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Mixed convection flow near a non-orthogonal stagnation point towards a stretching vertical plate

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Abstract

An analysis of the steady two-dimensional mixed convection flow of an incompressible viscous fluid near an oblique stagnation point on a heated or cooled stretching vertical flat plate has been studied. It is assumed that the plate is stretched with a velocity proportional to the distance from a fixed point and the temperature of the plate is constant. Both the cases of the assisting and opposing flows are considered. It is shown that the velocity increases as the shear parameter γ increases with the increase of the straining parameter a/c. These flows have a boundary layer structure near the stagnation region. It is also found that the flow has an inverted boundary layer structure when the stretching velocity of the surface exceeds the stagnation velocity of the free stream ($a/c < 1$). It is shown that the position of the point x_s of zero skin friction (shear stress on the wall) is shifted to the left or to the right of the origin and it depends upon the balance between obliqueness, straining motion and buoyancy effects.

Keywords: Non-orthogonal flow; Stagnation point; Mixed convection; Stretching sheet

1. Introduction

The study of the flow field and heat transfer due to a stretching surface in a quiescent or moving fluid is relevant to several practical applications in the modern industry. To cite a few practical examples, industrial processes such as the extrusion of metals and plastics, cooling and/or drying of paper and textiles, glass blowing and material handling involve boundary layers on continuous moving surfaces in a free stream under prescribed boundary conditions. The control of the cooling rate of the sheets is very important for the desired material structure. As the sheets move through a cooling tank or in open air, they are cooled by the boundary layers induced on their surface due to the viscous force. It should be mentioned that the problem of boundary layer flow adjacent to a continuous moving sheet is physically different from that of the classical Blasius flow past a stationary flat plate and that the two problems cannot be mathematically transformed from one to the other. It seems that Crane [1] was the first to give a similarity solution in closed analytical form for steady two-dimensional incompressible boundary layer flow caused by the stretching of a sheet, which moves in its own plane with a velocity varying linearly with distance from a fixed point. Subsequently, several investigators [2–6] have studied various aspects of this problem such as the effect of the mass transfer, wall temperature and magnetic field. Mixed convection boundary-layer flows, characterized by the modification of the convective flow and thermal fields by buoyancy forces, are frequently encountered in transport processes occurring both in nature and in industry. The wall skin friction and the heat transfer rate from the surface are affected by the buoyancy forces, and considerable error in their estimation

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Nomenclature

a	positive constant characterizing the outer flow
b	positive or negative constant characterizing the outer flow
A	constant
c	positive constant characterizing the stretching sheet
C_1, C_2	constants of integration
g	gravitational acceleration
k	thermal conductivity
Gr	Grashof number
ℓ	characteristic length
Nu	Nusselt number
$\bar{p}$	dimensional pressure
p	dimensionless pressure
Pr	Prandtl number
$\bar{q}_{\mathrm{w}}$	heat transfer from the plate
Re	Reynolds number
T	fluid temperature
$\bar{u}, \bar{v}$	dimensional velocity components along $\bar{x}$- and $\bar{y}$-axes
u, v	dimensionless velocity component
$\overline{\mathbf{V}}_{\mathbf{e}}$	dimensional free stream velocity
$\bar{u}_{\mathrm{e}}, \bar{v}_{\mathrm{e}}$	dimensional free stream velocity components along $\bar{x}$- and $\bar{y}$-axes
$\bar{u}_{\mathrm{w}}(\bar{x})$	dimensional velocity of the stretching sheet
$\bar{x}, \bar{y}$	dimensional Cartesian coordinates along the stretching surface and normal to it, respectively
x, y	dimensionless Cartesian coordinates
x_{s}	point of zero skin friction or shear stress on the wall

Greek symbols

α	thermal diffusivity of the fluid
β	coefficient of thermal expansion
γ	shear flow parameter
λ	mixed convection parameter
μ	dynamic viscosity
υ	kinematic viscosity
θ	dimensionless temperature
ρ	density of fluid
$\bar{\tau}_{\mathrm{w}}$	dimensional skin friction or shear stress on the wall
τ_{w}	dimensionless skin friction or shear stress on the wall
ψ	dimensionless stream function

Subscripts

w	refers to the wall
∞	refers to the far stream

can result if the buoyancy effects are not taken into account in the analysis (see Ramachandran et al. [7]). The effect of buoyancy forces on the steady boundary layer induced by a stretching sheet in the vertical direction has been considered by Ingham [8], Daskalakis [9], Chen [10,11], Chamkha [12], Ali [13], Partha et al. [14] and Ishak et al. [15].

Stagnation-point flow is a topic of considerable importance in fluid mechanics in the sense that they appear in virtually all flow fields of engineering and scientific interest. In some situations flow is stagnated by a solid wall, while in others a free stagnation point or line exists interior to the fluid domain. After Weidman and Putkaradze [16], these flows may be characterized as inviscid or viscous, steady or unsteady, two-dimensional or three-dimensional, symmetric or asymmetric, normal or oblique, homogeneous or two-fluid, and forward or reverse. The two-dimensional stagnation point flow impinging obliquely on a fixed plane wall has been investigated by a number of authors such as Stuart [17], Tamada [18], Takemitsu and Matunobu [19], Dorrepaal [20,21], Labropulu et al. [22], Tilley and Weidman [23], while Reza and Gupta [24], Lok et al. [25] and Mahapatra et al. [26] considered a similar problem on a stretching surface. It was found in these papers without or with heating plates that the stream function splits into a Hiemenz [27] flow and a tangential component. The main consequence of the free stream obliqueness is the shift of the stagnation point toward the incoming flow. This shift increases when decreasing the free stream incidence.

