VARIABLE STARS WITH PERIODS GREATER THAN ONE DAY IN GLOBULAR CLUSTERS

Thesis by

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Dr. William A. Baum, who measured, photoelectrically, important comparison stars in two other clusters.

ABSTRACT

Nineteen variable stars with periods greater than one day were observed in six globular clusters. The observational techniques and the accuracy of the measurements are discussed and light curves for 15 variables are exhibited.

Color-magnitude diagrams for six clusters are presented and the distance modulus to each is derived.

The resulting period-luminosity relation for type II cepheids indicates there are actually three separate lines in the period-luminosity diagram which these variables occupy. Means of identifying stars on these different lines are presented. The factor of two in the period which these lines are from each other is discussed from the standpoint of the nature of cepheid variation. Finally a hypothesis is advanced to explain the connection between type I and type II cepheids.

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VARIABLE STARS WITH PERIODS GREATER THAN ONE DAY IN GLOBULAR CLUSTERS

Introduction

To picture the universe one must know the distance between objects in it and the most used method for measuring these distances is by observing stars called cepheid variables. A cepheid is any of a group of stars which pulsate with periods ranging from around two hours to one hundred days and show a continuous range of physical properties throughout. The relationship of the property of intrinsic brightness to period yields the distance of a cepheid whose apparent brightness and period are known. This use of cepheids to determine the distance scale of the universe has made the nature of their variation and period-luminosity relation one of the most important subjects of astronomy.

The dimensions of our own galaxy depend on measures of cepheids with periods less than one day (RR Lyrae stars). The distances to the nearby galaxies depend on cepheids of periods greater than one day (long period cepheids). Since 1944 it has become evident that just as there are two types of stars, there are type I and type II long period cepheids. Because type II stars are defined as the kind of stars that occur in globular clusters and type I as the kind that occur in conjunction with gas and dust, there is no sample of both types of cepheids coexistent, in our galaxy, in which we can study the relationship between them. In nearby galaxies where both type I and type II cepheids can be observed together, the RR Lyrae stars (exclusively type II) are too faint and hence their relationship to the long period type I, or classical cepheid, remains unknown*. In particular, there is no certainty that the distance scale in our own galaxy derived from RR Lyrae stars is identical with the distance scale to nearer galaxies derived from classical cepheids.

The procedure for checking the equivalence of these two distance scales is clear. Step one is to calibrate accurately the brightness of the type II long period cepheids relative to the RR Lyrae stars. Step two is to determine the brightness of these long period type II cepheids relative to the classical cepheids. This will give the brightness of classical cepheids in terms of the RR Lyrae stars and therefore unify the two distance scales.

The primary objective of this investigation was to realize step one and firmly connect the long period type II cepheids with the RR Lyrae stars. Secondarily, it was hoped that more data on type II long period cepheids would clarify their relationship to the classical cepheids.

In our galaxy globular clusters, uniquely, contain both RR Lyrae and long period cepheids, all at effectively

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* The small Magellenic cloud is a recent exception that will be mentioned in Part V.

the same distance from the observer. Moreover, since they are prototypes of type II population, for this investigation they furnish an ideal group of objects in which to ascertain the connection between RR Lyrae stars and type II cepheids. It was also felt that further study of a number of globular clusters would yield valuable information on the properties of type II stars and systems of stars.

A few stars with periods around one hundred days, presumably regular or semi-regular red variables, were present in these clusters and were studied from the standpoint of their connection with the sequence of red variables known in the galactic field and neighborhood of the sun.

Extensive observations of variables in globular clusters have been made in the past and catalogued by Helen Sawyer Hogg of David Dunlap Observatory. This work was used as the basis for selecting globular clusters containing a maximum number of long period cepheids. Further, the older observing techniques were extended in the following four ways.

1) Some stars in each globular cluster were measured photoelectrically thus ensuring that all apparent magnitudes were systematically correct to a few hundreths of a magnitude.

2) Photovisual observations were made along with all

photographic observations to give checks on the correctness of the photographic observations and to give colors as well as magnitudes of the objects observed.

3) Fine grain, blue-sensitive emulsions almost doubled the usual accuracy of the photographic measures.

4) The variable diaphragm photometer used to measure the star images on the plates gave results exact to the ultimate limit of accuracy set by the grain of the plate.

The following pages contain the results of these measures on 13 type II cepheids with periods ranging from 1.4 to 33.6 days, 3 red variables with periods around 100 days, and 3 irregular red variables. The fundamental aim of the investigation has been achieved, namely to find the exact brightness of the long period, type II cepheids with respect to the RR Lyrae stars. That the period-luminosity relation turns out to be more complicated and more exact than previously supposed is interesting in itself. But more important still is the possibility that this new relation will give basic insight into the problems of what is the actual physical process of cepheid variation and what is the connection between type I and type II cepheids.

PART II

Observational Procedure

1. Selection of clusters

The first requirement of the program was to obtain well defined light curves for the variables. This demanded as many consecutive nights of observation as possible in order to have evenly spaced measures of each star's brightness at all phases. The time available during one observing night was sufficient to observe about six clusters. The clusters M3, M5, M13, M14, M15 and M2 were chosen, then, because they were all accessible from the latitude of Mt. Wilson and because they contained the largest possible number of long period cepheids, all bright enough to register on short exposures. Adequate photoelectric calibration could not be obtained for M14 so it was dropped from the program and M10 substituted.

The observing list then included twelve known long period cepheids, two stars with periods around one hundred days and three irregular variables(1). During the course of the program one additional long period cepheid was discovered, a period was determined for one of the stars previously listed as irregular and a new irregular variable was found. These 19 variables were observed on a total of 65 nights starting in March, 1952 and extending through October 1952, a span of about 230 days. The clusters were distributed in right ascension from about 13^h to 21^h and all were observed from almost the beginning to the very end of the program. Good fortune in having clear weather and long observing runs generously assigned by the Mt. Wilson and Palomar Observatories made succesful completion of the program possible in one observing season.

2. The usable field of the telescope.

Almost all plates were taken at the newtonian focus of the 60-inch telescope on Mt. Wilson. At the usual focal ratio of f/5, the comatic aberration of such a reflector seriously distorts stellar images at small radial distances from the optical center of the plate. The effect of this on measured magnitudes was investigated by taking plates of selected areas 57 and 61. The deviation of each star measured from its known magnitude(2) was plotted against its distance from the center of the plate.

Stars more than a magnitude brighter than the plate limit are enlarged by coma and measure too bright, stars fainter than this are also enlarged but the weakened light fails to record and they measure too faint. The result is a set of correction curves rather asymptotically approaching zero with decreasing radius, negative corrections for bright stars and positive for faint. These correction curves are very sensitive to seeing, focus and guiding in the sense that small, dense images are more affected by distortion and give larger corrections. It is therefor difficult to correct comatic images and the only practical procedure is to stay within the coma free field.

The selected area plates showed, that for all observing conditions, the corrections were less than $\overset{m}{.}01$ inside a radius of 300". This is for the 60-inch used at full aperature. Since the coma size varies as the reciprocal of the f ratio squared, reducing the aper ture with a diaphragm extends the size of the coma-free field. It turned out that for the 60-inch diaphragmed to 40 inches, the corrections were less than $\overset{m}{.}01$ for stars inside a radius of 470", diaphragmed to 32 inches the usable field was 690".

All plates were taken with the 32-inch aperature giving a correction free field of 23 minutes of arc across. All variable stars and comparison stars fell within these limits and consequently all distance to center effects were avoided. In addition, whenever possible, the variable stars were exposed to about a magnitude above the plate limit where the cross over from negative to positive corrections would extend the correction free field still further.

Reducing the aper ture to 32 inches increased the exposure times by nearly a factor of four. The variables were so bright, however, and the exposure times so short, that this did not mean anything in comparison to the time consumed in setting the telescope from cluster to cluster.

In fact, with the 32-inch diaphragm some exposures were only thirty seconds, anything shorter introduces errors in pulling the plate slide.

The optical axis of the 60-inch wanders unpredictably about the plate center by amounts as large as 100". Constant care was taken that the optical axis stay on the plate center. At first this was done by taking collimation plates frequently but later the adjustment was made by observing the symmetry properties of the mirror during the knife edge test for focus.

Such precautions made it impossible to recognize distance-to-center effects on any of the plates. In fact there was probably a wide margin of safety in all cases.

3. Measuring magnitudes with the astrophotometer.

Star images on the photographic plates were converted to magnitudes by means of the variable diaphragm astrophotometer(3) at the California Institute of Techology. This instrument closes an iris diaphragm over the star image until the star blocks enough light to balance a comparison beam. If the magnitudes of a few of these stars are known they can be plotted against the diaphragm reading and a calibration curve results. This calibration curve is quite linear in most cases. Its slope is steeper for good seeing plates with small crisp images. This means that moderate seeing plates with average image sizes actually give the most accurate results. At about one-half a magnitude brighter than the plate limit, depending on image quality, the light blocked by the image becomes more dependent on density than diameter and the calibration relation begins to curve over. Very near the plate limit the measures become quite inaccurate due both to the almost vertical slope of the calibration curve and the random fluctuation of the diaphragm settings. The results, however, are still vastly more accurate than those with any other type of photometer.

If the photometer is given an hour or two to warm up, it will hold perfectly constant for an indefinitely long period of measurement (8 hours was the longest attempted). In this way the magnitudes of many hundred of stars may be found from a few standards with a minimum of intercomparison.

The accuracy of this photometer is truly amazing, in most cases it is better than $\overset{\mathrm{m}}{.}01$. Since the error due to the graininess of the photographic emulsion is from $\overset{\mathrm{m}}{.}03$ to $\overset{\mathrm{m}}{.}07$, the only accidental errors introduced in this step are occasional reading errors.

4. <u>Background effects</u>.

One precaution must be taken when using the astrophotometer, however, and that is to make sure that the plate has a uniform fog background. The photometer is very sensitive to non-uniform background when used in the usual way. Ordinary chemical fog has negligible effect but density fluctuations of the background are hard to discern visually against heavier background and caution must be used in such cases.

By adjusting the comparison beam of the photometer the diaphragm may be closed very tightly about the star with only a small loss of accuracy due to centering errors. This procedure strongly reduces the background effects and was used in this work in all doubtful cases.

General sky background and diffuse light of the cluster were also reduced by using the 32-inch diaphragm. Since the increased f ratio reduces the background surface brightness relative to the point like star images, this enables work much closer to the nucleas of the cluster to be done.

In general, like distance to center correction, the best thing to do with background effects is to avoid them. This was done throughout the work.

5. Selection of comparison stars.

In each cluster a sequence of non-variable stars had to be chosen whose magnitudes could be determined and used to calibrate each measure of a variable. They had to meet the following three conditions:

a) Comparison stars could not be more than 23' from the variable and still be in the correction free field of the 32-inch aperature.

b) The stars must cover a range of brightness equal to or greater than the range of the variable.

c) The stars must be sufficiently far from the cluster that its background light or crowding of stars did not affect the measures.

It was also good to have the comparison stars the same color as the variable but although this condition was met in general it was usually by accident. This was due to the fact that the cepheids of longer periods were brighter than any stars in the cluster, especially in the photographic wavelengths. The brighter comparison stars then had to be field stars and in some cases were very hard to find within a 23' radius of the variable. In two clusters the plates had to be offset from the cluster center because of this. In M3 the brightest sequence star and the cepheid were about equidistant from the plate center. In M5 the variables were about in the middle of the bright comparison stars.

In all cases then, the sequence and variables fell within the correction free field. In only two instances the sequence had to be extrapolated a few tenths of a magnitude as can be seen by referring to Table I. The linear calibration curve on the bright end ensured that no appreciable error was introduced.

TABLE I

Photoelectric Calibration Stars

Cluster	Star	P	V	P-V	No. of measures	Source
M3	206	10.95	9.86	+1.09	1-2	Baum(4)
	1437	12.27	11.77	+0.50	1	
	1402	13.15	12.67	+0.48	1-2	
	297	14.25	12.85	+1.40	1-2	
	237	14.58	14.07	+0.51	1-2	
	additional stars listed in reference(4)					
M5	1	11.32	11.14	+0.18	1	Baum
	2	11.91	11.53	+0.38	5	Baum & Whitford(6)
	3	12.76	11.70	+1.06	1	Baum
	4	12.86	12.38	+0.48	1	"
	5	13.06	11.82	+1.24	4	Whitford
	6	13.87	13.07	+0.80	1	Baum
	7	15.11	14.31	+0.80	1	"
	8	16.29	15.63	+0.66	1	"
M13	Unpublished results kindly made available by Johnson. A total of 39 stars measured twice each on the average and extending from P = 11.18 to P = 17.67, V = 10.74 to V = 17.20.					
M10	1	11.23	9.93	+1.30	2	Whitford
	2	12.59	12.23	+0.36	2	
	3	14.89	13.76	+1.13	2	
	4	15.76	14.82	+0.94	2	
M15	Published results(7) on 17 stars, two or more accordant measures. From P = 12.72 to P = 17.15, V = 12.18 to V = 16.49.					
M2	1	12.16	11.31	+0.85	1	Whitford
	2	14.86	14.03	+0.83	1	
	3	16.14	15.35	+0.79	1	

6. Photoelectric calibration.

Previous work on the variables investigated here relied on inherently inaccurate photographic transfers of the magnitude scales from standard areas which were inaccurate in themselves. With the advent of the photocell the probable error of a single transfer became less than $^{m}.05$ and drawing a near-linear calibration curve through several such stars reduces the errors still further. The confidence that can be placed in the accuracy of the photoelectric sequences used in this research, then, is the key reason why results from different clusters can be compared, and the final period-luminosity relation be trusted to have negligible systematic errors.

As a general procedure, long exposure photographs of the clusters were examined to see that no star less than five magnitudes fainter than the star to be measured was included in the diaphragm of the photoelectric cell when centered on the star. Then a similarly clear area of sky was chosen for comparison. This precaution is especially necessary in globular clusters because of the many faint stars in the area. In the one cluster in which this was not done, M13, the measures show a longer scatter than normal, undoubtedly because of the inclusion of some invisible stars in star or background measures.

The stars measured photoelectrically and used for calibration in each cluster are summarized in Table I. Baum measured, specifically for this program, the second brightest star listed in M3 and all the stars listed in M5 which included remeasures of two stars previously measured by Whitford. His equipment and reduction procedure are described elsewhere(4). Whitford, to aid this investigation, very kindly measured all the sequence stars in M10 and M2. Johnson had previously measured the stars in M15 and M13. Their observational technique is also described in previous publications(2). The only sequence with but a single night's transfer is the one in M2. The author worked at the telescope with Whitford during the calibration of M10 and M2 and reduced the observations taken on those nights. All observations taken the same night as the M2 transfer were in perfect order and consequently confidence can be placed in the M2 sequence.

7. Secondary sequences.

In all the clusters secondary sequences were set up from the photoelectric standards. These furnished stars of nearly equal magnitude steps in both the photographic and photovisual which were conveniently located in the cluster. When the color-magnitude diagrams were measured on 100-inch plates these sequences were well within the smaller correction free field when many of the original stars were not. At least three comparisons with the primary sequence were made to set up each secondary sequence which consisted of from ten to fifteen stars.

In M10 and M2 where the photoelectric calibration was sparser than might have been wished, transfer plates to clusters with more adequate sequences were reduced in the following way. Calibration curves derived from the well calibrated side of the transfer were shifted in magnitude to agree with the four standards in M10 or the three standards in M2. The secondary sequences were then read off. This process merely uses the shape of the calibration curve (sometimes non-linear) which is not so well defined by the fewer photoelectric stars and adjusts it to their mean magnitude which they do define well. In the case of M2, three of the longer exposure transfers were measured on the Ross photometer at Mt. Wilson which is less sensitive to differences of background inevitable on a long exposure transfer.

In the mean the transfers gave a rough confirmation of the zero point and probably solidified the scale slightly in these two clusters. The way in which the secondary sequences were set up makes them at least as accurate as the original calibrations since the process tends to smooth out small irregularities.

8. Photographic materials and techniques.

Four by five inch Eastman IIa-o plates behind a GG1 filter and 103a-D plates behind a GG11 filter were used in the photographic and photovisual work respectively.

A double slide plate holder on the telescope enabled two exposures to be put on one plate, one on each half. Since over five hundred plates were used this saved considerable expense. The plates were easy to handle, saving of telescope time and only the central portions of each half plate were in the coma free field anyway.

Blue sensitive plates were, as a rule, exposed first followed immediately by the photovisual observation. Plates taken later than August were developed in a mechanical agitator built by William Miller which gives improved homogeneity of development.

9. Color equation and definition of m_{pg} and m_{pv}.

The plate and filter combinations just described, record stars of varying color as being slightly brighter or fainter than on the standard, international system of magnitudes. During the past three years both Sandage and the author have tried to determine the connection to be applied, as a function of color, to magnitudes derived with this plate and filter combination. Numerous plates of Selected Areas 57, 61 and 68 were measured since they contain stars most nearly on the international system(2). Magnitude discrepancies were plotted as a function of color and the corrections or "color equations" that resulted were reported in an earlier paper(5).

Magnitudes derived with these plate and filter combinations were designated P' and V' in that paper in order to indicate that they were very close to the international system now being designated as P, V. Color index was then either P'-V' on our system or P-V on the international system. It has since been decided to designate P' as m_{pg}, V' as m_{pv} and P'-V' as color index (C.I.). In future work these new symbols will be used consistently although they are exactly the same as those in the first paper(5), being defined by the IIa-o + GG1 and 103a-D + GG11 plate and filter combinations.

The magnitude corrections as a function of color (i.e. the color equations) were difficult to determine since the differences between the two systems are so small, less than the accidental errors of measurement. As a result subsequent measures changed the original corrections, indicating that there was actually no correction at all for photovisual magnitudes. Since no stars bluer than P-V = 0 were present in the Selected Areas, the corrections were originally extrapolated into that region. Ten stars measured photoelectrically by Johnson in M13 had negative color indices. The present work indicated that there was no color equation for these stars, at least not to Johnson's approximation to the international system. The final color equations which were used, then are the following:

$$V = m_{pv}$$

$$\left.\begin{array}{l} P = m_{pg} \\ P-V = C.I. \end{array}\right\} \qquad P - V < 0$$

$$\left.\begin{array}{l} P = m_{pg} - 0.08\ C.I. + {}^{m}_{.}07 \\ P-V = 0.92\ C.I. + {}^{m}_{.}06 \end{array}\right\} \qquad P - V > 0$$

where m_{pv}, m_{pg} and C.I., are respectively, photographic magnitude, photovisual magnitude and color index with the plate and filter combination. P, V and P-V are the corresponding quantities on the international scale.

Since IIa-o plates were used here for photographic magnitudes instead of the usual 103a-o plates, a check color equation was derived for the new blue emulsion. As expected, the blue color equation was the same as for the 103a-o plates behind the same filter.

In the paper referred to earlier, results were transformed to the P,V system by the equations given. It seems wiser now not to so transform the results because:

1) The extremely small differences between the two systems do not warrant the labor of transformation.

2) The P,V system is essentially undefined for negative color indices anyway and a retransformation might have to be made.

The present results, therefore, are on the m_{pg}, m_{pv} system and all future publications will also be on this system. For all practical purposes the magnitudes can be regarded as P,V or international magnitudes.

10. Accuracy of results.

It has already been noted that the photoelectric calibrations in these clusters give a systematic accuracy of a few hundredths of a magnitude. Another important gain in accuracy is due to the small accidental errors realized in this investigation. Under good conditions the random errors in the photographic magnitudes were $\pm{}^{m}.03$, in the photovisual $\pm{}^{m}.04$ (these are standard deviations from the mean, probable errors are therefore less). This is an appreciable gain over the usual photographic accuracy.

The reasons for this much greater accuracy are listed below in the order of their estimated importance.

1) For photographic magnitudes, the blue sensitive Eastman IIa-o emulsion has much finer grain and greater acuity than the usual 103a-o. The plate is almost as fast as the 103a-o and gives results almost twice as accurate.

2) The variable iris-diaphragm photometer used to measure the plates eliminates the errors previously introduced in the measuring apparatus.

3) Careful development, particularly with Miller's mechanical agitator gives uniform and thorough development, the last reducing the troublesome Eberhard effect.

4) Numerous and accurate comparison stars left no uncertainty in the position of the calibration curve.

