

# THE MATHEMATICAL APPROACH FOR PROXIMITY ANALYSIS FOR 3D GIS

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## ABSTRACT

Next generation of GIS software would highly depend on 3D analysis in solving geographic problems. 3D analysis is a very important component for GIS as it defines as decision making tools for geographic features. One would like to query about geographic object from numerical calculation or propose optimum solutions for GIS applications. Such desired components in future software or system are to deal with the 3D analytical solutions. This paper presents a portion of the problems, which are 3D solid buffering for 3D GIS. This analytical solution is very important for 3D spatial analysis. The discussion related to the implementation of buffering model for solid object is the main concern in the research, where the primitive objects of point, line, and face will be taken in consideration for the development of 3D solid buffering. These primitives will be integrated in three-dimension in an initial research. The mathematics, the geometry, and the algorithms involved will be presented. The approach is verified by using simulated data sets and a commercial

GIS package for visualization purpose. Finally, The outlook of the 3D analytical solutions for 3D GIS is presented.

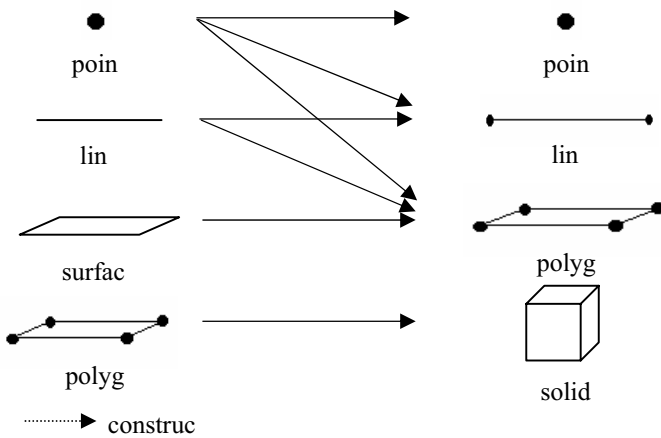
**Keywords:** 3D solid buffering, and 3D GIS

## **1.0 INTRODUCTION**

The spatial operation to generate proximity information of a spatial object, for instances, phenomenon along linear features or polygon features in a three-dimensional space is defined as buffering analysis. It is categorized as one of the 3D analytical solutions. In GIS, analytical tools are important for geographic analysis. User may want to know the shortest road from a place to another place, the distance between two places, or even the area of a housing estate. All these demands increase the need of the analysis tool in geographic information (Bernhardsen (1999); Gaiani, *et al.* (2001); Koh and Chen (1999); Zlatanova *et al.* (2002)). Therefore, the geometrical modeling for buffering zone of the 3D solid object will be discussed in this research. In the section 2, each of the buffering objects will be studied in detailed (i.e. how to construct them geometrically) based on the geospatial primitives (point, line, and polygon. The approach of the buffering object for each of the primitives will be mentioned. All the mathematics that follows the expected buffering result will be highlighted. In the section 3, 3D solid buffering will be discussed, together with the approach and mathematics. The experiment and discussions will be mentioned in section 4. Section 5 concludes the research and research finding.

## 2.0 PRIMITIVE OBJECTS

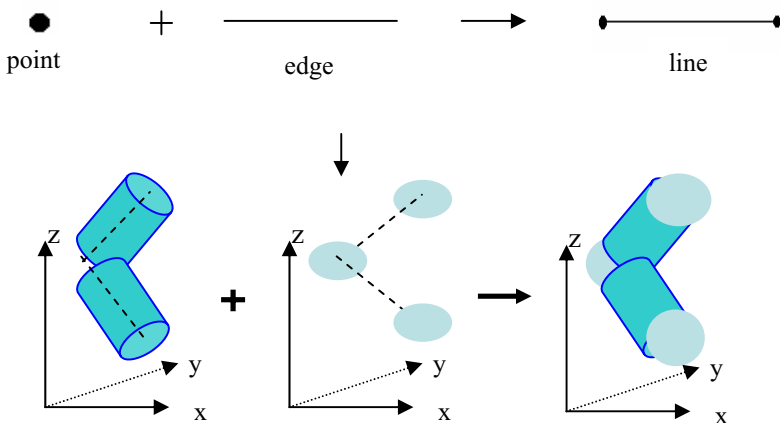
Any feature object appears in the real world is constructed by primitive object. The feature object could be a road, building, or just a simple traffic sign. They are represented by 3 kinds of primitives, i.e. point, line, and polygon. A point could construct the point itself, lines, faces and solid objects, whereas line is part of lines, faces or solid objects. From the Figure 1, a complete constructive model for primitive object is given.



