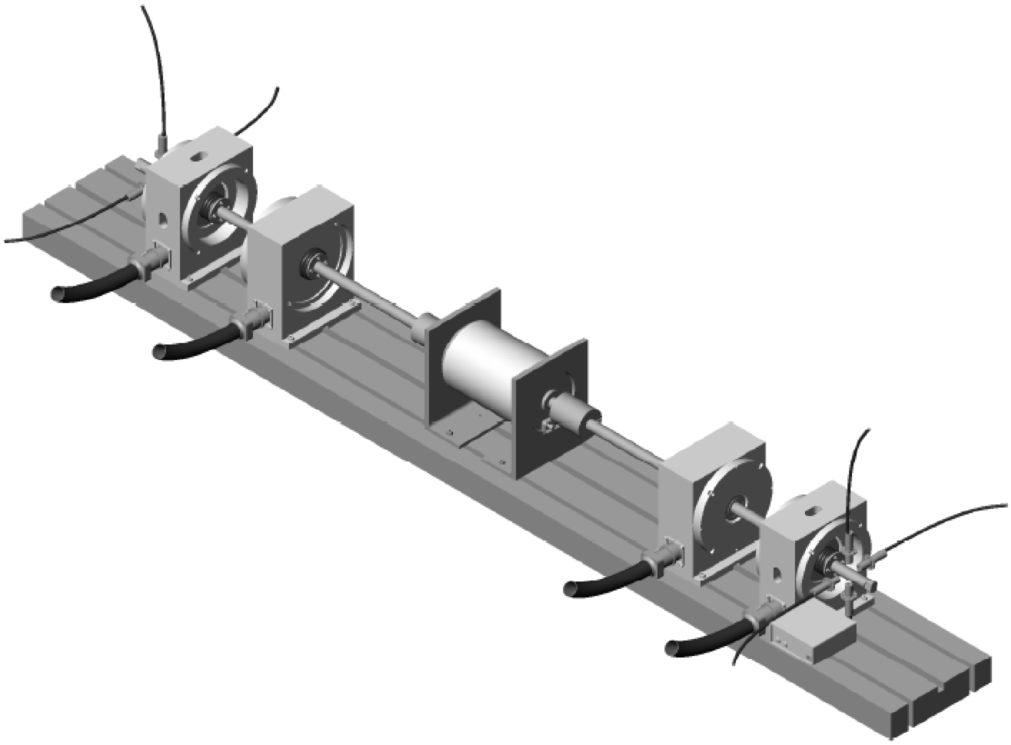
CHAPTER 1

INTRODUCTION

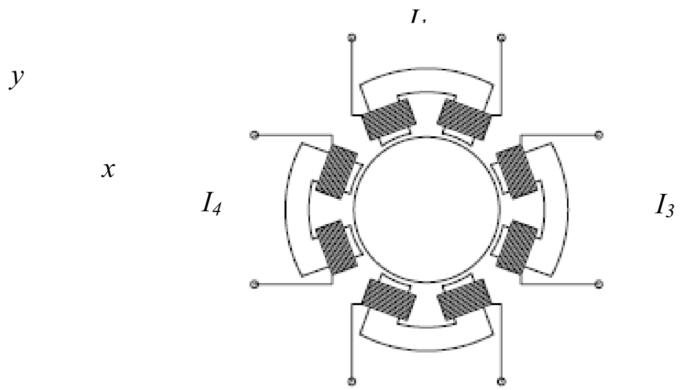
1.1 Introduction to Active Magnetic Bearing System

Bearings are one of the most essential components in all rotating machinery and the study on its mechanism and development is becoming more indispensable as the technology need pushes for more high-precision high-speed devices. By standard definition, bearing is the static part of machine (stator) that supports the moving part (rotor) of a system. While air and fluid bearings may be found in multi-degree-of-freedom ball and socket joint of machines, ball bearings, which allow for pure rotation, are by far the most popular and widely used in many industrial application mainly due to its low production cost and ubiquitariness (Wilson, 2004). Magnetic bearings are alternative to this traditional types or bearings, in which the bearings are constructed from permanents magnets, electromagnets or both in which the bearing in this combination is called hybrid magnetic bearing. An active magnetic bearing (AMB) system is then defined as a collection of electromagnets used to suspend an object via feedback control. For one degree-of freedom (DOF) system, usually AMB is synonymously called magnetic suspension system as used in ground transportation system where the vehicle is floated by the combination of controlled electromagnetic and permanent magnetic forces i.e. Maglev Train (Trumper *et al.*, 1997; Namerikawa and Fujita, 2004; Fujita *et al.*, 1998; Bleuler, 1992). For system with higher DOF, AMB system contains a suspended cylindrical rotor that rotates in varying speed depending on the applications. Thus, the obvious feature of AMB system is its non-contact suspension mechanism, which offers many advantages compared to conventional bearings such as lower rotating losses, higher operating speed, elimination of high-cost lubrication system and lubrication contaminations, suitability to operate at temperature extremes and in vacuum and having longer life span (Okada and Nonami, 2002; Knospe and Collins, 1996; Bleuler, 1992). Due to these significant reasons, AMB has been applied in a wide range of applications such as industrial machineries and medical equipment, power and vacuum technologies, and artificial heart, to quote a few applications (Knospe, 2007; Mohamed *et al*., 1997b; Shen *et al.,* 2000; Maslen *et al*., 1999, Tsiotras and Wilson, 2003; Kasarda, 2000; Lee *et al.* 2003).

