ t_r \in [\underline{I}' - \epsilon, \underline{I}']$ and $\exists\ t'_r \in [\overline{I}', \overline{I}' + \epsilon]$ such that $t'_r \notin I$ and $t_r \notin I$. By construction, the set of all such sub-outcome sets form a partition of I.

Next, by lemma 2.6.1, $f_{\mu(\mathbf{m}_{-i})|\theta}(t_r)$ and $f_{\mu(\mathbf{m})|\theta,m_i}(t_r)$ are continuous. This implies that for any t_r, both $Pr(\mu(\mathbf{m}_{-i}) = t_r|\theta) = 0$ and $Pr(\mu(\mathbf{m}) = t_r|\theta) = 0$. Therefore, for any closed region I', for all $Z \in \{L, I, H\}$, both $Pr(\mu(\mathbf{m}_{-i}) \in I', \mu(\mathbf{m}) \in Z|\theta) = Pr(\mu(\mathbf{m}_{-i}) \in (\underline{I'}, \overline{I'})), \mu(\mathbf{m}) \in Z|\theta)$ and $Pr(\mu(\mathbf{m}_{-i}) \in Z, \mu(\mathbf{m}) \in I'|\theta) = Pr(\mu(\mathbf{m}_{-i}) \in Z, \mu(\mathbf{m}) \in (\underline{I'}, \overline{I'})|\theta)$. Because $(\underline{I'}, \overline{I'})$ is open, this implies that we can leverage lemma 2.6.2 to describe each of the event probabilities.

Let $Z, Y \in \{L, I, H\}$. In a given equilibrium, index all sub-outcome sets for Z, Z_1 to Z_J and all such sub-outcome sets for Y, Y_1 to Y_K. Then $Z = \bigcup_{j=1}^{J} Z_j$ and $Y = \bigcup_{k=1}^{K} Y_k$. For any set V', let $int(V')$ describe the interior of V'. Since the sub-outcome sets form a partition of their respective outcome sets,

$$\begin{aligned} Pr\left(ZY|m_i, \theta\right) &= \sum_{j=1}^{J}\sum_{k=1}^{K} Pr\left(\mu(\mathbf{m}_{-i}) \in Z_j,\ \mu(\mathbf{m}_{-i}, m_i) \in Y_k|\theta\right) \\ &= \sum_{j=1}^{J}\sum_{k=1}^{K} Pr\left(\mu(\mathbf{m}_{-i}) \in int(Z_j),\ \mu(\mathbf{m}_{-i}, m_i) \in int(Y_k)|\theta\right) \end{aligned}$$

where the second equality follows from the above arguments and the fact that for any open set Z, $int(Z) = Z$. By lemma 2.6.2, this expression is equivalent to

$$Pr(ZY|m_i,\theta) = \sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{n=1}^{N} Pr(n|\theta) * \tag{A.26}$$

$$max\left\{0, F_{\mu(\mathbf{m}_{-i})|\theta,n}\left(min\left\{\overline{Z}_j, \kappa(\overline{Y}_k, m_i, n)\right\}\right) - F_{\mu(\mathbf{m}_{-i})|\theta,n}\left(max\left\{\underline{Z}_j, \kappa(\underline{Y}_k, m_i, n)\right\}\right)\right\}$$

We wish to show that (A.26) is continuous in m_i. By lemma 2.6.1, $F_{\mu(\mathbf{m}_{-i})|\theta,n}(t_r)$ is continuous. Therefore, if each of $\overline{Z}_j, \underline{Z}_j, \kappa(\overline{Y}_k, m_i, n)$, and $\kappa(\underline{Y}_k, m_i, n)$ is continuous in m_i, then so is (A.26). $\underline{Z}_j$ and $\overline{Z}_j$ are constant in m_j. By the definition of the mean, for any Y_k and m_i, $\kappa(\overline{Y}_k, m_i, n) = \frac{n*\overline{Y}_k - m_i}{n-1}$, and $\kappa(\underline{Y}_k, m_i, n) = \frac{n*\underline{Y}_k - m_i}{n-1}$, both of which are clearly continuous.

Since this construction describes each of the possible member of the receiver's type space when there is positive participation, this proves that $\sum_{Y\in\{L,I,H\}}\sum_{Z\in\{L,I,H\}} w(Z)P(YZ|m_i, \theta)$ is continuous in m_i. Since $M_i(v_i)$ is continuous by assumption, it also proves that $\sum_{Y\in\{L,I,H\}}\sum_{Z\in\{L,I,H\}} w(Z)P(YZ|M_i(v$ is continuous in v_i. The conditions of lemma 2.4.6 are thus satisfied. □

Lemma 2.6.4 For any G with a mean μ, in equilibrium, if M is strictly monotonic and continuously differentiable, then the conditions of proposition 2.4.9 are satisfied.

Proof of lemma 2.6.4. To see that the mean is regular, note that $\mu(\mathbf{m}_{-i}) = \frac{n\mu(\mathbf{m})-m_i}{n-1}$, which implies that $\mu(\mathbf{m}) < \mu(\mathbf{m}_{-i}) < m_i$ if and only if $\mu(\mathbf{m}) < m_i$ and $\mu(\mathbf{m}) > \mu(\mathbf{m}_{-i}) > m_i$ if and only if $\mu(\mathbf{m}) > m_i$.

Next, we want to show that $E[u_i|t_i, \rho_i = 1]$ is differentiable with respect to v_i everywhere in the regions specified in proposition 2.4.9, excluding, perhaps, N points.

Consider the case of proposition 2.4.9, part A.i and assume, for some $v' \in V$, that $K = [\underline{v}, v')$, $Q = (v', \overline{v}]$, $\Sigma(K) \subseteq L$, and $\Sigma(Q) \subseteq H$. By (2.4), $E[u_i|t_i, \rho_i = 1]$ is differentiable at some point v_i if and only if $\frac{d\sum_{Y\in\{L,I,H,\Phi\}}\sum_{Z\in\{L,I,H\}} w(Z)P(YZ|\theta,M(v_i))}{dv_i}$ exists. Moreover, by our assumptions, if $v_i \in Q$, then $\sum_{Y\in\{L,I,H,\Phi\}}\sum_{Z\in\{L,I,H\}} w(Z)P(YZ|\theta, M(v_i))$ reduces to

$$(1 - A(\mu^\Phi))Pr(\Phi|\theta) + Pr(LH|\theta, M(v_i)) + Pr(HH|\theta, M(v_i)) \tag{A.27}$$

The first term of (A.27) does not depend on m_i and for all $v_i \in Q$, $Pr(HH|\theta, M(v_i)) = Pr(H|\theta)$. Therefore, (A.27) is differentiable at v_i if and only if $\frac{dM(v_i)}{dv_i}\frac{\partial Pr(LH|m_i,\theta)}{\partial m_i}|_{m_i=M(v_i)}$ exists. By assumption, $\frac{dM(v_i)}{dv_i}$ exists, so all that is left is to evaluate the second derivative. To analyze this derivative we apply[4] lemma 2.6.2 to $Pr(LH|m_i, \theta)$. By assumption in this case, $infL = \sigma(\underline{v})$, $supL = infH = \sigma(v')$ and $supH = \sigma(\overline{v})$.

$$Pr(LH|M_i(v_i), \theta) = \sum_{n=1}^{N} Pr(n|\theta) * \left(F_{\mu(\mathbf{m}_{-i})|\theta,n}(min\{\overline{L}, \kappa(\overline{H}, m_i, n)\}) - F_{\mu(\mathbf{m}_{-i})|\theta,n}(max\{\underline{L}, \kappa(\underline{H}, m_i, n)\})\right)$$

Since $m_i \leq \overline{v}$, we obtain $\kappa(\overline{H}, m_i, n) \geq \overline{v} \geq \overline{L}$, and therefore:

$$Pr(LH|M_i(v_i), \theta) = \sum_{n=1}^{N} Pr(n|\theta) \left(F_{\mu(\mathbf{m}_{-i})|\theta,n}(\overline{L}) - F_{\mu(\mathbf{m}_{-i})|\theta,n}(max\{\underline{L}, \kappa(\underline{H}, m_i, n)\})\right)$$

[4]This is appropriate, even when we let $[\underline{v}, v') \in I$ by the arguments given in the proof of lemma 2.6.3

$$\frac{dPr(LH|M_i(v_i),\theta)}{dv_i} = -\sum_{n=1}^{N} Pr(n|\theta)* \tag{A.28}$$

$$f_{\mu(\mathbf{m}_{-i})|\theta,n}(max\{\underline{L},\kappa(\underline{H},m_i,n)\})\frac{\partial max\{\underline{L},\kappa(\underline{H},m_i,n)\}}{\partial m_i}\frac{dM_i(v_i)}{v_i}$$

We now show that for any n, the summand of (A.28) exists everywhere but at, at most, one point. From lemma 2.6.1, $f_{\mu(\mathbf{m}_{-i})|\theta,n}(max\{\underline{L},\kappa(\underline{H},m_i,n)\})$ exists, and $\frac{dM_i(v_i)}{v_i}$ exists by assumption. $\frac{\partial \underline{L}}{\partial m_i} = 0$ and $\frac{\partial\kappa(\underline{H},m_i,n)}{\partial m_i} = \frac{-1}{n-1}$. Therefore, $\frac{dmax\{\underline{L},\kappa(\underline{H},m_i,n)\}}{dm_i}$ is well defined unless $\underline{L} = \kappa(\underline{H}, M(v_i), n)$. Since we have assumed that M is strictly monotonic, and since $\kappa(\underline{H}, m_i, n)$ is strictly monotonic, this can happen at at most one value of v_i.

Since we have shown this for a given n, there can be up to N values at which $Pr(LH|M_i(v_i),\theta)$, and therefore $E[u_i|t_i,\rho_i = 1]$, are non-differentiable with respect to v_i. The proofs for the remaining cases follow the same steps. □

Proof of Proposition 2.6.6.

Let $N = \{1, 2\}$, assume players 0 and 2 play strategies as described in the statement of the proposition, and consider the best response of player 1. Since $W(v_2)$ is symmetric $Pr(\rho_2 = 1|\theta) = Pr(\rho_2 = 1|\theta)$ and $v_1^0 = v_1^\nu = 0$. Therefore, by lemma 2.4.6, player 1 must play a strategy such that his value sets are partitioned around 0. Now consider the incentive compatibility condition. By the assumption of the receiver's strategy, if $m_1 \in [\underline{v}, 0)$, then $\sum_{Y\in\{L,I,H,\Phi\}}\sum_{Z\in\{L,I,H\}} w(Z)P(YZ|m_1,\theta) = Pr(HH|m_1,\theta)$. By lemma 2.6.2 (and by definition of the mean), $Pr(HH|m_1,\theta) = Pr(\rho_2 = 1|\theta)(1 - F_{\mu(\mathbf{m}_{-i})|\theta,n=1}(-m_1))$. Naturally, when only one other player sends a message, the density of others' messages is equivalent to the density of $\tilde{M}(\cdot)$. That is, $f_{\mu(\mathbf{m}_{-i})|\theta,n=1}(t_r) = f_{S|\theta}(t_r)$. Therefore, when $m_1 \in [\underline{v}, 0)$, $\frac{d\sum_{Y\in\{L,I,H,\Phi\}}\sum_{Z\in\{L,I,H\}} w(Z)P(YZ|m_1,\theta)}{dm_1} = f_{S|\theta}(-m_i)$.

Similarly, if $m_1 \in (0, \overline{v}]$, then $\sum_{Y\in\{L,I,H,\Phi\}}\sum_{Z\in\{L,I,H\}} w(Z)P(YZ|m_1,\theta) = Pr(\Phi|\theta)+Pr(LH|m_1,\theta)+Pr(HH|m_1,\theta)$. However, $Pr(\Phi|\theta)$ and $Pr(HH|m_1,\theta)$ are invariant for all $m_1 \in (0, \overline{v}]$. Thus, $\frac{d\sum_{Y\in\{L,I,H,\Phi\}}\sum_{Z\in\{L,I,H\}} w(Z)P(YZ|m_1,\theta)}{dm_1} = \frac{dPr(LH|m_1,\theta)}{dm_1}$. By lemma 2.6.2, $Pr(LH|m_1,\theta)) = Pr(\rho_2 = 1|\theta)\left(F_{\mu(\mathbf{m}_{-i})|\theta,n=1}(-m_1) - F_{\mu(\mathbf{m}_{-i})|\theta,n=1}(0)\right)$. Thus, by the same argument as above, when $m_1 \in (0, \overline{v}]$, $\frac{d\sum_{Y\in\{L,I,H,\Phi\}}\sum_{Z\in\{L,I,H\}} w(Z)P(YZ|m_1,\theta)}{dm_1} = -f_{S|\theta}(-m_i)$.

We now wish to show that $M(v_1) = v_1$ is a best response to the proposed equilibrium. In this class of games, $E[v_r|\theta^h] = -E[v_r|\theta^l]$. Moreover, since $Pr(\theta^h) = Pr(\theta^l)$, $\frac{Pr(\theta^h|v_1)}{Pr(\theta^l|v_1)} = \frac{f_{v|\theta^h}(v_1)}{f_{v|\theta^l}(v_1)}$. Therefore, the left-hand side of our (IC) condition simplifies to $\frac{f_{v|\theta^h}(v_1)}{f_{v|\theta^l}(v_1)}$. From equation B.1, we know $f_{S|\theta}(M_i(v_1)) = \frac{f_{v|\theta}(v_1)F_c(X(v_1))}{Pr(\rho_1=1|\theta)}$, and therefore, by the above arguments, the right-hand side of (IC) simplifies to $\frac{f_{v|\theta^l}(-v_1)}{f_{v|\theta^h}(-v_1)}$. Since we are considering a class of game for which $f_{v|\theta^l}(v_1) = f_{v|\theta^h}(-v_1)$, the right-hand side also transforms to $\frac{f_{v|\theta^h}(v_1)}{f_{v|\theta^l}(v_1)}$, and we see that $M(v_1) = v_1$ is therefore incentive compatible given the strategies of the other players. This also establishes that $L = [\underline{v}, 0)$, $I = 0$, $H = (0, \overline{v}]$ for player 1. $I = 0$ follows from the continuity of $g(\cdot)$ which follows from lemma 2.6.1.

We must now check that the sender's net expected value to participation is symmetric around 0. This is simply a matter of checking that for every v_1, equation A.2 obtains the same value at v_1 and $-v_1$. By assumption, $A(\mu^\Phi) = a^I = 0.5$. Let $v_1 \in L$, then equation A.3 yields $\sum_{Y\in\{L,I,H,\Phi\}}\sum_{Z\in\{L,I,H\}}[w(Z)-w(Y)]\,P(YZ|v_1,\theta) = -0.5Pr(\Phi|\theta) - Pr(HL|v_1,\theta)$, while $\sum_{Y\in\{L,I,H,\Phi\}}\sum_{Z\in\{I$ $v_1,\theta) = 0.5Pr(\Phi|\theta) + Pr(LH|v_1,\theta)$. By symmetry of the environment we have $Pr(\theta|v_1) = Pr(\Theta\setminus\theta|-v_1)$, by symmetry of player 2's strategy we have $Pr(\Phi|\theta^h) = Pr(\Phi|\theta^l)$ and by the above arguments $Pr(HL|v_1,\theta) = Pr(LH(-v_1)|\Theta\backslash\theta)$. Therefore, we get $X(v_1) = W(-v_1) = Pr(\theta^l|v_1)\left(Pr(\Phi|\theta^h) + Pr(HL|v_1,\theta^h)\right)$ $0.5Pr(\Phi|\theta^h) - Pr(HL|v_1,\theta^h)$, which is what we wanted to show.

Finally, we need to check the consistency of the receiver's beliefs. By proposition 2.4.1, this equilibrium is consistent iff for each $t_r \in [\underline{v}, 0)$, $g(\theta^h|t_r) < 0.5$ and for each $t_r \in (0, \overline{v}]$, $g(\theta^h|t_r) > 0.5$. Choose any $t_r \in [\underline{v}, 0)$. By Bayes' rule, $g(\theta^h|t_r) < 0.5$ iff $\sum_{n=1}^{2} Pr(n)(f_{\mu(\mathbf{m})|\theta^h,n}(t_r) - f_{\mu(\mathbf{m})|\theta^l}(t_r,n)) < 0$. When $n = 1$ the summand is negative iff $f_{v|\theta^h}(t_r) < f_{v|\theta^l}(t_r)$, which is true by assumption. Thus, it only remains to be shown that $f_{\mu(\mathbf{m})|\theta^h,n=2}(t_r) - f_{\mu(\mathbf{m})|\theta^l,n=2}(t_r) < 0$. Decomposing this inequality using equation B.2 and what we already know about this equilibrium, we see that this inequality is true iff

$$\int_{\underline{v}}^{2t_r+\overline{v}} \left(f_{v|\theta^h}(\hat{\mu}_1)f_{v|\theta^h}(2t_r-\hat{\mu}_1) - f_{v|\theta^l}(\hat{\mu}_1)f_{v|\theta^l}(2t_r-\hat{\mu}_1)\right) F_c(W(\hat{\mu}_1))F_c(W(2t_r-\hat{\mu}_1))d\hat{\mu}_1 < 0$$

$F_c(W(\hat{\mu}_1))F_c(W(2t_r-\hat{\mu}_1))$ is positive, so we show simply that the parenthetical is negative at each point. Rearranging, this is true iff $\frac{f_{v|\theta^h}(\hat{\mu}_1)}{f_{v|\theta^l}(\hat{\mu}_1)} < \frac{f_{v|\theta^l}(2t_r-\hat{\mu}_1)}{f_{v|\theta^h}(2t_r-\hat{\mu}_1)} = \frac{f_{v|\theta^h}(\hat{\mu}_1-2t_r)}{f_{v|\theta^l}(\hat{\mu}_1-2t_r)}$. Where the equality follows from the assumption that $f_{v|\theta^h}(v) = f_{v|\theta^l}(-v)$. By assumption 2.3.2, the inequality is true iff $\hat{\mu}_1 < \hat{\mu}_1 - 2t_r$. Since $t_r < 0$, by assumption, this is true and the integrand is therefore negative at every point. This implies that $f_{\mu(\mathbf{m})|\theta^h,n=2}(t_r) - f_{\mu(\mathbf{m})|\theta^l,n=2}(t_r) < 0$ which completes the proof that for each $t_r \in [\underline{v}, 0)$, $g(\theta^h|t_r) < 0.5$. The proof that $g(\theta^h|t_r) > 0.5$ for each $t_r \in (0, \overline{v}]$ follows the same steps. □

Proof of Lemma 2.6.8. We complete the proof assuming that $M(v_i)$ is a reflection through the point $M(0)$, of which $M(v_i) = v_i$ is a special case. This assumption implies that for any v_i, $M(v_i) = 2M(0) - M(-v_i)$.

