EFFECT OF SINGLE AND PERIODIC DISTURBANCES ON INTERMITTENCY IN PIPE FLOW

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ABSTRACT

A set of experiments has been performed to investigate the behaviour of intermittency in pipe flow, particularly in response to periodical disturbances at fixed frequency (up to 360 cycles per minute) which were introduced in the flow.

The effects were studied at various values of Reynolds number in the transition range from Re = 2000 to 3000.

The main object was to relate the input frequency of the disturbance to the output frequency of the turbulent slugs emerging at the end of the pipe (Part III. C).

The propagation velocity of the interfaces separating regions of laminar and turbulent flow was studied by performing experiments with a single disturbance (Part III. B).

A single experiment has been performed also to investigate the frequency and the intermittency of the turbulent slugs of a flow with a fixed, geometrical perturbation at the inlet (Part III. A).

TABLE OF CONTENTS

TABLE OF CONTENTS (Cont'd)

LIST OF TABLES

LIST OF FIGURES

LIST OF FIGURES (Cont'd)

SYMBOLS

x	distance length along the pipe from inlet to exit
d	diameter of the pipe
L	length of the pipe
u	generical velocity
U or $\bar{u}$	mean velocity of the flow laminar condition
U_F	velocity of the front of the turbulent slug
U_R	velocity of the rear of the turbulent slug
T	temperature in °C
t	generical time or time elapsed from onset of disturbance
τ_L	interval of time of laminar flow
τ_T	interval of time of turbulent flow
$\tau_{L,T}$	interval of time of a laminar-turbulent couple
t_{tot} or t^*	total time of an experiment
ΔT_L	interval of time of laminar flow between two parts of a split slug
f or f_{in}	input frequency (driving frequency)
$\bar{f}$ or f_{out}	output frequency (response of the flow)
n	natural frequency of turbulent slugs
ν	kinematic viscosity
Re	Reynolds number
γ	intermittency factor
$1-\gamma$	fraction of laminar flow
ϵ	nondimensional slug frequency
U_F/U	nondimensional velocity of the front of turbulent slugs
U_R/U	nondimensional velocity of the rear of turbulent slugs

$\tau_L U/d$	nondimensional length of laminar region
$\tau_T U/d$	nondimensional length of turbulent region
$F(\tau U/d)$	normalized total distribution function $F = \dfrac{\Sigma_o^{\tau} \; m_\tau}{\Sigma_o^{\tau*} \; m_\tau}$
m_τ	number of intervals whose length is τ
$(\bar{\ })$	denotes mean value
$(\ )_T$	denotes turbulent
$(\ )_L$	denotes laminar

I. INTRODUCTION

A) Background

Bibliographic references are given at the end; here we can recall the main works from which the present has taken inspiration:

1) Coles (1961): Interface and intermittency in turbulent shear flow.

In the first part the author reviews the experiments on intermittency for pipe flow. He analyses the behaviour at high values of x/d and establishes the persistence of mixed laminar-turbulent regions at stations up to 10,000 diameters from the entrance. He also gives conclusions about the mean flow in the vicinity of a typical turbulent slug.

2) Rotta (1956): Experimenteller Beitrag zur Entstehung turbulenter Stromung im Rohr.

The author establishes the method for determining the intermittency factor and the frequency of the turbulent slugs on the basis of the difference in mean momentum between laminar and turbulent flow (the same procedure was used in our experiments; we have introduced only an electrical device to record the fluctuations of the jet).

3) Lindgren (1957): The transition process and other phenomena in viscous flow.

This can be considered the most comprehensive work on propagation and characteristics of turbulent slugs. The author focusses his attention on the evolution of a single turbulent slug observing it at successive stations relatively near to the entrance.

He determines the front and the rear velocity of turbulent slugs as functions of Reynolds number.

B) Present Work

In all the experiments of these authors, the transition was "natural," triggered by a fixed, geometrical disturbance.

The purpose of the present experiments was to study the response of the flow to other types of disturbance, in particular single pulses and periodic pulses. Apparatus to produce such pulses was constructed and various properties of the transition were measured, e.g., intermittency factor, frequency distribution and, in particular, the turbulent slug frequency resulting from given perturbation frequencies.

In addition, an experiment with natural transition was performed to serve as a reference for the present series.

C) Some Basic Principles

The phenomenon of intermittency occurs in almost all types of turbulent flow: near the boundaries of free shear flows (wakes, jets, mixing layer) and also in bounded flows such as Couette and pipe flows. As pointed out by Coles the research can easily be started by studying pipe flow and using water as a fluid.

Transition in pipe flow is associated with intermittent turbulence. At the lower (Reynolds number) end of the transition range, in the presence of predominantly laminar flow, slugs of turbulence occur. With increasing Reynolds number, the number of turbulent slugs increases. These persist along the pipe in a statistically stationary equilibrium, if the distance downstream from the pipe entrance is sufficiently great.

The relative degree of turbulence is related to the number and the size of those slugs; a measure of the relative degree of the transition is given by the intermittency factor, $\gamma = \frac{\text{time turbulent}}{\text{total time}}$.

The parameter that influences the transition is the Reynolds number defined as

$$Re = \frac{\bar{u}d}{\nu} ,$$

where

d = diameter of the pipe

ν = kinematic viscosity of the fluid (monotonically decreasing function of the temperature alone).

$\bar{u}$ = mean flow velocity.

This number is the only possible nondimensional quantity characterizing homogeneous, incompressible viscous flow, provided x/d is great enough for conditions to have reached a *stationary state*.

The mean flow velocity, $\bar{u}$, was determined by measuring volumetric flow rate (by collecting the flow in calibrated containers over measured time intervals).

The turbulent and laminar slugs issuing from the pipe exit have different free-fall trajectories, due to their different momenta. This may be illustrated as follows.

The laminar flow velocity profile is parabolic (Hagen-Poiseuille flow) $u/u_{max} = (1 - (r/R)^2)$. From the equation of continuity, $\bar{u} = \frac{u_{max}}{2}$ so that $u/\bar{u} = 2(1 - (r/R)^2)$.

Generalizing the Blasius one-seventh-power velocity distribution law for turbulent flow:

$$u/u_{max} = (1 - r/R)^{1/n}$$

From the equation of continuity $\bar{u} = u_{max} \frac{2n^2}{(n+1)(2n+1)}$ so that

$$\frac{u}{\bar{u}} = \frac{(2n+1)(n+1)}{2n^2} (1 - r/R)^{1/n}$$

We consider now the momentum:

$$I = \int_0^R u^2 \cdot 2\pi r \cdot dr = 2\pi \int_0^R u^2 \, rdr = \overline{u^2} A = \beta \bar{u}^2 A$$

where

I = momentum of the flow

$\beta = \overline{u^2}/\bar{u}^2$

may be calculated from the distribution law for the above velocity profiles.

We obtain, for laminar flow, $\beta = 4/3 = 1.333$

and for turbulent flow, $\beta = \frac{(n+1)^2(2n+1)^2}{2n^2(n+2)(2n+2)}$;

for $n = 7$ $\beta = 1.02$

If the mass flow rate is held fixed, $\dot{m}$ = const, then being $\mathring{Q}$ = const, since ρ = const, we obtain $\bar{u}$ = const, and we can compare the momentum of the jet when it is turbulent and when it is laminar, comparing the values of β. We conclude that the momentum is greater for laminar flow than it is for turbulent.

Due to this difference in mean momentum the trajectory of the jet varies between two extremal positions. When the flow is laminar the jet reaches further from the pipe exit than it does when the flow is turbulent.

This furnishes a possibility for collecting the laminar flow separately from the turbulent.

On this is based Rotta's procedure, which we followed, for separating the laminar and turbulent flow. (Ref. 10). The intermittency factor can be determined from the ratio of turbulent volume to total volume. The mean velocity is determined from the total volume flow collected from both sides of the collector.

It was found in our experiments that the mean velocities of the turbulent and laminar portions of the flow were not exactly the same, that of the laminar portion being slightly higher.

This was determined from the separate collections of the two kinds of flow, taking into account the intermittency factor, γ. At the lower Reynolds number the difference was very small, and increased to 4 or 5% at the higher end of the transition range. (See Tables 2, 3, 4.)

II. EXPERIMENTAL TECHNIQUE

A) Experimental Equipment

1) Pipe

In order to obtain a very large value of x/d (x = length of the pipe, d = diameter of the pipe) within the limitation of the length of the laboratory itself, the pipe was chosen to be of small diameter. Brass tubing of 3/8 inch outer diameter was used; the internal diameter was 0.182 inch (d = 0.462 cm). Each section of the tube was 12 feet long. Six sections (the last one was shorter than the others) plus an additional piece in the vicinity of the apparatus for the perturbation, gave a total length of 851 in. (L = 2160 cm).

Thus x/d = 4650 for the experiments on natural transition; this value is reduced to x/d = 3840 for the experiments with perturbation due to the shorter length between the perturbation station and the exit.

The 3/8 in. brass tubing was joined by drilling out 1/4 in. standard compression unions to 3/8 in. I.D., and using O rings as seals for the ends. The ends of each section were turned on a lathe to assure an exact fit and this, combined with the bored out unions, gave a smooth joint. The tubing was hung from the ceiling of the laboratory, by means of bolts six in. long mounted on a concrete wall. It was aligned with a transit and held straight by means of a slotted angle iron. To reduce vibrations transmitted from the building to the pipe, the space between the pipe itself and the support was filled with sand. The end of the pipe was chamfered to a sharp edge to prevent dripping, especially at low mass flow rate.

The upstream end of the pipe was connected, by means of a flange, to a stilling section of 1.5 inch diameter (see fig. 1a). The inlet to the pipe was rounded and smooth. The upstream end of the stilling section was connected to a valve for the regulation of the mass flow rate. A screen to reduce the entrance turbulence was located downstream of the valve; a thermometer was also located upstream of the inlet.

During the experiments with a fixed geometric disturbance, a plug of diameter d = 0.4 cm. was introduced in the pipe inlet; the free area was thus reduced to $A_{free} = 0.041\ cm^2$. (25% of the free area).

2) Feed System

One-inch diameter rubber hose connected the valve to a small reservoir at the top of a tower seven meters high arranged to provide a sufficiently high, constant hydraulic head. The water used was common city water whose characteristics are considered constant and sufficiently near "standard."

3) Mechanical Apparatus for Periodic Perturbations

At station P (154 in. downstream of the inlet) two holes were drilled in the pipe to connect the flow to an accumulator which was formed by placing a cylindrical box around the pipe (fig. 1b). Pressure perturbations could be applied to the accumulator (and thus to the pipe flow) by means of a piece of rubber tubing, connected to the bottom, which could be squeezed by means of a hammer mechanism. An air bleed valve was provided at the top of the accumulator.

The hammer mechanism (fig. 1c) consisted of a brass piece attached to the armature of a solenoid; it squeezed the rubber hose when the solenoid was energized. A spring brought it back to the original position. A stop with an adjustable position was located above the armature, and the support of the rubber hose also had an adjustable position.

This mechanism was mounted separate from the pipe to avoid vibrations. The solenoid was driven at different frequencies by a pulse amplifier which could operate properly up to 300 cycles per minute. At higher frequencies the armature could not move properly due to inertia. The hammer was brought closer to the rubber tubing but nothing definitive can be said as to whether or not a perturbation was induced in the flow and not absorbed by the rubber itself. Also, beyond 400 cycles per minute the frequency was not stable.

Thus the useful frequency range was at most 350 cycles per minute.

4) Electrical Apparatus for Periodic Perturbations. (Pulse Amplifier).

An electrical pulse amplifier was built especially for this research. This provided 32 volt pulses of constant, stable frequency, for driving the solenoid.

Pulse width was 0.1 seconds; but at high frequency this was reduced to 0.06 seconds. A circuit was also built to supply single pulses for the single-pulse experiments.

5) Apparatus for the Response of the Flow

As explained before, all the measurements are based on the varying position of the jet between the two extremal positions for laminar and turbulent flow.

Collector. To collect the fluid issuing from the end of the pipe, while keeping separate the laminar and turbulent flows, a simple rectangular plastic box was built (15 cm. high, 50 cm. long and 8 cm. wide). It was divided into four compartments by two knife-edge partitions. (The two compartments in front were used to collect laminar flow, the other two to collect turbulent flow.) The box was mounted on rails to allow movement along and transverse to the axis of the pipe.

Thus, runs of long duration could be made by collecting in alternate pairs of compartments; collecting in one pair while draining from the other into calibrated measuring containers where volume was determined. The marks on the glass containers were spaced at 20 cc. intervals and the total capacity of each was 2000 cc. A thermometer was located in the box to observe the final temperature of the fluid.

Grid. An electrical device was constructed for detecting and recording the position of the jet (in order to have a faster responding device than the collector previously described). This consisted of a grid of stainless steel wires spaced at 2 mm.; resistors were connected to the screws at the end of each wire, by means of which the tension of the wires could be regulated.

By placing this electrical grid horizontally in a plane penetrated by the jet, it could be precisely determined when the flow left the laminar position, and the amplitude and duration of each turbulent slug could be measured. The displacement of the jet increases with Reynolds numbers so two grids were built, the first with 17 wires for a distance of 2 cm. and the second with 5 wires for a distance of 1 cm.

Grid Circuit. As indicated on the circuit (fig. 2) diagram, the grid acts as a potentiometer (variable resistance) connected in parallel to the resistance provided by the water. This latter resistance is very large so it was necessary to place large resistors between the grid wires; 10 K Ω in one case (for a total of 170 K Ω) and 47 K Ω in the other case (for a total of 188 K Ω) in order to be able to detect the difference in potential due to the jump of the jet from one wire to the next.

The circuit was energized by a battery of 45 volts; the signal could be observed either on an oscilloscope or recorded on a visicorder. A calibration of the circuit was made to relate each position of the wire-water connection to a corresponding position of the signal beam on the recorder paper.

The connection to the water was completed either through the pipe or through a copper plate under the grid (in order to reduce the length of water involved in the circuit, and thus the resistance).

An electronic timer was used to provide a reference time scale on the recorder paper. It marked intervals of one second.

Counter. An EPUT/Timer was connected to the pulse amplifier to measure the frequency or the period of the input disturbance. The visicorder also was connected with the pulse-amplifier, so that the input frequency was also recorded on the paper.

B) Experimental Procedure

Before each experiment it was always necessary to let the water run for half an hour to reduce the difference between the inlet and outlet temperature (especially during the daytime). The mass flow rate was regulated with the valve to get the required value of Reynolds number. All the runs of the experiments lasted two minutes each and were repeated a second time, after measurement of mass flow rate.

1) Simple-pulse Experiment

Ten runs at increasing Reynolds number were performed; a minimum of five readings of volumes of laminar flow were taken, for each run; the value recorded on the tables is the averaged one. For every run the values of the inlet and outlet temperatures were recorded. When the flow was laminar (at the higher Reynolds numbers, occasional natural bursts of turbulence occurred), a single perturbation was introduced, and the time of travel of the resulting turbulence slug, between the perturbation station and the exit was measured either by means of a stop-watch or from the visicorder record.

2) Periodic Perturbation Experiments

A minimum of ten runs for each experiment was performed; and a minimum of five volume flow readings for non-perturbed and perturbed flow were taken in each run.

3) Natural Transition Experiment

With the fixed, geometric disturbance positioned at the inlet, runs were made at different Reynolds numbers. At each Reynolds number, flow rates were measured with and without grid in place. The grid output was recorded.

C) Sources of Errors

The temperatures were measured with thermometers having a range from 15 to 35°C and an accuracy of 1/10 of a degree. During the runs no variations greater than 2 to 3 tenths of a degree were noticed. The difference between inlet and outlet temperatures was at most 2°C, in only a few cases.

The average temperature was computed, on the basis of which the kinematic viscosity and Reynolds number were calculated.

In the range of temperature in which the experiments were performed a difference of one degree corresponds to an error of 3% in the determination of Reynolds number. Another source of error is in the determination of volumes; a precision of 10 cc. was possible and errors of 1% in Reynolds number could be introduced in this way. The total error in the determination of Reynolds number is within 5%.

Other sources of errors can be found in the following: The value of input frequency recorded on the visicorder paper is in good agreement (1-3%) with the average computed on the basis of the counter. This variation, even though not large, can introduce a shifting of the f_{in} value that can have a noticeable effect on the plotting of the response frequency, especially at high frequency. The output frequency was obtained from measurements on the visicorder record. Two types of error in this measurement are difficult to evaluate. Very small movements of the laminar jet might give connections to the first grid wire, which would be interpreted on the output as turbulent flow. Care was taken in positioning the grid to avoid this. Another possible source of error was that, at the higher Reynolds numbers, occasionally natural bursts of turbulence occurred. As a result, the apparent response frequency would be in error (higher).

In this connection, we may note a phenomenon which occurred during the experiments; for a long period of time (about 30 minutes) after starting the experiment, the flow came back to laminar when the perturbation was stopped. If the experiment was performed for a longer time, we noticed a tendency of the flow to persist turbulent even when not perturbed. To avoid this, we allowed conditions to "settle" by closing down the flow for about a minute. The reason for this behaviour is not understood; something like accumulation of air near the inlet might be an explanation.

In the measurement of laminar and turbulent intervals, τ_L and τ_T, on the visicorder record, one second corresponded to 10 millimeters of trace. The resulting error in $\Sigma\tau_L$ or $\Sigma\tau_T$ was 2 to 4%.

III. RESULTS AND DISCUSSION

A. Natural Transition; Flow with Disturbed Entry Condition

1) Introductory Remarks

Here again we repeat that intermittency is the main feature of transition from laminar to turbulent flow. Alternating slugs of laminar and turbulent fluid move along the pipe and persist even if the pipe is very long.

Previous experiments (ref. 1) performed at values of x/d as large as 10,000 have confirmed the existence of a statistically stationary mixed laminar-turbulent regime.

In order to study natural transition, a cylindrical plug was positioned in the inlet and centered coaxially with the pipe. The free area was thus reduced from $A = 0.167 \text{ cm.}^2$ to $A' = 0.126 \text{ cm.}^2$ (about 25%).

The effect of this fixed, geometric disturbance was to produce fully turbulent flow near the pipe entrance. Natural processes (which are not understood) then led to damping of the turbulence in certain regions, so that the flow far downstream was only intermittently turbulent.

Our measurements were made with a pipe of x/d = 4650. We have performed a set of experiments at various values of Re in the transition range to determine the average natural frequency of the turbulence $\bar{n}$ and the natural intermittency factor $\bar{\gamma} = \int_0^{t^*} \gamma dt / \int_0^{t^*} dt$.

2) Intermittency Factor and Natural Frequency

The intermittency factor $\bar{\gamma}$ is an average value of an on-off intermittency function γ, with $\gamma = 0$ for laminar flow and $\gamma = 1$ for turbulent flow; strictly speaking, and assuming that it is possible to discriminate between laminar and turbulent flow, the intermittency function γ depends on x, r, t, x/d and Re.

In all our experiments the pipe length and diameter were constant, and we averaged the value of γ in some sense with respect to radius, getting $\overline{\gamma} = \overline{\gamma}$ (t,Re); then we averaged with respect to time both the intermittency factor and the frequency getting $\overline{\gamma} = \overline{\gamma}(Re)$ and $\overline{n} = n(Re)$.

The latter quantity is made nondimensional by dividing by U/d, so that we will speak of a nondimensional slug frequency: $\overline{\epsilon} = \overline{n}d/U$.

The parameter with respect to which we study the variation of $\overline{\gamma}$ and $\overline{\epsilon}$ is the Reynolds number $Re = U.d/\nu$.

For Re < 2000 (approximately), the turbulent regions decay and disappear downstream of the inlet where our disturbance is positioned. No turbulent slugs have any chance to survive as far as the end of the pipe.

This value of Re = 2000 is in good agreement with the lower limit obtained by many other observers. It is also in good agreement with the values obtained in the present experiments by perturbing the flow with a single disturbance or with a periodic disturbance, as will become evident in later sections.

For Re > 2800 (approximately), laminar regions never emerge spontaneously from the original turbulent flow near the entrance, or at any rate do not remain laminar at large values of x/d. Far downstream, the pipe is completely filled by turbulent flow for values of

Re higher than this upper limit.

The range between these two extreme values of Re is the transition range ($2000 < Re < 2800$), and a regime of stationary mixed laminar-turbulent flow can exist in this range.*

For the most part, our experiments with natural transition do not break any new ground. Our purpose in these experiments was to show, by verifying the data of other observers, that our new measurements with controlled disturbances were carried out under essentially normal conditions for pipe flow. We also wanted to prove the instrumentation as well as to allow a direct comparison of the data obtained with natural and with artificial transition.

We have taken as the most important properties of the signal obtained from the electrical turbulence discriminator the relative duration and the absolute frequency of the alternations between laminar and turbulent flow.

The results obtained are given in table 1 and for the quantities $\overline{\gamma}$ and $\overline{\epsilon}$ are shown in figs. 3 and 4. Particular care was taken in the vicinity of the extrema of the transition range.

The intermittency curve in fig. 3 is in good agreement with the data collected by Coles (ref. 1), taking into account his remark that the viscosity (and hence the Reynolds number) was not accurately measured in some of the earlier unpublished experiments.

It is possible that the statistical properties of the flow

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* The upper limit may be a weak function of the parameter x/d, as will be discussed further.

still depend slightly on the nature of the initial disturbance and on x/d, even for x/d as large as 5000. A better test of such a dependence, if any, is provided by fig. 4, which shows the average dimensionless frequency $\overline{\epsilon}$ for natural transition.

To the extent that the velocity of the turbulent slugs in this Reynolds number range is nearly the mean fluid velocity U, (see fig. 13), the reciprocal quantity $1/\overline{\epsilon}$ is nearly the length of pipe (measured in diameters) occupied on the average by a laminar and a turbulent region together. This length increases, and $\overline{\epsilon}$ therefore decreases, towards the boundaries of the transition region, where either the laminar or the turbulent regions become relatively longer.

Our experiments at $x/d = 4650$ show a maximum in $\overline{\epsilon} = \overline{n}\, d/U$ of about 0.030 at Re = 2460. Previous experiments at lower values of x/d show some differences. At the lower Reynolds numbers, for example, the population of turbulent slugs may increase with increasing distance down the pipe, due to splitting of individual slugs, and at higher Reynolds numbers the population may decrease because of consolidation of adjacent slugs. (In these remarks we anticipate our data obtained with controlled disturbances, in order to explain the tendency for the maximum in $\overline{\epsilon}$ to shift to lower Reynolds numbers. (See Coles fig. 3, ref. 1) with increasing x/d.) Our experiments do not show that these processes are complete, but they must be nearly so. We also have the feeling that the inlet disturbance level was not very high in these experiments especially at the lower Reynolds numbers.

3) Normalized Distribution Function

From the records taken during the experiments we could determine the statistical distribution function both for the laminar intervals τ_L and the turbulent intervals τ_T. At low Reynolds numbers the turbulent intervals are very short and almost uniform (0.1 to 0.3 sec.); for Re > 2500 they rapidly increase in length as shown in fig. 6. The laminar intervals behave in the opposite way: At low Reynolds numbers they are very long (8 to 20 sec.); they decrease as Re increases, and for Re > 2500 become short and almost uniform (0.1 to 0.4 sec.), as shown in fig. 5.

At a given Reynolds number these figures show the relative fraction of laminar or turbulent intervals having a value of τ less than a given value, that is, having a length less than a given length; they are made nondimensional by multiplying by U/d.

4) Length of Laminar and Turbulent Intervals

For the particular values 0.1, 0.5, 0.9, of the normalized distribution functions of figs. 5 and 6 (i.e., for the 10^{th}, 50^{th}, and 90^{th} percentile), the relevant dimensionless length $\tau U/d$ is plotted against Re in fig. 7 for the laminar and turbulent intervals.

Also shown are the mean curves obtained from the intermittency and frequency data, using the definitions $1/\overline{n} = \overline{\tau}_L + \overline{\tau}_T = \overline{\tau}_T/\overline{\gamma}$ so that $\overline{\tau}_T\ U/d = \overline{\gamma}/\overline{\epsilon}$ and $\overline{\tau}_L\ U/d = (1-\overline{\gamma})/\overline{\epsilon}$.

We conclude from these data that in the lower transition range the turbulent intervals tend to have a uniform length of about 15 diameters, while the length of the laminar intervals is almost uniformly distributed over a wide range (except that very short values do not occur). In the upper transition region the picture is reversed; the length of the laminar intervals tends to be about 10 diameters while the much longer turbulent intervals are almost uniformly distributed (except that very short values do not occur and there is some increase of population density near 10-15 diameters).

As might be expected, the laminar and turbulent intervals are equal on the average and both have a value of about 20 diameters, at the particular Reynolds number 2460 where the dimensionless frequency $\overline{\epsilon}$ has its maximum value (see fig. 8).

Also the percentile values for turbulent and laminar intervals are equal at Reynolds number 2460 (fig. 7).