Figure 1.1 illustrates an example of the standard structure of six-DOF AMB system and the schematic arrangement of the rotor and magnetic coil (stator) of the system. The system is composed of a cylindrical rotor or shaft made of laminated or solid ferromagnetic material, sets of electromagnetic coils, power amplifiers, position sensors and digital controller. The shaft is coupled to an external driving mechanism such as pumps, electric motors or piezo actuators by a flexible coupling which provides the rotational motion that forms the sixth DOF of the system. The electromagnetic coils generate the magnetic forces by the current I_i and the position sensors monitor the gap between the rotor and stator in which the captured information is used by the digital controller to determine the control signal necessary to suspend the rotating rotor to the centre of the actuating bearings. The control signal is sent to the power amplifiers for necessary amplification of the current I_i such that forces produced are able to withstand the dynamic requirement of the rotor as well as the external mechanical load. In addition, with some changes in the configurations of the AMB, the electromagnetic coils are not only able to supply the radial forces, but also generate the forces for rotational motion consequently eliminating the need of external driving mechanism. This so-called self-bearing motor appears rather appealing for space-constraint application, however the design construction and formulation of the control system is considerably much more complex (Kasarda, 2000; Kanekabo and Okada, 2003; Bleuler, 1992).



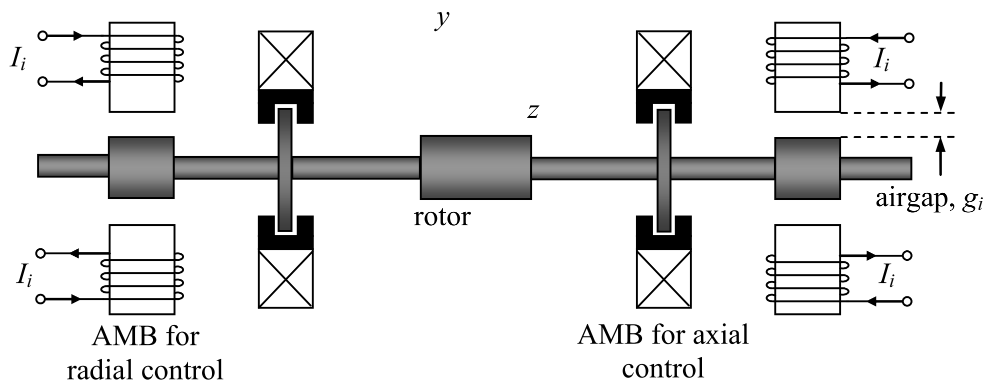
(a) Typical AMB system set-up



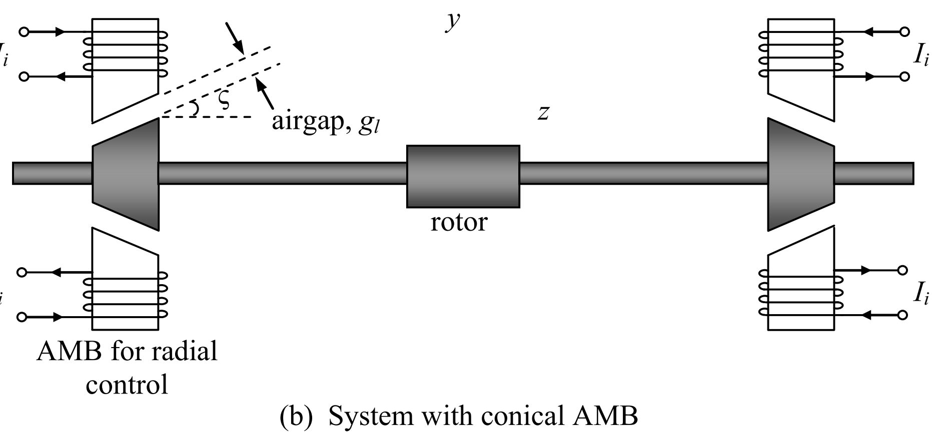
(b) Rotor and electromagnetic coils (stator) with respective coil currents, I_1, I_2, I_3 and I_4.

Figure 1.1 Active Magnetic Bearing System

In most of AMB system, there exist separate sets of electromagnetic coils that control the radial (*x*- and *y*- axes) and axial (*z*-axis) movement of the rotor due to negligible dynamic coupling between these axes of motions. Advantageously, separate control schemes are feasible to regulate the motions in radial and axial of the system. As illustrated in Figure 1.1 (a), at each end of the rotor, a set of electromagnetic coils is used for radial control where in each set, it contains two pairs of coil as shown in Figure 1.1 (b). Based on this figure, at this end of the system, the coil currents I_1 and I_2 supply the forces in y-direction while I_3 and I_4 supply the forces in x-direction. For the axial motion, one magnetic coil is located on each side of the rotor end. As an alternative to electromagnetic coil, in some AMB system where the rotor movement is very minimal, permanent magnets are sufficient to supply the regulating axial force and thus more favored to be used.



