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Early Detection of Breast Cancer Using Wave Elliptic Equation with High Performance Computing

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Abstract— **Breast cancer is the commonest female malignancy in Malaysia and all over the world. The incidence of breast cancer in Malaysia is estimated to be around 27 per 100,000 populations, with close to 3,000 new cases annually. The numerical solution is applied to solve a mathematical model in medicine field. The wave equation can be used as mathematical models in science and engineering fields especially for biological aspects of electromagnetic wave. This paper focuses on the implementation of parallel algorithm for the simulation of breast cancer tumor using two dimensional Helmholtz's wave equation on a distributed parallel computer system (DPCS). The numerical finite-difference method is chosen as a platform for discretizing the wave equations. The mathematical model of Helmholtz's model is used to visualize the growth of breast cancer. Parallel Virtual Machine (PVM) is emphasized as communication platform in parallel computer system. The performance of the parallel computing will be analyzed in terms of time execution, speed up, efficiency, effectiveness and temporal performance.**

Keywords- electromagnetic wave; breast cance; elliptic equation; finite-difference method; parallel algorithm

I. INTRODUCTION

Breast cancer happens when cells in the breast begin to grow out of control and spread throughout the body. Large collections of this out of control tissue are called tumors. Breast cancers found during screening examinations are more likely to be small and still confined to the breast. Finding a breast cancer as early as possible improves the likelihood that treatment will be successful.

In the research of early detection of breast cancer using wave equation on elliptic cylindrical coordinates with High Performing Computing, we considered mathematical modeling and simulation of breast cancer growth using two dimensional equations. The breast tumor mathematical model involves some parameters need to be counted on and obviously the problem need incalculable repetitive calculations on large amounts of data to give relevant and valid results. The Helmholtz's wave equation which being used on detecting the tumor cells growth and the biological aspects of electromagnetic waves, are used in solving the detection of breast cancer.

At first, the two dimensional elliptic equation will be solved sequentially using Gauss-Seidel and finite difference method. The method will then be transforms into Red Black Gauss-Seidel in order to parallelize the algorithm.

A. *Electromagnetic Concept's in Detection of Breast Cancer*

In today's technology, breast tumor detection can be done by using microwave imaging where, the dielectric properties of biological tissues are highly temperature dependent, which makes microwave imaging a promising method to control the effect during microwave hyperthermia treatment.

Gunnarsson T. (2007) has performed the approaches of microwave imaging using tomography methods where a cross-sectional slice of dielectric properties is generated. The major reason for this is that the potentially high dielectric contrast between cancerous tissues and normal breast tissues. When expose to microwaves, breast tumor exhibits electrical properties that are significantly different from a healthy breast tissues.

Besides that, when exposed to microwaves, the high water content of malignant breast tissues cause significantly microwave scattering than normal fatty breast tissues that have low water content.

II. MATHEMATICAL MODEL

Gunnarsson T. (2007) stated that a common wave equation is the scalar Helmholtz's equation describing the time harmonic electrical field in a situation, where the incidence field is a vertically polarized and the object properties is the homogenous along the vertical z-axis. The problem is transformed into a two dimensional problem, which can be defined as equation.

$$\left(\nabla^2 + k^2(r)\right)e(r) = 0 \tag{1}$$

where k, e(r), r, x, y is represent the wave number of the electromagnetic, total electric field, current in the electrical field, *x* direction of the space variable and *y* direction of the space variable respectively.

The Helmholtz's equation can then be transforms into matrix forms by using finite-different method. After applying the finite-difference approximation to the Helmholtz's equation, then

$$\left(\frac{r_{i+1,j} - 2r_{i,j} + r_{i-1,j}}{(\Delta x)^2} + \frac{r_{i,j+1} - 2r_{i,j} + r_{i,j-1}}{(\Delta y)^2} + \bar{k}^2\left(r_{i,j}\right)\right)e\left(r_{i,j}\right) = 0 \tag{2}$$

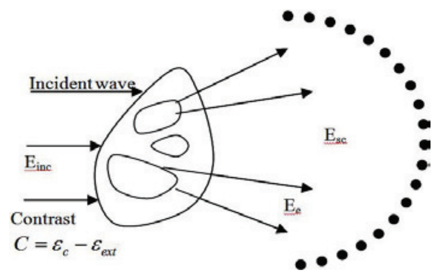

Figure 1 General Formulation of the Scattering Properties in Microwave Imaging

$$\left(\frac{r_{i+1,j}-2r_{i,j}+r_{i-1,j}}{h^2}+\frac{r_{i,j+1}-2r_{i,j}+r_{i,j-1}}{k^2}+\bar{k}^2\left(r_{i,j}\right)\right)e\left(r_{i,j}\right)=0 \tag{3}$$

where $\Delta x = h$ and $\Delta y = k$.