B. Response of Laminar Flow to a Single Disturbance

1) Introductory Remarks

As previously indicated in the general description of the experimental procedure, we performed a set of experiments in each of which the flow was given a single perturbation after it had exhibited a laminar behaviour for a long period of time. In general, the response to this single perturbation was that the jet at the pipe exit performed one or more oscillations and moved from the laminar to the turbulent position; after the turbulent slug or train of slugs had passed, the flow again took up the original laminar position.* This experiment differs

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* In many experiments the real slug, having a mean velocity consistent with the average velocity of the flow, was preceded by an extraneous slug or group of slugs. We cannot account for this phenomenon but we point out the numerical results later, with an attempted explanation of which is the most obscure part of the phenomena observed.

from the previous one in that our object is to study the dynamics of turbulent regions embedded in a laminar flow at a particular Reynolds number rather than the statistical properties of a mixed flow in equilibrium. Our assumption is that laminar flows in the transition range are unstable to finite disturbances, and our hope is that the response of such a flow to a single strong disturbance is a well-posed non-linear experimental problem for which reproducible and meaningful data can be obtained.

2) Front and Rear Velocity of Turbulent Slugs

By the very nature of the problem, the flow must now depend critically on time or on distance along the pipe. It is known (ref. 5 and ref. 1) that a turbulent region in such a flow grows in length at an almost constant rate for Reynolds numbers greater than about 2500. Time and distance, however, can only be called large or small in terms of the characteristic time $1/\overline{n}$ and the characteristic distance U/n defined by the statistical measurements of the preceding section.

For our measurements the pipe length was constant with x/d = 3850 where x is now the distance between the point of disturbance and the end of the pipe. The corresponding values of $\overline{n}x/U$, or the pipe length measured in statistical units, ranged from about 120 at Re = 2460, near the center of the transition region, to about 10 at Re = 2100 and Re = 2750 near the extrema. These values can give an idea of the relative length of the pipe at various experiments with different Reynolds numbers.

The quantity measured in the experiments was the elapsed times, t, from the moment of the disturbance to the occurrence of various changes in the flow at the pipe exit.

We note that the velocity x/t, formed from the pipe length and elapsed time, can be interpreted as an average phase velocity for the disturbance and made nondimensional with the main flow velocity U (see fig. 13; discussion will follow later on). However, we emphasize that the flow is not strictly conical* since the phase velocity is not constant for small times and since unobservable events known to occur inside the pipe, such as splitting, can affect the phase velocity locally. Consequently the plotted data will be left in the form Ut/d (see fig. 9).

Many experiments were performed to establish the lower limit of the transition, that is, the lowest Reynolds number at which turbulence can survive and emerge at the end of the pipe. This value has been found to be Re = 2000; turbulence at the pipe exit was never observed in a hundred attempts.

At larger Reynolds numbers, from 2000 to about 2500, the predominant phenomenon is splitting of the turbulent slugs produced by the disturbance. We can say that at low Reynolds number slugs increase to a given length and split, repeating this process. In other words the front and rear of a slug are not independent but interact in some unknown way. For splitting to occur, a slug has to be of sufficient length, which may depend on Reynolds number or possibly $\bar{n}x/U$. Thus, as the Reynolds number increases, the increase in the amount of turbulent flow is provided mainly by splitting rather than by increase in length of turbulent regions (see fig. 10). Some

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* i.e. flow properties dependent on x/t.

unknown mechanism prevents slug length from increasing. Since our experiments were carried out at fixed x/d, it is not clear whether the increase of splitting with Reynolds number is connected more precisely with the increase of $\bar{n}x/U$.

At higher Reynolds numbers, above about Re = 2550, splitting usually does not occur, and a single slug usually grows from a single disturbance. The front and the rear of the turbulent slug appear to be independent, and can be separated by distances that are related to the Reynolds numbers. Turbulence can be increased (as Reynolds number increases) by growing in length of a single turbulent slug. These ranges overlap, in fact growing is still present for Re = 2300 but it is damped by splitting that ceases around Re = 2550.

The ideas outlined above are based on observation and comparison of grid outputs of individual runs, together with a study of the statistical results, for example, those shown in fig. 13, where the nondimensional values of the front and rear velocity of turbulent slugs are plotted versus Reynolds number. (The Lindgren curve (ref. 5) is also shown for comparison.)

General features of fig. 13 are as follows. The branch U_F/U decreases to a minimum, then increases, changing curvature at about Re = 2580, then seems to tend to a definite limit (stabilization of turbulence growing). The other branch, U_R/U, after the point of minimum at about 2300 is divided into the following sub-branches:

Branch (a) is the locus of the rear velocity of the last slug of the group when splitting is present. This is joined to branch (d) (at Re = 2580 where the splitting range ends), which is the locus of the rear velocity of the slug when only growing is present.

Branch (b) is the locus of the rear velocity when the slug is single. This curve follows the increase of U_F/U after the minimum, up to Re = 2400, then decreases and continuously joins to curve (d) at Re = 2580.

Branch (c) is the locus of the rear velocity of the first slug of the group; for Re < 2400 it is almost coincident with (b), while for Re > 2400, instead of decreasing, it is continuously increasing up to Re = 2500; from this point, it is supposed that it joins branch (d) at Re = 2580 to form a single curve.

In fig. 14 we have plotted values of U_R/U_F versus Reynolds number (ref. 12) and also the values of U_F-U_R/U versus Reynolds number. We notice that, for values greater than about Re = 2600, U_R/U_F is linearly decreasing while the difference U_F-U_R/U seems to tend to a limiting value denoting a probable stabilization of growing at higher Reynolds numbers.

3) The Phenomenon of Splitting

We have already noted that splitting is present in the range 2100 < Re < 2500. The number of divisions of a single slug arising from one perturbation tend to increase as Reynolds number increases (see fig. 10). This may be because effective length of the pipe, $\overline{n}x/U$, is increasing in this range.

It was noted, when more than one slug travel together in a train separated by laminar regions, that the first slug has a higher velocity than in the case of no splitting; fig. 10 shows the effect of splitting on the front velocity of the first slug (data are expressed in the form d/Ut).

We have measured the interval of time ΔT_L during which the flow is laminar between two turbulent slugs when splitting phenomenon is present; the data are scattered, and in fig. 11 we have plotted the average results in the form $\Delta T_L \cdot U/d$ versus Re. We note a general decrease of ΔT_L as Reynolds number increases and as the number of slugs of the same group increases. All the data converge to a value of about 0.1 sec. for Re = 2500. This seems to be the shortest laminar interval possible between two turbulent slugs; the minimum possible length seems to be 10.5 diameters for Re = 2500.

In fig. 12 are plotted the mean values of the turbulent regions expressed in the form $U\Delta T_T/d$. The effect of growing that starts at about Re = 2300 and becomes predominant only after Re = 2550 is evident. We have also included the values from fig. 8 to show the big difference that exists in the mean values of the length of turbulent regions between experiments on natural transition and the present one on single disturbance. It appears that the slugs are bigger when they are embedded in a laminar flow. However, it is possible that the different kinds and magnitudes of disturbance in the two sets of experiments can have an effect on the length of turbulent slugs.

We have mentioned before the presence of a disturbance ahead of the group of slugs that constitute the true response to the perturbation. This phenomenon was not always present, but when it happened it was always at a precise time interval after the beginning of the disturbance, so that the scatter of data from the mean values reported in fig. 15 is very low. We are not able to give a satisfactory explanation of this phenomenon and it is also difficult to make conclusions about the influence it has on the turbulent slugs which follow. The velocity computed on the basis of the full length, x = 3850 diameters, is roughly twice that of the real slugs. However, we suspect that these extraneous slugs are produced somewhere along the pipe by a pressure wave which passes along the pipe; they then travel with the same velocity as the real slugs. We did not include them in our calculations of splitting and in determining the values of U_F/U and U_R/U.

C. Response of Laminar Flow to a Periodic Disturbance

1) Introductory Remarks

Previously we have pointed out that the real purpose of the present experiments is to attempt to control the intermittency phenomenon in pipe flow by means of a periodic disturbance (with given driving frequency).

We have already described the method of perturbation that permits a wide range of frequencies of the disturbance (input frequency) and the method for detecting the response of the disturbed flow (output frequency); we have also pointed out the limitation of these methods.

Our attention now is focussed on a narrow range of Reynolds numbers, so we have performed five experiments at the Reynolds numbers 2100, 2300, 2500, 2700, and 3000. The experiments were performed in an arbitrary order. For each experiment about fifteen runs were made and repeated twice, with a lag of time, in the same day. For the method and for the operations of taking results see page 11.

Our experiments were performed at input frequencies from about ten to about 350 cycles per minute. Only very few runs were made at higher frequencies than this, chiefly because of problems arising in the generation of such disturbances as discussed previously. It should perhaps be remarked that at the higher frequencies (e.g. up to 600) and Re up to 2300 it was not possible to produce fully turbulent flow, and sometimes even completely laminar flow was obtained. This could be due to a stabilization of the flow connected with the periodic disturbance, but on the other hand it is not certain that the flow was perturbed at those frequencies due to the mechanical limitations. For this reason, our attention was focussed on the range of low frequencies (up to 350 cycles per minute).

Two sets of experiments for each Reynolds number were performed, in an attempt to cover, with the second series, the points of major interest which appeared during the first series. We used this method also to check the reproducibility of the results. Almost always the points obtained were in surprisingly good agreement with the previous one (especially at Re = 2300). The scatter of the results estimated by plotting the values recorded in two runs of two minutes each is shown on the diagrams. Each experiment itself has been divided in two parts and separate readings taken in order to check the constancy of the output frequency during one run. The results are collected together with other data in tables 2, 3, 4.

In most of the experiments the scatter is very low; in all cases the values taken from the two parts of a single run are very close. Only in a few cases for the experiment at Re = 2500 do the values from the two different experiments performed at the same input frequency differ to such an extent that it is impossible to ascribe the difference to error or to fluctuation of the results. In such circumstances there exist a low and a high value; the low value is usually in good agreement with the general behaviour of the curve and it is considered the correct one, while the high value is due in our opinion to superposition of natural disturbances.

In figs. 16a,b,c, each at one fixed Reynolds number, the dimensionless output frequency $\bar{f}d/U$, based on the mean values of the output frequency in cycles per second, is plotted versus the dimensionless input frequency fd/U. We have preferred to represent the plot of the results as straight segments with discontinuities in slope, rather than to attempt to work out a continuous curve; this is suggested by the frequent almost perfect alignment of three or four results. In addition the resulting diagrams are all collected in one for better reference and easier comparison, see fig. 17. In fig. 18 we have plotted the values of $\epsilon_{out}/\epsilon_{in}$; (ratio of output frequency to input frequency) versus input frequency. All the curves show a similar behaviour; at low input frequencies $\epsilon_{out}/\epsilon_{in} > 1$ population is increased over the number of disturbances; at high input frequencies $\epsilon_{out}/\epsilon_{in} < 1$ a damping effect reduces the population.

Out of the five experiments, the three at Reynolds numbers 2100, 2300, 2500 show a similar type of behaviour notwithstanding some differences that we will mention.

The other two experiments, at Reynolds numbers 2700 and 3000 are similar between themselves but show new phenomena that need to be discussed separately.

Due to these fundamental differences we can divide the transition range $2100 < Re < 3000$ into two parts: first transition $2100 < Re < 2500$ and second transition $2500 < Re < 3000$. In the following sections, various features of the phenomena in each of these ranges are pointed out, with some attempt, where possible, to explain them.

2) First Transition Range

We have already pointed out that our original belief (that at every Reynolds number there should be a disturbance frequency high enough to produce a fully developed turbulent flow) seems to be wrong, certainly in the range of low Reynolds numbers. For all input frequencies, most of the response frequencies are confined to a band that is narrower for lower Reynolds numbers.

On the plots is shown a fan of straight lines through the origin with slopes 1:1, 2:1, 3:1, etc., corresponding to response frequencies of 1, 2, 3, etc., times the input frequency; these are for reference, to indicate the average number of slugs at the exit for each input pulse.

We consider now the line 1:1, which corresponds to a response of one turbulent slug to each input disturbance; for this condition, either there are no splitting and no damping effects, or they balance each other. The records of runs for this condition showed almost perfect regularity of the response, in one-to-one correspondence with the input (but some imperfection in the phase).

Analysing the diagrams for nondimensional output frequency versus input frequency in the range of low input frequencies we see that for Re = 2500 splitting is quadruple, in agreement with the experiments for response to a single disturbance; for Re = 2300 splitting is double instead of the triple splitting observed for single pulses, and for Re = 2100 splitting is between double and triple, as compared to double splitting in the single-pulse experiments. This non-monotonic behaviour with Re seems puzzling.

From fig. 17 we note that all the response curves have a decreasing slope in the range of low input frequencies, leading to a condition of maximum; as frequency increases in this range, something counteracts splitting more and more effectively. Output frequencies are greater than input frequencies in the low frequency range; this indicates that there splitting is more effective than any growing or damping effect.

In addition we note that the intersections of all curves of response frequency with the line 1:1 are very close to each other ($\overline{\epsilon}$ = 0.019 to 0.0208). As input frequencies increase, splitting is more and more counter-balanced by damping and growing, both reducing the frequency of the response. In the range of intermediate and high input frequencies, we have an alternation of the two opposite effects, leading to sequences of maxima and minima in a narrow band of output frequencies. This band for Re = 2500, is roughly centered about the value of the natural frequency.

The curve for Re = 2500 has two features not present in the other two curves; there is an additional maximum in the low input frequency range and another maximum at the natural frequency for $\overline{\epsilon} = 0.025$ (approximately). The value of the response for this last point is nearly equal to the input, meaning that the response is one to one when input frequency is the same as the natural frequency. (If we do not include this point, we find a slope that aligns four points up to the real maximum that is in agreement with the maximum of the other curves in this region.)

The general behaviour of the resulting plots shows the presence of several maxima and minima in the region observed. The first maximum as already mentioned, falls always in the low frequency region.

If we consider the input frequencies at which the maxima and the minima occur, we note a general tendency for a shifting of these frequencies towards higher values as Reynolds numbers increase.

The minima tend to lie in a very narrow range of input frequency and have almost equal output frequency, while the maxima are more spread, both in input and output values. (Only for Re = 2500 do all the maxima have the same value of output frequency, nearly equal to the value of the natural frequency.) For the other two experiments there is a slight tendency for an increase of the value of output frequency of the maxima, as the input frequency increases.

We have been able to find no evident relationships between these minima and maxima to the natural frequency.

At Re = 2500, the band of output frequencies is slightly higher than at the two lower values. This is possibly a consequence of the fact that at Re = 2500 the splitting (in the single-pulse experiment) is of higher order (up to four or five). Also of interest is the fact that, on the basis of the measured growth of single slugs, one would expect the flow at Re = 2500 to be fully turbulent (above a certain frequency), whereas it still is made up of readily distinguishable slugs (the relative duration of laminar and turbulent intervals is discussed later). It may also be pointed out that at this Reynolds number the effective length of the pipe is a maximum.

3) Behaviour at High Input Frequencies

As mentioned on page 26, only a few experiments were performed at high input frequencies (above 350); all gave a resulting very low output frequency response.

For experiments at Re = 2100, the results of three runs gave:

input freq.	550	output 5	cycles per minute
input freq.	750	output 3	cycles per minute
input freq.	1000	output 0	cycles per minute

These values are averaged over ten minutes of experiment, not recorded; between output perturbations the jet was always very steady, clear and immobile.

For experiments at Re = 2300, the results of a run at an input frequency 580 gave an output frequency response of 35 cycles per minute. At higher frequencies a completely laminar, steady behaviour was observed.

For experiments at Re = 2500, the results of a run at decreasing input frequencies from 1000 to 600 cycles per minute always gave completely laminar flow. In the range 350-600 many runs were attempted without getting any reliable result.

Our personal impression is that a kind of stabilization of the flow by periodic disturbances seems possible, especially at low Reynolds numbers, while at high values nothing definitive can be stated.

4) Determination of Total Duration of Laminar and Turbulent Intervals

From the records of each run of the performed experiments, we could estimate the length of the interval of time during which the flow was laminar and of the interval of time during which it was turbulent. The data have been reduced to a standard experiment of one minute; minute-intervals within one run (given input frequency) are kept separate to get some idea of the scatter.

To give an idea of the relative degree of the transition and of the total duration of laminar intervals, we have plotted in figs. 19, 20 and 21 the quantity $(1-\overline{\gamma})$, percentage of laminar flow, versus the nondimensional input frequency. For studying these curves, the following observations may be useful. First, it is evident that we can expect a response with a high output frequency when the time intervals τ_L and τ_T are both small. Second, on the basis of the present experiments we can add the important result that for values of input frequency corresponding to maxima of output frequency we always have points that are close to the line $\overline{\gamma} = 0.5$ in spite of the wide variation of $\overline{\gamma}_n$ for natural transition at these Reynolds numbers (see fig. 3 for $\overline{\gamma}_n$ versus Re).

The behaviour of the diagrams obtained for the three experiments at Re = 2100, 2300, and 2500 should be studied keeping in mind the corresponding figures 16a,b,c, for output versus input frequencies.

At very low input frequencies all values of Re give a steep decrease of the curve $1-\gamma$, which indicates development of turbulent flow in accordance with the high splitting order present in this range. At higher input frequencies, when damping effects become effective (zone of decreasing slope in the curve output frequency versus input frequency) we have an increase of laminar regions, up to a maximum. These features are common to all three Reynolds numbers.

For Re = 2100 the values of $1-\overline{\gamma}$ are relatively high everywhere; the flow has a strong tendency to remain laminar. The turbulence increases only in the vicinity of the maxima of output frequency. Corresponding to the minima in output frequency we have maxima of the curve $1-\overline{\gamma}$, so that we can conclude that those minima are due to the presence of big laminar intervals. In regions where the output frequencies increase the values of $1-\overline{\gamma}$ decrease, while in regions where the output frequencies decrease the values of $1-\overline{\gamma}$ increase, exhibiting a "normal behaviour." The maxima of the percentage laminar flow $(1-\overline{\gamma})$ have values increasing with the input frequency and indicate a tendency to establishment of laminar flow (note the alignment of the values of the maxima themselves) at very high input frequencies.

For Re = 2500 the values of $1-\overline{\gamma}$ are almost everywhere low; the flow has a strong tendency toward turbulent behaviour. At this Reynolds number (upper limit of first transition range) we have a behaviour opposite to the "normal" one, observed at the lower limit (Re = 2100); in regions where the output frequencies increase, the values of $1-\overline{\gamma}$ are increasing and in regions where the output frequencies decrease, the values for $1-\overline{\gamma}$ are decreasing too. This is due to the fact that, at this Reynolds number, the tendency of the flow seems to be turbulent, and to permit an increase of output frequencies the laminar region must increase in length and number. At input frequencies corresponding to the maxima in output frequencies we always have values of $1-\overline{\gamma} = 0.5$, and these also correspond closely to the maxima of the curve itself. Corresponding to the minima in output frequencies we have minima of $1-\overline{\gamma}$ indicating the presence of big turbulent regions.

For Re = 2300, i.e. between the other two Reynolds numbers, the values of $1-\overline{\gamma}$ are more scattered; the flow seems to have no particular tendency favoring laminar or turbulent behaviour. As the input frequency increases we have a decreasing of $1-\overline{\gamma}$ down to an absolute minimum that corresponds to a minimum of output frequency. We can now say that this minimum of output frequency is due to the presence of big turbulent regions and we note that this behaviour is opposite to the one for low Reynolds numbers. With further increase of frequency, $1-\overline{\gamma}$ increases continuously and a maximum is reached at a frequency corresponding to the second minimum of the output frequency. We can now say that this minimum is due to the presence of big laminar regions, a behaviour similar to that at low Reynolds numbers. Finally we note the tendency, as frequency increases still further, to reach a condition of fully laminar flow, as also occurred for low Reynolds number. (We do not exclude the possibility of a third maximum corresponding to another increase of turbulence before we get the laminar condition as a limit.) We conclude that, at low input frequencies, the flow behaves as at high Reynolds numbers, while at high input frequencies the flow behaves as at low Reynolds numbers. In between there is a zone of transition with values of $1-\overline{\gamma}$ always increasing. Irrespective of the different behaviours pointed out all the maxima in output frequencies give values very close to $1-\overline{\gamma} = 0.5$ as already remarked.

On the diagrams we have also traced the lines $1-\overline{\gamma}_n$ and $\overline{\epsilon}_n$ relative to values for natural transition. At low Reynolds numbers there seems to be an increase in turbulence at a frequency corresponding to the natural frequency; this is more noticeable for Re = 2300 (additional minimum) than for Re = 2100 (simple inflection). At high Reynolds numbers we note a <u>decrease</u> in turbulence (opposite behaviour) at a frequency corresponding to the natural frequency. At low Reynolds numbers the value of $1-\overline{\gamma}_n$ for natural intermittency, seems to be a limiting value for high input frequencies.

At the high Reynolds number (2500) the curve of percentage laminar flow, $1-\overline{\gamma}$, seems to oscillate around the natural value, indicating a possible stabilization of the flow by disturbances. In connection with this we may note again that for high Reynolds numbers the phenomena are more likely to have reached completion, since the effective length of the pipe is large, whereas at low Reynolds numbers, when the pipe is relatively short, it may be that statistically stationary conditions were not yet attained. This might explain why values of $1-\overline{\gamma}$ at low Re do not oscillate around the natural value, especially at low frequencies, but show a tendency to reach natural values at high frequencies.

Instead of collecting in one figure the curves for the values of $1-\overline{\gamma}$ we have computed the curves for the intermittency factor $\overline{\gamma}$ and collected them (fig. 22) for easier comparison. Here we see clearly that the curve for Re = 2300, at low frequencies, follows the one of high Reynolds numbers, while at high frequencies it follows the one for low Reynolds numbers. At low input frequencies, the high and low Reynolds number curves have opposite behaviour; at the high Reynolds number the maxima of $\overline{\gamma}$ occur at values of fd/U to which correspond minima at low Reynolds number, and vice versa. At high input frequencies there is a tendency for the maxima and minima to be in accordance for all curves at the same values of fd/U, irrespective of the very different values for $\overline{n}d/U$ of natural transition.

5) Second Transition Range

In this range, 2500 < Re < 3000, there are only two experiments performed at Re = 2700 and Re = 3000.

As the Reynolds number increases beyond the upper limit of the first transition range (Re > 2500) the turbulence develops very easily because of the phenomenon of growing of slugs. The intensity of this growing is related to Reynolds number and increases as it increases (see experiments for single disturbance, fig. 13). We no longer have, as in the first transition range, two contrary effects that were predominant alternatively in different zones of the interval of input frequencies observed.

In the first transition range, in fact, there was a laminar behaviour protected by an unknown damping effect, against which splitting acted to produce turbulence to a greater degree than the periodical disturbance itself would have done. Thus we had a wide band of output frequencies due to the action of both splitting and damping.

In the second transition range, on the other hand, there is no more splitting (a doubling effect was noted a few times). Now turbulence is promoted by the phenomenon of growing, that extends the regions of turbulence. The response frequency is reduced, due to overtaking of one turbulent slug with another, so that the entire flow is engulfed if enough length has been allowed. Due to the length of our pipe, x = 3850 diameters, the turbulent slugs had the chance to grow and join together, to bring the flow to a fully-developed turbulent state even when the input frequencies were low. The whole range of input frequency has been investigated, as was done for the other experiments, but always (except for the very lowest input frequencies) there was nothing but a continuous turbulent response to the periodic perturbations. The position of the jet did not hold steady but performed small oscillations, in the range of the turbulent position, on the grid; the laminar position was never reached.

At very low frequencies (up to 7) it was still possible to get some laminar regions. Careful experiments were performed for one run at Re = 2700. We noted that the output frequency response was twice the input for input frequencies of 1 and 2 cycles per minute, and then reduced to a value equal to the input frequency for 3, 4, 5, 6 cycles per minute (see fig. 23).*

We calculated, on the basis of the growing of a single slug, the maximum input frequency for which it was still possible to have a not completely turbulent response; this value is 6.3. Experiments performed around the value of 7 cycles per minute gave as response a fully turbulent flow; the last intermittent response was obtained for an input frequency of 6 cycles per minute.

Fig. 24 shows the nondimensional time of travel of the disturbance from the perturbation station to the pipe exit versus input frequency. From this we can conclude that the velocity of slugs decreases (larger travel times) when their population in the pipe increases. (At 1 and 2 cycles per minute there was an apparent

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* It may be useful to note that one minute is approximately the time for a slug to traverse the length of the pipe.

splitting of each slug, which was never observed at these Reynolds numbers in the single-perturbation experiments. The number of runs here was not enough to investigate this point thoroughly.)

IV. CONCLUDING REMARKS

The results of our experiments on natural transition are in general agreement with those of previous investigators, as regards intermittency factor and nondimensional frequency variation with Reynolds number. In addition, we have measured the distribution functions for lengths of laminar and turbulent regions (fig. 7). An interesting feature shown there is that at Re $\doteq$ 2450 the laminar and turbulent regions have the same distribution functions. (Re = 2450 is the value which corresponds to maximum average frequency (fig. 4).) Another result obtained from the measurements of length distributions is that, at the lower end of the transition range, turbulent intervals tend to a maximum value of 30 diameters, while, at the upper end of the transition range, laminar intervals tend to a maximum value of 10 diameters. The latter value was also obtained as a limiting value for the laminar interval between any two successive portions of a split slug, in the single-disturbance experiments.

