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# A Simulated Annealing Approach for Uncapacitated Continuous Location-Allocation Problem with Zone-Dependent Fixed Cost

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**Abstract** Location analysis is concerned with locating one or more service facilities while fulfilling some constraints such as the demand of the customers and minimizing the total cost. Despite the cost of transporting goods or services, there is a fixed cost associated with opening a given facility such as the cost of the land, taxes or trunking (or hauling) cost to supply product, services and labour. This cost may vary from one area to another. The aim of this study is to put forward a Simulated Annealing (SA) procedure for solving the uncapacitated continuous location-allocation problem in the presence of a zone-dependent fixed cost. Simulated Annealing is one of the meta-heuristic methods derived from the annealing process of a solid. Several parameters in SA will be tested such as initial starting points, initial temperature and cooling schedules. Data set for 50 customer problem taken from the literature is used. The problems of locating 2 to 15 facilities are solved by using C++. The computational results are presented with encouraging results.

**Keywords** Uncapacitated, location-alocation, zone-dependent, fixed cost, heuristics.

## 1 Introduction

Location problem is also known as a resource allocation problem. In general, one or more service facilities are going to be located in order to serve a distributed set of demand ("customer"). For example, location for routine services such as factories, warehouses, schools, etc, as well as for emergency services such as ambulances and fire stations.

Hsieha [1] says that the location-allocation (LA) problem is to select the locations of a number of supply centers to serve customers, and to decide the corresponding allocation of the customers (demand centers) with known demands to supply centers with the given optimization criterion.

Location science has been developed in three main areas [2]. The first one is called the continuous models which allow facility locations to be anywhere on the plane or a subset of the plane (there is an infinite number of possible sites). The second type is known as discrete models where the possible locations of the sources are fixed (only finite number of known sites which are feasible). The third model of location problems is based on graph and network theory. However, this paper will only concentrate on developing an algorithm of the continuous case. The location-allocation (LA) problem involves locating either single facility or multiple (two or more) facility. Multiple-facility location is conceptually and mathematically more complex [3]. The area is divided between several facilities and the problem becomes a combination of simultaneously finding both the proper allocation of customers to facilities and the best location for the facilities.

The main subject when dealing with location-allocation problem is the cost. Despite the cost of transporting goods or services, there is another cost, known as fixed cost, involves in solving LA problems. Fixed cost is a cost associated with opening a given facility which may vary from one area to another. A research done by Brimberg and Salhi on Continuous Location-Allocation Problem does incorporate zone-dependent fixed cost [4]. They produce an optimal method for solving the single facility Weber problem in the presence of a zone-dependent fixed cost. That method forms the basis for the heuristic proposed in this work. Simulated Annealing (SA) is a methodology used to approximate the solution of an NP problem. By selectively choosing state configurations (approximate solutions) and evaluating them against a user-defined cost function, a near optimal solution to the problem can be obtained. Like other approximation algorithms, there is no guarantee that the solution is optimal but applications of SA in various fields have shown it to be very accurate [5].

## 2 Problem Formulation

The uncapacitated multi-facility continuous location-allocation problem is also known as Multisource Weber problem. This problem provides one of the basic models in location theory [6]. Because there is no capacity constraints on the facilities, an optimal solution will have the demand at each customer served by the facility that is the closest to it (ties being broken arbitrary) [7]. In this work, we are interested in finding the location of M facilities in continuous space in order to serve customers at n fixed points as well as the allocation of each customer to the facilities so that the total cost of opening sites which may vary from one area to another and transportation goods/services to customers is minimized. We assume there is no restriction on the capacity of the facilities. The continuous fixed-charge location-allocation problem may be formulated as:

$$\text{Minimize} \sum_{i=1}^{M} \sum_{j=1}^{n} x_{ij}\, d\,(\,X_i,\ a_j\,) + \sum_{i=1}^{M} f\,(X_i) \tag{1}$$

