# Direct Design to Horizontal Sway Limitation for Unbraced Steel Frames

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## ABSTRACT

A method applicable to the design of unbraced multi-storey steel frames to specified limits on horizontal sway deflection is presented. This method of design is the extension and combination of the work by Wood and Roberts [1] and Anderson and Islam [2]. Only simple calculations are required by the method and its application is illustrated by worked examples. Regular and non-regular steel frames are considered. The method proposed is suitable for the design of unbraced multi-storey frames if the choice of sections is controlled by sway deflection.

## Notations

| | |
|---|---|
| $K_{bt}$ | is the top beam stiffness of substitute frame |
| $K_{bb}$ | is the bottom beam stiffness of substitute frame |
| $K_c$ | is the column stiffness of substitute frame |
| $K_u$ | is the upper adjacent column stiffness of substitute frame |
| $K_l$ | is the lower adjacent column stiffness of substitute frame |
| $\Delta$ | is the storey sway |
| $H$ | is the storey height |
| $F$ | is the wind shear |
| $K_c$ | is the Grinter substitute frame column stiffness |
| $E$ | is the elastic modulus of member |
| $\bar{s}$ | is taken as zero if effect of cladding is ignored |
| $\bar{\Phi}$ | is the sway index |

## INTRODUCTION

In unbraced frames, lateral stiffness is provided by the flexural rigidities of the beams and columns, connected by moment resisting joints. Thus, some degree of continuity is essential for the overall stability of the frame. The detailing of the connections may be expensive to ensure the frame stability. However, unbraced frames offer more flexibility in architectural planning than braced construction. For unbraced frames, the control of sway deflection can be a more important criterion than strength; design to ultimate resistance will often result in excessive sway under service loading. This form of deflection arises mainly from the wind, and its control may govern the member sections. There were many limits for the sway ($\Delta$):height (h) ratio, $\phi$, to be used in design. A survey by the Council on Tall Buildings [3] showed that the limiting value varied from 1/1000 to 1/200. However a limit of 1/300 has been commonly used and recommended by present British and European guides.

## DERIVATION OF DESIGN EQUATIONS

By analysing the frame using the stiffness distribution method suggested by Wood [4], Wood and Roberts [1] distribution coefficients, $k_t$ and $k_b$ (see Fig. 1) can be defined as

$$k_t = \frac{K_c + K_u}{K_c + K_u + K_{bt}} \tag{1}$$

$$k_b = \frac{K_c + K_u}{K_c + K_l + K_{bb}} \tag{2}$$

The authors [1] also obtained a non-dimensionless expression for sway index, $\overline{\Phi}$ where,

$$\overline{\Phi} = \left[1 + \frac{3(k_b + k_t - k_b k_t)}{4 - 3k_b - 3k_t + 2k_b k_t + \bar{s}(1 - k_b k_t/4)/3}\right] \tag{3}$$

The actual sway index is then obtained from :

$$\overline{\Phi} = \frac{\Delta / H}{FH/(12EK_c)} \tag{4}$$

To develop this into an expression for direct design the same devices used by Anderson and Islam [2] (i.e. assuming point of contraflexure at mid-height of column and also minimising the Σ second moment of area x length) are now applied to the substitute frame concept which Wood and Roberts [1] used as the basis for their analysis.

Imposing a central point of contraflexure on the column of a single 'cell' in Fig. 2(a) implies that the distribution coefficients, $k_t$ and $k_b$ at the top and the bottom of the column are equal. This arises because, from Fig. 3 it can be seen that

$$M_t = M_b \qquad \text{and} \qquad \theta_t = \theta_b$$

Therefore

$$k_b = k_t = k \tag{5}$$

Substituting Eqn. (5) into Eqn. (3), the sway index now becomes:

$$\overline{\Phi} = 1 + \frac{3(2k - k^2)}{4 - 6k + 2k^2}$$

$$= \frac{4 - 6k + 2k^2 + 6k - 3k^2}{4 - 6k + 2k^2}$$

$$= \frac{4 - k^2}{(4 - 2k)(1 - k)} = \frac{(2 - k)(2 + k)}{2(2 - k)(1 - k)}$$

$$\therefore \qquad \overline{\Phi} = \frac{2 + k}{2 - k} \tag{6}$$

and from Eqn. (4) and Eqn. (6),

$$\frac{\Delta / H}{FH / (12EK_c)} = \frac{2 + k}{2 - k} \tag{7}$$

As:

$$(\Delta/H) = [(FH)/(12EK_c)] \cdot \overline{\Phi} \tag{8}$$

Equation 8 refers to the point which lies on the diagonal of Fig. 1.