We want to show that for any disjoint $\theta, \theta' \in \Theta$ and for all n: A) $Pr(n|\theta) = Pr(n|\theta')$;

B) $Pr(LH|M(v^l), \theta, n) = Pr(HL|M(v^h), \theta', n)$; C) $Pr(LL|M(v^l), \theta, n) = Pr(HH|M(v^h), \theta', n)$; D) $Pr(HH|M(v^l), \theta, n) = Pr(LL|M(v^h), \theta', n)$; E) $Pr(HL|M(v^l), \theta, n) = Pr(LH|M(v^h), \theta', n)$.

A) By the assumption that $W(\cdot)$ is symmetric, for all i, $Pr(\rho_i = 1|\theta^h) = Pr(\rho_i = 1|\theta^l)$.

B) By Table 2.1, $Pr(LH|M(v^l), \theta, n) = Pr(HL|M(v^l), \theta', n)$ holds trivially since both sides are equal to 0.

To prove the remaining claims we first construct $f_{S|\theta^h}(M(v_j))$ and $f_{\mu^d(\mathbf{m}_{-i})|\theta,n}(t_r)$. By the assumption that $M(v_i) = 2M(0) - M(-v_i)$, $\frac{dM(v_i)}{dv_i} = \frac{dM(-v_i)}{dv_i}$ and $\frac{dM^{-1}(m_i)}{dm_i} = \frac{d^{-1}M(-m_i)}{dm_i}$, and by assumption $f_{v|\theta^h}(v_1) = f_{v|\theta^l}(-v_1)$. Therefore, by equation B.1,

$$\begin{aligned} f_{S|\theta^h}(M(v_j)) &= \frac{\frac{dM^{-1}(m_1)}{dm_1}|_{m_1=M(v_1)} f_{v|\theta^h}(v_1) F_c\left(X(v_1)\right)}{Pr(\rho_j = 1|\theta^h)} \\ &= \frac{\frac{dM^{-1}(m_1)}{dm_1}|_{m_1=M(-v_1)} f_{v|\theta^l}(-v_1) F_c\left(W(-v_1)\right)}{Pr(\rho_j = 1|\theta^l)} \\ &= f_{S|\theta^l}(M(-v_j)) \end{aligned}$$

Moreover, this implies that (utilizing a change of variable),

$$
\begin{aligned}
F_{S|\theta^h}(M(v_j)) &= \int_{M(\underline{v})}^{M(v_j)} f_{S|\theta^h}(M(s))dM(s) \qquad = \int_{M(\underline{v})}^{M(v_j)} f_{S|\theta^l}(M(-s))dM(s) \\
&= \int_{M(\underline{v})}^{M(v_j)} f_{S|\theta^l}(2M(0)-M(s))dM(s) = -\int_{2M(0)-M(\underline{v})}^{2M(0)-M(v_j)} f_{S|\theta^l}(M(t))dM(t) \\
&= \int_{M(-v_j)}^{M(\overline{v})} f_{S|\theta^l}(M(t))dM(t) \\
&= 1 - F_{S|\theta^l}(M(-v_j))
\end{aligned}
$$

Therefore, by definition of the pdf of an order statistic (and utilizing a change of variable),

$$
\begin{aligned}
&Pr(y_k(\mathbf{m}_{-i}) < M(v)|\theta^h, n) \times \frac{(k-1)!(n-1-k)!}{(n-1)!} \\
&= \int_{M(\underline{v})}^{M(v)} F_{S|\theta^h}(M(s))^{k-1}(1-F_{S|\theta^h}(M(s)))^{n-1-k} f_{S|\theta^h}(M(s))dM(s) \\
&= \int_{M(\underline{v})}^{M(v)} (1-F_{S|\theta^l}(M(-s)))^{k-1}(F_{S|\theta^h}(M(-s)))^{n-1-k} f_{S|\theta^h}(M(-s))dM(s) \\
&= \int_{M(\underline{v})}^{M(v)} (1-F_{S|\theta^l}(2M(0)-M(s)))^{k-1}(F_{S|\theta^h}(2M(0)-M(s)))^{n-1-k} f_{S|\theta^h}(2M(0)-M(s))dM(s) \\
&= -\int_{2M(0)-M(\underline{v})}^{2M(0)-M(v)} (1-F_{S|\theta^l}(M(t)))^{k-1}(F_{S|\theta^h}(M(t)))^{n-1-k} f_{S|\theta^h}(M(t))dM(t) \\
&= \int_{M(-v)}^{M(\overline{v})} (1-F_{S|\theta^l}(M(t)))^{k-1}(F_{S|\theta^h}(M(t)))^{n-1-k} f_{S|\theta^h}(M(t))dM(t) = \\
&Pr(y_{n-k}(\mathbf{m}_{-i}) > M(-v)|\theta^l, n) \times \frac{(k-1)!(n-1-k)!}{(n-1)!}
\end{aligned}
$$

Referencing Table 2.1, we see that this implies claims C and D. To see this, notice that letting $k = \frac{n-1}{2}$ (implies $n-k = \frac{n+1}{2}$), $k = \frac{n}{2}$ (implies $n-k = \frac{n}{2}$), or $k = \frac{n-2}{2}$ (implies $n-k = \frac{n+2}{2}$) implies the equivalence of each term of the relevant probabilities.

Finally, repeating the same process but utilizing the definition of the joint pdf of two order statistics

$$Pr\left(y_l(\mathbf{m}_{-i}) < M(0), y_{l+1}(\mathbf{m}_{-i}) > M(0)|\theta^h, n\right) \times \frac{(l-1)!(n-l)!}{(n-1)!} \tag{A.29}$$
$$= \int_{M(\underline{v})}^{M(0)} \int_{M(0)}^{M(\overline{v})} F_{S|\theta^h}(M(s))^{l-1}(1-F_{S|\theta^h}(M(r)))^{n-l-2} f_{S|\theta^h}(M(s)) f_{S|\theta^h}(M(r)) dM(s) dM(r)$$
$$= \int_{M(0)}^{M(\overline{v})} \int_{M(\underline{v})}^{M(0)} (1-F_{S|\theta^l}(M(t)))^{l-1} F_{S|\theta^l}(M(q))^{n-l-2} f_{S|\theta^l}(M(t)) f_{S|\theta^l}(M(q)) dM(t) dM(q) =$$
$$Pr\left(y_k(\mathbf{m}_{-i}) < M(0), y_{k+1}(\mathbf{m}_{-i}) > M(0)|\theta^h, n\right) \times \frac{(l-1)!(n-l)!}{(n-1)!}$$

where $k = n-1-l$. Therefore, (A.29) implies that $Pr\left(y_{\frac{n-1}{2}}(\mathbf{m}_{-i}) < M(0), y_{\frac{n+1}{2}}(\mathbf{m}_{-i}) > M(0)|\theta, n\right) = Pr\left(y_{\frac{n-1}{2}}(\mathbf{m}_{-i}) < M(0), y_{\frac{n+1}{2}}(\mathbf{m}_{-i}) > M(0)|\theta', n\right)$ and $Pr\left(y_{\frac{n}{2}}(\mathbf{m}_{-i}) < M(0), y_{\frac{n+2}{2}}(\mathbf{m}_{-i}) > M(0)|\theta, n\right) = Pr\left(y_{\frac{n-2}{2}}(\mathbf{m}_{-i}) < M(0), y_{\frac{n}{2}}(\mathbf{m}_{-i}) > M(0)|\theta', n\right)$. Therefore, by inspection of Table 2.1, E is also proved. □

Proof of lemma 2.6.7. Assume $L = [\underline{v}, v^d)$, $I = [v^d]$ and $H = (v^d, \overline{v}]$. Following the logic of lemma 2.4.6, in order for lemma 2.4.5 to hold at v^d, both $E[u_i(M(v^d), \rho_i = 1, t_i)|v^d] = \lim_{v_i \to v^{d-}} E[u_i|t_i, \rho_i = 1]$ and $E[u_i(M(v^d), \rho_i = 1, t_i)|v^d] = \lim_{v_i \to v^{d+}} E[u_i|t_i, \rho_i = 1]$ must hold. Let $v^l \in L$ and $v^h \in H$, then these conditions are equivalent to, respectively,

$$0 = \sum_{\theta \in \Theta} Pr(\theta|v^d) E[v_r|\theta] \sum_{Y \in \{L,I,H,\Phi\}} \sum_{Z \in \{L,I,H\}} w(Z) \left[P(YZ|M(v^d), \theta) - P(YZ|M(v^l), \theta)\right] \tag{A.30}$$
$$0 = \sum_{\theta \in \Theta} Pr(\theta|v^d) E[v_r|\theta] \sum_{Y \in \{L,I,H,\Phi\}} \sum_{Z \in \{L,I,H\}} w(Z) \left[P(YZ|M(v^d), \theta) - P(YZ|M(v^h), \theta)\right] \tag{A.31}$$

Based upon the equivalences revealed by Table 2.1, (A.30) and (A.31) reduce to, respectively,

$$0 = a^I \sum_{\theta\in\Theta} Pr(\theta|v^d)E[v_r|\theta] \big(Pr(HH|M(v^d),\theta) + Pr(LL|M(v^d),\theta) -$$

$$\big[Pr(\Phi|\theta) + Pr(LI|M(v^d),\theta) + Pr(HI|M(v^d),\theta) + Pr(LL|M(v^d),\theta) + Pr(HH|M(v^d),\theta)\big]\big)$$

$$0 = (1-a^I) \sum_{\theta\in\Theta} Pr(\theta|v^d)E[v_r|\theta] \big(Pr(HH|M(v^d),\theta) + \big(Pr(LL|M(v^d),\theta)\big) -$$

$$\big[Pr(\Phi|\theta) + Pr(LI|M(v^d),\theta)) + Pr(HI|M(v^d),\theta) + Pr(LL|M(v^d),\theta) + Pr(HH|M(v^d),\theta)\big] +$$

$$\frac{1}{(1-a^I)} \big(Pr(HI|M(v^d),\theta) + Pr(HH|M(v^d),\theta) - Pr(HH|M(v^h),\theta)\big)\Big)$$

There are two things to note from these conditions. First, $Pr(\Phi|\theta)+Pr(LI|M(v^d),\theta))+Pr(HI|M(v^d),\theta)+Pr(LL|M(v^d),\theta) + Pr(HH|M(v^d),\theta) = 1$, and second is $Pr(HI|M(v^d),\theta) + Pr(HH|M(v^d),\theta) - Pr(HH|M(v^h),\theta) = 0$. The first follows from Table 2.1 as these are the only non-zero event probabilities. The second can also be obtained from Table 2.1:

$$
\begin{aligned}
&Pr(HI|M(v^d),\theta,n) + Pr(HH|M(v^d),\theta,n)\\
&=1_{n\ odd}\Big[Pr(y_{\frac{n-1}{2}}(\mathbf{m}_{-i}) > M(v^d)|\theta,n)+\\
&\qquad \frac{1}{2}\Big(Pr(y_{\frac{n-1}{2}}(\mathbf{m}_{-i}) < M(v^d), y_{\frac{n+1}{2}}(\mathbf{m}_{-i}) \geq M(v^d)|\theta,n)\Big)\Big]+\\
&\ \frac{1}{2}1_{n\ even}\big[Pr(y_{\frac{n}{2}}(\mathbf{m}_{-i}) > M(v^d)|\theta,n) +\\
&\qquad Pr(y_{\frac{n-2}{2}}(\mathbf{m}_{-i}) \leq M(v^d), y_{\frac{n}{2}}(\mathbf{m}_{-i}) > M(v^d)|\theta,n) + Pr(y_{\frac{n-2}{2}}(\mathbf{m}_{-i}) > M(v^d)|\theta,n)\big]\\
&=1_{n\ odd}\Big[\frac{1}{2}Pr(y_{\frac{n-1}{2}}(\mathbf{m}_{-i}) > M(v^d)|\theta,n) + \frac{1}{2}Pr(y_{\frac{n-1}{2}}(\mathbf{m}_{-i}) > M(v^d), y_{\frac{n+1}{2}}(\mathbf{m}_{-i}) \geq M(v^d)|\theta,n)+\\
&\qquad \frac{1}{2}\Big(Pr(y_{\frac{n-1}{2}}(\mathbf{m}_{-i}) < M(v^d), y_{\frac{n+1}{2}}(\mathbf{m}_{-i}) \geq M(v^d)|\theta,n)\Big)\Big]+\\
&\ \frac{1}{2}1_{n\ even}\big[Pr(y_{\frac{n}{2}}(\mathbf{m}_{-i}) > M(v^d)|\theta,n) +\\
&\qquad\quad Pr(y_{\frac{n-2}{2}}(\mathbf{m}_{-i}) \leq M(v^d), y_{\frac{n}{2}}(\mathbf{m}_{-i}) > M(v^d)|\theta,n)+\\
&\qquad\quad Pr(y_{\frac{n-2}{2}}(\mathbf{m}_{-i}) > M(v^d), y_{\frac{n}{2}}(\mathbf{m}_{-i}) > M(v^d)|\theta,n)\big]\\
&=\frac{1}{2}1_{n\ odd}\Big[Pr(y_{\frac{n-1}{2}}(\mathbf{m}_{-i}) > M(v^d)|\theta,n) + Pr(y_{\frac{n+1}{2}}(\mathbf{m}_{-i}) > M(v^d)|\theta,n)\Big]+\\
&\ 1_{n\ even}Pr(y_{\frac{n}{2}}(\mathbf{m}_{-i}) > M(v^d)|\theta,n)\\
&=Pr(HH|M(v^h),\theta,n)
\end{aligned}
$$

where the last step assumes that $F_{\mu^d(\mathbf{m}_{-i})}(\cdot)$ is massless (as it would be, for example, if $M(\cdot)$ is continuously differentiable).

Given these observations, conditions (A.30) and (A.31) each reduce to

$$
0 = \sum_{\theta\in\Theta} Pr(\theta|v^d)E[v_r|\theta]\left[1 - \big(Pr(LL|M(v^d),\theta) + Pr(HH|M(v^d),\theta)\big)\right] \tag{2.11}
$$

Since the continuity conditions yield the same condition, as long as v^d satisfies equation 2.11, then lemma 2.4.5 is satisfied. □

Proof of Proposition 2.6.9.

We complete the proof assuming $M(v_i)$ that is any increasing function that is a reflection through the point $M(0)$, of which $M(v_i) = v_i$ is a special case. This implies that for any v_i, $M(v_i) = 2M(0) - M(-v_i)$.

Assume that the receiver and players 2 to N play strategies according to the statement of the proposition. Let $j \in \{2, ..., N\}$. Applying lemma 2.6.8 to lemma 2.6.7 implies that v^d is solved *uniquely* by $v^d = 0$. That is, by the assumption of the monotone likelihood ratio of value densities, $v^d = 0$ is the only value that can sustain an equilibrium in which I is a single point separating convex L and H. Therefore, by the assumed strategies: $L = [M(\underline{v}), M(0))$, $I = M(0)$, $H = (M(0), M(\overline{v})]$.

We wish to show next that $M(v_1)$ is a best response for the sender. As discussed above, by definition of the median, for any $Z \in \{L, H\}$, if $m_1 \in int(Z)$, then local changes in m_1 do not affect the outcome probabilities. Therefore, intra-type-space partition member, incentive compatibility holds trivially. It is only left to check, therefore, that incentive compatibility holds at points of transition between partition members. In the proof of lemma 2.6.7, we already showed that, by the choice of v^d, players with values v^d are indifferent between messages. Therefore we need only to show that for $v^l < 0$, $E[u_i(M(v^l), \rho_i = 1, t_i)|v^l] \geq E[u_i(M(v^h), \rho_i = 1, t_i)|v^l]$ and $E[u_i(M(v^l), \rho_i = 1, t_i)|v^l] \geq E[u_i(M(v^d), \rho_i = 1, t_i)|v^l]$, and for $v^h > 0$, $E[u_i(M(v^h), \rho_i = 1, t_i)|v^h] \geq E[u_i(M(v^l), \rho_i = 1, t_i)|v^h]$ and $E[u_i(M(v^h), \rho_i = 1, t_i)|v^h] \geq E[u_i(M(v^d), \rho_i = 1, t_i)|v^h]$. By applying lemma 2.6.8 to the definition of $E[u_i(\cdot)|v_i]$, it is quick to see that for v^l both conditions are satisfied if and only if $(1 - Pr(HH|M(v^l), \theta^h) - Pr(LL|M(v^h), \theta^h))(Pr(\theta^l|v^l) - Pr(\theta^h|v^l)) \geq 0$. The first term is plainly non-negative, and since $v^l < 0$, by symmetry of the environment, $Pr(\theta^l|v^l) - Pr(\theta^h|v^l) > 0$. Similarly, for v^h both conditions are satisfied if and only if $(1 - Pr(HH|M(v^l), \theta^h) - Pr(LL|M(v^h), \theta^h))(Pr(\theta^h|v^h) - Pr(\theta^l|v^h)) \geq 0$, which is also plainly true.