subject to

$$\sum_{i=1}^{M} x_{ij} = w_j \quad \forall\, j = 1, ..., n \tag{2}$$

$$x_{ij} \geq 0, \quad \forall\, i = 1, ..., M;\ \ j = 1, ..., n \tag{3}$$

where $d\,(\,X_i,\ a_j\,)$ represents distance between facility $i$ and customer $j$ and $f\,(X_i)$ represents the fixed cost for facility $i$. (1) denotes the objective function which is the total cost, (2) guarantees that the demand of every customer is satisfied and (3) refers to non-negativity of the decision variables. We shall describe Algorithm 1 introduced by Brimberg and Salhi [4] as this will be used to solve the zone-dependent single facility location problem in our SA heuristic.

### 2.1 Algorithm 1

This algorithm is used to determine a location $X$ of a new facility in order to minimize the transportation cost and the zone-dependent fixed cost. The area is divided into zones which are all rectangular in shape. The distance between the customer points to the facility point is calculated using rectangular distance. The customers are assumed to be homogeneous so that the weights may all be taken as one. Below is the procedure to solve the zone-dependent single facility location problem.

**Step 1** Solve the single-facility minisum problem to obtain $X_M^*$.

If $X_M^*$ belongs to a zone with the smallest fixed cost, stop $(X^* = X_M^*)$;

else set the current solution $X_c = X_M^*, LIST = \{\}, r =$ index of the zone containing $X_M^*$ (set $r = K + 1$, if $X_M^* \notin \bigcup_{k=1}^{K} P_k$), and proceed to Step 2.

**Step 2** {Process candidate polygons}

For each $P_k, k = r$, do the following:

If $f_k < f(X_M^*)$, determine the visible boundary, $E_k' \subset E_k$, and store $E_k'$ in $LIST$.

**Step 3** {Solve candidate polygons}

Repeat for each $E_k' \in LIST$ until $LIST = \{\}$ :

Show by comparing fixed costs of adjoining zones that $E_k'$ can be eliminated.

If $w(X_k^*) + f_k < Z(X_c)$, set $X_c = X_k^*$. Set $LIST = LIST - E_k'$.

### 2.2 Simulated Annealing Algorithm

Below is the procedure of a basic SA heuristic that is used in this study.

**Step 1** ***Select setting for the heuristic parameters and generate an initial solution and its cost. This solution is defined as the current solution***

Initial points are randomly chosen to be the location of the facilities. The number of initial points chosen depends on the number of facilities. For example, if the facility is 5 ($M = 5$), we randomly choose 5 points. Then, each customer is allocated to their nearest facility. This will give us five set of allocations. Find the cost of these allocations. Then, find the optimal location for each region by using Algorithm 1. Calculate the new cost for each region. Find the total cost. Set the total cost as $F(x)$. Initialize the SA parameters:

Select an initial temperature $T_0 = 1000°C$.
$T_{k+1} = T_k/2$.
Repetition counter , $n = 1$.
Iteration $= 1$ and $k = 0$.
Best solution $x* = x, f(x*) = f(x)$.

**Step 2** ***Obtain a neighbouring solution of the current solution using a local search technique***

For each allocation, find the neighbouring fixed points of the facility which are defined as the fixed points that lie within a certain radius from the facilities taken to be $r_i = d(i,j)/2$ where $d(i,j)$ is the distance between facility $i$ and its furthest allocated customer. If there are more than one neighbouring points then moves the facility randomly to one of the points. However, if there is no neighbouring point then the current facility will be kept at its current location.

**Step 3** ***Obtain the cost of the neighbouring solution and compare it to the cost of the current solution***

Then allocate the customers to the new point and find the cost for this allocation. Apply Algorithm 1 to the new allocation to find the optimal cost for each set. Calculate the total cost. Set the cost as $F(x')$.

If $F(x') < F(x)$
    Let $F(x) = F(x')$,
Else
    Check the probability acceptance $P(\delta)$.
    If random number $< P(\delta)$
        Accept this move
        Go to Step 2.
    Else
        Reject this move
        Check another neighbourhood point of the current solution.