By taking the limit of sway:height ratio recommended by the UK guides [1,2],

$\Delta / H \quad = \quad 1/300$ gives:

$$K_c = (25\, FH / E)\overline{\Phi} \tag{9}$$

or more generally :

$$K_c = f\,\overline{\Phi} \tag{10}$$

where the constant, $f$ in Eq. 10 may be adjusted to suite other deflection requirements.

## DESIGN EQUATION FOR INTERMEDIATE STOREY OF A FRAME

Fig. 4 shows a typical intermediate storey in a building frame indicating the column and beam stiffnesses. For an intermediate storey, the distribution coefficients $k_t$ and $k_b$ are given by Eq. 1 and Eq. 2. The requirement that these be equal can be satisfied by making the column stiffnesses $K_{cu}$, $K_c$ and $K_{cl}$ (Fig. 4) the same while insisting that the upper and lower beam stiffnesses $K_{bt}$ and $K_{bb}$ are also equal.

Allowing for continuity between the storeys, the distribution coefficient from Eq. 1 and Eq. 2 can be rewritten as:

$$k = \frac{2K_c}{2K_c + K_b} \tag{11}$$

where $K_b$ denotes the beam stiffness.

Substituting Eq. 11 into Eq. 6 gives :

$$\overline{\Phi} = \frac{(3K_c + K_b)}{K_b} \tag{12}$$

When this expression for sway index is substituted into Eq. 10 the following relationship is obtained :

$$K_c = \frac{f \cdot K_b}{(K_b - 3f)} \tag{13}$$

For multi-bay frames Anderson and Islam [2] made the 'weight', W, per storey of each equal bay width as approximately :

$$W = \Sigma(H I_c) + \Sigma(B I_b) = \Sigma(H^2 K_c) + \Sigma(B^2 K_b) \qquad (14)$$

where $B$ is the bay width.

Now if $K_c$ and $K_b$ refer instead to the equivalent Grinter-frame concept which Wood and Roberts [1] used to formulate the graphical method, we have:

$$K_c = \Sigma(I_c / H)$$
$$K_b = \Sigma(3I_b / B)$$

The 'weight' is now :

$$\therefore \quad W = H^2 \cdot K_c + \frac{B^2}{3} \cdot K_b \qquad (15)$$

For unequal bay widths, $B$ may approximately be taken as the average span.

Substituting $K_c$ of Eq. 13 into Eq. 15 and differentiating the expression with respect to $K_b$ and setting zero for a minimum leads to

$$\frac{dW}{d K_b} = \frac{B^2}{3} + H^2 f \left( \frac{1}{K_b - 3f} + \frac{-K_b}{(K_b - 3f)^2} \right) = 0$$

$$\frac{B^2}{3} + H^2 f \left( \frac{K_b - 3f - K_b}{(K_b - 3f)^2} \right) = 0$$

$$\therefore \quad H^2 f \left( \frac{K_b - 3f - K_b}{(K_b - 3f)^2} \right) = \frac{B^2}{3}$$

$$\therefore \quad 9 H^2 f^2 = B^2 (K_b - 3f)^2$$

$$\therefore \quad K_b - 3f = \frac{3Hf}{B}$$

$$\therefore \quad K_b = 3f \left( 1 + \frac{H}{B} \right) \qquad (16)$$

Substituting $K_b$ into Eq. 14, $K_c$ follows from:

$$K_c = \frac{f\ K_b}{K_b - 3f} = \frac{3f^2\left(1+\dfrac{H}{B}\right)}{3f\left(1+\dfrac{H}{B}\right)-3f}$$

$$\therefore \qquad K_c = \frac{f\left(1+\dfrac{H}{B}\right)}{\dfrac{H}{B}} = f\left(1+\frac{B}{H}\right) \tag{17}$$

## COLUMN AND BEAM TAPERING

The simple expression of Eq. 16 and Eq. 17 can be applied to each storey in turn. However, the increasing wind shear down the frame results in more than one design for each member. This can be avoided by 'tapering' the design to resist the increasing wind shear.