The main result of the single-disturbance experiments was a delineation of regions of splitting and growing of turbulent slugs (figs. 9 and 13). Splitting begins from the very beginning of the transition range (Re = 2000), and at higher Reynolds numbers becomes multiple. Growing of turbulent slugs begins at Re = 2300. After Re = 2600 only growing affects the slugs, and splitting no longer occurs.

In these experiments the front velocity was found to have the limiting value $U_F/U = 1.15$.

In the experiments with periodic input disturbance, it was anticipated that the natural frequency would play an important role.

This did not occur (figs. 16a,b,c); the output frequencies for all values of Re have a roughly similar behaviour (fig. 17) and are not related in any consistent way with the natural frequency.

The maxima and minima in the output curves are surprising. We have attempted to explain their occurrence in terms of the opposing actions of splitting on one hand, growing and damping on the other. The idea of "damping" (elimination of turbulent slugs) had to be introduced to account for some of the phenomena in the periodic-disturbance experiments (e.g. see fig. 18). "Damping" might occur either due to joining of adjacent turbulent slugs or to their disappearance. It is a feature only of the periodic case, where laminar regions on each side of a slug are necessarily of limited extent. In the single-disturbance experiments damping was never observed above Re = 2000, i.e., an input disturbance always resulted in one or more turbulent slugs at the exit. From the frequency output curves (fig. 17) it appears that splitting predominates at low input frequencies and increases output frequencies, growing (high Re) or damping (low Re) predominate at high frequencies, and produce the similar effect of making the output frequency low compared to the input. In the intermediate input frequency range, these various effects more or less balance, with alternating predominance, to produce the maxima and minima.

The above discussion is for 2000 < Re < 2600; at higher values of Re, splitting no longer occurs and growing leads to a fully turbulent behaviour (fig. 23).

When the maxima of output frequency occur, the intervals of laminar and turbulent flow are the smallest possible, and in this case the sum of all laminar regions is equal to the sum of all turbulent regions (intermittency factor = 0.5).

At the lower end of the first transition range, high input frequencies tend to make the flow go laminar, while at the upper end, high input frequencies tend to make it go turbulent. We might say that the tendency at the lower end of the first transition range is to a "normal" state of laminar flow, while at the upper it is to turbulent flow; high frequencies seem to promote the establishments of these "natural" states. For low input frequencies, or in the case of fixed geometric disturbance, we can think of the mixed flow as consisting of the following: at the lower end, turbulent slugs are imbedded more or less at random in a laminar flow, while at the upper end, laminar regions are imbedded more or less at random in turbulent flow.

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* A complete reference on the subject can be found in Refs. 1 and 5.

Run n°	T_i	T_e	$\bar{\nu}$	V_L	V_T	V_{Tot}	R_e	$\bar{u}$	γ	$d/\bar{u}$ 10^{-2}	f_{out}	ε	S_T/d	S_L/d
1	23,5	24,8	901	720	-	720	1840	36	0	1,28	0	0	0	∞
2		24,6	905	795	5	800	2000	39,5	0,0062	1,17	0	0	0	∞
3		24,6	905	800	35	835	2110	41,5	0,042	1,10	16,5	0,003	13,3	380
4		24,6	905	760	150	910	2300	45,5	0,165	1,01	62,5	0,0105	15,8	86
5		24,6	905	720	220	940	2380	47	0,234	0,98	130	0,0213	11	36,8
6		24,6	905	565	400	965	2450	48,5	0,424	0,95	187,5	0,0297	13,8	21
7		24,5	907	320	680	1000	2520	50	0,680	0,92	122,5	0,0188	36	17
8		24,5	907	130	890	1020	2580	51	0,87	0,905	100	0,0188	46,3	6,9
9		24,5	907	95	935	1030	2610	51,5	0,910	0,89	50	0,0074	123	12
10		24,5	907	60	990	1050	2650	52,5	0,940	0,88	46,5	0,00683	138	8,8
11		24,5	907	40	1020	1060	2680	53	0,960	0,87	37,5	0,00545	176	7,3
12		24,4	909	35	1065	1100	2770	55	0,97	0,84	12,5	0,00175	570	2,3
13	23,5	24,4	909	5	1095	1100	2780	55,5	0,995	0,83	-	0	∞	0

TABLE - 1.

T_i = inlet Temp.
T_e = exit Temp.
$\bar{\nu}$ = Kinematic viscosity.
$\bar{u}$ = mean velocity.

V_L = laminar volume.
V_T = turbulent volume.
V_{Tot} = Total volume.

$R_e = \bar{u} d / \nu$ Reynolds.
d = diameter.
f_{out} = output freq.

$\gamma = \frac{V_T}{V_{Tot}}$ intermitt fact
$\varepsilon = \frac{n d}{u}$ adim freq.
S = slug length.

Run	$\bar{T}$	f_{in}	f output	$\bar{f}_{out}$	Δf_{out}	$\frac{\Delta f_{out}}{f_{out}}$	$\frac{f_{out}}{f_{in}}$	non perturbed Re	non perturbed $\bar{u}_L$	perturbed Re	perturbed $\bar{u}$	$\bar{u}_T$	$\Sigma \tau_L$	γ
1	26,1	9	20 - 24 -	22	4	18	2,2	2100	40	2100	40		51	0,167
8	26	26	47 - 68 -	57,5	21	37	2,2	2086	39,8	2040	39,5	38,7	47	0,220
3	26,3	46	72 - 110 - 112 - 134	107	62	57	2,33	2100	40	2100	39,4	38,6	36	0,400
10	25,6	64	110 - 122 -	116	12	10,4	1,82	2050	39,5	2020	39	38,8	22	0,620
2	26,15	90	84 - 100 - 90 - 102	94	18	19,2	1,05	2100	40	2100	40		37	0,384
9	25,8	121	90 - 98 - 90 -	92,6	8	8,6	0,765	2050	39,5	2030	39	38,2	43	0,276
4	26,35	150	140 - 148 - 112 - 118	129	36	28	0,865	2100	40	2080	39,4	38,8	27	0,550
12	25,45	200	55 - 72 -	63,5	17	26,8	0,32	2060	39,5	2040	39	38,8	47,5	0,210
5	26,25	250	118 - 114 - 170 - 148	137,5	56	40,5	0,55	2100	40	2080	39	37,8	33	0,450
11	25,75	270	75 - 55 -	65	20	30,5	0,24	2060	39,5	2040	39	38,2	42	0,300
7	26,1	280	60 - 80 -	70	20	28,6	0,25	2100	40	2080	39			
6 - 7	26,1	320	40 - 26 -	33	14	42,5	0,105	2100	40	2080	39,5	37,7	50	0,160
6	25,9	360	48 -	48			0,133	2100	40	2080	39,5	37,8	49	0,185

TABLE 2

f_{in} = frequency input.
f_{out} = frequency output.
Δf_{out} = dispersion.

$\bar{u}_L$ = Laminar velocity.
$\bar{u}^*$ = Velocity of perturbed flow.
$\bar{u}_T = \frac{\bar{u}^* + (\gamma - 1)\bar{u}_L}{\gamma}$ = Turbulent velocity.

$\Sigma \tau_L$ = Total time of laminar flow.
γ = Intermittency factor.

Run	$\bar{T}$	f_{in}	f output	$\bar{f}_{out}$	Δf_{out}	$\frac{\Delta f_{out}}{f_{out}}$	$\frac{f_{out}}{f_{in}}$	non perturb. R_e	$\bar{u}_L$	perturb. R_e^*	$\bar{u}^*$	$\bar{u}_T$	$\sum \tau_L$	γ
1	25,8	31	40 - 40 - 60	47	20	42,5	1,52	2400	46	2370	45,5	44,1	20	0,665
10	25,3	40	72 - 74 -	73	2	2,7	1,82	2300	45	2260	44	42	42	0,3
2	25,5	54	106 - 104 -	105	2	1,9	1,94	2360	45,5	2310	44,5	44	26,5	0,557
3	25,2	76	108 - 108 - 104 - 128	112	20	17,6	1,47	2300	44,5	2260	44	42,7	40,3	0,328
4	25	106	140 - 140 - 108 - 96	121	44	36,4	1,14	2330	45,5	2300	45	44	40,1	0,33
11	24,5	110	112 - 120 -	116	8	6,9	1,05	2280	45	2260	44,5	44,1	32,4	0,465
6	25,2	136	96 - 94 -	95	2	2,1	0,7	2300	44,5	2260	44	44	12,4	0,795
12	24,2	152	124 - 128 -	126	4	3,2	0,83	2280	45	2230	44	43,5	19	0,68
7	26,2	178	150 - 148 -	149	2	1,33	0,84	2320	44,3	2300	44	43,8	31	0,485
13	24,4	200	108 - 118 -	113	10	8,9	0,565	2260	44,5	2230	44	43	39	0,35
5	24,8	230	61 - 74 -	68	13	19	0,296	2300	45	2280	44,5	42,5	47,3	0,212
14	24,5	250	94 - 82 -	88	12	13,6	0,35	2270	44,5	2220	44	43,5	34	0,45
8	25,3	275	104 - 132 -	120	28	23,3	0,44	2300	45	2270	44,5	42	28	0,53
15	24,8	350	40 - 36 -	38	4	10,5	0,11	2250	44,5	2230	44	42	49	0,183

TABLE - 3

Run	$\bar{T}$	f_{in}	f_{output}	$\bar{f}_{out}$	Δf_{out}	$\frac{\Delta f_{out}}{f_{out}}$	$\frac{f_{out}}{f_{in}}$	non perturb. Re	non perturb. $\bar{U}_L$	perturb. Re^x	perturb. $\bar{U}^*$	$\bar{U}_T$	$\Sigma \tau_L$	γ
1	25,4	30	132 - 136 - 168 - 170	151	28	13	5,04	2550	49,5	2480	48	47,2	20	0,665
11	25,8	39	168 - 166 -	167	2	1,2	4,3	2540	49	2480	48	47,5	20,5	0,660
3	25,1	52	140 - 132 - 156 - 140	142	24	16,8	2,74	2530	49	2450	47,5	46,8	19	0,685
13	25,45	64	158 - 162 - 162 - 150	158	12	7,6	2,47	2540	49,2	2460	47,5	47,5	22	0,630
2	25,3	72	164 - 168 -	166	4	2,4	2,31	2550	49	2470	47,5	47,5	21,5	0,640
12	25,35	80	182 - 190 - 160 -	177	30	17	2,21	2515	49	2440	47,5	46,5	27,5	0,540
4	24,8	102	148 - 162 - 144 - 172	156	28	18	1,53	2510	49	2460	48	47,3	28	0,535
14	24,9	125	64 - 76 - 60 - 48	62	28	45	0,495	2510	49	2440	47,5	47,5	10,5	0,835
6	24,3	156	182 - 170 -	172	12	6,8	1,13	2495	49	2420	47,5	46,2	31	0,485
16	24,1	160	184 - 178 - 178 - 186	182	8	4,4	1,13	2480	49	2400	48	47	36	0,410
5	24,5	197	142 - 130 - 142 - 144	140	14	10	0,71	2500	49	2420	47,5	47,2	20	0,665
15	24,4	218	158 - 156 - 168 - 164	172	12	7	0,79	2480	49	2420	47,7	47	21,5	0,640
7	24,2	235	176 - 182 - 152 - 156	166	30	18	0,71	2480	49	2420	47,8	46,8	27	0,550
17	23,9	260	60 - 65 - 60 - 75	65	15	23	0,25	2460	49	2380	47,5	-	-	-
8_I	24,9	285	112 - 110 -	111	2	1,8	0,38	2500	49	2460	48	47,4	15	0,750
8_{II}	24,9	310	164 - 156 -	160	8	2	0,515	2500	49	2460	48	47,3	25	0,585
9_I	24,9	330	144 - 136 - 144 -	141	8	5,7	0,43	2520	49	2465	48	-	-	-
9_{II}	25,1	340	150 - 174 - 176 -	166	26	15,7	0,49	2520	49	2465	48	-	-	-

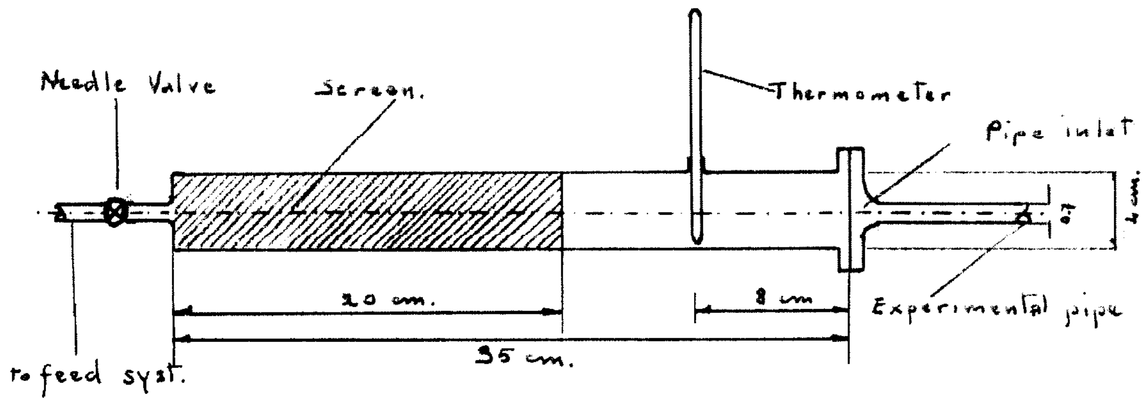
TABLE - 4.

Exp D Re 2100			Exp A Re 2300			Exp B Re 2500		
ε_{in}	ε_{out}	$\varepsilon_{out}/\varepsilon_{in}$	ε_{in}	ε_{out}	$\varepsilon_{out}/\varepsilon_{in}$	ε_{in}	ε_{out}	$\varepsilon_{out}/\varepsilon_{in}$
0,00194	0,00384	2	0,00516	0,00835	1,6	0,00475	0,0212	4,4
0,00475	0,0115	2,4	0,00685	0,0128	1,88	0,00635	0,026	4,1
0,0096	0,0208	2,2	0,0086	0,018	2,1	0,0079	0,022	2,78
0,0125	0,022	1,76	0,0128	0,019	1,48	0,0103	0,0243	2,40
0,0184	0,0184	1	0,0172	0,0207	1,20	0,0117	0,0268	2,30
0,023	0,016	0,7	0,0188	0,0197	1,05	0,0126	0,0284	2,26
0,0287	0,025	0,87	0,0232	0,0163	0,7	0,0157	0,0237	1,5
0,0384	0,0115	0,3	0,031	0,0267	0,86	0,0177	0,0177	1
0,048	0,0268	0,56	0,0344	0,0197	0,57	0,0196	0,0103	0,52
0,053	0,0134	0,25	0,0397	0,012	0,3	0,0251	0,0283	1,13
0,0512	0,0072	1,15	0,043	0,0155	0,36	0,0314	0,022	0,71
0,0666	0,0087	1,3	0,0474	0,0204	0,43	0,0346	0,0252	0,725
			0,051	0,00685	0,115	0,0386	0,0275	0,71
						0,0394	0,0182	0,46
						0,041	0,0097	0,236
						0,052	0,0228	0,44
						0,0545	0,0252	0,41

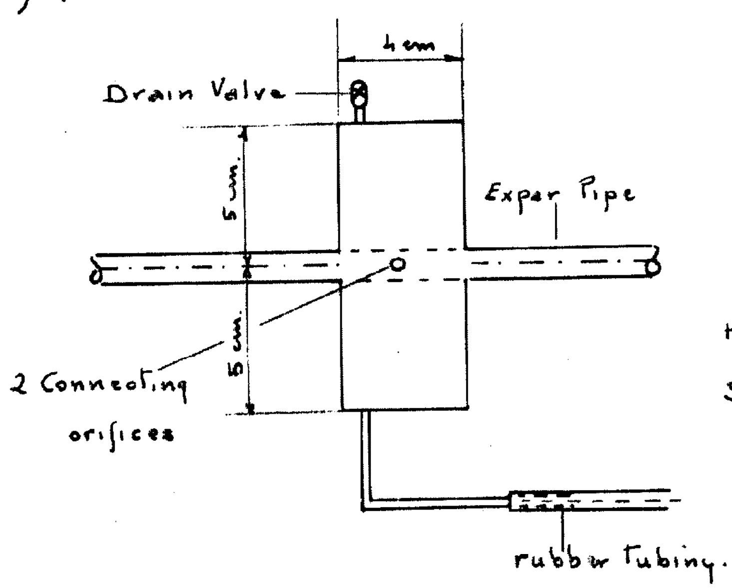
TABLE. 5

Details of Experimental Setup.

a) Stilling section



b) Accumulator



c) Hammer Mechanism.

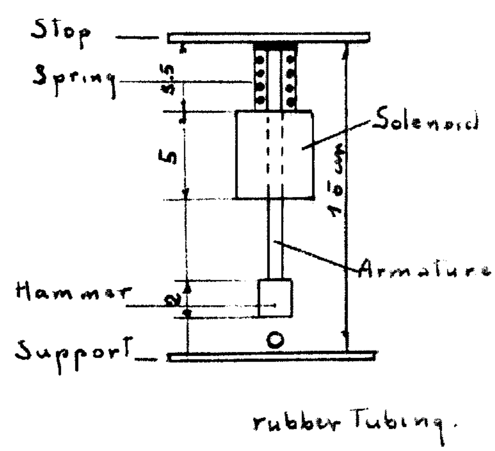


Fig-1

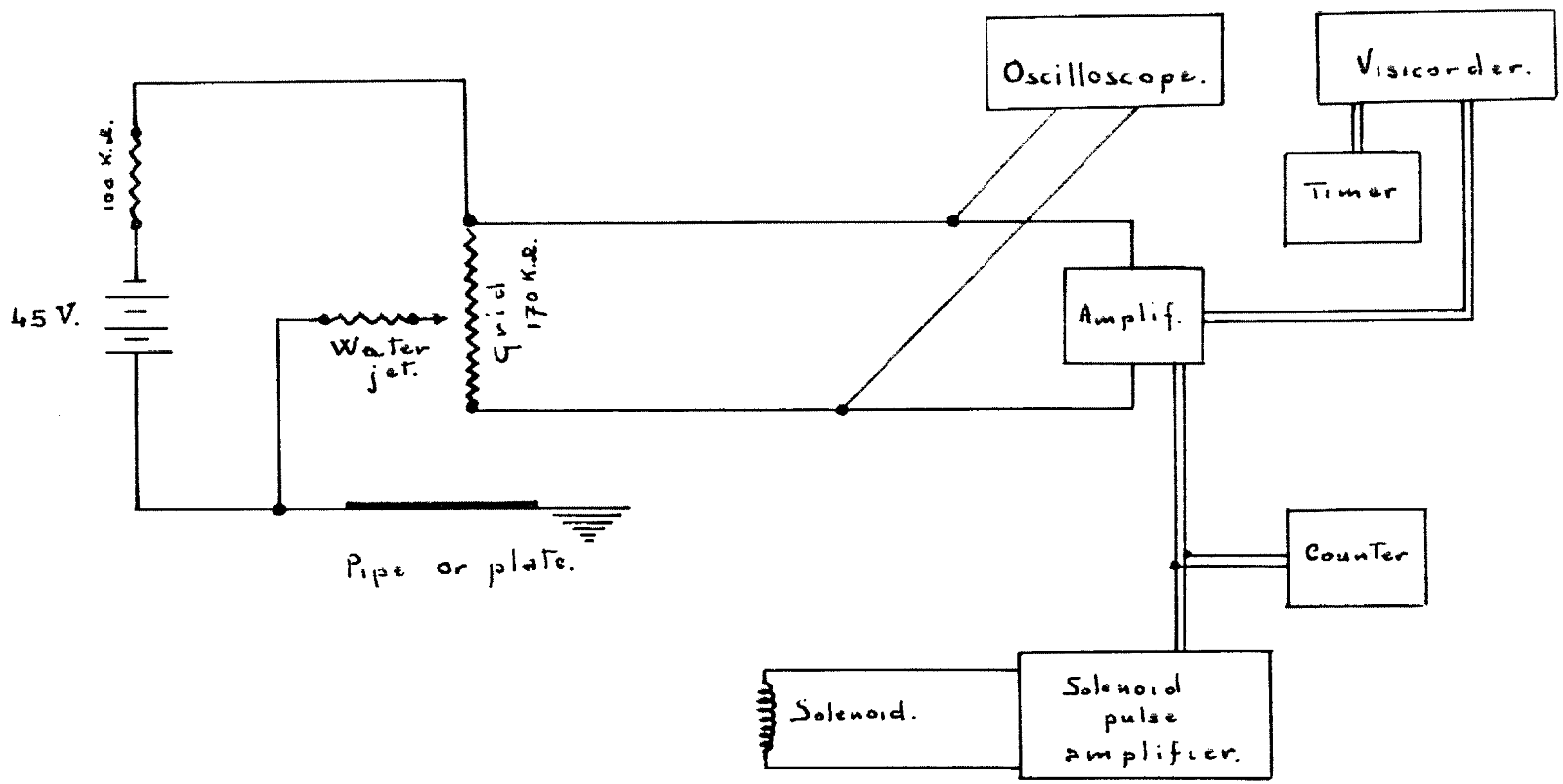


Fig. 2: Electrical circuit and instrumentation.

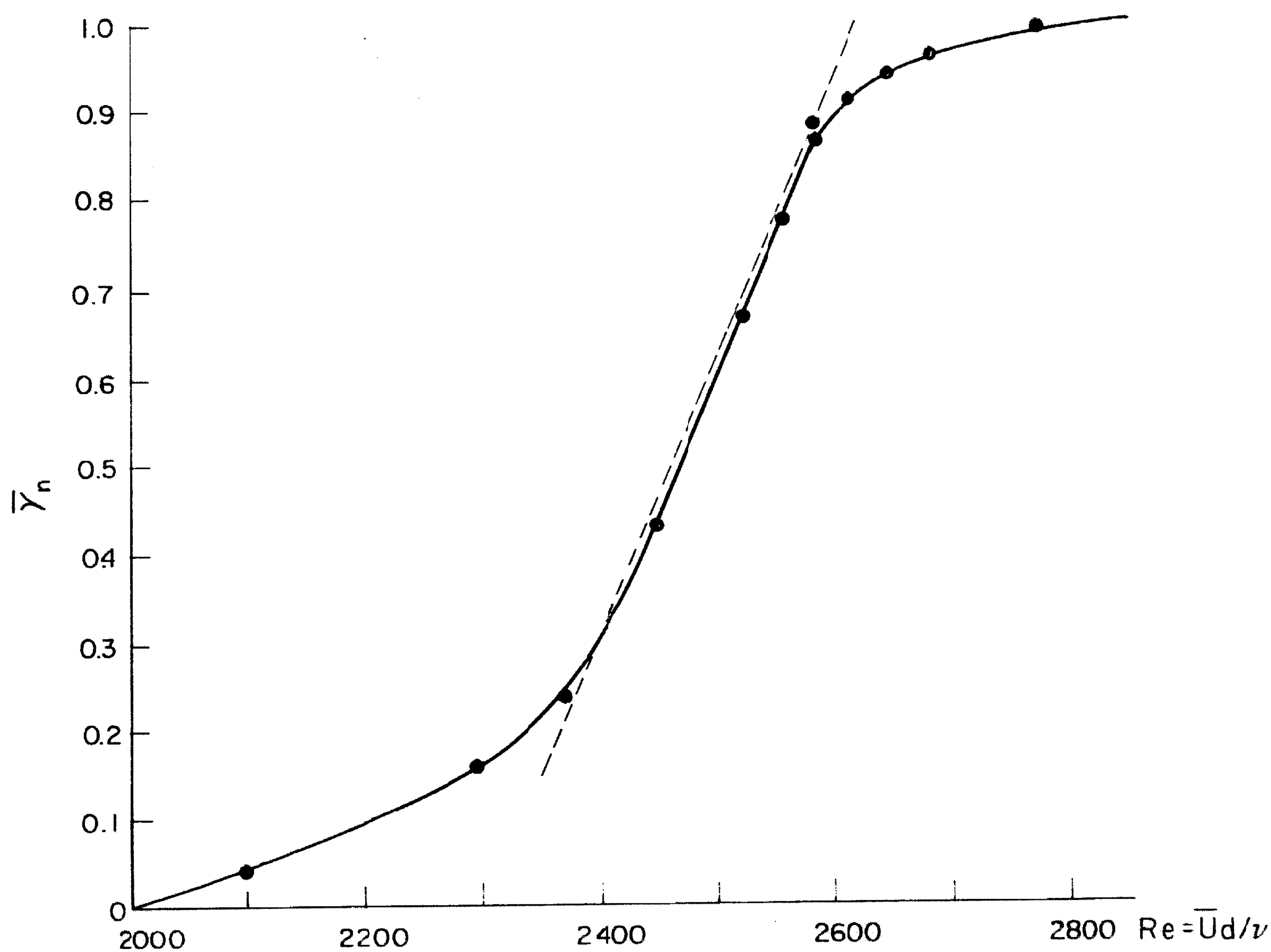


Fig. 3: Intermittency factor at x/d = 4650 (natural transition).

