The Strategic Behavior of Rational Novices

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ABSTRACT

There is a growing amount of experimental evidence that suggests people often deviate from the predictions of game theory. Some scholars attempt to explain the observations by introducing errors into behavioral models. However, most of these modifications are situation dependent and do not generalize. A new theory, called the *rational novice model*, is introduced as an attempt to provide a general theory that takes account of erroneous behavior. The rational novice model is based on two central principals. The first is that people systematically make inaccurate guesses when they are evaluating their options in a game-like situation. The second is that people treat their decisions similar to a portfolio problem. As a result, non optimal actions in a game theoretic sense may be included in the rational novice strategy profile with positive weights.

The rational novice model can be divided into two parts: the behavioral model and the equilibrium concept. In a theoretical chapter, the mathematics of the behavioral model and the equilibrium concept are introduced. The existence of the equilibrium is established. In addition, the Nash equilibrium is shown to be a special case of the rational novice equilibrium. In another chapter, the rational novice model is applied to a voluntary contribution game. Numerical methods were used to obtain the solution. The model is estimated with data obtained from the Palfrey and Prisbrey experimental study of the voluntary contribution game. It is found that the rational novice model explains the data better than the Nash model. Although a formal statistical test was not used, pseudo R^2 analysis indicates that the rational novice model is better than a Probit model similar to the one used in the Palfrey and Prisbrey study.

The rational novice model is also applied to a first price sealed bid auction. Again, computing techniques were used to obtain a numerical solution. The data obtained from the Chen and Plott study were used to estimate the model. The rational novice model outperforms the CRRAM, the primary Nash model studied in the Chen and Plott study. However, the rational novice model is not the best amongst all models. A sophisticated rule-of-thumb, called the SOPAM, offers the best explanation of the data.

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Chapter 1: Introduction

There is a growing amount of experimental evidence that suggests people often do not behave according to game theory. Some examples of such evidence can be found in centipede game experiments[1], voluntary contribution game experiments[2] and experiments with a version of the chain store paradox[3]. In the first price auction experiments, although game theory predicts behavior that is close to the data, the statistical variations in the data cannot be reconciled with the theory. The evidence against game theory is particularly strong in the centipede game and the voluntary contribution game because the subjects have been found to deviate even from dominant strategies.

Another example can be found in the studies of first price auctions. Although all these studies show that Nash equilibrium models usually offer good fits to individual bids, deviations were always observed. The bids usually spread over a range that roughly centers at the Nash equilibrium. Nash models failed to explain the bid spread. Usually the statistical model used to fit the data is nothing more than the Nash equilibrium bidding function with an ad hoc random variable added. The reason that Nash models are incapable of explaining the bid spread is that all agents are assumed to be rational. As a result, the agents are required to follow very restrictive equilibrium strategy profiles that are the solution of complex mathematical problems.

Palfrey and McKelvey in "An Experimental Study of the Centipede Game" suggest that imperfect agents may be the answer. They developed a model which assumes there is a mixture of rational and irrational agents. The irrational agents play a fixed non-optimal

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[1] See Palfrey and Mckelvey, "An Experimental Study of the Centipede Game".
[2] See Palfrey and Prisbrey, "Anomalous Behavior in Linear Public Goods Experiments: How much and Why?"
[3] See Schmidt, "Reputation Building By Error Prone Agents"

strategy. The rational agents adjust their strategies given the probability of encountering an irrational agent. In a number of similar studies, people are indeed found to be imperfect as oppose to exhibit the kind of perfect rationality defined in game theory. Subjects behave as if they were trying to play optimal strategies but they were not sure where the optimum was.

My goal is to develop a theory that can provide a coherent explanation of non game theoretic behavior across different economic environments. Irrationality will be modeled at a fundamental level. A new model, which is called the rational novice model, is introduced as an alternative approach to game theory modeling. It is not only able to explain deviations from game theoretic models, but is also able to provide quantitative predictions of the distribution of these deviations.

The rational novice model is based on two ideas. The central idea of the rational novice model is that people systematically produce inaccurate guesses when they are evaluating their options in a game-like situation. This fact is common knowledge and people respond to it. The second idea is that people approach their decisions as if they are solving portfolio problems. This is different from the traditional game theory in which agents try to find optimal pure strategies. Instead, each agent considers a portfolio of strategies and determines the exact fraction of the times each strategies being played. This idea of a portfolio of strategies is also different from that of mixed strategies in which strategies are played randomly. One of the consequences is that risk averse people hedge their actions to reduce the risks associated with making incorrect evaluations. Thus, there is no single "optimal" strategy.

This model will be different from the Palfrey and McKelvey's centipede model in two important aspects. First, there will be no distinction between rational and irrational agents. All agents are assumed to be rational and irrational at the same time. Irrationality is modeled by introducing noisy signals into the agents' decision making mechanisms. Furthermore, while Palfrey and McKelvey's definition of an imperfect agent is game specific, this model provides a general theoretical framework of imperfection which can be applied to any environment.

Another approach is to model agents as rational decision-makers with imperfect information. Harsanyi (1973) introduced a game theory model in which the payoff matrix is uncertain. McKelvey and Palfrey (1994) introduced a similar model called the quantal choice model. In the quantal choice model, each agent observes an erroneous payoff vector. Then each agent chooses a strategy that maximizes the observed payoff. The Quantal Response Equilibrium (QRE) is then defined as a fixed point of this process. Given the statistical nature of the observed payoff vectors, the QRE puts positive probabilities not only on best responses.

The Harsanyi setup and the quantal choice model can be interpreted as a special case of the rational novice model. In all three models, agents observe erroneous payoffs. The agents in the quantal choice model accept the observed payoffs as their real payoffs. On the other hand, the rational novice model assumes the agents understand that they are erroneous in calculating their payoffs and that they try to respond to it. They respond by treating their problem as a portfolio problem. The agents who do not care about errors (risk neutral agents) will be playing quantal responses. Thus, the quantal choice model is a special case of the rational novice model when everyone is risk neutral.

Rosenthal (1989) has also developed a model of agents with bounded rationality. Rosenthal assumes that the probability of playing any strategy is linear in the payoff. Thus, inferior strategies will still be played, but with smaller probabilities. The rational novice model is considerably different from the Rosenthal. The rational novice model bases the subjects' behavior on a maximization principle instead of an assumed response function.

The advantage of the rational novice model can be seen when applied to the first price auctions. In previous studies, statistical models were constructed by adding an ad hoc nuisance random variable to the Nash model. The rational novice model does not require such construction. Furthermore, the statistical variations in bids are natural consequences of the rational novice model.

This theory assumes a totally different level of rationality from the traditional game theory. Nash models assume agents have preferences that can be describe mathematically. Actions are chosen to maximize preferences. Let us call this the first level of rationality. In the rational novice model, the agents are not able to fully deduce the solution to the maximization problem in the first level of rationality. They are only capable of arriving at noisy guesses. Assuming that the agents realize their imperfection, a second level of rationality is added to the model. In this second level of rationality, the agents choose a portfolio of actions to reduce the risks associated with making the wrong guess.

The main goal of this study is to answer two questions. Is the rational novice model a plausible theory? Does the rational novice model offer a good explanation to experimental data? The first question can be answered by determining whether the rational novice model is consistent with intuition and whether it has desirable theoretical properties. The only counter intuitive element in the model is that it seems that the agents are required to solve a more complicated mathematical problem (in the second level of rationality) while they are assumed to be incapable to solving a simpler one (in the first level of rationality). However, as can be seen in the later chapters, though complicated, the portfolio problem is essentially the same across different applications. On the other hand, each different situation requires a different maximization problem in the first level of rationality. It is plausible that people can learn the solution to the portfolio problem over time and over different circumstances. Moreover, even if the agents are not able to perfectly solve the portfolio problem, it does not invalidate this approach. Recall that the second level of rationality is the response to the inability of behaving perfectly in the first level of rationality. As the theory moves from the first to the second level of rationality, a wider range of phenomena is explained. There may be a third and a fourth level of rationality that may even do better.

A number of theoretical properties are derived in chapter 3. The most important one is the existence of the equilibrium. It is also established that when the Nash equilibrium is a special case of the rational novice equilibrium when the errors are small. Chapters 4 and 5 are two applications of the rational novice model. In both the voluntary contribution game and the first price sealed bid auction, the rational novice model is shown to provide better explanations to experimental data than Nash models.

Chapter 2: The Rational Novice Behavior

The rational novice model can be divided into two parts. The first part describes the behavior of a single agent. This chapter is devoted to describe the behavior model.

The model is based on two central principles. The first principle is that when people are calculating the utilities associated with their available actions, they make independent errors in their calculations. These errors may come from the fact that people round off when they are making calculations. For lack of a better term, these errors are called internal errors to distinguish them from external errors that are deviations from optimal strategies. The second principle is that people consider portfolios of strategies instead of single strategies when they make their decisions. Risk averse people dislike the risk associated with internal errors and they diversify this risk away by playing a mixture of actions. These two ideas are best illustrated by a simple example. Consider an agent faced with a game against nature. Imagine there are two jars full of coins and the agent has to pick one out of the two. The jars are of different dimensions and shapes. Ideally the agent will count the number of coins in each jar and pick the one with more coins. However, if the agent is not allowed to count or he has to make the decision in a hurry, he has to make a rough guess of the number of coins in each jar. Furthermore, the agent is aware of the inaccuracy of his guesses. Now if the agent is to play this game ten times with identical jars which he cannot open until the end of the games and he is not sure which jar has more money, it is to his advantage to hedge his actions. That is, he may want to pick jar one seven times and jar two three times to hedge against the risks of wrong guesses. This scenario is also analogous to the stock market. Consider each jar as a stock. The diversification by picking a mixture of different jars is similar to the diversification of portfolios in the stock market. The difference between the rational novice model and the other models with imperfect agents is that in the rational novice model, agents do not only react to other agents' imperfection. The agents are also reacting to their own imperfection by minimizing their external errors according to some relatively simple rules.

2.1 The Basic Framework of the Behavioral Model

The two basic principles are summarized in the following:

The Basic Principles

- The Principle of Imperfect Agents: Agents make internal errors when they evaluate their actions in a game.

- The Principle of Multiple Strategies: Each agent chooses a portfolio (exact frequencies) of actions to play.

The following is the formal model. Consider an agent faced with a game against nature. The game is repeated T times. Define:

a) the set of actions, $A=\{a^1, a^2, \ldots, a^M\}$

b) the set of true values, $v_a \in \Re$ for all $a \in A$

c) a vector of best guesses, $\bar{g} = \{g_a\}_{a\in A}$

d) the error structure, $\tau: \Re^M \to P(\Re^M)$

e) the conditional belief of the true value, $P(\bar{v}|\bar{g})$

f) strategy frequency profile, $\bar{p} = (p^1, p^2, \ldots, p^M) \in \Delta^M$

g) the utility function, $u: \Re \to \Re$.

The following assumptions are made to facilitate the tractability of the model. These assumptions are independent of the two central principles.

1) The agents do not discount

2) Each agent is paired with a random set of opponents in each game. There are no reputation effects.

3) Agents are risk averse and have concave utility functions u().

The two central principles can be formally defined as follows:

- The Principle of Imperfect Agents

In each period, if the agent takes the action $a \in A$, he receives v_a. (Consider each action as a jar and v_a as the amount of money in jar a.) An imperfect agent makes errors in his evaluation. For each action a, the agent arrives at a best guess g_a. The best guesses $\bar{g} = \{g_a\}_{a \in A}$ are generated by a joint probability measure $\tau(\bar{v})$, which is a function of the true values $\bar{v} = \{v_a\}_{a \in A}$. The function $\tau: \Re^M \to P(\Re^M)$ is called the error structure. Given the best guesses $\bar{g}$, the agent is assumed to have a belief $P(\bar{v}|\bar{g})$ about the probability measure of the true values $\bar{v}$ conditioned on the best guesses $\bar{g}$. In our "pick the jar" example, the agent guesses there are 10 coins in jar a. However, he is not sure and he has some beliefs about the probabilities of the cases where there are 9, 10 or 11 coins in the jar.

- The Principle of Multiple Strategies:

The agents do not discount and they care about the total sum of values in all the periods, $\sum_{t=1}^{T} v_{a_t}$. If the agent is perfect, he makes his decision in the following manner. He makes an evaluation of each feasible action and chooses the action which gives the highest v_a and plays it through all T periods.

Since the agent does not discount, the total sum of values only depends on the frequencies of actions and not the exact sequence of the actions. Define the strategy frequency profile of the agent to be $\bar{p}=(p^1,p^2,\ldots,p^M)$ belonging to the simplex Δ^M.[1] If the total number of games played is T, then the number of times action a^m is played is Tp^m. (The agent is going to pick the jar, which is labeled by a^m, Tp^m times.) The total value $\sum_{t=1}^{T} v_{a_t}$ can be written as $T\sum_{m=1}^{M} p^m v_{a^m}$ or $T\bar{p}\cdot\bar{v}$.

Combining the two principles, the agent's preference can be represented by the following maximization problem :

(2.1) $\quad \underset{\bar{p}}{\text{Max}} \int u(T\bar{p}\cdot\bar{v})dP(\bar{v}|\bar{g})$ subject to $\bar{p}\in\Delta^M$.

2.2 Comments and Discussion

This behavior model is similar to the ones that are used in the portfolio selection theories in the finance literature. Notice that in this model, the agents choose the frequencies of their actions. They are assumed to know exactly how often they play a certain action. It is also assumed that the sequences of their actions are randomized. For example, an agent may want to play action a 2 times and action b once. It is equally likely that he plays (a,a,b), (a,b,a) or (b,a,a).

Since the agent chooses an action in each of the T games he plays, the only possible frequencies of each action are multiples of 1/T. Therefore, although it is convenient to work with the continuous simplex Δ^M, the situation is better represented by the following maximization problem :

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[1]In actuality, the agents' can only choose between some points in the simplex.

$\underset{\bar{p}}{\mathrm{Max}} \int u(T\bar{p}\cdot\bar{v})dP(\bar{v}|\bar{g})$ subject to $\bar{p}\in\Delta_T^M$, where Δ_T^M is the T-discrete simplex defined by

$$\Delta_T^M = \left\{ \bar{p} = (p_1, p_2, \ldots, p_M) \middle| p_i \in \left\{\tfrac{1}{T}, \tfrac{2}{T}, \ldots, \tfrac{T}{T}\right\} \ \forall i \text{ and } \sum_{i=1}^{M} p_i = 1 \right\}.$$

In a later section, it will be shown that when T gets large, the continuous formulation is a good approximation of the discrete one.

Chapter 3: Aggregate Behavior and the Rational Novice Equilibrium

3.1 An Overview

The previous chapter describes a model of behavior of an agent faced with a game against nature. This section extends the model to multi-player games. The central theme of the rational novice model of multi-player games is the concept of the rational novice equilibrium, which is a departure from the traditional game theoretic equilibrium concepts. Rather than defining an equilibrium concept that describes mutually dependent individual behavior of several players, the rational novice equilibrium defines a large population's average aggregate behavior that fulfills some self-consistent conditions.

Definitions:

a) A population of agents J

b) $A=\{a^1,a^2,\ldots,a^M\}$, the symmetric action set for the agents

c) $a_{-j}=\{a_1,a_2,\ldots,a_{j-1},a_{j+1},\ldots,a_n\}$, the action profile of the N-1 players excluding player j

d) $v^j_{a_j,a_{-j}}$, the payoff of player j when he is playing a_j

e) $v^j_a(p)=\sum\limits_{a_{-j}\in A\times A\times\ldots\times A} v^j_{a,a_{-j}}(p_{a_1}\cdot p_{a_2}\cdot\ldots\cdot p_{a_{j-1}}\cdot p_{a_{j+1}}\cdot\ldots\cdot p_{a_n})$, the average value of action a for j

g) $\tau:\Re^M\to P(\Re^M)$, the error structure

h) $u:\Re\to\Re$, the utility function

Consider the following environment. There is a population J of agents. In each period, each agent is matched with N-1 players to play a symmetric[1] N player game. The random matching process is one such that the probability of playing against any group of

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[1] The theory can be easily generalized to consider non-symmetric games.

N-1 players is the same for all agents. The draws are independent from period to period. Each player has the same action set $A=\{a^1,a^2,\dots,a^M\}$. The game is played for T periods.

The value an agent receives from playing an action now depends on the actions of the group he matches with. Let $v^j_{a_j,a_{-j}}$ be the value agent j receives when he plays a_i and his opponents play $a_{-j}=\{a_1,a_2,\dots,a_{j-1},a_{j+1},\dots,a_n\}$, where $a_1\in A$, $a_2\in A$ ... and so on. Let $p=\{p_{a^1},p_{a^2},\dots,p_{a^M}\}$ be the probabilities that all the actions $\{a^1,a^2,\dots,a^M\}$ are played. So the probability that the agent's opponents playing $a_{-j}=\{a_1,a_2,\dots,a_{j-1},a_{j+1},\dots,a_n\}$ is $p_{a_1}\cdot p_{a_2}\cdot\dots\cdot p_{a_{j-1}}\cdot p_{a_{j+1}}\cdot\dots\cdot p_{a_n}$. Define

$$v^j_a(p)=\sum_{a_{-j}\in A\times A\times\dots\times A} v^j_{a,a_{-j}}(p_{a_1}\cdot p_{a_2}\cdot\dots\cdot p_{a_{j-1}}\cdot p_{a_{j+1}}\cdot\dots\cdot p_{a_n}).$$

$v^j_a(p)$ is called the average value of action a for agent j. (This can be thought of as the average number of coins put in a jar by your opponents.) Since the game is symmetric $v^j_a(p)=v_a(p)$ for all j.