### Beam tapering

The assumptions used by Anderson and Islam [2] resulted in relationships between the stiffness of the upper and lower beam (Fig. 5). With second moments of area $I_{bu}$ and $I_{bl}$, then from Fig. 5(b) the `weight' is :

$$W = H^2 K_c + B^2 I_b$$

$$= H^2 K_c + \tfrac{1}{2} B^2 I_b + \tfrac{1}{2} B^2 I_b$$

$$= H^2 K_c + B^2 \frac{(I_{bu} + I_{bl})}{2}$$

$$\therefore \qquad I_b = \frac{(I_{bu} + I_{bl})}{2} \tag{18}$$

The relationship between the $I_{bu}$, $I_{bl}$ and the wind shear are:

$$I_{bl} = \frac{(F_2 h_2 + F_3 h_3)}{(F_1 h_1 + F_2 h_2)} \cdot I_{bu} \tag{19}$$

**Column Tapering**

Consider a column subjected to pure sway as shown in Fig. 6.

By slope-deflection equation:

$$M = 6EI\frac{\Delta}{h^2}$$

$$\therefore \qquad I = \frac{Mh}{6E\left(\frac{\Delta}{h}\right)}$$

but $$M = F\cdot\frac{h}{2}$$

$$\therefore \qquad I = \frac{F\,h^2}{12E\left(\frac{\Delta}{h}\right)}$$

$$\frac{I_C}{h} = K_C = \frac{Fh}{12E\left(\frac{\Delta}{h}\right)} \tag{20}$$

The rules of column tapering the limiting value of Δ/h for consecutive storeys, are as follows and shown in Fig. 7.

$$I_{cu} = I_c\cdot\frac{F_1h_1^2}{F_2h_2^2} \tag{21}$$

$$I_{cl} = I_c\cdot\frac{F_3h_3^2}{F_2h_2^2} \tag{22}$$

It is also necessary to consider the effect of the proposed taper, on the values of the distribution coefficients $k_t$ and $k_b$. By substituting Eq. 21 and Eq. 22 into Eq. 1 and Eq. 2 it can be shown that $k_t = k_b$ (see the derivation in Ref. [5]).

## DESIGN EXAMPLE

The method discussed above is demonstrated by designing a six storey frame subjected to unfactored wind loading as illustrated in Fig. 8. It is designed by both the tapered and untapered approaches to a limiting sway index of 1/300; $E$ is taken as 205 kN/mm$^2$. Attempts to derive specific equations to limit sway of the top storey were unsuccessful however, as the strength under vertical loading normally controls the design of the top storey therefore, the equations for sway are not significant.

Design for sway limitation is commenced at the sixth (top) storey for the untapered method. For the tapered approach, the columns are considered as continuous over two storeys and the design is commenced at the fifth storey. Eq. 17, 21 and 22 were used to calculate the second moment area for the internal columns. The required second moment of area for the external column is half of the internal column. The beam second moment of areas are then calculated from Eq. 18 and 19. A similar procedure is followed for the third storey. The results are compared with those obtained from the Anderson and Islam [2] method. Table 1 summarises the comparison. From Table 1 it can be seen that the results of the design of the columns are almost consistent for the upper storeys. Differences between results were endent for the lower storeys because the analyses were based on different assumptions. In this particular case, Anderson and Islam [2] had assumed fixed bases for the frames and separate analysis for the bottom two storeys were made.

Untapered direct design of beams resulted in similar section second moment of area for both the beams and the columns. Since the difference between the section second moment of areas are not large, the resulting beam or column sections will be the same as those given by Anderson and Islam method [2].

## CONCLUSIONS

The method proposed in this paper is suitable for the design of unbraced multi-storey frames if the choice of sections is controlled by sway deflection. However, it has a limitation in that a unique design cannot be obtained for frames of irregular storey height. A strategic and engineering judgement is needed for such cases in applying the proposed method.