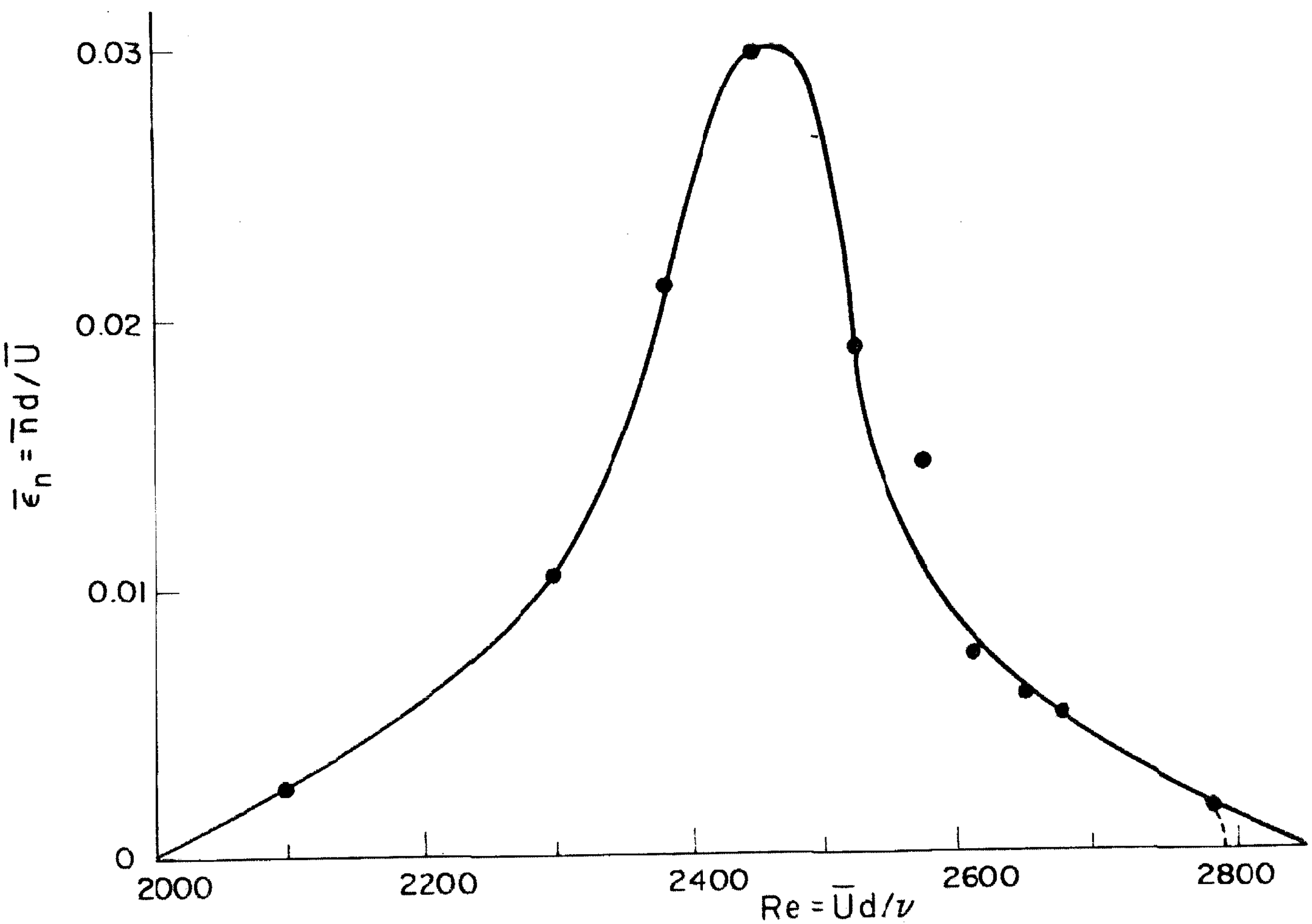


Fig. 4: Average frequency of turbulent slugs at $x/d = 4650$ (natural transition).

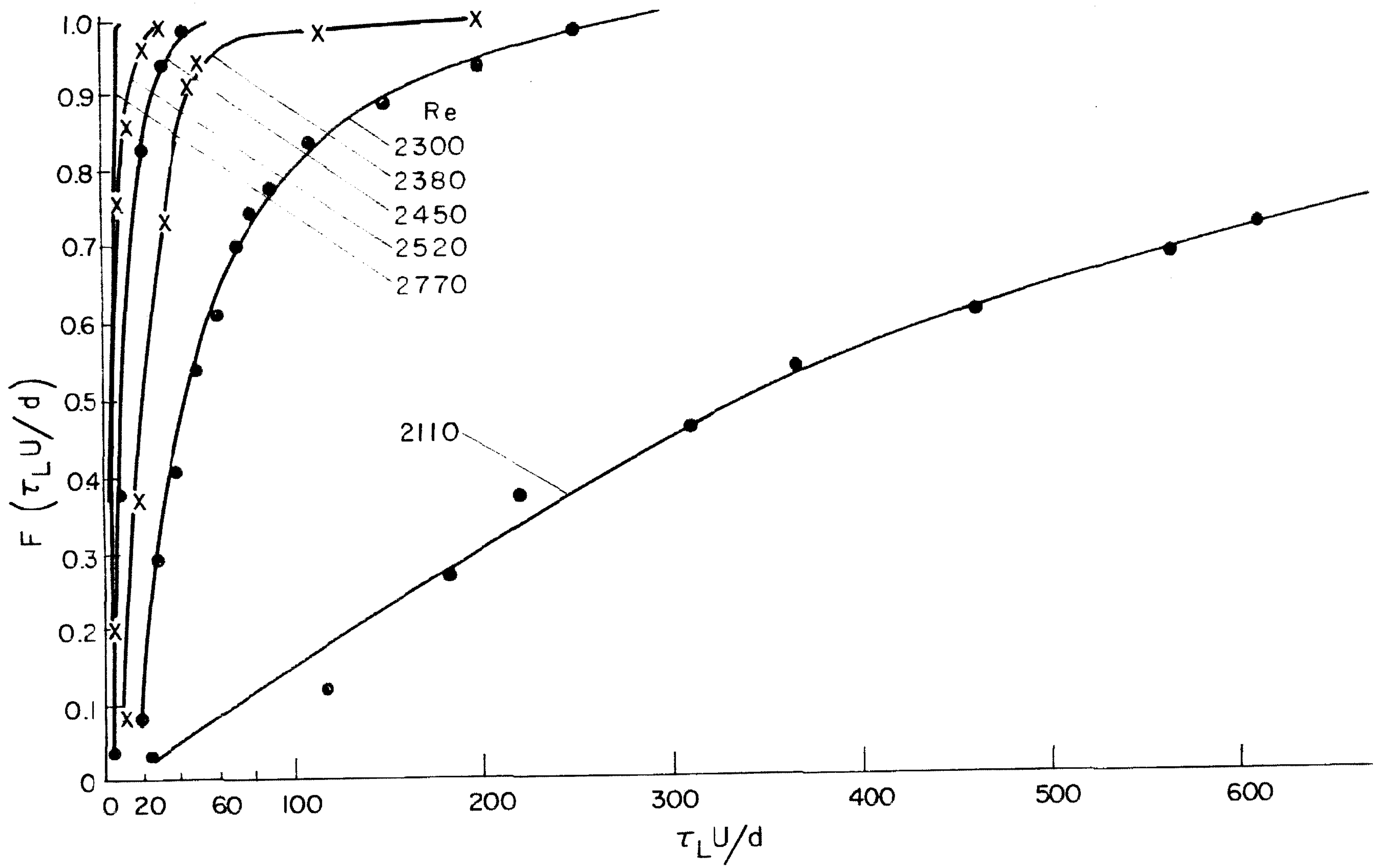


Fig. 5: Normalized distribution function $F(\tau_L u/d)$ for length of laminar regions (measured approximately in pipe diameters) at $x/d = 4650$ (natural transition).

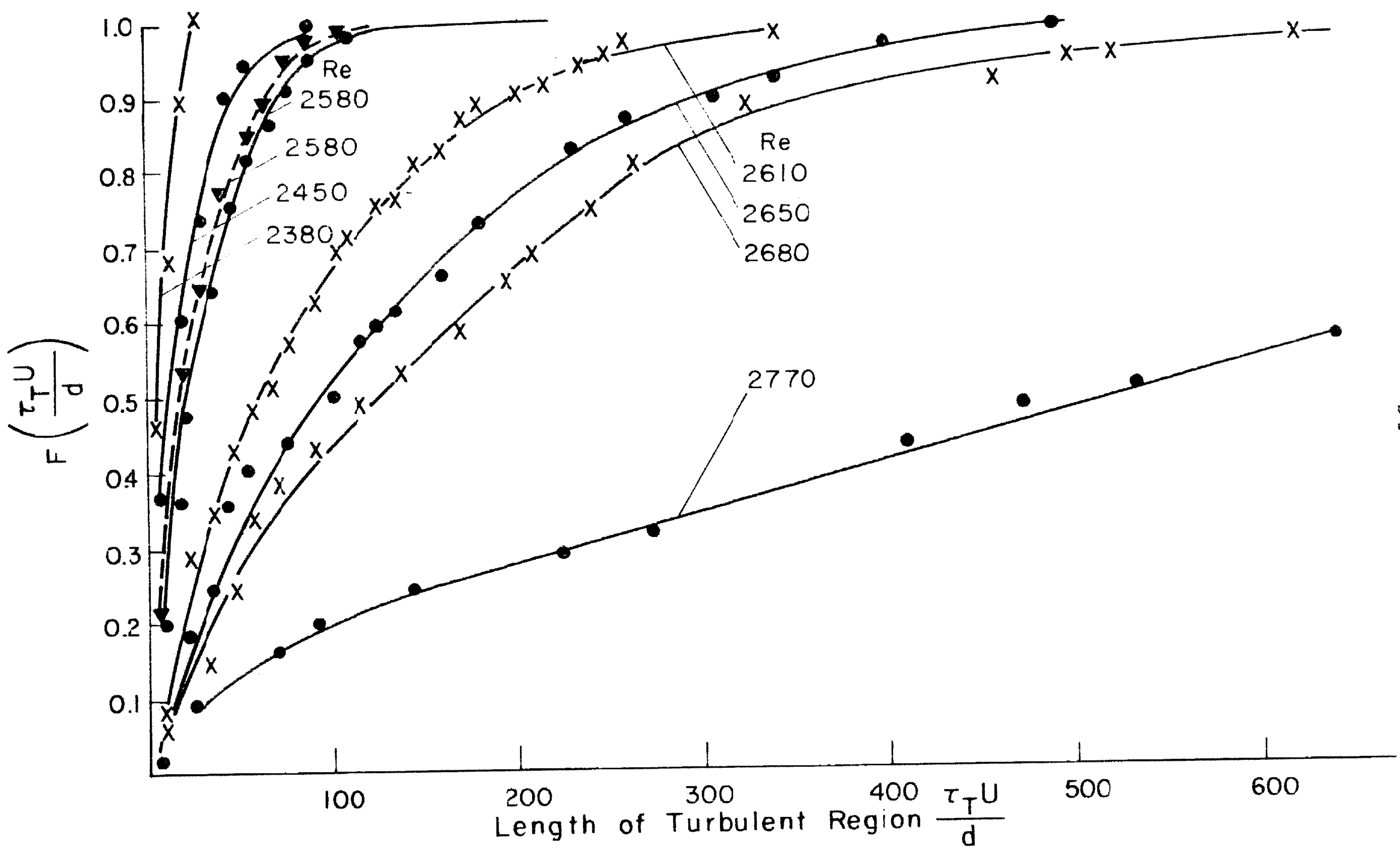


Fig. 6:
Normalized distribution function $F(\tau_T \bar{u}/d)$ for length of turbulent regions (measured approximately in pipe diameters) at $x/d = 4650$ (natural transition).

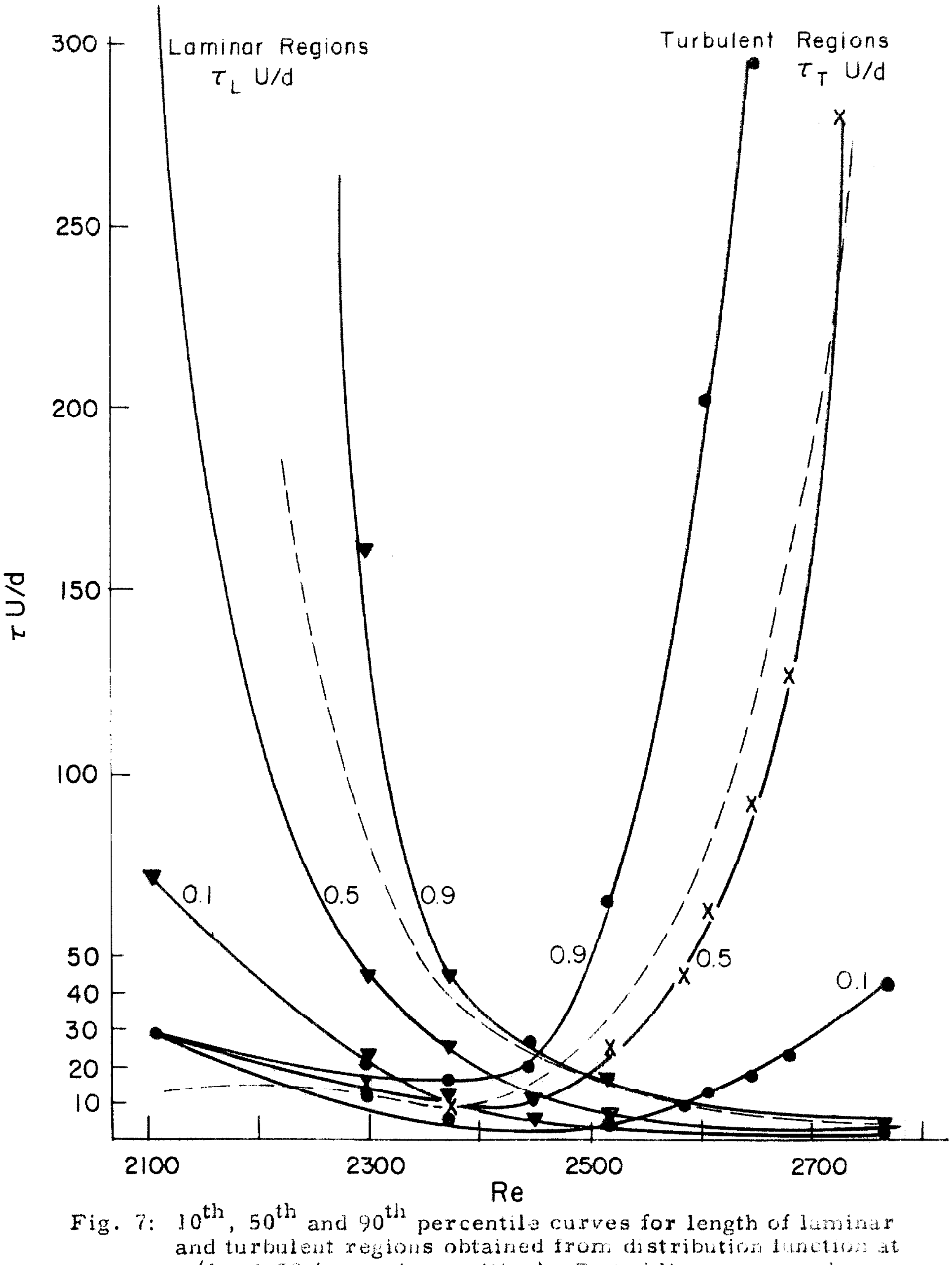


Fig. 7: 10th, 50th and 90th percentile curves for length of laminar and turbulent regions obtained from distribution function at x/d = 4050 (natural transition). Dotted lines mean values obtained from frequency, cf. Fig. 8

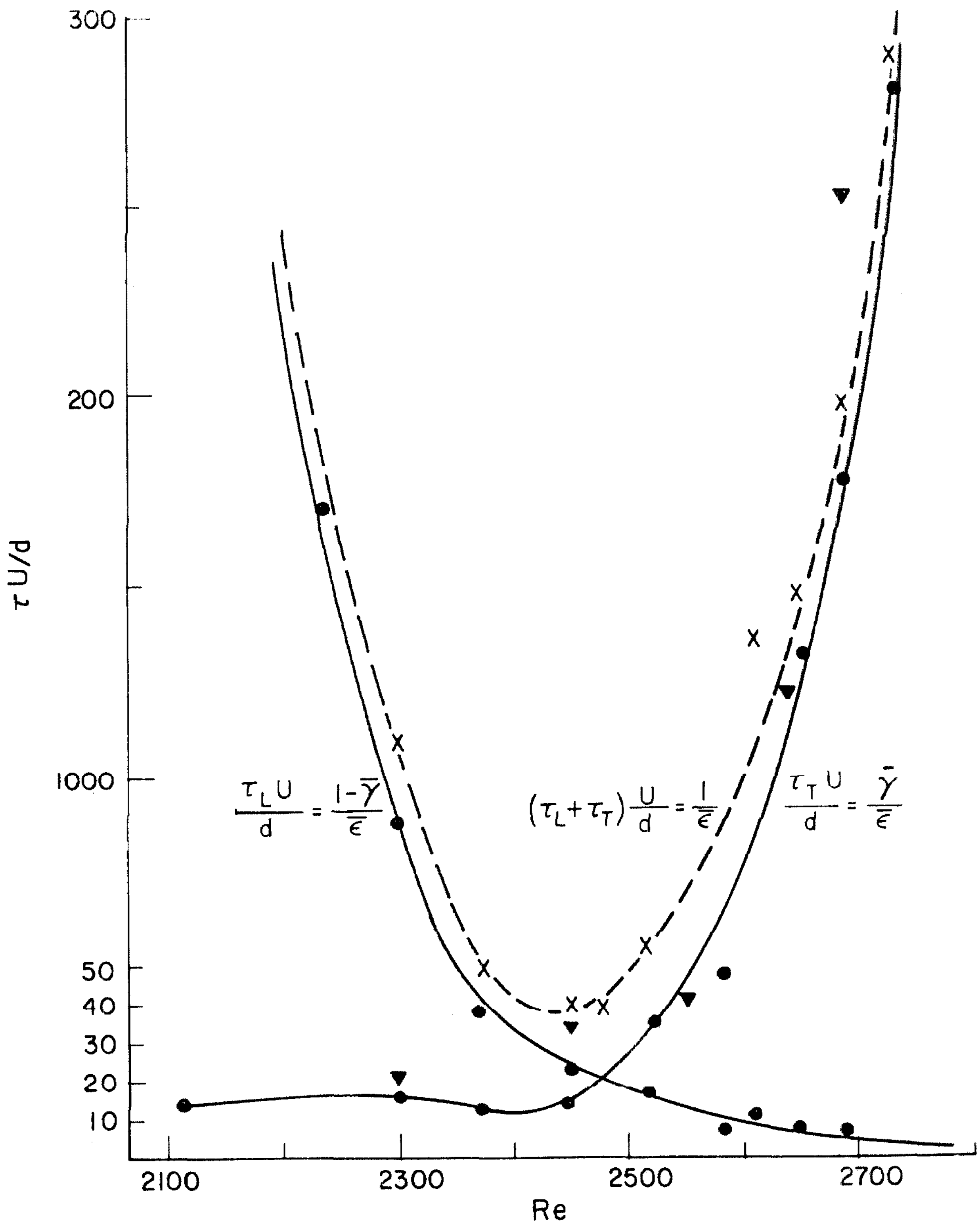


Fig. 8: Curves of mean values for length of laminar and turbulent regions (measured approximate in pipe diameters), and for a laminar and turbulent region together (dotted line). x/d = 4650 (natural transition
▼Rotta data for $\tau_T U/d$ at x/d = 322.

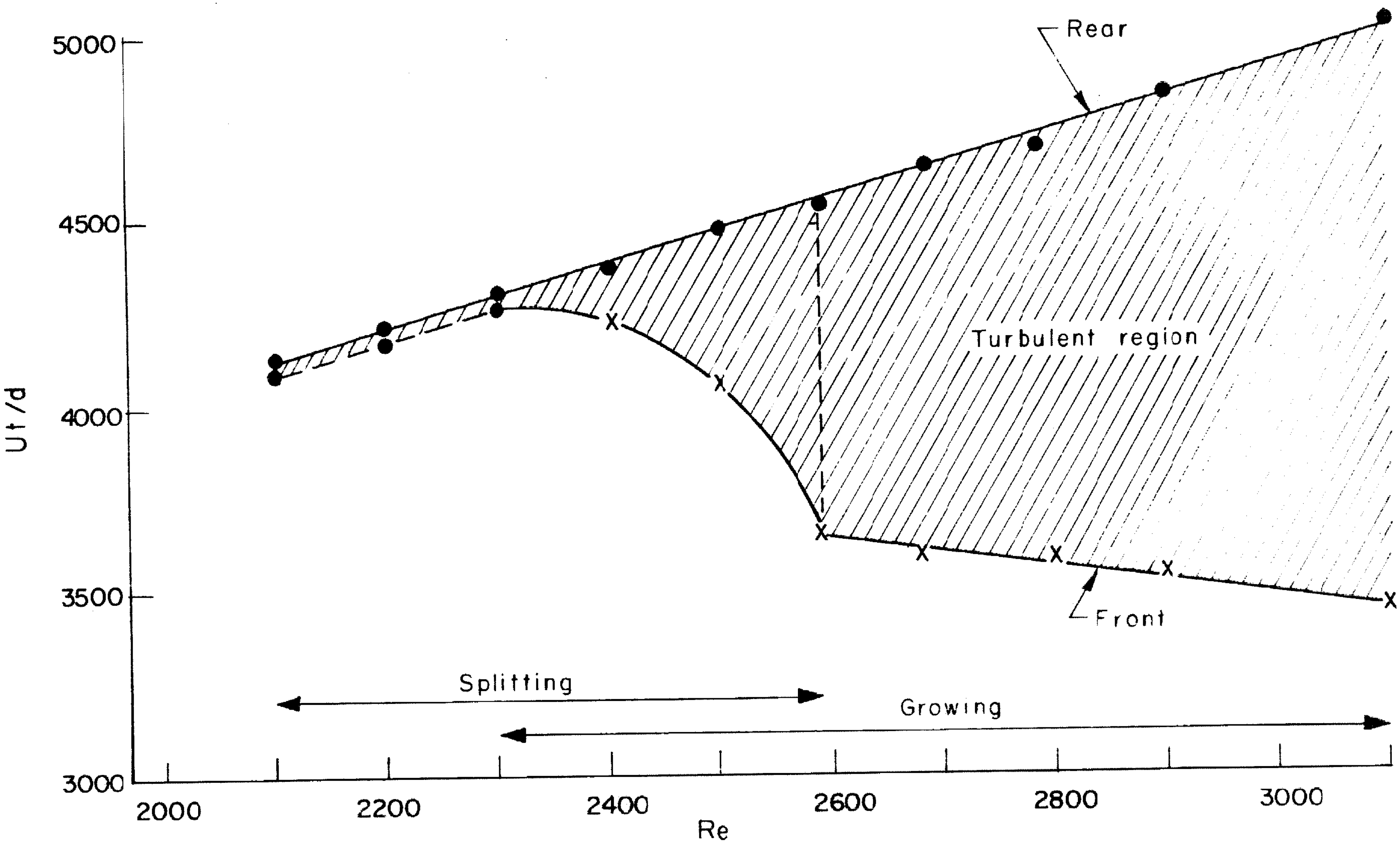


Fig. 9: Time elapsed for emerging of front and rear of a turbulent train of slugs at $x/d = 2350$ (nondimensional value). (Single disturbance.)