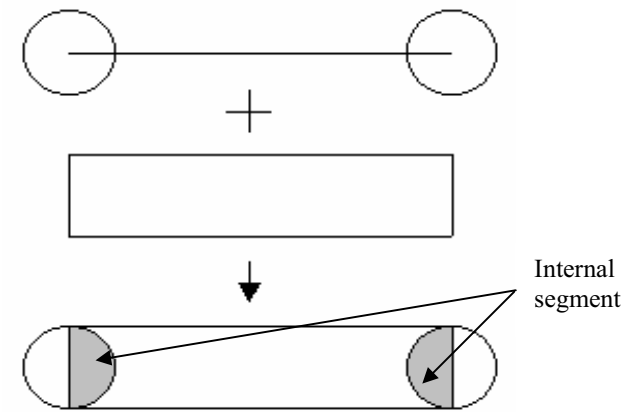
**Figure 1:** Geospatial object

Any spatial object could be used to produce spatial analysis, e.g. 3D buffering model. From the Figure 1, the buffering models of each feature object are different according to the primitive that constructs them. The buffering model for primitive point is sphere, whereas the primitive line is a cylinder. Since nodes appear in the line segment, adding the point buffering model into the line buffer model becomes a necessity. Figure 2 shows the method to create a line buffering output.

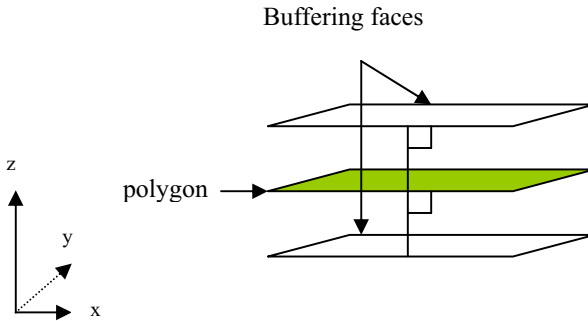
Refer to Figure 2, the buffering model for a line is a combination between spheres and cylinder. However, this combination creates an overlap segment (see Figure 3). This unnecessary segment is the internal segment and need to be removed. The complete modeling of line buffering object is discussed in Chen and Abdul-Rahman (2003). The third kind of primitive is polygon. A polygon feature is represented by some arc/node structure that determines an area's boundary. With at least three or more nodes joining together consecutively and return to the starting node define a polygon. The buffering model for polygon is constructed by upper and lower faces (see Figure 4). However, to perform geometrically a buffering operation towards a polygon in three-dimensional space, the joined components of a single polygon need to be identified, which are the nodes, and edges. Therefore, the point and line buffering model should be considered in constructed polygon buffering model. The combination of point, line and face buffering model create internal segments that need to be removed. The complete modeling of polygon buffering object, including the removal of internal segment is discussed in Chen and Abdul-Rahman (2003).



