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# Application of Running Average Function to Non-Dispersive Errors of Network-Based Real-Time Kinematic Positioning

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## Abstract

The GPS errors can be separated into a frequency-dependent or dispersive component (e.g. the ionospheric delay) and a non-dispersive component (e.g. the tropospheric delay and orbit biases). Dispersive and non-dispersive errors have different dynamic effects on the GPS network corrections. The former exhibits rapid changes with high variations due to the effect of free electrons in the ionosphere, whilst the latter change slowly and smoothly over time due to the characteristic behaviour of the tropospheric delay and the nature of orbit biases. It is found that the non-dispersive correction can be used to obtain better ionosphere-free measurements, and therefore helpful in resolving the long-range integer ambiguity of the GPS carrier-phase measurements. A running average is proposed in this paper to provide a stable network correction for the non-dispersive term. Once the integer ambiguities have been resolved, both dispersive and non-dispersive corrections can be applied to the fixed carrier-phase measurements for positioning step so as to improve the accuracy of the estimated coordinates. Instantaneous positioning i.e. single-epoch positioning, has been tested for two regional networks: SydNET, Sydney, and SIMRSN, Singapore. The test results have shown that the proposed strategy performs well in generating the network corrections, fixing ambiguities and computing a user's position.

**Keywords:** GPS, RTK, network-RTK, running average.

## 1. Introduction

Real-time kinematic (RTK) ambiguity resolution, a key step for precise GPS positioning, is complicated due to many error sources in the carrier-phase measurements. These errors can be grouped into station- and distance-dependent errors. Station-dependent errors such as receiver-biased errors, multipath effects and measurement noises, notably degrade the ambiguity resolution. The

effort to reduce this type of errors has been considerably undertaken in the past decade. Ambiguity resolution is also seriously affected by the presence of the distance-dependent errors: ionospheric delay, tropospheric delay and orbit biases. Due to the distance-dependent errors, reliable RTK ambiguity resolution is limited to relatively short inter-receiver distances, typically of the order of 10km or 25km at the maximum. However, there exists a strong demand to extend the baseline length, without sacrificing RTK performance. The use of multiple GPS reference stations i.e. a GPS network, makes it possible.

GPS networks have been deployed for many years, providing opportunities to mitigate distance-dependent errors in many ways. A good example is the network of the International GNSS Service (IGS), and its products (cf. <http://igsceb.jpl.nasa.gov>). To date the coverage of IGS is not dense enough to be sensitive to small-scale errors, and therefore does not meet the requirement of regional or local GPS users. Although the IGS products are improving, many countries have developed their own regional or local GPS networks. The inter-station distances in these networks are kept below 200km in order to model the distance-dependent errors adequately.

The concept and the technique of carrier-phase network-based RTK positioning were introduced by Wanninger (1995), based initially on utilising three reference stations of a GPS network. Estimated distance-dependent errors for each reference station are combined in order to interpolate and estimate the same types of errors for users within the network coverage. A variety of algorithms for estimating such 'network corrections' exist, but the popular algorithms are: Virtual Reference Station (VRS) implemented by Trimble (Lynn & Anil, 1995; Wanninger, 1997) and Area Correction Parameters which is also known as Flächen Korrektur Parameter (FKP) in German (Wubbena & Bagge, 1998). The major difference between the two methods is that they use different approaches to make corrections for the rovers. VRS

provides a rover with “error-removed” data for a virtual reference station in close proximity to the rover. Therefore the rover must send its approximate position to VRS. FKP is suitable for broadcasting the corrections to multiple users in the network because the FKP corrections are actually interpolation coefficients for the rovers’ position. Previous work has shown that the network-based technique is an efficient means of improving long-range ambiguity resolution, and enables high accuracy positioning with less dense GPS reference station networks than would be the case if single-baseline RTK techniques were used.

The network corrections can be separated into dispersive (ionosphere-related) and non-dispersive (troposphere- and orbit-related) components according to their dependency on GPS signal frequency. Euler *et al.* (2004) discussed the impact of incorrectly determined network integer ambiguity on the separated dispersive and non-dispersive corrections. Keenan *et al.* (2002) proposed a user standard correction transmission format that separates the network corrections. Dispersive and non-dispersive components have different dynamic effects. Typically dispersive components exhibit rapid changes, with high variations due to the effect of free electrons in the ionosphere (Hernandes *et al.*, 1999; Odijk, 2002). On the other hand, non-dispersive components change slowly and smoothly over time due to the characteristic behaviour of the tropospheric delay and the nature of orbit biases (Tajul *et al.*, 2005). Further attention should be given to the separation, and the dynamic effect, of the network corrections.