The agents are all assumed to be identical. The framework can be easily extend to heterogeneous agents by adding a type and making the error structure and the utility function depend on the type.

To discuss the subjective probabilities of the average values given the best guesses, the priors of the average values and the process that generates the best guesses need to be defined. Assuming the agents have no information on the average value other than the best guesses, the prior distribution of the average values will be uniform in $\Re^M$. The process that generates the best guesses is called the error structure. Let $\bar{v}(p)=\{v_a(p)\}_{a\in A}$. Define the error structure τ as follows:

$\tau: \Re^M \to P(\Re^M)$.

The guesses defined in the last section are random variables with the property that $\bar{g}^j = \{g_a^j\}_{a \in A}$ has probability measure $\tau(\bar{v}(p))$. The probability measure of the guesses $\bar{g}^j$ depends on the true values $\bar{v}(p)$.

Agent j's belief about his true values given his best guess $\bar{g}^j$ is denoted by $P(\bar{v}|\bar{g}^j)$. The utility function of agent j is defined by $u: \Re \to \Re$. Typically, if agent j receives value x from the games, his utility will be $u(x)$. Although the agents are all identical, they do not behave the same since they will have different draws of $\bar{g}^j$.

Definition 1:

Consider an environment (J, $u(\cdot)$, $\tau(\cdot)$, $P(\cdot|\cdot)$, T), $\bar{q} \in \Delta^M$ is a *discrete* rational novice equilibrium if

Condition (A): $\bar{g}^j$ has probability measure $\tau(\bar{v}(\bar{q}))$ for all j.

Condition (B): $\bar{p}(\bar{g}^j)$ is a selection of the solutions of the following problem for all j :

$$\operatorname*{Max}_{\bar{p}} \int u(T\bar{p} \cdot \bar{v}) dP(\bar{v}|\bar{g}^j) \text{ subject to } \bar{p} \in \Delta_T^M.$$

Condition (C): $\bar{q} = \int_{\Re^M} \bar{p}(\bar{g}) d\tau(\bar{v}(\bar{q}))(\bar{g})$.

Definition 2:

Consider an environment (J, $u(\cdot)$, $\tau(\cdot)$, $P(\cdot|\cdot)$, T), $\bar{q} \in \Delta^M$ is a *continuous* rational novice equilibrium if condition (A), (C) and the following condition hold.

Condition (B'): $\bar{p}(\bar{g}^j)$ is a selection of the solutions of the following problem for all j :

$$\underset{\bar{p}}{\text{Max}} \int u(T\bar{p}\cdot\bar{v})dP(\bar{v}|\bar{g}^j,\omega^j) \text{ subject to } \bar{p}\in\Delta^M.$$

Condition (C) states that in a rational novice equilibrium, $\overline{q}$ is the frequency of the actions played averaged over the random guesses and the distribution of the types. Condition (B) requires that each agent in the population behave according to the rational novice way. Condition (A) requires that the guesses that influence the behavior stated by condition (B) or (B') are distributed as a correct function of the equilibrium average frequencies $\overline{q}$ of the actions. As it has been mentioned earlier, the discrete rational novice equilibrium is a more faithful behavioral model since the agents can only choose their frequencies in multiples of 1/T. However, the continuous rational novice equilibrium, as will be shown later, is a good approximation of the discrete one when T gets large. Furthermore, the continuous rational novice equilibrium is easier to work with in many applications.

Although the frequencies of each agent are not probabilistic, since each agent draws his opponents randomly and the order of play is random for each opponent, the probability that an action is played in a certain period is equal to the average frequency that it is played.

Each agent's strategy $\bar{p}(\bar{g}^j)$ is a random variable that has a probability measure depending on $\overline{q}$. Hence the rational novice equilibrium $\overline{q}$ is the fixed point when averaging out individual strategies as functions of $\overline{q}$.

3.2 Theoretical Properties of the Rational Novice Equilibrium

The first two theorems establish the existence of the rational novice equilibrium.

Theorem 1: If

i) $u(x)$ is concave and continuous in x,

ii) A function $f_\tau : \Re^M \times \Re^M \to \Re$ exists such that $\tau(\bar{v})(B) = \int_B f_\tau(\bar{v}, \omega^j, x)dx$ for all Borel set B of $\Re$. (i.e. The density function of τ, f_τ exists.) $f_\tau(\bar{v}, x)$ is uniformly continuous in $\bar{v}$.

iii) $P(\bar{v}|\bar{g}^j)$ is weakly continuous in $\bar{g}^j$.

then a discrete rational novice equilibrium exists.

Proof:

Let $\varphi : \Re^M \to\to \Delta_T^M$ be a correspondence defined by

$$\varphi(\bar{g}) = \left\{\bar{p} \in \Delta_T^M : \bar{p} \text{ maximzes } \int u(T\bar{p} \cdot \bar{v})dP(\bar{v}|\bar{g})\right\}.$$

Let $\eta : \Delta^M \to\to \Delta^M$ be a correspondence defined by

$$\eta(\bar{q}) = \int_{\Re^M} \varphi(\bar{g})d\tau(\bar{v}(\bar{q}))(\bar{g}).$$

$\int_{\Re^M} \varphi(\bar{g})d\tau(\bar{v}(\bar{q}))(\bar{g})$ is the Gel'fand integral of the correspondence $\varphi(\bar{g})$. We need to show that $\eta(\bar{q})$ has a fixed point.

Define the set $F = \{f :: \Re^M \to \Delta_T^M$ such that f is weakly measurable and $\forall x \in \Re^M, f(x) \in \varphi(x)\}$. By the definition of Gel'fand integral, $\eta(\bar{q})$ can be written as

$$\eta(\bar{q}) = \left\{ \int_{\Re^M} f(\bar{g})d\tau(\bar{v}(\bar{q}))(\bar{g}) : f \in F \right\}.$$

Since u is continuous and P is weakly continuous in $\bar{g}$, $\int u(T\bar{p} \cdot \bar{v})dP(\bar{v}|\bar{g})$ is a continuous function in $\bar{p}$ and $\bar{g}$. Since Δ_T^M is a finite set and maximizing a continuous function over a finite set always yields a solution, $\varphi(\bar{g})$ is always nonempty. This guarantees that $\eta(\bar{q})$ is nonempty.

Consider any $f \in F$. Since $f{::}\,\Re^M \to \Delta_T^M$, f only has finitely many possible values. Let $f_1, f_2, \ldots, f_K$ be the possible values of f. Let $B_k = \{x \in \Re^M | f(x) = f_k\}$. Notice that B_k can be the empty set for some k. $\int_{\Re^M} f(\bar{g}) d\tau(\bar{v}(\bar{q}))(\bar{g})$ can be written as $\sum_{k=1}^{K} f_k \tau(\bar{v}(\bar{q}))(B_k)$. For all B_k, $\tau(\bar{v})(B_k) = \int_{B_k} f_\tau(\bar{v}, x) dx$. For any $\delta > 0$, there exists a ε such that $\int_{\Re} |f_\tau(\bar{v}, x) - f_\tau(\bar{v}', x)| dx < \delta$ if $|\bar{v} - \bar{v}'| < \varepsilon$, since $f_\tau(\bar{v}, \omega^j, x)$ is uniformly continuous in $\bar{v}$. It implies that if $|\bar{v} - \bar{v}'| < \varepsilon$, $\int_{B_k} |f_\tau(\bar{v}, x) - f_\tau(\bar{v}', x)| dx < \delta$ since $B_k \in \Re$. Hence if $|\bar{v} - \bar{v}'| < \varepsilon$, $\left| \int_{B_k} f_\tau(\bar{v}, x) dx - \int_{B_k} f_\tau(\bar{v}', x) dx \right| < \delta$. Therefore, $\tau(\bar{v})(B_k) = \int_{B_k} f_\tau(\bar{v}, x) dx$ is continuous in $\bar{v}$.

By definition (please see (e)), $\bar{v}(\bar{q})$ is linear in $\bar{q}$. Thus $\tau(\bar{v}(\bar{q}))(B_k)$ is continuous in $\bar{q}$ for all k. Therefore, $\int_{\Re^M} f(\bar{g}) d\tau(\bar{v}(\bar{q}))(\bar{g}) = \sum_{k=1}^{K} f_k \tau(\bar{v}(\bar{q}))(B_k)$ is a continuous function in $\bar{q}$ for all $f \in F$. Define a function $h: \Delta^M \to \Delta^M$ such that

$$h(\bar{q}) \equiv \int_{\Re^M} f(\bar{g}) d\tau(\bar{v}(\bar{q}))(\bar{g}).$$

$h(\bar{q})$ is a selection of $\eta(\bar{q})$ by the definition of $\eta(\bar{q})$. That is, $h(\bar{q}) \in \eta(\bar{q})$ for all $\bar{q}$. We have shown that $h(\bar{q})$ is continuous in $\bar{q}$. Since Δ^M is convex and compact and $h(\bar{q})$ is continuous in $\bar{q}$, $h(\bar{q})$ has a fixed point. Hence, $\eta(\bar{q})$ has a fixed point. QED

Theorem 2: If

i) $u(x)$ is concave and continuous in x,

ii) $\tau(\bar{v})$ is weakly continuous in $\bar{v}$,

iii) $P(\bar{v}|\bar{g}^j)$ is weakly continuous in $\bar{g}^j$.

then a continuous rational novice equilibrium exists.

Proof: The following lemma is needed for the proof.

Lemma 1: Let Y be a locally convex Hausdorff space. Let X and Z be topological spaces. Let $\varphi: X \rightarrow\rightarrow Y$ be an upper hemicontinuous correspondence with nonempty $\sigma(Y, Y')$-compact convex values. Let $P: Z \rightarrow P(X)$ which maps z into probability measures of x to be weakly continuous. Let $\eta: Z \rightarrow\rightarrow Y$ be a correspondence defined by :

$$\eta(z) = \int_X \varphi(x) dP(z)(x).$$

Further assume $\langle y, y' \rangle$ is bounded. Then $\eta(z)$ is upper hemicontinuous. Furthermore, $\eta(z)$ has nonempty $\sigma(Y, Y')$-compact convex values.

Proof of lemma 1: Define the support mapping of $\eta(z)$ evaluated at y' to be the real function $z \mapsto h_{\eta(z)}(y')$, where

$$h_{\eta(z)}(y') = \max\{\langle y, y' \rangle : y \in \eta(z)\}.$$

By the Castaing and Valadier theorem (Castaing and Valadier, "Convex Analysis and Measurable Multifunctions," Theorem II-20, p. 51), it is sufficient to show that $\eta(z)$ has nonempty $\sigma(Y, Y')$-compact convex values and $z \mapsto h_{\eta(z)}(y')$ is upper semicontinuous for each $y' \in Y'$.

To show that $\eta(z)$ has nonempty $\sigma(Y, Y')$-compact convex values, Strassen's Theorem is used. Strassen's Theorem (Correspondence Form) states that if (X, Σ, μ) is a probability space and if $\varphi: X \rightarrow\rightarrow Y$ has nonempty, $\sigma(Y, Y')$-compact convex values and

$\int_X \|h_{\varphi(x)}\| d\mu(x) < \infty$, then the Gel'fand Integral of φ, $\int_X \varphi(x) d\mu(x)$ is nonempty, $\sigma(Y, Y')$-compact and convex.

$h_{\varphi(x)}(y') = \max\{\langle y, y'\rangle : y \in \varphi(x)\}$. Since $\langle y, y'\rangle$ is bounded, $h_{\varphi(x)}(y')$ is bounded and so $\|h_{\varphi(x)}\|$ is bounded. Therefore, $\int_X \|h_{\varphi(x)}\| d\mu(x) < \infty$. Strassen's Theorem is applied and for each z, $\eta(z) = \int_X \varphi(x) dP(z)(x)$ is nonempty, $\sigma(Y, Y')$-compact and convex.

The only thing left to show is that $z \mapsto h_{\eta(z)}(y')$ is upper semicontinuous for each $y' \in Y'$. $h_{\eta(z)}(y')$ can be written as $h_{\int_X \varphi(x) dP(z)(x)}(y')$. We know that :

$$h_{\int_X \varphi(x) dP(z)(x)}(y') = \int_X h_{\varphi(x)}(y') dP(z)(x).$$

Because $\varphi(x)$ is upper hemicontinuous, $h_{\varphi(x)}(y')$ is upper semicontinuous. Therefore, $\int_X h_{\varphi(x)}(y') dP(z)(x)$ is also upper semicontinuous. Thus $h_{\eta(z)}(y')$ is also upper semicontinuous. QED.

Once again, let $\varphi: \Re^M \to\to \Delta^M$ be a correspondence defined by

$$\varphi(\bar{g}) = \left\{\bar{p} \in \Delta^M : \bar{p} \text{ maximzes } \int u(T\bar{p} \cdot \bar{v}) dP(\bar{v} | \bar{g}, \omega^j)\right\}.$$

Let $\eta: \Delta^M \to\to \Delta^M$ be a correspondence defined by

$$\eta(\bar{q}) = \int_{\Re^M} \varphi(\bar{g}) d\tau(\bar{v}(\bar{q}))(\bar{g}).$$

To show that a continuous rational novice equilibrium exists, it is sufficient to show that $\eta(\bar{q})$ has a fixed point. Since Δ^M is compact and convex, according to the Kakutani Fixed

Point Theorem, if $\eta(\overline{q})$ is upper hemicontinuous with nonempty convex compact values, then $\eta(\overline{q})$ has a fixed point.

Since u is continuous and P is weakly continuous in $\bar{g}$, $\int u(T\bar{p}\cdot\bar{v},\omega)dP(\bar{v}|\bar{g})$ is a continuous function in $\bar{p}$ and $\bar{g}$. Since Δ^M is a compact and convex subset of $\Re^M$, according to the Maximum Theorem, $\varphi(\bar{g})$ is upper hemicontinuous in $\bar{g}$ with compact values for each $\omega\in\Omega$. To show that $\varphi(\bar{g})$ has convex values, consider $\bar{p}_1\in\varphi(\bar{g})$ and $\bar{p}_2\in\varphi(\bar{g})$. Since u is concave, for all $\lambda\in[0,1]$,

$u(T(\lambda\bar{p}_1+(1-\lambda)\bar{p}_2)\cdot\bar{v})\geq\lambda u(T\bar{p}_1\cdot\bar{v})+(1-\lambda)u(T\bar{p}_2\cdot\bar{v})$. This implies,

$$\int u(T(\lambda\bar{p}_1+(1-\lambda)\bar{p}_2)\cdot\bar{v})dP(\bar{v}|\bar{g})\geq\lambda\int u(T\bar{p}_1\cdot\bar{v})dP(\bar{v}|\bar{g})+(1-\lambda)\int u(T\bar{p}_2\cdot\bar{v})dP(\bar{v}|\bar{g})$$

And since $\bar{p}_1$ and $\bar{p}_2$ maximize $\int u(T\bar{p}\cdot\bar{v})dP(\bar{v}|\bar{g})$,

$$\int u(T(\lambda\bar{p}_1+(1-\lambda)\bar{p}_2)\cdot\bar{v})dP(\bar{v}|\bar{g})\geq\lambda\int u(T\bar{p}\cdot\bar{v})dP(\bar{v}|\bar{g})+(1-\lambda)\int u(T\bar{p}\cdot\bar{v})dP(\bar{v}|\bar{g})$$

for all $\bar{p}\in\Delta^M$. Therefore,

$$\int u(T(\lambda\bar{p}_1+(1-\lambda)\bar{p}_2)\cdot\bar{v})dP(\bar{v}|\bar{g})\geq\int u(T\bar{p}\cdot\bar{v})dP(\bar{v}|\bar{g})\quad\forall\bar{p}\in\Delta^M.$$

Hence $\lambda\bar{p}_1+(1-\lambda)\bar{p}_2$ also maximizes $\int u(T\bar{p}\cdot\bar{v})dP(\bar{v}|\bar{g})$ on Δ^M. So $\lambda\bar{p}_1+(1-\lambda)\bar{p}_2\in\varphi(\bar{g})$ and $\varphi(\bar{g})$ has convex values.

Δ^M is locally convex. $\varphi(\bar{g})$ is upper hemicontinuous with nonempty compact convex values. τ is weakly continuous in $\bar{v}$ and hence $\tau(\bar{v}(\overline{q}))$ is weakly continuous in $\overline{q}$ since $\bar{v}(\overline{q})$ is continuous in $\overline{q}$. $\langle y,y'\rangle$ is bounded since $y\in\Delta^M$ and $y'\in\Delta^M$. (The dual of Δ^M is Δ^M itself.) Applying lemma 1, $\eta(\overline{q})$ is upper hemicontinuous with nonempty compact convex values for each $\omega\in\Omega$. Therefore, $\eta(\overline{q})$ has a fixed point. QED

The next theorem establishes that the continuous rational novice equilibrium is a good approximation of the discrete rational novice equilibrium when T is large.

Theorem 3: If

i) $u(x)$ is concave and continuous in x,

ii) A function $f_\tau : \Re^M \times \Re^M \to \Re$ exists such that $\tau(\bar{v})(B) = \int_B f_\tau(\bar{v}, \omega^j, x)dx$

for all Borel set B of $\Re$. (i.e. The density function of τ, f_τ exists.)