**Figure 2:** Line's properties and buffering model



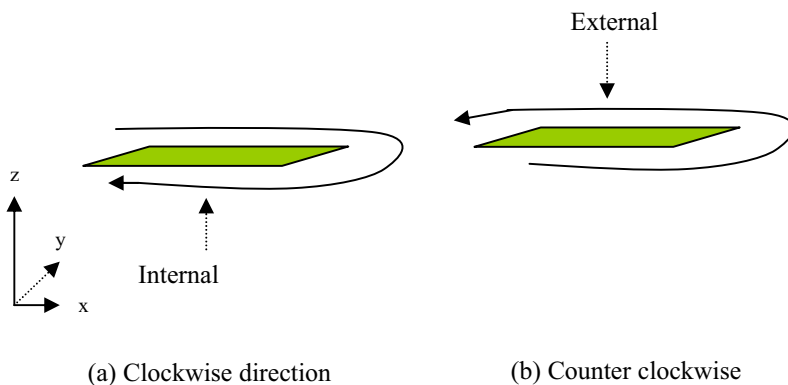
**Figure 3:** Internal segment of line buffering model



**Figure 4:** Two buffering faces

### 3.0 BUFFERING MODEL FOR SOLID OBJECT

A solid object constructed by faces could perform the buffering process. These faces connect each other, which shares the common vertices and lines, form a closed boundary. Therefore, the construction rule for a face is important in determine the internal and external aspect of a solid object. This is because the external aspect will be used in modeling the solid buffer zone. A series of points that construct a face in counter-clockwise direction is facing the external aspect; otherwise is internal. The main idea of differentiating the face's aspect is to determine the upper and lower faces for polygon buffering (see Figure 5).



**Figure 5:** Upper and lower faces of polygon buffering model

The buffering layers are expanded upper and lower starting from the center of polygon, respective to the buffer length. Both of them should be parallel to the main polygon. To define these polygons buffer, the vector's cross product is used. This vector produces a result that is perpendicular to the main polygon (v and u). For vectors u and v, the cross product is defined by (see Figure 6):

$$\begin{aligned}
 u \times v &= \hat{x}(u_y v_z - u_z v_y) - \hat{y}(u_x v_z - u_z v_x) + \hat{z}(u_x v_y - u_y v_x) \\
 &= \hat{x}(u_y v_z - u_z v_y) + \hat{y}(u_z v_x - u_x v_z) + \hat{z}(u_x v_y - u_y v_x)
 \end{aligned}$$

(Eq.1)

This can be written in a shorthand notation, which takes the form of a determinant (see Eq.1).

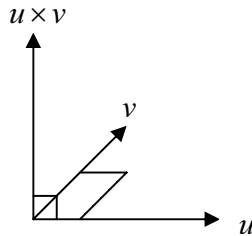


Figure 6: Vector (u x v)

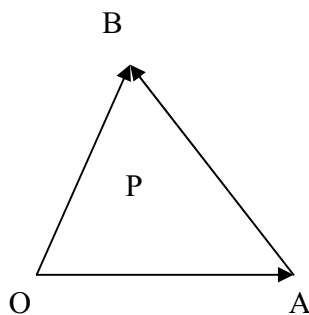
Here, the (u x v) is always perpendicular to both u and v, with the orientation determined by the right-hand rule.

$$u \times v = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ u_x & u_y & u_z \\ v_x & v_y & v_z \end{vmatrix} \qquad \text{(Simplified) }$$

from Eq.1)

Some of the mathematics is illustrated below (refer to Figure 7). The vector u (A) and v (B) need to be compute in the early step (see

Figure 7 and Equation 2 to 5). The purpose of computing those vectors is to find out the vector scale  $[|N_x|, |N_y|, |N_z|]$ . Later on the vector scale are used to compute the point sets that form the polygon.



**Figure 7:** Vector for polygon P

$$\vec{OA} \times \vec{OB} = [|n_1| \quad |n_2| \quad |n_3|]$$

(Eq.2)

$$|n_1| = \begin{vmatrix} Y_{oa} & Z_{oa} \\ Y_{ob} & Z_{ob} \end{vmatrix}$$

(Eq.3)

$$|n_2| = \begin{vmatrix} Z_{oa} & X_{oa} \\ Z_{ob} & X_{ob} \end{vmatrix}$$

(Eq.4)

$$|n_3| = \begin{vmatrix} X_{oa} & Y_{oa} \\ X_{ob} & Y_{ob} \end{vmatrix}$$

(Eq.5)



$$\vec{AB} = \sqrt{\left( |n_1|^2 \quad |n_2|^2 \quad |n_3|^2 \right)}$$

(Eq.6)

$$\hat{AB} = \frac{\left[ |n_1| \quad |n_2| \quad |n_3| \right]}{\vec{AB}}$$

$$= \left( |N_x| \quad |N_y| \quad |N_z| \right)$$

(Eq.7)

$$x_c = x_o + |N_x| \times r$$

(Eq.8)