In this paper, a running average function is proposed to improve non-dispersive corrections. In order to validate this proposition, tests of instantaneous ambiguity resolutions are conducted and compared with conventional network-RTK positioning. Test results with and without applying the function will be compared.

## 2. Methodology of Network-RTK

Network-RTK needs all GPS reference stations to transmit their raw GPS measurements to a control centre. The network algorithm at the control centre will select one of them as a master station and calculate the network corrections. Then the network corrections need to be distributed to users.

Because of the long distances between the stations in the network, the task of the network ambiguity resolution is challenging. Furthermore, the process needs to be done in real-time. Several discussions about this process can be found in Hu *et al.* (2005), Chen *et al.* (2004), Dai (2002) and Odijk (2002). For the static mode, the ambiguity resolution process can take advantage of long observation sessions. In the real-time mode, however, the degree of

freedom is less. Hence, all measurement errors need to be appropriately modelled, and a fast ambiguity search and validation methodology is required.

To assist network ambiguity resolution, the data from dual-frequency receivers are processed, choke-ring type antennas are used, as well as knowledge of the network baseline lengths and precise (predicted) ultra-rapid orbits from the IGS, low multipath environment is assumed, and the reference stations are static. The processing takes advantage of various linear combinations of carrier-phase and pseudorange measurements. Well-known linear combinations, such as the widelane and the ionosphere-free, are often used for network ambiguity resolution (Han, 1997; Sun *et al.*, 1999).

Once network ambiguities are fixed, the residuals are used to approximate the distance-dependent errors within the area. This “lump sum” approach is applied in order not to combine the residuals into a single network correction; hence they are separated according to whether they are dispersive or non-dispersive. The separation can be easily done via geometry-free and ionosphere-free combinations. Properties of these combinations can be found in Rizos (1997). The next step is to interpolate these residuals relative to the user’s approximate position, which in turn provides the user with the network correction. Dai (2002) discussed several interpolation methods that can be used for this purpose. A linear interpolation algorithm is adequate to perform this task for a local network. Therefore the linear combination method (LCM) (Han, 1997) is used in this study.

Due to the rapid changes and high variability of the ionosphere effect, interpolating the dispersive component has to be performed as frequently as possible (e.g. epoch-by-epoch). Conversely, rapid variations can be observed in the non-dispersive component because of remaining multipath and noises in the ionosphere-free measurements. Hence, a similar attempt to interpolate this component, as in the case of dispersive component, will have a tendency of increasing residuals. For this reason it is suggested in this paper that non-dispersive errors should not be interpolated on an epoch-by-epoch basis. In addition, a running average is applied to non-dispersive errors in order to obtain smooth non-dispersive correction. This smoothed result remains valid for many epochs (say 5 to 10 minutes) and the process should be continuously running for the next ‘windows’.

## 3. Tests for Local GPS Networks

Two local GPS networks in different geographical locations were tested in this study. One is the Sydney Network (SydNET) located in the mid-latitudes (latitude range 33°36’ – 34°08’S, and longitude range 150°34’ – 151°12’E), and the other is the Singapore Integrated

Multiple Reference Station Network (SIMRSN) located near the equator (latitudes  $1^{\circ}15' - 1^{\circ}30'N$ , and longitudes  $103^{\circ}40' - 103^{\circ}59'E$ ). It is expected that atmospheric effects are more severe in the equatorial area. Figures 1 and 2 show the locations of the stations within SydNET and SIMRSN, respectively.

To investigate the proposed network processing strategy, tests were conducted in post-processed, but ‘simulated’ RTK mode. For verification purposes, the data have been processed in static mode. Stations SPWD of SydNET and LOYA of SIMRSN were selected as the two networks’ master stations. Meanwhile, the station VILL of SydNET and NYPC of SIMRSN were treated as user stations. The selection is made to avoid severe multipath for the user station because the proposed network algorithm is not aimed at mitigating such effects at the moment. Other stations were considered to be reference stations (see Figures 1 and 2). It was assumed that the two networks had access to IGS ultra-rapid orbit data and were equipped with data transmission facilities. Reductions to the user’s and the master’s raw GPS measurements by the network correction were avoided in the first place, except for an *a priori* tropospheric model. The network correction (i.e. dispersive and non-dispersive terms) was generated by removing satellites in the master-to-reference combinations whose elevations were less than  $10^{\circ}$ . For master-to-user processing, it was further categorised by changing the satellites’ cut-off elevation angles from  $10^{\circ}$  to  $15^{\circ}$  and  $20^{\circ}$ .