In [2], iterative process of calculating model results and making biological comparisons can continue to the point at which the model suggest appropriate experiments to clarify portions of the biological mechanism not yet understood and to make realistic prediction. For this research, iterative method of Black Gauss Seidel is used to solve the equation (1).

III. Red-Black Gauss Seidel Iterative Method

This research is focused on the two dimensional elliptic equation:

$$\frac{\partial^2 r}{\partial x^2}+\frac{\partial^2 r}{\partial y^2}=0 \qquad (4).$$

After some rearrangement on equation (3), we will have the finite-difference approximation equation as follows

$$r_{i,j+1}=\frac{1}{\theta}\left[(2+2\theta)r_{i,j}-r_{i+1,j}-r_{i-1,j}-\theta r_{i,j-1}\right] \tag{5}$$

with assuming $\theta=\frac{h^2}{k^2}$.

Red Black Gauss Seidel algorithm is used to implement the parallel algorithm in solving the finite difference equation. The iterative method contains 2-sub domain, Ω^R and Ω^B. There is a communication between Ω^R and Ω^B [3].

1) *i. Grid calculation at Ω^R:*

$$r_i^{(n+1)}=\frac{1}{\theta}\left[(2+2\theta)r_i^{(n)}-r_{i+1}^{(n)}-r_{i-1}^{(n)}\right]-b_i\ ,\ i=1,3,5,\ldots,m \qquad (6).$$

2) *ii. Grid calculation at Ω^B:*

$$r_i^{(n+1)}=\frac{1}{\theta}\left[(2+2\theta)r_i^{(n)}-r_{i+1}^{(n)}-r_{i-1}^{(n+1)}\right]-b_i, i=2,4,6,\ldots,m-1 \tag{7}$$

where $b_i = r_i^{(n-1)}$.

I. The Visualization of The Breast Cancer

When visualizing the existence of breast cancer cells using data that were computed by C programming, it shows that the cells were being affected by cancer when the current of electric field (the *r*-terms) Gunnarsson T. (2007) has performed the approaches of microwave imaging using tomography methods where a cross-sectional slice of dielectric properties is generated. The major reason for this is that the potentially high dielectric contrast between cancerous tissues and normal breast tissues. When expose to microwaves, breast tumor exhibits electrical properties that are significantly different from a healthy breast tissues. Besides that, the high water content of malignant breast tissues cause significantly microwave scattering than normal fatty breast tissues that have low water content.

If the edges of a shape are straight line segments, then a solution is integrable or knowable in closed-form only if it is expressible as a finite linear combination of plane waves that satisfy the boundary conditions (zero at the boundary, i.e., membrane clamped), Alfred Clebsch (1862). In Fig. 2, it shows that the graph has formed the lower half of and elliptic shape where the electric force approaching 0.6 volts as the wave space increases.

As addition, we also able to visualize the breast cancer growth by using software of COMSOL Multiphysics as shown in Fig. 3.

Small tumors are successfully detected even when a significant mismatch exists between the average normal breast-tissue dielectric properties assumed in the beam former design and the actual average dielectric properties of the breast being scanned.

I. Performance analysis

In the PVM implementation of the modeling codes there is a master task and there are number of worker tasks. The main job of master task is to divide the model domain into sub domains and distribute them to worker tasks. The worker tasks perform time marching and communicate after each time step. The method of Gauss Seidel then is transforms into Red Black Gauss-Seidel in order to parallelize the algorithm. The performance of the parallel algorithm will be analyzed in terms