## REFERENCES


[1] Wood. R.H., and Roberts, E.H., A graphical method of predicting side-sway in the design of multi-storey buildings, *Proceeding of Institution of Civil Engineers,* Pt. 2, 59, p. 353, 1975.

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[4] Wood, R. H., Effective lengths of columns in multi-storey buildings, *The Structural Engineer*, 52(7), pp 235-244; (8), pp. 295-302; (9), 341-346, 1974.

[5] Saim, A.A., *Aspects of continuity in steel and composite frames*. PhD Thesis, Warwick University, 1998.

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**Table 1 Comparison of Results**

| Storey | Internal Column (cm$^4$) | | | External Column (cm$^4$) | | | Lower Beam (cm$^4$) | | | Upper Beam (cm$^4$) | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $I_c$ (ref [7-5]) | $I_c$ (Direct Design) | $I_c$ (After Tapering) | $I_c$ (ref [7-5]) | $I_c$ (Direct Design) | $I_c$ (After Tapering) | $I_{bl}$ (ref [7-5]) | $I_{bl}$ (Direct Design) | $I_{bl}$ (After Tapering) | $I_{bu}$ (ref [7-5]) | $I_{bu}$ (Direct Tapering) | $I_{bu}$ (After Tapering) |
| 5 | 5673 | 5663 | 5663 | 2836 | 2836 | 2831 | 7252 | 5666 | 7554 | 3626 | 5666 | 3778 |
| 3 | 13234 | 13215 | 13216 | 6617 | 6608 | 6608 | 14447<br>15993 | 13218 | 15105 | 10835<br>11995 | 13218 | 11332 |
| 2 | 16314 | 16991 | 16990 | 8157 | 8496 | 8495 | 16243 | 16995 | 18883 | 14862 | 16995 | 15107 |

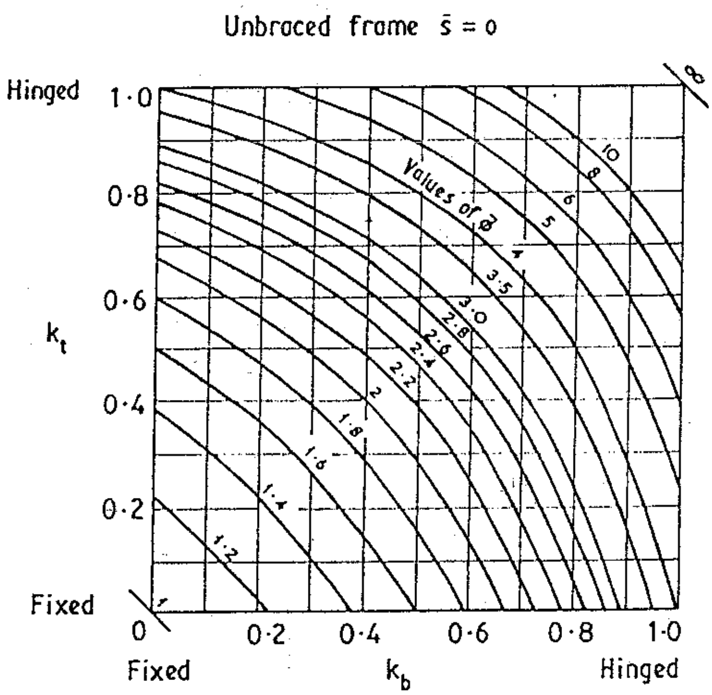
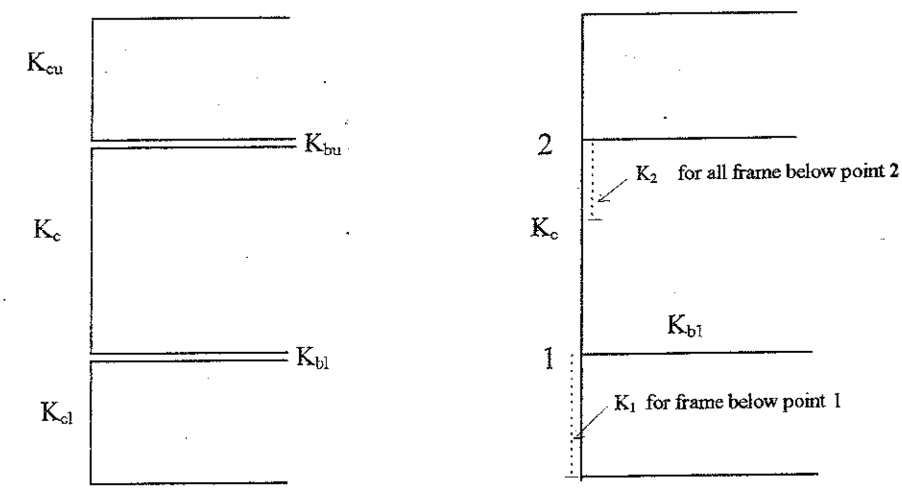