(a) System with cylindrical AMB



(b) System with conical AMB

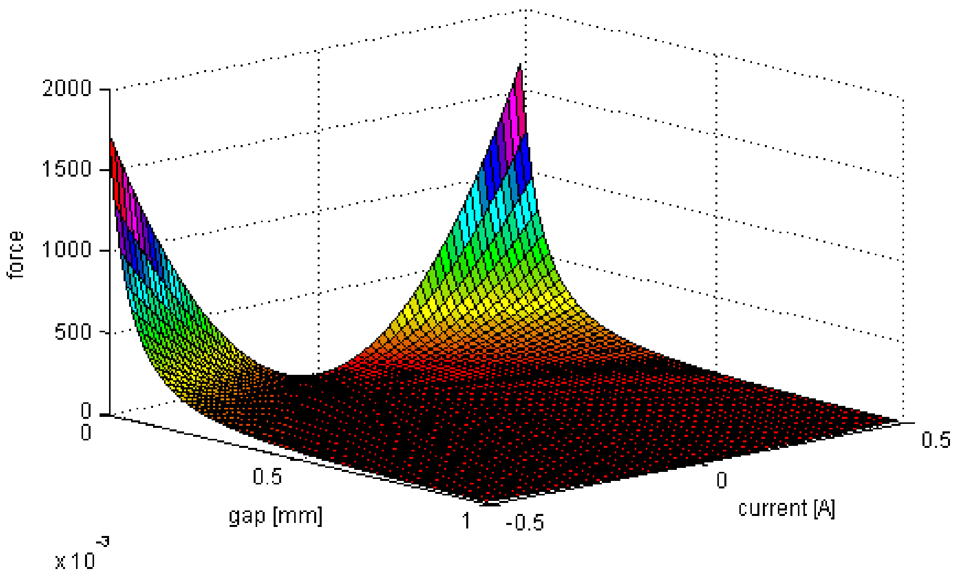
Figure 1.2 Configurations of Active Magnetic Bearing System

Figure 1.2 further illustrates two configurations of AMB system where in Figure 1.2 (a), the cylindrical rotor is used. This is similar to the aforementioned description of system in Figure 1.1 in which the axial motion is separately controlled by a pair of electromagnetic coil. In contrast to this configuration, conical magnetic bearing (Figure 1.2 (b)) where the rotor surface at the bearing end has small angle, ς, which makes the airgap between the rotor and bearing to be in slanted position. With this set-up, the electromagnetic coils supply both the axial and radial forces to the system and the most obvious advantage obtained is the elimination of a pair of axially-control electromagnetic coils. Nevertheless, this system experiences high coupling effect between the axes and formulation of reliable controller under wide operating is a very challenging task (Mohamed and Emad, 1992; Huang and Lin, 2004; Cole *et al.*, 2004).

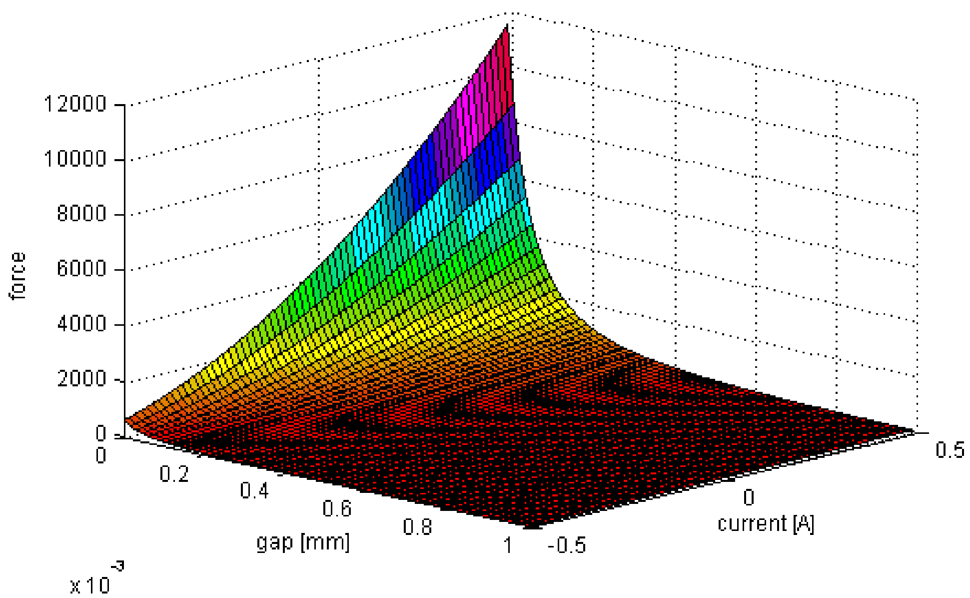
The various structural designs of AMB system are constructed to meet different kind of requirements of the real-world application in order to exploit the advantage of this non-contact lubrication-free technology. However, there are also numerous nonlinearities inherited in AMB system that cause the system instability. One of the most prominent nonlinearities is the relationship between the force-to-current and force-to-airgap displacement. The general equation that governs the magnetic force in AMB system is given as:

$$\begin{aligned} f_m &= \mu_o A_g N^2 \left(\frac{I_i}{g_i}\right)^2 \\ &= K \left(\frac{I_i}{g_i}\right)^2 \end{aligned} \tag{1.1}$$

where μ_o is the permeability of free space, A_g is the cross-section area of the airgap, N is the number of turn of the coil, and I_i and g_i is the current and airgap at i-th coil, respectively. By using the parameters given in (Mohamed and Emad, 1992; Lin and Gau, 1997), this relationship can be plotted and shown in Figure 1.3 (a). Noticeably, the relationship of the magnetic force in which the magnitude is proportional to the square of the input current and inversely proportional to the square of the rotor position causes sudden surge of the force magnitude as the airgap approaches zero. Theoretically, this so-called negative stiffness imperatively causes singularity error in many controller designs which practically translated to the saturation of magnetic actuator. As one of the techniques to overcome this difficulty, a small bias current, I_b, is usually introduced to the coil such that the linearity of the force-to-current about