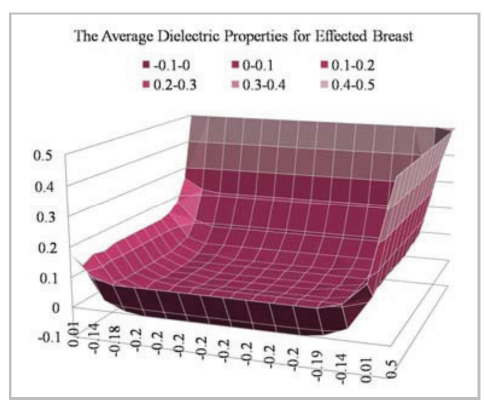

Figure 2 The Detection of Breast Cancer in Two Dimensional

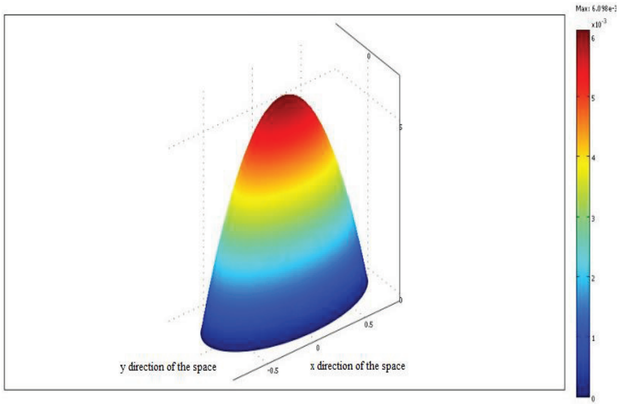

Figure 3 The Visualization of Breast Cancer in Two Dimensional

of the time execution, speed up, efficiency, effectiveness and temporal performance [4]. Fig. 4 shows the parallel algorithm.

A. The Execution Time versus Number of Processors

Execution time is the amount of time needed for a complete run of a computer program routine. The time required for a computer to decode and perform a compiled instruction. Fig. 5 shows that the execution time is decreasing with the increasing of the number of processors.

B. The Speed up versus Number of Processors

The Amdahl's law states that the speed of a program is the time to execute the program while speed up is defined as the time it takes to complete an algorithm with one processor divided by the time it takes to complete the same algorithm with N processors. The formula of speed up for a parallel application is given

$$\text{Speed up}(p) = \frac{\text{Time}(1)}{\text{Time}(p)} \quad (8)$$

where *Time*(1) is execution time for a single processor and *Time*(*p*) is execution time using *p* parallel processors.

Fig. 6 shows that the speed up increases when the number of processors is added. It is because the distributed memory hierarchy reduces the time consuming access to a cluster of workstations. According to Amdahl's Law, the speed up increases with the number of processors increase up to the certain level.

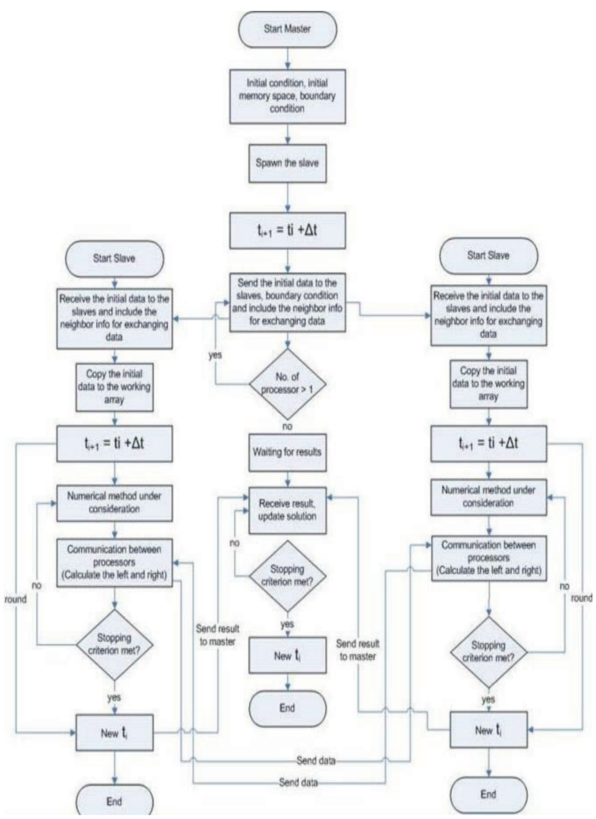

Figure 4 Parallel Algorithm

C. The Efficiency versus Number of Processors

The efficiency of a parallel program is a measure of processor utilization. Efficiency is defined as the speed up with N processors divided by the number of processors N. An efficiency of 100 percent means that all of the processors are being fully used all the time.

$$\text{Efficiency} = \frac{\text{Speed up}}{p} \quad (9)$$

where *p* is the number of processors.

Fig. 7 shows that the efficiency decreases with the increasing of number of processors. As known, efficiency is the ratio of speed up with number of processors. So, efficiency is a performance closely related to speed up. But in this case, it can be summarized as the system has reached its maximum efficiency at 12 number of processor

D. The Effectiveness versus Number of Processors

Effectiveness is used to calculate the speed up and the efficiency. The effectiveness is

$$\text{Effectiveness} = \frac{\text{Speed up}}{p.\text{Time}(t)} \quad (10).$$

Fig. 8 shows that the effectiveness is escalating with the increasing of the number of processors. The formula of the effectiveness is depending on the speed up, when the speed up increases, the effectiveness will also increase. In addition, the adding of the number of processors is to make the graph increasing.