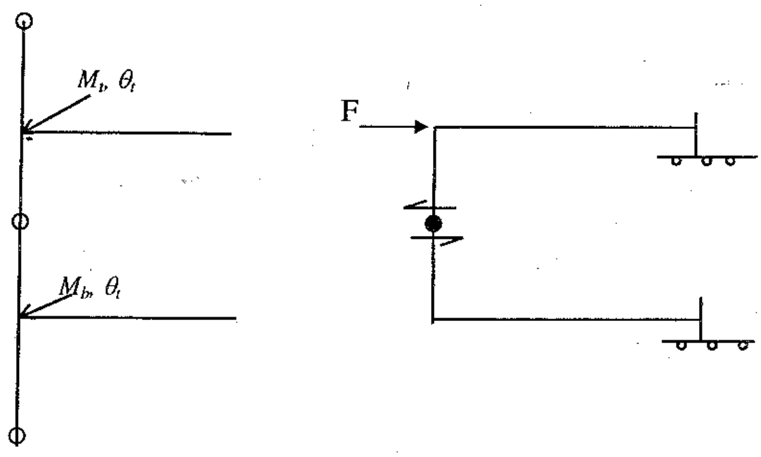

*Fig. 1 : Sidesway deflection for unbraced frame values of* $\overline{\Phi}$




(a) "Cell" with beam split up

(b) Forward intergration

*Fig. 2 : Illustrating stiffness concept*




(a) (b)