(a) Nonlinear magnetic force



(b) Nonlinear magnetic force with biased current $I_b = 0.8$A

Figure 1.3 Nonlinear relationship between magnetic force and current/airgap the centre of the system can be established to some degree which provides higher system bandwidth and easier controller design. Figure 1.3 (b) shows the effect when $I_b = 0.8$ A is added to the equation (1.1) where an almost linear relationship between force and current and no singularity point is observed when the gap is zero.

Another major nonlinearity existed in AMB system is vibration due to the mass unbalance of the rotor, or called imbalance. Imbalance is a common problem in all machineries with rotational shaft when the principle axis of inertia of the rotor does not coincide with its axis of geometry due to mechanical imperfections occurred in fabricating machine parts, as shown in Figure 1.4 (Herzog *et al*. 1996; Shafai *et al*. 1994; Huang and Lin, 2004). When the rotor is 'forced' to rotate around its center of inertia, G_m, instead of its centre of geometry, G, a centrifugal force caused by the acceleration of the inertia centre creates a synchronous transmitted force and furthermore manifested into synchronous rotor displacement. In the worst case scenario, since the imbalance effect is proportional to the rotor rotational speed, at high-speed operation the rotor whirls exceeding the allowable airgap and causes the rotor to partially or worse yet annularly rub the stator which result in permanent damage to the bearing system (Choi, 2002). Among the commonly considered design solution to prevent this to occur is to have a mechanical retainer bearing

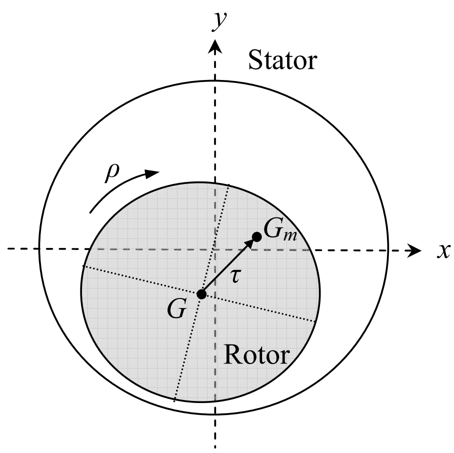


Figure 1.4 Illustration of unbalance rotor

installed as of the safety measures, however, the contact further exaggerates the nonlinear dynamic motion to cause a more chaotic motion (Knospe, 2007; Grochmal and Lynch, 2007; Li *et al*., 2006; Sahinkaya *et al*. 2004).

Other significant nonlinearities associated with the rotor dynamics are gyroscopic effect and bending modes for flexible rotor. Gyroscopic effect present in AMB system results in the coupling between the pitch (rotation around x-axis) and yaw (rotation around y-axis) motion and the magnitude is proportional to the rotor rotational speed. This imposes a more challenging task for stabilization of the system for high-speed application (Li *et al*., 2006; Hassan, 2002). In addition, in some applications where a long rotor is required, the excitation of flexible mode of the rotor becomes crucial which may result in an inherently unstable system (Li *et al*., 2006; Jang *et al*., 2005; Nonami and Ito, 1996).

In all AMB-related applications, the main objective is either asymptotically regulating the rotor to center position (zero airgap deviation) of the system or tracking a predefined rotor positions. However, with the presents of these nonlinearities, the AMB system is liable to exhibit unpredictable and irregular dynamic motions which complicate the design of effective system controller (Jang *et al*. 2005, Kasarda, 2000). Conventional feedback controller methods developed by assuming that the motions on each system axis are dynamically decoupled rarely meet the stringent system requirements which result in limited operational range of the system. Furthermore, nominal parameter values are commonly used in the system where in real application, the exact values are poorly known and subjected to variation which consequently result in deterioration of some controller performances on the system. The need for more advanced control strategies is thus becoming indispensable in order to achieve the desired system performance. In the following section, the various control methods that have been designed for AMB system is discussed.

1.2 AMB System Configuration and Control Strategies

The idea of active control magnetic bearing system has sparked interest as early as 1842 after Earnshaw (1842) proved that the levitation of ferromagnetic body and maintaining a stable hovering in six-DOF position is impossible to achieve by solely using permanent magnet (Matsumura and Yoshimoto, 1986). Ever since then, numerous control methods have been proposed by many research groups not only to stabilize the system, but also to improve the performance of the system under