5) The 32-inch diaphragm reduced sky background on the plates and removed the possibility of radial distortion of the images.

6) The optical axis of the telescope was kept carefully aligned.

7) Before measurement all plates were cleaned on the glass side with lens tissue, on the front with a camel's hair brush. This removed dust, dirt and stains that could possibly give spurious readings.

PART III

LIGHT CURVES OF 15 VARIABLES

The reason for dividing the discussion of the variables into period groups will become evident in Part V. It turns out that not only is the mean luminosity of the variable important, but also the shapes of the various light curves at the same period are very significant.

Considerable work has been done previously on some of these variables but it will not be referred to unless it is subsequent to Mrs. Hogg's bibliography(8).

A. The type II cepheids

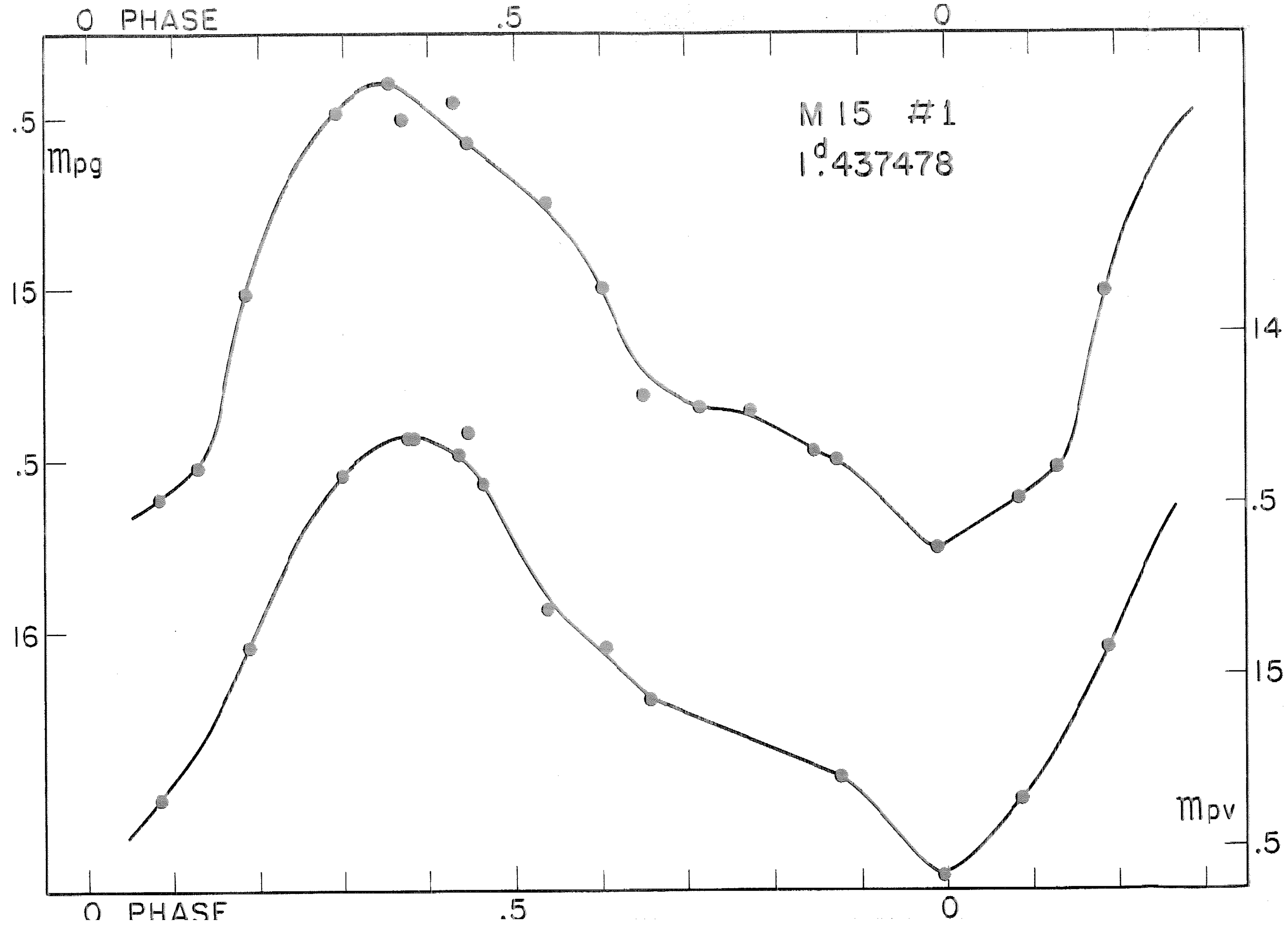
1. The one and a half day periods.

M15 #1

Figure 1 shows the light curve of this variable. Its position in M15 made it very easy to observe and the numerous, nearby calibration stars gave accurate magnitudes. It is the only long period variable in M15, however, and therefore observations were only made at preselected, evenly spaced phases. As a consequence it has the fewest number of points of any of the variables. Nevertheless the light curve and amplitude are well defined and the mean magnitude trustworthy.

This star is remarkably similar to its counterpart in M13. It differs by 1% in period, 1% in amplitude and 7% in absolute magnitude. (See Table III and IV.) The position of these two stars in their color magnitude diagrams is almost exactly the same. (See Appendix I.) The light curves have interesting similarities considering

Figure 1. Light Curve of Variable #1 in M15.


M 15 #1
$1^{d}.437478$
0 PHASE
.5
0
m_{pg}
.5
15
.5
16
m_{pv}
14
.5
15
.5
0 PHASE
.5
0

the range of shapes these light curves can exhibit.

M13 #1

This variable, shown in Figure 2, was well into the nucleus of the cluster and difficult to measure. By exposing to optimum density and using an extremely tight diaphragm to measure it, the severe crowding was overcome in the blue measures. In the visual, however, only plates of exquisite seeing could be used and this explains the fewer number of points and greater scatter on the m_{pv} curve. General procedure in such cases was to compare plates taken under poorer seeing conditions to plates taken during the best seeing and ascertain at what point crowding produced measurable errors. All plates poorer than this were discarded. A final check was a pair of 100-inch plates taken with seeing 3 and seeing 5.

The upper curve in Figure 2 gives visual evidence of the kind of accuracy that could be obtained when the star repeated exactly from cycle to cycle.

These two stars should be compared to nos. 43 and 60 in ω Centauri(9) and contrasted to no. 92.

2. The two day period.

M13 #6

Figure 3 shows one of the most interesting light curves obtained. Older measures of this star were bothered by a bright companion, but on all the plates taken here

Figure 2. Light Curve of Variable #1 in M13.

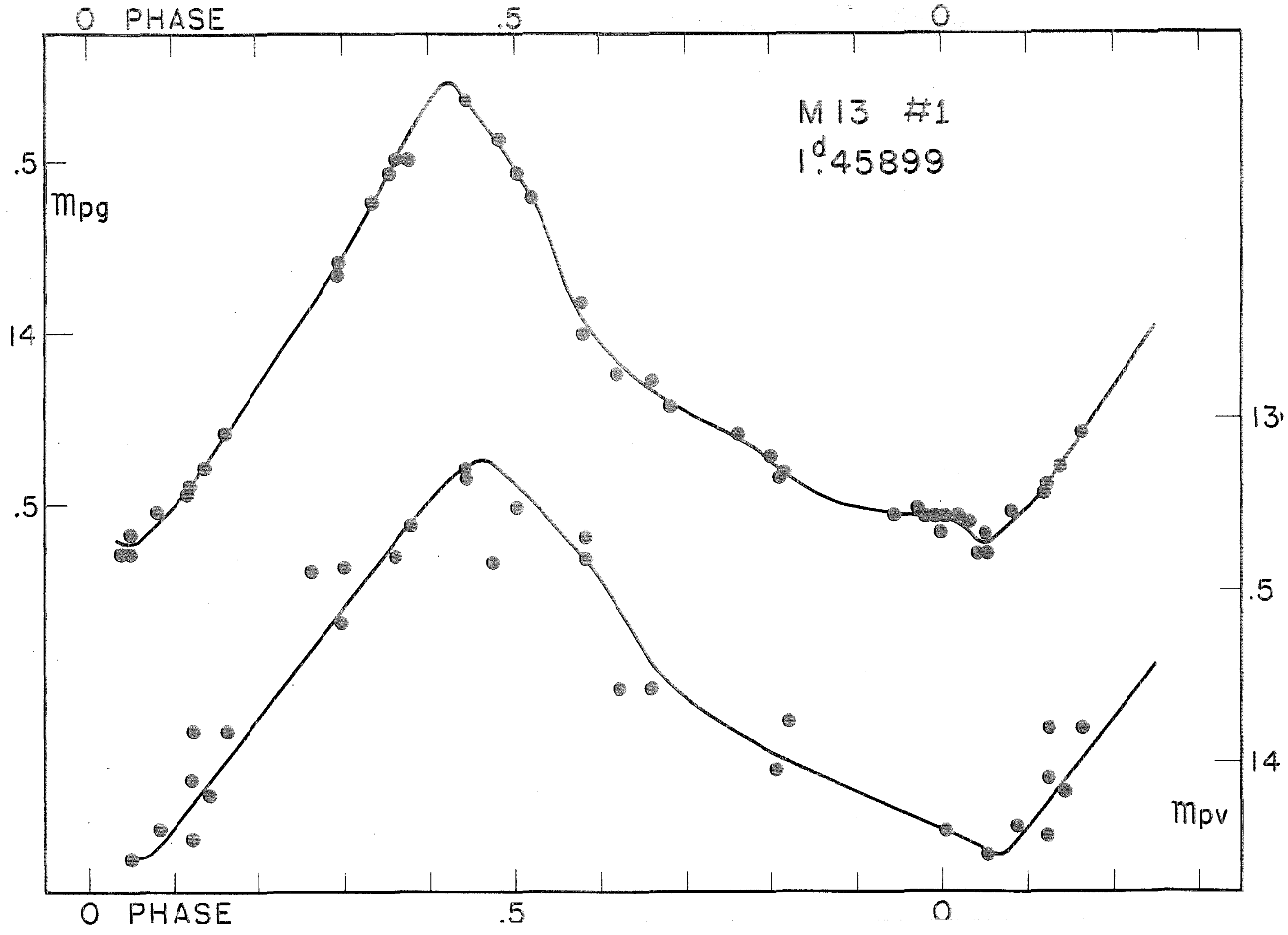
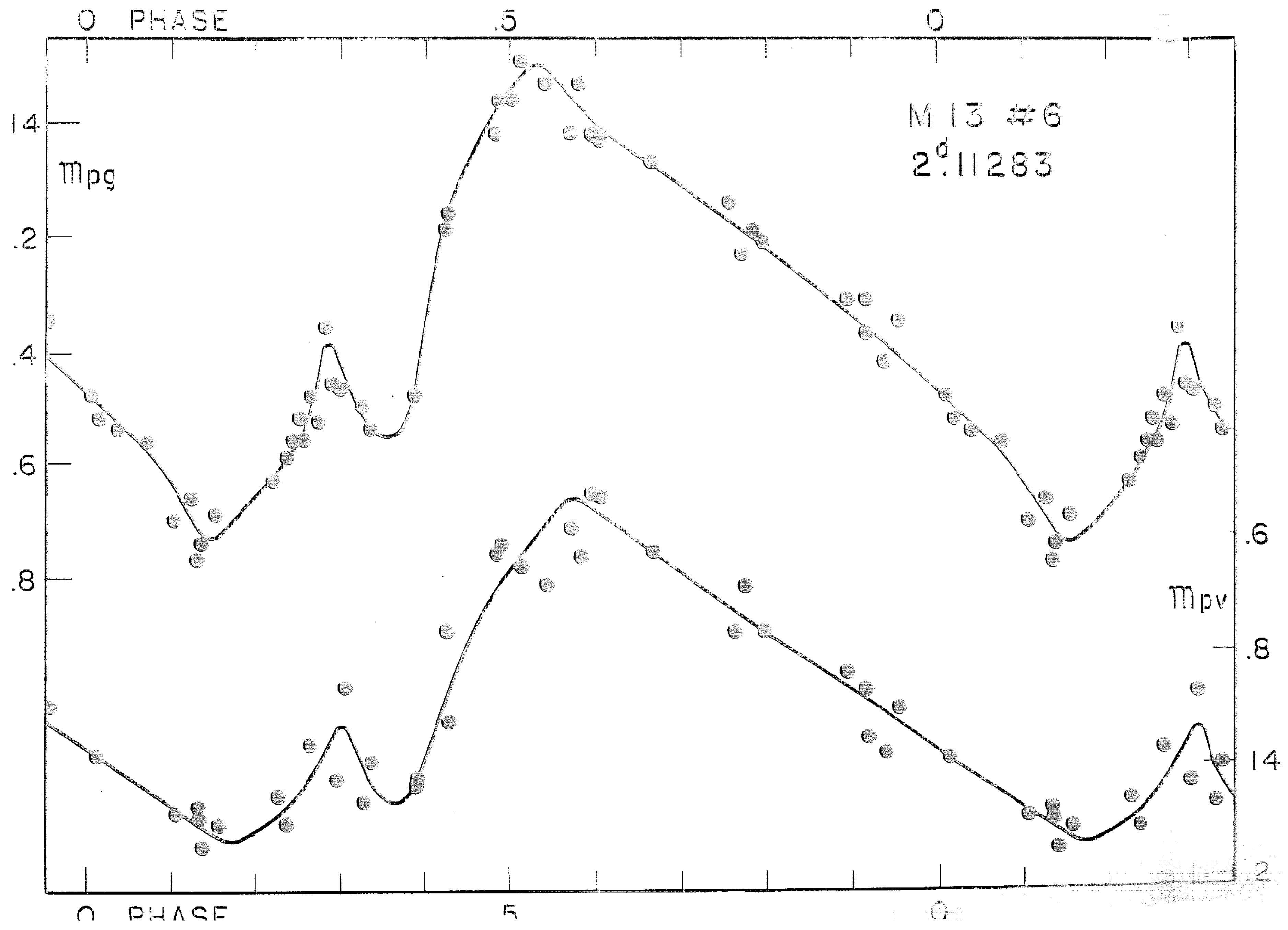

M13 #1
1ᵈ.45899
0 PHASE
.5
0
m_pg
.5
14
.5
13
.5
14
m_pv
0 PHASE
.5
0

Figure 3. Light Curve of Variable #6 in M13. Note the Small Amplitude.



0 PHASE
.5
0
14
.2
.4
.6
.8
mpg
M 13 #6
$2^d.11283$
mpv
.6
.8
14
.2
0 PHASE
.5
0

it had a wide clear area about it and there was not the slightest difficulty in measuring it.

There are about 40 observations plotted in Figure 3, covering a span of 200 days. The sharp hump on the rise to maximum is an unusual feature and since points at these phases were gotten from several different cycles the feature is a permanent part of the light curve.

A previous period of $\overset{d}{.}678758$ days(10) for this variable is erroneous and arises from the fact that most observations are taken one day apart (the fractional part of the reciprocal of the periods being equal).

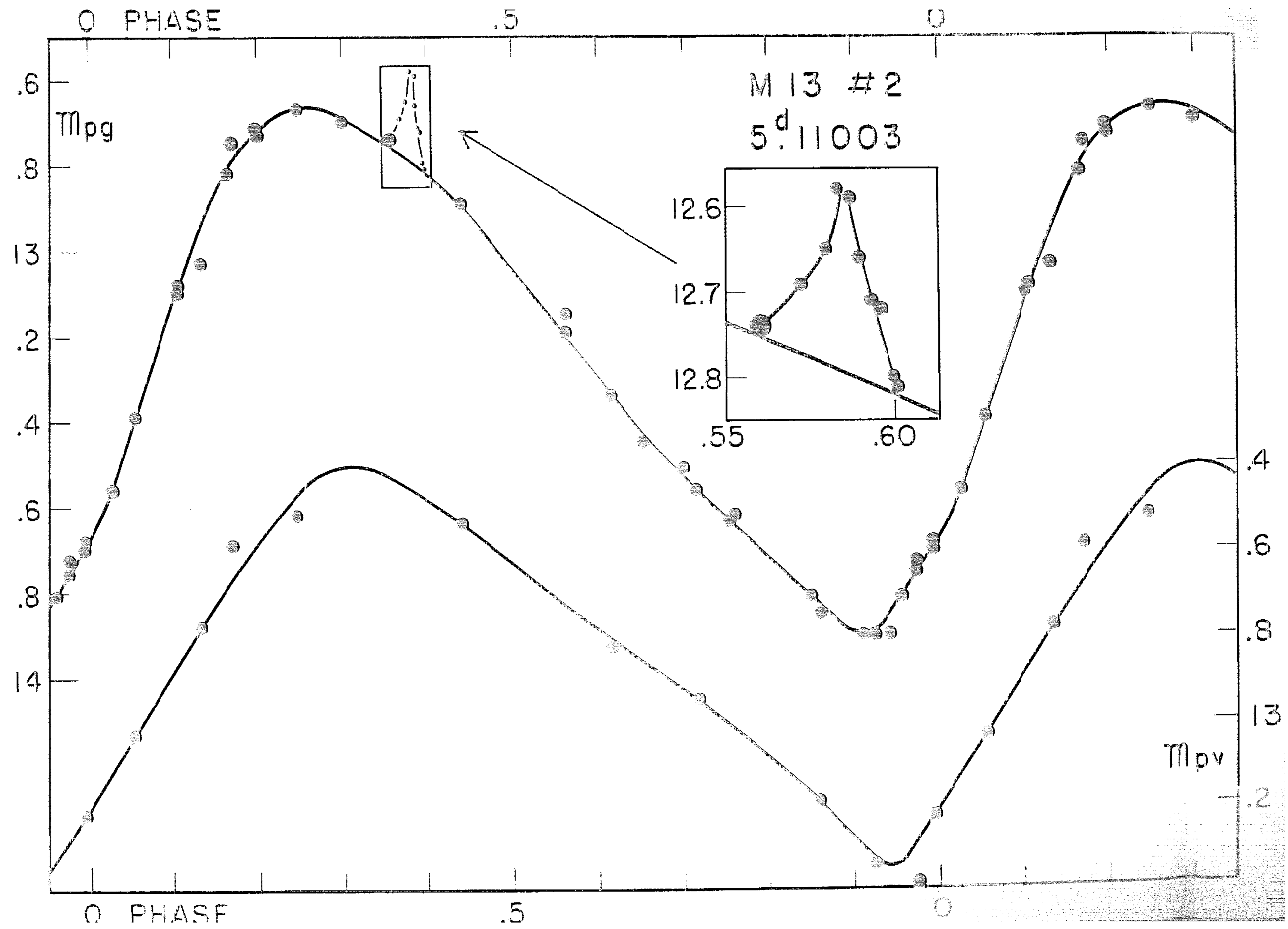
It is of extreme interest to compare this star to no. 61 in ω Centauri. The two, with almost the same period have exactly the same light curve, even to the sharp hump on the way to maximum and the long slow decline.

3. The five and seven day variables.

M13 #2

This variable is shown in Figure 4. Like #1 in the same cluster it was centrally located and somewhat crowded. In both cases the fact that the brighter globular stars are very red made it very much more difficult to escape the crowding effects in the photovisual wavelengths. This again is the explanation for the fewer points on the m_{pv} curve. It was possible, however, to again make sure that the measures plotted were systematically free of crowding effects.

Figure 4. Light Curve of Variable #2 in M13. The insert shows a "flare" that was observed in a series of plates taken about fifteen minutes apart. The flare also shows in the photovisual but the whole level of brightness was above the lower curve due to crowding and was not plotted.


M 13 #2
5.d11003
0 PHASE
.5
0
m_{pg}
.6
.8
13
.2
.4
.6
.8
14
12.6
12.7
12.8
.55
.60
.4
.6
.8
13
.2
m_{pv}
0 PHASE
.5
0

The flare near maximum was discovered accidently. A series of plates about fifteen minutes apart over a span of about three hours was taken in order to check on some suspected variables of periods around $0\overset{d}{.}2$. It was this set of consecutive observations which happened to include this interesting phenomenon. The flare also showed in the set of photovisual plates but since the entire star was crowded on these exposures, the whole level of brightness was above the lower curve in Figure 4 and was therefore not plotted. It is established that this flare actually occurred and it is certainly well defined as shown in the insert. It is impossible to say whether it is a permanent feature of the light curve, however, since no other observations fell in this narrow range of phase. It would be important to check this point by obtaining more observations at this phase.