$f_\tau(\bar{v}, x)$ is uniformly continuous in $\bar{v}$.

iii) $P(\bar{v}|\bar{g}^j)$ is weakly continuous in $\bar{g}^j$.

then there exist a continuous rational novice equilibrium $\bar{q}$ which is the limit of a subsequence of discrete rational novice equilibrium $\bar{q}_T$ as T gets large.

Proof: The following lemma is needed for the proof.

Lemma 2: Define a real continuous function $f: K \to \Re$ and a sequence of real continuous functions $f_n: K \to \Re$. Assume for each n, $f_n(x) = 0$ has a solution x_n. If K is compact and $f_n \to f$ uniformly then f(x)=0 has a solution when is a limit of a converging subsequence of $\{x_n\}$.

Proof: Since K is compact, $\{x_n\}$ has a converging subsequence. Without loss of generality, it can be assumed that $\{x_n\}$ converges. Let $x = \lim_{n\to\infty} x_n$. It is sufficient to show that f(x)=0. Assume otherwise.

Assume $|f(x)| > 0$. Thus there exists e>0 such that $|f(x)| > e$. Using the definition of x_n, we have $f_n(x_n) = 0$ for all n. By adding and subtracting $f_n(x)$ and f(x), we have

$$f_n(x_n) - f_n(x) + f_n(x) - f(x) + f(x) = 0 \quad \text{for all n.}$$

Therefore $\quad f_n(x_n) - f_n(x) + f_n(x) - f(x) = f(x)$,

hence $\quad |f(x)| = |f_n(x_n) - f_n(x) + f_n(x) - f(x)|$, and $\quad |f(x)| \le |f_n(x_n) - f_n(x)| + |f_n(x) - f(x)|$ for all n.

Since $f_n(x)$ is continuous in x and $x = \lim_{n\to\infty} x_n$, there exist some N such that if n>N $|f_n(x_n) - f_n(x)| < e/2$. Since $f_n \to f$ uniformly, there exist some M such that if n>M, $|f_n(x) - f(x)| < e/2$. Thus $|f(x)| \le e$ and contradicts $|f(x)| > e$. QED

By theorem 1, a discrete rational novice equilibrium exists. Furthermore, there exists a continuous function in $\overline{q}_T$, $h_T(\overline{q}_T) = \int_{\Re^M} \overline{p}_T(\overline{g}) d\tau(\overline{v}(\overline{q}_T))(\overline{g})$. For all $\overline{g}$, $\overline{p}_T(\overline{g}) \in \varphi(\overline{g}) = \{\overline{p} \in \Delta_T^M : \overline{p} \text{ maximzes } \int u(T\overline{p} \cdot \overline{v}) dP(\overline{v}|\overline{g})\}$.

Similarly, a continuous rational novice equilibrium exists. In addition, there exists a continuous function in $\overline{q}$, $h'(\overline{q}) = \int_{\Re^M} \overline{p}(\overline{g}) d\tau(\overline{v}(\overline{q}))(\overline{g})$. For all $\overline{g}$, $\overline{p}(\overline{g}) \in \varphi'(\overline{g}) = \{\overline{p} \in \Delta^M : \overline{p} \text{ maximzes } \int u(T\overline{p} \cdot \overline{v}) dP(\overline{v}|\overline{g})\}$.

A discrete rational novice equilibrium and a continuous rational novice equilibrium is given by $\overline{q}_T = h_T(\overline{q}_T)$ and $\overline{q} = h'(\overline{q})$ respectively. Let $f_T(x) = x - h_T(x)$ and $f(x) = x - h'(x)$. Since $\int u(T\overline{p} \cdot \overline{v}) dP(\overline{v}|\overline{g})$ is continuous in $\overline{p}$, for all $\varepsilon > 0$, there exists a $\overline{T}$ such that if T>$\overline{T}$, for all $\overline{g}$, $|\overline{p}_T(\overline{g}) - \overline{p}(\overline{g})| < \varepsilon$. Therefore,

$$\int_{\Omega} \int_{\Re^M} |\overline{p}_T(\overline{g}) - \overline{p}(\overline{g})| d\tau(\overline{v}(x))(\overline{g}) \le \int_{\Re^M} \varepsilon d\tau(\overline{v}(x))(\overline{g}).$$

Hence $\int_{\Re^M} |\overline{p}_T(\overline{g}) - \overline{p}(\overline{g})| d\tau(\overline{v}(x))(\overline{g}) < \varepsilon$. So $|h_T(x) - h'(x)| < \varepsilon$. Therefore, $h_T(x) \to h'(x)$ uniformly. Hence $x - h_T(x) \to x - h'(x)$ uniformly. Apply lemma 2, we have the solution of $x - h(x) = 0$ (which is a continuous rational novice equilibrium) is the limit of $x - h_T(x) = 0$ (which is a discrete rational novice equilibrium) as T gets large. QED

The next theorem addresses the issue of the relation between the rational novice model and the Baysian Nash equilibrium model. Consider a symmetric N player game in which agent j is paid $v_{a_j,a_{-j}}$ when j is playing a_j and his opponents play the joint action $a_{-j} = \{a_1, a_2, \ldots, a_{j-1}, a_{j+1}, \ldots, a_n\}$. Each player has M moves. The Baysian Nash equilibrium is the vector of mixed strategies $(\bar{s}_1, \bar{s}_2, \ldots, \bar{s}_N)$ where $\bar{s}_j = (s_j^1, s_j^2, \ldots, s_j^M)$ is a solution to

$$\underset{\bar{s}=(s^1,s^2,\ldots,s^M)}{\text{Max}} \sum_{a} \sum_{a_{-j}} v_{a,a_{-j}} (s^{a_1}, s^{a_2}, \ldots, s^{a_{j-1}}, s^{a_{j+1}}, \ldots, s^{a_n}) s^a \text{ subject to } \bar{s} \in \Delta^M.$$

Since the game is symmetric, $\bar{s}_1 = \bar{s}_2 = \ldots = \bar{s}_N = \bar{s}$.

Theorem 4: If

i) the error structure $\tau(\bar{v})$ assigns probability one to $\bar{v}$ and zero to all other possible $\bar{g} \in \Re^M$, and

ii) $P(\bar{v}|\bar{g}^j)$ assigns probability one to $\bar{g}^j$ (i.e., the agents believe their guesses are correct),

then any continuous rational novice equilibrium frequency $\bar{q}$ is equal to a mixed strategy $\bar{s}$ of the Nash equilibrium of the single period game. Notice that the rational novice equilibrium $\bar{q}$ may not be unique.

Proof: Consider a rational novice equilibrium that satisfies conditions (A), (B) and (C). Since $\tau(\bar{v})$ assigns probability one to $\bar{v}$, condition (A) implies $\bar{g}^j = \bar{v}(\bar{q})$ for all j. Since $P(\bar{v}|\bar{g}^j)$ assigns probability one to $\bar{g}^j = \bar{v}(\bar{q})$, condition (B) becomes for all j, $\bar{p}(\bar{v}(\bar{q}))$ is the solution of

$$\underset{\bar{p}}{\text{Max}}\ u(T\bar{p} \cdot \bar{v}(\bar{q})) \text{ subject to } \bar{p} \in \Delta^M.$$

Since u is increasing in the first argument, this is equivalent to

$$\underset{\bar{p}}{\text{Max}}\ T\bar{p}\cdot\bar{v}(\bar{q}) \text{ subject to } \bar{p}\in\Delta^{M}.$$

which is equivalent to

$$\underset{\bar{p}}{\text{Max}}\ \sum_{a} p_a \sum_{a_{-j}\in A\times A\times\ldots\times A} v_{a,a_{-j}}(\bar{q}_{a_1}\cdot\bar{q}_{a_2}\cdot\ldots\cdot\bar{q}_{a_{j-1}}\cdot\bar{q}_{a_{j+1}}\cdot\ldots\cdot\bar{q}_{a_n})$$

So condition (C) becomes

$$\bar{q}\in\bar{p}(\bar{q})$$

which is equivalent to $\bar{q}$ is the solution of

$$\underset{\bar{p}}{\text{Max}}\ \sum_{a} p_a \sum_{a_{-j}\in A\times A\times\ldots\times A} v_{a,a_{-j}}(\bar{q}_{a_1}\cdot\bar{q}_{a_2}\cdot\ldots\cdot\bar{q}_{a_{j-1}}\cdot\bar{q}_{a_{j+1}}\cdot\ldots\cdot\bar{q}_{a_n})$$

which is the definition of a Nash behavior. QED

Notice that even though the continuous rational novice equilibrium $\bar{q}$ is equal to the Baysian Nash equilibrium in the above case, it is not necessary that each agent is playing a Baysian Nash equilibrium. For example, consider a 2x2 game in which an agent chooses strategy A or B. Suppose that the Nash equilibrium is the mixed strategy playing A with probability 1/2 and B with probability 1/2. In a rational novice equilibrium of the same game, only the population average frequencies of A and B are 1/2. The agents are not required to play 1/2 A and 1/2 B. For example, a rational novice equilibrium may be one in which 1/2 of the agents play A all the time and 1/2 of the agents play B all the time.

The previous example also illustrates the fundamental difference between the rational novice equilibrium concept and the Nash equilibrium concept. The structure of errors and beliefs in the rational novice model is identical to a signaling game in which each player receives a signal about their payoffs. Hence, each agent is faced with a maximization problem as if he is a Nash player in the appropriate signaling game.

However, in a Nash equilibrium, each player is reacting to his opponents while in a rational novice equilibrium, each player is reacting to a population of opponents.

All of the above theorems assume the population is homogeneous. It is the author's conjecture that the same theorems hold if the population is heterogeneous.

Chapter 4: The Voluntary Contribution Game

4.1 Overview

In this chapter, the rational novice model is applied to a Voluntary Contribution game. There are two reasons why the voluntary contribution game is chosen. The first one is that experimental data available for analysis suggests that people are deviating from Nash dominant strategies. The rational novice model offers a potential explanation. The second reason is that in the particular voluntary contribution game that is studied in this paper, the rational novice behavioral model can be tested independently from the rational novice equilibrium concept. Recall that there are two parts to the rational novice model: the individual behavioral model and the rational novice equilibrium concept. In this particular environment, any valid rational novice individual behavioral profile is a valid rational novice equilibrium. Therefore, we can directly test the behavioral model without putting in play the aggregate principles of the rational novice equilibrium. In a later chapter, the full strength of the rational novice model will be tested.

With the permission of Jeffrey Prisbrey and Thomas Palfrey[1], the rational novice model is tested using the data of their voluntary contribution game experiments. The data analysis consists of fitting the rational novice model to the data and making comparisons between the rational novice model and statistical models employed in the Prisbrey and Palfrey study.

The following section will be a description of the voluntary contribution game. The rational novice model will be applied and a solution representing the behavioral profile will be solved.

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[1]The author would like to thank Jeffrey Prisbrey and Thomas Palfrey here for their permission to use their data of voluntary contribution game experiments.

4.2 The Behavior Model of the Voluntary Contribution Game

Consider a series of T voluntary contribution games. In each game, the population is randomly divided into groups of n. In the beginning of each game, each participant receives a token that is worth r dollars.[2] For each agent, there are two possible actions in the game. The agent can choose to contribute or not to contribute his token. If he contributes his token, everyone in his group receives s dollars. In traditional decision theory, it is a dominant strategy for each participant not to contribute if $r > s$ and contribute if $r \leq s$. Each game can be represented by the following payoff table:

	if k other participants contribute
not contribute	$ks + r$
contribute	$(k+1)s$

Another variation of the game is that each agent receives N tokens, given N>1. The agent is allowed to contribute any number of tokens up to N. This scenario is essential the same as if he plays the single token game N times under the treatment of the rational novice model. A more detailed discussion is provided in the next section.

In several experimental studies[3], subjects have been observed to contribute even when their dominant strategy is not to. They also do not contribute when it is a dominant strategy to contribute. In Palfrey and Prisbrey 1992, a series of voluntary contribution

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[2]In some experimental subjects receive N tokens and each can contribute none, some or all. We analyze that situation below.

[3] For example, see Palfrey and Prisbrey 1992, Isaac, Walker and Thomas 1984, and Saijo and Yamaguchi 1992.

experiments are reported. A substantial amount of contribution was observed in their experiments. Only 45 percent of their observations are consistent with subjects playing Nash strategies. But most of the players played with only a little anomalous variation in their choices. Furthermore, assuming each subject was employing the same cut point strategy, the cut point that minimizes the classification errors is very close to the one predicted by a Nash equilibrium. Palfrey and Prisbrey concluded the results track quite closely the predictions of non-cooperative theories. We want to see whether the rational novice model provides a better explanation for the anomalous variations.

Each participant is assumed to have a discount factor of 1. That is, the situation is identical to the one in which each participant is playing T voluntary contribution games simultaneously instead of in sequence. This assumption is reasonable when the experiments are carried out in a relatively short time.

Each participant is going to choose a frequency p of playing the option of keeping the token. If the game is going to be played T times, then each participant is going to keep the token pT times and contribute (1-p)T times. p can be different for each participant. Each participant is assumed to have a single period utility function equal to the monetary payoff. Let n denote the action of keeping the token. Let c denote the action of contributing. The respective single period values of the actions are given by:

$$v(n) = ks+r, \text{ and}$$
$$v(c) = (k+1)s,$$

where k is the expected number of contributors. The rational novice model assumes the agents to make errors when they evaluate their options. The internal error structure is assumed to be the following. Each agent makes guesses about the value of their two strategies: keeping the token (denoted by n) and contributing (denoted by c). The best guesses can be written as

$$g(n) = ks + r + \varepsilon,$$

$$g(c) = (k+1)s + \delta,$$

where ε and δ are random variables that are independently and identically distributed. $E(\varepsilon)$ and $E(\delta)$ are assumed to be zero. Let $E(\varepsilon^2) = \sigma_\varepsilon^2$ and $E(\delta^2) = \sigma_\delta^2$. For reasons that will be apparent later, $\varepsilon - \delta$ is assumed to be normally distributed. $E(\varepsilon - \delta)=E(\varepsilon)-E(\delta)=0$ and $E(\varepsilon - \delta)^2=E(\varepsilon^2)+E(\delta^2)-2E(\varepsilon\delta)=\sigma_\varepsilon^2 + \sigma_\delta^2$. $E(\varepsilon\delta)=0$ since ε and δ are independent.

Given their guesses, the agents have beliefs on the true value of their options. The value conditioned on their guesses are given by:

$$v(n|g(n)) = g(n) - \varepsilon, \text{ and}$$

$$v(c|g(c)) = g(c) - \delta.$$

The above belief structure assumes that every agent's belief is accurate. The value of playing frequency p in a single game is given by

$$v(p)=p(g(n)-\varepsilon)+(1-p)(g(c)-\delta).$$

The value of playing T games is $V(p)=Tv(p)$. Each participant solves the following maximization problem:

$$\max_p \ u(V(p)) \text{ subject to } 0 \le p \le 1 .$$

u() is assumed to be $EV(p)-\alpha\left[EV(p)^2-(EV(p))^2\right]$. That is, a linear combination of the expected value and variance of $V(p)$. The reason to make this choice is that the resulting individual behavior is relatively easy to solve and similar across different games. If a more complex utility function, such as the log or the constant relative risk averse utility function, is used, then computation of behavior becomes intractable.

Participants are assumed to be either risk neutral or risk averse. Therefore, they either dislike or do not care about variance. This assumption can be represented by setting $\alpha \geq 0$. The expectation and the variance of the value of playing T games is

$$EV(p)=T\{pg(n)+(1-p)g(c)\}, \text{ and}$$
$$EV(p)^2-(EV(p))^2=T^2\left\{\sigma_\varepsilon^2 p^2+\sigma_\delta^2(1-p)^2\right\}.$$

The participant's problem can be written as:

$$\underset{p}{\operatorname{Max}}\ T\{pg(n)+(1-p)g(c)\}\text{-}\alpha T^2\left\{\sigma_\varepsilon^2 p^2+\sigma_\delta^2(1-p)^2\right\} \text{ subject to } 0\leq p\leq 1 .$$

A variation of the voluntary contribution game allows the agents to receive and contribute more than one token. Let the number of tokens received in each game be N. In the situation in which agents receive a single token per period for T periods, the rational novice model assumes that the agents are making their T decisions simultaneously. It is equivalent to the situation where an agent has to decide how many tokens out of N to contribute. Let z be the number of tokens withheld by an agent. The expected value is

$$EV(p) = zg(n)+(N-z)g(c),$$
$$= N\left\{\left(\tfrac{z}{N}\right)g(n)+\left(1-\tfrac{z}{N}\right)g(c)\right\}.$$

This situation is equivalent to a single token voluntary contribution game in which T=N and p=z/N. We are not going to distinguish the difference between the single token case and the multiple token case in our following discussion.