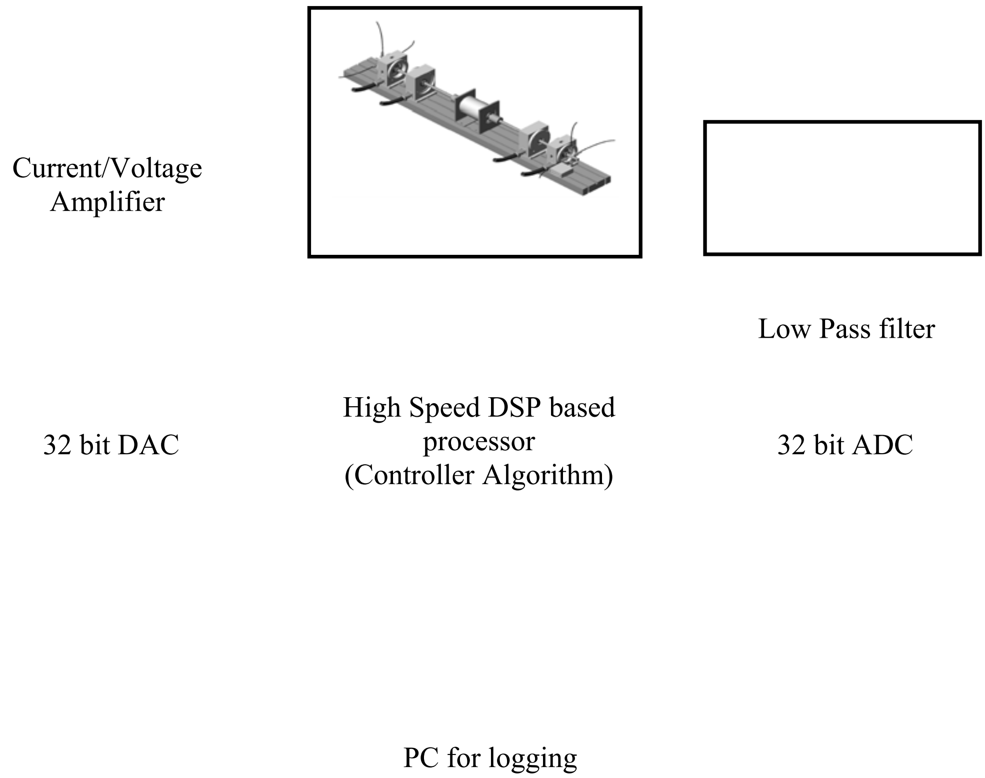


Figure 1.5 Hardware configuration for closed-loop control of AMB system

wide operational condition. Figure 1.5 illustrates the hardware set-up for the closed-loop control of AMB system. The measurement of the four gap deviations forms as the feedback information used by control algorithm executed in a fast Digital Signal Processor (DSP) based processor. The calculated control signal is further amplified to perform the required vibration control, positioning or alignment of rotor of the system.

The electromagnets can be controlled by either the coil current (current-based control) or the voltage (voltage-based control). In voltage-based control approach, two design steps are usually adapted in which in the first step, a low-order current controller is designed such that desired electromagnetic force is produced. Then, tracking this current trajectory signal is used as the control objective for the design of input voltage controller. The common assumption in this approach is the combination of the processor and voltage amplifiers is able to fulfill the timing of the two-stage nature of the controller which usually is very difficult to meet (Bleuler *et al.*, 1994b). Another important drawback is due to the inclusion of dynamic of the power amplifier and circuit constraints, the linearization of amplifier and system dynamics usually involved in the controller formulation which further limit the system performance (Hassan, 2002; Charara *et al.* 1996). Some nonlinear control methods such as differential flatness (Levine, *et al.*, 1996), backstepping-type control (DeQueiroz, *et al.* 1996a; DeQueiroz, *et al.* 1996b) and feedback linearization and passivity-based control (Tsiotras and Arcak, 2005) are proposed but the difficulty of overcoming singularity problem results in more complicated controller structures. In the current-based control method, since there is a direct relationship between the coil input current and the magnetic force shown by equation (1.1), the abovementioned challenges in voltage-based control design can be relaxed and becomes more advantages to AMB control system (Bleuler *et al.*, 1994a).

The current-based control scheme can be classified into three modes of operations of power amplifiers as shown in Table 1.1 (Sahinkaya and Hartavi, 2007; Hu *et al.*, 2004). The configuration of the tabulated coil currents is based on a single pair of electromagnet in which one of the coils produces the opposite force of the other coil. For Class-A control, a bias current, I_b, is applied to both coil and a differential control current, I_c, is added to the bias current in one coil and subtracted from the opposite coil depending on the net force required. The bias current is set to half of the maximum allowable current, I_{max}. This mode of operation, also named as Constant Current Sum (CCS) control, is the most widely used method in controlling AMB system due to the fact that high bearing stiffness and good dynamic range can be achieved (Grochmal and Lynch, 2007; Sahinkaya and Hartavi, 2007). In Class-B mode of operation, or also known as Current Almost Complementary (CAC) condition, a small bias current is supplied to both magnetic coils and at one instant of

Table 1.1 Mode of operation for a pair of electromagnet

Mode of Operations	**Input Current**
Class A	$I_1 = I_b + I_c$, $I_2 = I_b - I_c$, where $\lvert I_c \rvert < I_b$, $I_b = 0.5 I_{max}$
Class B	$I_1 = I_b + I_c$ and $I_2 = I_b$, or $I_1 = I_b$ and $I_2 = I_b + I_c$.
Class C	$I_1 = I_{c1}, I_2 = 0$, or $I_1 = 0, I_2 = I_{c2}$,