Next we want to show that $X(v_1)$ is symmetric around $v_1 = 0$. We assumed that $A(\mu^\Phi) = a^I = 0.5$. Let $v_1 < 0$. Then by equation A.3 and Table 2.1,

$$\sum_{Y \in \{L,I,H,\Phi\}} \sum_{Z \in \{L,I,H\}} [w(Z) - w(Y)]\, P(YZ|M(v_1), \theta) = -0.5Pr(\Phi|\theta) - Pr(HL(M(v_1))|\theta)$$

Furthermore, this implies that $\sigma(-v_1) \in H$, and thus

$$\sum_{Y\in\{L,I,H,\Phi\}} \sum_{Z\in\{L,I,H\}} [w(Z) - w(Y)]\, P(YZ|M(-v_1), \theta) = 0.5Pr(\Phi|\theta) + Pr(LH(M(-v_1))|\theta)$$

Therefore, by equation A.2,

$$X(v_1) = -\sum_{\theta\in\Theta} Pr(\theta|v_1)E[v_r|\theta]\,(0.5Pr(\Phi|\theta) + Pr(HL(M(v_1))|\theta))$$

By lemma 2.6.8 (and letting $\theta' \in \Theta \setminus \theta$) this becomes

$$X(v_1) = -\sum_{\theta\in\Theta} Pr(\theta|v_1)E[v_r|\theta]\,(0.5Pr(\Phi|\theta) + Pr(LH(M(-v_1))|\theta'))$$

By the assumed symmetry of W for the other senders, $Pr(\Phi|\theta) = Pr(\Phi|\theta')$ and by the symmetry of our environment both $E[v_r|\theta'] = -E[v_r|\theta]$ and $Pr(\theta|v_1) = Pr(\theta'| - v_1)$. Therefore,

$$\begin{aligned} X(v_1) &= \sum_{\theta\in\Theta} Pr(\theta'| - v_1)E[v_r|\theta']\,(0.5Pr(\Phi|\theta') + Pr(LH(M(-v_1))|\theta')) \\ &= W(-v_1) \end{aligned}$$

Finally, we want to show that the assumed strategy for the receiver is a best response to the strategies of the senders. The choice of $a^I = 0.5$ is part of a best response as a^I is irrelevant to the receiver's expected utility. Moreover, it is plain that since $W(\cdot)$ is symmetric, $Pr(\mu^\Phi|\theta^h) = Pr(\mu^\Phi|\theta^l)$, and thus $A(\mu^\Phi) = a^I = 0.5$. We want to show that[5]

$$f_{\mu^d(\mathbf{m})|\theta^h}(t_r) - f_{\mu^d(\mathbf{m})|\theta^l}(t_r) \begin{cases} < 0, \text{ if } t_r < M(0) \\ > 0, \text{ if } t_r > M(0) \\ = 0, \text{ if } t_r = M(0) \end{cases} \tag{A.32}$$

[5]This is obtained by applying Bayes' rule to $g(\theta^h|t_r)$

By our definition of the median,

$$f_{\mu^d(\mathbf{m})|\theta^h}(t_r) - f_{\mu^d(\mathbf{m})|\theta^l}(t_r) = \sum_{n=1}^{N} Pr(n|\theta)($$

$$1_{n\ odd}\frac{n!}{(\frac{n-1}{2})!(\frac{n-1}{2})!}\left[F_{S|\theta^h}(t_r)^{\frac{n-1}{2}}(1-F_{S|\theta^h}(t_r))^{\frac{n-1}{2}}f_{S|\theta^h}(t_r) - F_{S|\theta^l}(t_r)^{\frac{n-1}{2}}(1-F_{S|\theta^l}(t_r))^{\frac{n-1}{2}}f_{S|\theta^l}(t_r)\right] +$$

$$1_{n\ even}\frac{1}{2}\frac{n!}{(\frac{n-2}{2})!(\frac{n}{2})!}$$

$$\left\{\left[F_{S|\theta^h}(t_r)^{\frac{n-2}{2}}(1-F_{S|\theta^h}(t_r))^{\frac{n}{2}}f_{S|\theta^h}(t_r) - F_{S|\theta^l}(t_r)^{\frac{n-2}{2}}(1-F_{S|\theta^l}(t_r))^{\frac{n}{2}}f_{S|\theta^l}(t_r)\right] +\right.$$

$$\left.\left[F_{S|\theta^h}(t_r)^{\frac{n}{2}}(1-F_{S|\theta^h}(t_r))^{\frac{n-2}{2}}f_{S|\theta^h}(t_r) - F_{S|\theta^l}(t_r)^{\frac{n}{2}}(1-F_{S|\theta^l}(t_r))^{\frac{n-2}{2}}f_{S|\theta^l}(t_r)\right]\right\})$$

Combining terms within the curly brackets yields

$$\left\{F_{S|\theta^h}(t_r)^{\frac{n-2}{2}}(1-F_{S|\theta^h}(t_r))^{\frac{n-2}{2}}f_{S|\theta^h}(t_r)\left(1-F_{S|\theta^h}(t_r)+F_{S|\theta^h}(t_r)\right) -\right.$$

$$\left.F_{S|\theta^l}(t_r)^{\frac{n-2}{2}}(1-F_{S|\theta^l}(t_r))^{\frac{n-2}{2}}f_{S|\theta^l}(t_r)\left(1-F_{S|\theta^l}(t_r)+F_{S|\theta^l}(t_r)\right)\right\}$$

Let $\zeta(l,t_r) = \left(F_{S|\theta^h}(t_r)(1-F_{S|\theta^h}(t_r))\right)^l f_{S|\theta^h}(t_r) - \left(F_{S|\theta^l}(t_r)(1-F_{S|\theta^l}(t_r))\right)^l f_{S|\theta^l}(t_r)$

Then by the above simplification,

$$f_{\mu^d(\mathbf{m})|\theta^h}(t_r) - f_{\mu^d(\mathbf{m})|\theta^l}(t_r) = \tag{A.33}$$
$$\sum_{n=1}^{N} Pr(n|\theta)\left[1_{n\ odd}\frac{n!}{(\frac{n-1}{2})!(\frac{n-1}{2})!}\zeta\left(\frac{n-1}{2},t_r\right) + 1_{n\ even}\frac{n!}{2(\frac{n-2}{2})!(\frac{n}{2})!}\zeta\left(\frac{n-2}{2},t_r\right)\right)\right]$$

We now show that for all $l > 0$,

$$\zeta(l,t_r)\begin{cases} < 0, \text{ if } t_r < M(0) \\ > 0, \text{ if } t_r > M(0) \\ = 0, \text{ if } t_r = M(0) \end{cases}$$

as this will imply each of the conditions of A.32. To prove this we need to establish two facts. First, that $F_{S|\theta^l}(M(0)) > \frac{1}{2}$ and second that $F_{S|\theta^l}(m_i)$ first order stochastically dominates $F_{S|\theta^h}(m_i)$. To see the first, assume the second and note simply that $1 = F_{S|\theta^h}(M(0)) + (1 - F_{S|\theta^h}(M(0))) = F_{S|\theta^h}(M(0)) + F_{S|\theta^l}(M(0)) < 2F_{S|\theta^l}(M(0))$. To prove first order stochastic dominance, it is easiest to show that $f_{S|\theta^l}(m_i)$ and $f_{S|\theta^h}(m_i)$ satisfy the monotone likelihood ratio property. By equation B.1,

$$\begin{aligned}\frac{f_{S|\theta^h}(m_i)}{f_{S|\theta^l}(m_i)} &= \frac{\frac{dM^{-1}(m_i)}{dm_i} f_{v|\theta}(M^{-1}(m_i))F_c\left(W(M^{-1}(m_i))\right)}{\frac{dM^{-1}(m_i)}{dm_i} f_{v|\theta}(M^{-1}(m_i))F_c\left(W(M^{-1}(m_i))\right)} \\ &= \frac{f_{v|\theta}(M^{-1}(m_i))}{f_{v|\theta}(M^{-1}(m_i))}\end{aligned} \tag{A.34}$$

Therefore, by assumption 2.3.2, and since $M(v_i)$ is assumed to be increasing, $\frac{f_{S|\theta^h}(M(v_i))}{f_{S|\theta^l}(m_i)}$ is strictly increasing. This implies that $F_{S|\theta^l}(m_i)$ first order stochastically dominates $F_{S|\theta^h}(m_i)$. (A.34) also implies that for $v_i < 0$, $f_{S|\theta^h}(M(v_i)) < f_{S|\theta^l}(M(v_i))$ and for $v_i > 0$, $f_{S|\theta^h}(M(v_i)) > f_{S|\theta^l}(M(v_i))$.

From the proof of lemma 2.6.8, we know that $F_{S|\theta^h}(M(v)) = 1 - F_{S|\theta^l}(M(-v))$ and that $f_{S|\theta^h}(M(v)) = f_{S|\theta^l}(M(-v))$. Therefore, for any l, it is obvious that $\zeta(l, M(0)) = 0$. This implies that $f_{\mu^d(\mathbf{m})|\theta^h}(M(0)) - f_{\mu^d(\mathbf{m})|\theta^l}(M(0)) = 0$, and therefore that $g(\theta^h|M(0)) = \frac{1}{2}$.

To see that $\zeta(l, t_r) < 0$ if $t_r < M(0)$, recall that for any $z \in [0,1]$, $z(1-z)$ is increasing on $[0, \frac{1}{2})$, maximized at $\frac{1}{2}$, and decreasing on $(\frac{1}{2}, 1]$. Now let t_r^l be such that $F_{S|\theta^l}(t_r^l) = \frac{1}{2}$. Since $F_{S|\theta^l}(M(0)) > \frac{1}{2}$, $t_r^l < M(0)$. Therefore, for $t_r \in [M(\underline{v}), t_r^l]$, $f_{S|\theta^h}(t_r) < f_{S|\theta^l}(t_r)$. Moreover, since $F_{S|\theta^l}(m_i)$ first order stochastically dominates $F_{S|\theta^h}(m_i)$, for $t_r \in [M(\underline{v}), t_r^l]$, $F_{S|\theta^h}(t_r)(1 - F_{S|\theta^h}(t_r)) < F_{S|\theta^l}(t_r)(1 - F_{S|\theta^l}(t_r)$. Together, these facts imply that for $t_r \in [M(\underline{v}), t_r^l]$, $\zeta(l, t_r) < 0$.

Next, notice that for $t_r \in (t_r^l, M(0))$, $F_{S|\theta^l}(t_r)(1 - F_{S|\theta^l}(t_r)$ is decreasing, while, since $F_{S|\theta^h}(M(0)) < \frac{1}{2}$, $F_{S|\theta^h}(t_r)(1 - F_{S|\theta^h}(t_r))$ is increasing. This implies that for $t_r \in (t_r^l, M(0))$, $\zeta(l, t_r)$ is increasing. However, since we know that $\zeta(l, M(0)) = 0$, it must be that $\zeta(l, t_r)$ is bounded below 0 on this region. Therefore, for all $t_r \in [M(\underline{v}), M(0))$, $\zeta(l, t_r) < 0$. This in turn implies that $f_{\mu^d(\mathbf{m})|\theta^h}(M(0)) - f_{\mu^d(\mathbf{m})|\theta^l}(M(0)) < 0$ and therefore that $g(\theta^h|M(0)) < \frac{1}{2}$. The proof that for all $t_r \in (M(0), M(\overline{v})]$, $\zeta(l, t_r) > 0$ and therefore that $g(\theta^h|M(0)) > \frac{1}{2}$ follows the same steps. □

Proof of lemma 2.6.10. Assume an equilibrium and let $Q(v_i) = E[u_i|t_i, \rho_i = 1, \underline{m}] - E[u_i|t_i, \rho_i =$

$1, \overline{m}]$. This is the amount that player i prefers to send $\underline{m}$ compared to $\overline{m}$; if $Q(v_i) > 0$, then player i prefers to send $\underline{m}$ and if $Q(v_i) < 0$, then player i prefers to send $\overline{m}$. Expanding,

$$Q(v_i) = (1 - Pr(\theta^l|v_i))E[v_r|\theta^h] \left\{ \sum_{Y \in \{L,I,H,\Phi\}} \sum_{Z \in \{L,I,H\}} w(Z) \left[P(YZ|\underline{m}, \theta^h) - P(YZ|\overline{m}, \theta^h) \right] \right\} + Pr(\theta^l|v_i)E[v_r|\theta^l] \left\{ \sum_{Y \in \{L,I,H,\Phi\}} \sum_{Z \in \{L,I,H\}} w(Z) \left[P(YZ|\underline{m}, \theta^l) - P(YZ|\overline{m}, \theta^l) \right] \right\}$$

For simplicity of notation, we let

$$\alpha = E[v_r|\theta^h] \left\{ \sum_{Y \in \{L,I,H,\Phi\}} \sum_{Z \in \{L,I,H\}} w(Z) \left[P(YZ|\underline{m}, \theta^h) - P(YZ|\overline{m}, \theta^h) \right] \right\}$$

and

$$\gamma = E[v_r|\theta^l] \left\{ \sum_{Y \in \{L,I,H,\Phi\}} \sum_{Z \in \{L,I,H\}} w(Z) \left[P(YZ|\underline{m}, \theta^l) - P(YZ|\overline{m}, \theta^l) \right] \right\}$$

Thus, $Q(v_i) = \alpha + Pr(\theta^l|v_i)(\gamma - \alpha)$.

Next, assume that $\alpha > \gamma$. This implies that $Q(v_i) > 0$ (player i prefers to send $\underline{m}$) if and only if $Pr(\theta^l|v_i) < \frac{\alpha}{\alpha - \gamma}$. Because $\frac{\alpha}{\alpha-\gamma}$ does not depend on v_i, and because $Pr(\theta^l|v_i)$ is strictly decreasing by assumption 2.3.2, this results in a simple equilibrium messaging strategy. Let v^t solve $Pr(\theta^l|v^t) = \frac{\alpha}{\alpha-\gamma}$. Then,

$$M(v_i) = \begin{cases} \underline{m}, \text{ if } v_i < v^t \\ \overline{m}, \text{ if } v_i > v^t \end{cases}$$

If we had instead assumed that assume that $\alpha > \gamma$, then the rule would be the same but with the inequalities reversed. Moreover, by the informativeness criterion, it cannot be the case that $\alpha = \gamma$ in equilibrium, as this would imply that senders choose the same message regardless of their values. Therefore, this messaging strategy proves that in equilibrium the value set is partitioned by a simple partition formed around v^t; for all values below v^t, players choose one message and for all values above v^t, players choose another message. □

Proof of lemma 2.6.10. Assume a fixed participation level n. We want to show, for some number of high messages, $\overline{n}$, that $g(\theta^h|n,\overline{n})$ is increasing in $\overline{n}$. Utilizing Bayes' rule, this is true if and only if

$$Pr(n,\overline{n}|\theta^h)Pr(n,\overline{n}+1|\theta^l) < Pr(n,\overline{n}+1|\theta^h)Pr(n,\overline{n}|\theta^l) \tag{A.35}$$

Since $Pr(n,\overline{n}|\theta^h) = Pr(\overline{n}|\theta^h,n)Pr(n|\theta^h)$, (A.35) reduces to

$$Pr(\overline{n}|n,\theta^h)Pr(\overline{n}+1|\theta^l,n) < Pr(\overline{n}+1|\theta^h,n)Pr(\overline{n}|\theta^l,n)$$

Define the probability that a sender sends the high message in state θ, given that he sends a message, as $\overline{p}(\theta) = Pr(\overline{m}|\theta,\rho=1)$. Then, $Pr(\overline{n}|n,\theta) = \binom{n}{\overline{n}}\overline{p}(\theta)^{\overline{n}}(1-\overline{p}(\theta))^{n-\overline{n}}$, and (A.35) becomes

$$\begin{aligned}&\binom{n}{\overline{n}}\overline{p}(\theta^h)^{\overline{n}}(1-\overline{p}(\theta^h))^{n-\overline{n}}\binom{n}{\overline{n}+1}\overline{p}(\theta^l)^{\overline{n}+1}(1-\overline{p}(\theta^l))^{n-\overline{n}-1}\\ &< \binom{n}{\overline{n}+1}\overline{p}(\theta^h)^{\overline{n}+1}(1-\overline{p}(\theta^h))^{n-\overline{n}-1}\binom{n}{\overline{n}}\overline{p}(\theta^l)^{\overline{n}}(1-\overline{p}(\theta^l))^{n-\overline{n}}\end{aligned}$$

which condenses nicely such that it is true if and only if $\frac{\overline{p}(\theta^l)}{1-\overline{p}(\theta^l)} < \frac{\overline{p}(\theta^h)}{1-\overline{p}(\theta^h)}$. This, by the above discussion on $Pr(m|\theta)$, is true, and therefore, $Pr(\theta^h|n,\overline{n})$ is increasing in the number of sent high messages, for a given level of participation. □

Proof of proposition 2.6.12. For this proof we consider cases along two dimensions, the first is the position of v^t in V as it compares to v^I, and the second is the relationship between $Pr(\rho_i = 0|\theta^h)$ and $Pr(\rho_i = 0|\theta^l)$. By assumption 2.3.3, v^I is an interior point.