Whether it is a sporadic flare or a regular "hump" on the light curve is important. Flares are known to occur in red dwarf stars but they are noticed because of their large amplitudes. If flares like this occur in giant or super giant stars it would be extremely interesting, but any chance, single observation would be regarded as an unexplained accidental error.

M10 #3

The cepheid shown in Figure 5 was discovered during the course of this investigation. It has a rather small amplitude which may explain why it was not discovered by Oosterhoff who made an extensive search of the cluster for RR Lyrae stars, of which there are none known.

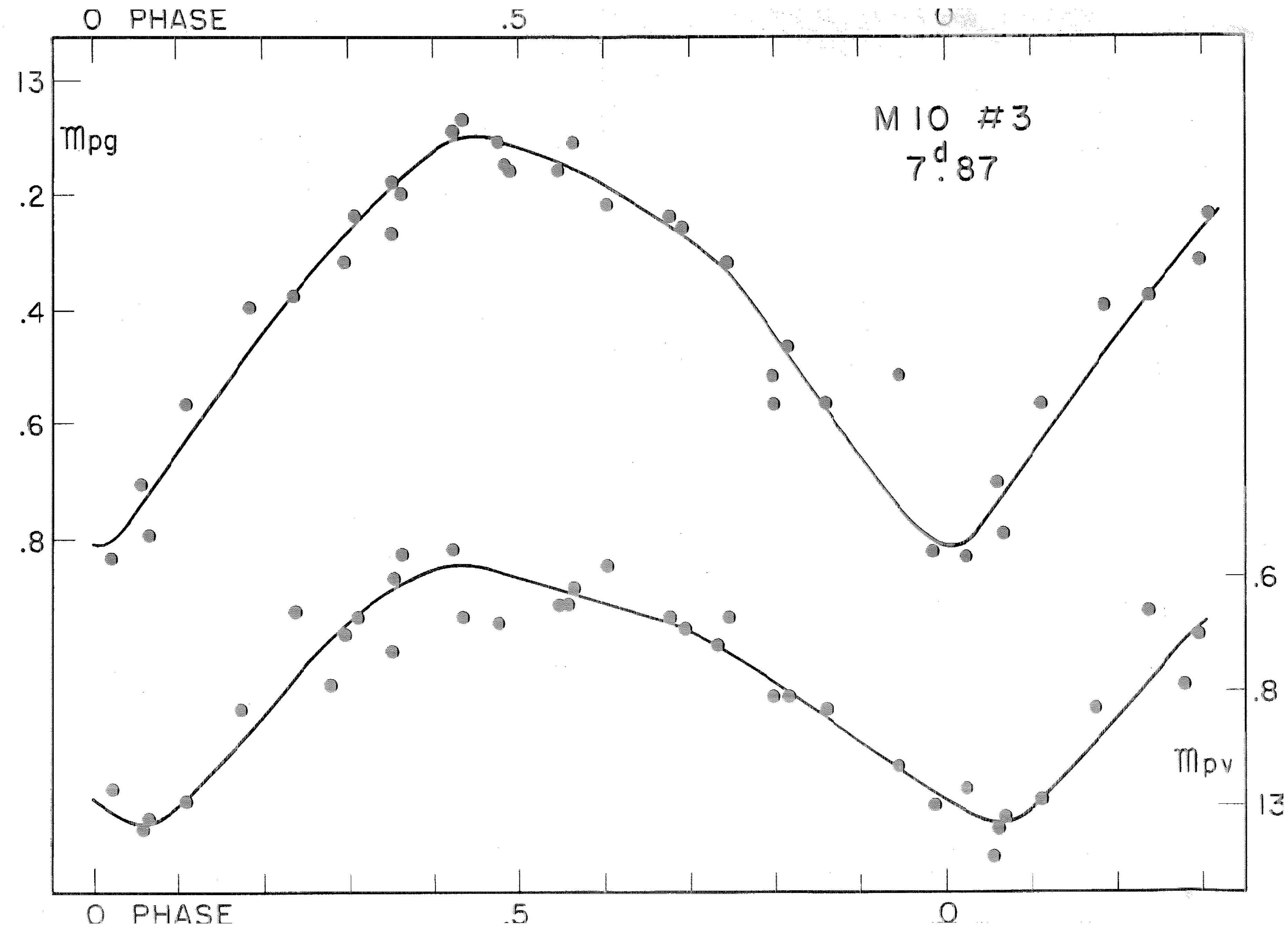
The light curves of this variable and the previous one in M13 are very much alike. A $4^{d}.4$ period in ω Centauri is also quite similar and this type of light curve seems to be characteristic of variables around this period.

The scatter about the curves in Figure 5 is larger than should be expected. The implication is that the star does not repeat exactly from cycle to cycle, although this could be checked more exactly with a more accurate period than it was able to derive from the span covered here.

4. Fifteen through nineteen day periods.

Until the discovery of M10 #3 there was no cepheid known in a globular cluster with a period between seven and fourteen days. There is still almost as large a gap between 7.9 and 14 days which is probably significant in terms o f the nature of cepheid pulsation.

Figure 5. Light Curve of Variable #3 in M10, Discovered During this Investigation.


M10 #3
7ᵈ.87
0 PHASE
.5
0
13
.2
.4
.6
.8
mpg
.6
.8
13
mpv
0 PHASE
.5
0

M3 #154

Figure 6 shows the results for a star which turned out to be a very important one for this investigation. It was one of the most difficult of all the stars to measure since it was almost exactly in the center of the cluster. Luckily it was excessively bright which enabled exposures as short as 20 or 30 seconds to be made. The usual measuring techniques for such cases, then, gave clear measures. There was no trouble in the photovisual since the variable was quite clear on those plates. This fortunate circumstance gave a check on the blue magnitude which was good to better than one-tenth of a magnitude.

The hump near the minimum of the light curve is defined by observations obtained from two different cycles and is, therefore, probably a permanent feature of the light curve. Features such as this are known in some classical cepheids but on the rise to maximum, not at minimum.

M2 #1

The results for a star of almost exactly the same period are shown in Figure 7. This variable was an easy object to measure and apparently repeats very well from cycle to cycle as judged from the very small scatter of points.

The above two stars show obvious and important differences in their light curves. The one in M3 is "peaked up", more like a classical cepheid through the

Figure 6. Light Curve for Variable #154 in M3. Points defining hump at minimum come from two separate cycles.

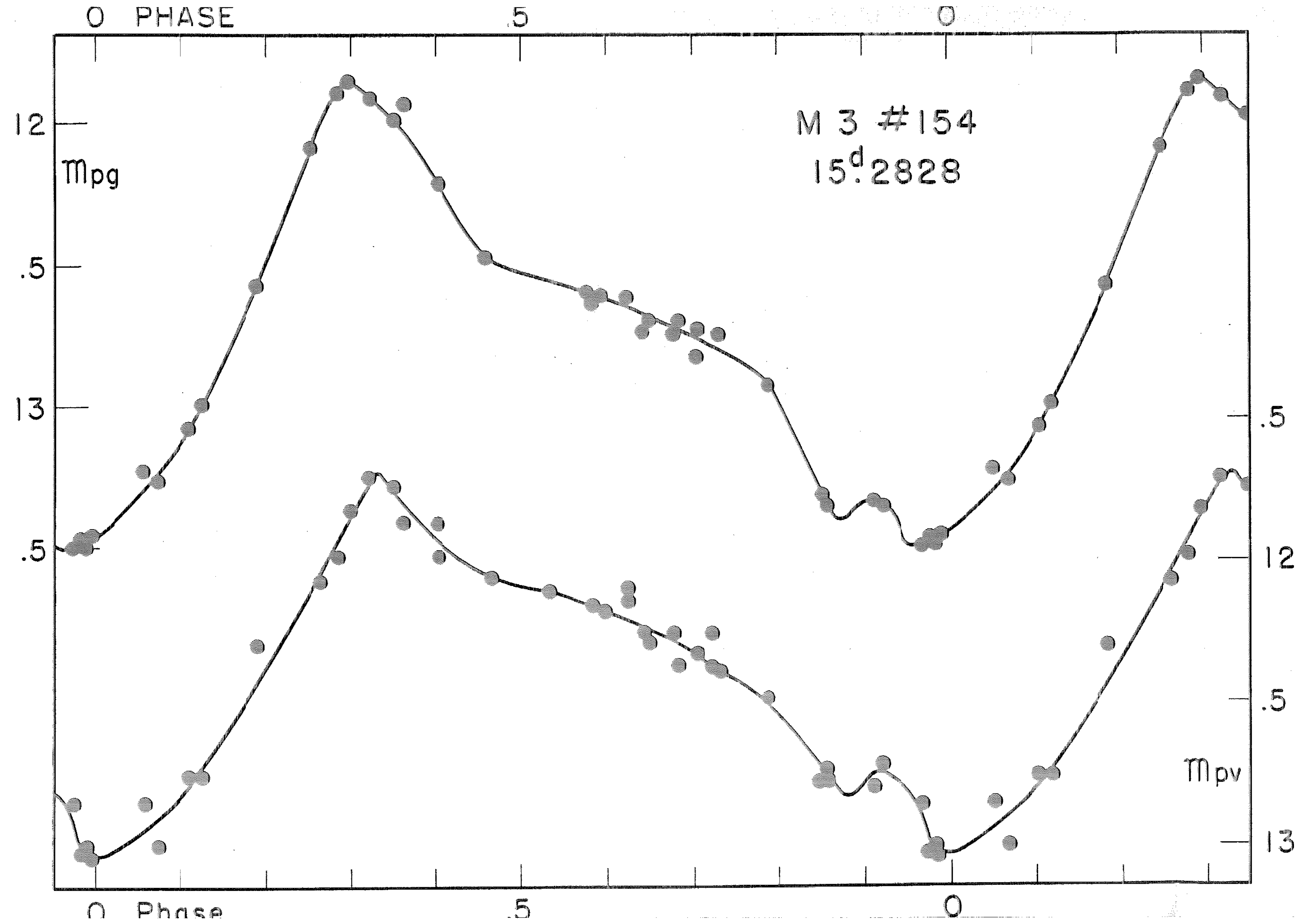
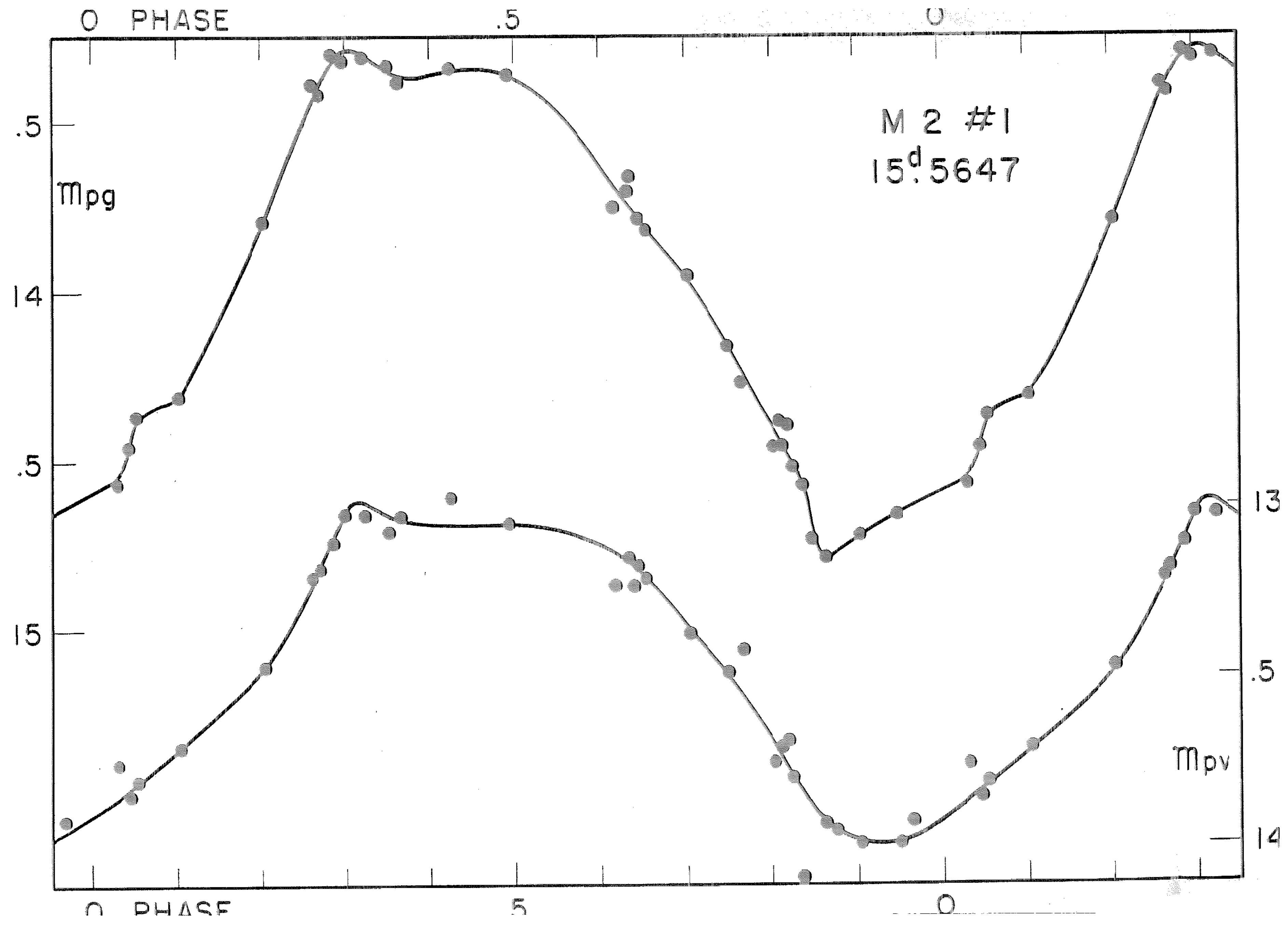

O PHASE
.5
0
M 3 #154
15.d2828
12
mpg
.5
13
.5
.5
12
.5
mpv
13
O Phase
.5
0

Figure 7. Light Curve for Variable #1 in M2.


0 PHASE
.5
0
M 2 #1
15ᵈ.5647
.5
14
.5
15
mpg
13
.5
14
mpv
0 PHASE
5
0

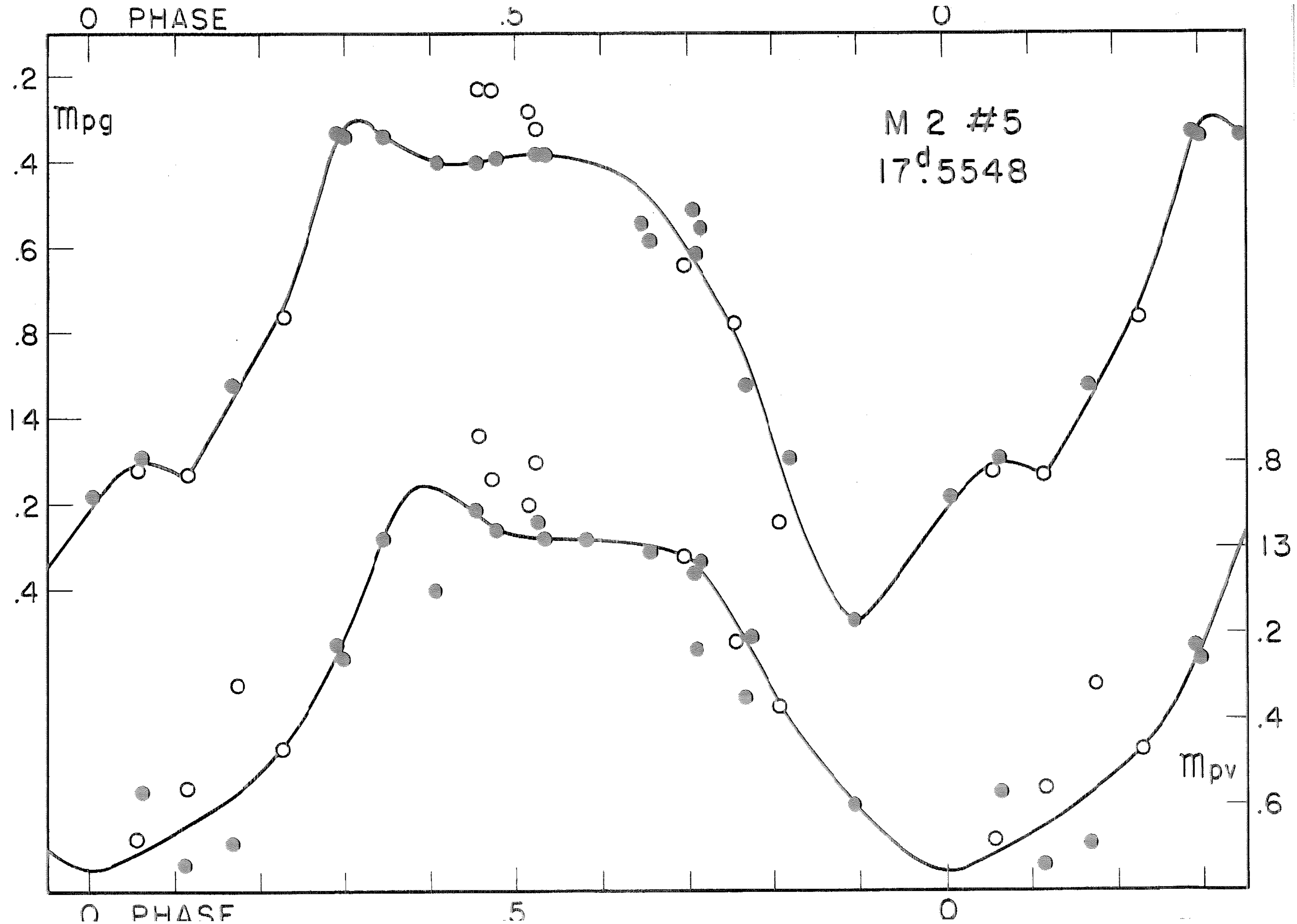
usual type II cepheid. Yet it has a "still stand" characteristic of the so-called W Virginis type. The one in M2 is more like a W Virginis star (the prototype for these periods) but still is appreciably different from W Virginis. Even at this point it is becoming clear that the designation "W Virginis" has a very limited classification value.

M2 #5

Figure 8 shows the first of the four variables which had double periods indicated. Of the four, it was the most poorly defined case of a double period and since it was the first one measured it was only a hint of the phenomenon to be discovered in subsequent light curves.

Two points from cycle 6 (first observed cycle called 1) and two points from cycle 8 are plotted with open circles in Figure 8. They fall too high, by from one to two-tenths of a magnitude in both photographic and photovisual so there is no doubt the star actually was brighter, at these times, than the mean curve drawn. The remainder of the points are plotted as filled circles and are from cycles 1, 2, 3, 4, 5, 7, 9 and 11. The only points which actually contradict the alternating cycle behavior established in the other three stars are two points from cycle 4. Although more data would be desirable, it can be stated that this variable definitely tends to alternate regularly between cycles of different amplitude.

Figure 8. Light Curve of Variable #5 in M2. Open circles are two points from cycle 6 and two points from cycle 8. The remaining cycles plotted in filled circles are 1, 2, 3, 4, 5, 7, 9, 11. Only two points from cycle 4 contradict the tendency to alternate cycles.


M 2 #5
17^d.5548
0 PHASE
.5
0
m_pg
.2
.4
.6
.8
14
.2
.4
.8
13
.2
.4
.6
m_pv
0 PHASE
.5
0

This variable has the same period as, and is generally quite similar to, W Virginis itself which is now suspected of showing variations from cycle to cycle(11).

M10 #2

This cepheid gives the first clear picture of what is actually happening in the stars with a period double their listed period. In Figure 9 the filled circles are all from cycles 1, 3, 5 and 7 and the open circles from cycles 2, 4 and 6. The even cycles fall about $\overset{m}{.}15$ fainter at maximum and about $\overset{m}{.}3$ brighter at minimum, than the odd cycles. If the light curves are plotted on an intensity scale instead of the logarithmic magnitude scale, the respective differences from maximum and minimum are about equal. This means that the star pulsates with a period of 18.75 days, but that it rides up and down on a variation of 37.5 days which has an amplitude of about $\overset{m}{.}3$ or $\overset{m}{.}4$. The situation is pictured schematically in Figure 10.