All the above investigations on flows impinging obliquely on a plane wall are, however, without considering the effect of the thermal buoyancy forces on the flow and thermal fields. Amaouche and Boukari [28] has studied the interaction of a buoyancy induced convection flow and free stream impinging at some angle of incidence on an inclined heated flat plate. Also, Lok et al. [29] have considered recently the mixed convection flow near the non-orthogonal stagnation point on a vertical surface, which is subject to a constant heat flux.

The aim of this paper is to investigate the steady mixed convection flow of an incompressible viscous fluid impinging obliquely on a heated or cooled vertical stretching surface. Using similarity variables, the full governing partial differential equations are transformed into a system of three non-linear ordinary differential equations which are then solved numerically using Keller's box method for both assisting and opposing flows. Representative results for the velocity profiles and the location of the point of zero skin friction (or shear stress on the wall) are obtained for several values of the governing parameters, which are presented in tables and figures. To the best of our knowledge this problem has not been studied before and the results reported here are new.

2. Basic equations

Consider a two-dimensional flow impinging on a stretching vertical surface in a viscous and incompressible fluid, the oblique velocity of the inviscid fluid is $\overline{\mathbf{V}}_\mathrm{e}(\bar{u}_\mathrm{e}, \bar{v}_\mathrm{e})$, which consists of irrotational stagnation point flow and a uniform shear flow parallel to the stretching surface. It is assumed that the constant temperature of the plate is T_w, while the uniform temperature of the ambient fluid is T_∞. Fig. 1 illustrates such a flow field for a vertical, heated stretching surface with the upper half of the flow field ($x > 0$) being assisted and the lower half of the flow field ($x < 0$) being opposed by the buoyancy force. The reverse trend will occur if the plate is cooled below the ambient temperature T_∞. The reported results are thus true for both the heated ($T_\mathrm{w} > T_\infty$) and cooled ($T_\mathrm{w} < T_\infty$) surface conditions when the appropriate (assisting and opposing) flow regions are selected (see [30]). The coordinate system $\mathrm{O}\bar{x}\bar{y}$ has its origin O located at the center of the sheet with the positive $\bar{x}$-axis extending along the sheet in the upward direction, while the $\bar{y}$-axis is measured normal to the surface of the sheet and is positive in the direction from the sheet to the fluid, the flow being confined to $\bar{y} > 0$. The continuous stretching surface is assumed to have the velocity $\bar{u}_\mathrm{w}(\bar{x}) = c\bar{x}$, where $c > 0$ is a constant. Under these assumptions along with the Boussinesq approximation, the steady two-dimensional flow of a viscous and incompressible fluid is described by the following equations:

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \tag{1}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{x}} + \upsilon\bar{\nabla}^2\bar{u} + g\beta(T - T_\infty) \tag{2}$$

$$\bar{u}\frac{\partial \bar{v}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{v}}{\partial \bar{y}} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{y}} + \upsilon\bar{\nabla}^2\bar{v} \tag{3}$$

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \alpha\bar{\nabla}^2 T \tag{4}$$

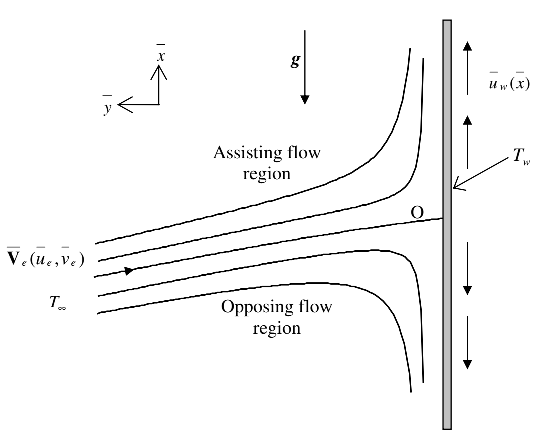

Fig. 1. Physical model and coordinate system.

where $\bar{u}$ and $\bar{v}$ are the velocity components along the $\bar{x}$- and $\bar{y}$-axes, respectively, g is the acceleration due to gravity, $\bar{p}$ is the pressure, ρ is the density, υ is the kinematic viscosity, α is the thermal diffusivity, β is the coefficient of thermal expansion, $\bar{\nabla}^2$ is the Laplacian in Cartesian coordinates $(\bar{x}, \bar{y})$. Following Amaouche and Boukari [28], Eqs. (1)–(4) are solved under the following boundary conditions

$$\begin{aligned} &\bar{v} = 0, \quad \bar{u} = \bar{u}_\mathrm{w}(\bar{x}) = c\bar{x}, \quad T = T_\mathrm{w} \quad \text{at} \quad \bar{y} = 0 \\ &\bar{u}_\mathrm{e} = a\bar{x} + b\bar{y}, \quad \bar{v}_\mathrm{e} = -a\bar{y}, \quad T = T_\infty \quad \text{as} \quad \bar{y} \to \infty \end{aligned} \tag{5}$$

where a and b are constants characterizing the outer inviscid flow.