Given the maximization problem of our agents, the first-order conditions are:

$$\text{if } T(g(n)-g(c))-2\alpha T^2\{p(\sigma_\varepsilon^2+\sigma_\delta^2)-\sigma_\delta^2\}\leq 0 \text{ then p=0}$$
$$\text{or if } T(g(n)-g(c))-2\alpha T^2\{p(\sigma_\varepsilon^2+\sigma_\delta^2)-\sigma_\delta^2\}\geq 1 \text{ then p=1}$$
$$\text{otherwise } T(g(n)-g(c))-2\alpha T^2\{p(\sigma_\varepsilon^2+\sigma_\delta^2)-\sigma_\delta^2\}=0.$$

The solution for p is given by:

$$(*)\qquad p=\begin{cases} 0 \\ \dfrac{2\alpha T\sigma_\delta^2+g(n)-g(c)}{2\alpha T\sigma_\delta^2+2\alpha T\sigma_\varepsilon^2} \\ 1 \end{cases} \text{if } \begin{cases} \dfrac{2\alpha T\sigma_\delta^2+g(n)-g(c)}{2\alpha T\sigma_\delta^2+2\alpha T\sigma_\varepsilon^2}\leq 0 \\ 0<\dfrac{2\alpha T\sigma_\delta^2+g(n)-g(c)}{2\alpha T\sigma_\delta^2+2\alpha T\sigma_\varepsilon^2}<1. \\ 1\leq\dfrac{2\alpha T\sigma_\delta^2+g(n)-g(c)}{2\alpha T\sigma_\delta^2+2\alpha T\sigma_\varepsilon^2} \end{cases}$$

The second-order condition for the non-corner solution is $-2\alpha T^2\{\sigma_\varepsilon^2+\sigma_\delta^2\}\leq 0$ which is satisfied automatically given our assumption that $\alpha\geq 0$.

Notice that the solution now allows non-Nash behavior. Furthermore, the comparative statics of the solution are consistent with intuition. First, p increases as g(n)-g(c) increases. In other words, when participants guess that the tokens have higher values than the public good, they are more likely to increase their frequency of keeping the token. Furthermore, p converges to $\frac{\sigma_\delta^2}{\sigma_\delta^2+\sigma_\varepsilon^2}$ as α or σ_δ^2 approaches infinity. p=$\frac{\sigma_\delta^2}{\sigma_\delta^2+\sigma_\varepsilon^2}$ is the frequency when the risk (variance of V(p)) is at a minimum. It is consistent with our intuition that the participants will play more to minimize the risk (i.e., closer to $\frac{\sigma_\delta^2}{\sigma_\delta^2+\sigma_\varepsilon^2}$) when they are making bigger errors or they care about their errors more.

4.3 Rational Novice Equilibrium of the Voluntary Contribution Game

Each agent can be completely characterized by $\alpha, \sigma_\varepsilon^2$ and σ_δ^2. Hence, a type of an agent is a triplet $\omega=(\alpha,\sigma_\varepsilon^2,\sigma_\delta^2)$. The following assumptions are made about the distribution of types :

- α is identical for every agent.
- σ_ε^2 and σ_δ^2 are independently and identically distributed with log normal distributions.

The rational novice equilibrium frequency $\overline{q}$(r-s) can be calculated by directly applying the definition of a rational novice equilibrium. Notice that the rational novice equilibrium is dependent on the environment r-s. The ability to track behavior over different environments enables us to estimate the model. According to condition (C), $\overline{q}$(r-s) can be calculated by integrating the rational novice behavior given by (*) over the guesses and the distribution of types.

Notice that the rational novice behavior in the voluntary contribution game does not depend on $\overline{q}$. Hence the conditions of the rational novice equilibrium are met as long as the agents are diversifying to hedge against their guesses. The requirement of adjusting to the average strategy of the population is met trivially. This special feature allows us to test the rational novice behavior model independent of the rational novice equilibrium concept.

$\overline{q}(r\text{-}s)$ is given by $\underset{\alpha,\sigma_\delta^2,\sigma_\varepsilon^2}{E}(\underset{\varepsilon-\delta}{E}(p|\alpha,\sigma_\delta^2,\sigma_\varepsilon^2))$. The first expectation of $\underset{\alpha,\sigma_\delta^2,\sigma_\varepsilon^2}{E}(\underset{\varepsilon-\delta}{E}(p|\alpha,\sigma_\delta^2,\sigma_\varepsilon^2))$ is taken over the guess $g(n)-g(c)$. Recall $g(n)-g(c)=r-s+\varepsilon-\delta$. Recall that $\varepsilon-\delta$ is assumed to be independently and normally distributed. The mean of $\varepsilon-\delta$ is zero and the variance of $\varepsilon-\delta$ is $\sigma_\varepsilon^2+\sigma_\delta^2$. $\underset{\varepsilon-\delta}{E}(p|\alpha,\sigma_\delta^2,\sigma_\varepsilon^2)$ can be calculated as follows. To simplify the notation, let $x=2\alpha T\sigma_\varepsilon^2$, $y=2\alpha T\sigma_\delta^2$, $\xi=\varepsilon-\delta$, $\sigma^2=\sigma_\varepsilon^2+\sigma_\delta^2$ and a=r-s.

$$\underset{\varepsilon-\delta}{E}(p|\alpha,\sigma_\delta^2,\sigma_\varepsilon^2)=\int_{-x-a}^{y-a}\frac{x+a+\xi}{x+y}dP(\xi|\sigma_\varepsilon^2,\sigma_\varepsilon^2)+P(y<a+\xi).$$

We have

$$\underset{\varepsilon-\delta}{E}(p|\alpha,\sigma_\delta^2,\sigma_\varepsilon^2)=\int_{-x-a}^{y-a}\frac{x+a+\xi}{x+y}d\Phi\left(\frac{\xi}{\sigma}\right)+1-\Phi\left(\frac{y-a}{\sigma}\right),$$

$$=\frac{x+a}{x+y}\left\{\Phi\left(\frac{y-a}{\sigma}\right)-\Phi\left(\frac{-x-a}{\sigma}\right)\right\}+\frac{\sigma}{\sqrt{2\pi}(x+y)}\left\{e^{-\frac{(x+a)^2}{2\sigma^2}}-e^{-\frac{(y-a)^2}{2\sigma^2}}\right\}+1-\Phi\left(\frac{y-a}{\sigma}\right).$$

Since a constant multiplied by a random variable distributed log normally is also distributed log normally (with a different mean), x and y are distributed log normally and independently with the same mean and variance. Let $\tilde{u}$ be the mean of log(x) and log(y) and $\tilde{\sigma}^2$ be the variance of log(x) and log(y). The rational novice equilibrium $\overline{q}$ can be written as :

$$\overline{q}(a)=\int_0^\infty\int_0^\infty\left(\frac{x+a}{x+y}\right)\left\{\Phi\left(\frac{y-a}{\sigma}\right)-\Phi\left(\frac{-x-a}{\sigma}\right)\right\}+\frac{\sigma}{\sqrt{2\pi}(x+y)}\left\{e^{-\frac{(x+a)^2}{2\sigma^2}}-e^{-\frac{(y-a)^2}{2\sigma^2}}\right\}+1-\Phi\left(\frac{y-a}{\sigma}\right)d\Phi\left(\frac{\log(x)-\tilde{u}}{\tilde{\sigma}}\right)d\Phi\left(\frac{\log(y)-\tilde{u}}{\tilde{\sigma}}\right).$$

Notice that since the solution to individual behavior (equation (*)) is a non decreasing function of r-s, so $\underset{\varepsilon-\delta}{E}(p|\alpha,\sigma_\delta^2,\sigma_\varepsilon^2)$ is also a non decreasing function of r-s. Hence, $\overline{q}$ is also a non decreasing function of r-s regardless of the distributions of α, σ_δ^2 and σ_ε^2.

4.4 Data Analysis : Techniques and A Description of the Data

The availability of the Palfrey and Prisbrey data enables me to study the application of the rational novice model to the voluntary contribution game empirically. The model was estimated using maximum likelihood techniques. The pseudo R^2 is used as a benchmark of how well the model fits the data. The rational novice model was tested against the Nash equilibrium model and compared to a Probit model that is analyzed in the Palfrey and Prisbrey paper.

The subjects were randomly matched into groups for the experiments. Each subject received 9 tokens per game[4]. In each game, each subject could contribute up to 9 tokens. Each token is worth r dollars to the owner and each token contributed is worth s dollars to all in the group. This game has the same rational novice structure as one in which each subject receives one token and plays the game for 9 times as explained in the last section. The experimental parameters r and s varied from period to period.

The rational novice model is estimated as follows. For each subject and each set of parameters (r,s), the frequency of not contributing is calculated. A typical data point looks like (p,r,s) where p is the frequency of a subject withholding the tokens. A data point is generated from each subject in each period. Recall, the rational novice frequency is given by:

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[4]Palfrey and Jeffrey have run a number of experiments. The subjects received different number of tokens in different experimental sessions. In this study, only the experimental sessions in which each subject received 9 tokens are analyzed.

$$(*) \qquad p = \begin{cases} 0 \\ \dfrac{2\alpha T\sigma_\delta^2 + g(n) - g(c)}{2\alpha T\sigma_\delta^2 + 2\alpha T\sigma_\varepsilon^2} \\ 1 \end{cases} \text{ if } \begin{cases} \dfrac{2\alpha T\sigma_\delta^2 + g(n) - g(c)}{2\alpha T\sigma_\delta^2 + 2\alpha T\sigma_\varepsilon^2} \le 0 \\ 0 < \dfrac{2\alpha T\sigma_\delta^2 + g(n) - g(c)}{2\alpha T\sigma_\delta^2 + 2\alpha T\sigma_\varepsilon^2} < 1. \\ 1 \le \dfrac{2\alpha T\sigma_\delta^2 + g(n) - g(c)}{2\alpha T\sigma_\delta^2 + 2\alpha T\sigma_\varepsilon^2} \end{cases}$$

Since the distributions of g(n)-g(c), σ_ε^2 and σ_δ^2 are known and α is assumed to be a constant, the log likelihood function of p can be calculated as follows. Letting $x = 2\alpha T\sigma_\varepsilon^2$, $y = 2\alpha T\sigma_\delta^2$, $\xi = \varepsilon - \delta$, $\sigma^2 = \sigma_\varepsilon^2 + \sigma_\delta^2$, and $z = 2\alpha T$. Notice that σ^2 can be expressed as $(x+y)/z$. (*) becomes :

$$(*) \qquad p = \begin{cases} 0 \\ \dfrac{y + a + \xi}{y + x} \\ 1 \end{cases} \text{ if } \begin{cases} \dfrac{y + a + \xi}{y + x} \le 0 \\ 0 < \dfrac{y + a + \xi}{y + x} < 1. \\ 1 \le \dfrac{y + a + \xi}{y + x} \end{cases}$$

x and y are distributed with the same log normally density. Let the mean and variance of log(x) and log(y) be $\overline{u}$ and $\overline{\sigma}^2$ respectively. The probability of observing p=0 is $\text{prob}(\xi \le -y - a) = \int_0^\infty \int_0^\infty \Phi\left(\frac{-y-a}{\sigma}\right) d\Phi\left(\frac{\log(x) - \bar{u}}{\bar{\sigma}}\right) d\Phi\left(\frac{\log(y) - \bar{u}}{\bar{\sigma}}\right)$. The probability of observing p=1 is $\text{prob}(\xi \le x - a) = \int_0^\infty \int_0^\infty \Phi\left(\frac{x-a}{\sigma}\right) d\Phi\left(\frac{\log(x) - \bar{u}}{\bar{\sigma}}\right) d\Phi\left(\frac{\log(y) - \bar{u}}{\bar{\sigma}}\right)$. The density of p for 0<p<1 is $\int_0^\infty \int_0^\infty \frac{1}{\sqrt{2\pi}} e^{-\left(\frac{y(p-1) + xp - a}{\sigma}\right)^2} d\Phi\left(\frac{\log(x) - \bar{u}}{\bar{\sigma}}\right) d\Phi\left(\frac{\log(y) - \bar{u}}{\bar{\sigma}}\right)$.

Each data point represents the decision a subject made in one period. In each period, each subject received 9 tokens and he could choose to contribute any number of them. Let p_i be the fraction of tokens a subject keeps for himself. Let r_i and s_i be the relevant private good value and public good value respectively. Since the number of tokens received is 9, p_i is only feasible when it is in increments of 1/9. However, the continuous model is used as an approximation when the likelihood function is calculated.

The log likelihood function L_i of the data point $(p_i, a_i = r_i - s_i)$ with parameter set $(\overline{u}, \overline{\sigma}^2, z)$ can be written as:

$$L_i = \begin{cases} \int_0^\infty \int_0^\infty \Phi\left(\frac{-y-a_i}{\sigma}\right) d\Phi\left(\frac{\log(x)-\bar{u}}{\bar{\sigma}}\right) d\Phi\left(\frac{\log(y)-\bar{u}}{\bar{\sigma}}\right) & p_i = 0 \\ \int_0^\infty \int_0^\infty \frac{1}{\sqrt{2\pi}} e^{-\left(\frac{y(p-1)+xp-a_i}{\sigma}\right)^2} d\Phi\left(\frac{\log(x)-\bar{u}}{\bar{\sigma}}\right) d\Phi\left(\frac{\log(y)-\bar{u}}{\bar{\sigma}}\right) & \text{if} \quad 0 < p_i < 1. \\ \int_0^\infty \int_0^\infty \Phi\left(\frac{x-a_i}{\sigma}\right) d\Phi\left(\frac{\log(x)-\bar{u}}{\bar{\sigma}}\right) d\Phi\left(\frac{\log(y)-\bar{u}}{\bar{\sigma}}\right) & p_i = 1 \end{cases}$$

The total log likelihood of a data set is $L(\overline{u}, \overline{\sigma}^2, z) = \sum_i L_i$. The estimates and maximum log likelihoods are listed in Table 1.

The maximum likelihood technique requires the data to be distributed independently. The rational novice model cannot guarantee independence when a subject has to face the same private good value and public good value pair more than once. Therefore, if the same subject received the same private good value and public good value in two different periods, the data from both periods would be discarded.

With the estimated $(\overline{u}, \overline{\sigma}^2, z)$, the rational novice equilibrium frequency $\overline{q}(a)$ can be calculated. Although $\overline{q}(a)$ itself has no statistical significance, it is a useful aid to give a sense of how accurate the rational novice model is on average. To give a sense of how close $\overline{q}(a)$ and the actually average frequency is, the pseudo R^2 is calculated. The pseudo R^2 is defined as pseudo $R^2 = 1 - \frac{ssr}{\overline{ssr}}$,

where

$$ssr \equiv \sum_i \left(p_i - \overline{q}((r-s)_i)\right)^2, \text{ and}$$

$$\overline{ssr} \equiv \sum_i \left(p_i - \tfrac{1}{K}\sum_k p_k\right)^2.$$

If the model explains the data set perfectly, the sum of residuals ssr will be zero and the pseudo R^2 will be one. The closer to one the pseudo R^2 is, the better explanatory power the model has. The pseudo R^2 is listed in Table 3. To convey graphically how well the model is fitting, the average frequency of the subjects playing not contributing is calculated for each a=r-s. This is compared to the estimated average frequency $\overline{q}(a)$ in Figure 1.

The Nash equilibrium is a special case of the rational novice equilibrium. The Nash equilibrium corresponds to the special case when $(\overline{u} = -\infty, \overline{\sigma}^2 = 0)$. (All the agents are making no mistakes when $\sigma_\delta^2 = \sigma_\varepsilon^2 = 0$.) The likelihood for $(\overline{u} = -\infty, \overline{\sigma}^2 = 0)$ is zero if the data deviates from the Nash equilibrium theory in any amount. Instead of testing $(\overline{u} = -\infty, \overline{\sigma}^2 = 0)$ which will result in automatic rejection of the Nash theory, the hypothesis $(\overline{u} = -c, \overline{\sigma}^2 = 0)$ is tested. Some latitude is given to the Nash equilibrium. I call the hypothesis $(\overline{u} = -c, \overline{\sigma}^2 = 0)$ the relaxed Nash model. c is chosen such that -c is a lot smaller than the maximum likelihood estimate of $\overline{u}$. c=25 is used. For larger values of c, the likelihood of the relaxed Nash model becomes very small. The hypothesis

$(\bar{u}=-c, \bar{\sigma}^2=0)$ is tested using the likelihood ratio test. The appropriate restrictions and statistics are listed in Table 2.

In the Palfrey and Prisbrey paper, a Probit analysis was performed to estimate the agents' responses. In this paper, a similar Probit analysis is carried out and compared to the rational novice model. Only the simplest Probit model in the Palfrey and Prisbrey paper is analyzed in this paper. The dependent variable is the investment decision. The independent variables are the constant and the marginal rate of substitution between the private and the public good. The Palfrey and Prisbrey study uses the marginal rate of substitution between the public good and the private good, the inverse of the variable used here. The estimates and the maximum log likelihood are listed in Table 1. Two methods are used to compare the rational novice model to the Probit model. The pseudo $R^2$of the Probit model, which is used as a benchmark for the explanatory power of the model, is calculated and is listed in Table 3. The pseudo R^2 is an adequate benchmark for the performance of a model but it does not enable an investigator to reject one model statistically in favor of another.

It would be useful if one model were nested in the other but unfortunately that is not true here. Recall that r=the private good value and s=the public good value. The probit model can be written in the following way. A subject does not contribute if $a+b\left(\frac{s}{r}\right)+\varepsilon>0$ where a,b are parameters of the model and ε a normally distributed random variable with mean zero. The condition can be rewritten as $ar+bs+r\varepsilon>0$. The rational novice model is given by (*). If $\alpha>0$ then these are clearly non nested models. In the special case that $\alpha=0$, a subject does not contribute if $r-s+\xi>0$. This is only equivalent to the probit model if a=-b and the distribution of ξ depends on s. So although a special case of the rational novice model is equivalent to a special case of the probit model, the two models are non nested. The above arguments apply to the case in which each subject is given one token. Since both the rational novice model and the Probit model treat all the decisions on the 9 tokens to be simultaneous, the above arguments should extend to the multiple token case. In our analysis, each subject receives 9 tokens in each period. Therefore, for each data point, the likelihood function of the Probit model can be written as:

$$L_i = \binom{9}{z_i} \Pr ob(a + b\left(\frac{s_i}{r_i}\right) + \varepsilon > 0)^{z_i} \Pr ob(a + b\left(\frac{s_i}{r_i}\right) + \varepsilon \leq 0)^{9-z_i},$$

where z_i is the number of tokens the subject does not contribute. This likelihood function is not nested into the one of the rational novice model stated earlier.