First, assume $v^t \geq v^I$. We want to show that $X(v_i)$ is increasing for $v_i > v^t$. Let $v_i' > v_i > v_t$, and $Q(\theta,m) = E[v_r|\theta]\sum_{Y\in\{L,I,H,\Phi\}}\sum_{Z\in\{L,I,H\}}[w(Z)-w(Y)]\,P(YZ|\theta,m)$, then by equation A.2

$$X(v_i') - X(v_i) = \left[Pr(\theta^h|v_i') - Pr(\theta^h|v_i)\right]\left[Q(\theta^h,\overline{m}) - Q(\theta^l,\overline{m})\right] \tag{A.36}$$

The first bracketed term is clearly positive, so it simply remains to be shown that the second bracketed term is positive.

By lemma 2.6.11, the only terms in the summand of $Q(\theta, m)$ that are potentially non-zero are $Pr(LH|\theta, \overline{m})$, $Pr(IH|\theta, \overline{m})$ (the coefficients on which are both positive) and each of the likelihoods when no other sender sends a message.

Assume $Pr(\rho_i = 0|\theta^h) < Pr(\rho_i = 0|\theta^l)$. This implies that the receiver's best response is to play $A(\mu^\Phi) = 0$ as no messages are more likely to be sent in the low state. Thus, the only remaining non-zero terms in the summand of $Q(\theta, \overline{m})$ have positive coefficients, implying that $X(v_i') - X(v_i) > 0$. Next, assume $Pr(\rho_i = 0|\theta^h) \geq Pr(\rho_i = 0|\theta^l)$, and consider the receiver's posterior when $t_r = (0, 1)$. We wish to show that $g(\theta^h|t_r = (0, 1)) > \nu$, implying that a receiver of this type will prefer the risky product. Using Bayes' rule, $g(\theta^h|t_r = (0, 1)) > \nu$ iff

$$E[v_r|\theta^h]Pr(t_r = (0, 1)|\theta^h) > -E[v_r|\theta^l]Pr(t_r = (0, 1)|\theta^l)$$

Collecting terms, and substituting for the definition of $Pr(t_r = (0, 1)|\theta^h)$, this is true if and only if

$$\frac{E[v|\theta^h]Pr(\overline{m}|\theta^h)}{E[v|\theta^l]Pr(\overline{m}|\theta^l)} > \left(\frac{Pr(\rho = 0|\theta^l)}{Pr(\rho = 0|\theta^h)}\right)^{N-1} \tag{A.37}$$

By the assumption that $Pr(\rho_i = 0|\theta^h) \geq Pr(\rho_i = 0|\theta^l)$, the right-hand side is weakly less than 1. By the assumption that $v^t \geq v^I$, and the left had side is strictly greater than 1, completing the proof that $X(v_i)$ is increasing. To see the final inequality, note that for any $v_i > v^I$, $E[v|\theta^h]f_{v|\theta^h}(v_i) > E[v|\theta^l]f_{v|\theta^l}(v_i)$. Given that $Pr(\overline{m}|\theta) = \int_{v^t}^{\overline{v}} f_{v|\theta}(v)F_c(X(v))dv$,

$$\frac{E[v|\theta^h]Pr(\overline{m}|\theta^h)}{E[v|\theta^l]Pr(\overline{m}|\theta^l)} = \frac{\int_{v^t}^{\overline{v}} E[v|\theta^h]f_{v|\theta^h}(v)F_c(X(v))dv}{\int_{v^t}^{\overline{v}} E[v|\theta^l]f_{v|\theta^l}(v)F_c(X(v))dv} > \frac{\int_{v^t}^{\overline{v}} E[v|\theta^l]f_{v|\theta^l}(v)F_c(X(v))dv}{\int_{v^t}^{\overline{v}} E[v|\theta^l]f_{v|\theta^l}(v)F_c(X(v))dv} = 1 \tag{A.38}$$

Thus, if $v^t \geq v^I$, then $X(v_i)$ is increasing when $v_i > v^t$. The proof that $v^t \geq v^I$ implies that $X(v_i)$ is decreasing when $v_i < v^t$ mirrors the steps taken thus far. □

Proof of corollary 2.6.12. This corollary follows from the final step of the proof of proposition 2.6.12. In the exposition of the inequality in (A.38), we note that because $v^t > v^I$, $\int_{v^t}^{\overline{v}} E[v|\theta^h]f_{v|\theta^h}(v)F_c(X(v))dv >$

$\int_{v^t}^{\overline{v}} E[v|\theta^l] f_{v|\theta^l}(v) F_c(X(v))dv$. However, v^t does not need to be greater than v^I for this inequality to remain true. Indeed, when $v^t = v^I$, the integrand on the left-hand side is strictly greater than the integrand on the right-hand side at every interior point. Therefore, because the integrands are continuous, there exists some value of v^t, call it $\hat{v}^t$, such that $\hat{v}^t < v^I$ and $\int_{\hat{v}^t}^{\overline{v}} E[v|\theta^h] f_{v|\theta^h}(v) F_c(X(v))dv = \int_{\hat{v}^t}^{\overline{v}} E[v|\theta^l] f_{v|\theta^l}(v) F_c(X(v))dv$. Thus for a value of v^t epsilon greater than $\hat{v}^t$, (A.37) continues to hold, and it therefore remains that $X(v_i)$ is increasing on $v_i > v^t$. Furthermore, since this $v^t < v^I$, by proposition 2.6.12, $X(v_i)$ is decreasing on $v_i < v^t$.

The proof that there exists a $v^t > v^I$ such that the described comparative statics of $X(v_i)$ hold, follows a similar set of steps. □

Proof of proposition 2.6.13. In addition to the conditions of the statement of the proof, assume that $X(v_i) = X(-v_i)$ and $A(\mu^\Phi) = 0.5$. To prove that this is an equilibrium, we will show that no player has an incentive to deviate. By the symmetry, of the environment and $X(v_i) = X(-v_i)$, lemmaA.35, tells us that the receiver's best response cutoff strategy is around equality in the number of sent low messages and sent high messages (for any n).

From lemma 2.6.10, we know that $v^t = v^I$ iff $\alpha = -\gamma$ (as defined in that proof). By the previous observation regarding receiver's s strategies, $X(v_i) = X(-v_i)$, $A(\mu^\Phi) = 0.5$, and $E[v_r|\theta^h] = -E[v_r|\theta^l]$, this is plainly true. By a similar observation, it is clear that $X(v_i) = X(-v_i)$. Furthermore, $X(v_i) = X(-v_i)$ implies that $Pr(\rho_i = 0|\theta^h) = Pr(\rho_i = 0|\theta^l)$, and therefore $A(\mu^\Phi) = 0.5$ is a best response. □

Appendix B

Constructions for Chapter 2

In this appendix we construct a number of densities which are needed for section 2.6.2. We begin by stating the definitions we need to proceed. Note that for any sender, i, $t_r \in T_r$, and $\mathbf{m}_{-i}$, the mean $\mu(\cdot)$ is invertible in i's message in the sense that there is at most one m_i such that $\mu(\mathbf{m}_{-i}, m_i) = t_r$. We describe this message as $\mu^{-1}(t_r, \mathbf{m}_{-i})$. By definition of the mean, $\mu^{-1}(t_r, \mathbf{m}_{-i}) = (|\mathbf{m}_{-i}| + 1)t_r - \mathbf{m}_{-i} \cdot \mathbf{1}$, where $\mathbf{1}$ is a column vector of 1s.

Next, choose any $t_r \in V$ and set of participants, $\mathbf{n}$. Let $n = |\mathbf{n}|$, and rename the players in $\mathbf{n}$ as $1, 2, ..., n$, and consider the case in which the messages of players $1, 2, ..., i-1$ are known to player i but the messages of players $i+1, ..., n$ are not. Given that the aggregate must equal t_r, the set of feasible messages for player i is constrained. We describe this constrained set as $\eta(t_r, m_1, ..., m_{i-1})$. Given the messaging strategy $M(\cdot)$, $\eta(t_r, m_1, ..., m_{i-1}) = \{m_i \in V : \exists v_{i+1}, ..., v_n,$ s.t. $m_i = \mu^{-1}(t_r, m_1, ..., m_{i-1}, M(v_{i+1}), ..., M(v_n))\}$. It will prove useful to identify the maximal and minimal elements of this set and thus we let $\overline{\eta}(\cdot) = max\ \eta(\cdot)$ and $\underline{\eta}(\cdot) = min\ \eta(\cdot)$.

We are now prepared to construct $F_{S|\theta}(m_i)$. This is not simply a transformation of variable via $M(\cdot)$ because the distribution also depends on the relative likelihood of participation for each type. Instead, we are looking for the probability that $M(v_i) < m_i$ for some m_i, *conditional on participation.* That is, for any $M(\cdot)$ with inverse $M^{-1}(\cdot)$, $F_{S|\theta}(m_i) = Pr(M(v_i) \leq m_i | \theta, \rho_i = 1)$.

Therefore, $F_{S|\theta}(m_i)Pr(\rho_i = 1|\theta) = Pr(v_i \leq M^{-1}(m_i), c_i \leq X(v_i)|\theta)$, and

$$\begin{aligned} F_{S|\theta}(m_i)Pr(\rho_i = 1|\theta) &= \int_V \int_C f_{v|\theta}(v_i) 1_{v_i \leq M^{-1}(m_i)} 1_{c_i \leq X(v_i)} f_c(c_i) dc_i dv_i \\ &= \int_{\underline{v}}^{M^{-1}(m_i)} f_{v|\theta}(v_i) F_c\left(X(v_i)\right) dv_i \end{aligned}$$

and,

$$f_{S|\theta}(m_i) = \frac{dF_{S|\theta}(m_i)}{dm_i} = \frac{\frac{dM^{-1}(m_i)}{dm_i} f_{v|\theta}(M^{-1}(m_i)) F_c\left(W(M^{-1}(m_i))\right)}{Pr(\rho_i = 1|\theta)} \tag{B.1}$$

Next, we construct $f_{\mu(\mathbf{m})|\theta,\mathbf{n}}(t_r)$ via a continuous change of variable. For some $\mathbf{m}$, let $\hat{\mu} = \mu(\mathbf{m})$ and for $k = 1, ..., n-1$, let $\hat{\mu}_k = m_k$. Thus, $m_n = \mu^{-1}(\hat{\mu}, \hat{\mu}_1, ..., \hat{\mu}_{n-1})$, and the absolute value of the Jacobian determinant is

$$\left| det \begin{pmatrix} \frac{\partial m_1}{\partial \hat{\mu}_1} & \cdots & \frac{\partial m_1}{\partial \hat{\mu}_{n-1}} & \frac{\partial m_1}{\partial \hat{\mu}} \\ \vdots & \ddots & \vdots & \vdots \\ \frac{\partial m_{n-1}}{\partial \hat{\mu}_1} & \cdots & \frac{\partial m_{n-1}}{\partial \hat{\mu}_{n-1}} & \frac{\partial m_{n-1}}{\partial \hat{\mu}} \\ \frac{\partial m_n}{\partial \hat{\mu}_1} & \cdots & \frac{\partial m_n}{\partial \hat{\mu}_{n-1}} & \frac{\partial m_n}{\partial \hat{\mu}} \end{pmatrix} \right| = \left| \frac{\partial m_n}{\partial \hat{\mu}} \right| = n$$

By the above definition of $\mu^{-1}(\cdot)$, given any set of values $\mathbf{m}_{-i}$, in the state θ, the likelihood that the aggregate message takes the value t_r is $f_{S|\theta}\left(\mu^{-1}(t_r, \mathbf{m}_{-i})\right)$. Therefore, implementing the change of variable,

$$f_{\mu(\mathbf{m})|\theta,\mathbf{n}}(\hat{\mu}) = n \int_{\underline{\eta}(\hat{\mu})}^{\overline{\eta}(\hat{\mu})} ... \int_{\underline{\eta}(\hat{\mu},\hat{\mu}_1,...,\hat{\mu}_{n-2})}^{\overline{\eta}(\hat{\mu},\hat{\mu}_1,...,\hat{\mu}_{n-2})} f_{S|\theta}(\hat{\mu}_1)...f_{S|\theta}(\hat{\mu}_{n-1}) f_{S|\theta}\left(\mu^{-1}(\hat{\mu}, \hat{\mu}_1, ..., \hat{\mu}_{n-1})\right) d\hat{\mu}_1...d\hat{\mu}_{n-1} \tag{B.2}$$

Appendix C

Proofs for Chapter 3

Proof of Lemma 3.4.1. Inequalities IC.1 and IC.2 follow almost immediately from the fact that in equilibrium senders choose a message that maximizes their expected utility given their types. Choose an arbitrary sender, i, and arbitrary cost, c_i. Let $v_i' > v_i > \hat{v}$, $t_i = (v_i, c_i)$, and $t_i' = (v_i', c_i)$. Let $M(\cdot)$ be a symmetric equilibrium messaging strategy, and assume that $M(\hat{v}) \in I$. We need to show that:

$$\sum_{Y \in \{L,I,H,\Phi\}} \sum_{Z \in \{L,I,H\}} w(Z) \left[P(YZ|M(t_i'), \theta^l) - P(YZ|M(t_i), \theta^l) \right] > 0 \tag{C.1}$$

Once we have established this fact, the lemma follows by algebraic manipulation of the utility maximization condition.

To see the inequality holds, we first observe that for a given θ, all but two positively weighted event likelihoods are constant between $M(t_i')$ and $M(t_i)$. That is, for a given θ,

$$\begin{aligned} &\sum_Y \sum_Z w(Z) \left[P(YZ|M(t_i'), \theta) - P(YZ|M(t_i), \theta) \right] \\ &= \left(a^I Pr(LI|M(t_i'), \theta) + Pr(LH|M(t_i'), \theta) \right) - \left(a^I Pr(LI|M(t_i), \theta) + Pr(LH|M(t_i), \theta) \right) \end{aligned}$$

For any level of participation n, $Pr(LH|M(t_i), \theta)$, the probability of the event in which $\mu(\mathbf{m}_{-i}) \in L$ and $\mu(\mathbf{m}) \in H$, is simply the probability that $\mu(\mathbf{m}_{-i}) \in (\frac{M(\hat{v}) - M(t_i)}{n-1}, M(\hat{v}))$. This a property of the mean; any value of the aggregate message that excludes sender i's message in that range, will be overwhelmed by sender i's message to an extent that the aggregate message will be greater than $M(\hat{v})$, and as a result, the receiver's action changes from choosing the safe product to the risky product.

Similarly, $Pr(LI|\theta, M(t_i))$ is the probability that $\mu(\mathbf{m}_{-i}) = \frac{M(\hat{v})-M(t_i)}{n-1}$. This is the value of the aggregate message that excludes sender i's message that results in the aggregate message equaling $M(\hat{v})$ exactly[1]. Combining these likelihoods, $Pr(LI|M(t_i),\theta) + Pr(LH|M(t_i),\theta) = Pr(\mu(\mathbf{m}_{-i}) \in [\frac{M(\hat{v})-M(t_i)}{n-1}, M(\hat{v}))|\theta)$.

From here it is clear that if we increase sender i's message, then the range in which sender i changes the outcome increases strictly and the likelihood therefore increases weakly. Therefore, we obtain

$$Pr(LH|M(t_i'),\theta) \geq Pr(LI|M(t_i),\theta) + Pr(LH|M(t_i),\theta)$$

Applying this to inequality C.1 completes the proof. □

Proof of Lemma 3.4.2. Choose an arbitrary sender, i, and arbitrary cost, c_i. Let $v_i' > v_i > \hat{v}$, $t_i = (v_i, c_i)$, and $t_i' = (v_i', c_i)$. Assume $M(\cdot)$ is a symmetric equilibrium messaging strategy. By the incentive compatibility conditions

$$\frac{Pr(\theta^h|t_i)}{Pr(\theta^l|t_i)} \leq \frac{\sum_Y \sum_Z w(Z)\left[P(YZ|M(t_i),\theta^l) - P(YZ|M(t_i'),\theta^l)\right]}{\sum_Y \sum_Z w(Z)\left[P(YZ|M(t_i),\theta^h) - P(YZ|M(t_i'),\theta^h)\right]}$$

Rearranging terms, we obtain

$$\begin{aligned} &E[u|t_i,\rho_i = 1, M(t_i)] \\ \leq &Pr(\theta^h|t_i)\sum_Y \sum_Z w(Z)P(YZ|M(t_i'),\theta^h) - Pr(\theta^l|t_i)\sum_Y \sum_Z w(Z)P(YZ|M(t_i'),\theta^l) \\ < &E[u|t_i',\rho_i = 1, M(t_i')] \end{aligned} \tag{C.2}$$

[1]We assumed at the outset of the proof that $M(\hat{v}) \in I$. If this is not the case, the result of the proof is unchanged.

where the second inequality follows from that $Pr(\theta^h|t_i)$ is strictly increasing in v_i.

The proof when $v_i' < v_i < \hat{v}$ that $E[u|(v_i, c_i), \rho_i = 1]$ is decreasing follows a similar set of steps. □

Proof of Proposition 3.4.3. Choose an arbitrary sender, i, and arbitrary cost, c_i. Let $v_i' > v_i > \hat{v}$, $t_i = (v_i, c_i)$, and $t_i' = (v_i', c_i)$. Assume $M(\cdot)$ is a symmetric equilibrium messaging strategy. First, we observe that for a given θ, all but two positively weighted event likelihoods are constant between $M(t_i')$ and $M(t_i)$. That is, for a given θ,

$$\begin{aligned}&\sum_Y \sum_Z w(Z)\left[P(YZ|M(t_i'),\theta) - P(YZ|M(t_i),\theta)\right] \\ &= \left(a^I Pr(LI|M(t_i'),\theta) + Pr(LH|M(t_i'),\theta)\right) - \left(a^I Pr(LI|M(t_i),\theta) + Pr(LH|M(t_i),\theta)\right)\end{aligned} \tag{C.3}$$

Next, we utilize this insight to rearrange terms of inequality C.2 from the proof of lemma 3.4.2, yielding

$$\begin{aligned}&\sum_\theta Pr(\theta|t_i)E[v_r|\theta]\left[a^I Pr(LI|M(t_i),\theta) + Pr(LH|M(t_i),\theta)\right] \\ \leq &\sum_\theta Pr(\theta|t_i)E[v_r|\theta]\left[a^I Pr(LI|M(t_i'),\theta) + Pr(LH|M(t_i'),\theta)\right] \\ < &\sum_\theta Pr(\theta|t_i')E[v_r|\theta]\left[a^I Pr(LI|M(t_i'),\theta) + Pr(LH|M(t_i'),\theta)\right]\end{aligned} \tag{C.4}$$

where the second inequality follows from the fact that $Pr(\theta^h|t_i)$ is strictly increasing in v_i. These inequality therefore tell us that this part of a sender's expected utility in particular is increasing in his value, v_i.