time, the control current is added to only one of the coils to produce the desired control force. Although a possible lower power losses can be attained due to smaller I_b, the bearing stiffness is reduced quite significantly which make the system to be suitable for low vibration application. In this control mode, a possibly large feedback gain is required to achieve the required bearing stiffness and likely will result in current saturation. Tsiotras and Wilson (2003) and Tsiotras and Arcak (2005) have shown that the control of AMB system with saturated input and low bias current is nontrivial and a challenging nonlinear control problem. Another mode of operations is the Class-C control where the bias current is totally eliminated and the two coils are alternatively activated at an instant of time. This is equivalently called Current Complementary Condition (CCC) where only one coil is energized depending on the direction of the required force needed. Under this mode of operation, the nonlinearity effects are severe and controller singularity problem occurred when the gap deviation approaching zero is one of the most crucial design problems which result in controller complexity. Apart from this design issue, the lacks of robustness against changes in operating condition as well as poor dynamic performance are also major shortcomings of this approach (Sahinkaya and Hartavi, 2007; Charara *et al*. 1996; Levine *et al*. 1996).

Due to many possible combinations of design configurations and actuating schemes exists in the control of AMB system, there exists abundance of control design techniques that have been proposed to meet the control objectives which are stabilization of the system and fulfilling specific application-related system performances. The control strategies can be essentially divided into three main groups: the linear control, nonlinear control and the control approach based on mimicking human's decision making process and reasoning or known as Intelligent Control (IC) method. The linear and nonlinear control strategies are model-based approaches where a mathematical model representing the AMB system as a class of a dynamical system is a required for the development of the control. As an alternative, due to the complexity in formulating the control law especially for the nonlinear control techniques, the adaptation of the IC methods in AMB control has found growing interest especially Fuzzy Logic (FL), Genetic Algorithm (GA) and Neural Network (NN), or the fusion of any of the method with existing mathematical-based methods.

The conventional Proportional-Derivative (PD), Proportional-Integral (PI) and Proportional-Integral-Derivative (PID) control for AMB system are among the earliest controllers considered for the control of AMB system due to its simplicity in the design as well as hardware implementation (Bleuler *et al.*, 1994b) and until today, the controller still receives considerable attention in some specialized application. In the work done by Allaire *et al*. (1989) and William *et al.* (1990), discretized PD controller is designed based on linearized model at a nominal operating point. The main emphasis of the work by Allaire *et al.* (1989), however, is the design construction of AMB system to accommodate the variation of the load capacity in thrust motion and the PD controller is used to achieve closed-loop stability. Due to apparatus limitation, mechanical shims are used to gauge the airgap and the controller is manually adjusted. William *et al.* (1990) has continued the study where the relationship between the characteristic of the developed PD controller to the stiffness and damping properties of AMB system is established. Other than stiffness and damping curves, the rotor vibratory response is also used to show the effectiveness of the control algorithm where from the experimental result, due to time delay in feedback response and hardware limitation, the high frequency response does not agree with the theoretical result. To overcome the difference, the so-called Proportional-Derivative-Derivative (PDD) and Proportional-Integral-Derivative-Derivative (PIDD) are proposed and applied into the system which yields quite a satisfactory result.

In more recent year, Hartavi *et al*. (2001) has studied the application of PD controller on 1 DOF AMB system where the electromagnetic model is developed based on Finite Element Method (FEM), initially proposed by Antilla *et al.* (1998). Good system stability is achieved, however, only under limited range of operating condition. Polajzer *et al*. (2006) has further proposed a cascaded decentralized PI-PD for control of the airgap and independent PI current controller to achieve high bearing stiffness and damping effect of a four DOF AMB system. The controller is designed based on simplified linearized single-axis model where the effect of magnetic nonlinearities and cross-coupling effect are ignored. A considerable improvement has been achieved in term of its static and dynamic response in comparison to PID control developed in previous work. In an AMB system where the rotor is flexible, the control of vibration due to bending mode of the rotor is crucial. For the AMB system developed by Okada and Nonami (2002), a hybrid-type magnetic bearing is used and PD controller is proposed to perform the inclination control such that the system with flexible rotor is able to step through the bending modes occurred at five critical rotational speeds. The five bending modes are analyzed from the finite element model of the rotor that is transformed into a linear state equation and the controller parameters are designed based on the linearized model. With the central rotor position is controlled separately to provide sufficient stiffness, the system with the proposed PD controller for inclination control is able to run up to 6300 rpm rotational speed.

Due to limited performance of PD, PI or PID controller and design procedure to incorporate various design requirements, other linear controller methods have been proposed to fully exploit the possible active potentials of the AMB system in permitting to a much higher degree of rotor vibration and position control (Bleuler *et al*., 1994a; Huang and Lin, 2003). Another most popular linear control method used by researchers is the Linear Quadratic Regulator (LQR) control which is based on optimal control theory (Anderson and Moore, 1990). LQR design method is designed by selecting the so-called weighting matrices that minimizes a pre-defined linear quadratic cost function. Matsumura and Yoshimoto (1986) are considered as among the earliest researchers that have applied the LQR-type controller in AMB system. In their study, an LQR controller is designed and cascaded with and integral term such that the steady-state error of the airgap deviation is eliminated. This optimum servo-type control is formulated based on a linearized 5-DOF AMB system at a constant biased current, where the deviations of rotor position from this equilibrium are treated as system states to be regulated and the input to the system is the electromagnetic voltages. The digital simulation results show that the system achieve stability condition at zero speed and 90000 rpm, however, at this high rotational speed, the coupling effect influence the control performance significantly. The method is further applied into a system where the integral servo-type control is to perform both the radial and thrust control for a cylindrical AMB system (Matsumura *et al*., 1987). Through this study it is verified that multi-axial control of AMB system is difficult to achieve with the proposed type of controller. Since both of these works are based on a linearized model at one operating point, Matsumura *et al*. (1999) has used a different linearization technique called exact linearization approach such that the linear model can represent a wider range of the nonlinear model. The design LQR controller for this newly linearized model confirms to achieve wider range of stabilization area. The control method of this highly-cited work (Matsumura and Yoshimoto, 1986) is also further adapted in a new type of horizontal hybrid-type magnetic bearing (Mukhopadhyay *et al.* 2000). In this work, the new type AMB system is developed by using a rotor made from strontium-ferrite magnet and both the top and bottom stators are made from Nd-Fe-B material where the combination of this permanent magnet configuration is proven to provide high bearing stiffness to produce repulsive force for rotor levitation. The force-to-airgap relationship is established by using finite element analysis (FEA) where the relationship is integrated with the dynamic model of the AMB system. The optimum integral servo-type control is designed to stabilize the system and tested on the system up to the 800 rpm rotor speed.

