# Volterra Algorithm for Modelling Sea Surface Current Circulation from Satellite Altimetry Data

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**Abstract.** This paper was utilized a new approach for modelling sea surface current from JASON-1 satellite altimetry data. This was based on utilizing of the Volterra series expansion in order to transform the time series satellite altimetry data into a real ocean surface current. Thus,the basic equation of hydrodynamic has been solved by second order Volterra model. Then, the Volterra kernel inversion used to obtain the sea surface current velocity. The finite element model of Lax-Wendorff schemes used which was based on triangular space-time elements to map the spatial current variation in the South China Sea over different monsoon periods. In situ sea surface current measurements were collected along the east coast of peninsular Malaysia by using electromagnetic current meters. The study shows that the maximum current magnitude of 1.2 m/s was occurred during the northeast monsoon period as compared to other monsoon periods. The main noticeable feature was an existence of anticlockwise eddy in the Gulf of Thailand. The results also shows a good correlation between in situ current measurements and the Volterra-Lax-Wendrof scheme with high $R^2$ of 0.91. It can be said that Volterra-Lax-Wendrof scheme can be used as numerical scheme for modelling sea surface current from altimetry data.

**Keywords:** JASON-1 satellite altimetry data, Volterra model, Lax-Wendorff schemes, Finite element model, Sea surface current.

## 1 Introduction

Satellite altimetry systems have been recognized as an excellent tool for coastal hydrodynmic investigations [8]. Indeed, satellite altimeters provide long-term and continuous coverage of wave and wind fields of the world oceans. One of the first satellites that used altimetry technology in the measurement of sea levels was the Seasat 1978-64A. In the 1990s with more complex and advanced imaging and altimetry systems, radar remote sensing has become more significant tool and has been

O. Gervasi et al. (Eds.): ICCSA 2008, Part II, LNCS 5073, pp. 119–128, 2008.

utilized by the ERS- 1, Geos-3 and TOPEX/Poseidon satellites. In 1992 TOPEX/Poseidon satellite altimetry mission was launched and its mission was ended in 2002. Since then it was replaced by Jason-1. Both satellite missions provide the most precise altimetry data when compared to others. Although satellite altimetry records are still quite short as compared to the tide gauge data sets, this technique appears quite promising for sea level change problem. In fact, it provides sea level measurement with large coverage. A precision of about 1 mm/year of measurement global change can be obtained. At present, few studies have been utilized different altimeter sensors for the sea surface investigations in the South China Sea [8]. Hu et al. [4] investigated the sea surface height with a period of 3-6 months using six years TOPEX/POSEIDON altimeter data. They reported that the sea surface height variations are associated strongly with current and eddies features. Imawaki et al. [3] introduces a new technique for acquiring high-resolution mean surface velocity by combined use of TOPEX/POSEIDON and ERS-1/2 altimeter data and drifter data obtained from 1992 through 2001 to detect the undulation of mean-surface dynamic topography down to the oceanic current. Indeed, the ocean current associated with geoid by the amount of sea surface dynamic topography is different compared to the mean–surface height acquired from satellite altimeter**.** In order to improve the hydrodynamic equation solutions of the mean sea-surface to invert accurate pattern of sea surface current we need to obtain altimeter data that is both more accurately and more widely distributed in time. Furthermore, we need to improve our knowledge of the long wavelength geoid so that its errors are substantially smaller than the corresponding values of the sea surface topography. The main contributions of this work is to design numerical scheme to reduce the impact of the Coriolis parameter in estimating sea surface current. As a matter of fact, at low latitude zone such as Malaysia where the existance of weak Coriolis parameter which leads to weak geostrophic current. According to this prospective, this study attempt to implement the second order of the Volterra model and Lax-Wendroff scheme to suppress the numerical solution of geostrophic continuity equation.

## 2 Data and Study Area

The study area is covered the South China Sea (SCS) where it is considered as an equatorial, semi-enclosed sea with a complex topography that includes large shallow regions. According to Wyrtki [9],the SCS is located between the Asian continent, Borneo, the Philippines, and Taiwan (Fig. 1 ). The northeastern part adjoins a deep sea basin, while the southern part is a shelf sea with depths less than 200 m. The SCS has two features that have very important and interesting effects on the general circulation: (1) the Coriolis force decreases to zero at the equator where both nonlinear and frictional forces become very important; and (2) the monsoon regime exerts a strong effect on the SCS circulation. Neither of the above phenomena are studied in coastal regions of the world ocean at the present time [9].