Figure 10.

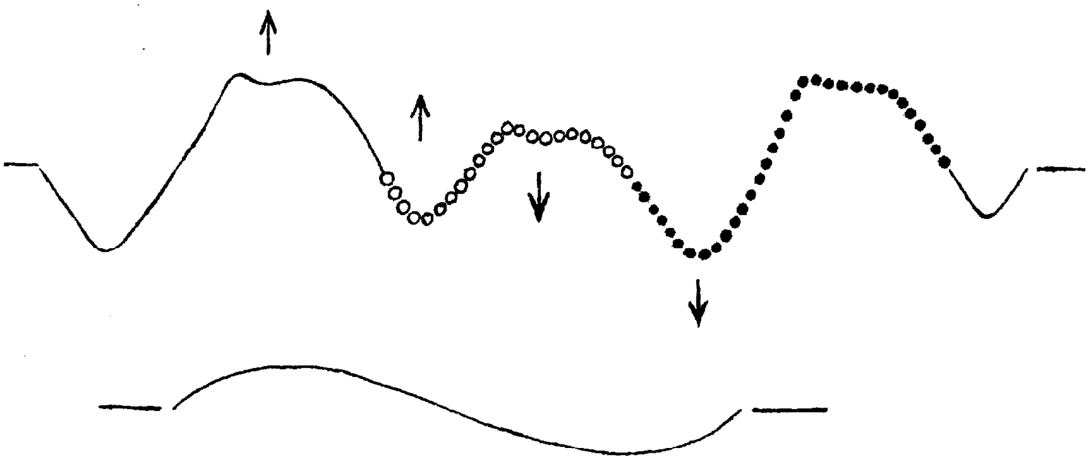
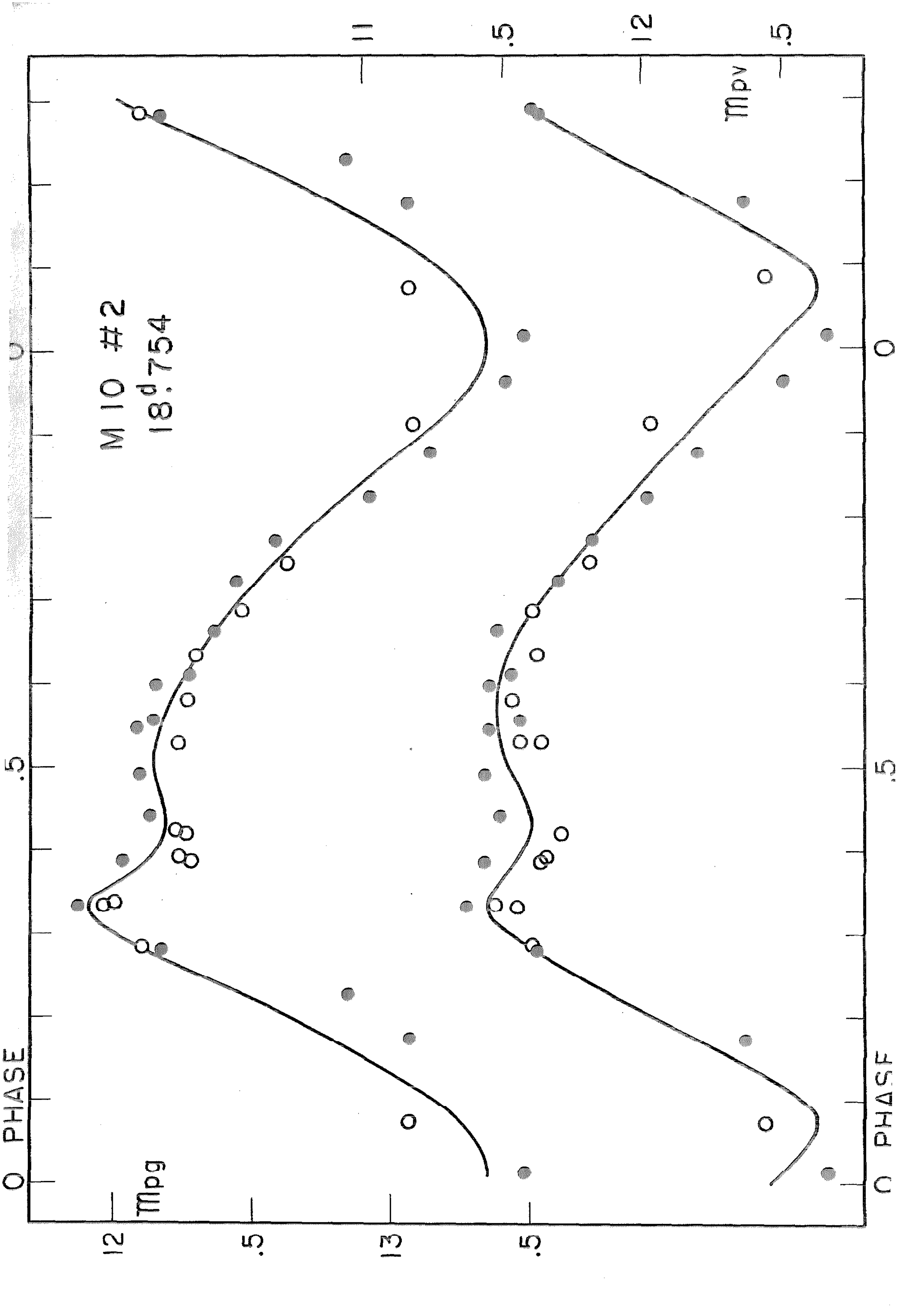


Figure 9. Light Curve for Variable #2 in M10. Filled circles are from cycles 1, 3, 5 and 7. Open circles are from cycles 2, 4 and 6.



M10 #2
18d.754
0 PHASE
.5
0
0 PHASE
.5
0
Mpg
12
.5
13
.5
11
.5
12
.5
Mpv

The top curve shows in an exaggerated way how the variable has a deep minimum, high maximum, shallow minimum, shallow maximum, deep minimum, etc.. The arrows show how these features are moved up and down by the underlying, in phase variation of twice the period. The parts outlined in open circles and filled circles are plotted at the same phase in Figure 9. (i.e. they are superposed.) This shows why the open circles in Figure 9 fall above the filled circles at minimum and vice versa at maximum.

M2 #6

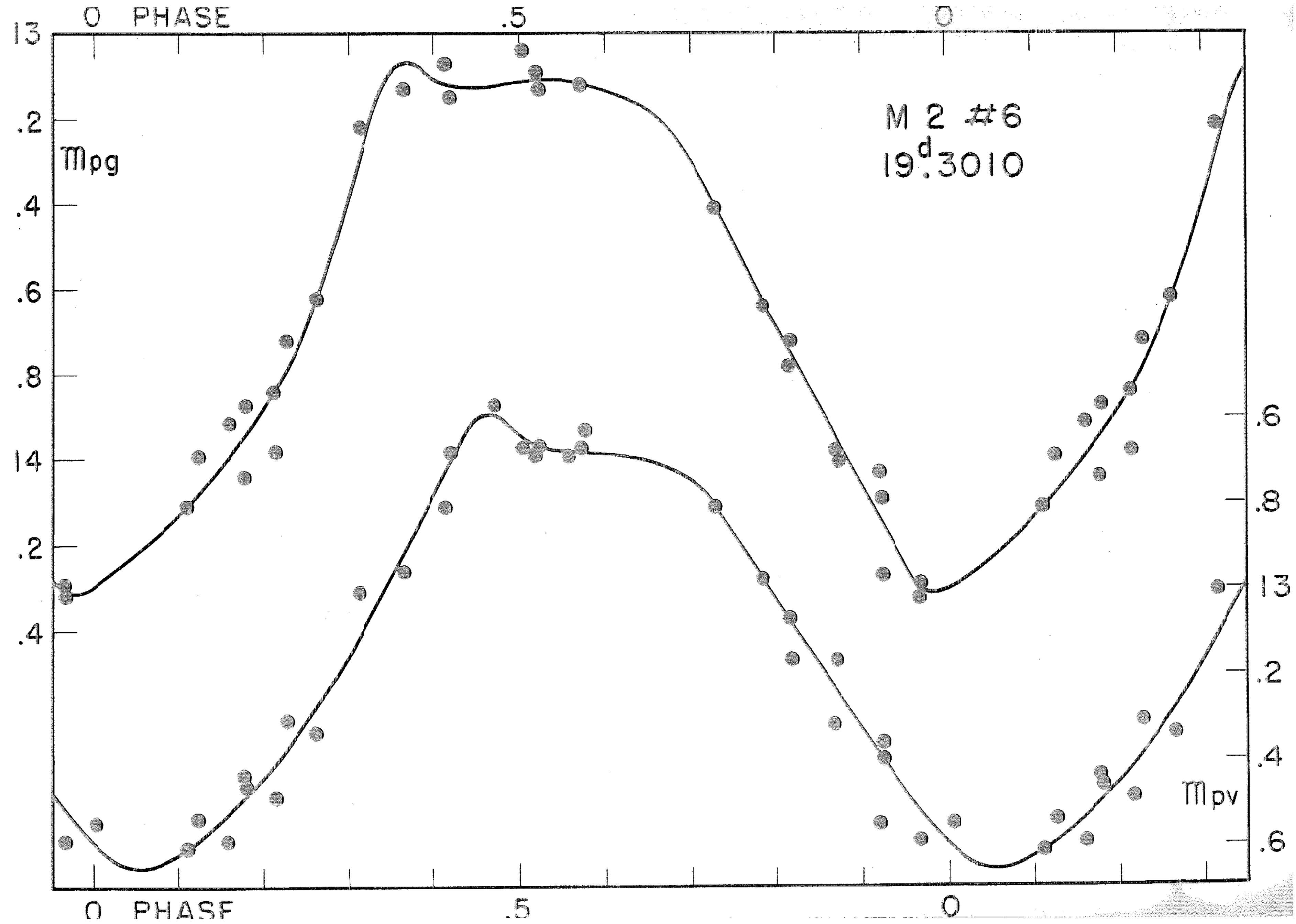
The last of the group of what have been loosely called W Virginis stars in the past (13-19 days) is shown in Figure 11.

There is nothing of note about this variable. The scatter of the points is a little larger than should normally be expected but no evidence for a double period was found.

5. The 25 through 33 day periods.

This is the group of stars which Joy(12) called RV Tauri type. This problem will be discussed in the following section but it is already evident from the following three examples that they are type II cepheids just as much as the group discussed in the previous section.

Figure 11. Light Curve for Variable #6 in M2.


M 2 #6
19.d3010
0 PHASE
.5
0
Mpg
Mpv
13
.2
.4
.6
.8
14
.2
.4
.6
.8
13
.2
.4
.6

M5 #42

The star exhibited in Figure 12 was in a very easy position to observe on plates of any exposure and repeats most exactly from cycle to cycle. The light curve is therefore exceedingly well defined.

As Figure 12 shows, the light curve bears an important similarity to M3 #154. The high peak and deemphasis of the still stand make it resemble a classical cepheid more than the following variable which has almost the same period and is a few minutes of arc away in the same cluster.

M5 #84

This cepheid was nearer the cluster center than #42 but was perfectly clear for exposures of two minutes or less. Most exposures were around one minute.

Figure 13 shows the striking difference in light curves between this star and the preceding one.

Even more important, however, was the fact that it gave definitive evidence of the double period. The filled circles are from cycles 1, 3, 5, 7 and 9; the open circles from cycles 2, 4, 6 and 8. This was the clinching evidence that the mechanism explained in section 4 under M10 #2 was actually in operation in these stars.

M2 #11

This was the last of the four cepheids to show a double period and is exhibited in Figure 14.

Figure 12. Light Curve of Variable #42 in M5.

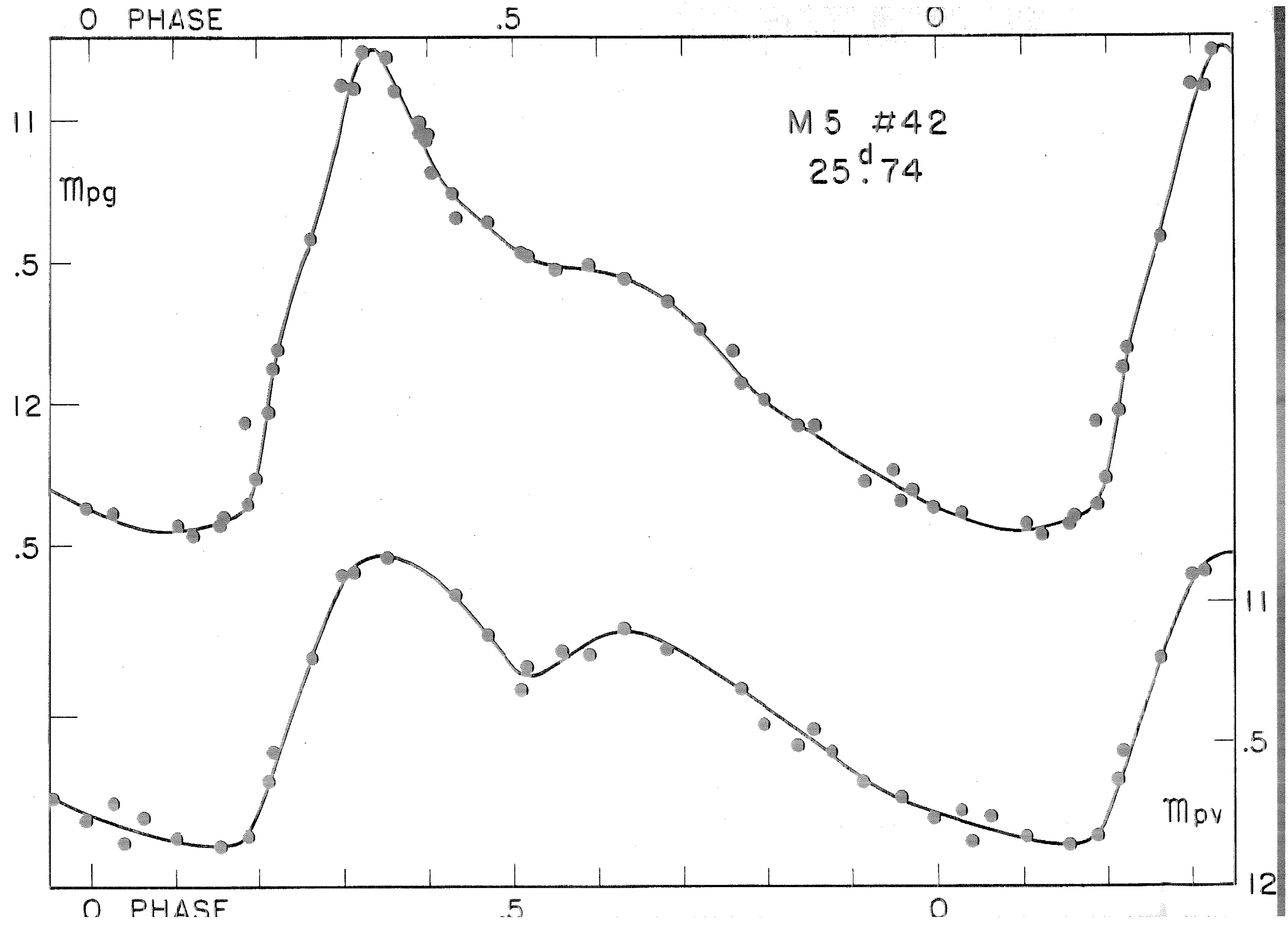

M5 #42
$25^{d}.74$
0 PHASE
.5
0
m_{pg}
11
.5
12
.5
m_{pv}
11
.5
12
0 PHASE
.5
0

Figure 13. Light Curve for Variable #84 in M5. The filled circles are from cycles 1, 3, 5, 7 and 9. The open circles from cycles 2, 4, 6 and 8.

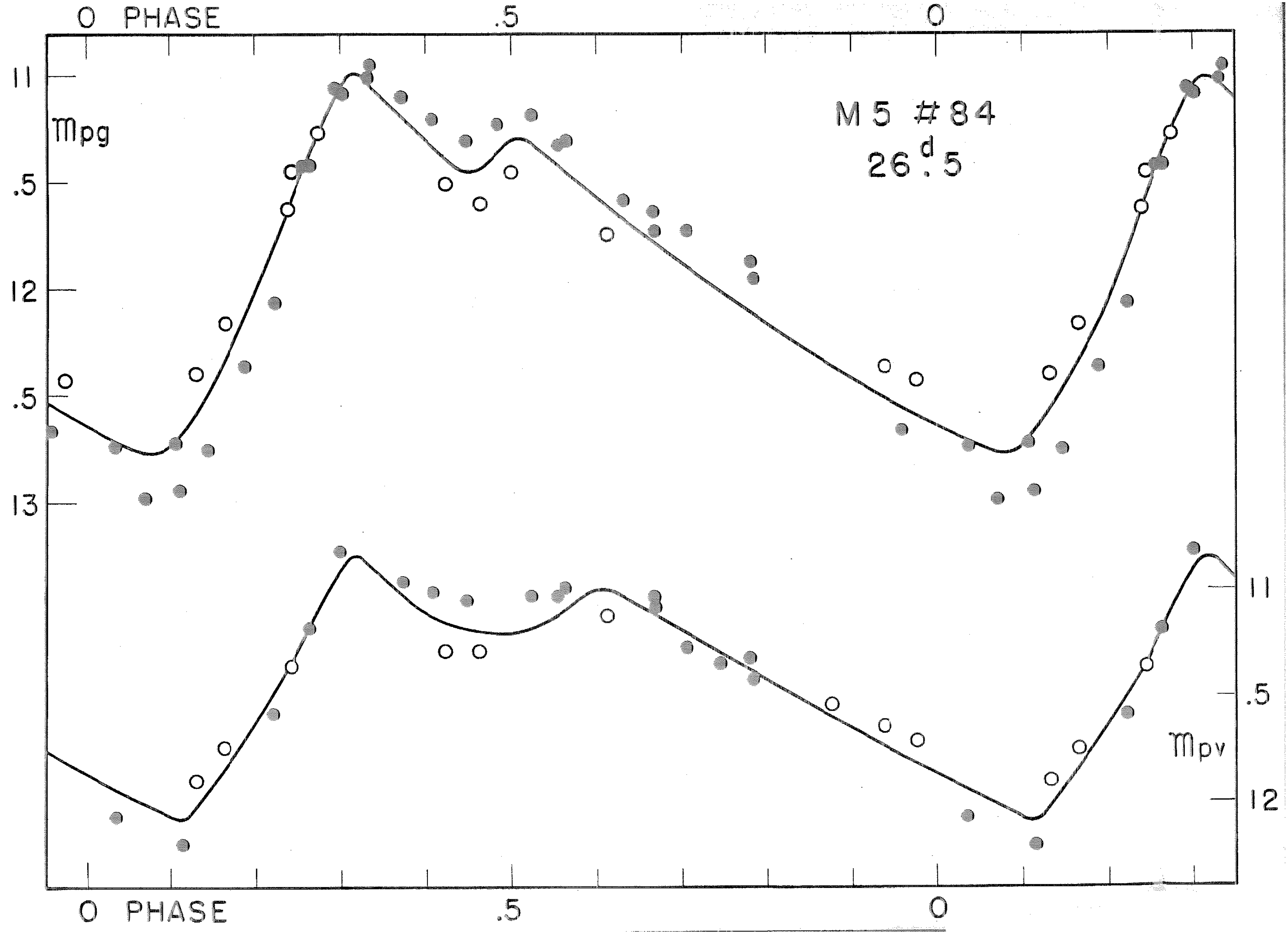
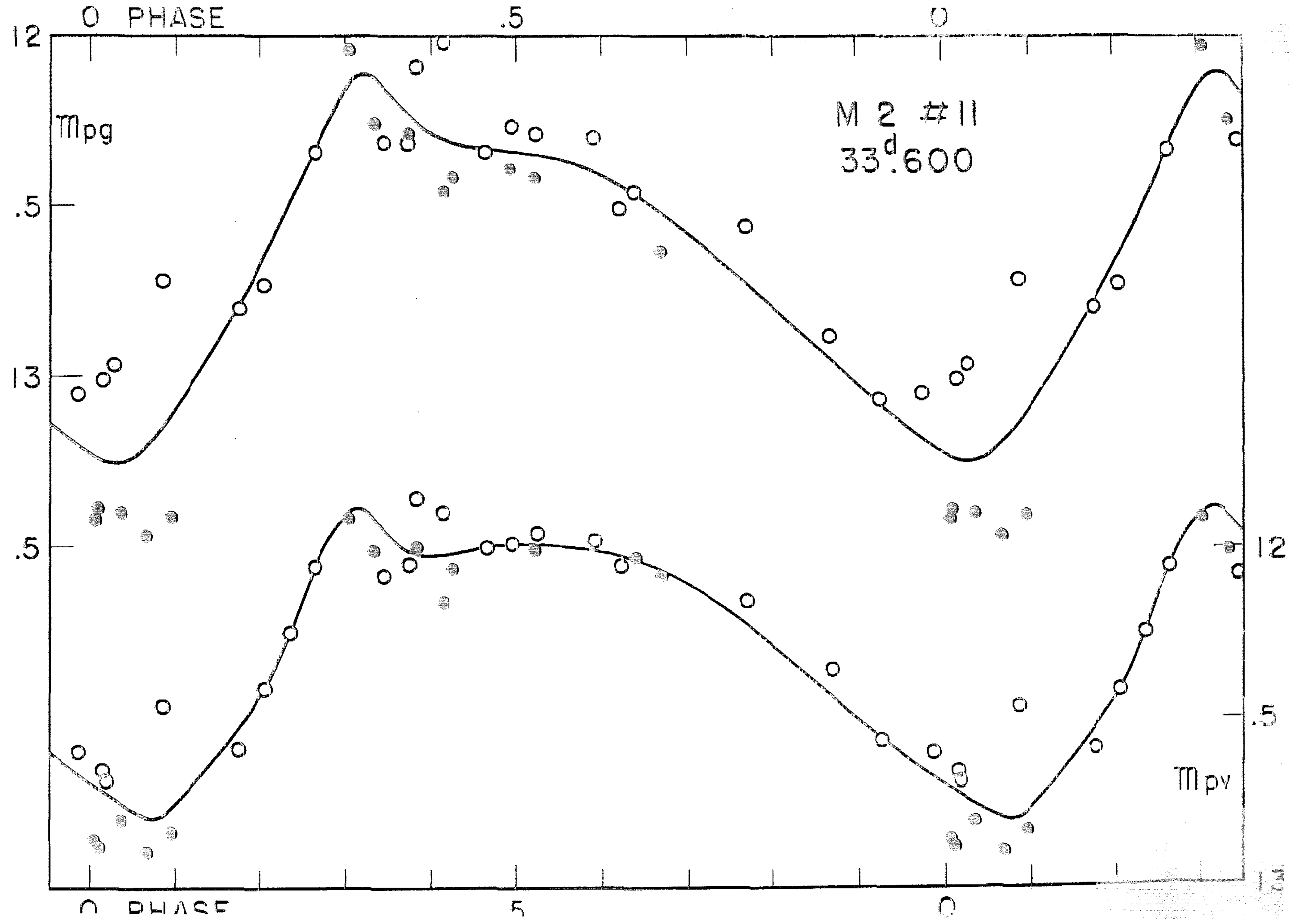