Finally, we apply this result to $X(v_i)$. To do this, we need to expand its terms:

$$X(v_i) = \sum_\theta Pr(\theta|v_i)E[v_r|\theta]\sum_Y \sum_Z \left[w(Z) - w(Y)\right] P(YZ|\theta, M(v_i))$$

where

$$\sum_Y \sum_Z [w(Z) - w(Y)]\, P(YZ|m_i, \theta) = \{ \tag{C.5}$$

$$\begin{aligned} & & a^I Pr(LI|m_i,\theta) + \quad & Pr(LH|m_i,\theta) \\ - \quad & a^I Pr(IL|m_i,\theta) & & + (1-a^I) Pr(IH|m_i,\theta) \\ - \quad & Pr(HL|m_i,\theta) - \quad (1-a^I)Pr(HI|m_i,\theta) & & \\ & -A(\mu^\Phi)Pr(\Phi L|m_i,\theta) \; + (a^I - A(\mu^\Phi))Pr(\Phi I|m_i,\theta) \; + (1-a^\Phi)Pr(\Phi H|m_i,\theta)\} & & \end{aligned}$$

Similarly to equation C.3, only two of these positively weighted event probabilities is non-constant between $M(t_i)$ and $M(t_i')$, and these are $a^I Pr(LI|m_i,\theta)$ and $Pr(LH|m_i,\theta)$. Of the remaining terms, by properties of the average aggregate messaging mechanism, only $(1-a^I)Pr(IH|m_i,\theta)$ and $(1-a^\Phi)Pr(\Phi H|m_i,\theta)$ are non-zero.

Therefore, we can re-write $X(v_i)$:

$$\begin{aligned} X(v_i) = \quad & \sum_\theta Pr(\theta|t_i) E[v_r|\theta] \left[a^I Pr(LI|M(t_i),\theta) + Pr(LH|M(t_i),\theta)\right] \\ & + \sum_\theta Pr(\theta|t_i) E[v_r|\theta] \left[(1-a^I)Pr(IH|\theta) + (1-a^\Phi)Pr(\Phi H|\theta)\right] \end{aligned}$$

We know from inequalities C.4 that the first term is increasing. To see that the second term is increasing as well, let $C(\theta) = (1-a^I)Pr(IH|\theta) + (1-a^\Phi)Pr(\Phi H|\theta)$ and note that $C(\theta) > 0$ for both θ. The second term is thus

$$E[v_r|\theta] \left[Pr(\theta^h|t_i)C(\theta^h) - Pr(\theta^l|t_i)C(\theta^l)\right]$$

which is plainly increasing since $Pr(\theta^h|t_i)$ is strictly increasing in v_i. □

Proof of proposition 3.4.4. From Chapter 1, section 2.5 it follows immediately that

$$X(v_i) = \begin{cases} -a^{\Phi}E[v_r|v_i], & \text{if } M(v_i) \in L \\ (1-a^{\Phi})E[v_r|v_i], & \text{if } M(v_i) \in H \end{cases} \tag{C.6}$$

In order to proceed from here we must first establish that $Pr(\Phi|\theta^h) = Pr(\Phi|\theta^l)$ if and only if $F_c(X(v_i)) = F_c(X(-v_i))$ for all v_i. That $Pr(\Phi|\theta^h) = Pr(\Phi|\theta^l)$ if $F_c(X(v_i)) = F_c(X(-v_i))$ for all v_i is clear. For the other direction, assume that $F_c(X(v_i)) \neq F_c(X(-v_i))$ for at least one v_i. Since $E[v_r|v_i] = -E[v_r| - v_i]$, this assumption implies that $a^{\Phi} \neq 0.5$. Moreover, if we let $\epsilon(v) = F_c(X(v)) - F_c(X(-v))$, equation C.6 says that $\epsilon(v)$ does not change sign on either of the half-spaces created by $\hat{v}$. Thus:

$$\begin{aligned} 1 - Pr(\Phi|\theta^h) &= \sum_v F_c(X(v))Pr(v|\theta^h) = \sum_v F_c(X(v))Pr(-v|\theta^l) \\ &= \sum_v \left(F_c(X(-v)) + \epsilon(v)\right) Pr(-v|\theta^l) \\ &= 1 - Pr(\Phi|\theta^l) + \sum_v \epsilon(v)Pr(v|\theta^l) \end{aligned}$$

where

$$\begin{aligned} \left|\sum_v \epsilon(v)Pr(v|\theta^l)\right| &= \left|\sum_{v<\hat{v}} \epsilon(v)Pr(v|\theta^l) + \sum_{v>\hat{v}} -\epsilon(-v)Pr(v|\theta^l)\right| \\ &= \left|\sum_{v<\hat{v}} \epsilon(v)Pr(v|\theta^l) - \epsilon(v)Pr(-v|\theta^l)\right| \\ &= \left|\sum_{v<\hat{v}} \epsilon(v)\left[Pr(v|\theta^l) - Pr(v|\theta^h)\right]\right| > 0 \end{aligned}$$

Therefore, it cannot be that $Pr(\Phi|\theta^h) = Pr(\Phi|\theta^l)$, which proves our claim that $Pr(\Phi|\theta^h) = Pr(\Phi|\theta^l)$ if and only if $F_c(X(v_i)) = F_c(X(-v_i))$.

In order for $a^{\Phi} \in (0,1)$, the receiver must be indifferent between products when he receives the null message. Therefore, $a^{\Phi} \in (0,1)$ only if $Pr(\Phi|\theta^h) = Pr(\Phi|\theta^l)$. The previous argument says that this may only occur if $F_c(X(v_i)) = F_c(X(-v_i))$ for all v_i. Equation C.6 shows clearly that $X(v_i)$ is strictly decreasing in a^Φ for all $v_i < \hat{v}$ and strictly increasing in a^Φ for all $v_i > \hat{v}$. Therefore, as we change a^Φ, $X(v_i)$ increases point-wise on half of the domain, and decreases point-wise on the other half. δ is therefore simply the smallest amount, in either direction, that a^Φ must move away from 0.5 until $X(v_i) = c_i$ for any v_i, c_i. At such a point, $F_c(X(v_i)) \neq F_c(X(-v_i))$, $Pr(\Phi|\theta^h) \neq Pr(\Phi|\theta^l)$, and therefore $a^\Phi \notin (0,1)$, in equilibrium.

It is trivial to see that if $a^\Phi \in (0.5-\delta, 0.5+\delta)$ then both the sender's and the receiver's best responses are identical to what they would have been if $a^\Phi = 0.5$. □

Appendix D

Instructions and Slides

Messaging Game – INSTRUCTIONS – 2011.02.03 – 2x5x1

Thank you for participating in this experiment. During the experiment we require your complete, un-distracted attention, so we ask that you follow these instructions carefully. You may not use the computer except as specifically instructed. Do not chat with other students or engage in activities such as using your cell phones or headphones or reading books.

For your participation, you will be paid in cash at the end of the experiment. Different participants may earn different amounts, and what you earn depends partly on your decisions, partly on the decisions of others, and partly on chance. It is therefore important that you listen carefully and fully understand the instructions before we begin.

The entire experiment will take place through computer terminals, and all interaction between participants will take place through the computers. It is important that you not talk or in any way try to communicate with other participants during the experiment except according to the rules described in the instructions.

[pause]

In this experiment, you will make choices over a sequence of **90** matches. In each match, you will be randomly paired with a group of participants. You will make decisions and receive a payoff that will depend on your decisions and on the decisions of the other participants in your group in that match.

At the end of the experiment, you will be paid the sum of what you have earned in all matches, plus the show-up fee of $**5**. Everyone will be paid in private and you are under no obligation to tell others how much you earned. Your earnings during the experiment are denominated in *points*. At the end of the experiment you will be paid *$1.00 for every* ***500 points*** you have earned.

[pause]

This is an experiment studying rating behavior regarding product quality. In this experiment we have a game with two products, a “new” product and an “old,” known product. The quality of the **old** product is assumed to be well-known and those who consume it therefore always receive **100** points. However, the quality of the **new** product is uncertain. In each match the new product is either of “high” quality “low” quality. If its quality is “low”, then the new product will give fewer points on average, and if its quality is “high”, then the new product will give more points on average. The slide at the front of the room displays the likelihood of each number of points, depending on the quality of the new product.

[slide 1, **explain**]

Thus, if you consume the new product, then you may receive more or fewer points than if you had consumed the old product, which, again, gives 100 points. Notice also that no matter what the quality of the new product, you can still receive any value; it is just more likely that you receive low values when the quality is low, and it is just *more likely* that you receive high values when the quality is high.

In each match, the computer randomly matches you into **groups of three**. Since there are **{12}** participants in today's session, this will result in **{4}** groups. You are not told the identity of the other participants with whom you are matched.

Next, the computer determines the quality of the new product for the match. There is a 50% chance that the computer will choose the new product's quality to be high, and there is a 50% chance that the computer will choose the new product's quality to be low.

Next, the computer randomly assigns a role to each member of the group: **2** members will have the “*sender*” role and one member will have the “*receiver*” role. Senders act first, and receivers act second. If you are a sender, you will get a clue about the new product's quality. That is, you will see an example quantity of points chosen as if you had consumed the new product.

[**while pointing to the board, “so if the new product is of high quality, then your clue is more likely to be a high value, and if the new product is of low quality, then your clue is more likely to be a low value”]**.

[slide 2]

The slide at the front of the room shows an example screen-shot of a sender's clue. In this case, the sender has received a value of **125.** Based on the previous slide, we know that this clue implies that the new product is more likely to be of high quality than it is to be of low quality.

Note that each sender's clue is independent of other senders' clue; that is, the clue that one sender receives does not affect the clue of another sender. This means that any **two** senders can have different clues, and because of the way clues are distributed [as seen on the slide] it is possible that one sender could have a high clue, like 175, while the another have a low clue, like 25.

[so if you are a sender, with a clue of 175, you can be fairly sure that the new product has high quality, but you cannot be certain. And remember, even if it is high quality, there is still a 15% chance that the *another* sender got a clue that is less than 100 [point to board]]

Next, if you are a sender, the computer will ask you for a message to send to the receiver. You can choose any message from the following list:

[read and point to each option. "**As you can see, there is a radio button for each possible message choice: 0, 25, 50, 75, 100, 125, 150, 175, 200**".

[slide 3]

This slide shows a screen-shot of a subject choosing a message. In this case, the sender has chosen a message of **125**.

It is important to note that the message chosen at this stage is *not immediately sent*. Instead, the message you choose is the message that you will send *if* you send a message. After you choose this potential-message, the computer presents each sender a "cost". This cost is the amount of points you would need pay in order to send your message. Note, that the message you choose in no way affects the cost that you receive.

After you see your cost, you *then* decide whether to send your message or to abstain. If you choose abstain, then the message you chose is not sent, and you do not pay your cost.

To reiterate: if you are a sender, you are shown an example point value of the new product; you choose your message; you view a cost of sending your message; and *then* you decide if you want to send the message you chose. Your cost is not affected by the message you choose.

[slide 4]

The potential costs to sending a message are listed on the x-axis of this table **[read and point {1, 2.5, 5, 7, 7.5, 10, 12.5, 15, 20, 25}**. On the y-axis is the likelihood that a sender obtains each cost. As you can see, each sender has an equal chance of receiving each cost. Since there are 9 total options, this means there is a 1/9 chance of receiving each cost, each match.

Note that each sender's cost is independent of other senders' cost; that is, the cost of one sender does not affect the cost of another sender.

[slide 5]

In this slide the sender has received a cost of **5** and now must decide between sending his message for a cost of **5** (on the left), and abstaining (on the right).

[slide 6]

While the receiver is waiting for the senders to act, his screen will look like this.

[**pause**]

After each sender makes his/her choice, it is the receiver's turn to act. First, the computer **averages** all the messages that were sent by the senders into one message. This **average** message is then presented to the receiver, and the receiver then chooses which product to consume: the new or the old.

If there are no messages sent to the receiver, then the receiver will simply see **"No messages were sent."**

[slide 7]

In this example, the receiver received an average message of **125**.

[slide 8]

and in this example, no messages were sent, so the receiver simply sees the message **“No messages were sent.”**

After observing the message, the receiver chooses which product to consume. If *you* have the role of the receiver and choose the old product, then you receive **100** points for sure.

[slide 9]

In this slide, the example receiver chooses the old product and receives 100 points.

However, if you are the receiver and choose the new product, then you receive an unknown quantity of points that is correlated with the quality of the new product, as I described before.

[slide 10]

In this example, the receiver chose the new product and received **150** points.

[slide 11]

In this example, the receiver chose the new product and received **50** points.

[Pause and Emphasize]

Now this is very important. At the end of each round all players in a group (both senders and receivers) receive the number of points obtained by the receiver. This means that if the receiver obtains 150 points, then the senders also receive 150 points (minus whatever costs they already paid). Similarly, if the receiver obtains only 50 points, then the senders also receive 50 points (minus whatever costs they already paid). It is therefore in the senders' best interest to attempt to maximize the receiver's point total.

[slide 12]

In our example, because the receiver got **150** points, everyone in the group received **150** points. In this slide you see the payoff information for a sender. Payout information is reflected both in the center of the screen and in the summary table at the bottom of the screen.[**point and go over each item of the history screen. Eg: You can see the match number, your role, your signal of the new product's quality, your cost, your message (if you sent one), what the receiver saw, the receivers decision, what the receiver got, the true quality of the new product, followed by the your payoff for that round]**. This information in the history table stays there throughout the remaining matches and you can see the result of every completed match.

[slide 13]

Here we see the summary information for a receiver. Since the receiver did not see a signal about the new product's quality, did not have a cost, and did not send a message, these fields are marked with an “N/A” in the history panel [**point**].

[pause]

This completes the description of a match. At the end of a match, groups are randomly reshuffled, roles are randomly re-assigned, and in each new group the quality of the new product is chosen anew. There are **30** matches in this part of the experiment, and each proceeds in exactly the same way.

[pause, briefly]

There are additional parts of the experiment that are nearly identical to the part I just described. I will explain the differences when you come to those parts of the experiment.

[slide 14]

The slide at the front of the room will summarize the game I just described

[read the slide].

You may refer to this slide throughout the experiment.

[pause]

We will now go through **5** practice matches. You are not paid for your performance in these matches; they are simply meant to familiarize you with the game before you begin the paid matches.

[AUTHENTICATE CLIENTS]

please double-click on the icon labeled **MC** on your computer's desktop. Then click "run". When the computer prompts you for your name, type your First and Last name. Then click SUBMIT and wait for further instructions.

[after all clients in]

If there are any questions during the practice matches, please raise your hand. At this time please slide out the dividers between your computers.

[confirm]
[start game]
[remember, you manually advance matches until match 4. (do match 4, **do not advance to match 5**)]

[after the practice matches]

This concludes the practice matches. Are there any questions before we continue to the paid matches? If there are any questions during the paid matches, please raise your hand and I will assist you in private.

[Do with Caution:]
[in the menu: setup> auto advance matches]
[in the menu: setup> set match delay to 2000 milliseconds]
[click next match]

[make sure to go back to manual before match 34]
[after match 34 (**Do not advance to match 35!**)]

This concludes the first part of the experiment. The next part of the experiment is the same as the first part of the experiment but with one difference: in the second part of the experiment there are **6** people in each group: **5** senders and **1** receiver. This means that in each match, there will be **{2}** group(s). The game is otherwise exactly the same as in the first part of the experiment.

[slide 15 with new info]

The second part of the experiment consists of **30** matches. Are there any questions before we begin the second part of the experiment? **[click next match. Go to automatic]**

[make sure to go back to manual before match 64]
[after match 64 (**Do not advance to match 65!**):]

This concludes the second part of the experiment. The next part of the experiment is the same as the first and second parts but with one difference: in the third part of the experiment there are **2** people in each group: **1** sender and still only **1** receiver. This means that in each match, there will be **{6}** group(s). The game is otherwise exactly the same as in the first part of the experiment.

[slide 16 with new info]

The second part of the experiment consists of **30** matches. Are there any questions before we begin the second part of the experiment?

[after the third phase is over]

This was the last match of the experiment.

[CLICK ON WRITE OUTPUT]

I will now pass out your payout sheets. Please fill out your information and enter $**5** where it says "show up fee". I will tell you how much you earned in total in private.

[hand out sheets]

I will now pay each of you in private in the back of the room. Remember you are under no obligation to reveal your earnings to the other participants.

Please do not use either the mouse or the keyboard. Please remain seated and keep the dividers pulled out until I call you to be paid. Do not converse with the other participants or use your cell phone. Thank you for your cooperation.

Please come to the back of the room when I call your name

[print out second pay sheet.]

[CALL all the participants in sequence by their name]

Welcome.
Please wait patiently for the experiment to begin, and do not use the computers until you are asked to do so.