4.5 Data Analysis : Results

The rational novice model is estimated using the Palfrey and Prisbrey data sets[5]. The techniques of the estimation and the log likelihood function are discussed in the last section. The estimates and log likelihood statistics are listed in Table 1. Figure 1 and 2 display graphical representations of the estimation of the rational novice model with the whole data set and with the first five periods discarded. In the figures, the rational novice equilibrium frequencies are calculated using the maximum likelihood estimates and are compared to the data. The estimates $(\overline{u}, \overline{\sigma}^2, z)$ represent the distribution of the errors σ_δ^2 and σ_ε^2 in the population. (Recall σ_δ^2 and σ_ε^2 have the same distribution.) The distribution of σ_δ and σ_ε is plotted in figure 3 for the whole data set and in figure 4 when the first 5 periods are discarded. The errors σ_δ and σ_ε are plotted in the unit of the U.S. dollar. σ_δ or σ_ε is smaller than 0.61 dollars 80 percent of the time and less than 1.26 dollars 90 percent of the time when the whole data set is used for estimation. If the first 5 periods are

[5]I thank Thomas R.Palfrey and Jeffrey E. Prisbrey for letting me use their data.

discarded, then σ_δ or σ_ε is smaller than 2.3 dollars 80 percent of the time and smaller than 5 dollars 90 percent of the time. As it can be seen, with typical public good values from 3 to 15 dollars and private good values ranging from 1 to 20 dollars, most of the time the subjects were making small errors (in relation to their public or private good values).

The following are the major results. The first result deals with the statistical accuracy of the rational novice model. There is no absolute measure of how accurate a model is. However, the pseudo R^2 defined in the previous section is a benchmark of explanatory power that a model has. The interpretation of the actual numbers is left to the reader.

Result 1 : The rational novice model is a fairly accurate statistical model.

Support : The pseudo R^2 of the rational novice model is 0.67 when all the data is used and 0.75 when the first 5 periods are discarded from all the experiments.

It is also interesting to see that the explanatory power of the rational novice model increases when the data from the first 5 periods of the experiments are discarded. It looks as if the subjects were "learning" to play the rational novice equilibrium.

Since the Nash model is a special case of the rational novice model, the rational novice model will always have better statistical accuracy. The important question is whether the rational novice model is a significant improvement over the Nash model. The answer to this question according to result 2 is yes. The Nash model can be rejected in favor of the rational novice model that were estimated.

Result 2 : The relaxed Nash model can be rejected as a special case of the rational novice model.

Support : Using the likelihood ratio test, the relaxed Nash models (c=25) can be rejected at 5 percent significance. The test statistics are listed in Table 2.

Rejecting the Nash model should come as no surprise. It is more interesting to see if the rational novice model can do better than the Probit model. Although we did not conduct any statistical tests, it is our conjecture that the rational novice model performs better than the Probit model.

Conjecture : The rational novice model is better than the Probit model.

Support : The pseudo R^2s of the rational novice model are higher both when all the data are used and when the first 5 periods are discarded.

4.6 Comments and Discussions

This research introduces a new approach to game theory modeling. This new approach is aimed at explaining deviations from traditional game theory predictions. Some theorists attribute deviations from game theory to errors people make. Both the Harsanyi model and the McKelvey and Palfrey quantal response model assume subjects observe erroneous payoffs. The rational novice model takes one step further by adding a principle of optimal response to the risks associating with erroneous payoffs. The rational novice model is able to explain human errors as a function of the economic environment. A combination of bounded rationality and risk diversification is utilized to develop the rational novice equilibrium model. The general framework of this new model is discussed and the equilibrium for the voluntary contribution game is solved in this framework.

The voluntary contribution game represents a paradox in game theory modeling. On one hand, in traditional game theory, people are predicted not to contribute when their marginal rate of substitution between the private and the public good is greater than 1. On the other hand, experimental investigations have shown that people indeed do contribute when the traditional game theory predicts otherwise. In a series of experiments reported in Palfrey and Prisbrey 1992, substantial contributions are observed to be at odds with the traditional game theory. Palfrey and Prisbrey's analysis show that the observations are mostly consistent with non-cooperative theories but there are still deviations that cannot be explained. Some subjects even deviated from the Nash strategy when their marginal rate of substitution between the private and the public good was less than one. Palfrey and Prisbrey employ a Probit analysis which explains the data fairly well.

The rational novice model is proposed as an alternative model to explain non-Nash behavior in voluntary contribution games. Instead of assuming each agent has an additional incentive to deviate from the Nash strategy, deviations come naturally when the agents diversify against the risk of miscalculating the values of their options.

Result 1 suggests that the rational novice model is a fairly accurate model in explaining the data. The estimates show that assuming that agents were making small errors is enough to explain the variation in the data. Result 2 indicates that the rational novice model is not only a more accurate model than the Nash model, the improvement over the Nash model is significant according to the likelihood ratio tests. This should not come as a surprise since we know that Nash models fail to explain why people deviate from dominant strategies while the rational novice model offers an explanation. It is also my conjecture that the rational novice model performs better than the Palfrey and Prisbrey Probit model.

In summary, the rational novice model is a reasonably accurate model of agents' behavior in voluntary contribution games. Additional data analysis is underway to provide a statistical test to distinguish between the Probit model and the rational novice model. The power of this new approach is that the general framework of the rational novice behavioral model is applicable to any finite game.

Chapter 5: First Price Sealed Bid Private Value Auctions

5.1 Overview

The first price sealed bid auction has been studied. In "Theory and Individual Behavior of First-Price Auctions," Cox, Smith and Walker 1988 (CSW) concluded that game theory could adequately explain auctions generated under laboratory conditions. Kagel, Harstad and Levin (KHL) in a 1987 study concluded that one Nash model, namely the Risk Averse Symmetric Nash Equilibrium (RASNE), was the best model amongst the ones they investigated although when applied to individual bids, sophisticated discounting models could not be statistically ruled out. Most of the previous studies were under a linear environment. Chen and Plott conducted a series of experiments under nonlinear conditions and found that the Constant Relative Risk Averse Model (CRRAM), another Nash model, offers a good explanation of individual behavior. However, it failed to outperform a sophisticated piece wise linear rule-of-thumb.

In the Chen and Plott study, the solution of the CRRAM is defined by a differential equation that does not have a closed form solution. The authors had to employ complicated computer techniques to calculate a numerical solution. It is unlikely that all the subjects were able to arrive at the same solution simultaneously. The fact that the CRRAM can explain the data reasonably well but not perfectly indicates that the subjects may be trying to behave rationally without complete success.

The rational novice model was developed as an attempt to provide a better theoretical model. Agents are no longer modeled as perfect rational decision makers. Instead, random noise is introduced into agent's decision making process. The size of the noise defines the amount of rationality an agent has. A small amount of noise results in strategy profiles close to that of Nash models while a large amount of noise leads to less "perfect" behavior. One natural consequence of this approach is that statistical variations are no longer a random unexplainable phenomenon. Given a structural framework of this noise, the fluctuations in behavior can be predicted.

The objective here is to solve the first price auction under the rational novice model and compare that solution to the solutions of other Nash models. The specific Nash model focused here is the CRRAM. The CRRAM is a very extensively studied model (see Chen and Plott). It explains the data reasonably well. In the Chen and Plott study, the CRRAM tracks the observed nonlinear behavior with success. It was also found that the CRRAM is the best amongst a number of other Nash models (mostly ones that assume homogeneous agents).

No closed form solution was found for the rational novice model. Numerical solutions were found using computational techniques. The specific rational novice model developed here is a one parameter model. The model depends on a single parameter that describes the average of the risk attitude in the population. This parameter is estimated from the data obtained from the Chen and Plott experiments. Notice that the rational novice model is a one parameter model while the CRRAM has as many parameters as the number of agents. In the Chen and Plott study, there were twelve subjects in each experimental session. Thus, the CRRAM has twelve parameters. Just in a pure data analysis point of view, it is more likely that the CRRAM will perform better than the rational novice model. If it turns out that the rational novice model can explain the model better, it will be strong evidence that the rational novice model has captured certain aspects of behavior that elude the Nash models.

5.2 First Price Sealed Bid Auction

Consider an N-person first price sealed bid auction. Let $j=\{1,...,N\}$ index agents. The value of the unit being auctioned to the j^{th} agent is a random variable v_j. v_j has the distribution H(x) and it is independent across j and time. v_j is realized in the beginning of each auction and is the j^{th} agent's private information. Each agent then submits a bid and the highest bidder buys the unit at the price of his bid. Let b_j be the bid of the j^{th} agent. If the j^{th} agent is the highest bidder, then his profit from this auction will be v_j-b_j. All the other agents receive no profit.

The N-person first price auction is well studied. Cox, Smith and Walker developed the Constant Relative Risk Averse Model (CRRAM) to explain a series of first price auction experiments they have conducted. In "Nonlinear Behavior In Sealed Bid First Price Auctions," Chen and Plott make comparisons between three Nash models under conditions different from the CSW study.[1] The three Nash models are the Risk Neutral Nash Equilibrium Model (RNNE), the Risk Averse Symmetric Nash Equilibrium Model (RASNE) and CRRAM. Under RNNE, all agents are identical and risk neutral. Under RASNE, agents are identical but risk averse. Under CRRAM, agents are heterogeneous and risk averse. Risk aversion is characterized by a one parameter utility function. It is assumed that the distribution of risk aversion is common knowledge. In both the CSW study and Chen and Plott study, the CRRAM was found to be the best model.

The next section provides a brief overview of the three variations of the Nash model studied in the Chen and Plott paper. The only differences among these three variations are the assumptions made on the risk attitudes of the subjects.

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[1]The Chen and Plott study was made with nonuniform distributed private values while the CSW study used uniform distributions. The nonlinearity in the Chen and Plott study enables the researchers to separate the models with relative ease.

5.2.1 Nash Equilibria

In the Chen and Plott paper, three Nash equilibrium models are studied. Although only one of the three models will be compared to the rational novice model in the data analysis section, all three are listed here for completeness.

1) The Risk Neutral Nash Equilibrium Model

Agents are assumed to be identical and risk neutral. They are expected utility maximizers with the utility function u(x)=x. With uniform distributed private values, the equilibrium bidding function is $\frac{n-1}{n}v$ where n=number of bidders in the auction and v=private value.

2) The Risk Averse Symmetric Nash Equilibrium Model

Agents are assumed to be identical and risk averse. They are expected utility maximizers with the utility function $u(x)=x^r$. r is called the risk aversion parameter. Since all the agents are assumed to be identical, r is the same for every agent. With uniform distributed private values, the equilibrium bidding function is $\frac{n-1}{n-1+r}v$. When the agents are more risk averse (r decreases), bids increase.

3) The Constant Relative Risk Averse Model

Agents are assumed to be risk averse and have utility function $u(x)=x^r$. The risk parameter r is assumed to distribute according to some publicly known distribution G(r).

All three Nash models are deterministic and do not allow for fluctuations in behavior. For example, the RNNE predicts a perfect linear relationship between the bids and the private values. However, in previous empirical studies, investigators found that the bids and private values did not correlate perfectly. Ad hoc random variables are usually added to the equilibrium bidding functions to "accommodate" observed fluctuations in previous analyses.

5.3 The Rational Novice Equilibrium

In this section, a rational novice equilibrium will be solved for the first price sealed bid auction. The central principle of the rational novice model is that each agent evaluates their strategic situation imperfectly and responds to the imperfection in a risk averse (or risk loving) manner. In the first price sealed bid auction, a lower bid increases the profit and decreases the probability of winning the auction. Each agent must find an optimal balance between possible profit and the chance of winning. In the rational novice model, each agent is incapable of evaluating the probability of winning correctly. Each of them only receives a noisy signal of what that probability is.

This model predicts that if the auctions are played repeatedly with different opponents, each agent will diversify his actions and employ a spread of bids instead of just a single optimal one.

The formal theory is as follows.

5.3.1 The Rational Novice Model

Consider a symmetric game in which each agent has K actions to choose from. Agents are chosen from a large population to play the game. Agents are assumed to be imperfect. Imperfection is defined by a sequence of random variable $\{\varepsilon_k : k = 1...K\}$. When each agent tries to evaluate the value of his k^{th} choice, he received a noisy signal. Let x_k be the true value of the k^{th} action and g_k be the signal the agent received. It is assumed :

$$g_k = x_k + \varepsilon_k .$$

We assume that ε_k has zero mean and is distributed independently across k. ε_k is also assumed to have a normal distribution. Let σ_k^2 be the variance of ε_k. Each agent bases his decision on his signals $\{g_k : k = 1...K\}$. Having only observed his noisy signals, the true value of the k^{th} action is a random variable given by :

$$(5.1) \quad x_k | g_k = g_k + \xi_k ,$$

where ξ_k is also independently and normally distributed with mean 0 and variance σ_k^2. The game is repeated T times. Each agent chooses what his actions will be in the T games. The agent is assumed not to discount. Therefore, the order of play is not important. The agent only cares about how many times each action is played. Let p_k be the fraction of times that the agent is playing action k. In T games, the total number of times the k^{th} action played is $p_k T$. The total value of the T games to the agent is $T\sum_{k=1}^{K} p_k (x_k | g_k)$. Notice that p_k is not a probability and not a mixed strategy. p_k is the exact fraction of the times an action is played in T games.

Assuming the agent maximizes a weighted sum of the expected value and the variance, the maximization problem is :

$$(5.2)\quad \underset{\{p_k\}_1^K}{\mathrm{Max}}\ E\left(T\sum_{k=1}^{K} p_k(x_k|g_k)\right)-\alpha \mathrm{Var}\left(T\sum_{k=1}^{K} p_k(x_k|g_k)\right)\quad \text{subject to } \sum_{k=1}^{K} p_k = 1^2,$$

where E(.) denotes taking expectation over the random variable $x_k|g_k$. Var(.) is the variance operator. Substituting (3.1) into (3.2), we have

$$(5.3)\quad \underset{\{p_k\}_1^K}{\mathrm{Max}}\ T\sum_{k=1}^{K} p_k g_k - \alpha T^2 \sum_{k=1}^{K} p_k^2 \sigma_k^2 \quad \text{subject to } \sum_{k=1}^{K} p_k = 1 \text{ and } p_k \geq 0 \text{ for all k.}$$

Let λ be the Lagragian multiplier. The first order condition of (3.3) is given by :

$$(5.4)\quad Tg_k - 2\alpha\sigma_k^2 T^2 p_k = \lambda \quad \text{for k=1...K.}$$

Notice that for a risk averse agent, $\alpha > 0$. Thus the second order condition is always satisfied. Solving (5.4), we have

$$p_k = \begin{cases} 0 & \text{if } \dfrac{1}{2\alpha T}\left(g_k\sigma_k^{-2} - \dfrac{\sum_k g_k\sigma_k^{-2} - 2\alpha T}{\sum_k \sigma_k^{-2}}\right) \leq 0 \\ \dfrac{1}{2\alpha T}\left(g_k\sigma_k^{-2} - \dfrac{\sum_k g_k\sigma_k^{-2} - 2\alpha T}{\sum_k \sigma_k^{-2}}\right) & \text{if } \dfrac{1}{2\alpha T}\left(g_k\sigma_k^{-2} - \dfrac{\sum_k g_k\sigma_k^{-2} - 2\alpha T}{\sum_k \sigma_k^{-2}}\right) > 0 \end{cases}.$$

The summation is taken over k's where p_k>0.

p_k is called the optimal behavioral strategy of the agent. The optimal behavioral strategy p_k of each agent depends on his signal which is random. Thus, p_k is also random in nature. Therefore, it is more interesting to look at the population-average of the optimal behavioral strategy. α and σ_k^2 are assumed to be identical across agents. To simplify the

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[2] p_k is also constrained to be non-negative. In the following discussion, we assume that this constraint is not binding.

problem, an approximation was made. For any continuous function f(x), the Taylor expansion implies Ef(x)=E(f(Ex)+f'(Ex)(Ex-x)+...). When the central higher moments of x is small, Ef(x) can be approximated by f(Ex). The population-average rational novice strategy $\overline{p}_k$ is obtained by taken the expectation of p_k with respect to g_k. When the errors are small, the population-average of the optimal behavioral strategy, $\overline{p}_k$, is approximated by

$$(5.5)\quad \overline{p}_k = \begin{cases} 0 & \text{if } \frac{1}{2\alpha T}\left(x_k - \frac{\sum_k x_k\sigma_k^{-2} - 2\alpha T}{\sum_k \sigma_k^{-2}}\right) \le 0 \\ \frac{1}{2\alpha T}\left(x_k - \frac{\sum_k x_k\sigma_k^{-2} - 2\alpha T}{\sum_k \sigma_k^{-2}}\right) & \text{if } \frac{1}{2\alpha T}\left(x_k - \frac{\sum_k x_k\sigma_k^{-2} - 2\alpha T}{\sum_k \sigma_k^{-2}}\right) > 0 \end{cases}.$$

Each individual optimal behavior is not a mixed strategy, and thus does not describe a random distribution of actions. However, since the signals are random, the population-average of the optimal behavioral strategy $\overline{p}_k$ describes the distribution of actions. Hence, when a large group of agents is brought in and made to play the game, the observed probability of the k^{th} action being chosen should be $\overline{p}_k$.