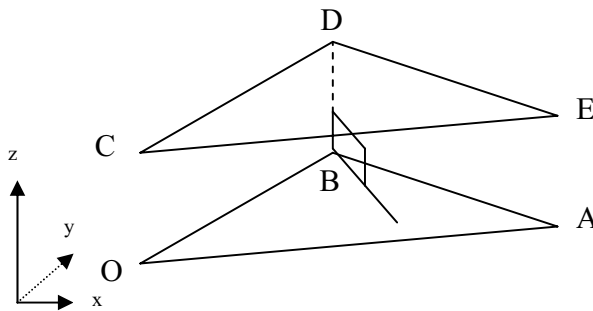
$$y_c = y_o + |N_y| \times r$$

(Eq.9)

$$z_c = z_o + |N_z| \times r$$

(Eq.10)

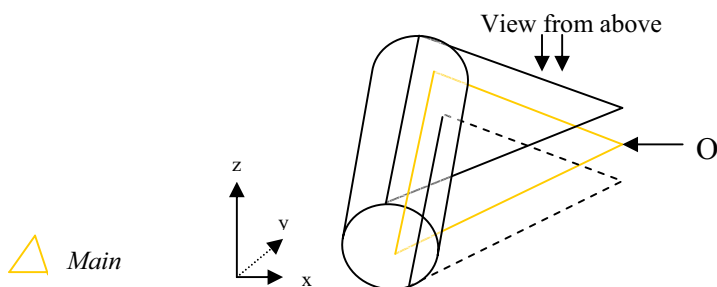
where  $x_c, y_c, z_c$  are the processed coordinates for the point C.



**Figure 8:** Upper polygon buffer CDE

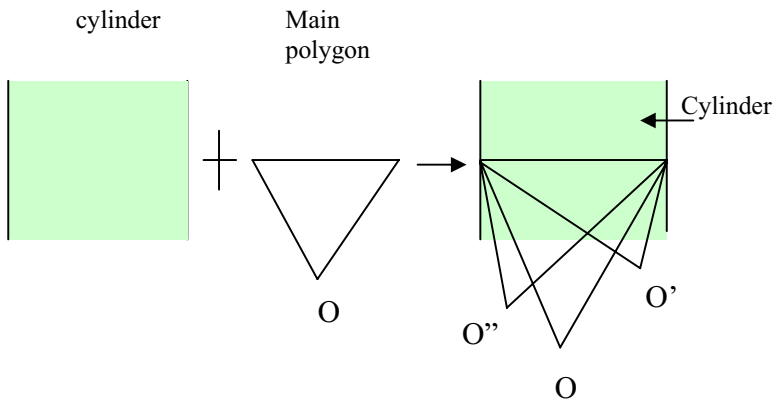
Later on, point D and E will be calculated (see Figure 8) by using the same equations start from 2 to 10. From the Figure 7, three points (C, D, and E) create an upper layer of face for a polygon buffer model. Recall that a polygon is a combination of point, line and surface. The polygon buffering zone should implement point and line buffer model. After the upper polygon buffer is defined, the line (cylinders) and point (sphere) buffering objects need to be combined. However, the combination may produce internal segment between line and the polygon buffer. Same situation also occurs between point and line buffer. Therefore, the unnecessary internal segment needs to be removed.

To remove the internal segment between point and line buffer, the same approach as shown in Figure 2 is used. On the other hand, to remove the internal segment between line and polygon buffer, the approach is shown in Figure 9.