M 5 # 84
26.d5
0 PHASE
.5
0
m pg
11
.5
12
.5
13
m pv
11
.5
12
0 PHASE
.5
0

Figure 14. Light Curve for Variable #11 in M2. Filled circles are from cycles 1, 3 and 5 and open circles from cycles 2, 4 and 6.


0 PHASE
.5
0
12
.5
13
.5
m_{pg}
M 2 #11
$33^{d}.600$
12
.5
13
m_{pv}
0 PHASE
.5
0

Independent confirmation of the double period for this star comes from two sources. Both Joy and Mrs. Hogg(12) changed the period of this star, in 1949, to 67.2 days from the previous 33.6 days.

The old period is indicated in Figure 14 because the star shows the same kind of alternation from cycle to cycle that the previous three did. For those three the only period known, the shorter one, was used. Actually it is an academic point as to which of the two periods is correct for such a star. Perhaps it is best to take the original periods, considering them to be the "dominant period" since they have the largest amplitude. It may be in such a case that neither is the fundamental period of vibration.

In all the four stars with double periods, and in this star in particular it was somewhat difficult to estimate the correct position of the mean light curve. The mean magnitudes for these stars, consequently, have slightly higher accidental error.

B. The type II red variables.

The following stars are clearly quite different from the type II cepheids. This is demonstrated by two of their properties: one, their position in the period luminosity diagram as shown in Figure 17 and two, their place in the color magnitude diagrams shown in appendix I.

Apparently type II cepheids with noticeable double periods have, in the past, been called RV Tauri stars$^{(12)}$. When an unknown number of these are removed from the already ill-defined RV Tauri group, its classification value becomes almost nil. The type II red variables treated here do not, in any case, much resemble the RV Tauri group. It is felt rather that the stars in the following section are a continuation of the long period, or Me variables what are known to assume more and more the characteristics of population II as the periods become shorter and approach 150 days. It is reasonable to expect such variables in globular clusters if the variables are actually type II and the following stars are the only possible candidates.

1. Two periods near 100 days.

M13 #11

The variable shown in Figure 15 was originally listed as irregular$^{(1)}$. The observations here enabled a period of $92\overset{d}{.}5$ days to be derived. About two cycles were observed and they fitted this period. There is no guarantee, of course, that the star has varied with this period in the past or will continue to in the future.

M3 #95

Figure 16 shows the results for this much worked on variable in M3. The observations here started out as the filled circles a little past minimum, they went through

Figure 15. Light Curve of Variable #11 in M13. This star was previously listed as irregular.

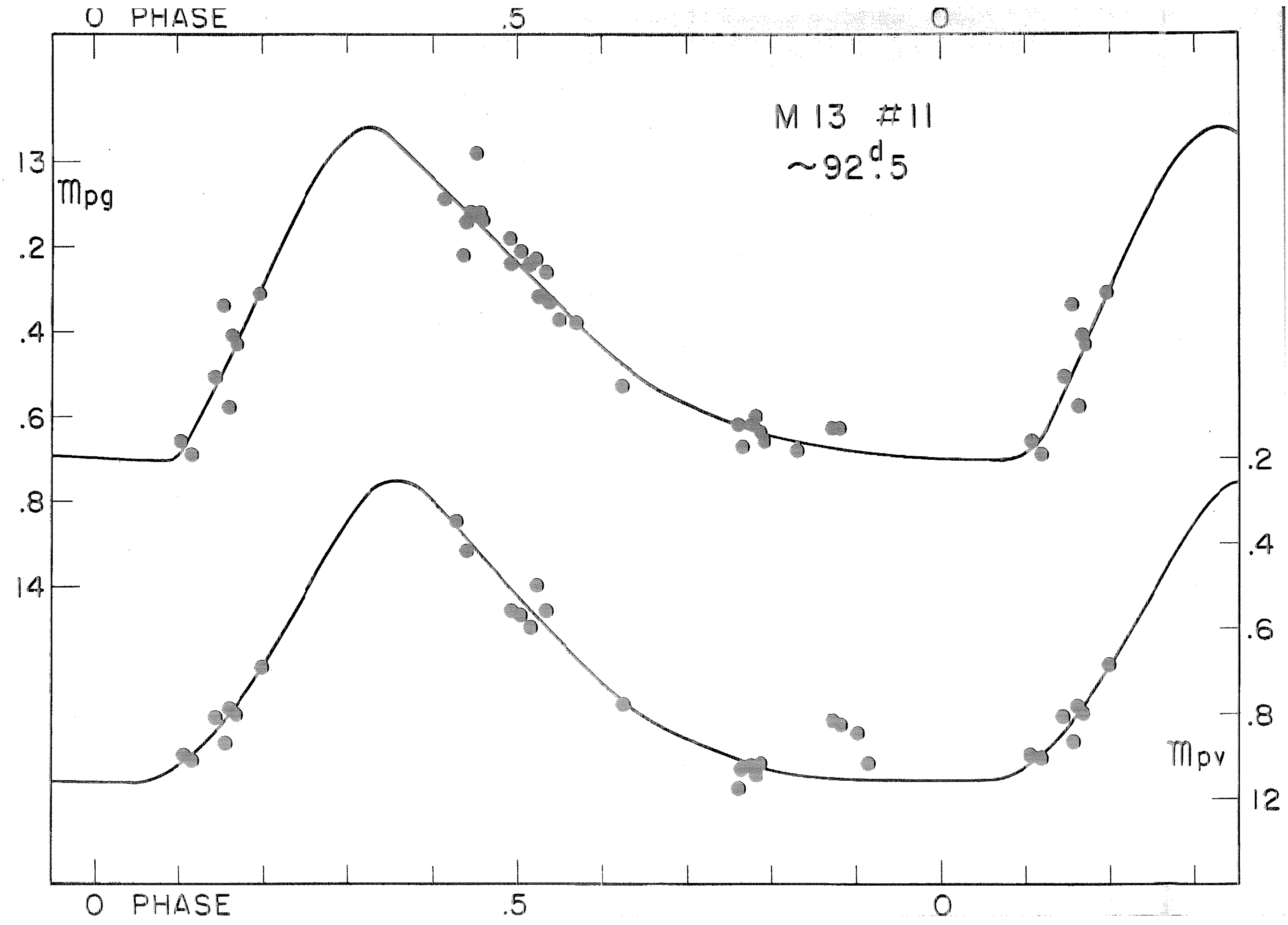
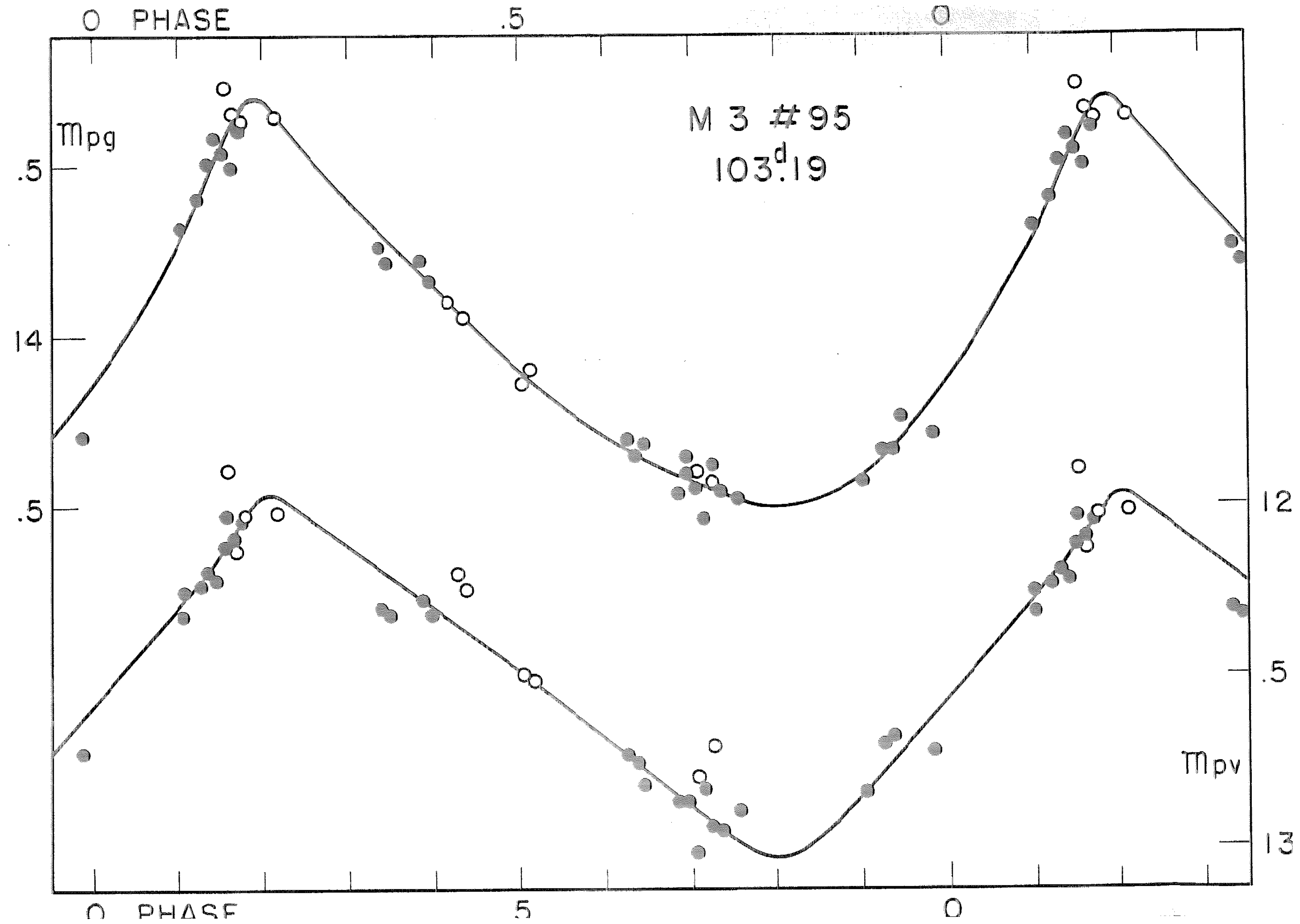

O PHASE
.5
0
M 13 #11
~92ᵈ.5
13
mpg
.2
.4
.6
.8
14
.2
.4
.6
.8
mpv
12
O PHASE
.5
0

Figure 16. Light Curve for Variable #95 in M3. Open circles are consecutive observations shifted by .110 in phase to agree with earlier points.


0 PHASE
.5
0
M 3 #95
$103^{d}.19$
m_pg
.5
14
.5
12
.5
m_pv
13
0 PHASE
5
0

one complete cycle as shown in Figure 16, then from maximum, the remaining points to minimum were displaced by .110 in phase. These points were moved back on to the mean curve but are indicated by open circles in Figure 16.

Points taken from observations by Sandage a year previous followed this curve fairly well but are not plotted. The period for this star is very well established and since there are spectra available(12) it is an important object. Phase shifts such as observed, if this is what it was, may be common in such stars, marking them as cyclic rather than periodic. More information, however, is needed on this score.

2. Irregular variables.

M5 #50

A period of 106.0 days is given for this star by Bailey. However Mrs. Hogg later found it to be irregular(12) and it was certainly not periodic during the 200 days span observed here.

M10 #1

Listed as irregular, observed here as irregular.

M13 #10

This star was almost impossible to observe because of crowding. The few points that were obtained did not fit any periodic variation.

M13 #12

This irregular variable was discovered during the program.

C. Table of data, in apparent magnitudes, for the variables observed.

Table II lists all the data derived from measurement of the light curves which were just presented. The designation of each variable follows Mrs. Hogg(1) except for the new variables discovered. They are assigned numbers following the last listed variables in the cluster. The periods are also from the above reference. No double periods are indicated but are given in Table IV.

The mean magnitudes were obtained from the light curves transformed to an intensity scale and correspond to the magnitude of mean luminosity.

Table II

Globular Cluster	Var. no.	Period	App. Photographic mag.		
			max.	min.	mean
		d			
M3	154	15.2828	11.86	13.50	12.65
	95	103.19	13.31	14.50	13.97
M5	42	25.74	10.76	12.46	11.68
	84	26.5	11.00	12.77	11.59
	50	Irr.	13.6	14.0	13.8
M13	1	1.45899	13.27	14.61	14.01
	6	2.11283	13.90	14.73	14.29
	2	5.11003	12.67	13.90	13.17
	11	92.5	12.92	13.71	13.39
	10	Irr. ?	13.4	13.7	13.51
	12	Irr.	13.32	13.67	13.50
M10	3	7.87	13.10	13.82	13.37
	2	18.754	11.91	13.34	12.51
	1	Irr.	12.8	13.2	13.0
M15	1	1.437478	14.39	15.75	15.02
M2	1	15.5647	13.29	14.79	13.83
	5	17.5548	13.30	14.47	13.71
	6	19.3010	13.07	14.31	13.52
	11	33.600	12.12	13.25	12.60
ω Cent	43	1.1568	13.41	14.55	
	92	1.3451	14.10	14.58	
	60	1.3495	13.32	14.48	
	61	2.2735	13.72	14.48	
	48	4.4743	13.09	13.95	
	29	14.7243	12.44	13.50	
	138	74.6	12.5	13.6	
	53	87.0	13.30	13.87	
	148	90.0	12.9	13.8	
	152	124.0	12.8	13.7	
	42	149.4	12.5	14.9	
	2	484.0	13.06	16.12	

Table II (cont'd)

Globular Cluster	Var. no.	App. Photovisual mag.		
		max.	min.	mean
M3	154	11.70	13.03	12.32
	95	11.97	13.03	12.47
M5	42	10.84	11.86	11.31
	84	10.86	12.09	11.36
	50	11.8	12.1	11.95
M13	1	13.12	14.27	13.69
	6	13.54	14.14	13.85
	2	12.41	13.35	12.78
	11	11.26	11.96	11.70
	10	12.2	12.4	12.30
	12	11.85	12.11	11.98
M10	3	12.58	13.03	12.76
	2	11.44	12.62	11.83
	1	11.2	11.8	11.6
M15	1	14.30	15.58	14.89
M2	1	12.99	14.01	13.39
	5	12.86	13.75	13.24
	6	12.59	13.66	13.03
	11	11.88	12.80	12.23

PART IV

THE ABSOLUTE MAGNITUDE SCALE IN EACH OF THE SIX CLUSTERS

Success of the program depended, in part, on finding the average apparent magnitude of the RR Lyrae stars in each cluster. The usual value of zero absolute photographic magnitude was assumed for these stars and this gave the absolute magnitude of the long period cepheids. The exact absolute magnitude of the RR Lyrae stars was not important for this investigation, any change resulting from future work can merely be applied to all the absolute magnitudes given here. For the same reason, reddening and absorption do not affect these results. Only the difference between the apparent magnitudes of the RR Lyrae stars and the long period cepheids, in each cluster, is important. The absolute value for the RR Lyrae stars is quite arbitrary.

During the work on the color-magnitude diagrams it became evident that the mean color index of the RR Lyrae stars was $+\overset{m}{.}1$. Hence if we assume M_{pg} equals zero, then M_{pv} must equal $-\overset{m}{.}1$.

1.0 Criterion of zero point.

Two methods were used to determine the apparent magnitude of the RR Lyrae stars. The first depended upon the position of these stars in the Hertzsprung-Russell diagram (magnitude plotted against color index for each cluster star). The horizontal branch in such a diagram of a globular cluster starts at negative color indices and stops abruptly at color index zero. Then, at the same magnitude, between color indices of $0^{m}.0$ and $0^{m}.2$ only the RR Lyraes are encountered (mean magnitude and color). Sometimes non-variable stars are present at the same magnitude for a little to the red of $0^{m}.2$. The fact that the RR Lyraes at average color and magnitude fall precisely in this "gap" in the horizontal branch of the globular cluster color-magnitude diagram was discovered in M3(13) and confirmed in M92(5). Since the stars on either side of this gap form a very narrow sequence, the mean apparent magnitude of the RR Lyrae variables can be accurately taken as the average apparent magnitude of non-variable stars on either side of the gap.

The second method was to simply measure the light curves of enough RR Lyrae stars and derive their mean apparent magnitude.

It was simpler to measure a color-magnitude diagram than a number of RR Lyrae stars and, in clusters where the gap was well defined, just as accurate. In clusters where the horizontal branch was poorly defined both methods were used whenever possible.

2.0 Measurement of color-magnitude diagrams.

With the 100-inch telescope diaphragmed to 58 inches to reduce coma, a plate in each color was taken of all clusters excepting M10 and M3. This set of plates was obtained on a night during which the seeing was five or better and yielded measures which would be hard to improve on.

In each cluster an annular area concentric with the cluster was selected for measurement. A limiting magnitude in the photovisual was selected at about a magnitude fainter than that of RR Lyrae stars. The area was adjusted so that it contained from three to four hundred stars, if all the stars in it, down to the limit, were measured. All the diagrams therefore contain about the same relative number of stars and the population of various sections can be compared in different clusters.

The annular area was chosen as close to the cluster as possible in order that the largest possible percentage of stars measured would be cluster members. At the same time the area was far enough from the cluster nucleus so that background light of the cluster would not affect the measures.