5.3.2 Rational novice Behavior in First Price Sealed Bid Auctions

Consider a population of identical agents. Let us assume that each agent's private value is restricted to integers from 0 to $\overline{v}$. Also assume that the bids are also restricted to integers. Each agent is going to adopt a different bidding profile for each private value v. Assume that the agents only care about their expected profit in the first price sealed bid auctions. When the private value is v, the bid b is worth $x_b^v = (v-b)G(b)$ to the agent. G(b) is the probability of winning when bid b is submitted.

The choice of σ_k^2 is less obvious. The choice of a constant σ_k^2 would make the problem technically easier to solve but represent a less "realistic" scenario. My guideline to pick σ_k^2 is that σ_k^2 should be roughly increasing with the true value x_b^v. The choice of σ_k^2 is :

$$\sigma_k^2 = \sigma^2 (v-b)^2 G(b).$$

where σ^2 is a convenient scaling factor. The choice of σ_k^2 is made based on two reasons. The first is an intuitive one. One would expect the size of the error to be dependent of the potential value. Consider the extreme case where the bid is close to the private value. Since the potential profit is the difference between the two, when the bid is close to the private value, one would expect the agent to consider this an unworthy bid no matter how he evaluates G(b). On the other hand, when the potential profit is larger, there is more room for the agent to make errors. The second reason is a practical one. A few other choices of σ_k^2 were used as trials and they do not provide reasonable results. For example, if σ_k^2 is constant (i.e., σ_k^2 does not depend on the private value and the bid), the model predicts underbidding when agents are risk averse. This is consistent with neither other models nor experimental observations.

For each value v, the bid is restricted to a value from 0 to v-1. This restriction is made solely because of this particular choice of $\sigma_k^2 = \sigma^2 (v-b)^2 G(b)$. The reason is that at the point b=v, σ_k^2 is zero. Consequently, an interior solution does not exist. Therefore, this restriction is put in place mainly to facilitate the derivation of a solution.

Substituting $x_b^v = (v-b)G(b)$ and $\sigma_k^2 = \sigma^2 (v-b)^2 G(b)$ into equation (5.5), the population-average rational novice behavior is given by :

$$(5.6)\quad \overline{p}_b^v = \begin{cases} 0 & \text{if } \frac{1}{\gamma}\left(\frac{1}{v-b} - \frac{1}{(v-b)^2 G(b)} \frac{\sum_k \frac{1}{v-k} - \gamma}{\sum_k \frac{1}{(v-k)^2 G(k)}}\right) \le 0 \\ \frac{1}{\gamma}\left(\frac{1}{v-b} - \frac{1}{(v-b)^2 G(b)} \frac{\sum_k \frac{1}{v-k} - \gamma}{\sum_k \frac{1}{(v-k)^2 G(k)}}\right) & \text{if } \frac{1}{\gamma}\left(\frac{1}{v-b} - \frac{1}{(v-b)^2 G(b)} \frac{\sum_k \frac{1}{v-b} - \gamma}{\sum_k \frac{1}{(v-k)^2 G(k)}}\right) > 0 \end{cases}$$

where $\gamma = 2\alpha\sigma^2 T$. The summation over k is taken over k's where $\overline{p}_k^v > 0$.

The rational novice equilibrium of the first price auction is a collection of bids' distribution, one for each possible private value. Equation (5.6) characterizes this distribution. This is called the *rational novice distribution of bids.*

The probability of winning G(b) is a function of the rational novice distribution of bids. Therefore, if we want to fully characterize the rational novice equilibrium, we have to solve for the G(b) for each possible b. Let q_v be the probability that v is drawn as the private value. If we draw an agent randomly from the population, let $\overline{p}_b$ be the probability that he will submit the bid b. $\overline{p}_b$ is given by :

$$\overline{p}_b \quad = \sum_{v=b}^{\overline{V}} q_v \overline{p}_b^v .$$

Let $\overline{p}_{b<B}$ be the probability that in the population, a bid will be less than B. $\overline{p}_{b<B}$ is given by :

$$\overline{p}_{b<B} \quad = \sum_{b=0}^{B-1} \overline{p}_b .$$

In an N-person auction, G(B) is given by :

$$(5.7)\quad G(B) \quad = \left(\overline{p}_{b<B}\right)^{N-1} + \sum_{m=2}^{N} \binom{n-1}{m-1} \left(\overline{P}_B\right)^{m-1} \left(\overline{p}_{b<B}\right)^{N-m} \frac{1}{m} \quad \text{for } B=0 \text{ to } \overline{v}-1.$$

The first term is the probability that every other bidder submits a lower bid. The second term is the probability that the agent wins the auction if he ties with m-1 other agents. When there is a tie, the unit is randomly assigned to one of the agents.

Equation (5.6) and (5.7) fully characterize the rational novice equilibrium of the private value first price auction.

5.3.3 Comparative Statics

In this section, I would like to examine the effects of risk aversion on the rational novice equilibrium. In past studies, it was shown that risk aversion generally will cause the agents to bid more in the first price auction. Since in the rational novice equilibrium it is no longer true that agents follow a single-value bidding function, the corresponding behavior is a little more complicated to describe.

Essentially, there are three aspects of the effects of risk aversion I would like to study. The first one is the *spread* of the bids. Intuitively, a more risk averse population will spread the bids in over wider range. This concept is difficult to quantify. The measure I use in this paper is the number of bids with a positive probability of being played.

The second aspect is that when risk aversion goes to zero, we expect to see the risk neutral Nash equilibrium. The third aspect is that of the sizes of the bids. In a Nash model, agents bid more when risk aversion is present. Does the same still hold in the rational novice model? Since the agents in the population are now bidding according to a distribution, it is a little more complicated to determine whether the agents are bidding more or less. One method is to look at how the *expected bid* changes when risk aversion changes. Unfortunately, the analysis of the expected bid is inconclusive. The expected bid can go either way when risk aversion changes. Another way is to look at the bid at which $\overline{p}_b^v$ is at a maximum.

The following subsections describe the effects of risk aversion on the bid spread and the bid submitted with maximum probability.

5.3.3.1 Bid Spread

The rational novice equilibrium is characterized by a distribution of bids given by equation (5.6). The model only has one parameter γ which describes the characteristics of the population. Recall $\gamma = 2\alpha\sigma^2 T$. Thus, the effects of risk aversion (α), relative size of the errors (σ^2) and the number of auctions carried out (T) are inseparable in the model.

Recall the rational novice distribution of bids is given by :

$$(5.6) \quad \overline{p}_b^v = \begin{cases} 0 & \text{if } \dfrac{1}{\gamma}\left(\dfrac{1}{v-b} - \dfrac{1}{(v-b)^2 G(b)} \dfrac{\sum_k \frac{1}{v-k} - \gamma}{\sum_k \frac{1}{(v-k)^2 G(k)}}\right) \le 0 \\ \dfrac{1}{\gamma}\left(\dfrac{1}{v-b} - \dfrac{1}{(v-b)^2 G(b)} \dfrac{\sum_k \frac{1}{v-k} - \gamma}{\sum_k \frac{1}{(v-k)^2 G(k)}}\right) & \text{if } \dfrac{1}{\gamma}\left(\dfrac{1}{v-b} - \dfrac{1}{(v-b)^2 G(b)} \dfrac{\sum_k \frac{1}{v-b} - \gamma}{\sum_k \frac{1}{(v-k)^2 G(k)}}\right) > 0 \end{cases}$$

while the summation with index k is over the bids with positive probability.

Claim 1: For a fixed v, the number of bids with positive probability (i.e., $\overline{p}_b^v > 0$) is non decreasing in γ.

Proof: It is sufficient to show that when γ increases, all the bids with positive probability remains so. Let B be the set of bids with positive probability given v. So, for all $b \in B$,

$$(5.8)\quad \frac{1}{\gamma}\left(\frac{1}{v-b}-\frac{1}{(v-b)^2 G(b)}\frac{\sum_{k\in B}\frac{1}{v-k}-\gamma}{\sum_{k\in B}\frac{1}{(v-k)^2 G(k)}}\right)>0.$$

Let $\gamma^{+}>\gamma$. Assume when γ increases to γ^{+}, some bid b has probability 0. Let $B^{-}=B-\{b\}$.

If bid b has probability 0, the follow must be satisfied.

$$(5.9)\quad \frac{1}{\gamma^{+}}\left(\frac{1}{v-b}-\frac{1}{(v-b)^2 G(b)}\frac{\sum_{k\in B^{-}}\frac{1}{v-k}-\gamma^{+}}{\sum_{k\in B^{-}}\frac{1}{(v-k)^2 G(k)}}\right)\le 0.$$

Since γ and γ^{+} are positive, (3.8) and (3.9) implies

$$\frac{1}{v-b}-\frac{1}{(v-b)^2 G(b)}\frac{\sum_{k\in B}\frac{1}{v-k}-\gamma}{\sum_{k\in B}\frac{1}{(v-k)^2 G(k)}}>\frac{1}{v-b}-\frac{1}{(v-b)^2 G(b)}\frac{\sum_{k\in B^{-}}\frac{1}{v-k}-\gamma^{+}}{\sum_{k\in B^{-}}\frac{1}{(v-k)^2 G(k)}}$$

which implies

$$\frac{\sum_{k\in B^{-}}\frac{1}{v-k}-\gamma^{+}}{\sum_{k\in B^{-}}\frac{1}{(v-k)^2 G(k)}}>\frac{\sum_{k\in B}\frac{1}{v-k}-\gamma}{\sum_{k\in B}\frac{1}{(v-k)^2 G(k)}}.$$

Since $B^{-}=B-\{b\}$, $\sum_{k\in B^{-}}\frac{1}{v-k}=\sum_{k\in B}\frac{1}{v-k}-\frac{1}{v-b}$ and $\sum_{k\in B^{-}}\frac{1}{(v-k)^2 G(k)}=\sum_{k\in B}\frac{1}{(v-k)^2 G(k)}-\frac{1}{(v-b)^2 G(b)}$.

Therefore,

$$\frac{\sum_{k\in B}\frac{1}{v-k}-\frac{1}{v-b}-\gamma^{+}}{\sum_{k\in B}\frac{1}{(v-k)^2 G(k)}-\frac{1}{(v-b)^2 G(b)}}>\frac{\sum_{k\in B}\frac{1}{v-k}-\gamma}{\sum_{k\in B}\frac{1}{(v-k)^2 G(k)}}.$$

After some algebra, one arrives at:

$$(5.10)\quad (\gamma^{+}-\gamma)\sum_{k\in B}\frac{1}{(v-k)^2 G(k)}+\frac{1}{v-b}\sum_{k\in B}\frac{1}{(v-k)^2 G(k)}-\frac{1}{(v-b)^2 G(b)}\sum_{k\in B}\frac{1}{v-k}+\frac{\gamma}{(v-b)^2 G(b)}<0.$$

However, (5.8) alone implies

$$\frac{1}{v-b}\sum_{k\in B}\frac{1}{(v-k)^2 G(k)}-\frac{1}{(v-b)^2 G(b)}\sum_{k\in B}\frac{1}{v-k}+\frac{\gamma}{(v-b)^2 G(b)}>0.$$

Since $(\gamma^{+}-\gamma)\sum_{k\in B}\frac{1}{(v-k)^{2}G(k)}>0$, we arrive at a contradiction. Therefore all the bids with positive probability will remain so if γ increases. QED.

The bids spread out more when γ increases. Recall that $\gamma = 2\alpha\sigma^2 T$. γ increases with both risk aversion (α) and the potential size of errors (σ^2). Claim 1 is consistent with our intuition. When people become more risk averse, they will spread their bids out more to hedge against their potential errors. The same thing will happen if their risk attitude remains constant while the sizes of their potential errors increase.

5.3.3.2 Bids Submitted with Maximum Probability

Let us first examine the case where γ is very small. It is expected that the rational novice equilibrium will resemble the risk neutral Nash equilibrium when γ goes to zero.

Claim 2: When γ approaches zero and when there is only one bid with positive probability, that bid is the risk neutral Nash equilibrium.

proof: Consider a continuous function defined by $f(b,\gamma)=\frac{1}{v-b}-\frac{1}{(v-b)^2G(b)}\frac{\sum_k\frac{1}{v-k}-\gamma}{\sum_k\frac{1}{(v-k)^2G(k)}}$.

When $f(b,\gamma)$ is greater than zero, $f(b,\gamma)$ gives the values of $\overline{p}_b^v$. Let $f(b^*,\gamma)$ be the maximum of $f(b,\gamma)$. b^* is obtained by differentiating f with respect to b and setting it equal to zero. It yields the following :

$$(v-b^*)G(b^*)^2=\left(2G(b^*)-(v-b)G'(b^*)\right)\frac{\sum_k\frac{1}{v-k}-\gamma}{\sum_k\frac{1}{(v-k)^2G(k)}}.$$

Since only bid b has positive probability, $\sum_k\frac{1}{v-k}=\frac{1}{v-b}$ and $\sum_k\frac{1}{(v-k)^2G(k)}=\frac{1}{(v-b)^2G(b)}$. Letting γ go to zero, we have

$$(v-b^*)G(b^*)^2 = \left(2G(b^*)-(v-b)G'(b^*)\right)\frac{\frac{1}{v-b^*}}{\frac{1}{(v-b^*)^2G(b^*)}}.$$

which implies

$$G(b^*) = \left(2G(b^*)-(v-b)G'(b^*)\right)$$

Simplifying the above, we have:

(5.11) $(v-b)G'(b^*) = G(b^*)$.

The risk neutral Nash equilibrium is given by maximizing (v-b)G(b). Differentiating with respect to b, we have

$$(v\text{-}b)G'(b)=G(b)$$

which is exactly (3.11). QED

In claim 2, we have established that in the limit where everyone is risk neutral, the rational novice equilibrium becomes the Nash equilibrium. The following claim examines the effects of risk aversion in the neighborhood of the risk neutral case.

Claim 3: The bid with maximum probability is increasing in γ.

Proof : Recall in the proof of claim 2, it is shown that the bid with the maximum probability is given by

$$(v-b^*)G(b^*)^2 = \left(2G(b^*)-(v-b)G'(b^*)\right)\frac{\sum_k \frac{1}{v-k}-\gamma}{\sum_k \frac{1}{(v-k)^2G(k)}}.$$

Rearranging terms, we have :

$$\frac{(v-b^*)G(b^*)^2}{\left(2G(b^*)-(v-b)G'(b^*)\right)} = \frac{\sum_k \frac{1}{v-k}-\gamma}{\sum_k \frac{1}{(v-k)^2G(k)}}.$$

Taking derivatives with respect to γ on both sides, we have :

$$\frac{\left[2(v-b^{*})G(b^{*})G'(b^{*})-G(b^{*})^{2}\right]\left[2G(b^{*})-(v-b^{*})G'(b^{*})\right]-\left[(v-b^{*})G(b^{*})^{2}\right]\left[3G'(b^{*})-(v-b^{*})G''(b^{*})\right]}{\left(2G(b^{*})-(v-b)G'(b^{*})\right)^{2}}\frac{db^{*}}{d\gamma}=\frac{-1}{\sum_{k}\frac{1}{(v-k)^{2}G(k)}}$$

In the neighborhood of risk neutrality, $(v-b)G'(b^{*})=G(b^{*})$. Substituting into the above, we have

$$\frac{\left[2G(b^{*})^{2}-G(b^{*})^{2}\right]\left[2G(b^{*})-G(b^{*})\right]-\left[G(b^{*})^{3}/G'(b^{*})\right]\left[3G'(b^{*})-(v-b^{*})G''(b^{*})\right]}{\left(G(b^{*})\right)^{2}}\frac{db^{*}}{d\gamma}=\frac{-1}{\sum_{k}\frac{1}{(v-k)^{2}G(k)}}$$

Simplifying the above, we have

$$\left(G(b^{*})-\left[G(b^{*})/G'(b^{*})\right]\left[3G'(b^{*})-(v-b^{*})G''(b^{*})\right]\right)\frac{db^{*}}{d\gamma}=\frac{-1}{\sum_{k}\frac{1}{(v-k)^{2}G(k)}}.$$

The above implies

$$\text{(5.12)}\quad \left(-\left[G(b^{*})/G'(b^{*})\right]\left[2G'(b^{*})-(v-b^{*})G''(b^{*})\right]\right)\frac{db^{*}}{d\gamma}=\frac{-1}{\sum_{k}\frac{1}{(v-k)^{2}G(k)}}.$$

The Nash equilibrium is obtained by maximizing (v-b)G(b). Therefore, the 2nd derivative of (v-b)G(b) should be negative. Thus, $(v-b)G''(b)-2G'(b)<0$. Hence $2G'(b)-(v-b)G''(b)>0$.

So (5.12) implies $\frac{db^{*}}{d\gamma}>0$. QED.

In a way, the above result shows that risk aversion leads to overbidding. This is consistent with both the traditional Nash models and the observed behavior in experiments.

5.4 Numerical Analysis

Equation (5.6) and (5.7) characterize the rational novice equilibrium of the private value first price auction. No closed form solution is found for the rational novice bid distribution. However, numerical methods are developed to calculate this distribution under the experimental conditions that we are about to analyze. The only relevant experimental condition that affects the rational novice equilibrium is the distribution of the private value q_v.