Fig. 1 SydNET network



Fig. 2 SIMRSN network

#### 4. Test Results and Analysis

Figures 3 (SydNET) and 4 (SIMRSN) show the original master-to-user double-differenced residuals of dispersive and non-dispersive effects for all satellite combinations. Associated network corrections are also highlighted in these figures.

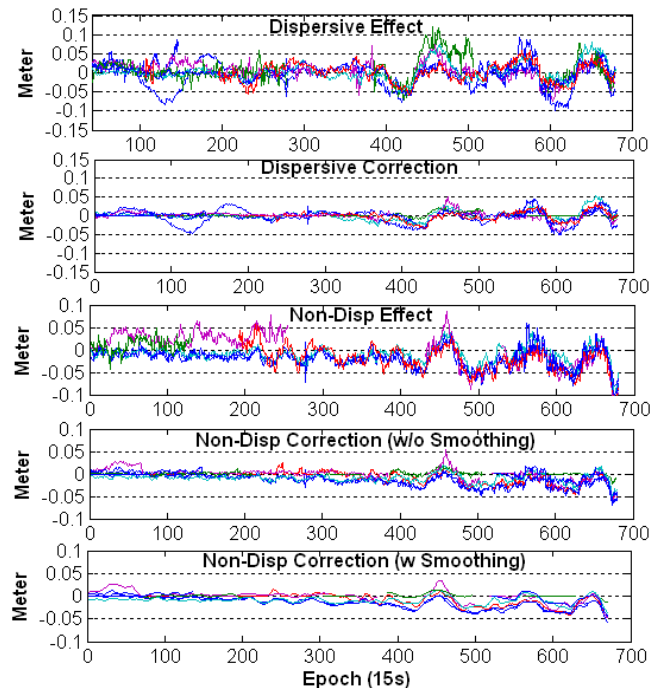


Fig. 3 SydNET Test. Top two: residuals of DD dispersive effect (top) and dispersive correction (bottom). Bottom three: residuals of DD non-dispersive effect (top), original correction (middle) and smoothed correction (bottom) for non-dispersive. Baseline: SPWD-VILL (~43km) in Sydney. Day of Year (DoY): 131/05 and the observation period of 3hrs (10.00pm-1.00am, local time)

As can be seen in Figure 3, both dispersive and non-dispersive corrections have performed reasonably well. The magnitude of the corrections is approximately almost the same or half the magnitude of the original residuals. Inspecting the residual patterns, it is obvious that the network corrections exhibit some trends. In Figure 4, however, there are less accurate corrections even though the baseline length in this network is shorter. This can be explained by the stronger atmospheric activity in the equatorial region. Therefore, this complicates the master-to-reference ambiguity resolution, which in turn results in lower quality network corrections.

The non-dispersive correction performed well in both tests when the smoothing function is applied. The magnitudes and trends of the smoothed corrections are in the range of the non-dispersive residuals. It can be noticed from both figures that network corrections for some epochs are not available, especially for low elevation satellites. Figures 5 and 6 indicate the number of satellites in view and the available corrections for the VILL and NYPC stations.