The apparent magnitude of the RR Lyrae stars in each cluster is given in Table III. Values derived from each of the two methods are listed and the adopted value used to derive the absolute magnitudes used in Part V are in the last column.

The individual color-magnitude diagrams are collected in Appendix I together with a short discussion of each one.

TABLE III

APPARENT MAGNITUDE OF RR LYRAE STARS

Cluster	RR Lyraes*		Gap & color color limits of gap		Adopted apparent modulus and reddening		Estimated uncertainty
	m_{pg}	m_{pv}	m_{pg}	m_{pv}	m_{pg}	m_{pv}	
M3			15.72	15.62	15.72	15.62	± .05
			.00 to .18		no reddening		
M5			15.24	14.98	15.24	14.98	± .05
			.16 to .36		+.16 reddening		
M13	14.83	14.57			14.83	14.57	± .07
	(2 vars.)				(+.16 reddening)?		
M10			15.22	14.72	15.22	14.72	± .10
			.40 to .60		+.40 reddening		
M15	15.90	15.79	15.89	15.79	15.90	15.79	± .05
	(4 vars.)		.00 to .20		no reddening		
M2	16.17	16.05	16.21	16.05	16.19	16.05	± .10
	(3 vars.)		.05 to .27		+.05 reddening		

* Light curves transformed to intensity scale and magnitude corresponding to mean luminosity derived.

PART V

PERIOD-LUMINOSITY RELATION

1. Mean luminosity of each variable.

Now most mean magnitudes in the past have been taken as the median magnitude (max. + min. divided by 2). The significant quantity for a variable star, however, is the magnitude corresponding to the star's mean luminosity. This quantity corresponds to the mean energy output of the star and is a basic physical property.

All light curves, then, in both colors, were plotted on an intensity scale. These intensity curves were planimetered so that the area above the mean intensity line was exactly equal to the area below. The mean intensity was then converted back to magnitude and this is the quantity which is listed under the heading "mean" magnitude in Tables II and IV.

Since the magnitude scale is logarithmic the process of converting to intensity stretches the maxima of the curves relative to the minima. Therefore the mean magnitude from the intensity curve is always brighter than the straight mean of the magnitude curve. The discrepancy is greatest when the amplitude is the largest and also is larger for stars of broad maxima.

The median magnitude is a first approximation to the intensity mean as can be seen by inspecting Table IV. The accuracy of the observations in this work, however, required

TABLE IV

Cluster	Var. no.	log period	Absolute M_{pg} median	Absolute M_{pg} mean	Absolute M_{pv} median	Absolute M_{pv} mean	Amplitude	color index	log dbl Per.
M3	154	1.184	-3.04	-3.07		-3.40	1.64	+.33	
	95	2.014	-1.81	-1.75		-3.25	1.19	+1.50	
M5	42	1.411	-3.63	-3.56		-3.77	1.70	+.21	
	84	1.423	-3.35	-3.65		-3.72	1.77	+.07	1.724
	50	2.025	-1.4	-1.4		-3.1	.4	+1.7	
M13	1	0.164	- .89	- .82		- .98	1.34	+.16	
	6	0.325	- .51	- .54		- .82	.83	+.28	
	2	0.708	-1.54	-1.66		-1.89	1.23	+.23	
	11	1.966	-1.51	-1.44		-2.97	.79	+1.53	
	10		-1.28	-1.32		-2.37	.3	+1.05	
	12		-1.33	-1.33		-2.69	.35	+1.36	
M10	3	0.896	-1.76	-1.85		-2.06	.72	+.21	
	2	1.273	-2.59	-2.71		-2.99	1.43	+.28	1.574
	1		-2.2	-2.2		-3.2	.4	+1.0	
M15	1	0.158	- .83	- .88		-1.00	1.36	+.13	
M2	1	1.192	-2.15	-2.36		-2.76	1.50	+.39	
	5	1.244	-2.30	-2.48		-2.91	1.17	+.42	1.545
	6	1.286	-2.50	-2.67		-3.12	1.24	+.44	
	11	1.526	-3.50	-3.59		-3.92	1.13	+.32	1.827
ω Cent	43	.063	- .67	- .74			1.14		
	92	.129	- .31	- .31			.48		
	60	.130	- .75	- .81			1.16		
	61	.356	- .55	- .58			.76		
	48	.651	-1.13	-1.15			.86		
	29	1.168	-1.68	-1.78			1.06		
	138	1.873	- .60				1.1		
	53	1.940	-1.07				.57		
	148	1.954	-1.30				.9		
	152	2.093	-1.40				.9		
	42	2.174	- .95				2.4		
	2	2.685	- .06				3.06		

that the intensity means be used. The median magnitudes are included for purposes of comparison.

Both Tables III and IV contain data in the photographic wavelengths for 6 cepheids and 6 red variables in ω Centauri. Martin's beautiful work in ω Centauri(9) was the only previous work of sufficient accuracy to be included with the present data. The weakest point of his work, of course, was the magnitude scale (the zero point is irrelevant). The 12 stars listed in Tables II and IV, however, are at most only 1.7 magnitudes brighter than the RR Lyrae stars. It was felt that any scale errors would not become appreciable in so short a range. This turned out to be the case as will be shortly demonstrated in section 2, and these stars gave important confirmation of the nature of the lower part of the period-luminosity diagram.

Only no. 1 in ω Centauri was not included and that was for two reasons. One, the scale that far from the RR Lyrae's could not be trusted. Two, the median magnitude of the star was very uncertain because of alternating cycle behavior such as observed in four of the variables treated here.

Absolute magnitudes and other data are collected in Table IV. These absolute magnitudes are obtained from Table II by subtracting the apparent moduli listed in Table III.

The median absolute magnitudes are included, as mentioned before, in order to compare them with the intensity means. The difference between median and mean can be as large as $.^{m}3$ but is usually much smaller, also, the only magnitudes which were available for the ω Centauri variables were median. The shape and amplitude of these cepheid light curves were compared to curves in the rest of the table and corrections to the median magnitudes were estimated. Comparison of these two columns for ω Centauri shows the corrections do not make much difference but the estimated intensity means are still probably a bit more significant.

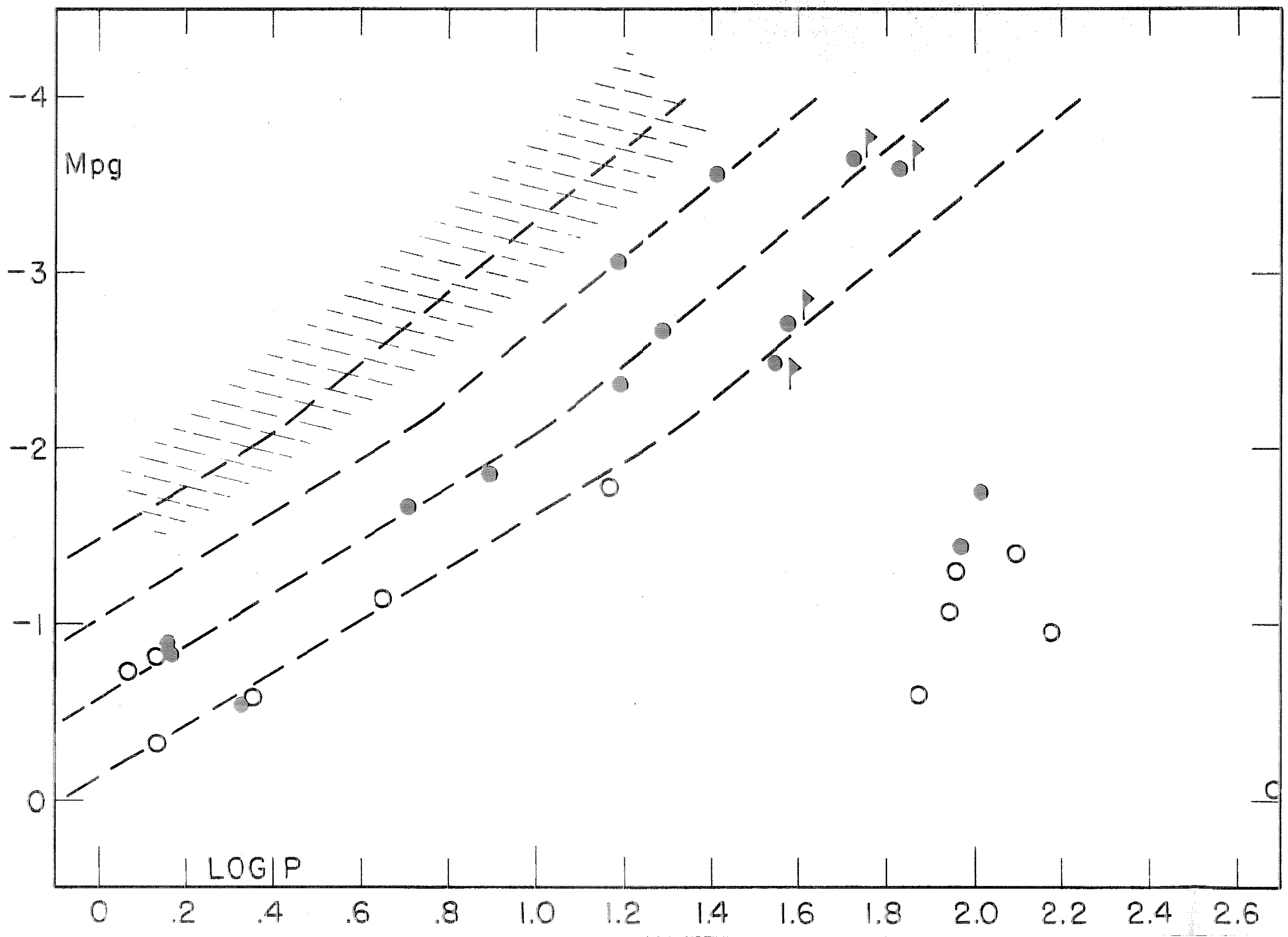
The color indices are normalized, that is the reddening in each cluster is subtracted from the apparent color index.

2. Period-luminosity diagram

Figure 17 exhibits the culmination of the entire project. It summarizes the results on the luminosities and periods of all the variables investigated. Analysis of it will be made in the following section but first the following explanation should be made.

The vertical scale is absolute photographic magnitude and the horizontal scale is log of the period. If the RR Lyrae stars were plotted they would occupy a horizontal line at M_{pg} equal to zero, starting at the extreme left hand edge of the diagram (about $.^{d}8$) and running further

Figure 17. Period-luminosity Diagram. Open circles are variables in w Centauri, closed circles are variables investigated in the present work. RR Lyraes are at M_{pg} = 0. Dashed lines are a factor of 2 in the period from each other. The four variables with double periods are flagged and plotted with the longest period. The shaded area represents the classical cepheids with the Magellenic cloud zero point of -1.5.


Mpg
-4
-3
-2
-1
0
LOG P
0
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
2.6

to left to about $^{d}.2$. The bottom most dashed line in the diagram almost exactly intersects the beginning of the RR Lyraes at M_{pg} equals zero.

All the magnitudes were taken from table IV and the open circles are stars from ω Centauri, the closed circles variables investigated in this project.

The four variables that turned out to have double periods are indicated in the last column of Table IV. It was finally decided to plot them with the long period since that is closer to the fundamental period of the star. Those points are flagged in the diagram.

Dashed lines are drawn through the points in the diagram. These lines are exactly factors of 2 in the period apart. It is obvious, therefore, that plotting the four stars with the double periods at their longer period merely makes them fall one line to the right of where they would fall if the original periods were used. These are the points which show the largest scatter from the line and it is felt that for these stars, where a mean light curve had to be estimated between cycles of different amplitude, the accidental error of the luminosity is the greatest.

The uncertainty for the stars which repeated well is due mainly to the uncertainty of the zero point in each cluster. It is estimated that in no case could this have been in error by more than a tenth of a magnitude. The four variables which had alternating cycles could not have errors in the luminosity greater than one-tenth. The total uncertainty for these four stars then is well under $\pm\overset{m}{.}15$, the uncertainty for the rest well under $\pm.10$. Since the dashed lines are from $\overset{m}{.}5$ to $\overset{m}{.}6$ apart in magnitude there is no doubt that the variables actually do fall on these lines of factor 2 apart in the period.

Finally the relationship for the classical cepheids is indicated by the shaded area. The determination of the zero point for classical cepheids rested, in the past, on proper motions of these stars in our own galaxy. This older, zero absolute magnitude for 1 day periods, would cause the classical cepheids to fall along the bottom, dashed line in the diagram. While this investigation was in progress, however, Thackery found the RR Lyrae stars(14) one and one-half magnitudes below the classical cepheids of one day in the small magellanic cloud. The classical cepheids are represented in the diagram with this new zero point of $-1\overset{m}{.}5$.

3. Analysis of the diagram.

The fact that the type II cepheids fall along discrete lines in the period luminosity diagram, lines that are translated from each other by exactly factors of 2 in the period, is a startling result. Such a situation has never been suspected, although looking back through the literature now, it is apparent that the relationship was present. It only required the accurate putting together of various isolated results to enable the total picture to emerge.

The first crucial question which must be answered is the following. Do stars of the same period which fall on different lines in the P-L plane show differences from each other which are correlated with the particular line on which they fall? The answer is an emphatic yes and the differences are three fold:

1) Shape of the light curve.

The four variables that fall on the second from the bottom line between log P equals 0 and 0.2 all have remarkably similar light curves. They are of large amplitude with high, sharp peaks. The three of them that range from 1.3 to 1.45 days all have light curves that show a sudden change to a more gentle slope at a point where the curve has descended from maximum to about 2/3 of the way to minimum. In contrast, the variable of 1.3 day period that falls right below them on the bottom line has a very small amplitude, symmetric light curve.

The next two variables that fall on the bottom line are of almost exactly the same period (2.11 and 2.27 days). They have very unusual light curves which are identical. Both curves show a gentle rise to maximum, a conspicuous bump, then a sharp rise to maximum and a long, straight decline to minimum.

Of the two variables with periods 4.5 and 5.1 days the one that falls on the upper line has a larger amplitude and a somewhat more peaked light curve than the one that falls on the bottom most line.

The two variables that define the third line from the bottom, the line that falls closest to the classical cepheids, have light curves which actually look much more like classical cepheids that the type II cepheids which fall on the two lines below. The curves have high peaks and inconspicuous still stands at about half-way between maximum and minimum. One of the variables (M3 #154, period $15\overset{d}{.}3$) that falls on the third line has a light curve that resembles η Aq. or XX Sgr., two classical cepheids with about half that period. The other variable that falls on this third line (M5 #42) has a light curve that resembles Z Lac. a classical cepheid, again of about half the period.

The two lines that fall below the variables just discussed are populated with variables that have much broader maxima, or what is possibly more exact, conspicuous secondary maxima. They look quite different from the variables just discussed. Yet there are differences between these bottom two lines also. The 14.7 day variable (w Cent #29) has a maximum which is .35 phase unit s broad, but the

15.5 day variable on the next line above (M2 #1) has a maximum only .2 phase units broad. This is in the previously established sense of the light curves getting more peaked as we go up the lines in the diagram at a certain period. The rest of the variables in this part of the diagram fall into the same pattern.

2) Amplitude of the light curve.

It is difficult to find any exact period amplitude relation, the point scatter being rather large. Two things are clear, however. One, the period amplitude relation is different for stars of less than 10 days period than for stars of greater than 10 day periods. Two, at any given period the amplitude increases as we go up the lines in the diagram.

For stars between 1 and 2.3 days, the mean amplitude on the line next to the bottom is $1^m.25$, the mean amplitude on the bottom line is $0^m.69$. For stars between 4 and 8 days the mean is 0.97 opposed to 0.86 on the bottom line. (This last does not mean much -- there are only three stars involved.)

For the 9 variables with periods between 14.7 and 33.6 days the mean amplitude on the bottom line is 1.22, on the next highest line 1.41, and on the third line up 1.67. It is suggestive that Eggin(16) finds the same order of amplitude differences between his classical cepheids of type A, B and two types of C, although the connection between those classes and the stars here is by no means clear.

3) <u>Color indices</u>.

Like the amplitude relation the color indices show large scatter. But the same two things are evident. Namely, there is a different color index-period relation for variables of less than 10 days and greater than 10 days. For stars with periods less than 10 days the color seems to get redder with longer period. The spectra of these stars(12) run in this same sense, as do the spectra of the classical cepheids(15). For stars with periods greater than 10 days the variables here indicate <u>bluer</u> color with increasing period. This is the same thing that is very roughly indicated by the spectra of galactic classical cepheids of somewhat shorter period. The eight stars in the upper part of the diagram, if mean color indices of the variables falling near each other are taken, giver period-color lines which are again factors of 2 apart. The variables falling on separate lines in the P-L plane giving separate lines in the color period diagram. This is shown in Figure 18, section 4.

Finally the spectra available though showing too much scatter to differentiate between lines, show that the stars measured in this investigation are in the mean equal to classical cepheids of the same luminosity (as pictured in Figure 17) and hence comparable to classical cepheids of 1/2, 1/4 or 1/8 the period as the case may be.

4. Interpretation of the period-luminosity law.

In any star which is in equilibrium, the downward pull of its gravity just counterbalances the outward thrust of its gas and radiation pressure. If this is a stable equilibrium any small displacement of material in the star will cause the material to oscillate back and forth about the equilibrium point until it is damped out or a new displacement occurs. In the first case we have a "free" oscillation, in the second a "forced" or "driven" oscillation.

Now it may be that the whole star is pulsating as pictured in the current adiabatic pulsation hypothesis or it may be that only the outer portions of the star actually pulsate. In either case there exists a differential equation, involving such parameters as density, temperature, velocity, etc., which exactly describes this oscillation. The lowest frequency, or fundamental, of course, satisfies this equation but also higher frequencies or overtones are possible. Indeed the star may oscillate with any one of these higher frequencies or any combination of them depending on the boundary conditions.

In the case of driven oscillations, those oscillations where the forcing term is in resonance with one of the natural frequencies of the system will be the most important.

In all cases, however, it is possible that stars which are physically quite similar, and hence have the same differential equation of oscillation, can vary with different periods determined by only slightly different boundary conditions.

It is suggested then that the stars found here to lie on the different lines in P-L plane are merely pulsating in overtones of the fundamental frequency of vibration.