The numerical method is an iterative procedure that will provide the solution for $\{G(B):B=0\ldots\overline{v}-1\}$. The procedure starts with a guess of the function $\{G(B):B=0\ldots\overline{v}-1\}$. In this study the starting guess is of the form :

$$G_{guess}(x)=\begin{cases}\dfrac{ax}{b} & \text{if } x<b\\ a+(1-a)\left(\frac{x-b}{1000-b}\right) & \text{if } b\le x^{\pm}\end{cases}$$

Then, the following steps are carried out.

<u>Step 1</u> : Set $G_1(b)$=$G_{guess}(b)$ for b=0,1,...,$\overline{v}-1$.

<u>Step 2</u> : Calculate $\overline{p}_b^v$ as a function of $\{G_1(b):b=0\ldots\overline{v}-1\}$ using the following :

$$\overline{p}_b^v=\begin{cases}0 & \text{if } \dfrac{1}{\gamma}\left(\dfrac{1}{v-b}-\dfrac{1}{(v-b)^2G_1(b)}\dfrac{\sum_k\frac{1}{v-k}-\gamma}{\sum_k\frac{1}{(v-k)^2G_1(k)}}\right)\le 0\\ \dfrac{1}{\gamma}\left(\dfrac{1}{v-b}-\dfrac{1}{(v-b)^2G_1(b)}\dfrac{\sum_k\frac{1}{v-k}-\gamma}{\sum_k\frac{1}{(v-k)^2G_1(k)}}\right) & \text{if } \dfrac{1}{\gamma}\left(\dfrac{1}{v-b}-\dfrac{1}{(v-b)^2G_1(b)}\dfrac{\sum_k\frac{1}{v-b}-\gamma}{\sum_k\frac{1}{(v-k)^2G_1(k)}}\right)>0\end{cases}.$$

<u>Step 3</u> : Calculate $G_2(b)$ by using the following equations :

$$\overline{p}_b=\sum_{v=b}^{\overline{v}}q_v\overline{p}_b^v$$

$$\overline{p}_{b<B}=\sum_{b=0}^{B-1}\overline{p}_b$$

$$G_2(B)=\left(\overline{p}_{b<B}\right)^{N-1}+\sum_{m=2}^{N}\binom{n-1}{m-1}\left(\overline{P}_B\right)^{m-1}\left(\overline{p}_{b<B}\right)^{N-m}\frac{1}{m}\quad\text{for B=0 to }\overline{v}\text{-1.}$$

Step 4 : Calculate $\Delta = \sum_{b=0}^{\bar{v}-1}(G_1(b)-G_2(b))^2$. When Δ is smaller than a certain threshold value, stop. Otherwise, set $G_1(b)=G_2(b)$ for b=0,1,...,$\bar{v}-1$ and go back to step 2.

When Δ=0, the above procedures will ensure that $G_1(b)$ (and $G_2(b)$) is a solution to equation (5.6) and (5.7). Analysis revealed that this process converges quite fast. Under our experimental parameters the process usually converges in fewer than 10 iterations with a threshold of 0.1.

5.5 Data Analysis: Techniques and A Description of the Data

In the Chen and Plott study, several experiments were conducted in which non-uniform distributed private values were assigned to the subjects. Six experiments were conducted in the Chen and Plott study. There were twelve subjects in each experiment. Some subjects participated in more than one experiment. The experiments were conducted in periods. In each period, the subjects were randomly divided into groups of three who would bid against each other in a sealed bid auction. After the subjects were randomly assigned, the private value would be revealed to the appropriate subject via a computer link. The subject then entered a bid into the computer.

Although the private values were known only to the appropriate subject, the distribution of the values was common knowledge. The private values were drawn from distribution of the following form. In three of the six experiments, the values are drawn from either a range from 0 to 999. Let q_v be the probability of v being drawn. The probability of drawing the private value v is given by:

$$q_v = \begin{cases} a/500 & \text{if} \quad v < 500 \\ (1-a)/500 & \quad\quad 500 < v \end{cases}.$$

In the other three of the six experimental sessions, an offset of 500 was added to all the private values. That is, the values are drawn from a range from 500 to 1499. And the corresponding probabilities are :

$$q_v = \begin{cases} a/500 & \text{if} \quad v < 1000 \\ (1-a)/500 & \quad\quad 1000 < v \end{cases}.$$

There were two values of a being used in the experiments. In four of the six experiments, a=0.8. In the other two, a=0.2. In table 4, a summary of the different parameters of the experiments is reported.

Since the rational novice model depends only on (v-b), a linear transformation is performed on the data to bring the range of the values and the bids down to [0,999]. Then the same estimation procedure is carried out.

In the Chen and Plott investigation, the Nash equilibrium is studied. The Nash equilibrium under this experimental environment exhibits nonlinear bidding functions. Subjects were observed to have employed nonlinear strategies. One Nash model, the Constant Relative Risk Averse Model (CRRAM) offers a good explanation. The CRRAM manages to outperform nearly all other models except one, the Sophisticated Ad Hoc Mode (SOPAM) which basically is a piece-wise linear model.

All the models studied in the Chen and Plott study predict single-valued bidding strategies as functions of the private value. The econometrics model used in the Chen and Plott paper is artificially constructed by adding a normally distributed error to the Nash bidding strategy. The variance of this error is independent of the value and the bid. On the other hand, the rational novice model explicitly characterizes the distribution of bids. It is the goal of this paper to determine whether a distribution of bids based on theory (the rational novice model) can perform better than an arbitrary distribution assigned solely for the sake of fitting errors. In this section, the rational novice model is estimated and compared to the results in the Chen and Plott paper. The rational novice model is found to be a better model than the Nash model. Since a visual inspection of the data will show that the sizes of the errors change with the value (usually increasing in the value), the reason that the rational novice model does better may be that the additional statistical assumptions in the Chen and Plott model are inadequate.

The maximum likelihood technique is used to estimate all the relevant models. The observations are in the form $\{(v_i, b_i): i = 1 \ldots n\}$ where (v_i, b_i) is the observed value and bid of a subject in an experimental auction. Given the parameter γ, section 5.4 has outlined a method to compute the numerical solution for the corresponding rational novice equilibrium by solving equation (5.6) and (5.7). The rational novice model is estimated by the following maximum likelihood procedures.

For each value of γ, the likelihood function can be calculated by:

1) applying the technique in section 5.4 to solve for a equilibrium (i.e., calculating G(b) for each possible b,) and

2) calculating the log likelihood function by

(5.8) $l(\gamma) = \sum_{i=1}^{n} \log\left(\overline{p}_{b_i}^{v_i}\right)$, where $\overline{p}_{b_i}^{v_i}$ is calculated by equation (5.6).

The Golden Section Search method is then used to find the value of γ that gives the maximum of the log likelihood function. γ is estimated from the Chen and Plott nonlinear first price auction experiments. The results of the estimation are listed in Table 4. The rational novice model characterized by $\bar{p}_h^v$ only allows bids in the range from 0 to v-1. The model is unable to explain bids greater than or equal to the private value. In theory, this restriction does not pose a conflict with the Nash models since bidding the private value is always a dominated strategy. However, in practice we do observe people bidding their value or sometimes even above that. In the experiment, there are a few instances that the subjects bid greater than or equal to their private values. Those data are ignored in the estimation procedures.

In the Chen and Plott study, five models are estimated and compared to the CRRAM. They are the two Nash models (RNNE and RASNE as described in section 5.2.1) and three ad hoc rules of thumb. The three rules of thumb are:

1) The Markdown Model (MM) in which the bid=factor x private value.
2) The Simple Ad hoc Model (SIMAM) in which the bid is a linear function of the private value. And,
3) The Sophisticated Ad hoc Model (SOPAM) in which the bid is a piece wise linear function of the private value in the form:

$$\text{bid}=\text{bid}=\begin{cases} \alpha+\beta\text{value} & \text{if} \quad v<500 \\ \alpha+\beta\text{value}+\gamma(\text{value}-500) & \quad\quad 500\le v \end{cases}.$$

The two Nash models are shown to be inferior to the CRRAM. Therefore, they will be ignored in the analysis. The rest of the models, CRRAM, MM, SIMAM and SOPAM, are compared to the rational novice model.

All of the above behavioral model can be generalized into the form: bid=f(value). Given the data set $\{(v_i, b_i): i=1\ldots n\}$, the econometrics model estimated is :

$$b_i = f(v_i) + \varepsilon_i$$

where ε_i are independent and normally distributed with mean zero. The appropriate parameters are estimated by the maximum likelihood method. Since a number of data points are ignored in the estimation of the rational novice model, the same group of data is ignored in the estimation of the Chen and Plott models.

As in the Chen and Plott study and the last chapter, Voung's Model Selection Test was used to compare the other models to the rational novice model. Recall that the Voung's Selection Test is defined as follows:

Let us consider the general case where one wants to compare two nonnested models f and g. Let l_t^f be the maximum log likelihood of data point t under the model f and l_t^g be the maximum log likelihood of data point t under the model g. Let T be the number of data points. Define :

$$LR=\sum_t (l_t^f - l_t^g) \text{ , and}$$

$$\hat{w}^2 = \tfrac{1}{T}\sum_t \left(l_t^f - l_t^g\right)^2 - \left(\tfrac{1}{T}LR\right)^2.$$

Consider the following three hypotheses:

H_0 : f and g are equivalent,

H_f : f is better than g, and

H_g : g is better than f.

Voung's theorem 5.2 states that

a) under H_0 : $\dfrac{LR}{\sqrt{T}\hat{w}} \to n(0,1)$

b) under H_f : $\dfrac{LR}{\sqrt{T}\hat{w}} \to +\infty$

c) under $H_g : \frac{LR}{\sqrt{T}\hat{w}} \rightarrow -\infty$.

The above theorem provides a simple directional test for model selection. Choose a critical value c determined by the desirable significance level. Calculate the statistics $\frac{LR}{\sqrt{T}\hat{w}}$. If $\frac{LR}{\sqrt{T}\hat{w}} \leq |c|$, one cannot discriminate between f and g. If $\frac{LR}{\sqrt{T}\hat{w}} > c$, one rejects the null hypothesis that the models are equivalent in favor of f. If $\frac{LR}{\sqrt{T}\hat{w}} < -c$, then one chooses g rejecting the same null hypothesis.

Voung's Selection Test does not take into account the complexity of the models. However, there is an additive correction factor that compensates for that. Let m be the number of parameters in model f and n be the number of parameters in model g. The correction factor used in the Chen and Plott study is $K(n,m,T)=(m-n)\log(T)/2$. The revised Voung statistics is $\frac{LR}{\sqrt{T}\hat{w}}+K(n,m,T)$. As one can see, the correction factor $K(n,m,T)$ adjusts the Voung statistics in favor of the less complicated model. For example, if model g has more parameters than f (i.e., m>n), then the Voung statistics are corrected in the favor of f ($K(n,m,T)>0$).

It was shown in Voung's paper that a number of different correction factors are all asymptotically equivalent. Furthermore, they were all shown to be equivalent to not applying a correction factor. Hence, there is no method to choose a correction scheme optimally. For consistency, the correction factor used in the Chen and Plott study is employed here. The results both with and without the application of this correction factor are reported.

Voung's Selection Test was performed at two levels. The test was performed on the pooled data of all six experiments to determine what is the best overall model. The test was also performed on individual experiments so that we can find out if difference experimental conditions will affect the relative performances of the models.

5.6 Data Analysis: Results

The techniques discussed in the previous section were used to estimate the rational novice model for the six experiments conducted in the Chen and Plott study. The estimates of the log likelihood function suggest that the rational novice model is a better overall model than the CRRAM.

Table 4 reports all the relevant statistics of the six Chen and Plott experiments. The first three rows contain the parameters of the experiments. Parameter a and the offset are explained in the previous section. A number of data points were ignored in the estimation. (The reason is also explained in the previous section.) The amount of data ignored are reported in row 4. Recall that $\gamma = 2\alpha\sigma^2 T$. Both γ and γ/T are reported in table 4. The estimates of γ/T provide a sense of how risk averse the subjects are. Also reported are the log likelihood and the Voung statistics of all the models. The Voung statistics are calculated with respect to the rational novice model. If the statistics are greater than 1.65, then the Voung test chooses the rational novice model over the other one. If the statistics are smaller than -1.65, then the Voung test chooses the other model. If the absolute value of the statistics is less than 1.65, then the Voung test cannot distinguish between the rational novice model and the other model. The Voung statistics with the correction factor (n-m)log(T)/2 are also reported. This correction factor is also explained in the previous section.

The following summarizes the important results.

Result 1: The rational novice model is a better overall model than the CRRAM.
Support: In the pooled data set, the CRRAM can be rejected at five percent significance in favor of the rational novice model independent of whether the correction factor K(n,m,T) is applied.

Result 1 alone will indicate that the rational novice is a better model than the CRRAM. However, if the test is carried out on individual experimental sessions, the result is less clear.

Result 2: In the six experiments, the rational novice model sometimes performs better than the CRRAM and sometimes does not.
Support: In two out of the six experiments, the CRRAM can be rejected at five percent significance in favor of the rational novice model. In the other two experiments, the rational novice model can be rejected at five percent significance in favor of the CRRAM. In the remaining experiments, the two models are not distinguishable.

The first result indicates that the rational novice model explains the data better than the CRRAM. Even with the less unambiguous result 2, the rational novice model seems to provide as much explanatory power as the CRRAM. This is quite remarkable considering the fact that the rational novice model is a one parameter model while the CRRAM has twelve.

Result 3: The rational novice model is a better model than the MM.

Support: In the pooled data set and four out of the six individual experimental sessions, the MM can be rejected at five percent significance in favor of the rational novice model. In the remaining two experiments, the Voung Model Selection Test chooses MM over the rational novice model in one experiment and cannot distinguish the two in another. The correction factor does not affect this result.

The third result should come at no surprise. In the Chen and Plott study, it is shown that MM is not as good as the CRRAM. Since result 1 and 2 have already shown that the rational novice model performs as well as the CRRAM, it should do better than MM.

Result 4: The rational novice model is a better model than the SIMAM.

Support: In the pooled data set and two out of the six individual experimental sessions, the SIMAM can be rejected at five percent significance in favor of the rational novice model. In the remaining four experiments, the Voung Model Selection Test chooses MM over the rational novice model in one experiment and cannot distinguish the two in the others. If the correction factor is applied, then the SIMAM can be rejected in four of the six experiments as well as the pooled data set.

There is still some ambiguity since the rational novice model does not outperform the SIMAM in all experiments. However, considering the fact that the rational novice model does better in most experiments (with correction factor applied) and in the pooled data set, it would seem that it is the better model. Similarly, the CRRAM is shown to be marginally better than the SIMAM in the Chen and Plott study. Thus there is no surprise since the performance of the rational novice model is close to that of the CRRAM Notice that SIMAM has 24 parameters while the rational novice model only has one.

Result 5: The rational novice model may be not good a model as the SOPAM.

Support: In two out of the six experiments, the SOPAM can be rejected at five percent significance in favor of the rational novice model. In the other four, the test chooses the SOPAM. If a correction factor is applied, then the Voung test only chooses SOPAM over the rational novice model in three experiments and cannot distinguish between the two in the remaining one. In the pooled data set, the SOPAM is chosen if no correction factor is applied. Otherwise, the rational novice model is chosen.

Result 5 is very ambiguous. If one considers the pooled data set, then the result depends on the application of the correction factor. Otherwise, the SOPAM explains the data better than the rational novice model in most but not all experiments. In experiment 5 and 6, the rational novice model does better.

Although SOPAM has no theoretical foundation, it explains the data better than the CRRAM. It seems that people may, after all, follow some sophisticated ad hoc rule of thumb than behaving rationally. The SOPAM has 36 parameters. The CRRAM has 12 and the rational novice model has one. Even without the application of a correction factor that penalizes high number of parameters, the Voung Model Selection Test rejects the SOPAM in favor of the rational novice model in two experiments. So if one only considers that two experiments or the pooled data set with a correction factor, one may conclude that people are more likely to behave in some rational ways than just acting on rules of thumb.

On the other hand, the SOPAM provides a better explanation in four of the six experiments. It is frustrating that one cannot find a coherent picture. There are two possible explanations of this confusing result. Since the Voung Selection Test is only asymptotically consistent, we may not have enough data to distinguish between the two models. Furthermore, since a number of different correction factors are asymptotically equivalent, with a finite number of data points, our choice of the correction factor may be over or under compensating for the complexity of the model. When the difference between two models is large (such as the difference between the MM and the rational novice model), these factors may not manifest themselves.

5.7 Summary and Comments

This research applies the rational novice model to the first price sealed bid auction. Auctions have been a well-studied subject in both the fields of theoretical modeling and the experimental studies. In a previous experimental study of nonlinear bidding behavior (Chen and Plott), it was found that "people do not exhibit the full extent of the kind of rationality that game theory assumes. The CRRAM is not as accurate as the SOPAM nor is the rational expectation hypothesis supported by the data."

The goal of this study is to develop and apply an alternate way of game theory modeling that does not require a very high degree of rationality. The rational novice equilibrium is proposed as this alternate approach. It is likely that the rational novice model will offer a better fit of the data since it does not have to make a wild guess at the distribution of the bids.

The rational novice model takes into account rationality as well as the potential of making mistakes. Nash models assume a very high degree of rationality. Since the data do not indicate the full extent of rational behavior, it is more likely that in addition to rational deduction, there are some other unknown phenomena going on. The rational novice model offers a better explanation. The agents are modeled with the ability of rational deduction as well as the capability of responding to mistakes. The comparative statics show that when the agents are more risk averse, they are likely to increase their bids as well as increase the size of their bid spreads. The first deduction is consistent with Nash models. The second is a new result that tells a better story. The rational novice model is a model not only capable of predicting the average bids, but also able to characterize the distribution of the bid spread.