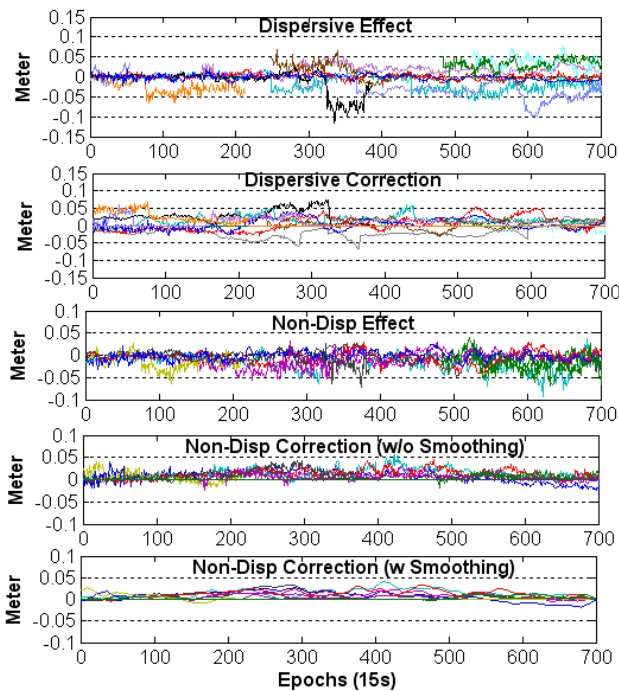


Fig. 4 SIMRSN Test. Top two: residuals of DD dispersive effect (top) and dispersive correction (bottom). Bottom three: residuals of DD non-dispersive effect (top), original correction (middle) and smoothed correction (bottom) for non-dispersive. Baseline: LOYA-NYPC (~14km) in Singapore. DoY: 166/03 and the observation period of 3hrs (8.00am-11.00am, local time)

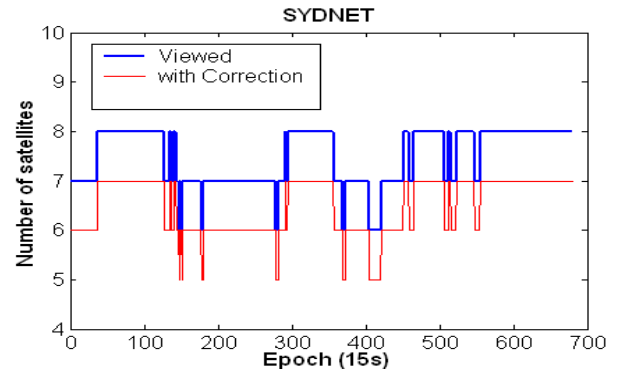


Fig. 5 Number of satellites in view (at 10° elevations and above) and available corrections for the station VILL in SydNET

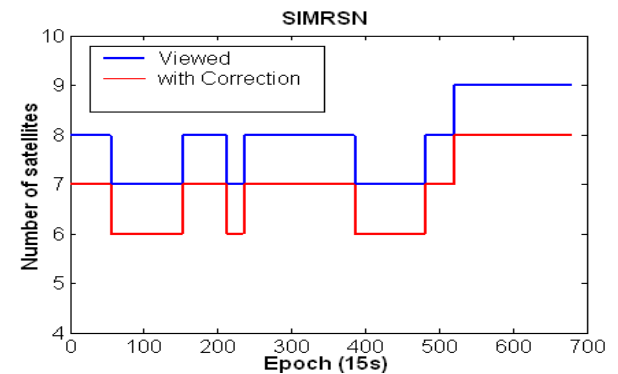


Fig. 6 Number of satellites in view (at 10° elevations and above) and available corrections for the station NYPC in SIMRSN

During the period of the tests, instantaneous (single-epoch) integer ambiguity resolution was attempted using both single-base and network-based modes of processing. Tables 1 and 2 show the statistics of L1 DD ambiguity resolution for SydNET and SIMRSN respectively. In the tables, the first column is the satellite cut-off elevation angles used in the processing. The second column is the number of DD L1 ambiguities which have been initialised during the period of the tests. The other columns indicate the percentile ambiguity resolution statistics (correct, rejected, wrong) for single-base and network-based techniques. As seen in the tables, the network-based technique performs better, i.e. a higher percentage for the correct fix rates and lower percentages for the rejected fix rates and wrong fix rates, compared to the single-base mode. It also can be noted that, the higher the cut-off elevation angle the better the results for both techniques.

Table 1 Statistics of single-epoch ambiguity resolution for the baseline SPWD-VILL in SydNET

cut-off	Case Initialize	Single-Base			Network-Based		
		Correct %	Reject %	Wrong %	Correct %	Reject %	Wrong %
10°	4103	84.5	5.8	9.7	91.5	3.0	5.6
15°	3916	87.8	2.9	9.3	94.6	1.4	4.0
20°	3345	93.6	0.5	5.9	98.1	0.4	1.5

Table 2 Statistic of single-epoch ambiguity resolution for the baseline LOYA-NYPC in SIMRSN

cut-off	Case Initialize	Single-Base			Network-Based		
		Correct %	Reject %	Wrong %	Correct %	Reject %	Wrong %
10°	4665	96.4	2.1	1.5	98.7	0.8	0.5
15°	3584	97.4	2.4	0.2	99.3	0.7	0
20°	3033	98.5	1.4	0.2	99.6	0.4	0

Figures 7 and 8 highlight the F-ratio validation values for both tests. The figures show that the network-based technique, in most cases, results in higher ratio values than the single-base mode. For this ratio test the critical threshold value is set to 3.