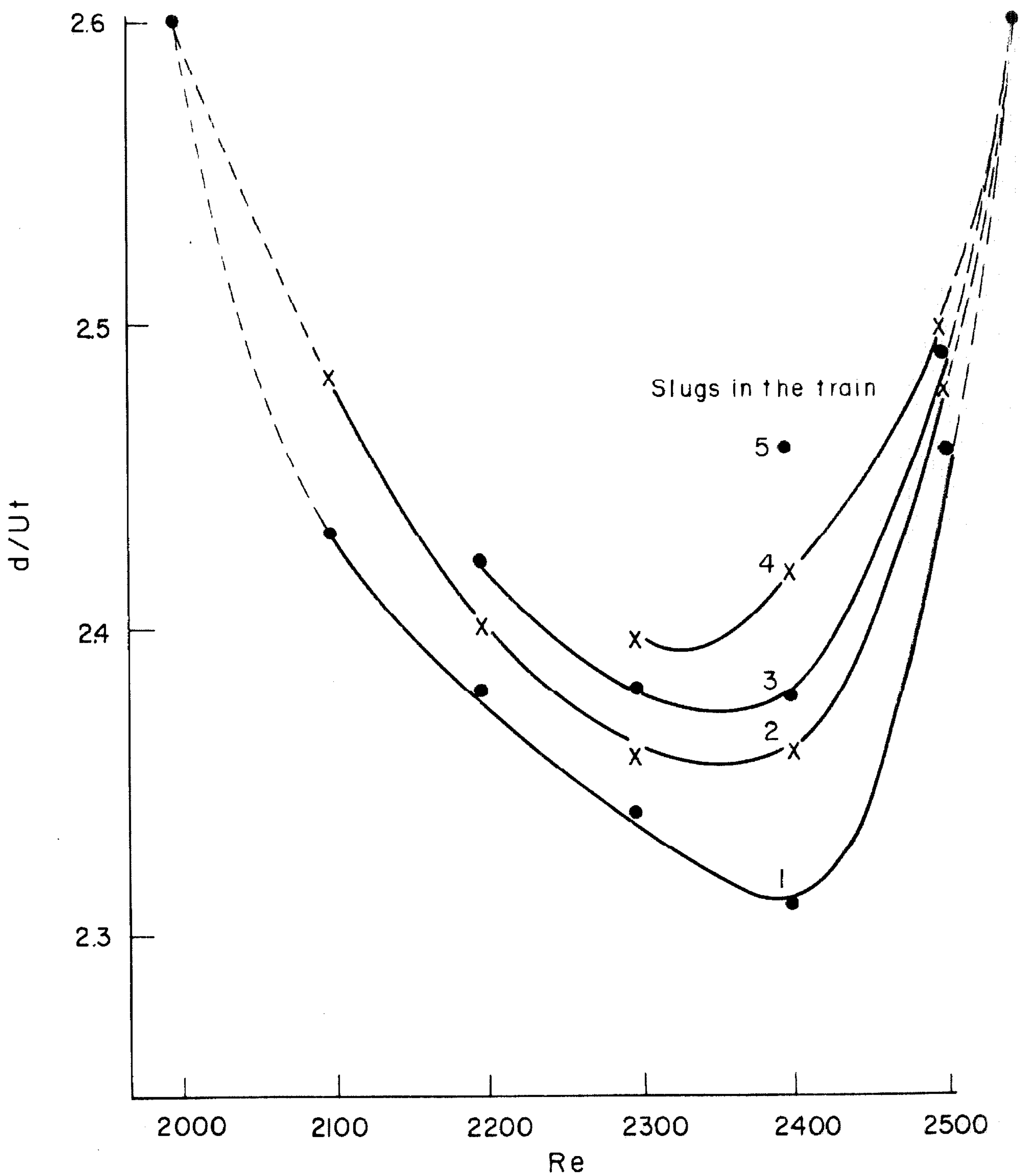


Fig. 10: Effect of splitting on the front velocity of the first slug of a train (t = arrival time of front.) (Single disturbance). x/d = 3850.

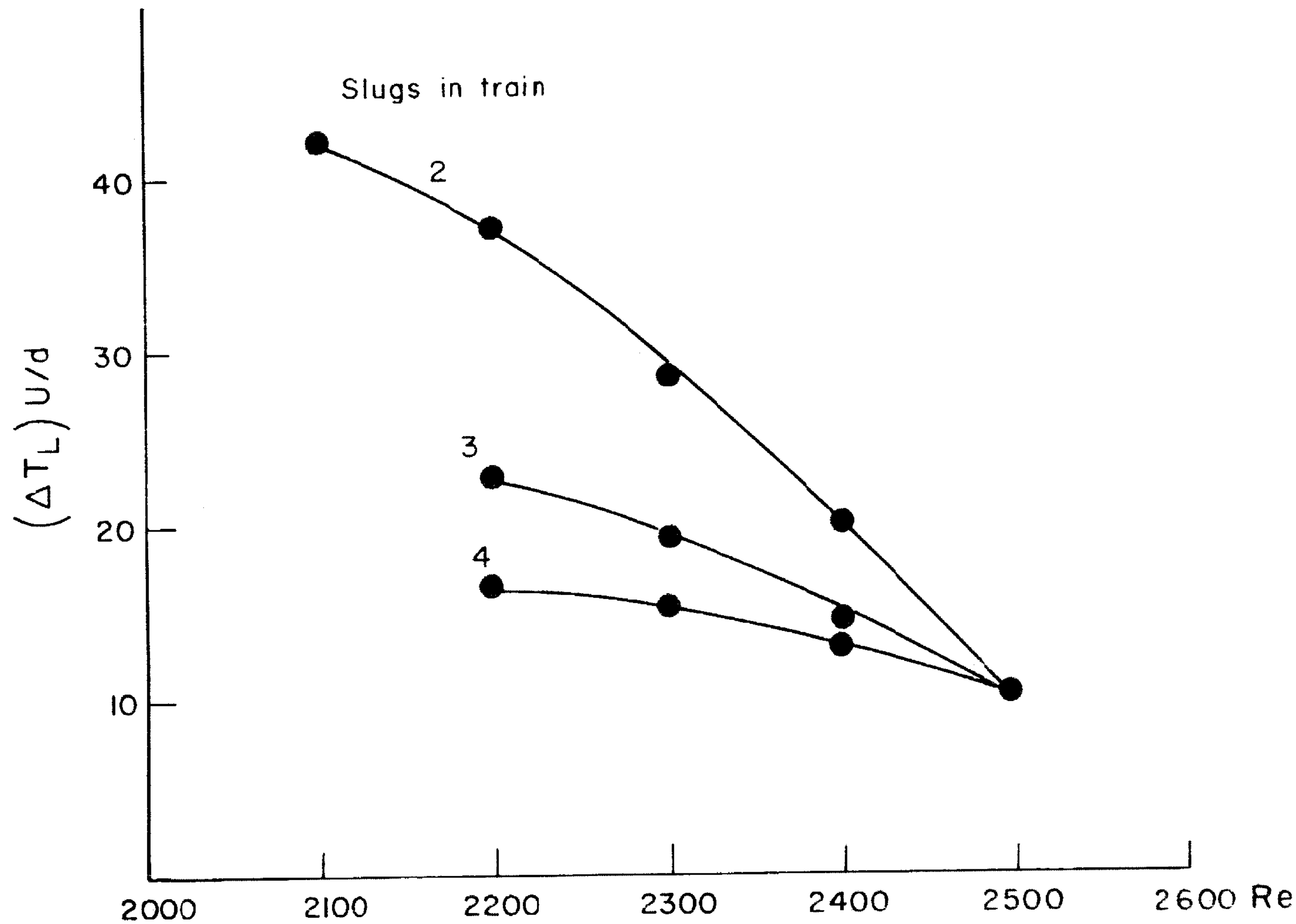


Fig. 11: Values of $\Delta T_L U/d$, nondimensional length of laminar interval between two turbulent slugs of the same train. (Single disturbance.) $x/d = 3850$.

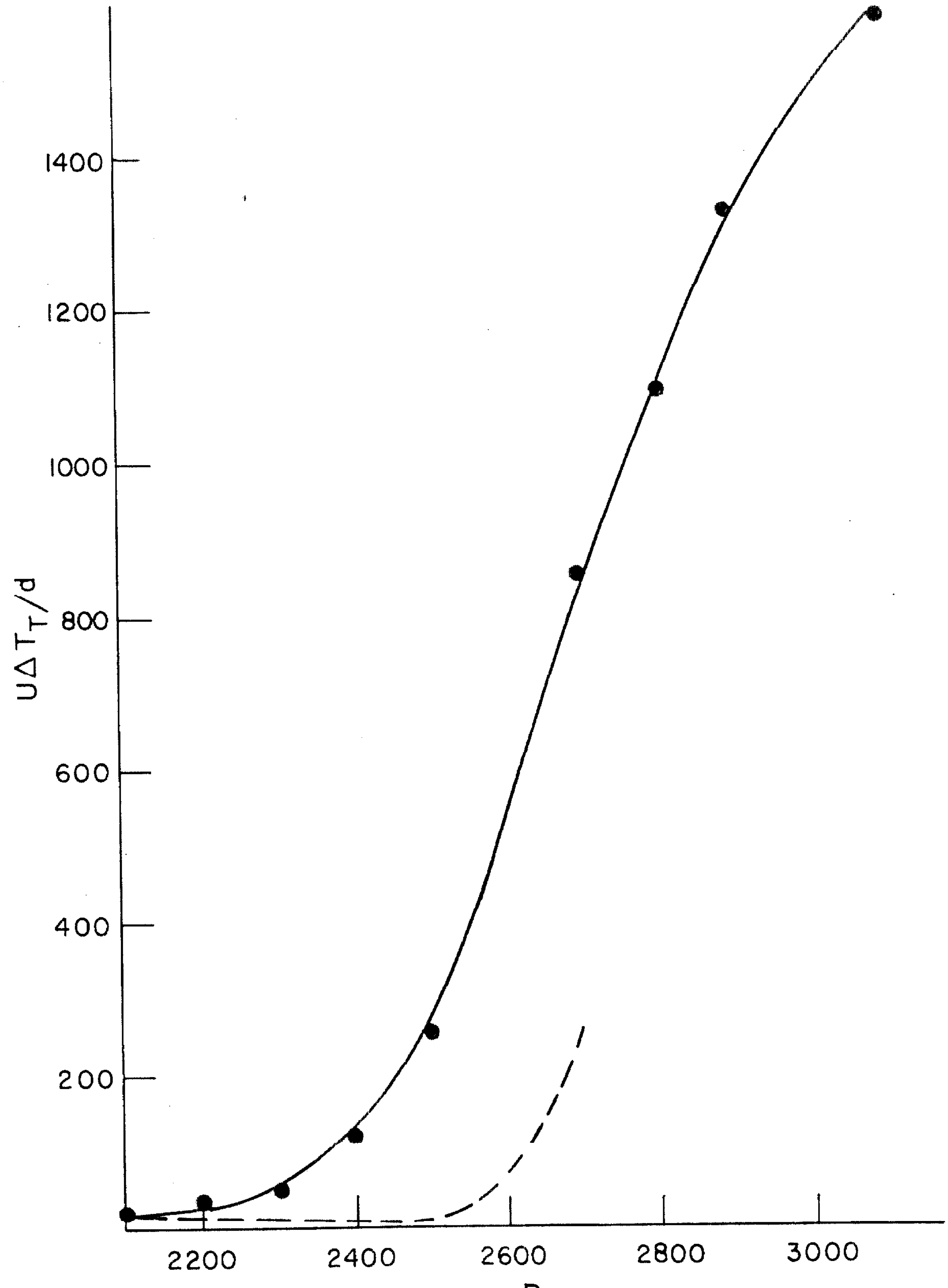


Fig. 12: Mean values for length of turbulent regions (measured in pipe diameters). Dotted line shows values computed for natural transition. (Single disturbance.) $x/d = 3850$.

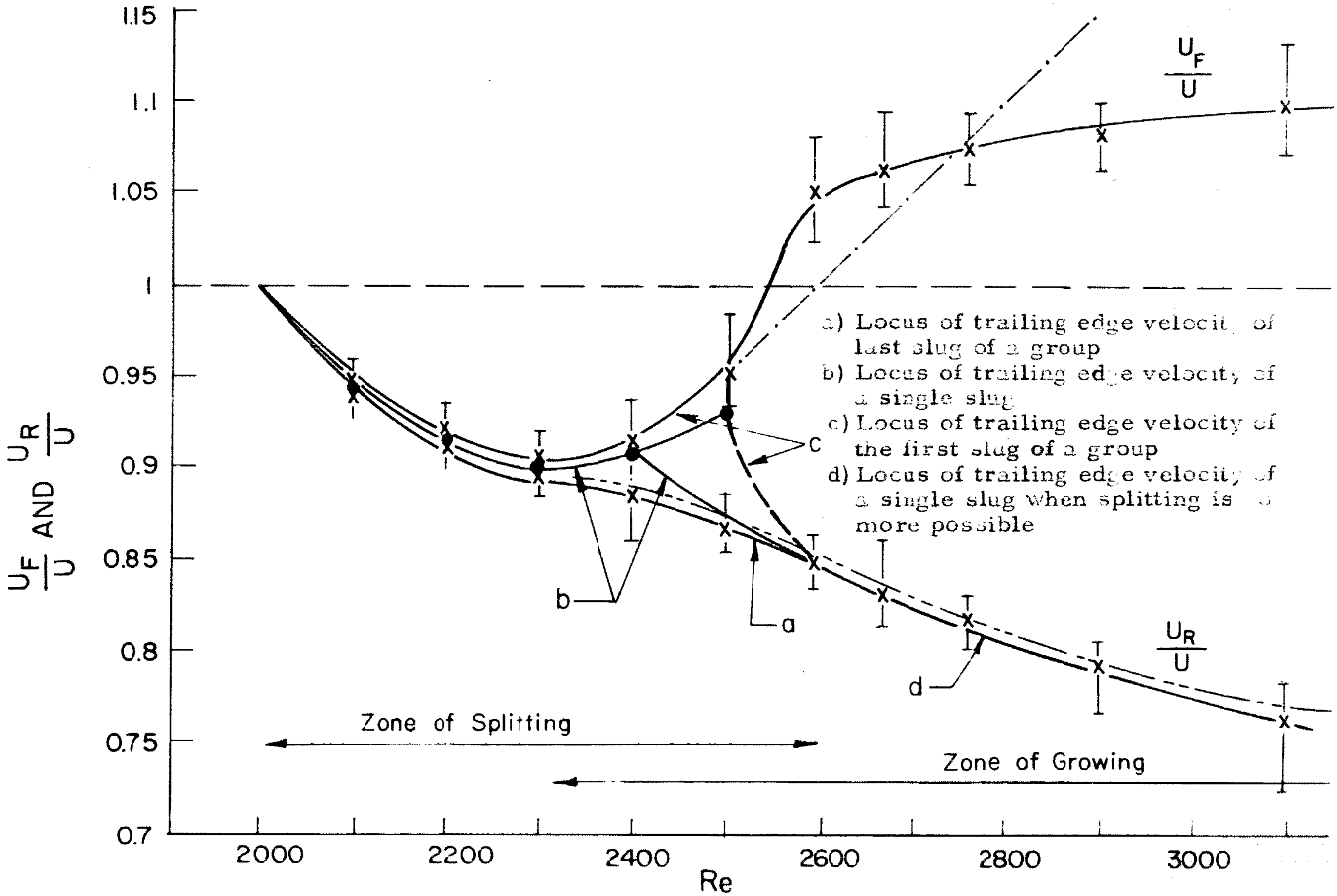


Fig. 13: Front and rear velocity (nondimensional) of a turbulent slug versus Re. (Single disturbance.) Dotted line - data of Lindgren. x/D = 3850.

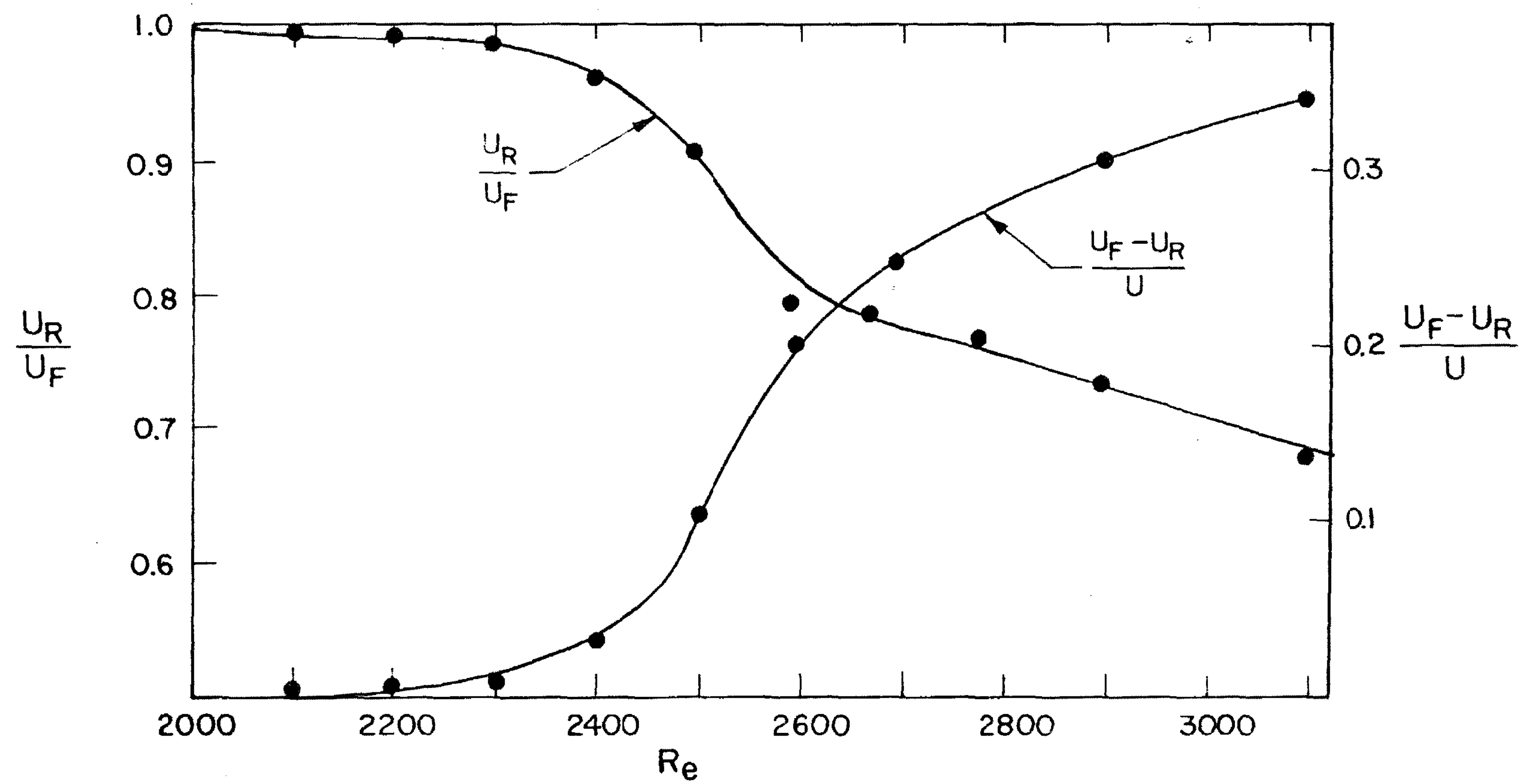


Fig. 14: Ratio and difference of nondimensional front and rear velocities. x/d = 3850.

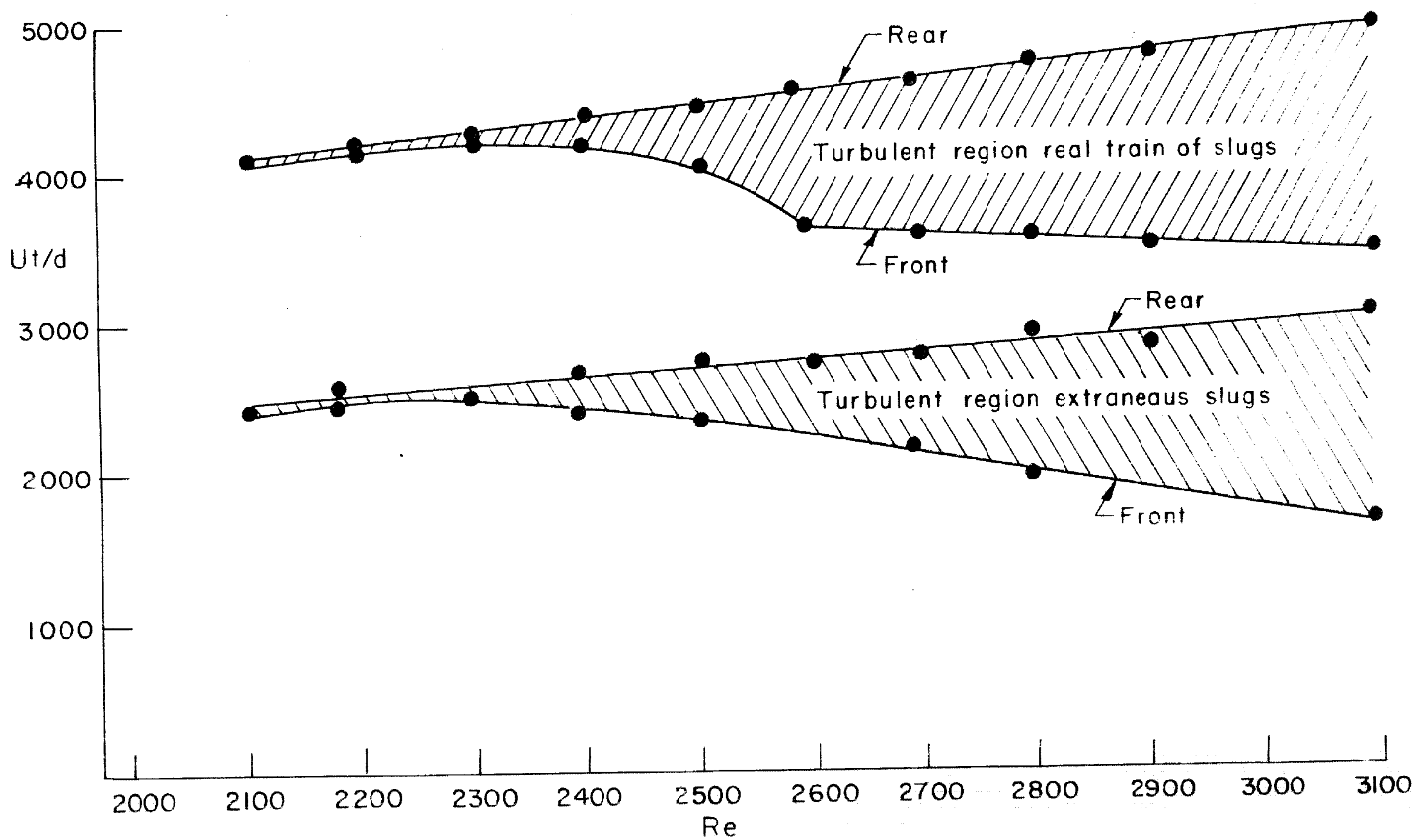


Fig. 15: Comparison between real train of slugs and extraneous slugs arising from same disturbance (Single disturbance.) x/d = 3850.

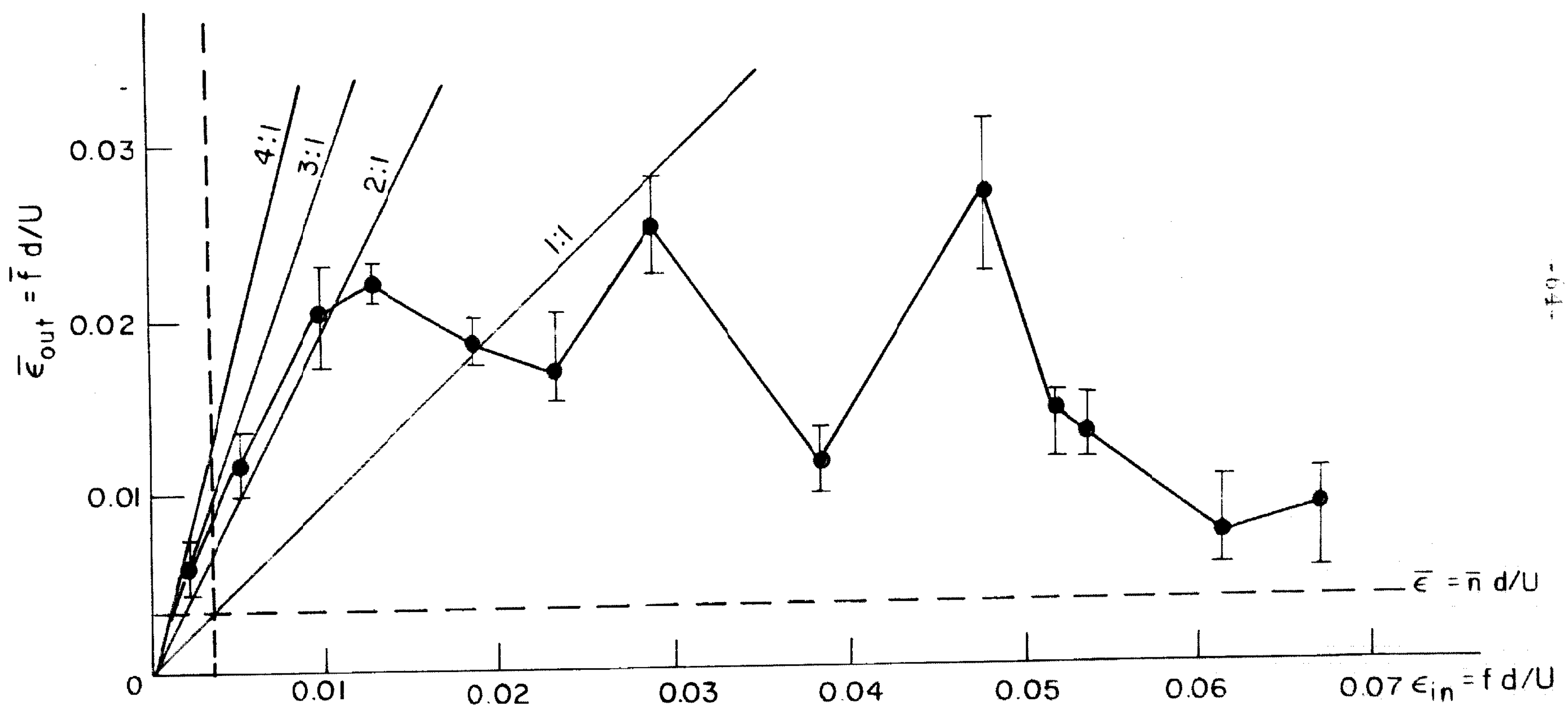


Fig. 16a: Nondimensional output frequency (ϵ_{out}) versus input frequency (ϵ_{in}). Dotted lines show nondimensional frequency from natural transition ($\epsilon_n = \bar{n}d/u$).
$\bar{f}$ = mean output frequency per sec. f = input frequency per sec.
Experiment at Re = 2100. Mean velocity u = 40 cm./sec. x/d = 3850.