*Fig. 3 : Distribution under sway*

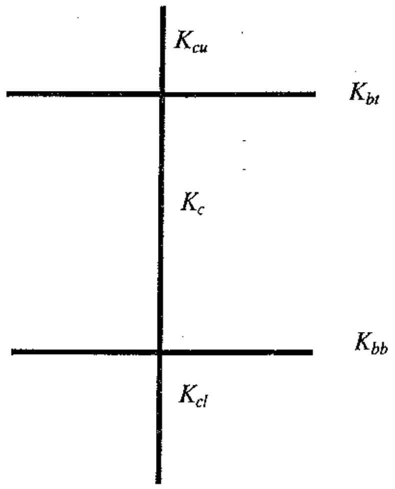



*Fig. 4 : Typical intermediate storey of a frame*

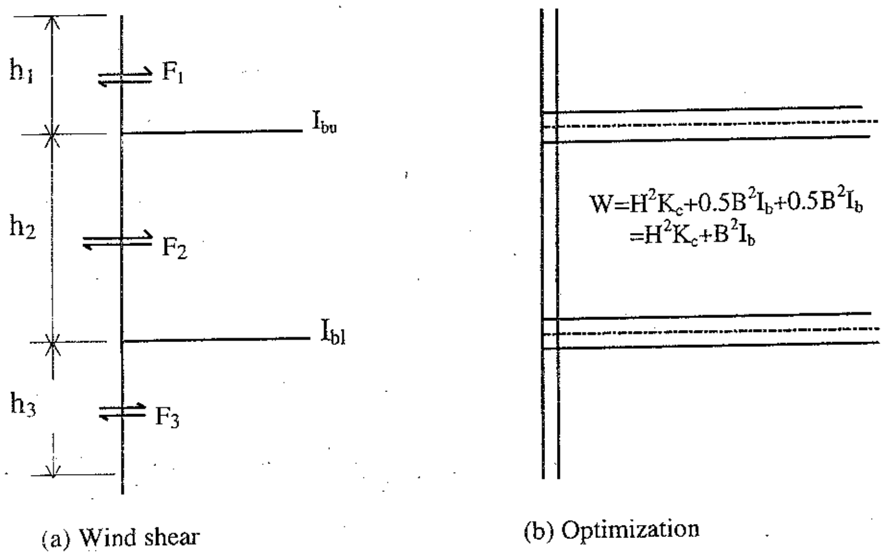



*Fig. 5 : Relationship between upper and lower beam*

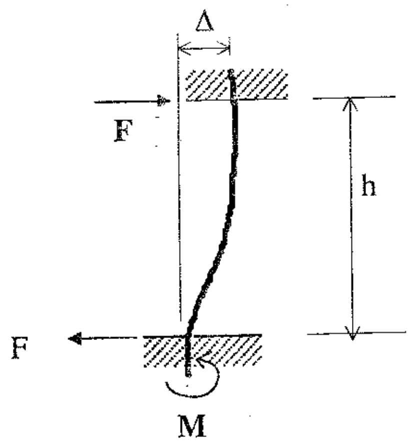



*Fig. 6 : Column subject to pure sway*

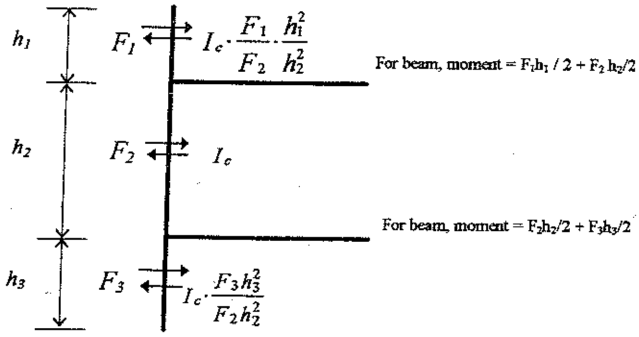



*Fig. 7: Column tapering*

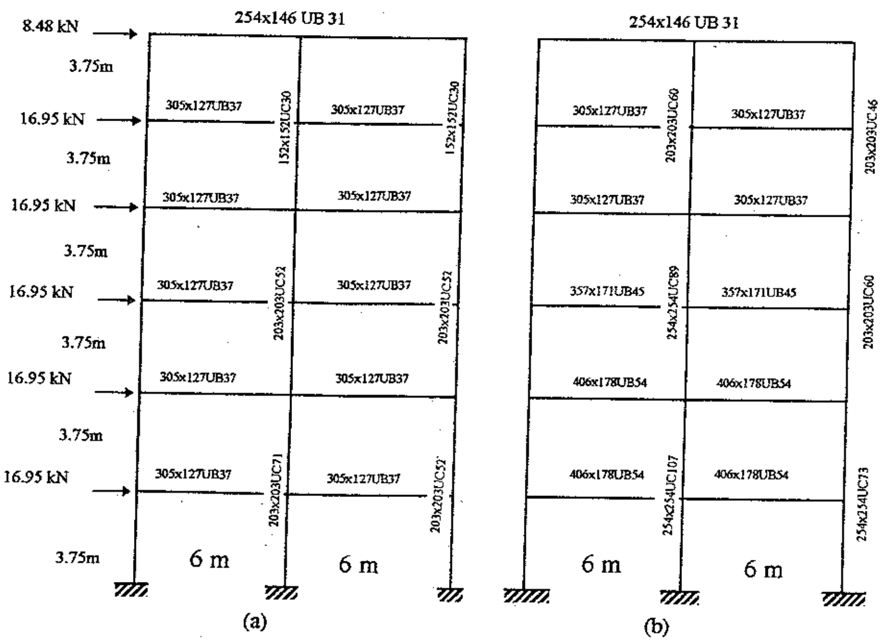



*Fig. 8 : Designs for six-storey two-bay frame[ref. 6]: (a) initial design; (b) design*