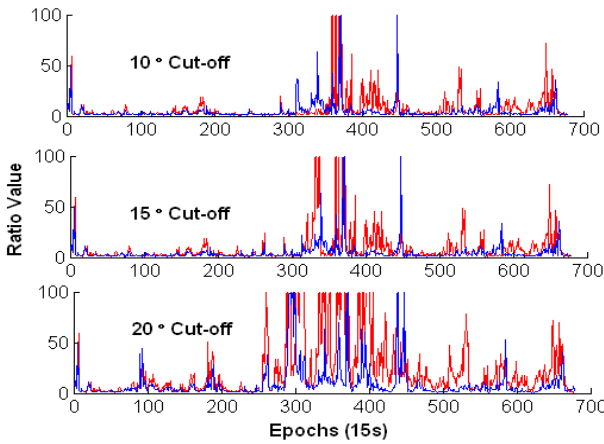


Fig. 7 F-Ratio values of single-base (blue line) and network-based (red line) techniques using various elevation cut-off angles in SydNET (SPWD-VILL)

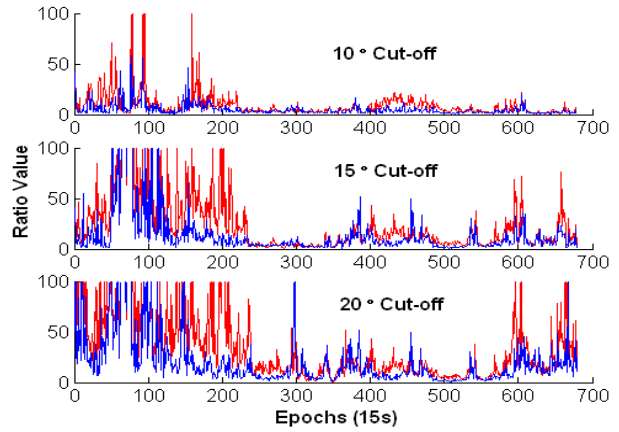


Fig. 8 F-Ratio values of single-base (blue line) and network-based (red line) techniques using various elevation cut-off angles in SIMRSN (LOYA-NYPC)

Further analysis is possible by checking the critical ratio value against the correct and wrong ambiguity results given in Tables 1 and 2. The analysis provides percentages for the ambiguities passed and were correctly accepted, passed but incorrectly rejected (type I error), failed and correctly rejected, failed but incorrectly accepted (type II error), as given in Tables 3 and 4 for SydNET and SIMRSN respectively. It is noted that the results of the network-based technique in both tables give higher percentages for correctly accepted ambiguity using the critical value, and lower percentages in making a type I error, compared to the single-base results. The same conclusion can be made for the correctly rejected wrong ambiguity and the type II error, except in the case of SydNET. Inspecting Table 1, this is only from the percentage calculation. It should be mentioned that the results differ only by applying the network correction or not. Hence, the network correction evidently strengthens the ambiguity resolution and the validation test.

Table 3 Statistics of ambiguity validation for SydNET

cut-off	Single-Based				Network-Based			
	Passed %		Failed %		Passed %		Failed %	
	Acc	Rjct	Acc	Rjct	Acc	Rjct	Acc	Rjct
10°	47.8	52.2	18.3	81.7	58.7	41.3	29.4	70.6
15°	47.5	52.5	19.4	80.6	61.7	38.3	28.5	71.5
20°	66.6	33.4	13.9	86.1	85.1	14.9	20.0	80.0

Table 4 Statistics of ambiguity validation for SIMRSN

Cut-off	Single-Based				Network-Based			
	Passed %		Failed %		Passed %		Failed %	
	Acc	Rjct	Acc	Rjct	Acc	Rjct	Acc	Rjct
10°	55.6	44.4	5.0	95.0	74.6	25.4	4.5	95.5
15°	82.1	17.9	0	100	90.6	9.4	0	100
20°	90.3	9.7	0	100	96.6	3.4	Nil	Nil

After removing the ambiguity biases, the DD L1 measurements are still contaminated by residual distance-dependent errors and station-dependent errors. These biases, together with geometry of the satellites, impact on the positioning results. Based on the fact that the user is static and is a part of the network stations, station-dependent errors such as multipath are assumed to be at a minimum level. During these tests, the geometry of the satellites for both stations was good, with geometric dilutions of precision (GDOP) less than 5 (see Figure 9).