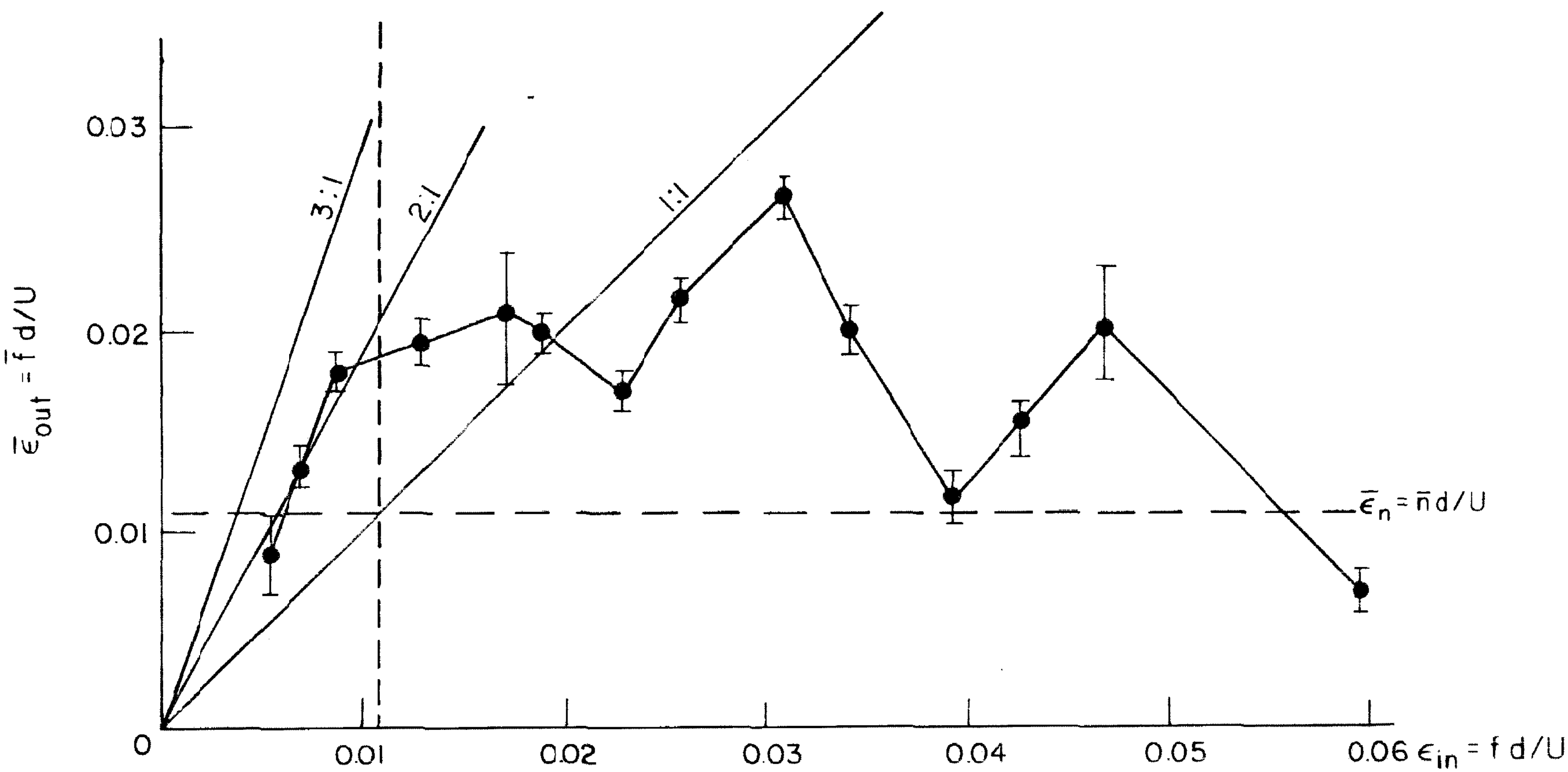


Fig. 16b. Experiment at Re = 2300. Mean velocity $\bar{u}$ = 45 cm./sec.

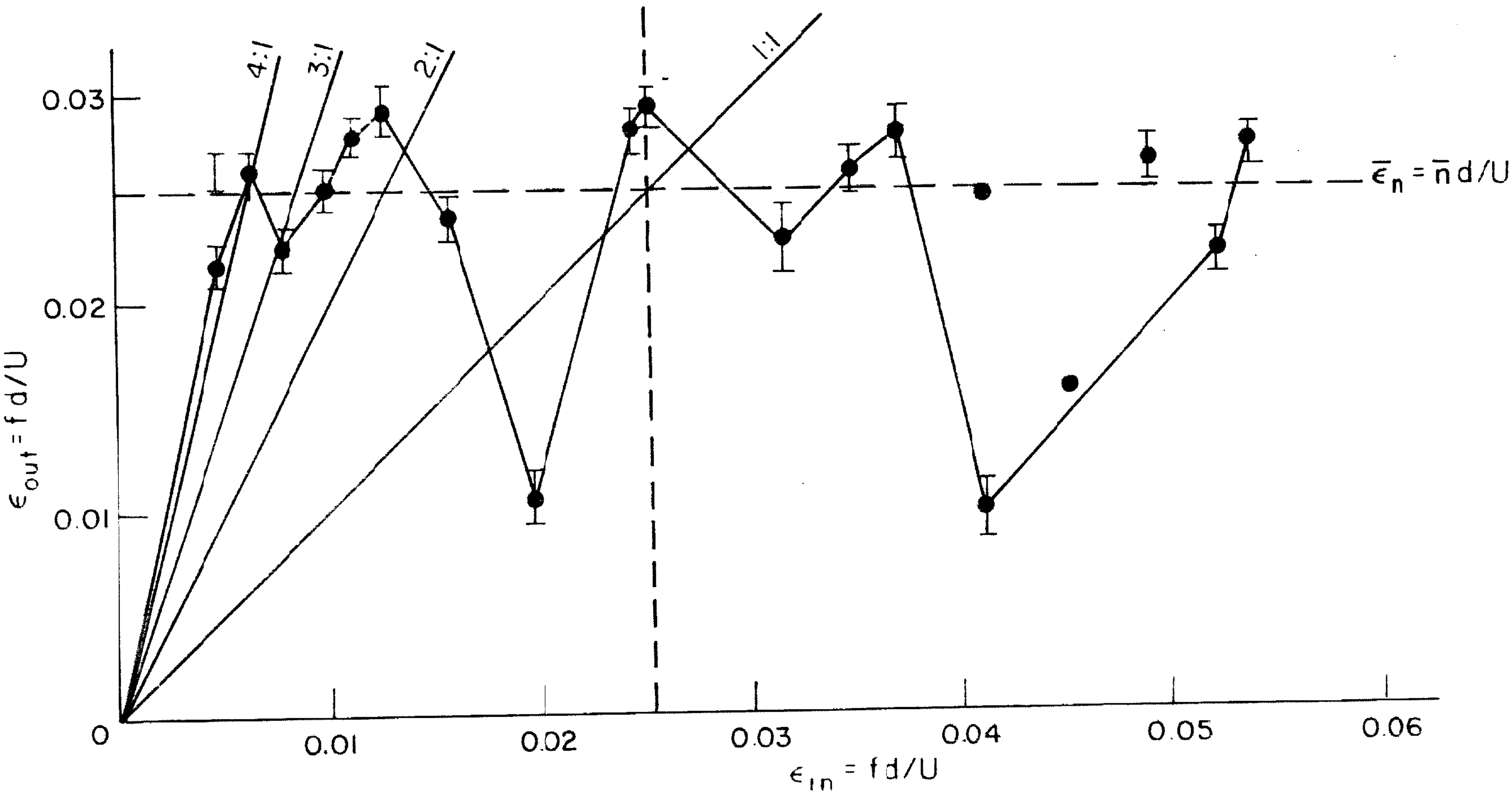


Fig. 16c. Experiment at Re = 2500. Mean velocity $\overline{u}$ = 49 cm./sec.

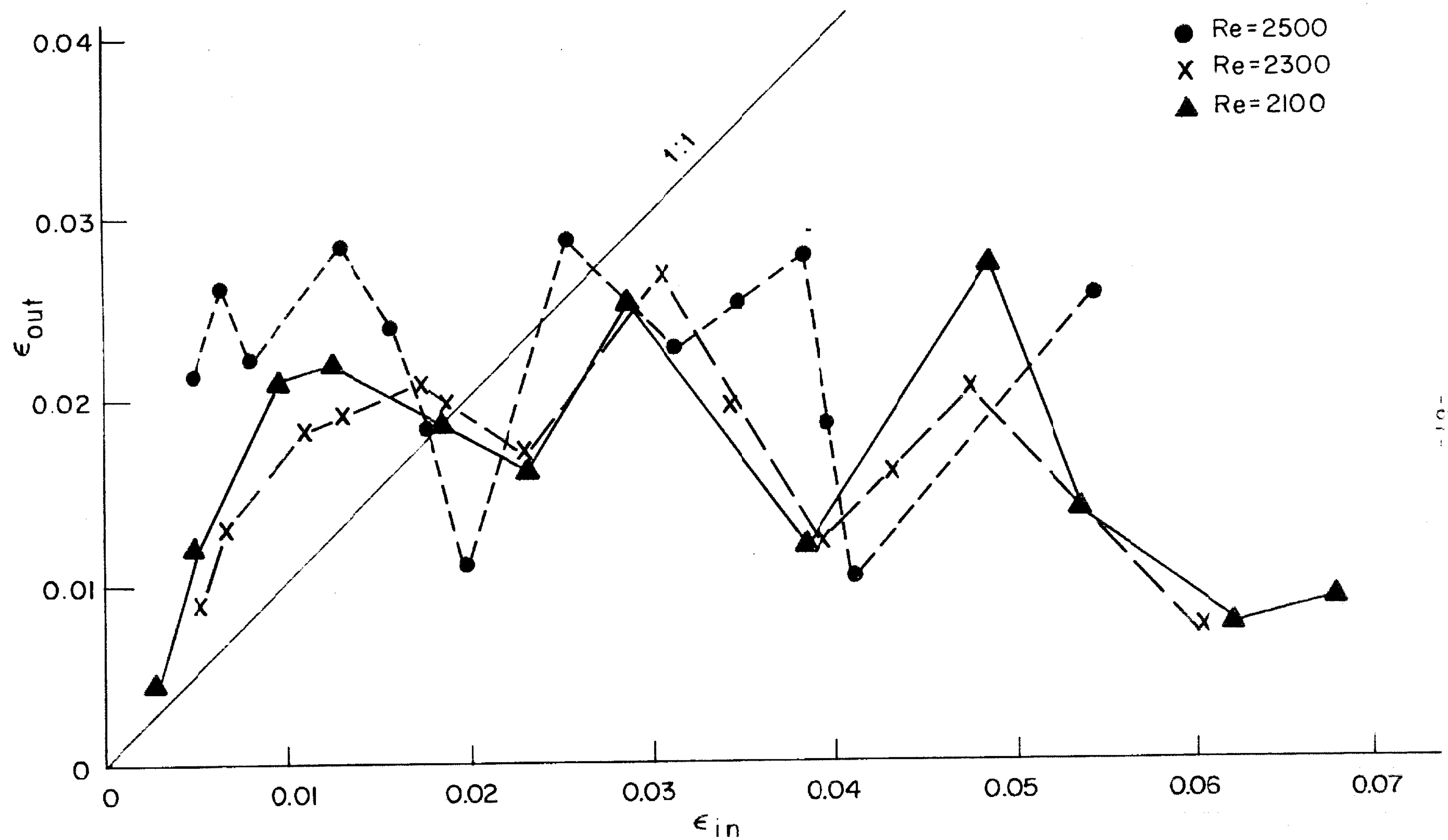


Fig. 17: Nondimensional output frequencies versus input frequencies. Comparison for different Reynolds numbers. $x/d = 3850$.

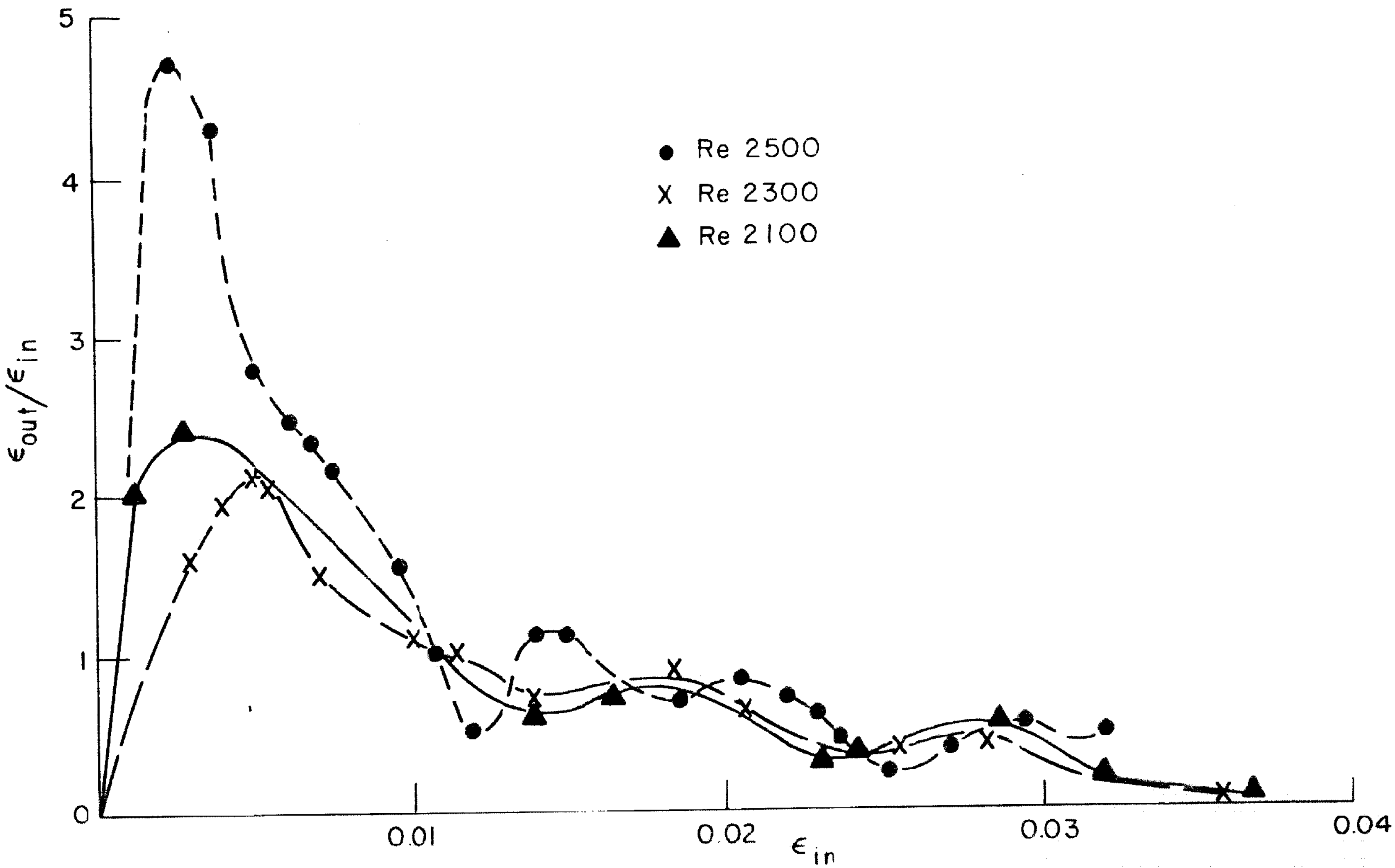


Fig. 18: Values of the ratio of output frequency to input frequency versus nondimensional input frequency at various Reynolds numbers. x/d = 3850.

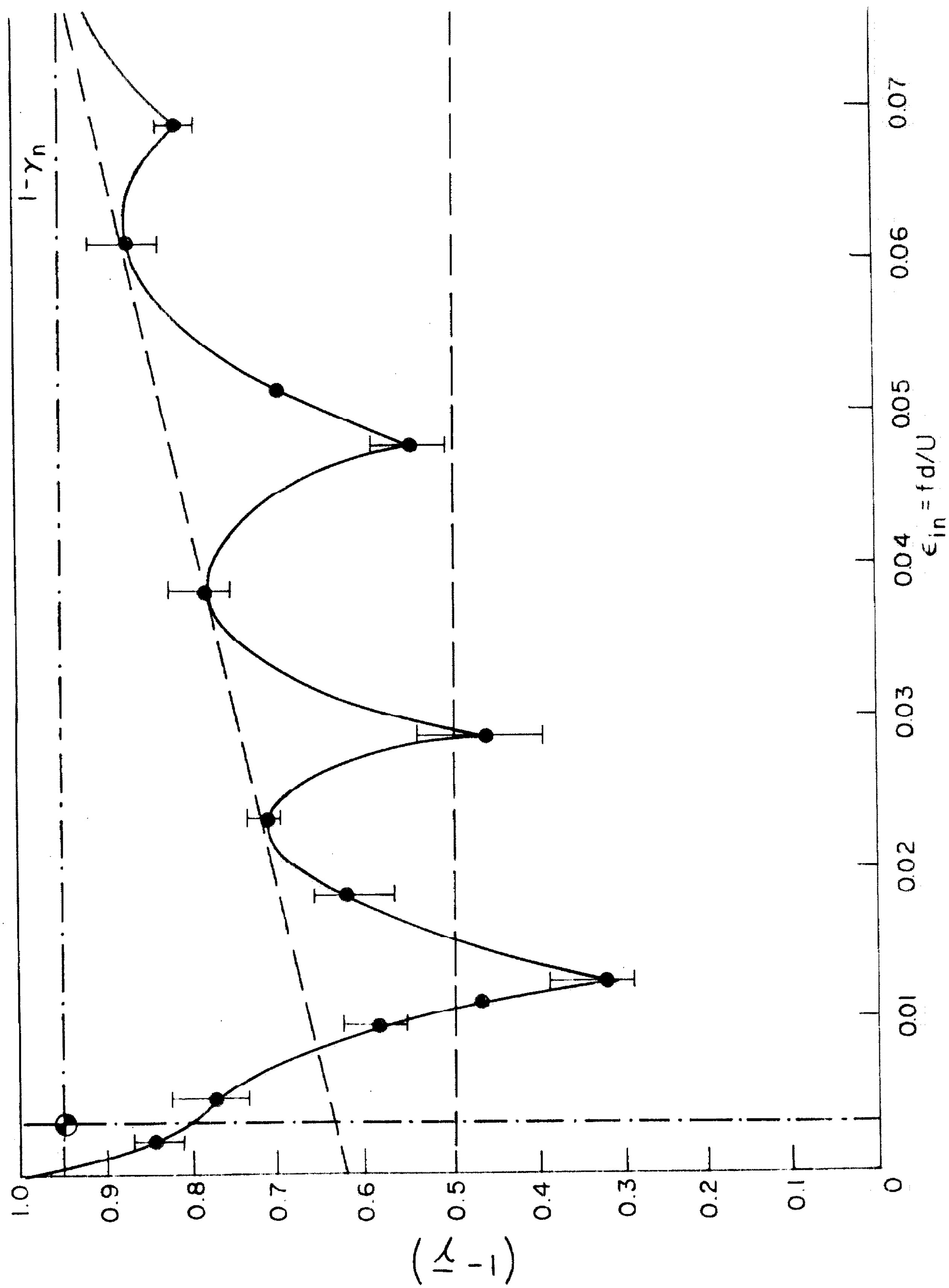


Fig. 19 (above): Percentage of laminar flow versus nondimensional input frequency at $Re = 2100$. ⊕ Point of natural transition.

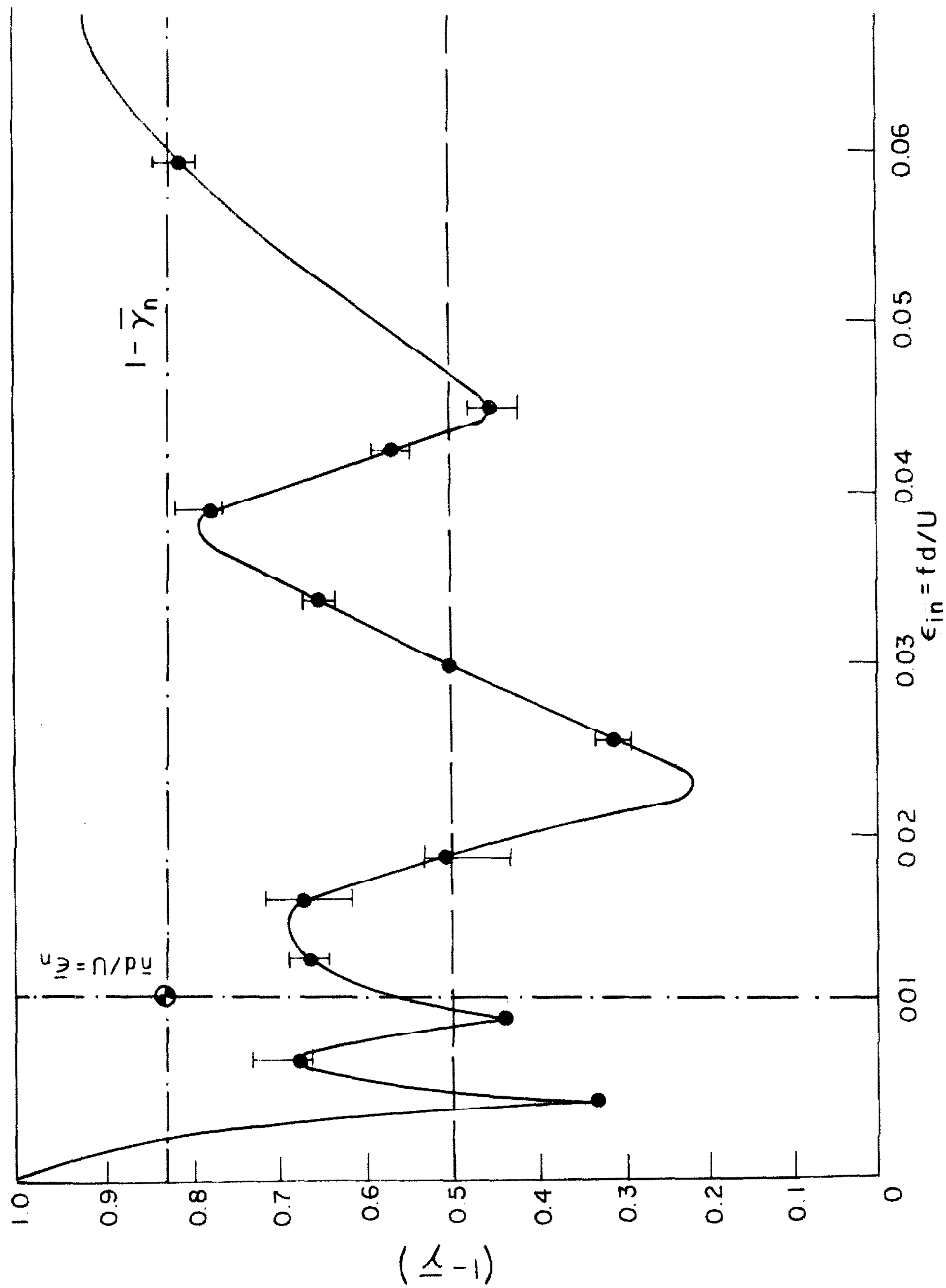


Fig. 20 (above): Percentage of laminar flow versus nondimensional input frequency at Re = 2300. ⊕ Point of natural transition.