New Product Point Probabilities

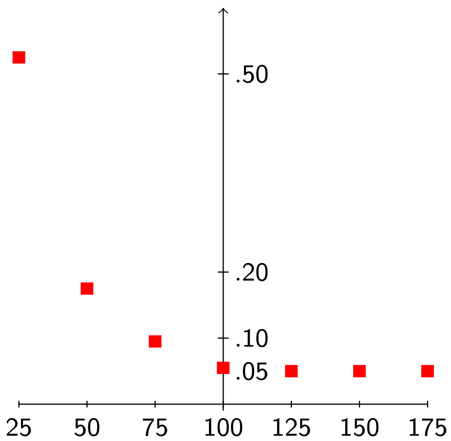


Figure: Low Quality

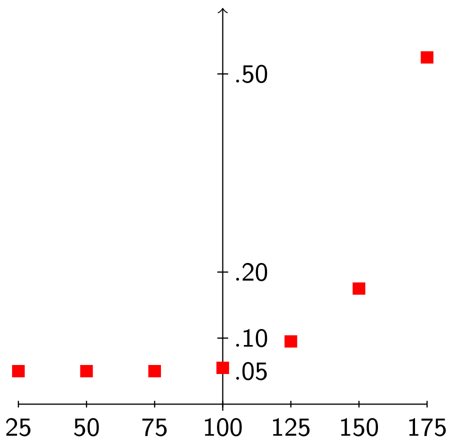
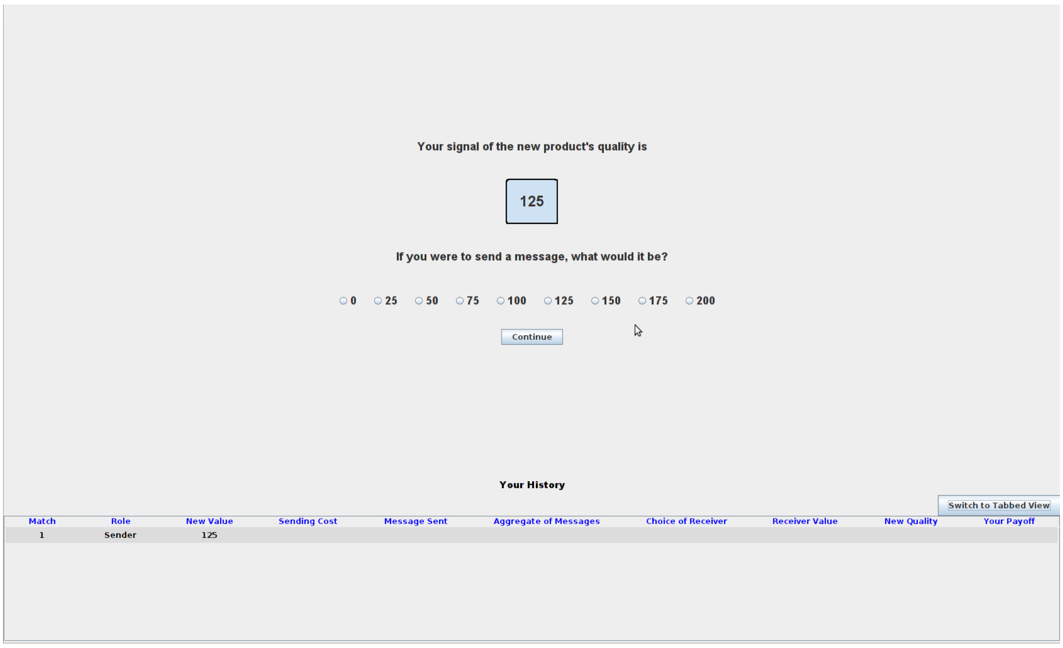
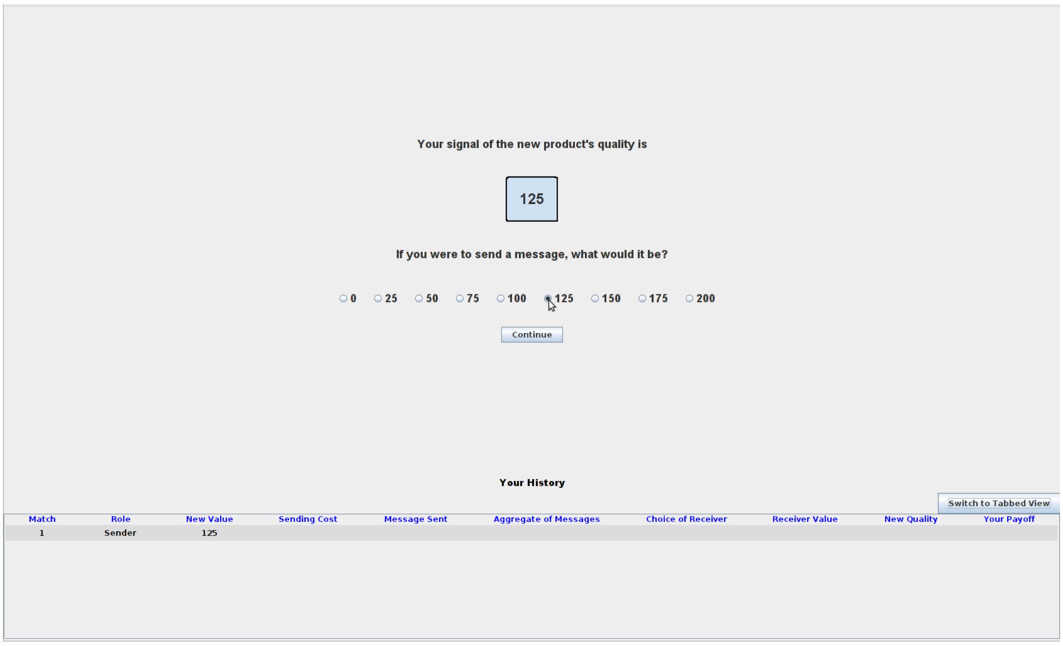


Figure: High Quality


Your signal of the new product's quality is
125
If you were to send a message, what would it be?
0 25 50 75 100 125 150 175 200
Continue
Your History
Switch to Tabbed View
Match Role New Value Sending Cost Message Sent Aggregate of Messages Choice of Receiver Receiver Value New Quality Your Payoff
1 Sender 125


Your signal of the new product's quality is
125
If you were to send a message, what would it be?
0 25 50 75 100 125 150 175 200
Continue
Your History
Switch to Tabbed View
Match Role New Value Sending Cost Message Sent Aggregate of Messages Choice of Receiver Receiver Value New Quality Your Payoff
1 Sender 125

Potential Costs and their Likelihoods

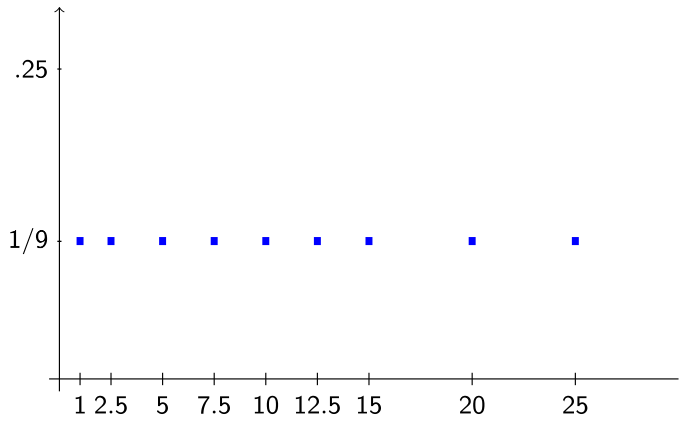
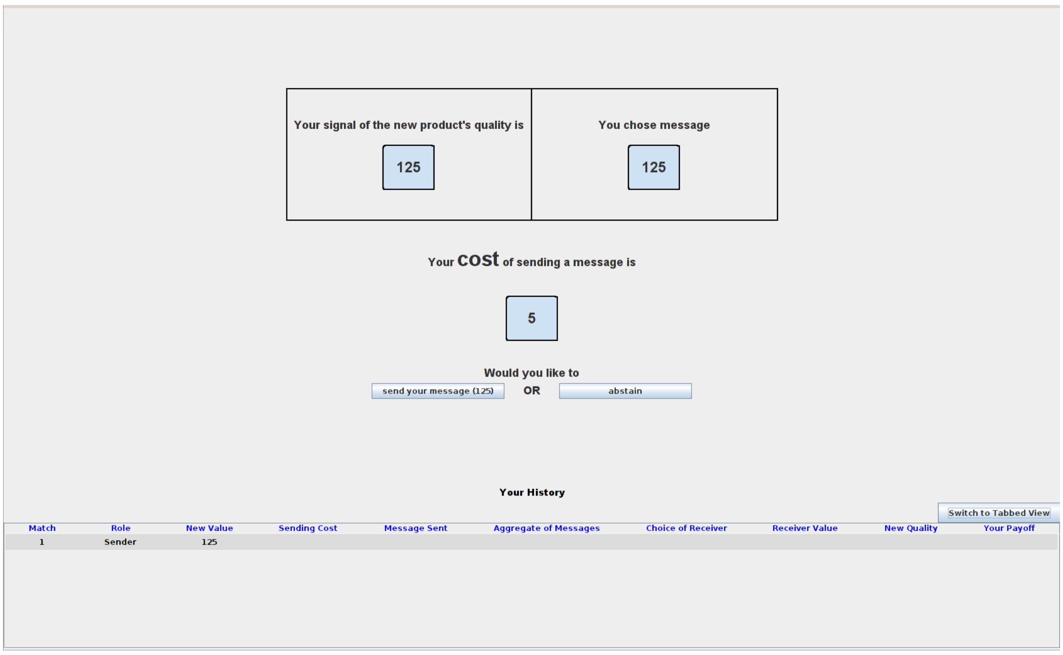
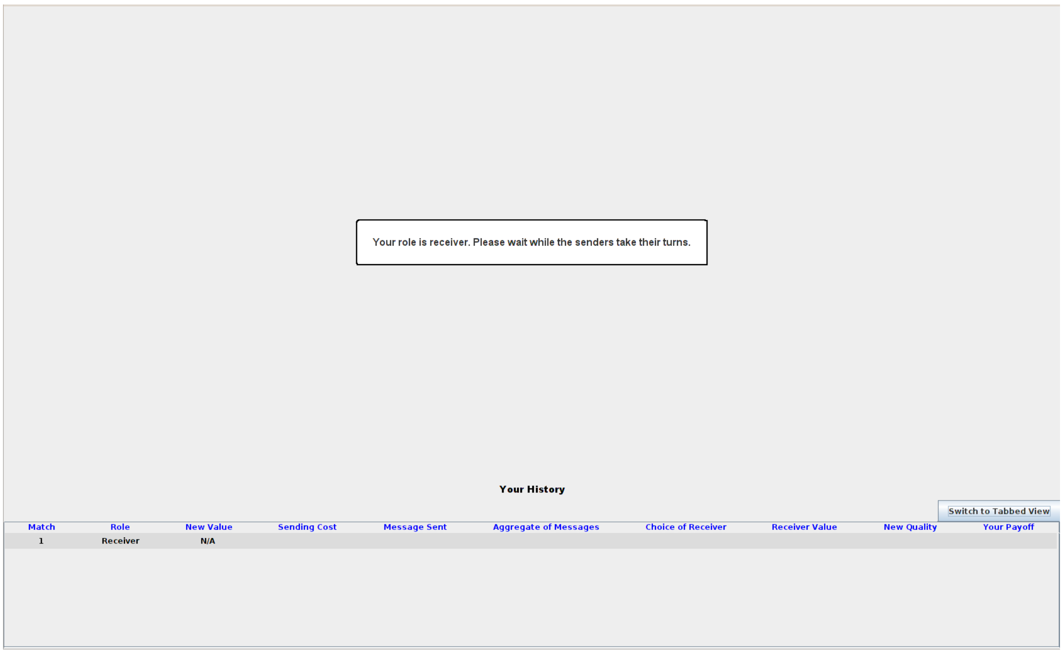
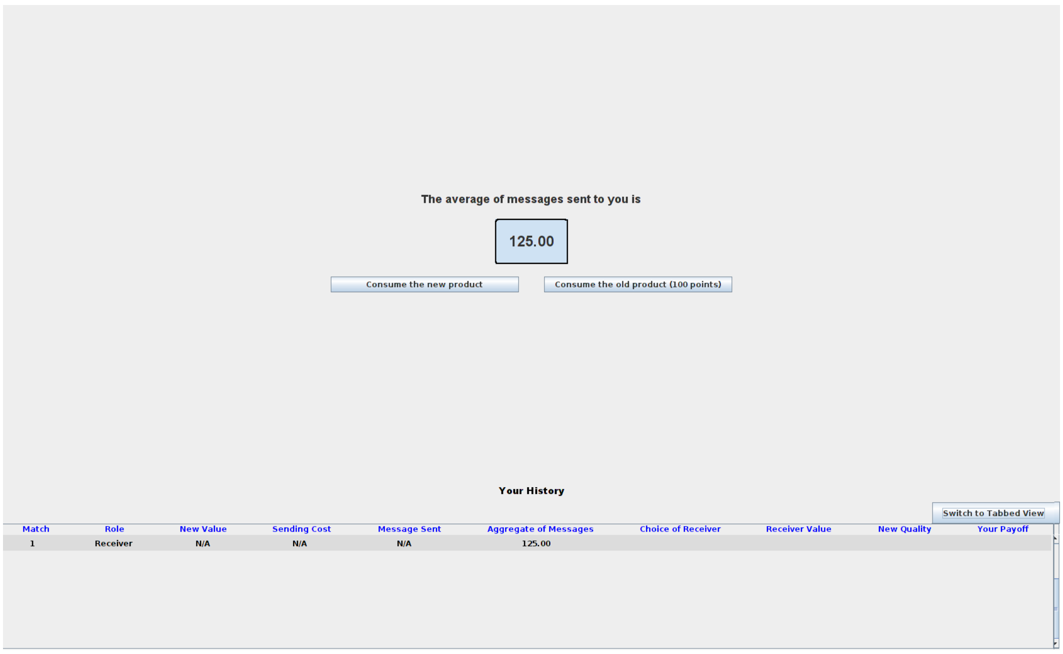
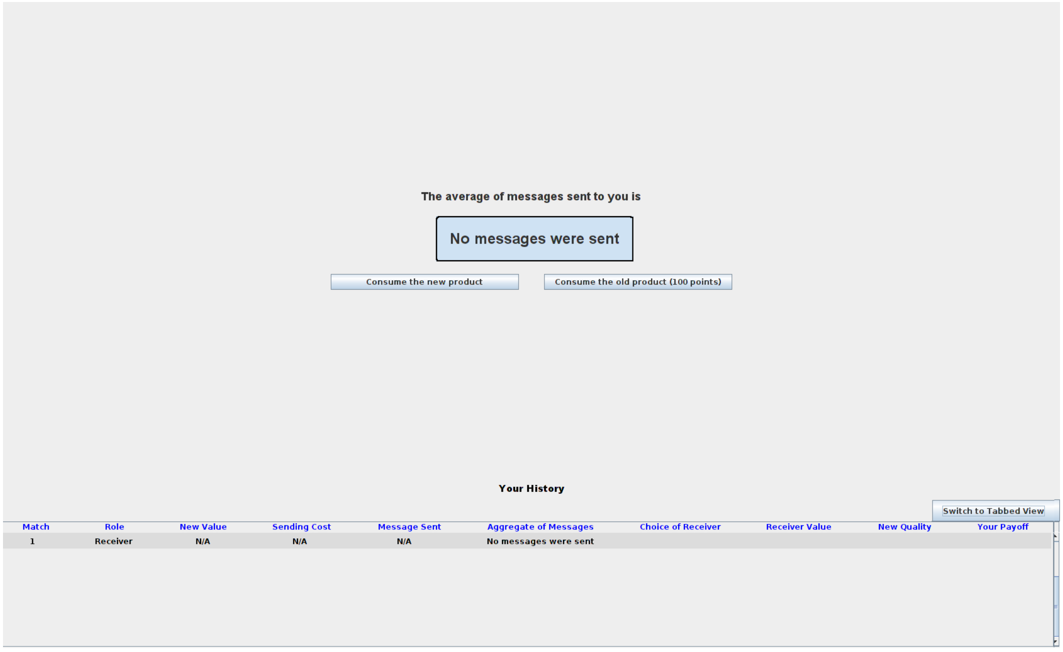


Figure: Likelihoods of Costs to Sending a Message

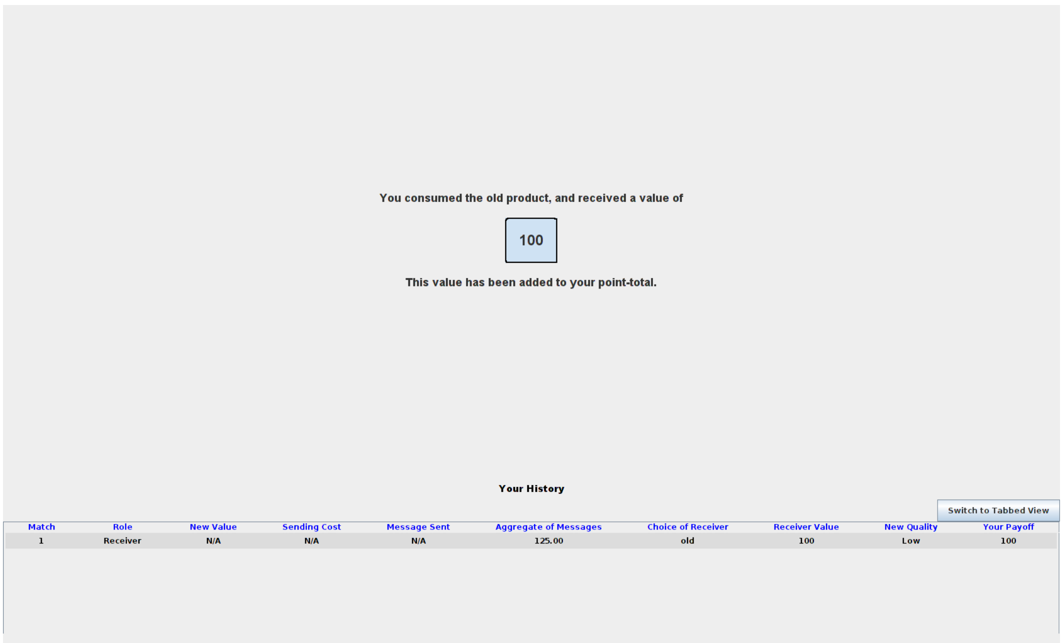



Your role is receiver. Please wait while the senders take their turns.
Your History
Switch to Tabbed View
Match | Role | New Value | Sending Cost | Message Sent | Aggregate of Messages | Choice of Receiver | Receiver Value | New Quality | Your Payoff
1 | Receiver | N/A