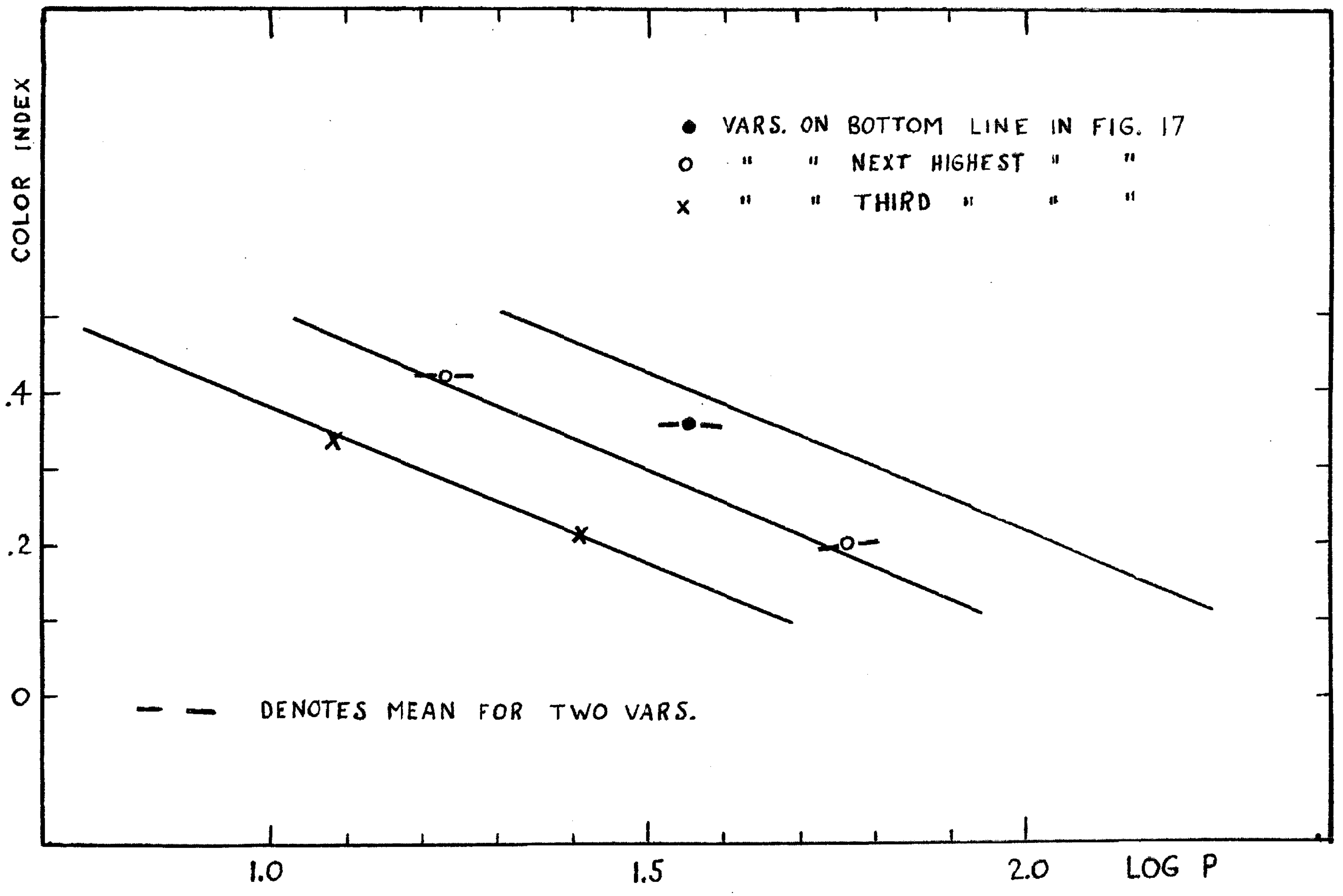
It can not be stated definitely that the lowest line in Figure 17 represents the fundamental period of oscillation. It seems logical, however, from the following standpoint. The RR Lyrae stars and the type II cepheids should form a continuous sequence in the period luminosity diagram. This is obvious from the similarity in spectra and luminosity between the longest period RR Lyrae stars and the shortest period type II cepheids. It is precisely the lower line in Figure 17 which forms a continuous junction with the RR Lyrae stars at log period $\approxeq$ -.1. Now it is conceivable that all RR Lyrae stars are varying in overtones of their fundamental and that they would extend to the right of log period equals zero, at magnitude zero if plotted at their fundamental period. This would allow a line of even lower frequency of vibration for the type II cepheids to join smoothly to the RR Lyrae stars. Since there are no RR Lyrae or type II cepheid stars known, that populate these possible lines of lower frequency in the P-L plane, however, it must be assumed that the lowest line in Figure 17 actually represents the fundamental period of vibration.

The fact that the stars on the upper P-L lines are actually varying in overtones of the fundamental line on the bottom can be pretty definitely established by the following circumstance.

Not much can be said about the stars of periods less than 11 days since it is hard to determine which way the colors go with increasing period and stars with known colors are not well distributed between the different lines. For the stars brighter than M_{pg} equal to -2, however, every case shows that the stars get bluer as the period gets longer at least to the limit of $M_{pg} = -3.7$ of the stars investigated here. It is then possible to see from Figure 18 that the color-period lines are again a factor of 2 apart in the period. This means that variables of the same absolute magnitude have the same color regardless of which line they fall on. In turn this implies the stars are the same kind of stars at any level of absolute magnitude and that it is merely the particular overtone in which they are varying which determines what period they will have.

This fact, however, is really unquestionably established by the four variables that had double periods. Here the star actually <u>is</u> the same star that can be considered as lying on one line or the other according as the longer or shorter period is used. It is merely being suggested here

Figure 18. Color-period relation for 8 variables between M_{pg} = -2.3 and -3.7 in Figure 17.


● VARS. ON BOTTOM LINE IN FIG. 17
o " " NEXT HIGHEST " "
x " " THIRD " " "
— — DENOTES MEAN FOR TWO VARS.
COLOR INDEX
.4
.2
0
1.0
1.5
2.0
LOG P

that the shorter period is the overtone of the long period i.e. the second permitted frequency. This is almost demanded by the fact that all these variables are from globular clusters whose H-R diagrams are so similar as to signify they are physically the same kind of stars. If two of these variables then have the same absolute magnitude and different period, it cannot be due to the fact that they are say built on different models. It must be that they are varying in different permitted frequencies both of which satisfy the same differential equation of oscillation. This remark holds absolutely of course for the stars which showed double periods.

It is interesting in this connection to note that the two stars on the uppermost populated line are from the clusters M3 and M5. These two clusters are almost identical to each other and although they occupy only very slightly different regions in the H-R diagram than other clusters, they show a more noticeable difference in distribution of stars within their array when compared to other clusters. The implication here is that the slight physical difference between these stars and stars in the other clusters does not appreciably affect the model, which determines the differential equation of oscillation, but affects rather the boundary conditions which determine the dominant mode of vibration.

In one of the same clusters, M5, is a star (#84) which falls on the third line with its short period and on the adjacent line with its long period. Since the other cepheid in M5 (#42) falls on the third line with no indication of a double period, and differs by only $^{m}.09$ in luminosity the indication is that a very small physical difference between stars can change the mode of vibration.

The shapes of the light curves can also be roughly predicted on this hypothesis of overtone variation. The empirical data of Figure 17 indicates the overtones occur in exact factors of 2. The data on the double periods indicates that they are in phase. In can easily be seen how adding together varying amounts of in-phase variations of double or quadruple the frequency can be made to give sharp rises, still stands, and humps. In fact it is necessary to add in the higher frequencies to get the sharp peaked light curves observed in the third from the lowest line in Figure 17. Of course it is just on this line that the existence of high frequency overtones is demanded by the previous discussion.

The fact that the overtone ratios come out to be exactly factors of 2, for at least the first three modes of vibration is strong evidence against the present theory of cepheid variation where the entire star performs adiabatic pulsations.

All stellar models (such as homogeneous, inverse square, etc.) which have been investigated on that hypothesis, give non-integral overtone ratios. From this it seems certain that no intermediate model can give succesive overtone ratios of exactly 2.

What the present observations might seem to indicate is that only the outer portions of cepheids pulsate. There may be a discrete outer bound given by gravity and an inner bound given, perhaps, by the beginning of the hydrogen connective zone. (Heat escaping from this convection zone may give rise to a driving force which couples with the free oscillation period of the material above and excites the overtone pulsations.) In any event, the important point is that given a definite inner and outer boundary for the pulsation, all successive overtones must be exactly a factor of 2 apart in period.

The following procedure is then suggested. A model cepheid should be constructed in which only the outer portion, between the two boundaries, pulsates. If this model satisfies all the observational tests of cepheid variation and no other model can be found which does the same, then the observations here would definitely prove that this is, in truth, the explanation of cepheid variation. For now, however, the observations only indicate this model as a possibility.

At first it seemed an accident that the classical cepheids fell on the highest line of Figure 17. This line was not populated by stars investigated here but merely obtained from the lower line by translation with a factor 2 as the lower lines had been.

Two circumstances indicate, however, that the classical cepheids may be varying in an even higher overtone of the same scheme that was just outlined. First is the resemblance of the light curves of the two type II cepheids on the line nearest the classical cepheids to the light curves of some classical cepheids previously mentioned. In general the classical cepheids have an even sharper rise to maximum which could be due to larger amounts of the high frequency overtones.

Secondly, and most important, the spectra available for these type II cepheids corresponds, in the mean, to classical cepheids of from 2 to 8 times shorter a period. This now points to the identity of cepheids at the same absolute magnitudes whether they be classical or type II. The only apparent explanation for the shorter periods of the classical cepheids is that they are pulsating in higher overtones in general.

Eggen has found evidence for splitting classical cepheids into four groups correlated with amplitude of variation. Undoubtedly the groups are correlated with the groups found here. What the connection is, it is impossible to say at this point.

The idea that classical and type II cepheids, members of populations which apparently are quite different especially in the giant branches, are so intimately related may not be so hard to accept if viewed in the following light: The RR Lyraes occupy a very small region in the H-R plane and stars immediately adjacent on either side are non-variable. If one says that the luminosity and temperature which places a star in the H-R diagram more or less uniquely determines its structure and general physical properties, then the fact that no non-variable stars occur in the region of the RR Lyraes implies that this region of the H-R diagram is inherently unstable and stars occupying it will be pulsating stars.

If we go slightly higher in luminosity than the RR Lyraes, at about the same color index, we encounter globular cluster variables with periods of around one day. Presumably the same remarks apply to this region of the H-R plane. Continuing in this way it is possible to trace out a more or less continuous region in the H-R plane which represents a physical state of instability that will make any star falling in it pulsate. This region as determined from the work here would run roughly vertical in the H-R plane starting at the magnitude of the RR Lyrae stars (about $M_{pg} = 0$) and color index .0 to .2, continuing up within color limits of about + .1 to + .5 to absolute magnitude -3.7 (as far as it was able to investigate here). From the data available for classical cepheids this region probably gradually turns toward redder color index as it goes still higher in luminosity.

Any stars falling within this path of instability in the H-R diagram would become cepheids, whether they were type I or type II stars. Now the most significant difference between the two populations is the quite different regions of the H-R plane above $M = 0$ which they occupy. The type II population as exemplified by the globular clusters would contain stars as bright as -2 or perhaps as bright as -3.7 which could fall into the region of instability and become cepheids. But probably no higher luminosities would be present in the population II and this would explain why no cepheids brighter than -3.7 are observed. It would be necessary to go to the highly luminous type I population to get cepheids which are brighter. This is precisely what is observed in the type I cepheids of enormously greater luminosity. It is well known how the type I H-R diagram has a scattering of stars to redder color index at very high luminosity and these undoubtedly intersect the region of instability at levels of high luminosity. By the same token type I populations avoid the region of the RR Lyraes and therefore we might expect to find no type I RR Lyrae stars or cepheids near that absolute magnitude. Again this seems to be the observed case.

5. Conclusion

The major aim of this investigation was to find accurate luminosities of the type II cepheids in terms of the brightness of the RR Lyrae stars. The importance of this project was assigned by the possibility of using the resultant relation for determining a segment of the distance scale of the universe.

The results show that the period-luminosity relation derived here, can indeed be used to determine distances with great accuracy. The only unexpected qualification is that the light curves of the variables must be determined with moderate accuracy in order that the particular line in the period-luminosity diagram on which it falls may be identified.

The observational results presented in sections 2 and 3 seem to promise a better insight into the nature of cepheid variation and the connection between type I and II cepheids. Some of the obvious implications of this new data have been discussed and a tentative explanation for the connection between the two types of cepheids was advanced. Much detailed work, however, is required before the nature of cepheid variation can be said to be known. And thorough testing of the idea of the relation of the two types of cepheids is necessary before it can be dignified with the name of theory.

Some of the more obvious attacks on the data will be made immediately. First, a frequency analysis of the light curves should yield quantitative results of real significance and show whether the light curves can be made up of the indicated frequencies. Secondly, the stellar model and differential equation of oscillation which governs it should be ascertained. The data derived here should materially assist in finding these two relations. Ultimately a model for a cepheid must be produced which will fit the observations presented in Figure 17.

That the observational tests such a model must meet are essentially simple and exact has been shown here. This may mean the model for, and nature of, cepheid variation are also essentially simple and exact. If this is the case, and hopefully it is, an important problem in astronomy may be on the verge of solution.

APPENDIX I

Color-magnitude Diagrams for Six Globular Clusters.

Figure 19. The measures plotted here for M3 are those of Allan R. Sandage, who kindly contributed them in aid of the program.

There are 379 stars in the diagram measured about four times each in the photovisual and photographic.

Mean magnitude and color of the variables measured here, enable them to be placed in this diagram. They are indicated by circled crosses. The one at the left is the 15 day cepheid and the one at the right that falls just off the extension of the giant branch is the 103 day period.

M3 has the largest number of RR Lyraes stars known in any cluster and also the highest relative population on the red side of the gap.

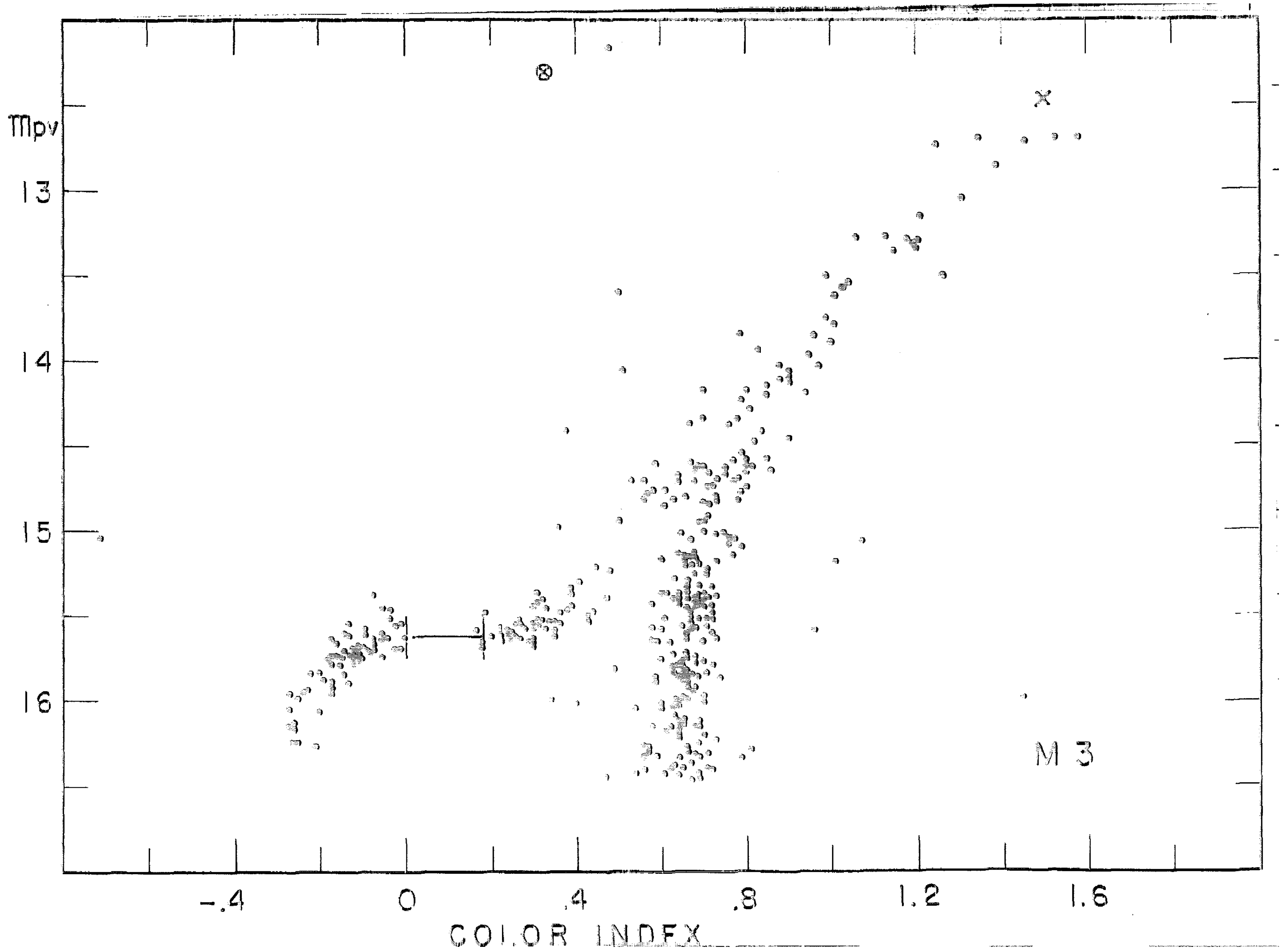
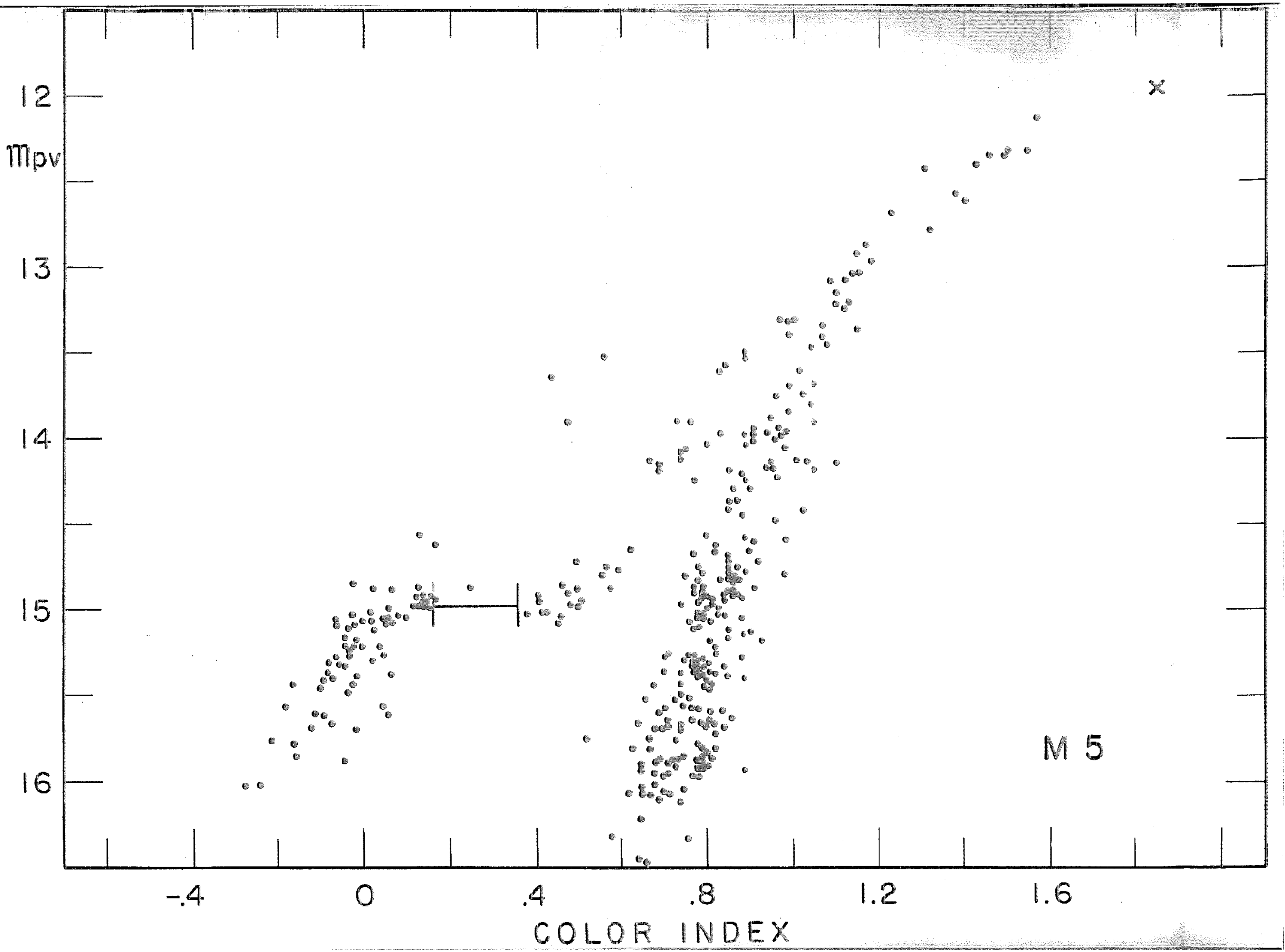

m_pv
13
14
15
16
-.4
0
.4
.8
1.2
1.6
COLOR INDEX
M 3

Figure 20. The diagram for M5 was obtained from the measurement of two 100-inch plates, one in the photographic and one in the photovisual.

The two cepheids are too bright to be shown on the diagram but the irregular variable is plotted at its position right off the end of the giant sequence.

The cluster is reddened by about .16 of a magnitude.

The cluster contains a large number of variable stars and consequently shows a well populated red side to the gap.



12
Mpv
13
14
15
16
-.4
0
.4
.8
1.2
1.6
COLOR INDEX
M 5
x

Figure 21. Stars in the diagram of M13 were measured once on each of two 100-inch plates. Stars brighter than 14.5 and a few stars at the red end of the horizontal sequence were measured also on three 60-inch plates in the red and blue.