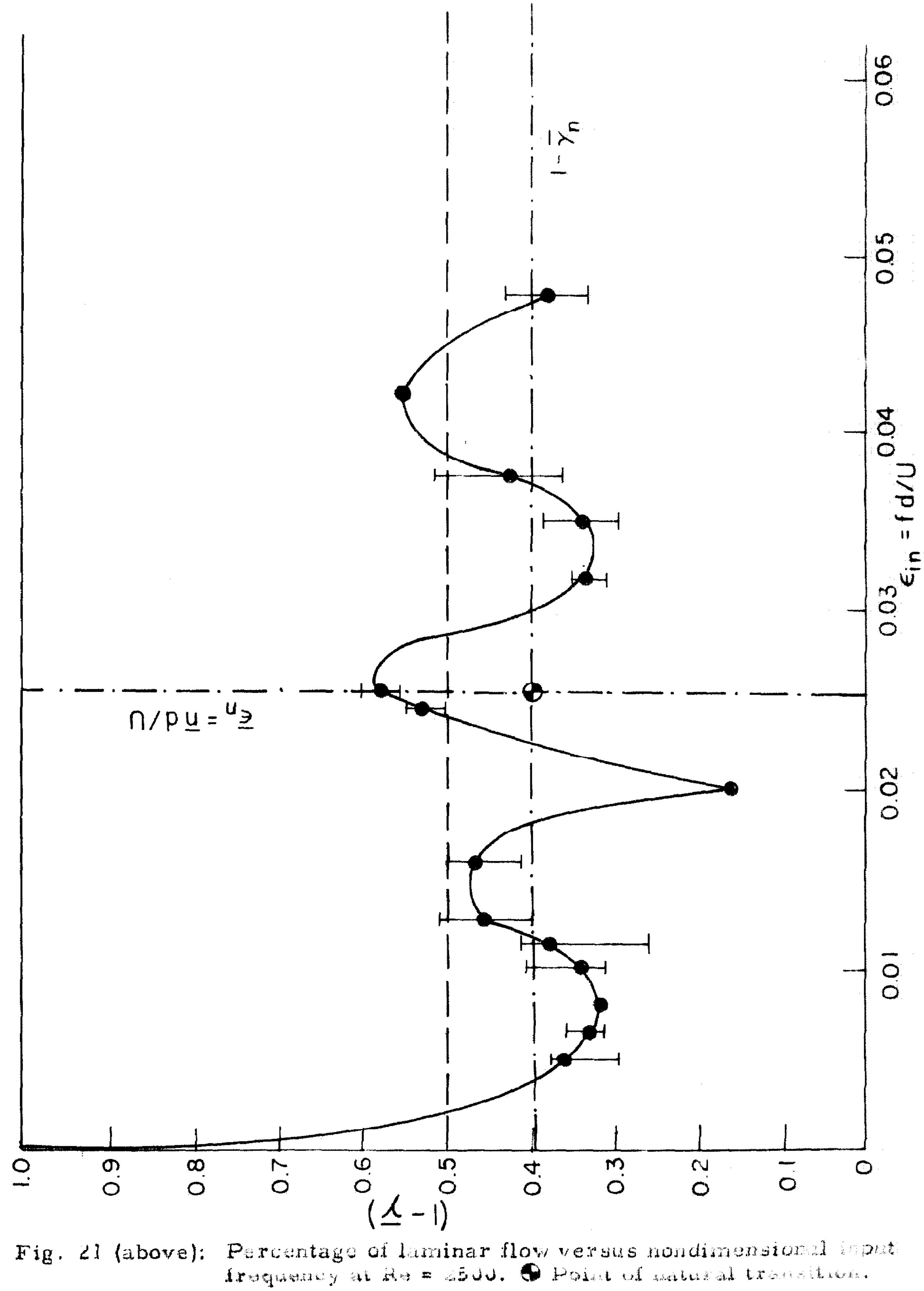


Fig. 21 (above): Percentage of laminar flow versus nondimensional input frequency at Re = 2500. ⊕ Point of natural transition.

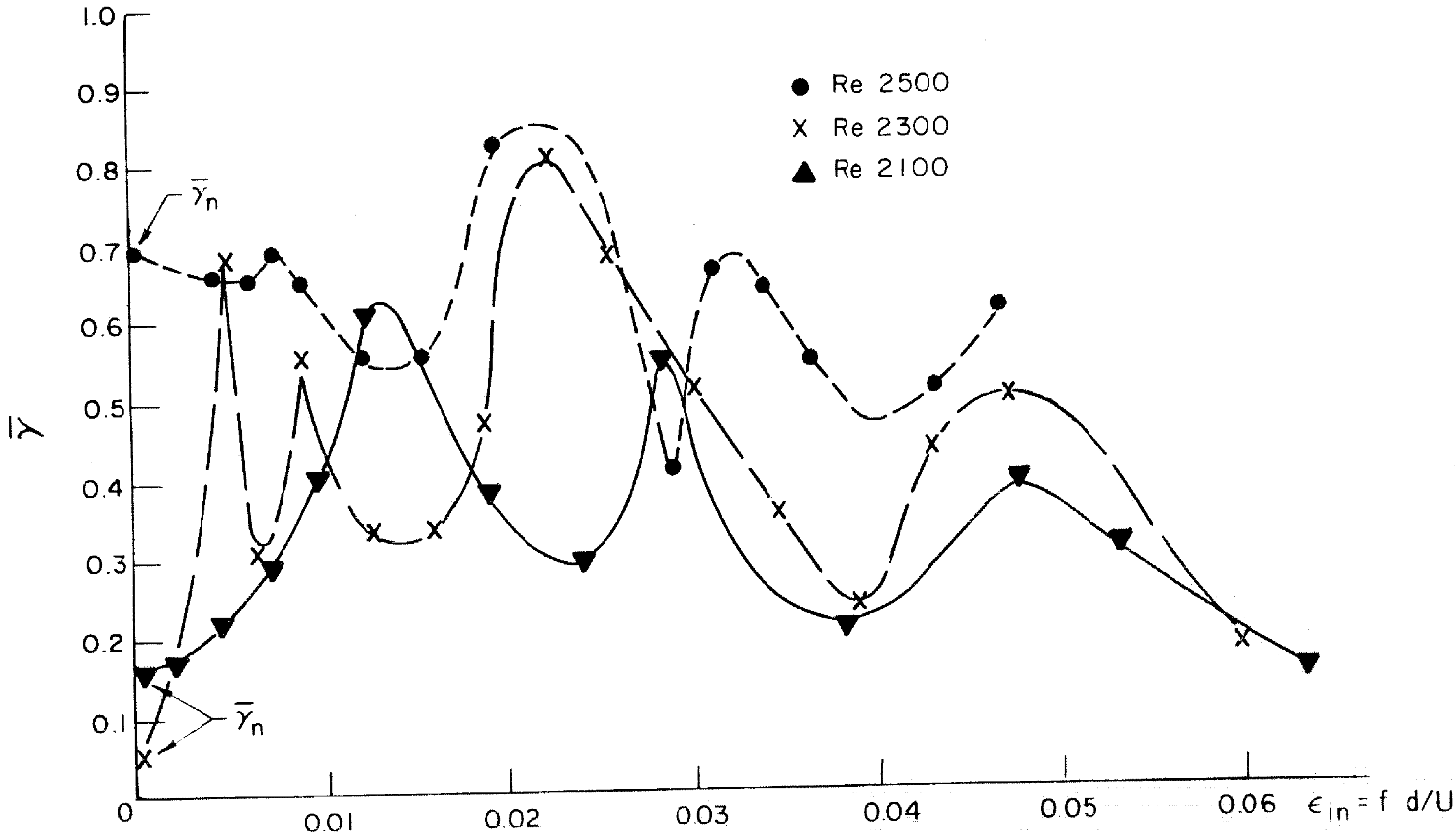


Fig. 22: Intermittency factor versus input frequency. Parameter Reynolds number. x/d = 3850.

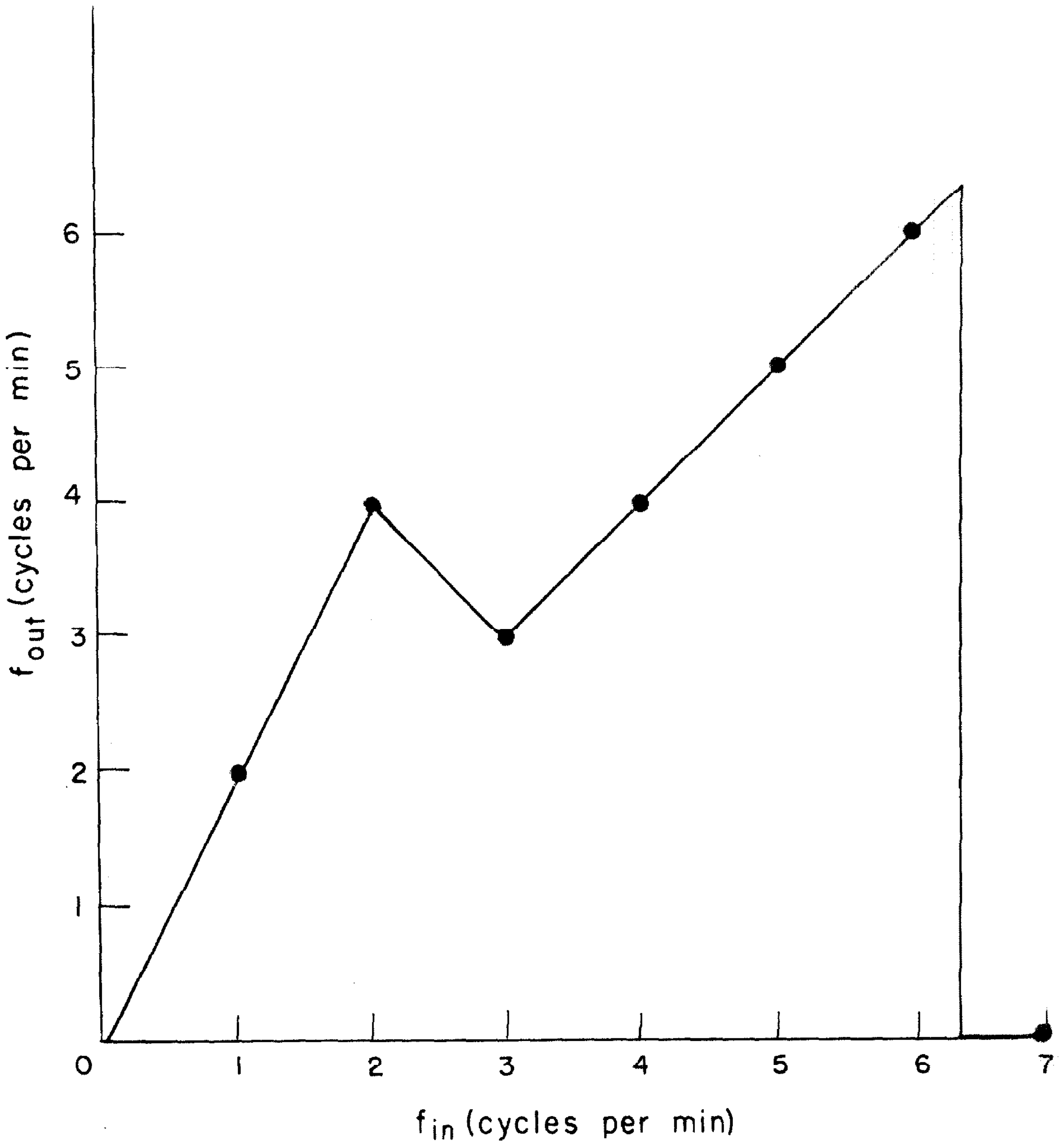


Fig. 23: Output frequency response versus input frequency. Re = 2700. Second transition range. Very low input frequency. x/d = 3850.

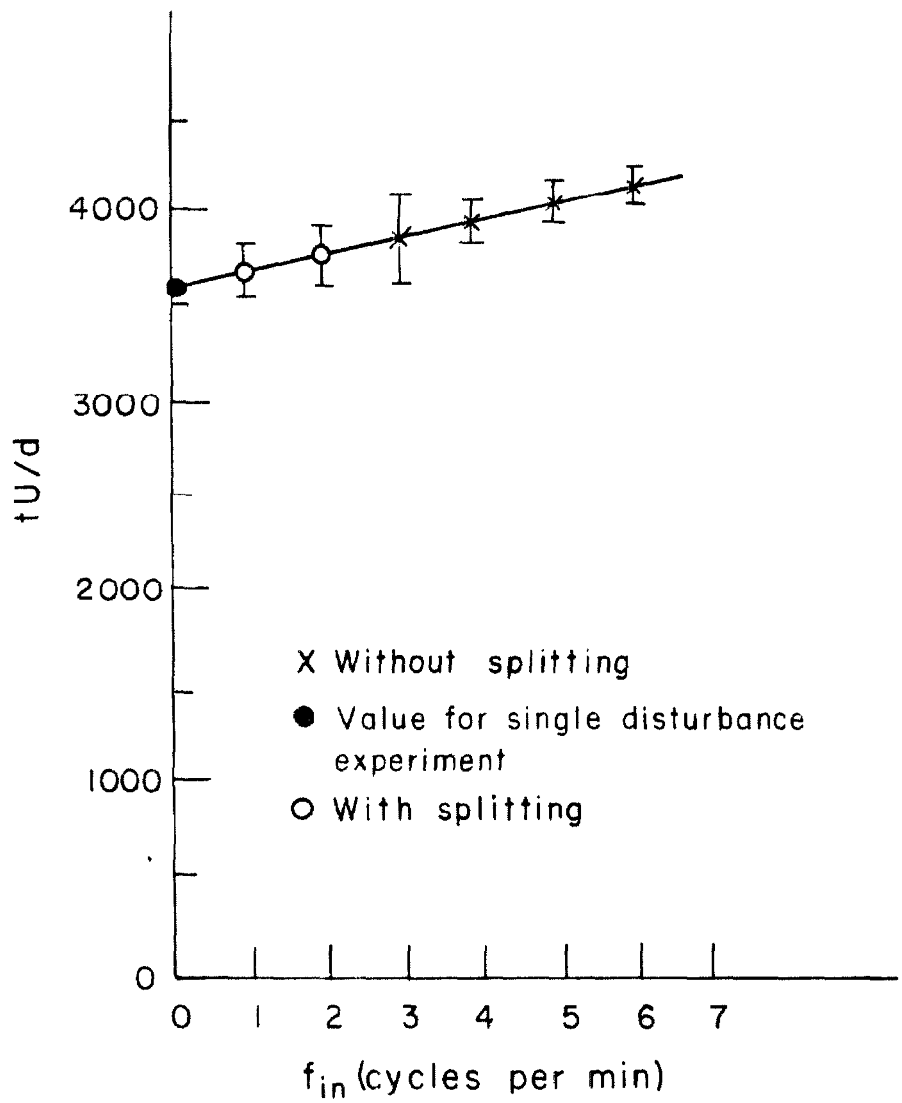


Fig. 24: Time from onset of disturbance to response versus input frequency. Re = 2700, x/d = 3850.

APPENDIX I

DISTRIBUTION OF LAMINAR AND TURBULENT INTERVALS

From the data recorded during the periodic perturbation experiments for Re = 2100, 2300 and 2500, we have been able to determine the normalized distribution function of laminar (τ_L) and turbulent (τ_T) time intervals (see page 18). These data are not included in this thesis, but it may be of interest to summarize the main results, qualitatively, as follows. For this discussion, we quote values of the time interval τ_{max}, which corresponds to a percentile $F(\tau) = .95$ on the distribution curves. Thus it is a fair representation of the extent of time intervals for the condition for which it is quoted.

At Re = 2100 turbulent values of τ_{max} are short (0.6 to 1.0 sec.) compared to the laminar ones (6 to 7 sec.). The flow has a tendency to stay laminar even if perturbed. This is typical of all conditions except in the vicinity of maxima and minima on the frequency output versus input curve (fig. 17). At the maxima the laminar values of τ_{max} are very much shorter (0.4 to 0.6 sec.) with a corresponding increase for the turbulent ones (up to 1.8 sec.). At minima points the situation is reversed--the laminar values of τ_{max} are long and the turbulent ones short.

At Re = 2500 the turbulent values of τ_{max} are 1 to 2 sec. while the laminar ones are about 0.4 to 0.5 sec. Here the tendency toward increased turbulence is evident. At the maxima of output frequency, the turbulent values of τ_{max} are reduced to 0.6 to 0.8 sec. while the laminar ones remain almost constant (0.4 to 0.5 sec.)

Corresponding to the minima, there is an increase of the turbulent value of τ_{max} to 2 or 3 sec. Thus the behaviour at this value of Re is opposite to that at 2100.

At Re = 2300 there is a change from "high Re" to "low Re" behaviour as input frequency is increased (fd/U = 0.025 is approximately the change-over value); this corresponds to a similar change of behaviour already described in regard to intermittency factors. For $fd/U < 0.025$, the laminar values of τ_{max} are small (0.6 to 0.9 sec.) and the turbulent ones are long (1.5 sec.). For $fd/U > 0.025$, the laminar values of τ_{max} are long (2 to 4 sec.) while the turbulent ones are 1 to 1.5 sec. In between there is a transition: the laminar values of τ_{max} are increasing and the turbulent values of τ_{max} are decreasing irrespective of the fact that there is a maximum in output frequency in this range.

APPENDIX II

A few examples of visicorder record paper are collected here to give a general idea of the various kinds of behaviour of the flow. The photographic copies have been reduced to one half, except those on page 83, which are reduced to one fourth. Symbol L means position for laminar flow. Symbol T means position for turbulent flow. Along the top is shown the time scale: one interval corresponds to one second. Along the bottom are recorded the input frequencies.

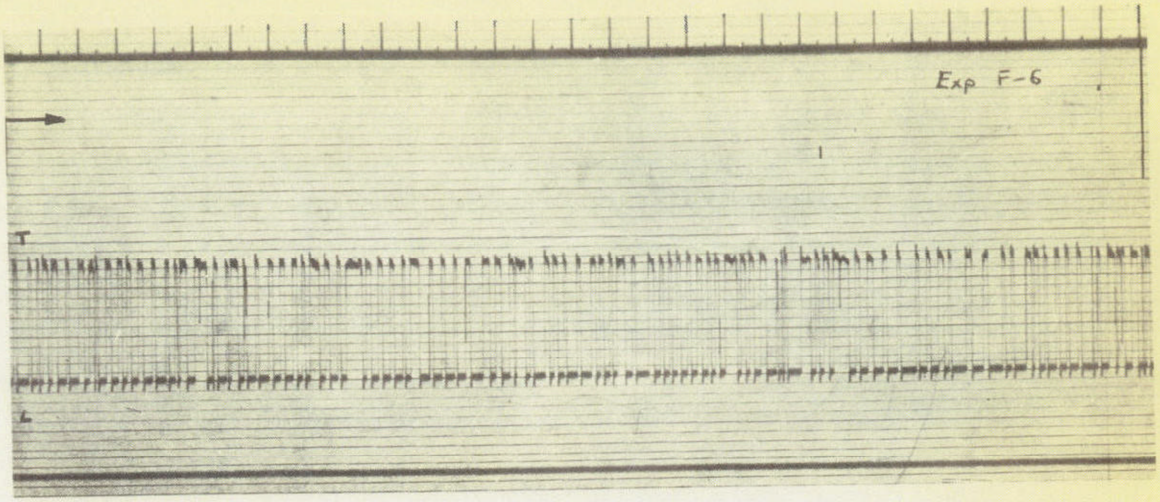


Fig. 1: Natural transition: $x/d = 4650$ Re = 2450
Intermittency factor $\overline{\gamma} = 0.425$
Nondimensional frequency $\overline{\epsilon} = 0.0297$

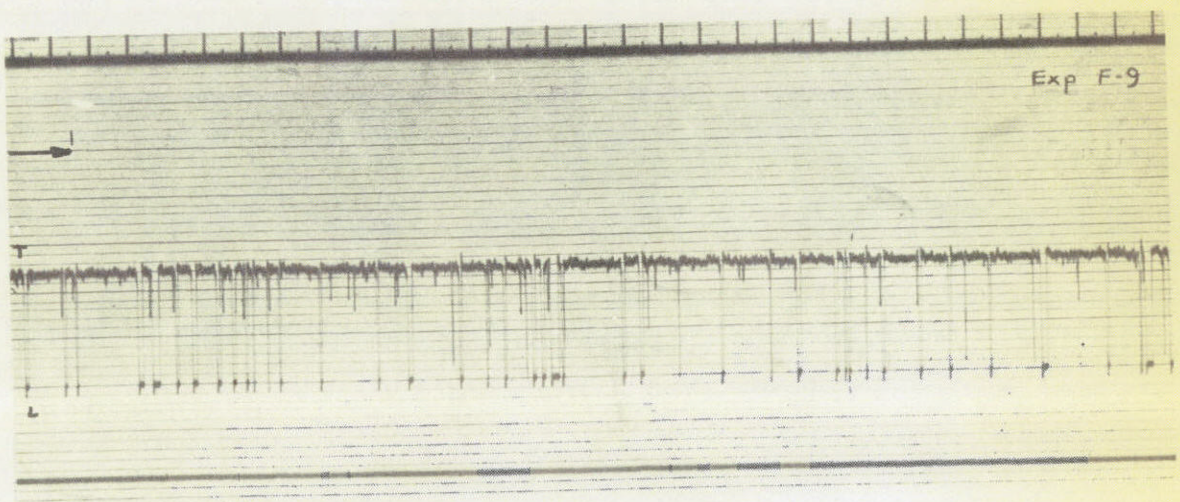


Fig. 2: Natural transition: $x/d = 4650$ Re = 2610
Intermittency factor $\overline{\gamma} = 0.915$
Nondimensional frequency $\overline{\epsilon} = 0.0074$

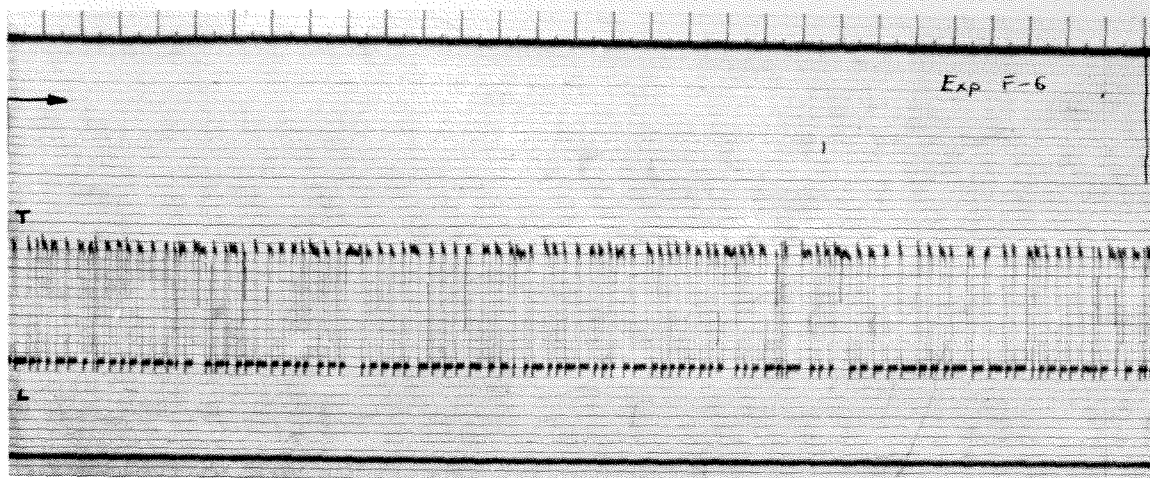


Fig. 1: Natural transition: x/d = 4650 Re = 2450
Intermittency factor $\overline{\gamma}$ = 0.425
Nondimensional frequency $\overline{\epsilon}$ = 0.0297

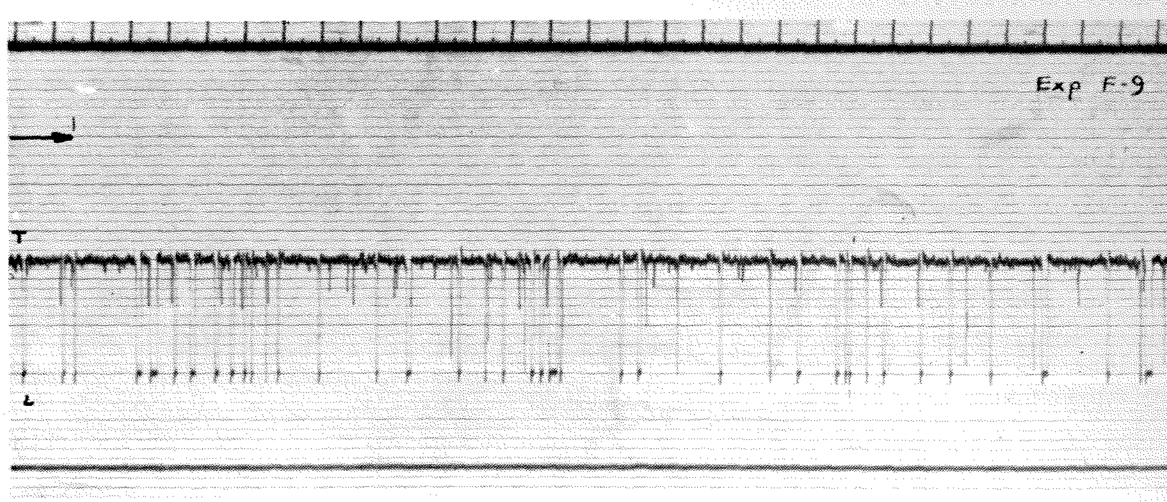


Fig. 2: Natural transition: x/d = 4650 Re = 2610
Intermittency factor $\overline{\gamma}$ = 0.915
Nondimensional frequency $\overline{\epsilon}$ = 0.0074