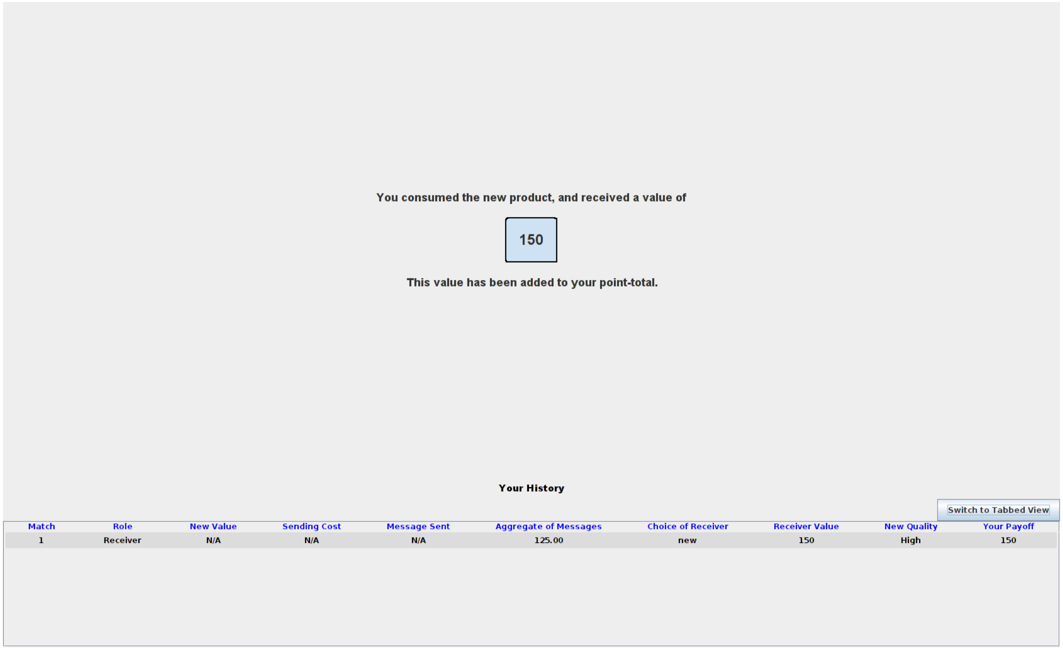

The average of messages sent to you is
125.00
Consume the new product
Consume the old product (100 points)
Your History
Switch to Tabbed View
Match | Role | New Value | Sending Cost | Message Sent | Aggregate of Messages | Choice of Receiver | Receiver Value | New Quality | Your Payoff
1 | Receiver | N/A | N/A | N/A | 125.00

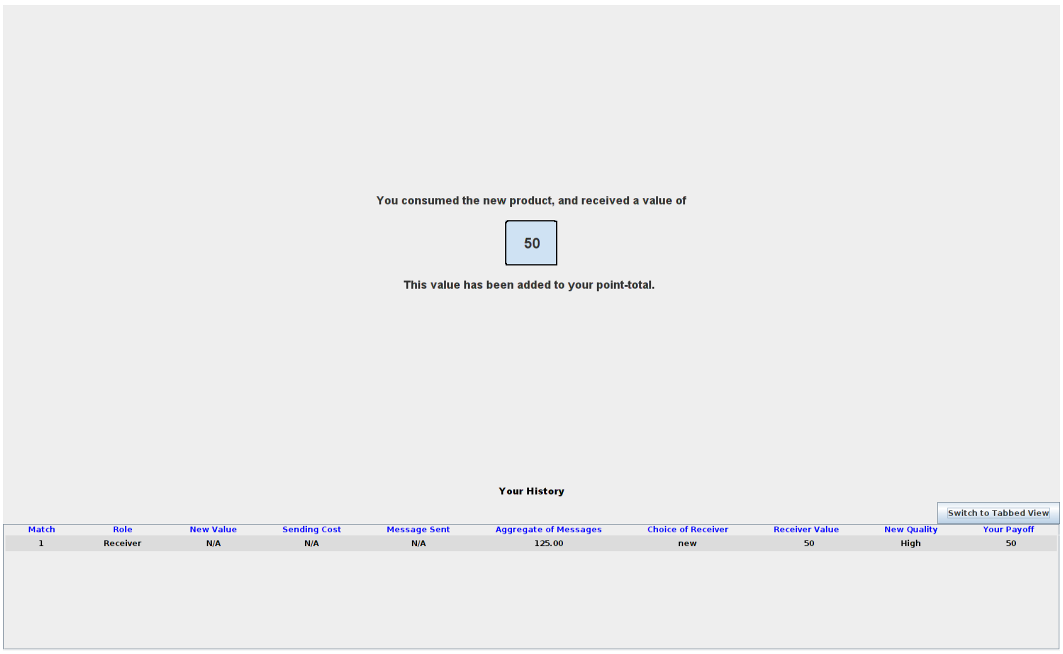


Match	Role	New Value	Sending Cost	Message Sent	Aggregate of Messages	Choice of Receiver	Receiver Value	New Quality	Your Payoff
1	Receiver	N/A	N/A	N/A	No messages were sent				



Match	Role	New Value	Sending Cost	Message Sent	Aggregate of Messages	Choice of Receiver	Receiver Value	New Quality	Your Payoff
1	Receiver	N/A	N/A	N/A	125.00	old	100	Low	100


You consumed the new product, and received a value of
150
This value has been added to your point-total.
Your History
Switch to Tabbed View
Match | Role | New Value | Sending Cost | Message Sent | Aggregate of Messages | Choice of Receiver | Receiver Value | New Quality | Your Payoff
1 | Receiver | N/A | N/A | N/A | 125.00 | new | 150 | High | 150


You consumed the new product, and received a value of
50
This value has been added to your point-total.
Your History
Switch to Tabbed View
Match | Role | New Value | Sending Cost | Message Sent | Aggregate of Messages | Choice of Receiver | Receiver Value | New Quality | Your Payoff
1 | Receiver | N/A | N/A | N/A | 125.00 | new | 50 | High | 50

The receiver consumed the new product, and received a value of

150

This value has been added to your point-total.

Your History

Switch to Tabbed View

Match	Role	New Value	Sending Cost	Message Sent	Aggregate of Messages	Choice of Receiver	Receiver Value	New Quality	Your Payoff
1	Sender	125	5	125	125.00	new	150	High	145

You consumed the new product, and received a value of

150

This value has been added to your point-total.

Your History

Switch to Tabbed View

Match	Role	New Value	Sending Cost	Message Sent	Aggregate of Messages	Choice of Receiver	Receiver Value	New Quality	Your Payoff
1	Receiver	N/A	N/A	N/A	125.00	new	150	High	150

- Groups of 3 are chosen randomly each match.
- 2 products: "old" and "new".
  - Old product gives 100 points.
  - New product gives 25, 50, 75, 100, 125, 150, or 175 points.
  - 50% chance new product is "high quality," and 50% chance the new product is "low quality".
  - Higher quality means higher chance of more points.
- In each group: 2 "senders" and 1 "receiver".
  - Senders get a clue about the quality of the new product.
  - Senders choose a message: 0, 25, 50, 75, 100, 125, 150, 175, or 200
  - Senders have a cost for sending their message.
  - Possible costs are 1, 2.5, 5, 7.5, 10, 12.5, 15, 20, and 25 points.
  - After a sender sees his cost, he decides whether or not to send his message to the receiver.
  - Next, the receiver views the *average* of all sent messages.
  - If no messages are sent, he sees "No messages were sent."
  - The receiver then decides which product to consume.
- Receiver obtains the number of points he obtained from his choice.
- A sender receives the number of points obtained by the receiver, minus his cost if he sent his message.
- The exchange rate is $1 for every 500 points.

- Groups of 6 are chosen randomly each match.
- 2 products: "old" and "new".
  - Old product gives 100 points.
  - New product gives 25, 50, 75, 100, 125, 150, or 175 points.
  - 50% chance new product is "high quality," and 50% chance the new product is "low quality".
  - Higher quality means higher chance of more points.
- In each group: 5 "senders" and 1 "receiver".
  - Senders get a clue about the quality of the new product.
  - Senders choose a message: 0, 25, 50, 75, 100, 125, 150, 175, or 200
  - Senders have a cost for sending their message.
  - Possible costs are 1, 2.5, 5, 7.5, 10, 12.5, 15, 20, and 25 points.
  - After a sender sees his cost, he decides whether or not to send his message to the receiver.
  - Next, the receiver views the *average* of all sent messages.
  - If no messages are sent, he sees "No messages were sent."
  - The receiver then decides which product to consume.
- Receiver obtains the number of points he obtained from his choice.
- A sender receives the number of points obtained by the receiver, minus his cost if he sent his message.
- The exchange rate is $1 for every 500 points.

- Groups of 2 are chosen randomly each match.
- 2 products: "old" and "new".
  - Old product gives 100 points.
  - New product gives 25, 50, 75, 100, 125, 150, or 175 points.
  - 50% chance new product is "high quality," and 50% chance the new product is "low quality".
  - Higher quality means higher chance of more points.
- In each group: 1 "sender" and 1 "receiver".
  - Senders get a clue about the quality of the new product.
  - Senders choose a message: 0, 25, 50, 75, 100, 125, 150, 175, or 200
  - Senders have a cost for sending their message.
  - Possible costs are 1, 2.5, 5, 7.5, 10, 12.5, 15, 20, and 25 points.
  - After a sender sees his cost, he decides whether or not to send his message to the receiver.
  - Next, the receiver views the sent message directly.
  - If no message is sent, he sees "No messages were sent."
  - The receiver then decides which product to consume.
- Receiver obtains the number of points he obtained from his choice.
- Sender receives the number of points obtained by the receiver, minus his cost if he sent his message.
- The exchange rate is $1 for every 500 points.

Appendix E

Experimental Software Payout Bug

Tables 3.1 includes a row labeled "Expected", which lists the expected value of each column, given empirical play. These numbers deviate from the row labeled "empirical" because the experimental code contained a bug which resulted, on average, in players receiving values for the risky product that were too high. The bug worked by replacing the lowest possible value, 25, with the most recent draw from the risky product. Thus, in the one-sender treatment, senders' draws were made from the correct distributions (see Figure E.1a), but receiver draws were from a distribution with far less weight on 25 than the theoretical distributions (Figure E.1b). Since multiple senders' draws were made in sequence, in the multiple sender treatments the bug affected the draws of both senders and receivers. The resulting empirical densities are summarized in Figure E.1.

The bug was potentially more harmful than solely through the obvious effect of the loss of the author's money. This is because if players adapted their beliefs about the payoff structure (or their beliefs about beliefs about the payoff structure, etc.) then the game would have diverged such that we could not calculate equilibria to the game, or even best responses to empirical play.

The simplest method to test for whether or not first order beliefs were altered from the theoretical distributions is by observing receiver behavior. Since all subjects played both roles, it is safe to assume that all players' beliefs were reflected in the actions of receivers. Taking the empirical density functions of receivers' draws from the risky product as the true value densities, we obtain new expected values for the risky product in each state. These in turn change the receivers' decision calculus. As we showed in the previous chapter, for a risk neutral receiver, choosing the risky

N	$E[v\|\theta^l]$	$E[v\|\theta^h]$	$\hat{\nu}$	ν_1^*	ν_2^*
1	100.28	152.08	-0.01	0.51	0.48
2	108.13	157.99	-0.16	0.47	0.56
5	102.91	165.34	-0.05	0.49	0.71

Table E.1: Expected Utilities When Receivers' Empirical Distributions of Risky Draws are Taken as the True Distributions of Values. $\hat{\nu}$ is the value of ν resulting from empirical distributions. Both columns labeled ν^* are values of ν, obtained through different methodologies, that minimize the sum of squared errors in subjects' actions, given empirical distributions.

product is the optimal action whenever their posterior belief about the high state, given the aggregate message, is greater than $\nu = \frac{100-E[v|\theta^l]}{E[v|\theta^h]-E[v|\theta^l]}$. Using the *theoretical* distributions of the risky product, we had $E[v|\theta^h] = 200 - E[v|\theta^l]$ and $\nu = 0.5$. Expected values in the each state of the world and the resultant νs generated by the *empirical* distributions are displayed in Table E.1.

Table E.1 reveals that the bug affected payoffs enough such that the expected value of the risky product in the low state of the world was higher than the value of safe product in all treatments. As a result, in every treatment, the receivers' optimal action was to always choose the risky product. That is, the frequency with which the receivers ought to have selected the risky product was 1, no matter the value of the aggregate message, including the null message. However, this is not what we observe. As was discussed above, receivers' behavior was consistent with a cutoff strategy around the aggregate message of 100, meaning the rate at which subjects chose the risky product when the aggregate message was less than 100 was nearly 0. In addition, the rate at which players selected the risky product upon receipt of the null message was between 6% and 9% in all treatments. Subjects' play therefore does not indicate that they fully adjusted their beliefs to incorporate the altered empirical value distributions. Moreover, there is no evidence that subjects updated their beliefs even partially. This is evident in Figures 3.4 and 3.7. In those figures, the theoretical ν is represented by the dotted horizontal line at 0.5. Receipt of any value of the aggregate message for which the posterior belief in the high state (represented by the orange dashed line) exceeds ν, will result in selection of the risky product by a value-maximizing receiver. Any adjustments towards the empirically adjusted beliefs would result in a reduction in ν and correspond in the figures to a downward shift in the horizontal dotted line. However, the data show no evidence that subjects' play was guided by a ν less than 0.5. Indeed, as the figures make clear, subjects' behavior is almost perfectly consistent with a ν value of 0.5.

The final two columns of Table E.1 display values of ν that, if believed to be the true ν by subjects, minimize the sum of squared errors in subjects' actions. ν_1^* and ν_2^* differ in their methodologies. To calculate ν_1^* we used empirical messaging frequencies and the *empirical* distributions of risky values to generate expected densities of aggregate messages which yielded Bayesian posteriors for each feasible value of the aggregate message. To obtain ν_2^*, we also used empirical messaging frequencies, but we used the *prior* distribution of risky values. ν_2^* therefore assumes that players did not update their beliefs about value distributions, while ν_1^* assumes that posteriors are equal to empirical rates of play. In all treatments, the ν_1^* are approximately equal to the prior, theoretical value of 0.5, and the ν_2^* range from 0.48 to 0.71. These results suggest that, if anything, players adjusted their belief about the distributions of risky values in the *opposite direction* of what we would expect, given the nature of the bug.