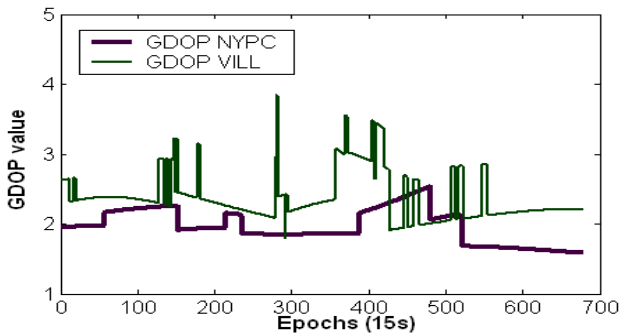


Fig. 9 GDOP values for VILL (SydNET) and NYPC (SIMRSN) during the tests

To reduce distance-dependent errors remaining in the measurements after the removal of the ambiguity biases, dispersive and non-dispersive corrections are applied. Figures 10 and 11 show the DD L1 residuals (for 10° cut-off elevation only) with and without applying the corrections for SydNET and SIMRSN respectively. It can be seen that the network corrections have reduced the magnitude of the residuals compared with the results without the corrections.

Figures 12 and 13 show the results of single-epoch positioning (with and without corrections) after differencing the known positions for VILL and NYPC respectively (for 10° cut-off elevation only). Their corresponding statistics are given in Tables 5 and 6 for each cut-off elevation on both stations. It can be observed from Figures 12 and 13 that the differences in Easting and Northing are at the centimetre level, while the height differences reach the decimetre level, mostly due to residual tropospheric biases.

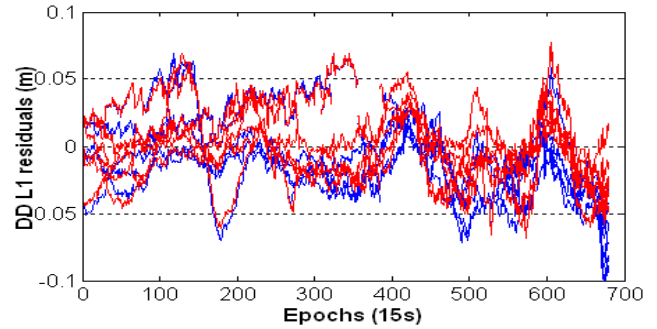


Fig. 10 DD L1 residuals for SPWD-VILL (SydNET), red is with correction and blue is without correction.

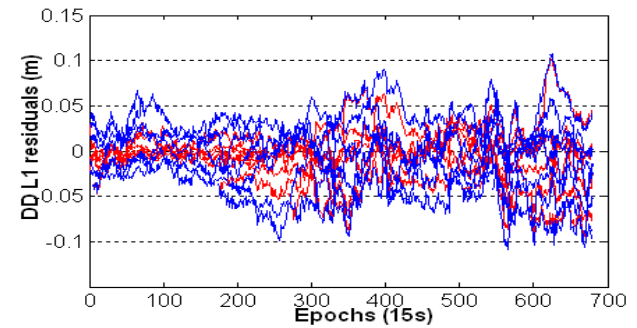


Fig. 11 DD L1 residuals for LOYA-NYPC (SIMRSN), red is with correction and blue is without correction

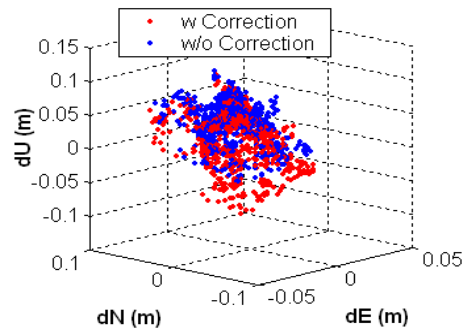


Fig. 12 Differences of calculated L1 positions compared to the known position VILL (SYDNET), red is position calculated with (w) correction and blue is without (w/o) correction