We introduce now the following dimensionless variables

$$\begin{aligned} &x = \bar{x}/\ell, \quad y = \bar{y}/\ell, \quad u = \bar{u}/(c\ell), \quad v = \bar{v}/(c\ell), \\ &p = \bar{p}/(\rho c^2\ell^2), \quad \theta = (T - T_\infty)/(T_\mathrm{w} - T_\infty) \end{aligned} \tag{6}$$

where $\ell = (\upsilon/c)^{1/2}$ is a characteristic length of the flat plate. Substituting (6) into Eqs. (1)–(4), gives

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{7}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \nabla^2 u + \lambda\theta \tag{8}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \nabla^2 v \tag{9}$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{Pr}\nabla^2\theta \tag{10}$$

where Pr is the Prandtl number and λ is the mixed convection parameter, which is defined as

$$\lambda = \frac{g\beta(T_\mathrm{w} - T_\infty)}{c^2\ell} = \frac{g\beta(T_\mathrm{w} - T_\infty)\ell^3/\upsilon^2}{(c\ell)^2\ell^2/\upsilon^2} = \frac{Gr}{Re^2} \tag{11}$$

Here $Gr = g\beta(T_\mathrm{w} - T_\infty)\ell^3/\upsilon^2$ is the Grashof number and $Re = (c\ell)\ell/\upsilon$ is the Reynolds number. It should be noted that $\lambda = 0$ corresponds to forced convection flow, while $\lambda \neq 0$ corresponds to the mixed convection flow, where $\lambda > 0$ refers to the heated sheet and $\lambda < 0$ refers to the cooled sheet, respectively. Further, we introduce the stream function ψ defined as

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \tag{12}$$

and eliminate pressure p from Eqs. (8) and (9). Thus, we have

$$\frac{\partial \psi}{\partial x}\frac{\partial}{\partial y}(\nabla^2\psi) - \frac{\partial \psi}{\partial y}\frac{\partial}{\partial x}(\nabla^2\psi) + \nabla^4\psi + \lambda\frac{\partial \theta}{\partial y} = 0 \tag{13}$$

$$\frac{\partial \psi}{\partial y}\frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \theta}{\partial y} = \frac{1}{Pr}\nabla^2\theta \tag{14}$$

and the boundary conditions (5) become

$$\begin{aligned} &\psi = 0, \quad \frac{\partial \psi}{\partial y} = x, \quad \theta = 1 \quad \text{at } y = 0 \\ &\psi = \frac{a}{c}xy + \frac{1}{2}\gamma y^2, \quad \theta = 0 \quad \text{as } y \to \infty \end{aligned} \tag{15}$$

where $\gamma = b/c$ is a shear flow parameter.

Physical quantities of interest are the skin friction or shear stress on the wall and the Nusselt number which are defined as

$$\bar{\tau}_{\mathrm{w}} = \mu\left(\frac{\partial \bar{u}}{\partial \bar{y}} + \frac{\partial \bar{v}}{\partial \bar{x}}\right)_{\bar{y}=0}, \quad Nu = \frac{\ell \bar{q}_{\mathrm{w}}}{k(T_{\mathrm{w}} - T_{\infty})} \tag{16}$$

where k is the thermal conductivity, μ is the dynamic viscosity and $\bar{q}_{\mathrm{w}}$ is the heat transfer from the wall which is given by

$$\bar{q}_{\mathrm{w}} = -k\left(\frac{\partial T}{\partial \bar{y}}\right)_{\bar{y}=0} \tag{17}$$

Using the dimensionless variables (6) and the definition of the stream function (12), the dimensionless skin friction or shear stress on the wall and the Nusselt number can be written as

$$\tau_{\mathrm{w}} = \left(\frac{\partial^2 \psi}{\partial y^2} - \frac{\partial^2 \psi}{\partial x^2}\right)_{y=0}, \quad Nu = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0} \tag{18}$$

where $\tau_{\mathrm{w}} = \bar{\tau}_{\mathrm{w}}/(\mu c)$.

The boundary conditions (15) suggest that Eqs. (13) and (14) have the solution of the form

$$\psi(x,y) = xF(y) + G(y), \quad \theta(x,y) = H(y) \tag{19}$$

where the functions $F(y)$ and $G(y)$ refer to the normal component and tangential component of the flow, respectively, see Dorrepaal [20]. Thus the velocity components (u, v) that obtained from Eqs. (12) and (19) are

$$u = xF'(y) + G'(y), \quad v = -F(y). \tag{20}$$

Substituting (19) into Eqs. (13) and (14) results in, after one integration, the following ordinary differential equations

$$F''' + FF'' - F'^2 + C_1 = 0 \tag{21}$$

$$G''' + FG'' - F'G' + \lambda H + C_2 = 0 \tag{22}$$

$$\frac{1}{Pr}H'' + FH' = 0 \tag{23}$$

where C_1 and C_2 are constants of integration. The boundary conditions (15) become

Table 1
Values of A for different values of a/c

a/c	A
0.1	0.7917
0.2	0.6407
0.5	0.3286
0.8	0.1145
1.0	0.0000
1.2	−0.0998
1.5	−0.2297
2.0	−0.4104
3.0	−0.6931
5.0	−1.1053

$$F(0) = 0, \quad F'(0) = 1, \quad F'(\infty) = \frac{a}{c} \tag{24a}$$

$$G(0) = G'(0) = 0, \quad H(0) = 1, \quad G''(\infty) = \gamma, \quad H(\infty) = 0 \tag{24b}$$

Table 2
Values of $H'(0)$ for different values of a/c and Pr

a/c	$H'(0)$			
	$Pr = 0.05$	$Pr = 0.5$	$Pr = 1.0$	$Pr = 1.5$
0.1	−0.0813	−0.3824	−0.6022	−0.7768
	(−0.081)	(−0.383)	(−0.603)	(−0.777)
0.2	−0.0990	−0.4073	−0.6245	−0.7971
	(−0.099)	(−0.408)	(−0.625)	(−0.797)
0.5	−0.1356	−0.4728	−0.6933	−0.8652
	(−0.136)	(−0.473)	(−0.692)	(−0.863)
1.0	−0.1784	−0.5643	−0.7979	−0.9772
	(−0.178)	(−0.563)	(−0.796)	(−0.974)
2.0	−0.2411	−0.7119	−0.9788	−1.1782
	(−0.241)	(−0.709)	(−0.974)	(−1.171)
3.0	−0.2901	−0.8336	−1.1322	−1.3521
	(−0.289)	(−0.829)	(−1.124)	(−1.341)

( ) Results by Mahapatra and Gupta [32].