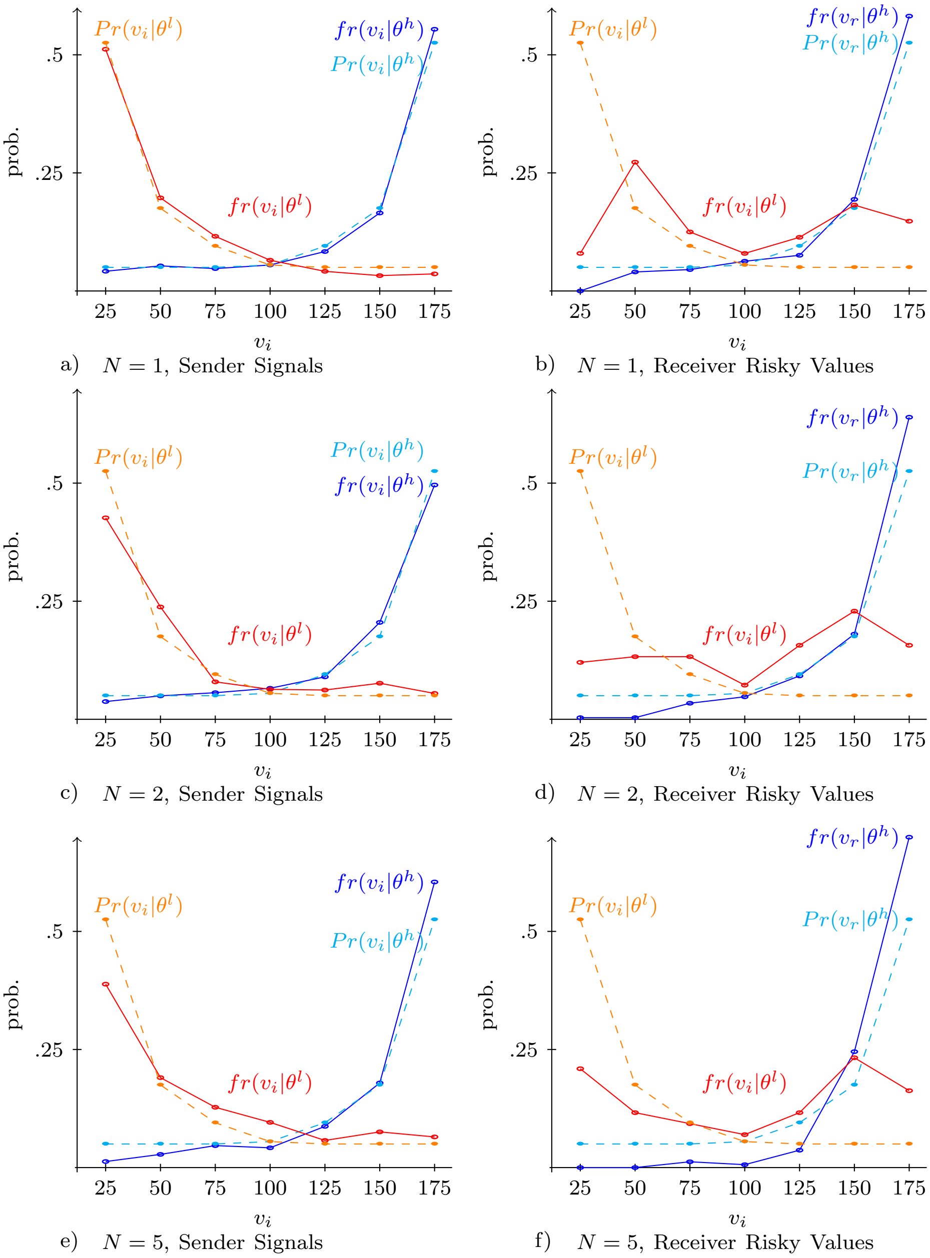


Figure E.1: Frequencies of sender signals (v_i) and receiver draws (v_r) from the risky product versus theoretical densities, separated by state of the world

Appendix F

Proofs and Constructions for Chapter 4

This appendix adopts the notation developed in the previous two chapters. In particular, we utilize the language developed there to describe outcome-equivalent events. Moreover, for brevity, for any $X \in \{\emptyset, L, I, H\}$, we let $A^*(X|\theta) = A(\mathbf{m})E[v_r|\theta] + (1 - A(\mathbf{m}))\hat{v}$ for all $\mathbf{m} \in X$. We will also be utilizing the following definition of cursed expected utility from Eyster and Rabin (Eyster and Rabin 2005):

$$E_{CE}[u|t_k, a_k] = \sum_{t_{-k} \in T_{-k}} p_k(t_{-k}|t_k) \sum_{a_{-k} \in A_{-k}} \overline{\sigma}_{-k}(a_{-k}|t_k) u_k(a_k, a_{-k}; t_k, t_{-k}) \tag{F.1}$$

and the following piece of notation:

Definition F.0.2. *The average likelihood, for type t_k, of event XY, given m_k, averaged over states of the world*

$$\overline{Pr}(XY|m_k, t_k) = \sum_{\theta} Pr(\theta|t_k) Pr(XY|m_k, \theta) \tag{F.2}$$

First, it is plain to see that since cursed players may only update their beliefs based on their types, and since receivers in our game do not have types[1], fully cursed receivers act only on their prior beliefs. Since in our game the ex-ante expected utilities of choosing both products are equal,

[1] strictly speaking, types are information players have at the outset of the game, about player types and/or parameters of the game.

the fully cursed receiver can rationalize any behavior.

Lemma F.0.3. *The fully cursed sender of type t_k and message m_k has expected utility:*

$$E_{CE}[u|t_k, m_k] = \sum_{\theta} Pr(\theta|t_k) \sum_{X\in\{\emptyset,L,I,H\}} \sum_{Y\in\{L,I,H\}} \overline{Pr}(XY|m_k, t_k)A^*(Y|\theta) \tag{F.3}$$

Proof. What follows is a proof by construction as we transform (F.1) into (F.3).

First, we introduce the state of the world, θ into equation F.1 :

$$\begin{aligned} &\sum_{\theta} Pr(\theta|t_k) \sum_{t_{-k}\in T_{-k}} p_k(t_{-k}|\theta, t_k) \sum_{a_{-k}\in A_{-k}} \overline{\sigma}_{-k}(a_{-k}|t_k)u_k(a_k, a_{-k}; t_k, t_{-k}, \theta) \\ =&\sum_{\theta} Pr(\theta|t_k) \sum_{t_{-k}\in T_{-k}} p_k(t_{-k}|\theta) \sum_{a_{-k}\in A_{-k}} \overline{\sigma}_{-k}(a_{-k}|t_k)u_k(a_k, a_{-k}; t_k, t_{-k}, \theta)\end{aligned}$$

In our game, a sender's utility only depends on other sender's types through their actions and through their types' correlation with θ. Therefore, since all quantities have been made conditional on θ, (F.1) reduces to

$$\begin{aligned} &\sum_{\theta} Pr(\theta|t_k) \sum_{a_{-k}\in A_{-k}} \overline{\sigma}_{-k}(a_{-k}|t_k)u_k(a_k, a_{-k}; t_k, \theta) \sum_{t_{-k}\in T_{-k}} p_k(t_{-k}|\theta) \\ =&\sum_{\theta} Pr(\theta|t_k) \sum_{a_{-k}\in A_{-k}} \overline{\sigma}_{-k}(a_{-k}|t_k)u_k(a_k, a_{-k}; t_k, \theta)\end{aligned}$$

Now we separate the other senders' action space by outcome equivalence

$$\sum_{\theta} Pr(\theta|t_k) \sum_{Y\in\{L,I,H\}} \sum_{\substack{a_{-k}\in A_{-k}:\\ \mu(\mathbf{m})\in Y}} \overline{\sigma}_{-k}(a_{-k}|t_k)A^*(Y|\theta) \tag{F.4}$$

By definition (Eyster and Rabin (Eyster and Rabin 2005)):

$$\overline{\sigma}_{-k}(a_{-k}|t_k) \equiv \sum_{t_{-k}\in T_{-k}} p_k(t_{-k}|t_k) \cdot \sigma_{-k}(a_{-k}|t_{-k}) \tag{F.5}$$

From Rabin (2005), $\overline{\sigma}_{-k}(\cdot)$ is "the average strategy of other players, averaged over the other players types." aka "the probability that players $j \neq k$ play action profile a_{-k} when they follow strategy σ_{-k}."

Therefore,

$$\begin{aligned}\sum_{\substack{a_{-k}\in A_{-k}:\\ \mu(\mathbf{m})\in Y}} \overline{\sigma}_{-k}(a_{-k}|t_k) &\equiv \sum_{\substack{a_{-k}\in A_{-k}:\\ \mu(\mathbf{m})\in Y}} \sum_{t_{-k}\in T_{-k}} p_k(t_{-k}|t_k)\cdot\sigma_{-k}(a_{-k}|t_{-k}) \\ &= Pr(Y|t_k, m_k) \\ &= \sum_{\theta} Pr(\theta|t_k)Pr(Y|\theta, m_k) \end{aligned} \tag{F.6}$$

Finally, we need only notice that $\{\emptyset, L, I, H\}$ represents a partition of possible events when we exclude player k's message. We thus obtain

$$\begin{aligned}\sum_{\substack{a_{-k}\in A_{-k}:\\ \mu(\mathbf{m})\in Y}} \overline{\sigma}_{-k}(a_{-k}|t_k) &= \sum_{X\in\{\emptyset,L,I,H\}} \sum_{\theta} Pr(\theta|t_k)Pr(XY|\theta, m_k) \\ &= \sum_{X\in\{\emptyset,L,I,H\}} \overline{Pr}(XY|m_k, t_k)\end{aligned}$$

Substituting back into our transformed expression for cursed expected utility, we obtain

$$E_{CE}[u|t_k, m_k] = \sum_{\theta} Pr(\theta|t_k) \sum_{X\in\{\emptyset,L,I,H\}} \sum_{Y\in\{L,I,H\}} \overline{Pr}(XY|m_k, t_k)A^*(Y|\theta)$$

which is what we wanted to show. □

This is in contrast to the uncursed expected utility:

$$E[u|t_k, m_k] = \sum_{\theta} Pr(\theta|t_k) \sum_{X\in\{\emptyset,L,I,H\}} \sum_{Y\in\{L,I,H\}} Pr(XY|m_k, \theta)A^*(Y|\theta)$$

Appendix G

Code Presentation and Discussion

G.1 Chapter 2

The code used to find fixed points of the game described in Chapter 2 was written in Python. It was written with flexibility in mind so it could, for example, be adapted for use in the experimental environment of Chapter 3 and under the alternative hypotheses of Chapter 4.

The code consists of two basic objects, states and environments. An environment specifies the variables of the game such as the number of senders, the type-space, and the probability distributions over types. A state is initialized with an environment object and strategies for both senders and receivers. A state contains methods for computing aggregate densities (more on this below) as well as methods for computing best responses given the initialization variables (more on this below as well).

The algorithm starts with an initial state, calculates best responses to the strategies of the initial state given its environment, and then generates a new state using the initial environment and a convex combination of initial strategies and best response strategies. The new state then becomes the "initial" state. This process is repeated until the best response strategies are sufficiently close to the initial strategies, at which point the algorithm terminates and outputs the final strategies. We varied the initial state, but most often the initial state consisted of the sincere messaging strategy, a participation strategy that satisfied the extreme participation conjecture, and a cutoff strategy around v^I for the receiver such that $A(v^I) = A(v^\Phi) = 0.5$. Most often we accepted the best response strategies as the new strategies ($\alpha = 1$ in the convex combination) but we altered α as needed to locate fixed points. Finally, to calculate "sufficiently close" between initial strategies and best responses, we took the norm of the differences between vectors of strategies. If the min of the absolute value of this norm was less than 11^{-4}, the program terminated.

The best response message strategy is, for each type, the message that maximizes a sender's expected utility given the state's initial variables. To calculate this, for each message we computed the likelihood the sender was each type of pivotal. These likelihoods were then weighted by the sender's posterior expected utility given the outcome specified by that pivotality type. The sum of these weighted likelihoods represent the part of expected utility that depends on the sender's message. Therefore, the maximizing message also maximizes expected utility. In the event of indifference (to at least 4 decimal places), the program selected the message nearest to the message specified by the initial strategy (provided that was a pure strategy).

The best response participation strategy is the likelihood a sender of each type participates given the environment and given the initial strategies. The method we used used, as both input and output, the vector of $F_C(X(v_i))$ for each v_i. This is without loss of generality for our purposes. In this case, $X(v_i)$ (the net expected utility of participation, excluding the cost of participation) used the expected utility of participation given that the sender played the expected utility maximizing message (calculated as described above).

We calculated the state conditional densities of aggregate messages both when considering all N senders as well as when considering only $N-1$ senders (since the former is necessary for calculating a receiver's best response and the latter is necessary for computing a sender's best response). To calculate these we first calculated the densities of aggregate messages conditional on both the state as well as on each level of participation (more on this below), and calculated the likelihood of each level of participation given the initial strategies. The likelihood of a given aggregate message was then the sum, over levels of participation, of likelihoods of the aggregate message weighted by the likelihood of that level of participation.

Calculating the densities of aggregate messages conditional on level of participation represented the computationally intensive portion of the algorithm. To achieve these densities, we utilized a recursive function that took as arguments five scalars, a state object, and two vectors of densities with length equal to the number of feasible aggregate messages given the level of participation (this is finite given that we estimate continuous spaces with discrete spaces). This recursive function is recreated below. The idea of the functions was to compute the likelihood of every permutation of sent-messages in each state of the world in a computationally parsimonious way. It worked by looping through the set of feasible messages for each participant, keeping track of the sum of all sent-messages as well as the product of the likelihoods of those sent messages in each state. Once the function reached the final participant, it calculated the average value of the sent messages (the sum divided by total participation), and added the final products of sent-message likelihoods to the entry of the respective density vectors that represented that average value. Because the loops over feasible sent-messages were nested (achieved by calling the function recursively), we ensured that every permutation was accounted for.

```
def aggregate_densities(prods, theSum, player, z, st, densities):
    """
        prods is a 2-tuple of scalars. It tracks the interim likelihood of the
            sent-messages
        theSum is the interim sum of sent-messages. This is a sufficient
            statistic for the average of sent-messages, given that we know the
            level of participation.
        z is the total level of participation
        player is the number of player the function is currently considering.
            player starts at 1 and increases to z.
        st is a state object. It is used for access to the environment object
            which provides us with the feasible message and aggregate message
            spaces
        densities is a 2-tuple of vectors with length equal to the number of
            feasible aggregate messages. Both start as vectors of zeros.

        Calculates the likelihood of each feasible aggregate message, given
            the state, st, and given the total level of participation, z.
            Called as: aggregate_densities((1, 1), 0, 1, z, self, densities)
    """

    if player == z:
        for i in range(len(st.env.spaces["messages"])):
            em = st.env.spaces["messages"][i]
            theSumTemp = theSum + em
            theMean = theSumTemp*1./z

            mean_ind = appxEqualIndex(theMean, \
```

```
            st.env.spaces["aggregateMessagesByN"][z-1])

        if not type(mean_ind) is int:
            print "ERROR: %s not in %s" % \
                (theMean, st.env.spaces["aggregateMessagesByN"][z-1])

        densities[0][mean_ind] += st.fm_lVector[i]*prods[0]
        densities[1][mean_ind] += st.fm_hVector[i]*prods[1]

    return densities
else:
    for i in range(len(st.env.spaces["messages"])):
        em = st.env.spaces["messages"][i]
        theSumTemp = theSum + em
        prodsTemp = (prods[0]*st.fm_lVector[i], \
                     prods[1]*st.fm_hVector[i])

        densities = fmu_helper(prodsTemp, theSumTemp, player+1, z, st,\
                               densities)
    return densities
```

G.2 Chapter 3

The code for this chapter was also written in Python. Much of the coding work needed for this chapter was done by the code created for the chapter 2. For this chapter, we extended the environment object described above to specify the experimental parameters. We called this an "experiment" object and it was initialized with one parameter, the number of senders in the game. Because the experiment object extended the environment object, we were able to re-use the state object without alteration, and finding fixed points of the game described by the experimental parameters was simply a matter of running the algorithm described above with experiment objects (one for each treatment) instead of environment object.

To calculate best responses to empirical play, we first distilled the experimental data into the rates subjects of each type played each action. These rates were then passed to the state object in lieu of strategies. Finally, we used the methods built into the state object to output best responses to the experimental rates of play (these methods are described above).

G.3 Chapter 4

Coding for this chapter presented two challenges: modifying the code from the chapters 2 and 3 to incorporate cursed beliefs; and implementing a search algorithm for both QRE and CE-QRE.

To tackle the first challenge, we extended the previously-described state object and altered the method that calculated the likelihoods of pivotality. We called this new object "stateCE". The alteration between states was straightforward and followed the discussion in the theory section of this chapter. Calculating fully cursed best response was simply a matter of finding the best response to empirical rates of play using the stateCE object.

To find the best fit χ and λ in the CE-QRE model, we defined a grid of feasible values for each, and implemented a full search over all vertexes of the the resultant lattice. Since, QRE is a special case of CE-QRE, the search for best fit λ in the QRE model followed the same steps, but for the degenerate case of $\chi = 0$. For both cases, the search domain for $\hat{\lambda}$ was between 0 and 10 at intervals of 10^{-3}. In the CE-QRE search, the search domain for $\hat{\chi}$ was between 0 and 1 at intervals of 10^{-3}. For a given $\hat{\chi}$, the expected utility of a given action was calculated using the appropriate convex combination of expected utilities for those actions generated by the state and stateCE objects (both assumed other players played the empirical rates of play). The quantal response strategies were then calculated in the standard way.

Included below is a function that takes as arguments: the matrix of expected utilities for each type, for each action, given the players are fully cursed; the matrix of expected utilities for each type, for each action, given the players are Bayesians; the sizes of the step the search takes between λs and χs, and the maximum λ value. The function returns a matrix of likelihoods of playing each action, for each type, for each λ, and for each χ. This returned-matrix was subsequently used to calculate the maximum likelihood λ-χ pair, given subjects' actions in the experiment.

```
def cqre_get_sigmas( EUBayes_a_t, EUCurse_a_t, l_step, l_max, x_step=1 ):
    """
      returns the likelihood of playing each action, for each type,
          for each lambda, for each chi!
      x_step=1 corresponds to the un-cursed special case
```

```
"""
l_range = range(int( l_max/l_step ))
num_l   = len( l_range )
x_range = range(int( 1./x_step ))
num_x   = len( x_range )

dims = list( EUBayes_a_t.shape )
dims_range = map(lambda x: range(x), dims)

# creates all the possible indices of types. This alleviates the
#   need to know the dimensionality of the type-space.
indices = list( itertools.product( *dims_range[0:-1] ))

# creates a matrix the shape of EUs, for every lambda, for every chi
sigmas_all = numpy.zeros( [num_x] + [num_l] + dims )

for x in x_range:
    chi = x * x_step # x_step=1 to be the uncursed special case

    EUChiCursed = chi*EUCurse_a_t + (1 - chi)*EUBayes_a_t

    for l in l_range:
        lam = l * l_step

        # creates a matrix the shape of EUs,
        #   less the final, actions, dimension
        # num_signals_s, num_costs_s
        denoms = numpy.zeros(( dims[0:-1] ))

        # denominators
        for t in indices:
            max_a = numpy.max( EUChiCursed[t] )
            denoms[ t ] = lam*max_a + log(sum(map( lambda x: \
                        exp( lam*EUChiCursed[t][x] - lam*max_a ), \
                        range( dims[-1] ) )))

        # strategies
        for t in indices:
            for a in range( dims[-1] ): # actions
                # loops over all actions, including abstention
                ixa = tuple( list(t) + [a] )
                chixlxixa = tuple( [x] + [l] + list(ixa) )
                num_s = lam*EUChiCursed[ ixa ]

                log_sigma = num_s - denoms[t]
                sigmas_all[ chixlxixa ] = exp( log_sigma )

return sigmas_all
```

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