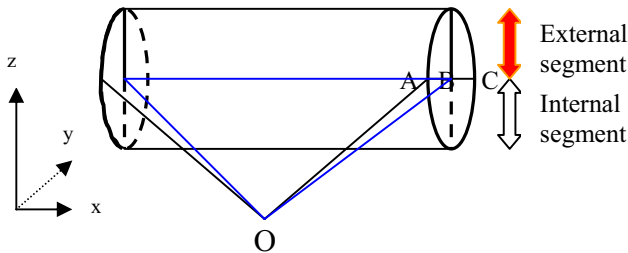


**Figure 9:** Combination of cylinder and surface buffer

The cylinder and the point O from polygon (refer to the Figure 10), which is the point opposite to the cylinder and it is being a part of the main polygon. Either the third point that creates a polygon is O, O', or even O'' from the Figure 11, the representation remains the same as in Figure 10.

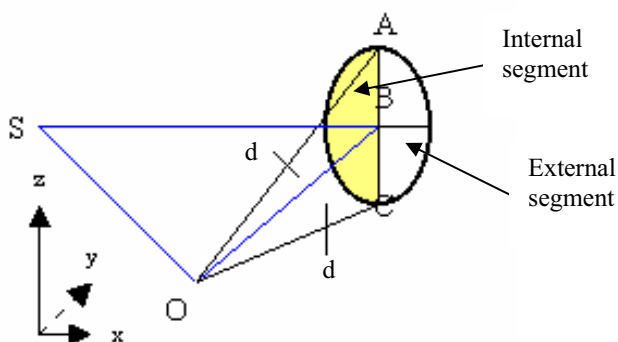


**Figure 10:** View from above



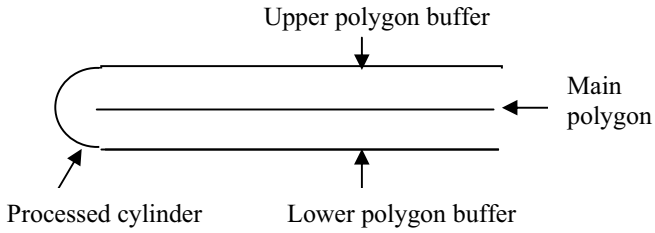
**Figure 11:** Method to remove internal segment

The cylinder is divided into two segments, which are the internal and external part. The main problem is to determine where the internal is and the external segment. Refer to Figure 12,  $d$  is the distant limit. The distance between cylinder segment and  $O$  (from the polygon) that exceeds  $d$  are identified as external segment.



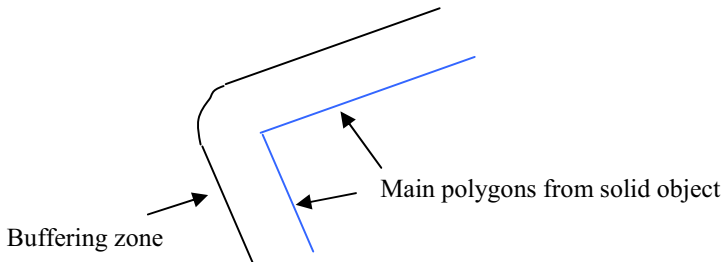
**Figure 12:** Calculate distance between  $OA$ ,  $OB$ , and  $OC$

The distance of  $OA$ , and  $OC$  will be computed. Both of them are in the same situation because the surface of circle is perpendicular to the line  $AC$ . Points  $A$  and  $C$  are the upper and lower point of the circle (see Figure 12). Therefore, there is a condition that needs to be fulfilled in order to remove the internal segment of the cylinder. That is the distance of either  $OA$  or  $OC$  is the benchmark of that condition. If any distance from the circle to point  $O$  is less than  $OA$  or  $OC$ , then it should be the internal part of the cylinder and need to be removed. Finally, Figure 13 shows the processed result.