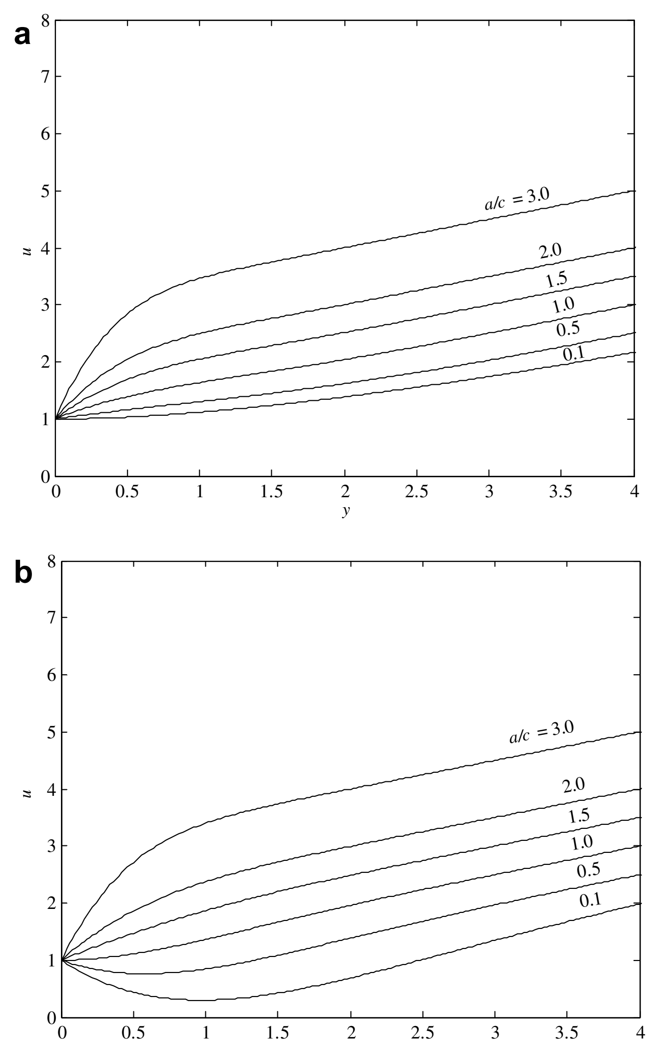

Fig. 2. Velocity profiles $u(x, y)$ for different values of a/c when $x = 1$, $Pr = 0.72$ and $\gamma = 0.5$: (a) heated sheet ($\lambda = 1$); (b) cooled sheet ($\lambda = -1$).

Primes denote differentiation with respect to y. Taking the limit $y \to \infty$ in Eq. (21) and using the boundary condition $F'(\infty) = a/c$, we get $C_1 = a^2/c^2$. From an analysis of the boundary layer Eq. (21) it is found that $F(y)$ behaves as $F(y) = (a/c)y + A$ as $y \to \infty$, where $A = A(a/c)$ is a constant accounts for the boundary layer displacement. By taking the limit $y \to \infty$ in Eq. (22) and using the boundary conditions (24), we get $C_2 = -A\gamma$. Thus, Eqs. (21) and (22) become

$$F''' + FF'' - F'^2 + \frac{a^2}{c^2} = 0 \tag{25}$$

$$G''' + FG'' - F'G' + \lambda H - A\gamma = 0 \tag{26}$$

The value of $A = A(a/c)$ is determined by solving numerically Eq. (25) subject to the boundary conditions (24a). Some values of A for several values of a/c are given in Table 1.

Employing (19), the non-dimensional skin friction and the Nusselt number given by Eq. (18) can now be written as

$$\tau_w = xF''(0) + G''(0), \quad Nu = -H'(0) \tag{27}$$

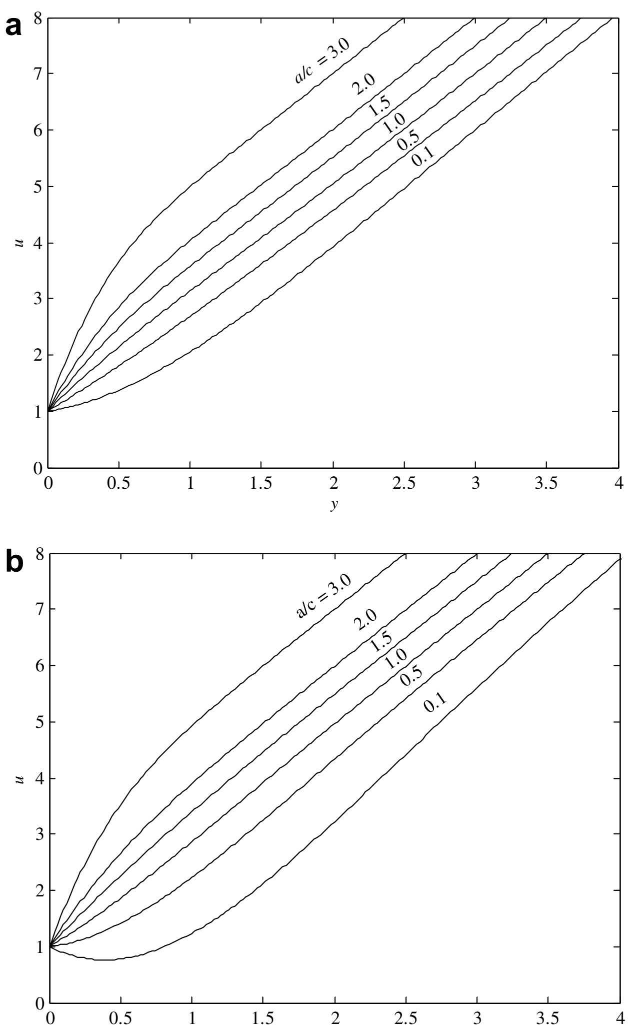

Fig. 3. Velocity profiles $u(x,y)$ for different values of a/c when $x = 1$, $Pr = 0.72$ and $\gamma = 2$: (a) heated sheet ($\lambda = 1$); (b) cooled sheet ($\lambda = -1$).