The cepheids fall between color index .3 and .5. The irregular and long periods fall along the giant branch. Two RR Lyrae stars are indicated by open crosses.

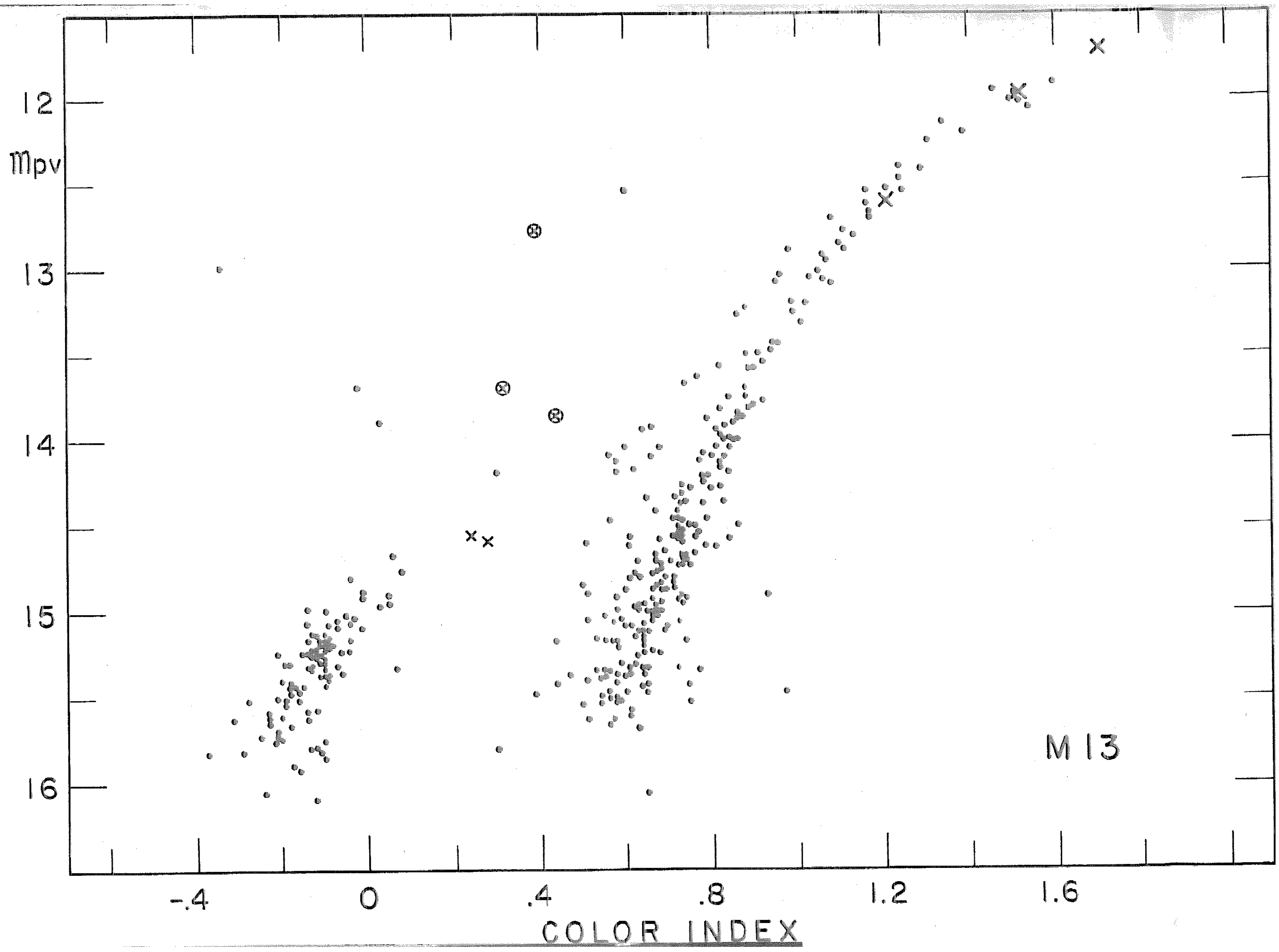

12
13
14
15
16
mpv
-.4
0
.4
.8
1.2
1.6
COLOR INDEX
M 13

Figure 22. The stars in M10 were measured once each on 60-inch plates.

The cluster shows redding of about + .4.

The two open circles to the left are cepheids, the one to the right is an irregular variable.

No RR Lyrae stars are known in this cluster.

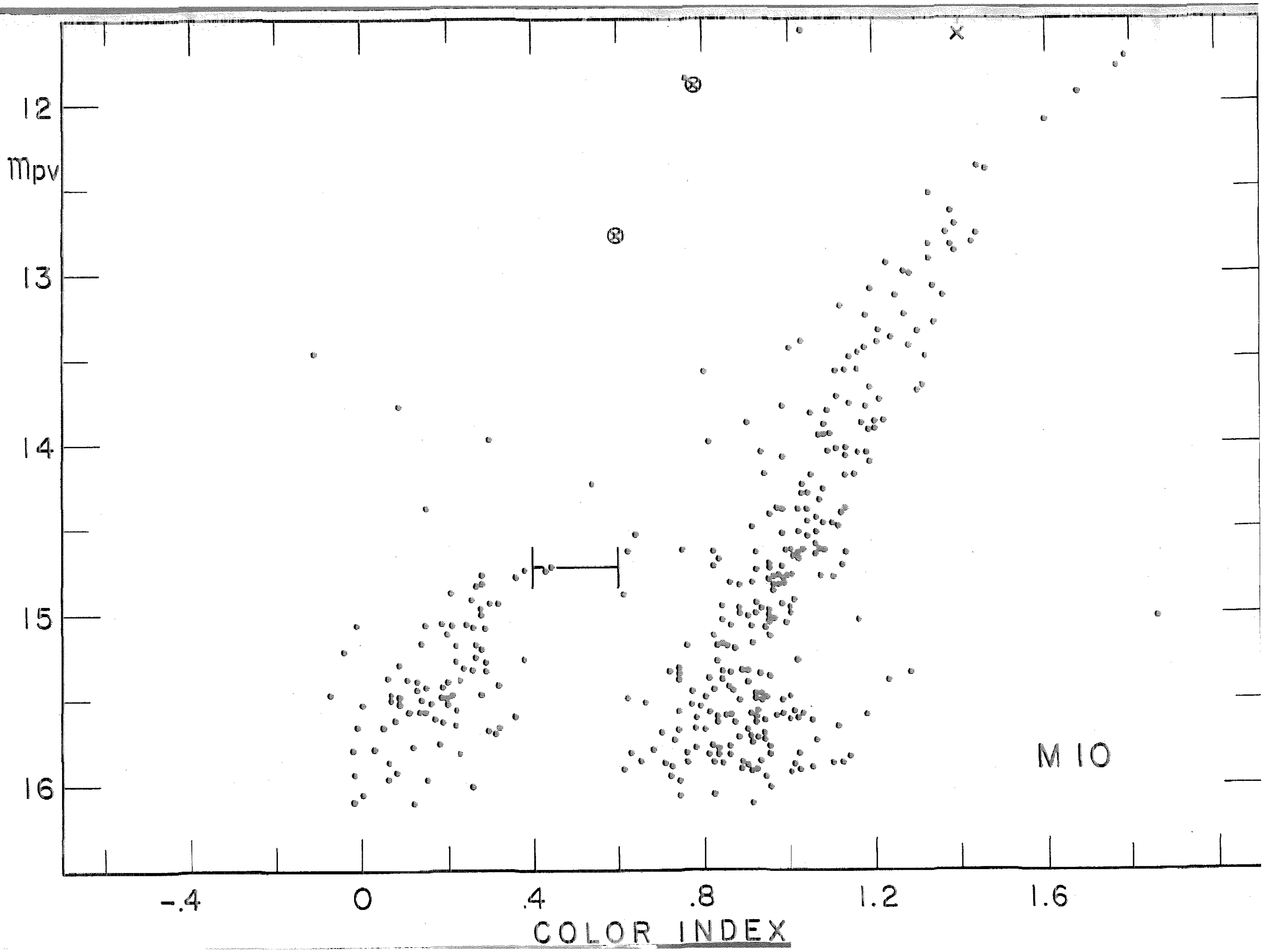
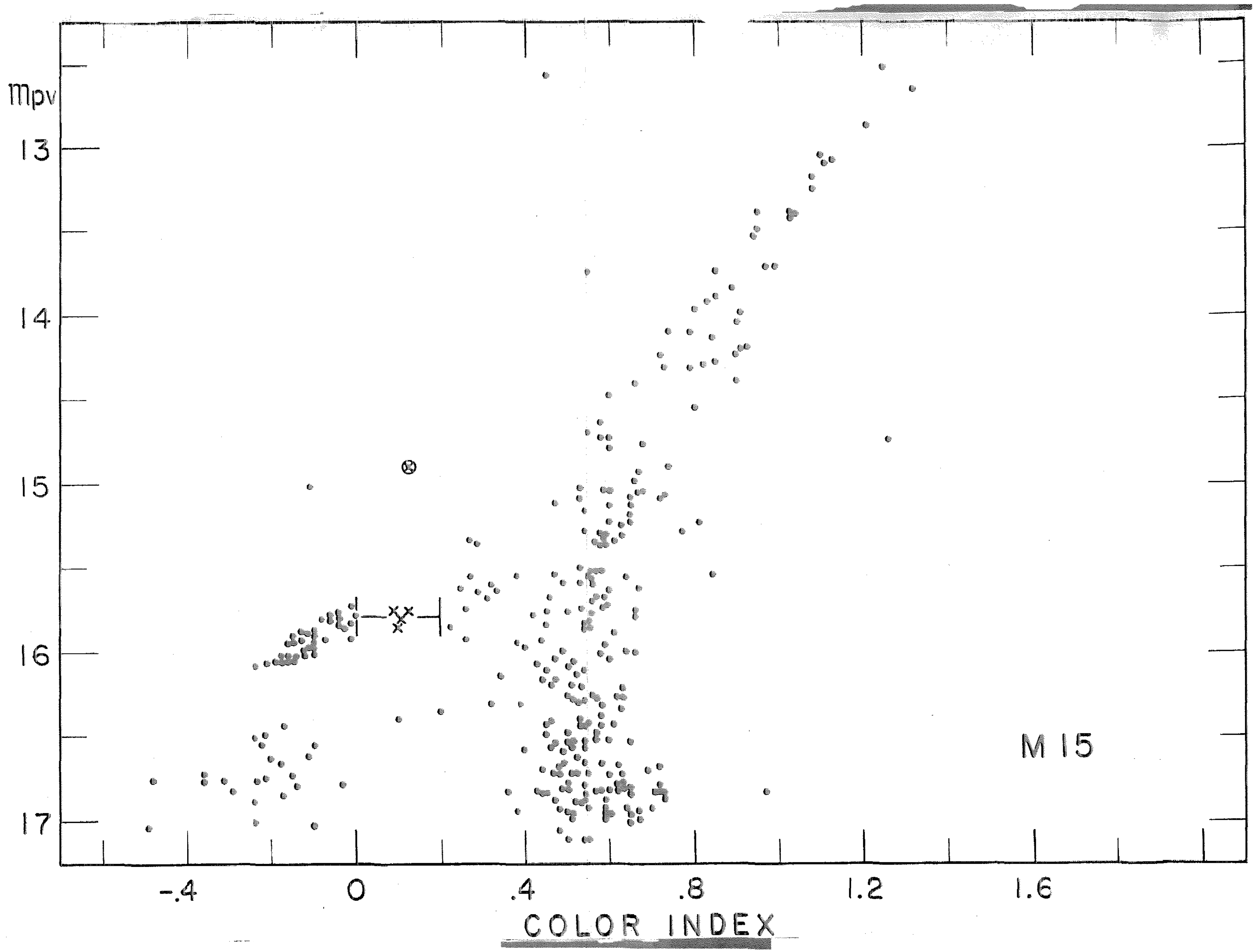

M 10
mpv
12
13
14
15
16
-.4
0
.4
.8
1.2
1.6
COLOR INDEX

Figure 23. The stars in M15 were measured once each on 100-inch plates. The fine quality plates enabled stars very close to the center of the cluster to be measured and therefore many fewer field stars occur than in previous diagrams(7).

The one and one-half day cepheid is shown with a circled cross. Four RR Lyrae stars are indicated by open crosses.

A moderately large number of RR Lyrae stars are present in this cluster, correlating with the moderate population on the red side of the gap.


mpv
13
14
15
16
17
-.4
0
.4
.8
1.2
1.6
COLOR INDEX
M 15

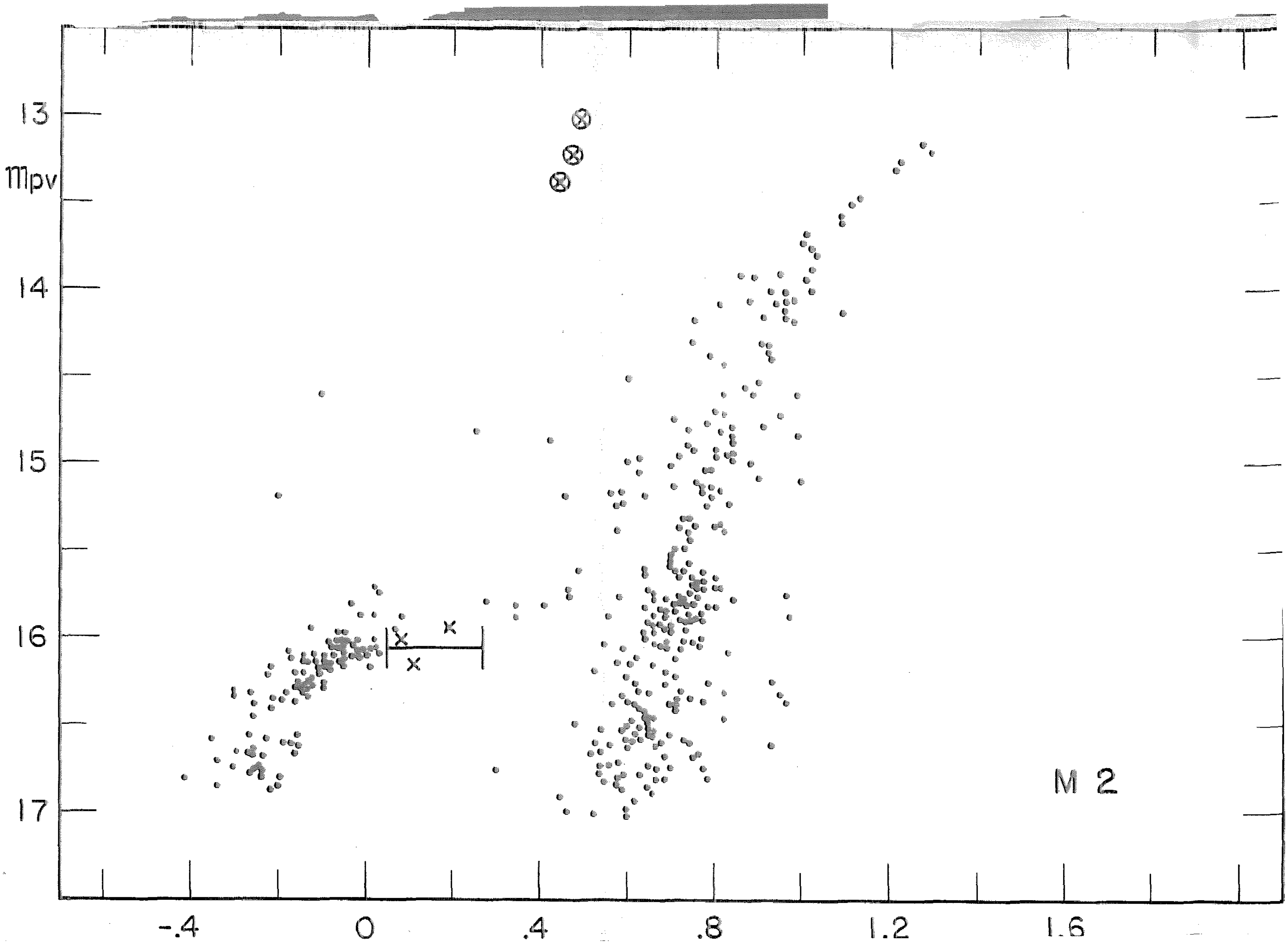
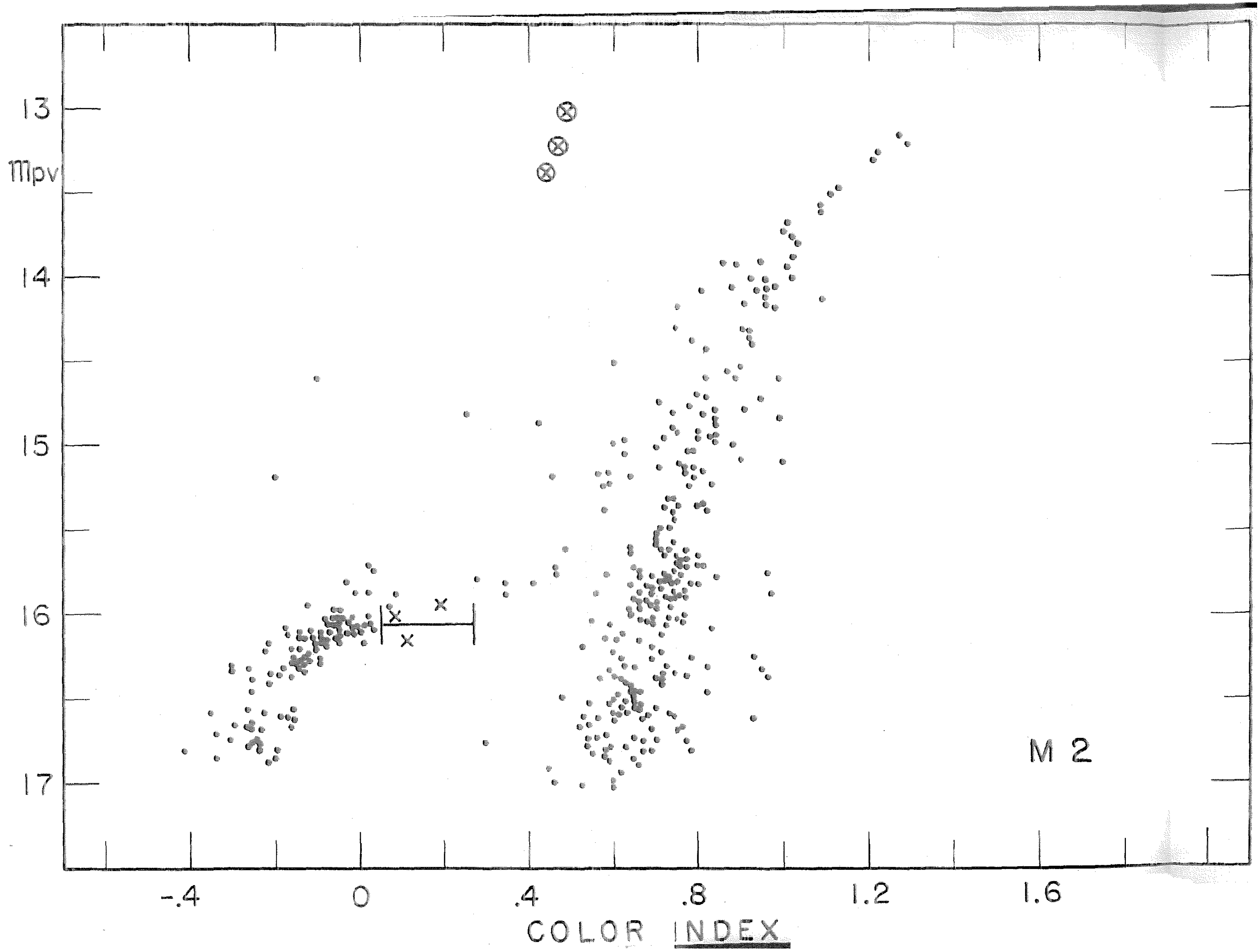

13
mpv
14
15
16
17
-.4
0
.4
.8
1.2
1.6
M 2

Figure 24. Color-magnitude diagram for M2.

Three RR Lyrae stars are shown with open crosses. The three faintest cepheids appear at the top of the diagram with circled crosses.


13
14
15
16
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