**Step 4** ***Update counters and parameters***

$n = n + 1$
If $n > 4$, update $k = k + 1$
    $T_{k+1} = T_k/2.$
    $n = 1$
iteration = iteration +1

**Step 5** ***Repeat steps 2-4 until the number of rejected solution equals to 3 (stopping criterion is met)***

## 3 Computational Results

The proposed heuristics are written in C++. For testing purposes, we used the 50-fixed point test problem given in the literature [8]. The algorithm is applied to the test problems to solve for 2 to 15 open facilities. We will start the SA procedure from different initial points in order to explore different search area. Some changes will also be made to other SA parameters namely the initial temperature and cooling schedule, in order to see the effect of these parameter values to the final solution. The results in term of the number of

Table 1: The Results for Various Initial Solution

| **Number of facility, $M$** | **Initial Solution 1** | | **Initial Solution 2** | | **Initial Solution 3** | | **Initial Solution 4** | |
|---|---|---|---|---|---|---|---|---|
| | Iteration | Cost | Iteration | Cost | Iteration | Cost | Iteration | Cost |
| 2 | 29 | 196.83 | 26 | 196.83 | 33 | 197.35 | 26 | 196.85 |
| 3 | 40 | 174.72 | 38 | 174.31 | 30 | 174.31 | 34 | 174.53 |
| 4 | 34 | 161.28 | 33 | 161.07 | 33 | 160.14 | 34 | 160.45 |
| 5 | 38 | 154.98 | 36 | 159.02 | **29** | **153.39** | 36 | 153.55 |
| 6 | 39 | 163.62 | 39 | 165.63 | 31 | 166.62 | 38 | 160.01 |
| 7 | 36 | 174.27 | **45** | **160.04** | 42 | 161.05 | 36 | 167.74 |
| 8 | **49** | **167.85** | 41 | 174.79 | 39 | 173.01 | 39 | 172.48 |
| 9 | 45 | 180.70 | 44 | 175.56 | 39 | 180.15 | **42** | **174.57** |
| 10 | 40 | 185.32 | **38** | **181.54** | 43 | 184.83 | 43 | 182.69 |
| 11 | 43 | 187.17 | **43** | **186.37** | 46 | 188.15 | 39 | 191.74 |
| 12 | **41** | **195.11** | 38 | 195.88 | 46 | 198.36 | 40 | 200.53 |
| 13 | 41 | 210.56 | 38 | 208.98 | 49 | 203.76 | 50 | 210.40 |
| 14 | **43** | **215.11** | 49 | 224.33 | 39 | 219.99 | 43 | 215.29 |
| 15 | 41 | 221.88 | 41 | 218.98 | **36** | **215.92** | 52 | 236.87 |

Table 2: The Results for Various Initial Temperature

| **Number of facility, $M$** | $T_0 = 1000$ | | $T_0 = 500$ | | $T_0 = 250$ | |
|---|---|---|---|---|---|---|
| | Iteration | Cost | Iteration | Cost | Iteration | Cost |
| 2 | 31 | 196.78 | 25 | 196.78 | 22 | 196.78 |
| 3 | 34 | 174.53 | 24 | 174.53 | 18 | 174.31 |
| 4 | 29 | 158.93 | 29 | 158.93 | 23 | 158.93 |
| 5 | 34 | 154.91 | 33 | 154.37 | 29 | 153.41 |
| 6 | 38 | 165.27 | 37 | 164.23 | **29** | **156.43** |
| 7 | 42 | 163.54 | 35 | 166.76 | 30 | 167.70 |
| 8 | 33 | 170.36 | 31 | 169.46 | 26 | 173.29 |
| 9 | 45 | 175.93 | 33 | 179.82 | 27 | 177.11 |
| 10 | 43 | 182.56 | 36 | 182.70 | 25 | 182.70 |
| 11 | 45 | 189.88 | 37 | 189.62 | 33 | 190.04 |
| 12 | 47 | 196.60 | 35 | 196.63 | 33 | 197.00 |
| 13 | 43 | 203.47 | 43 | 203.39 | 32 | 203.52 |
| 14 | 42 | 221.27 | 39 | 221.27 | 25 | 221.21 |
| 15 | 42 | 219.62 | 34 | 219.62 | 33 | 219.62 |