where values of $F''(0)$, $G''(0)$ and $H'(0)$ can be calculated from Eqs. (25), (26) and (23) for some values of the parameters λ and γ.

In particular, the dividing streamlines $\psi = 0$ and the curve $u = \partial\psi/\partial y = 0$ intersect the wall at the stagnation point where $\tau_w = 0$. Therefore the location of the point x_s of zero skin friction or shear stress on the wall is given by

$$x_s = -\frac{G''(0)}{F''(0)} \tag{28}$$

3. Results and discussion

Eqs. (23), (25) and (26) subject to the boundary conditions (24) have been solved numerically for some values of the parameter a/c, Prandtl number Pr, mixed convection parameter λ and shear flow parameter γ using Keller's box method in conjunction with Newton's linearization which is described in the book by Cebeci and Bradshaw [31]. Both assisting and opposing flow cases are considered. We notice that Eqs. (23) and (25) describe the flow and heat transfer of an orthogonal stagnation point flow towards a stretching sheet, which has been solved numerically by Mahapatra and Gupta [32] using the finite difference method with Thomas algorithm. In order to verify the accuracy of the present results, we have compared the values of the heat

Table 3
Values of the point x_s of zero shear stress on the wall ($\tau_w = 0$) for different values of λ and a/c when Pr = 0.72 and $\gamma = 1$: heated stretching sheet

λ	x_s					
	$a/c = 0.1$	$a/c = 0.2$	$a/c = 0.5$	$a/c = 1.2$	$a/c = 1.5$	$a/c = 2.0$
0.1	0.3564	0.5926	1.2652	−3.2453	−1.2612	−0.5967
0.2	0.4410	0.6741	1.3575	−3.3842	−1.3087	−0.6158
0.5	0.6948	0.9185	1.6343	−3.8009	−1.4513	−0.6733
1.0	1.1178	1.3258	2.0958	−4.4953	−1.6888	−0.7691
1.5	1.5408	1.7331	2.5572	−5.1897	−1.9264	−0.8648
2.0	1.9638	2.1404	3.0186	−5.8842	−2.1640	−0.9606
3.0	2.8098	2.9550	3.9414	−7.2731	−2.6392	−1.1521
5.0	4.5019	4.5843	5.7871	−10.0508	−3.5896	−1.5352
10.0	8.7320	8.6574	10.4013	−16.9951	−5.9656	−2.4929

Table 4
Values of the point x_s of zero shear stress on the wall ($\tau_w = 0$) for different values of λ and a/c when Pr = 0.72 and $\gamma = 1$: cooled stretching sheet

λ	x_S					
	$a/c = 0.1$	$a/c = 0.2$	$a/c = 0.5$	$a/c = 1.2$	$a/c = 1.5$	$a/c = 2.0$
−0.1	0.1872	0.4297	1.0806	−2.9676	−1.1661	−0.5584
−0.2	0.1026	0.3482	0.9884	−2.8287	−1.1186	−0.5392
−0.5	−0.1513	0.1039	0.7115	−2.4120	−0.9761	−0.4818
−1.0	−0.5743	−0.3035	0.2501	−1.7176	−0.7385	−0.3860
−1.5	−0.9973	−0.7108	−0.2113	−1.0231	−0.5009	−0.2902
−2.0	−1.4203	−1.1181	−0.6727	−0.3287	−0.2633	−0.1945
−3.0	−2.2663	−1.9327	−1.5956	1.0602	0.2119	−0.0029
−5.0	−3.9583	−3.5619	−3.4413	3.8379	1.1623	0.3802
−10.0	−8.1884	−7.6351	−8.0554	10.7822	3.5383	1.3378

transfer rate, $H'(0)$ for various values of a/c and Pr with those of Mahapatra and Gupta [32]. The results are in very good agreement as can be seen from Table 2. Therefore, we are confident that the present results are accurate.