**Figure 13:** View from above

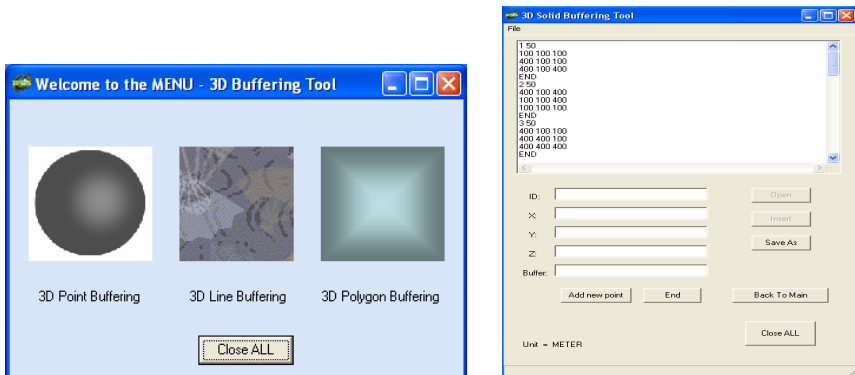
The remaining external segment (see Figure 13) will be used for another process, which the adjacent polygon buffer is combined (see Figure 14).



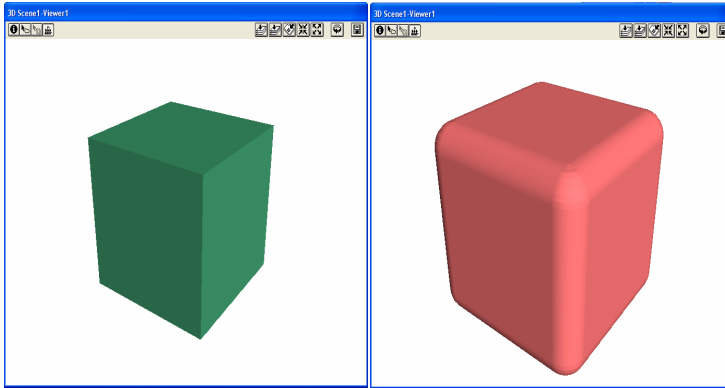
**Figure 14:** Final buffering zone

#### 4.0 EXPERIMENT AND DISCUSSIONS

In this project, the C++ language is used to create a software interface module called *3D Solid Buffering Tools*. It is tested using simulated dataset. The representation of 3D solid buffering module / software will display the result using a commercial package called 3D Analyst. The input is in ASCII format and the result is presented in the shapefile (\*.shp) format. Figure 15 shows the snap shots of the interface of the software, whereas Figure 16 and 17 present the output of the research.

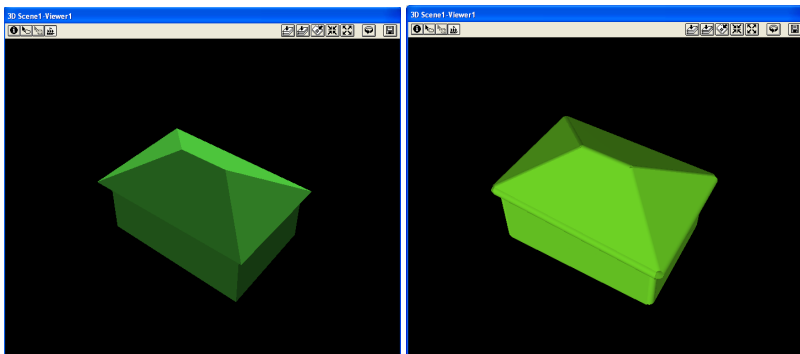


**Figure 15:** Software interface



(a) Input

(b) Solid

**Figure 16:** Solid buffering for simple object**Figure 17:** Solid buffering for complex object

## **5.0 CONCLUDING REMARKS**

The paper presents an approach for integrating all geometrical modeling of primitive buffering object in solving new spatial operations, i.e. 3D solid buffering. The integration is expected to be providing complete modeling for measurement-based 3D GIS analysis. The methodology was tested using C++ program and the result gives a positive output respected to the algorithm. Obviously, the initial outcomes of this experiment certainly will be utilized in the 3D GIS software development in the very near future, i.e. the 3D analytical operations.

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