In a quite similar scope of work, Lee and Jeong (1996) has designed centralized and decentralized LQR controller with integrator to perform a control on a vertical conical AMB system. For the centralized control, the coupling effect between the axial and thrust motions is considered and this effect is ignored on the decentralized controller design. The relationship between the current and voltage is emphasized where the mathematical model of the electromagnetic coil dimension and its dynamics are included in the design procedure where it is illustrated that the coupling effect between the axial and radial axes of motion is quite insignificant for the particular AMB system which result both the centralized and decentralized controller produce comparatively similar performances. In a rather different approach, Zhuravlyov (2000) has explored the design of LQR controller for not only regulating the rotor position but also to reduce the copper losses in the coils. Two-stage LQR based controller is developed such that the first controller is meant to stabilize the rotor to the reference position with magnetic force is the system input. For the second stage, another LQR controller is developed to produce the coil current and voltage which produces the optimized bearing force while at the same time, the copper losses in the coil is also minimized. Instead of taking the real value of the system matrix, this approach has used the complex state-space system such that the frequency content of the system can be incorporated. The study also shows that the real implementation of the controller is difficult especially when the second stage controller requires a switching term to achieve the desired objective, and controller simplification is needed for practical purposes.

The works in the development of controller based on μ-synthesis have also been reported by many researchers. Fujita *et al.* (1995) has proposed the μ-synthesis controller that is designed based on a few set of active electromagnetic suspension model. The combination of the nominal model, four set of model structures and possible model parameter values are used to determine uncertainty weighting function which form a sufficient representation of the range where the real system is assumed to reside. A special so-called D-K iteration is then used to tune the controller parameter to achieve robust stability as well as robust performance. Nonami and Ito (1996) have used μ-synthesis method for stabilization of five-axis control of AMB system with flexible rotor. The modeling of the system is performed by using FEM technique and the resulted high order system is truncated by removing the flexible mode for the purpose of controller design. It is shown that the controller can achieve robust performance for this system and the it is noted that by value of the structured singular value, μ, in the D-K iteration contribute to achieving good robust performance.

Namerikawa and Fujita (1999) have further included more nonlinearities in the AMB model by specifically classifying linearization errors, unmodelled dynamics, parametric variations and gyroscopic effect as the uncertainties in the system. These uncertainties are represented structurally in matrices and Linear Fractional Transformation (LFT) technique is used to uniformly represent the AMB as a class of uncertain system for controller development. Instead of using standard μ test, a so-called mixed μ test is adapted to reduce the design conservatism. Losch *et al.* (1999) have designed and implemented the μ-synthesis controller in a feed pump boiler equipped with active magnetic bearings. They have proposed a systematic and formalized way for deriving the controller design parameters based on model uncertainties, control requirements and known system limitations. A new method for determining suitable uncertainty weighting function has been proposed in which the effectiveness of the designed controller is demonstrated by the robust performance of the pump.

In different scope of research, Fittro and Knospe (2002) has designed the μ-synthesis controller for specifically solve the rotor compliance minimization problem – to reduce the maximum displacement that may occur at a particular rotor location collocated at the region the disturbance frequency is not specified. Although the controller produces a significant improvement compared to PD controller, the results obtained however has suggested that a more accurate plant mode is required to yield a more accurate result in minimizing the rotor compliance.

Another robust linear control design that has received considerable attention in the control of AMB system is H_∞ technique. Since the linear model does not always express the exact representation of the system due to various uncertainties present in the system, H_∞ control technique offer a nice procedure to construct the uncertainties into a proper structure for control design process. Fujita *et al*. (1990) has worked on verifying the well-established H_∞ controller on an experimental set-up of a one DOF magnetic suspension system. The main objective is to achieve robust system stabilization when the system is subjected to external disturbance. Various model uncertainties are also considered by formulating frequency weighting function which is included in the design procedure. Fujita *et al.* (1993) further develop H_∞ controller for five DOF AMB system by using the Loop Shaping Design Procedure

(LSDP). The so-called unstructured multiplicative perturbation which describes the plant uncertainties with the frequency weighting function is established which reflects the magnitude of uncertainties present. After specifying the uncertainty and performance weightings, by using the LSDP the shaping function is designed where the H_∞ controller is developed and tested experimentally which shows that some minor online adjustment on the shaping functions is still required to achieve a more favourable system response in term of regulating the airgap at various frequencies.