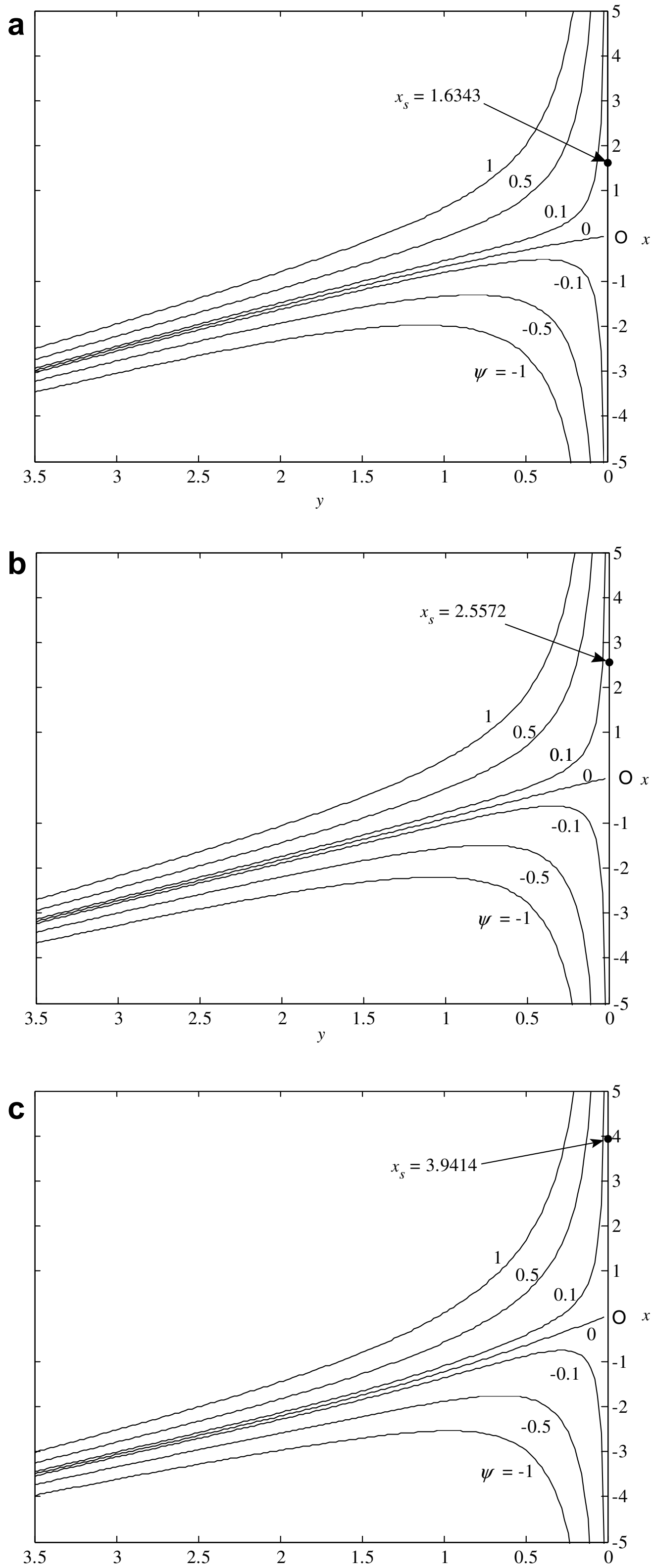

Fig. 4. The streamlines for $a/c = 0.5$ (<1), $Pr = 0.72$, $\gamma = 1$ and some values of λ when the vertical sheet is heated: (a) $\lambda = 0.5$; (b) $\lambda = 1.5$; (c) $\lambda = 3.0$.

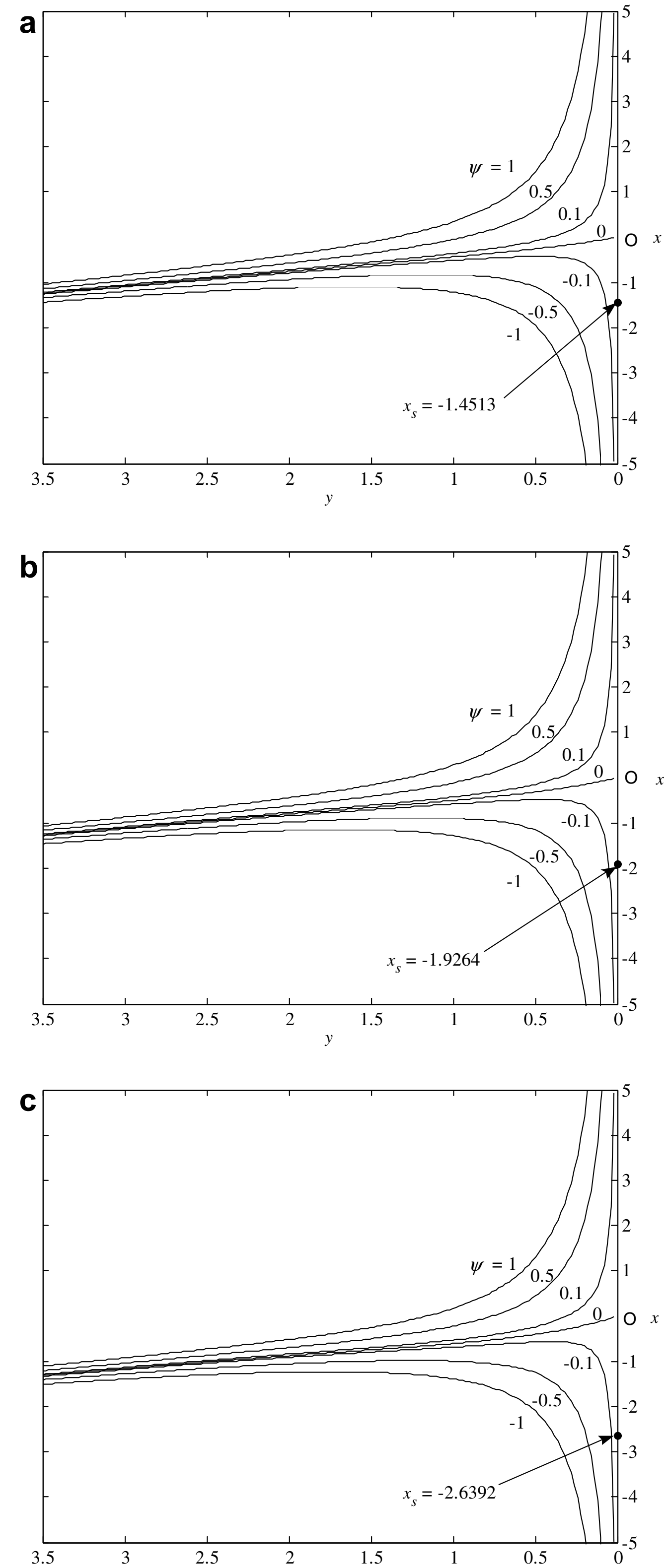

Fig. 5. The streamlines for $a/c = 1.5$ (>1), $Pr = 0.72$, $\gamma = 1$ and some values of λ when the vertical sheet is heated: (a) $\lambda = 0.5$; (b) $\lambda = 1.5$; (c) $\lambda = 3.0$.