E. The Temporal Performance versus Number of Processors

Temporal performance is a parameter to measure the performance of a parallel algorithm which is

$$\text{Temporal} = \frac{1}{\text{Time}(t)} \quad (11).$$

Fig. 9 shows that the temporal performance graph is proportional to the number of processors increase. It is because the execute time is decreasing versus the number of processors.

The analysis from the aspect of execution time, speed up, efficiency, effectiveness and temporal performance show that the performance of the parallel algorithm is improved by the increasing of the number of processors. Besides, we can also conclude that communication and execute times is always affecting the performance of parallel computing. Parallel computing is becoming more popular since that it is a low cost supercomputer and save a lot of time in solving a large problem. The Red Black Gauss Seidel which is effective is found to be well suited for parallel implementation on the PVM where data decomposition is run synchronously and concurrently at every time level.

As a conclusion, we had identified the two-dimensional elliptic equation can be applied in medical field. The elliptic equation is derived using the numerical finite difference method. Besides, it also able to visualize the breast cancer detection in two-dimensional space using wave elliptic equation.

The performance of the parallel computer had been analyzed using the graphs from the aspect of execution time, speed up, efficiency, effectiveness, and temporal performance. From the analysis, parallel processing can be concluded as a better price and performance tools in solving a mathematical problem. The parallel computing with PVM system is a well suite performance tools in solving the grand challenge of mathematical problem [5]. From the analysis, the parallel computing with PVM system can be concluded as a well suite performance tools in solving the grand challenge of mathematical problem. It can save a lot of execute time compare to the sequence processing.

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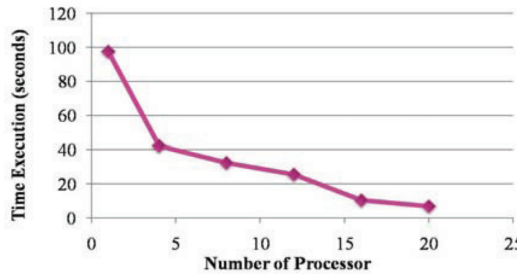

Figure 5 Time execution versus Number of Processors

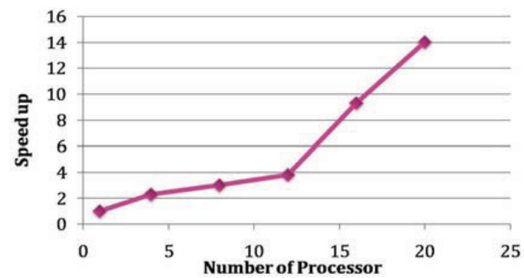

Figure 6 Speed up versus Number of Processors

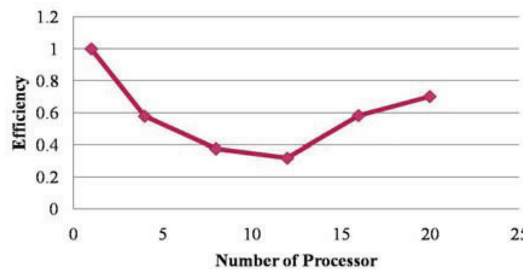

Figure 7 Efficiency versus Number of Processors

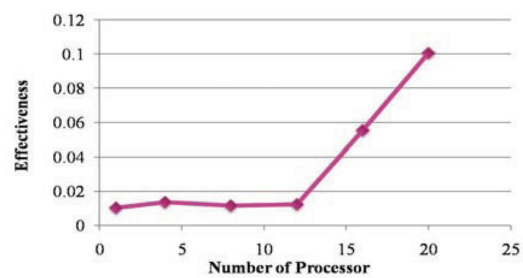

Figure 8 Effectiveness versus Number of Processors

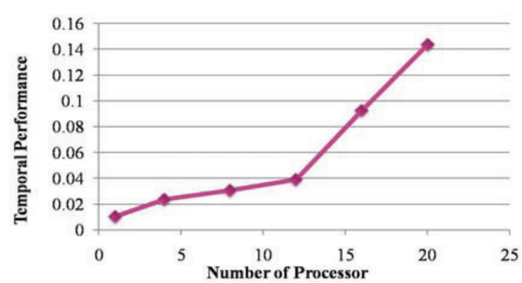

Figure 9 Temporal Performance versus Number of Processor