Table 3: The Results for Various Cooling Schedule

| Number of facility, $M$ | $T_{k+1} = 0.25 * T_k$ | | $T_{k+1} = 0.5 * T_k$ | | $T_{k+1} = T_k - 200$ | |
|---|---|---|---|---|---|---|
| | Iteration | Cost | Iteration | Cost | Iteration | Cost |
| 2 | 19 | 196.78 | 31 | 196.78 | 24 | 196.78 |
| 3 | 21 | 175.63 | 34 | 174.53 | 28 | 175.63 |
| 4 | 23 | 158.93 | 29 | 158.93 | 26 | 158.93 |
| 5 | 19 | 153.76 | 34 | 154.91 | 26 | 153.65 |
| 6 | 23 | 164.24 | 38 | 165.27 | 28 | 163.46 |
| 7 | 22 | 166.01 | 42 | 163.54 | 25 | 166.34 |
| 8 | 23 | 169.86 | 33 | 170.36 | 28 | 169.87 |
| 9 | 26 | 177.52 | 45 | 175.93 | 27 | 175.72 |
| 10 | 26 | 182.70 | 43 | 182.56 | 27 | 182.56 |
| 11 | 23 | 190.47 | 45 | 189.88 | 25 | 190.33 |
| 12 | 27 | 196.35 | 47 | 196.60 | 27 | 196.82 |
| 13 | **28** | **203.39** | 43 | 203.47 | 28 | 203.52 |
| 14 | 27 | 221.27 | 42 | 221.27 | 24 | 221.27 |
| 15 | 26 | 219.72 | 42 | 219.62 | 24 | 219.62 |

iterations taken and the cost for various SA parameters are computed and given in Table 1 to Table 3.

From our limited experiments, it can be seen in Table 1 that for small problem, $M \leq 5$, changes in the initial starting points converge to more or less the same solution. On the other hand, for $6 \leq M \leq 15$, changes in the initial starting point do affect the final results. Better result may be obtained from other initial solution.

From Table 2, for $M \leq 5$, no significant difference can be seen in term of solution quality when we change the initial temperature. However, the changes in initial temperature do affect the number of iteration taken to obtain the best solution.

From our observation, for small problem, $M \leq 5$, it is better to start from a low temperature because it requires less iteration to get to the best solution. It can also be seen that, for $6 \leq M \leq 15$, the changes in initial temperature do give significant difference in solution quality and computational effort. Hence, it is important to choose the right initial temperature and cooling schedule in order not to miss the best solution.

It can also be seen from Table 3, for $M \leq 5$ and $10 \leq M \leq 15$, even though the solution quality is more or less the same, the best solution can be reached faster when the temperature is decreased rapidly. However, for $6 \leq M \leq 9$, the changes in cooling schedule do give a difference in solution quality and also computational effort.

## 4 Conclusion and Suggestion

A Simulated Annealing heuristic is proposed to solve the uncapacitated continuous location-allocation problem with zone-dependent fixed cost which appears to have been scarcely investigated in the past. The problem is solved using different starting locations, initial temperature and cooling schedule. The effect of the parameter values to the solution quality and computational effort varies depending on the problem size. There are several suggestions for future research. One of them is the move. Instead of changing the location for all the facilities, only consider changing the location of a subset of the facilities. For example, only consider changing the location of the facility with small number of customers allocated to it. This is because opening a facility with few customers is not profitable. Another possible approach would be to develop algorithm base on other meta-heuristic such as Tabu Search (TS) or Genetics Algorithm (GA). The proposed heuristic can also be applied to solve capacitated continuous location-allocation problem with zone-dependent fixed cost.

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