Figs. 2 and 3 shows the variation of $u(x, y)$, the vertical component of the velocity, which is given by Eq. (20), with y at a fixed value of $x(=1.0)$ and $Pr = 0.72$ for several values of a/c when $\lambda = 1$, and $\gamma = 0.5$ (small shear in the free stream) and $\gamma = 2$ (large shear in the free stream), respectively. It is found that the velocity increases with the

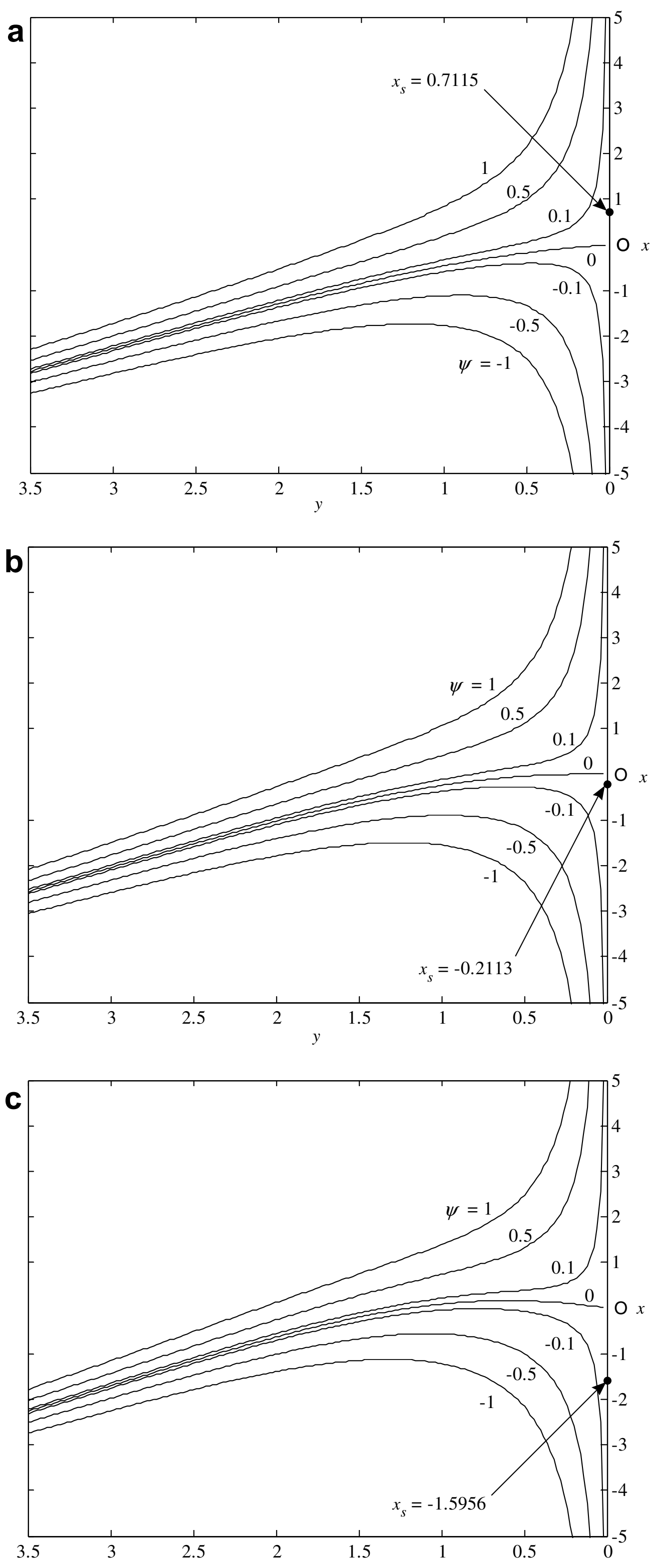

Fig. 6. The streamlines for $a/c = 0.5$ (<1), $Pr = 0.72$, $\gamma = 1$ and some values of λ when the vertical sheet is cooled: (a) $\lambda = -0.5$; (b) $\lambda = -1.5$; (c) $\lambda = -3.0$.