A simplified H_∞ controller has been designed by Mukhopahyay *et al.* (1997) for repulsive type magnetic bearing where an AMB configuration with permanent magnet in the radial axis is used to increase the bearing stiffness. The result from the study shows with the combination of the proper placement of the permanent magnet and controller design the radial disturbance is able to be attenuated for an 8 kg non-rotating rotor.

A continuous and discrete time H_∞ controller have been proposed by Font *et al*. (1994) to regulate the rotor to the center position of an electrical drive system by using AMB. The first six bending modes of the rotor is included in the system model such that the design controller can achieve robust stability towards the frequency excitation occurred at these modes. For the continuous controller, instead of using the truncated method, an aggregation method to reduce the order of the system is adapted where this technique offers the advantage of retaining the most important poles in the reduced order system. Satisfying closed-loop behaviors have been obtained, however, the power amplifier introduces severe constraint on the control capability.

Namerikawa and Fujita (2004) and Namerikawa and Shinozuka (2004) have used the H_∞ controller design technique for disturbance and initial-state attenuation (DIA) on magnetic bearing and magnetic suspension system, respectively. In the design procedure of the proposed H_∞ DIA controller, the selections of the frequency weighting related to the disturbance input, system robustness and the regulated variables are performed iteratively for the construction of linearized generalized system plant. A so-called weight matrix N obtained from this procedure is found to indicate the relative importance between attenuation of disturbance and intial-state uncertainty which further affects the calculated controller gains. Four H_∞ DIA controllers have been designed under different values of frequency weighting to assess the variation of matrix N on the system performance where it is shown the system overshoot is inversely proportional to the magnitude of N. In (Namerikawa and Fujita, 2004), the non-rotational AMB is used which implies that no gyro-scopic effect and imbalance present.

In a more recent work, Tsai *et al.* (2007) has proposed H_∞ control design for four-DOF vertical AMB system with gyroscopic effect. The well-known Kharitonov polynomial and Nyquist Stability Criterion are employed for the design of the feedback loop and it is confirmed experimentally that the controlled current produced is much less compared to the current produced by LQR or PID control methods. The performance of the system is verified in the range 6500 rpm to 13000 rpm rotor rotational speed.

Linear controller based on Q-parameterization theory has also been widely tested and applied in AMB system starting with the work from Mohamed and Emad (1992). In this work, a Q-parameter controller based on linearized conical AMB model is proposed which can meet various system requirements such as disturbance rejection, rotor stability and tolerances towards plant parameter variations. In the design procedure, these requirements are treated as constraints and can be classified by the doubly co-prime factorization matrices and the sets of stabilizing controllers which include the free design parameter Q. The search of the desired Q-parameter that produces the desired controller gain becomes an optimization problem where Q's are chosen through a customized optimization program. In this work, the controller is designed for imbalance-free rotor at speed $p = 0$, and good transient and force response is achieved until $p = 15000$ rpm. Since the order of the controller equal to the order of the plant and the order of the weighting function describing the constraint, the works are further extended by Mohamed *et al.* (1997a) where the linear system is transformed into three single-input-single-output (SISO) systems with the inclusion of the rotor imbalance. This simplification results in solving a set of linear equation rather that finding the solution from the complex optimization problem, where good rotor stabilization is achieved at three pre-defined rotor speed.

The Q-parameterization controller in discrete form is proposed by Mohamed *et al.* (1999) to specifically overcome the imbalance at various speed. The rotational speeds are scheduled in a table and appropriate gain adjustment according to the selected speed will be selected as the Q-parameter for the controller. This gain-scheduling method shows the elimination of imbalance at three rotor rotational speed is achieved with simpler design technique, however, a large look-up table is required to accommodate the operation at wider range of rotor rotational speeds.

With the linearization of force-to-current and force-to-airgap displacement relationship, AMB model belongs to a class of linear parameter varying (LPV) system which is suitable for LPV controller design. Zhang *et al.* (2002) has proposed a class of LPV controller that can maintain robust stability and performance at wide range of rotor speed. The augmented AMB system model is characterized as many sets of convex representation of system where the system matrix is considered as affine function of the rotor speed and treated as a set of structured uncertainty range. Due to the convexity property, H_∞ control rules is applied to each vertex yield stable closed-loop system and the LPV controller gain can be computed based on the convex representation of the system. The simulation result confirms that the robustness of the controller is obtained in which with the 3% uncertainty present, the nominal performance index, $\bar{\gamma}$, is well below 1 where the desired $\bar{\gamma}$ is only 1. However, in the experimental verification, due to the high computational time, some simplification is introduced in the controller algorithm to achieve acceptable system performance.

The synthesis of the LPV controller involves finding the solution of a single Lyapunov function that produces a stabilizing controller over a specified parameter range. When finding the solution is not possible, the normal approach is to formulate a few LPV controllers at many smaller parameter sub-regions which form a so-called switched LPV system. Lu and Wu (2004) have worked on this type of controller for AMB system and proposed hysteresis and average-dwell-time-dependent switching methods to maintain the system stability when the system switches from one sub-region to another. Both of the switching techniques lead to non-convex optimization problem that is difficult to be solved, however, the convexification of the hysteresis switching method is possible by using Linear Matrix Inequality (LMI) technique.