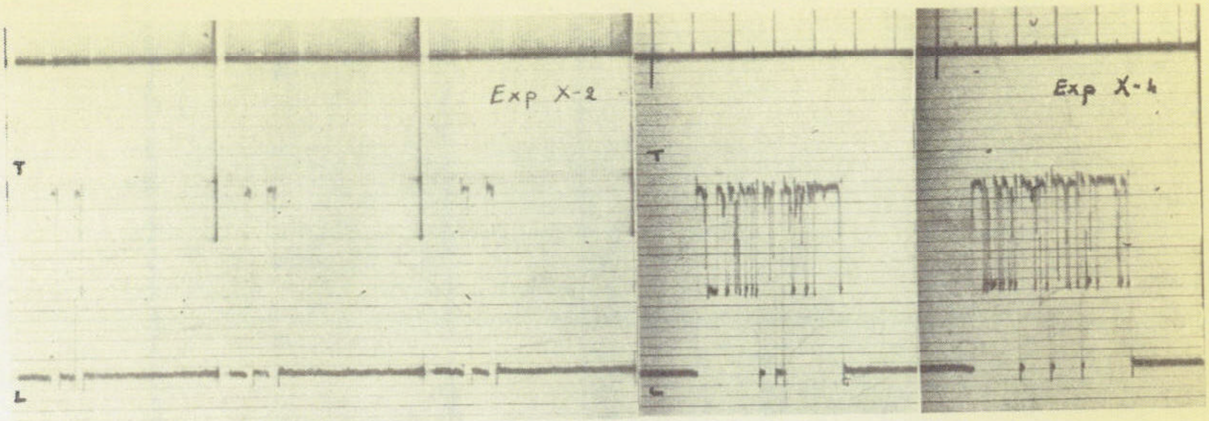


Fig. 3: Single disturbance: x/d = 3850
Splitting at Re = 2200; $u_F/u = 0.92$; $u_R/u = 0.91$
Splitting at Re = 2400; $u_F/u = 0.915$; $u_R/u = 0.885$

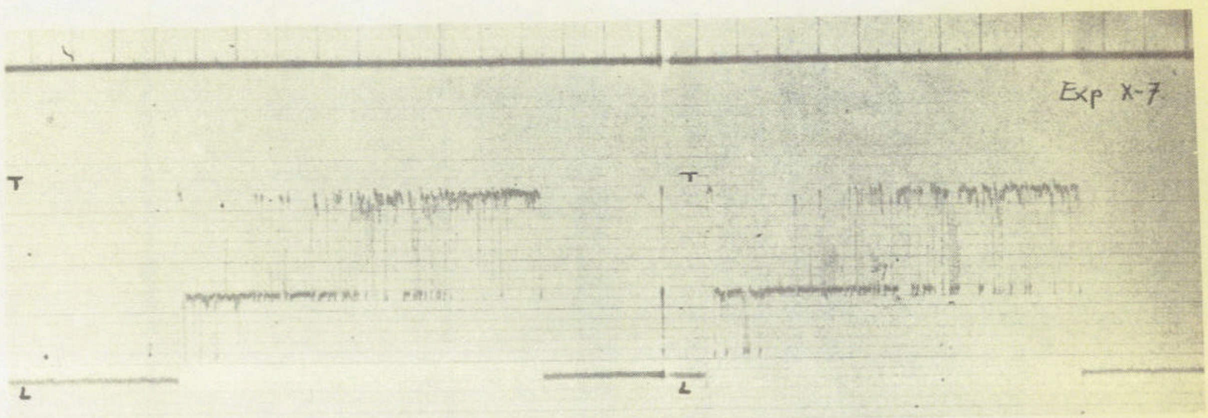


Fig. 4: Single disturbance: x/d = 3850
Growing at Re = 2780; $u_F/u = 1.07$; $u_R/u = 0.82$

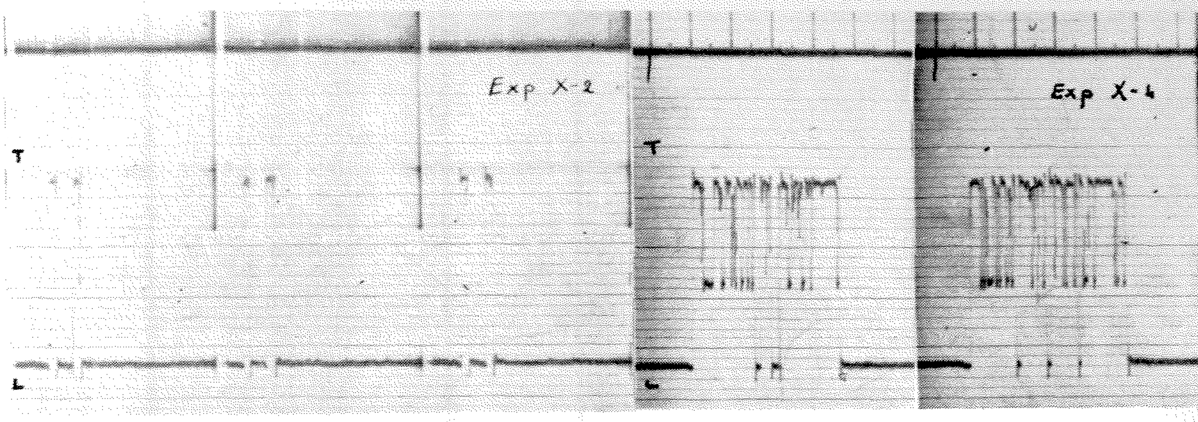


Fig. 3: Single disturbance: $x/d = 3850$
Splitting at $Re = 2200$; $u_F/u = 0.92$; $u_R/u = 0.91$
Splitting at $Re = 2400$; $u_F/u - 0.915$; $u_R/u = 0.885$

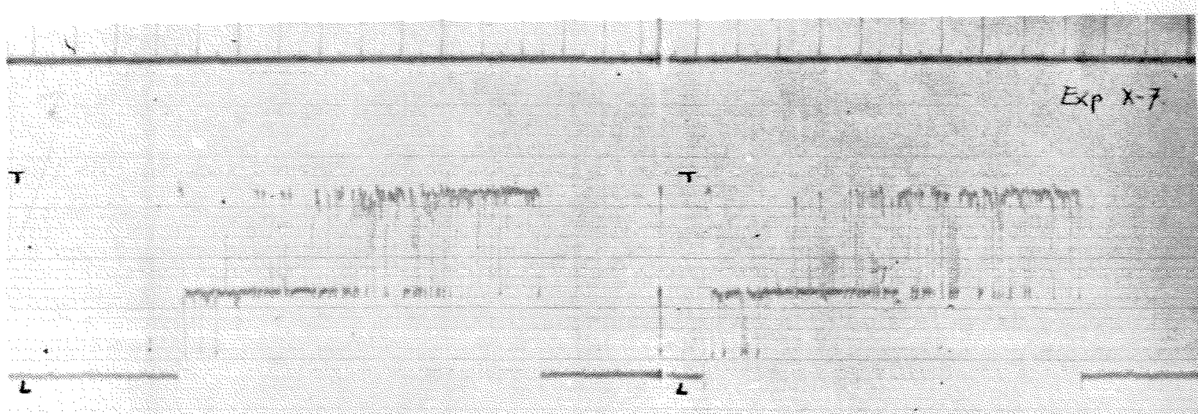


Fig. 4: Single disturbance: $x/d = 3850$
Growing at $Re = 2780$; $u_F/u = 1.07$; $u_R/u = 0.82$

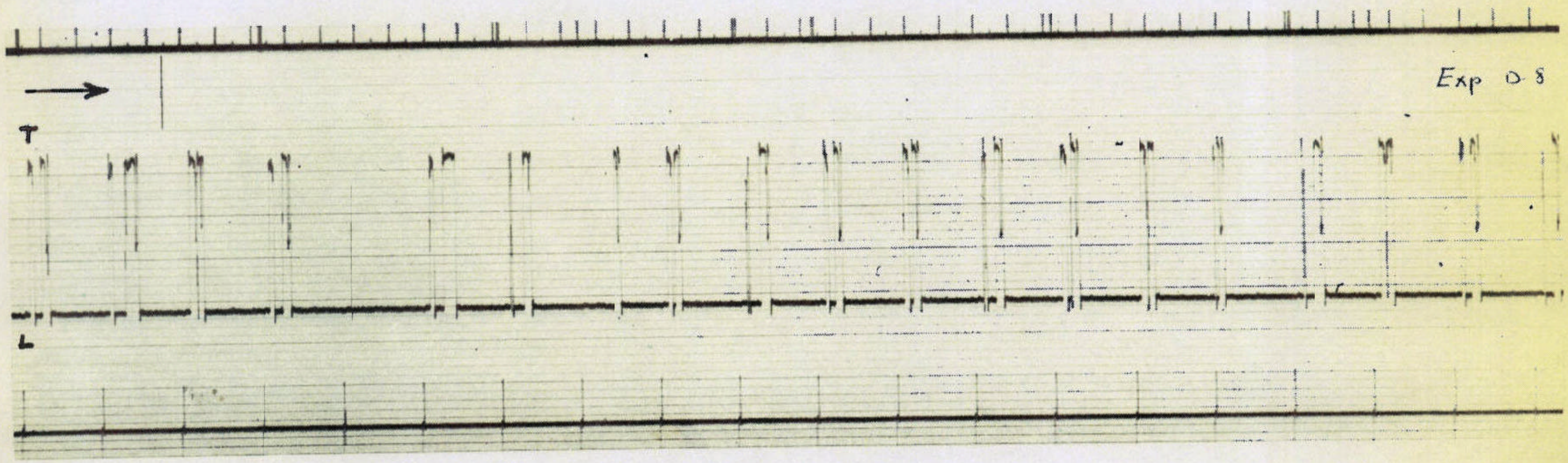


Fig. 5: Periodic disturbances: x/d = 3850 Re = 2100
Input frequency: 25 cycles per minute
Response output frequency: 50 cycles per minute
Almost regular doubling
Intermittency factor $\overline{\gamma} = 0.18$

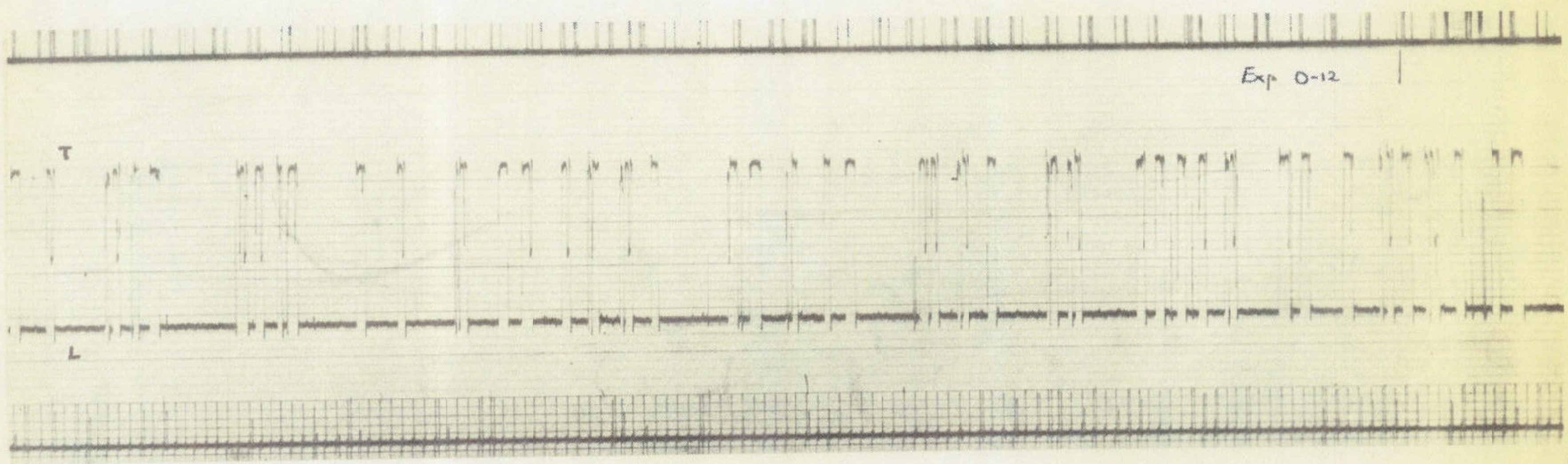


Fig. 6: Periodic disturbances: x/d = 3850   Re = 2100
Input frequency: 200 cycles per minute
Response output frequency: 80 cycles per minute
Point of minimum
Intermittency factor $\overline{\gamma} = 0.21$

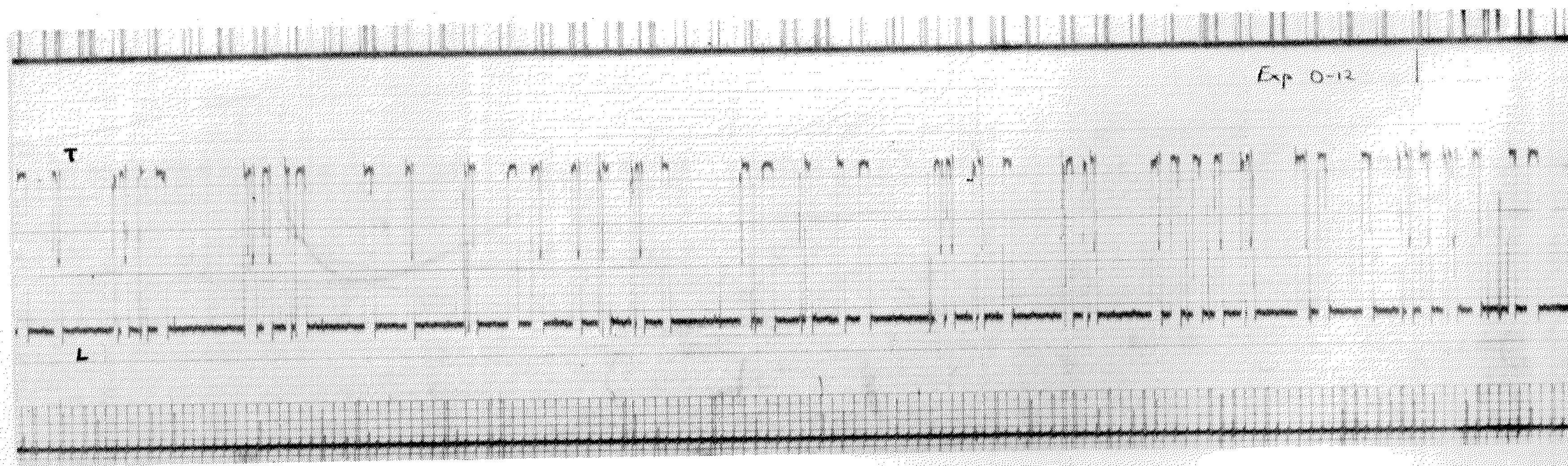


Fig. 6: Periodic disturbances: x/d = 3850 Re = 2100
Input frecuency: 200 cycles per minute
Response output frequency: 80 cycles per minute
Point of minimum
Intermittency factor $\overline{\gamma}$ = 0.21

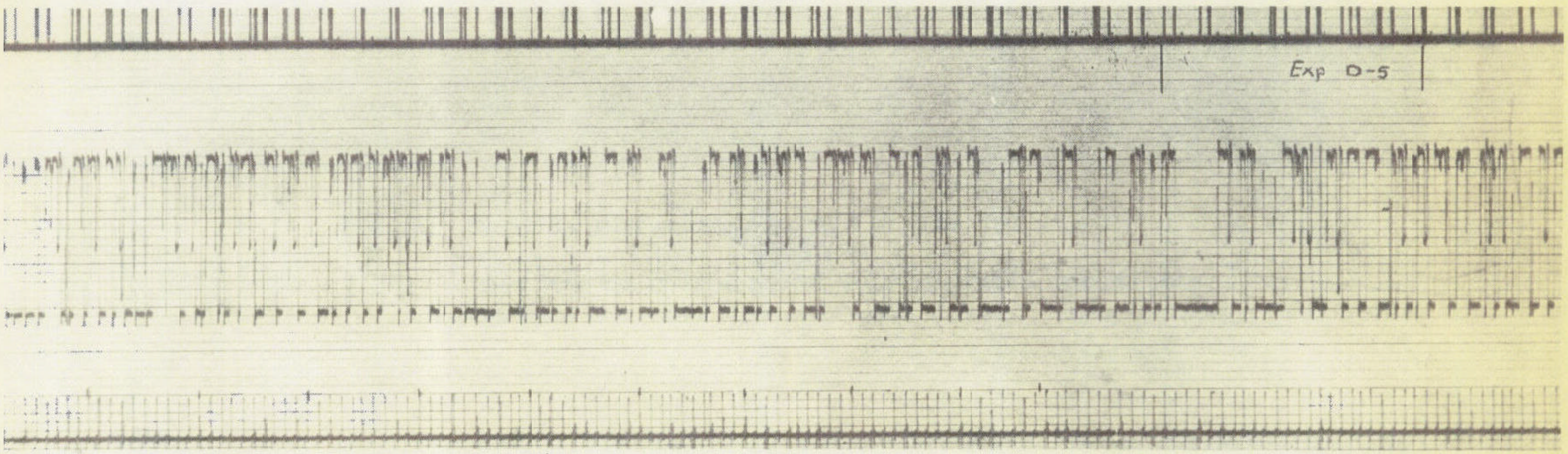


Fig. 7: Periodic disturbances: $x/d = 3850$ Re = 2100
Input frequency: 250 cycles per minute
Response output frequency: 140 cycles per minute
Point of maximum
Intermittency factor $\overline{\gamma} = 0.42$

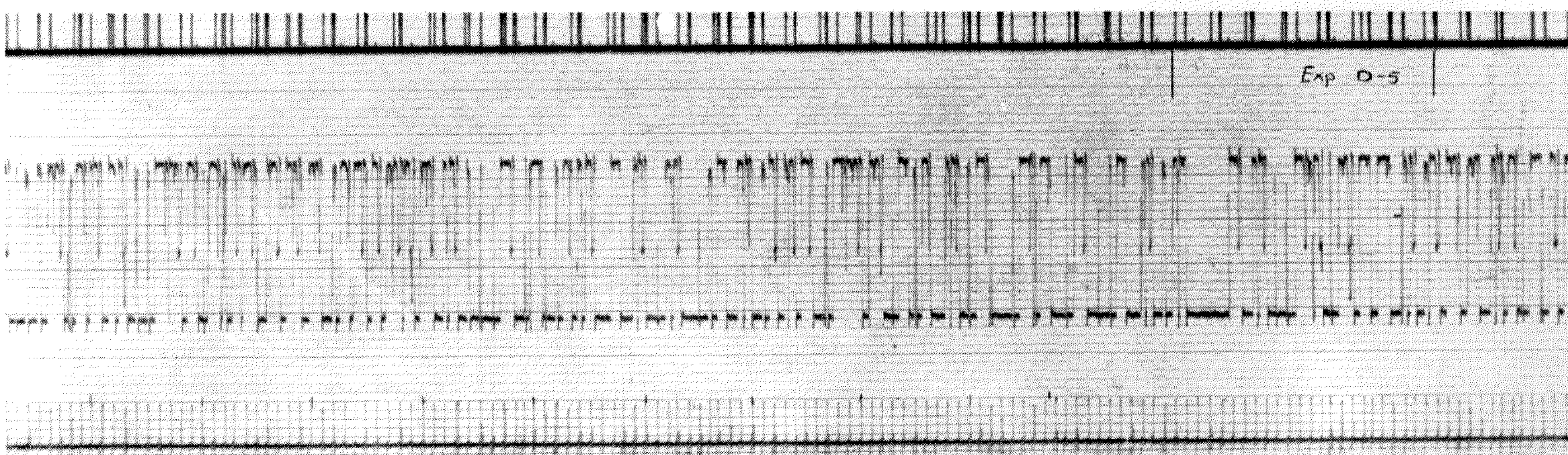


Fig. 7: Periodic disturbances: x/d = 3850 Re = 2100
Input frequency: 250 cycles per minute
Response output frequency: 140 cycles per minute
Point of maximum
Intermittency factor $\overline{\gamma} = 0.42$

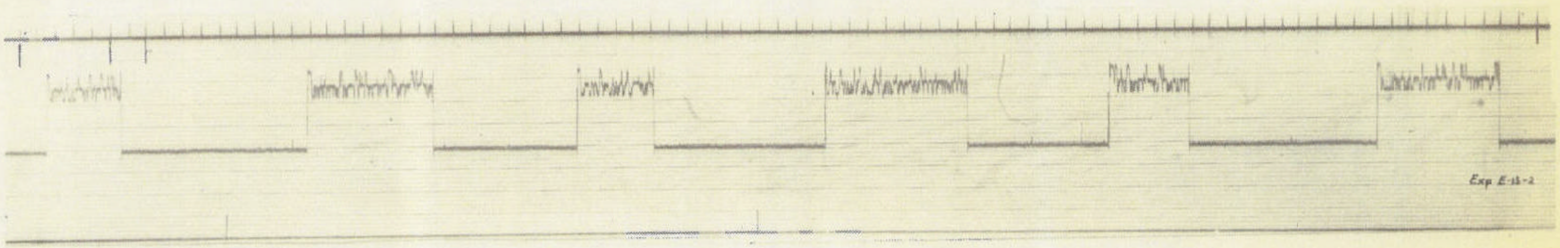


Fig. 8: Periodic disturbances: x/d = 3850
Second transition Re = 2700
Very low input frequency f_{in} = 2 cycles per minute
Double responses synchronized

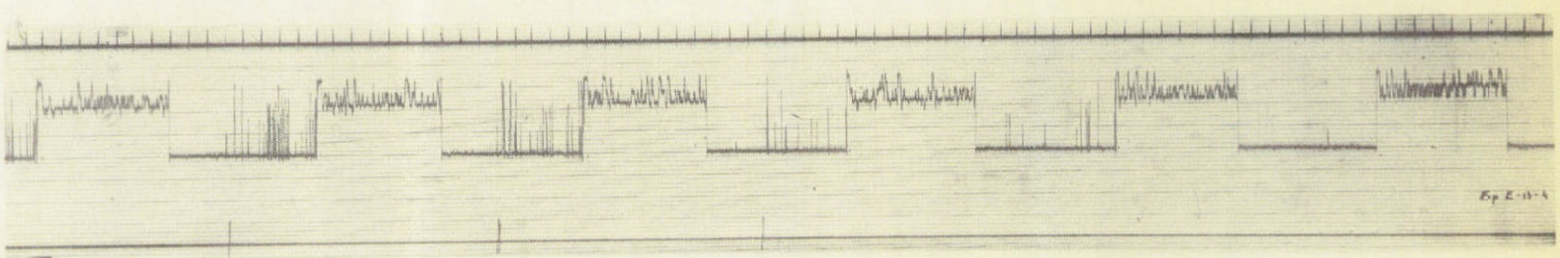


Fig. 9: Periodic disturbances: x/d = 3850
Second transition range Re = 2700
Very low input frequency f_{in} = 4 cycles per minute
Single responses synchronized

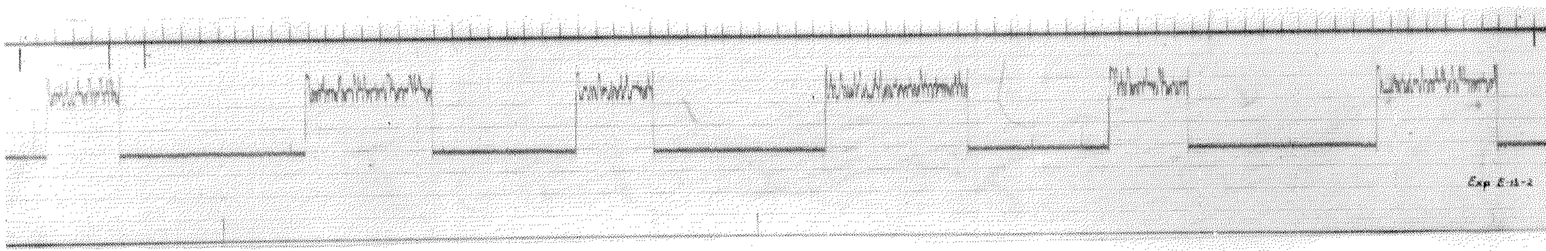


Fig. 8: Periodic disturbances: x/d = 3850
Second transition Re = 2700
Very low input frequency f_{in} = 2 cycles per minute
Double responses synchronized

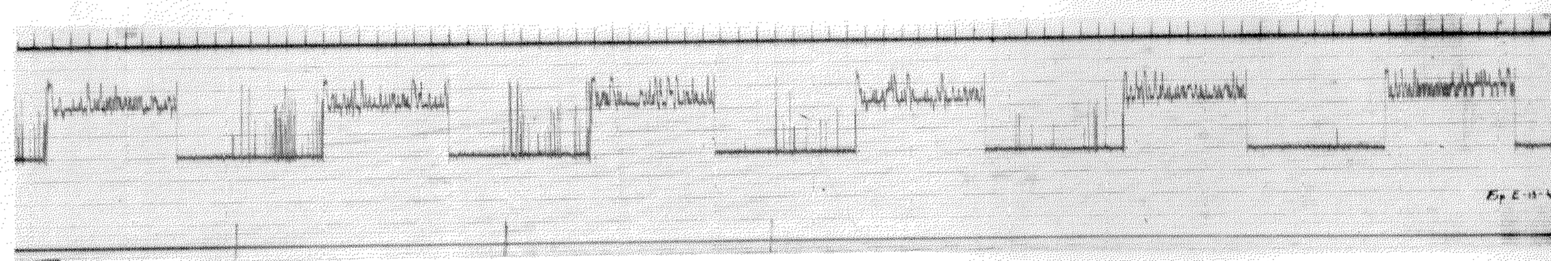


Fig. 9: Periodic disturbances: x/d = 3850
Second transition range Re = 2700
Very low input frequency f_{in} = 4 cycles per minute
Single responses synchronized