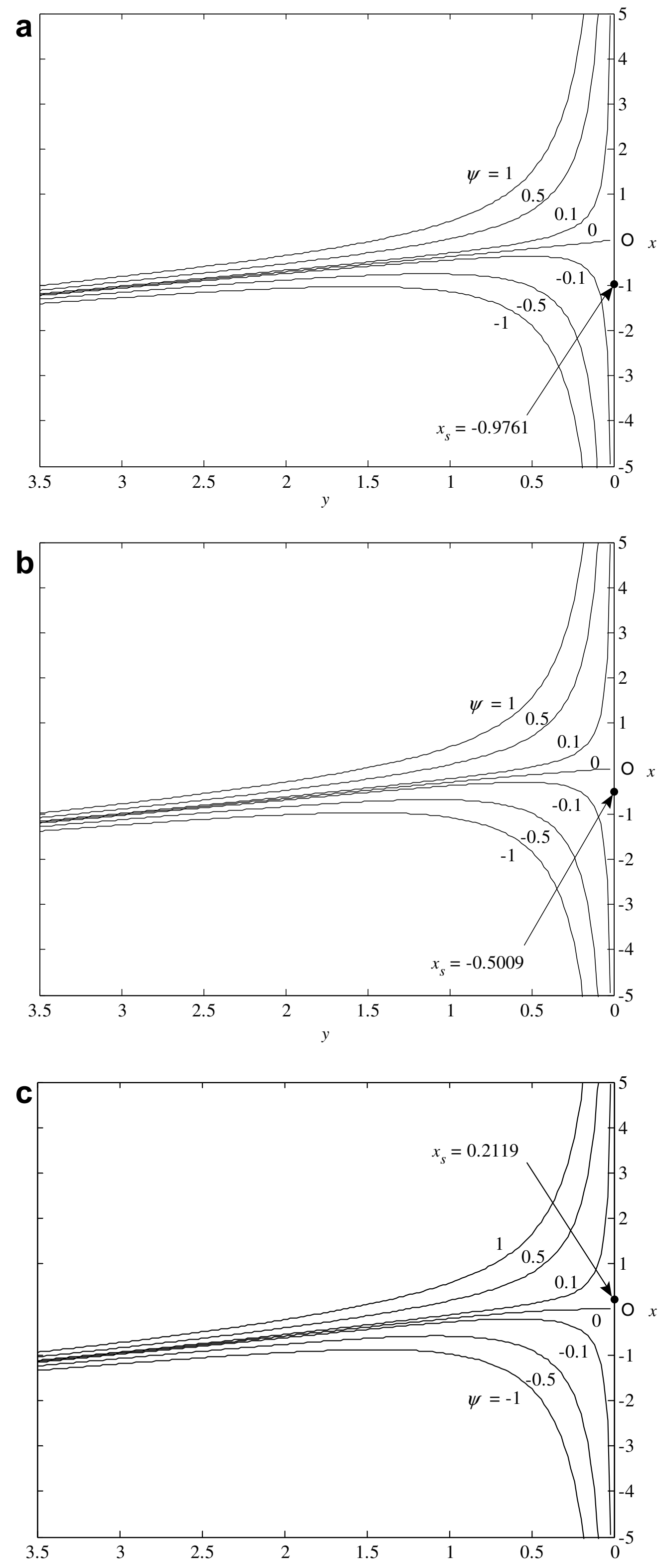

Fig. 7. The streamlines for $a/c = 1.5$ (>1), $Pr = 0.72$, $\gamma = 1$ and some values of λ when the vertical sheet is cooled: (a) $\lambda = -0.5$; (b) $\lambda = -1.5$; (c) $\lambda = -3.0$.

increase in a/c. A boundary layer structure is formed for both assisting and opposing flows when $a/c > 1$, however, an inverted boundary layer is observed when $a/c < 1$. It results from the fact that when $a/c < 1$, the stretching velocity $\bar{u}_w(\bar{x}) = c\bar{x}$ of the surface exceeds the stagnation velocity $\bar{u}_e = a\bar{x} + b\bar{y}$ of the external stream (see [26,32]). We can also see that there is not much difference between the cases of heated and cooled sheets when $a/c > 1$. However, a difference can be seen when $a/c < 1$, namely, the convexity of the velocity profiles for cooled sheet looks more expanded. Further, it is clearly seen from Figs. 2 and 3 that the slope of the velocity is higher for large value of shear (e.g. $\gamma = 2$) compared to that for smaller value (e.g. $\gamma = 0.5$).

Tables 3 and 4 show the values of the point x_s of zero skin friction or shear stress on the wall ($\tau_w = 0$) for some values of λ and a/c when $Pr = 0.72$ and $\gamma = 1$, for heated and cooled vertical sheets, respectively. For the case of heated sheet, assisting flow, it can be observed that for a fixed value of a/c, the location of the point x_s moves continuously to the right of the origin O (assisting flow region) with an increase in λ provided that $a/c < 1$, but it shifts to the left of the origin (opposing flow region) when $a/c > 1$. For the case of cooled sheet, the position of x_s moves from the right ($x_s > 0$, assisting flow region) to the left ($x_s < 0$, opposing flow region) when $a/c < 1$ with an increasing λ. However, when $a/c > 1$, x_s is shifted from the left ($x_s < 0$, opposing flow region) to the right ($x_s > 0$, assisting flow region). This is due to the balance between the obliqueness, straining motion and the buoyancy force. In order to illustrate these various situations, some streamline patterns are drawn in Figs. 4–7. Figs. 4 and 5 are for the heated stretching sheet assisting flow when $a/c < 1$ and $a/c > 1$, respectively. The case of the cooled stretching sheet is illustrated in Figs. 6 and 7.

4. Conclusion

In this paper, we have studied the steady two-dimensional oblique stagnation-point flow of an incompressible viscous fluid towards a vertical surface, which is stretched with a velocity proportional to the distance from a fixed point. The motivation is to determine the effect of buoyancy on the flow characteristics of the boundary layer by interaction of a buoyancy induced convection flow and a free stream impinging obliquely on a heated or cooled stretching flat plate of constant wall temperature for both cases of buoyancy assisting flow and buoyancy assisting flow, respectively. By using a proper scaling, an exact similarity solution of the Navier–Stokes and energy equations has been obtained. The present results show that the velocity profile at a given point of y increase with increase in a/c. For the case of cooled sheet, the convexity of the inverted boundary layer that formed looks more expanded than the case of heated sheet. It is also found that the mixed convection parameter affects the position of the point x_s of zero skin friction (shear stress on the wall). However, it needs to be mentioned that there exists no theoretical or experimental data available for comparison.

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