Merely characterizing the distribution of the bid spread will not be of any value if such characterization is not supported by empirical analysis. Results 1 through 5 reported in section 5 show the rational novice model also provides a better empirical fit to data gathered in the Chen and Plott study than the MM and the SIMAM. It is likely that the rational novice model performs better than the CRRAM but the evidence is inconclusive. In the case of the SOPAM, the rational novice does not seem to do as well. But again, the evidence is inconclusive.

In summary, the rational novice model is applied to the first price private value auction in this research. The results suggest that the rational novice model is likely to be more accurate than any Nash model (in particular, the CRRAM) as well as a number of ad hoc rules-of-thumb. However, the most sophisticated ad hoc model (SOPAM) seems to be a better model than the rational novice model. If one accepts the rational novice model as a possible explanation, then one would expect people not to exhibit the full extent of the kind of optimal behavior described by game theory, neither are they behaving in some random way without a discernible pattern. The rational novice model seems to have struck the appropriate middle ground between rational and irrational behavior and provides a good explanation of people's behavior.

There are three possible directions of future research. The first is to gather more data and find out whether the models can be better separated from one another. The second is to test if the rational novice model performs well in different games. This approach involves the application of the existing rational novice framework to other games. The third direction is to expand the theoretical framework of the rational novice model. For example, the rational novice framework can be expanded to handle extensive form games, imperfect perception or a number of other features which are not included in the present framework. One important such feature is learning which is not addressed in the rational novice model. Although multiple games are played, they are assumed to be played simultaneously. In actual life or in experiments, most often games are played sequentially and often people learn. One possible way to model learning is to assume people play sequential segments of simultaneous games and model the error structure as a function of the time specific segment.

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Table 1: Log Likelihood Estimations of Voluntary Contribution Games

Model	log likelihood	parameter estimates
Rational novice (whole data set N=773)	-800.8	$\bar{u}$ =-3.26 $\bar{\sigma}$ =3.30 z=1.67
Rational novice (first 5 periods discarded N=437)	-260.8	$\bar{u}$ =-4.72 $\bar{\sigma}$ =3.25 z=0.035
Probit (whole data set N=773)	-2496	constant coef=-1.39 mrs coef=0.81
Probit (first 5 periods discarded N=437)	-1185	constant coef=-1.88 mrs coef=1.35
Relaxed Nash (c=25) N=773	-19017	$\bar{u}$ =-25 (constraint) $\bar{\sigma}$ =0 (constraint) z=0

The total number of data points in the whole data set = 1280.
The total number of data points with first 5 periods discarded = 640.

The estimation of the rational novice model requires independence across data points. Some subjects received the same (private good value, public good value) pair more than once in an experiment. The rational novice model does not guarantee these decisions to be independent of each other. These data are discarded.

The number of data points after discarding the dependent data = 773.
The number of data points after discarding the dependent data and the first 5 periods = 437.

Since the Probit model is a discrete choice model while the rational novice model is a continuous model, the relative performance of the models can not be determined by comparing the log likelihoods of the two.

Table 2 : Test Statistics of Voluntary Contribution Games

Model	Test statistics	parameters constraint tested
Rational novice vs Relaxed Nash (c=25)	36432* (p-value=1.00)	$\bar{u}=-25$ $\bar{\sigma}=0$

The models are tested against the hypothesis that the Nash model is true.
A * indicates that the Nash equilibrium can be rejected at five percent significance.

Table 3 : Pseudo R^2 Statistics for Voluntary Contribution Games

Model	Pseudo R^2
Rational novice (whole data set)	0.67
Rational novice (first 5 periods discarded)	0.75
Probit (whole data set)	0.57
Probit (first 5 periods discarded)	0.68

Table 4 : Log Likelihood Estimation and Voung's Statistics in First Price Auctions

	Experiment						
	1	2	3	4	5	6	pooled
Number of Periods (T)	60	120	70	100	100	100	
Parameter a	0.8	0.2	0.8	0.2	0.8	0.8	
Offset	0	0	500	500	0	500	
Number of Data Points Discarded (percentage)	17 (2.36%)	38 (2.64%)	57 (6.79%)	39 (3.25%)	13 (1.08%)	100 (8.33%)	264
Number of Data Points Used	703	1362	783	1161	1187	1100	6296
Estimate of γ in rational novice model	1.905441	2.423428	2.055558	2.557898	2.155390	1.813302	-
(γ /T)	0.0318	0.0202	0.0293	0.0256	0.0216	0.0181	
Likelihood Estimates							
Rational novice	-3512	-7236	-4202	-6542	-5913	-5523	-32928
CRRAM	-3300	-7232	-4158	-6376	-6344	-5901	-33311
MM	-3666	-7289	-4295	-6362	-6358	-5979	-33949
SIMAM	-3548	-7207	-4209	-6303	-6309	-5908	-33484
SOPAM	-3100	-7035	-3961	-6227	-6220	-5758	-32301
w							
CRRAM	44.48	60.10	38.91	60.14	74.86	51.36	58.10
MM	50.49	69.43	56.04	57.85	73.10	55.59	62.31
SIMAM	45.48	60.09	41.63	60.06	78.51	51.59	59.33
SOPAM	64.87	74.10	49.94	61.31	73.99	57.57	65.36
Voung's Statistics							
CRRAM	-4.76	-0.05	-1.12	-2.76	5.76	7.37	6.59
MM	3.06	0.77	1.66	-3.12	6.08	8.21	16.39
SIMAM	0.81	-0.48	0.16	-3.98	5.04	7.47	9.37
SOPAM	-6.34	--2.71	-4.83	-5.14	4.15	4.09	-9.59
Voung's Statistics w/ correction*							
CRRAM	-3.95	-0.61	-0.18	-2.12	6.28	8.12	11.56
MM	3.78	1.34	2.31	-2.45	6.61	8.90	21.02
SIMAM	2.47	0.90	2.00	-2.63	6.08	9.03	19.54
SOPAM	-4.57	-1.00	-2.49	-3.13	5.82	6.21	4.46

* The correction factor for Voung's Statistics is K(n,m,T)=(n-m)log(T)/2 where n=number of parameters in the other model, m=number of parameters in the rational novice model and T=number of data points.

When the Voung Test is conducted at five percent significance, the critical value of the statistics is 1.65. That is, if the statistics is > 1.65, the other model is rejected in favor of the rational novice model. If the statistics is <-1.65, the rational novice model is rejected in favor of the other model. If the statistics is between -1.65 and 1.65, then the rational novice model is indistinguishable from the other model given the data.

Figure 1 : Subrational Model with all data

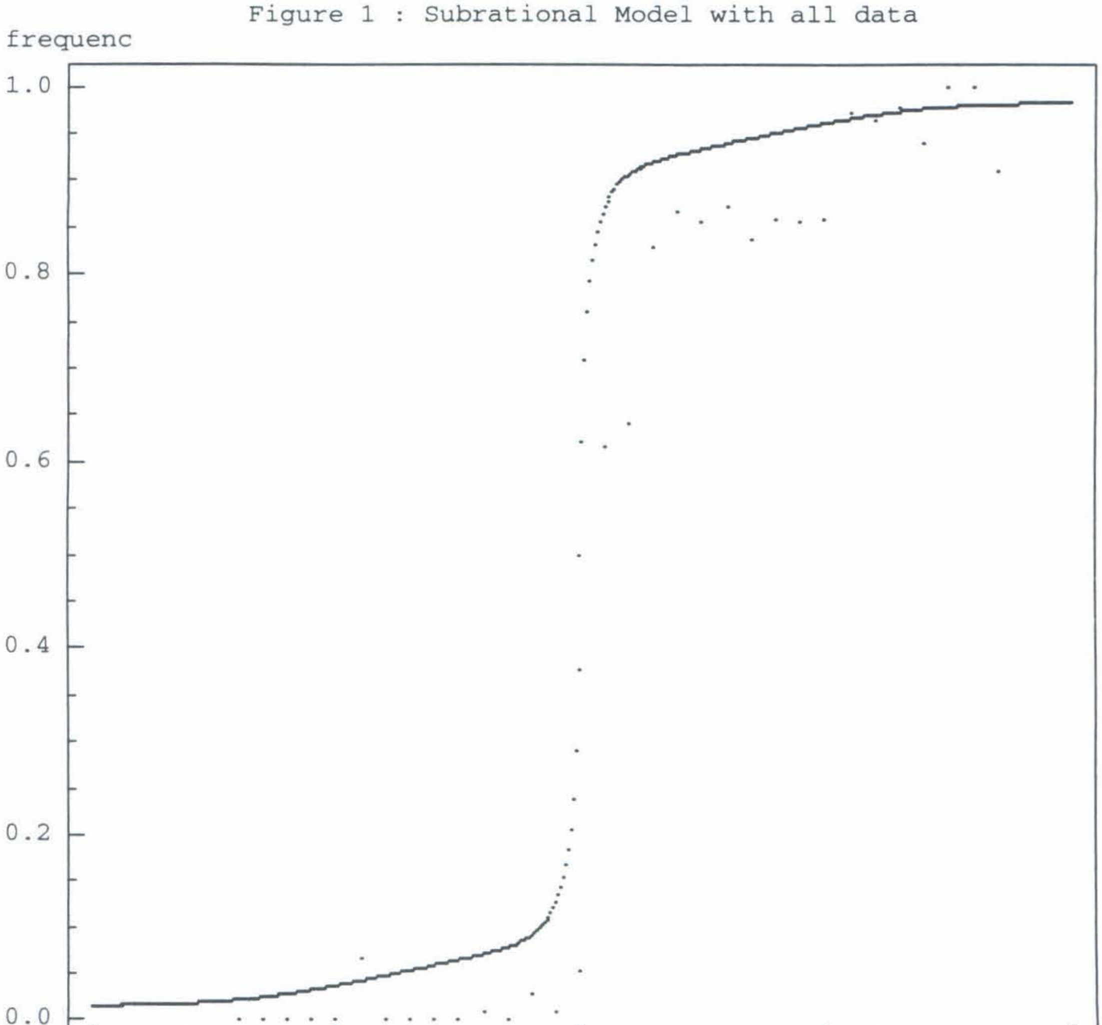

Figure 2 : Subrational Model with first 5 periods discarded

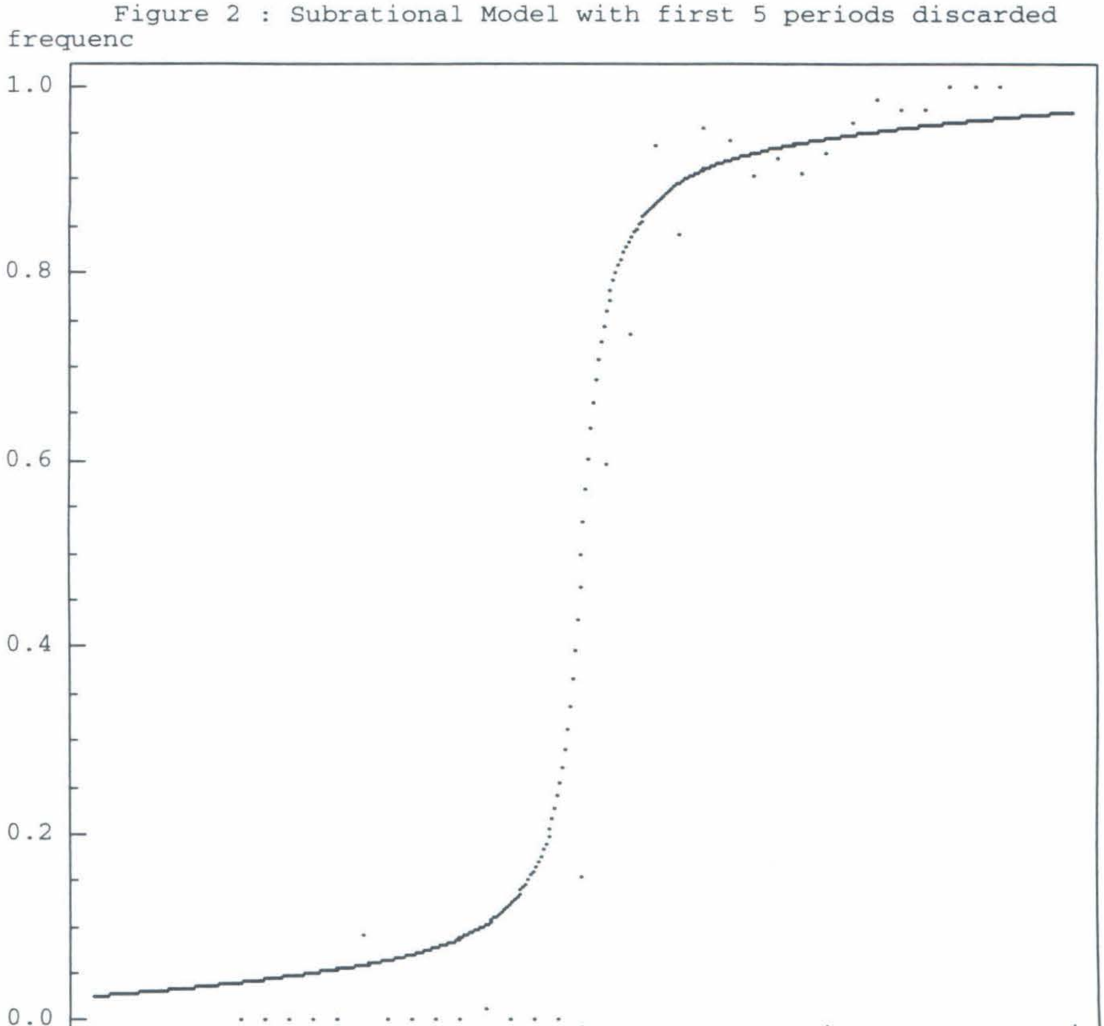

figure 3: Distribution of Errors (whole data set)

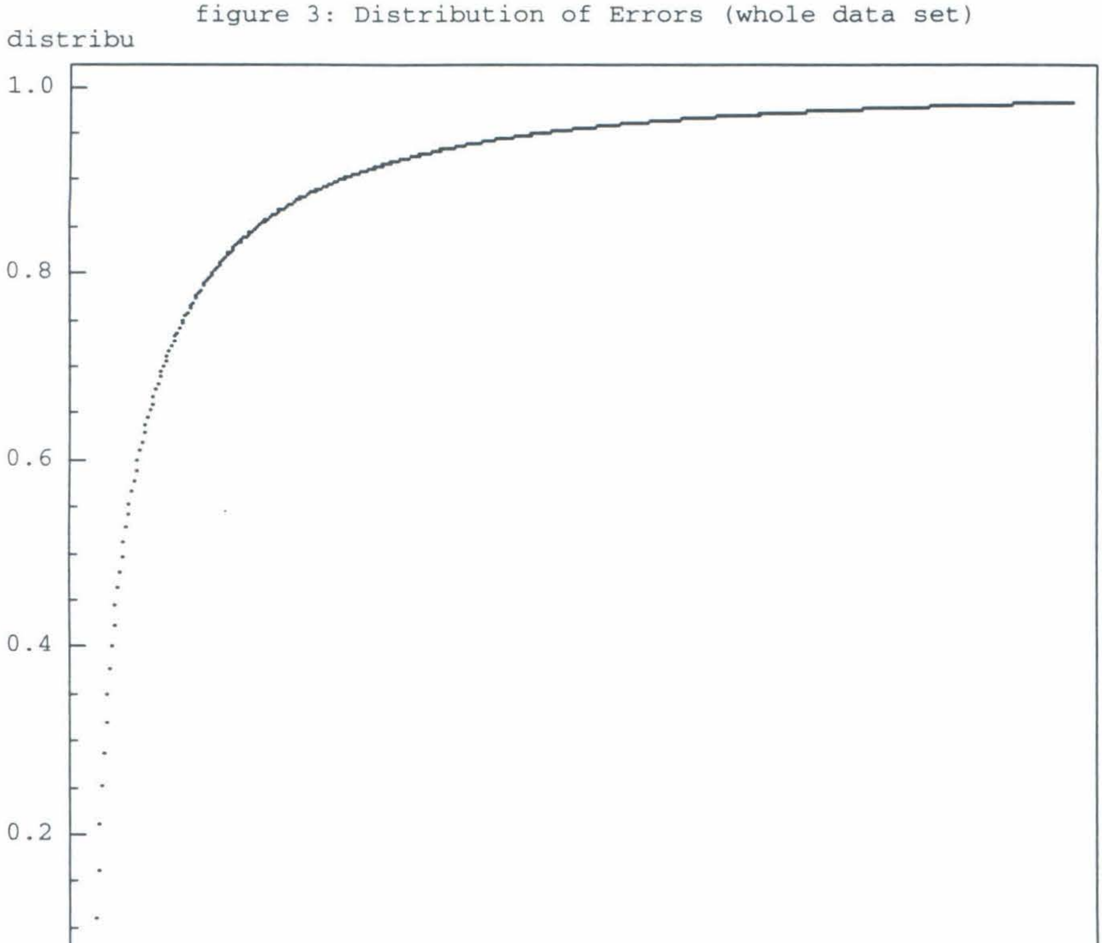

figure 4: Distribution of Errors (first 5 periods discarded)