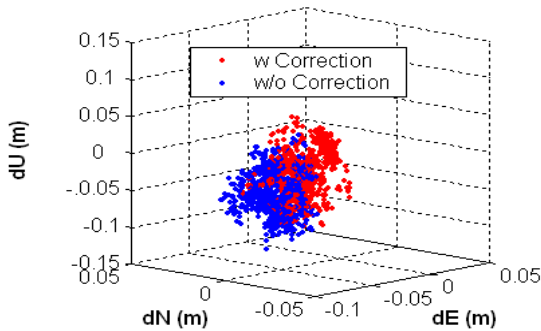


Fig. 13 Differences of calculated L1 positions compared to the known position of NYPC (SIMRSN), red is position calculated with correction and blue is without correction

From Tables 5 and 6 an improvement on the mean Up component (see column 5 of both tables) can be obtained once the corrections are applied. This result can be derived from the non-dispersive correction that reduces the residual tropospheric biases in the measurements. There are no significant differences found in the horizontal components in the case of VILL, but some improvements to the Easting component of NYPC is noticed. It is not clear why the large mean value on Easting component of NYPC were obtained. Perhaps it is because the known position is offset by the ‘true’ coordinate. Both examples do not indicate much deviation of the coordinate differences in Easting and Northing, however large a variation is noticed in the Up component despite applying the corrections (improvement up to 2.7cm in the case of NYPC at 20° cut-off elevation). In the case of station VILL (SydNET), the Up component variation increases slightly after the correction, but overall, the pattern is reasonable. It shows that applying the correction does not always guarantee better precision of the positioning results, especially in the Up component. It is dependent on the quality of the network corrections and other residual biases that still exist when performing the position computation.

Table 5 Position statistics for VILL (SydNET) with and without (w/o) corrections compared to known position

Cut-off	Corr	Mean (cm)			Deviation (cm)		
		dE	DN	dUp	dE	dN	dUp
10°	w/o	-1.5	-0.6	4.5	1.0	2.5	2.7
	With	-1.0	-0.2	1.8	1.2	2.7	3.9
15°	w/o	-1.5	-0.6	4.4	1.0	2.5	2.8
	With	-1.0	-0.1	1.3	1.1	2.8	3.8
20°	w/o	-1.2	-0.8	2.9	1.3	3.5	3.4
	With	-0.6	-0.6	-0.8	1.3	3.7	4.2

Table 6 Position statistics for NYPC (SIMRSN) with and without (w/o) corrections compared to known position

Cut-off	Corr	Mean (cm)			Deviation (cm)		
		dE	DN	dUp	dE	dN	dUp
10°	w/o	-4.7	0.5	-5.1	1.0	1.0	2.8
	With	-2.4	0.4	-2.8	1.3	0.7	2.8
15°	w/o	-4.5	0.4	-4.4	1.5	1.1	3.5
	With	-2.1	0.5	-1.8	1.8	0.8	2.5
20°	w/o	-4.1	0.4	-5.4	1.5	1.5	5.9
	With	-1.8	0.5	-1.8	1.7	0.9	3.2

## 5. Concluding Remarks

The ability to capture and model small-scale distance-dependent errors by the network GPS technique enables RTK ambiguity resolution even for longer inter-receiver distances. Information about these distance-dependent errors is included in the network corrections which can be separated into dispersive and non-dispersive components. This separation is useful for advancing network error modelling, and in order to provide more options for the network users’ processing strategy.

The dispersive effect that changes rapidly in time and space is modelled as frequently as possible. On the other hand, the slowly and smoothly varying non-dispersive effect is modelled less frequently than the dispersive effect. Furthermore, a running average is applied in order to smooth the non-dispersive correction. For the network user’s data processing, this study shows that the separation can be used to improve the IF measurements as well. Such improvement is important especially for real-time ambiguity resolution. The combination of dispersive and non-dispersive corrections is also useful for the user-side computation, if the high quality of both corrections can be assured.

Experiments with local GPS networks in two different geographical locations have demonstrated some advantages of the proposed strategy. Test results and analyses have shown that the proposed strategy performed reasonably well in generating the network correction, resolving the network ambiguities and computing the user’s position.

Since both cases show similar ambiguity levels their results can be directly compared. It must be noted that this processing strategy is only available if the user is provided with the measurements of the reference/master station and is able to recognise the network correction components.



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