In situ sea surface current measurements were acquired on October 6 – 12 2003, April 6-12 2004, and March 1-8 2005, respectively during JASON-1 satellite altimetry overpass. The Merged Sea Level Anomaly (MSLA) data were acquired from JASON-1 satellite altimetry between 102° E to 114° E and 1 ° N to 10° N. The in

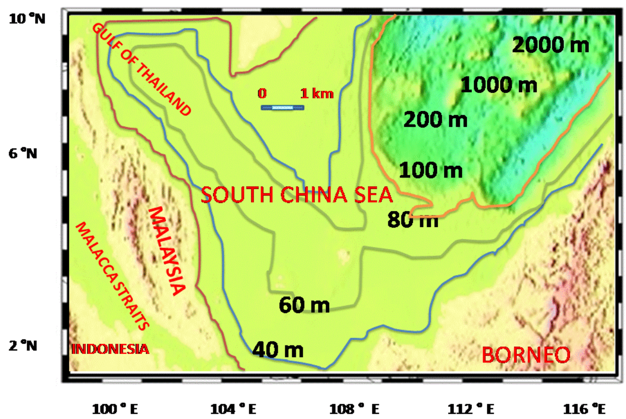


**Fig. 1.** Location of the South China Sea

situ sea surface current measurements howvever were collected between 102° 5'E to 105° 10'E and 2° 5'N to 6° 10'N (Fig. 2) using Valeport electro-magnetic current meter. In this context, the current meter was lowered down from the sea surface to water depth of 50 m.

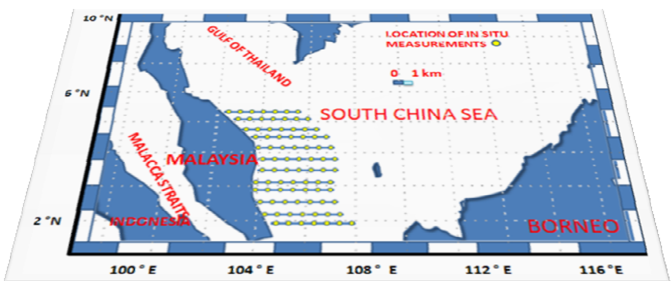


**Fig. 2.** Location of in situ measurements along east coast of Malaysia

## 3 Model

### 3.1 Volterra Model

According to Imawaki et al. [3] the instantaneous surface geostrophic velocity $U_g$ can be obtained from temporal mean velocity $\bar{U}(x,t)$ and anomaly of sea-surface geostrophic velocity $\bar{U}(x,t) = (u', v')$ by following formula:

$$U_g(x,t) = \bar{U}(x) + \bar{U}(u',v') \tag{1}$$

where $(u',v')$ are the sea surface geostrophic components in x and y directions which are estimated from the anomaly $\xi$ of sea surface dynamic topography which practically equivalent to the sea-surface height anomaly $\zeta_s$.

The Volterra model was used to express the geostrophic current velocity which were calculated from JASON-1 satellite altimetry as a series of nonlinear filters on the ocean surface current. This means that the Volterra model can be used to study the geostrophic current energy variation as a function of parameters such as the current direction, or the current waveform. A generalized, nonparametric framework to describe the input–output *x* and *y* geostrophic current components relation of a time-invariant nonlinear system. In discrete form, the Volterra series for input, $X(n)$, and output, $Y(n)$ as given by Inglada and Garello [2] can be expressed as:

$$\begin{aligned} Y(n) = h_0 + \sum_{i_1=1}^{\infty} h_1(i_1)X(n-i_1) + \sum_{i_1=1}^{\infty}\sum_{i_2=1}^{\infty} h_2(i_1,i_2)X(n-i_1)X(n-i_2) + \\ \sum_{i_1=1}^{\infty}\sum_{i_2=1}^{\infty}\sum_{i_3=1}^{\infty} h_3(i_1,i_2,i_3)X(n-i_1)X(n-i_2)X(n-i_3) + .........+ \\ \sum_{i_1=1}^{\infty}\sum_{i_2=1}^{\infty}............\sum_{i_k=1}^{\infty} h_k(i_1,i_2,.........i_k)X(n-i_1)X(n-i_2)...........X(n-i_k) \end{aligned} \tag{2}$$

where, $n$, $i_1$ , $i_2$ ,...,$i_k$ , are discrete time lags. The function $h_k(i_1 ,i_2 ,...,i_k)$ is the $k$th-order Volterra kernel characterizing the system. The $h_1$ is the kernel of the first order Volterra functional, which performs a linear operation on the input and $h_2$ , $h_3$ ,...,$h_k$ capture the nonlinear interactions between input and output AVISO sea level variation. The order of the non-linearity is the highest effective order of the multiple summations in the functional series. It might be the following Volterra model can be used to express the instantaneous surface geostrophic velocity $U_g$ as follows;

$$U_g(x,t) = N(\xi,\zeta_s) + \sum_{i=1}^{\infty}\int_R h_i(\bar{\tau},\xi)\prod_{j=1}^{i}\bar{U}_a(u'-\tau_j)(v'-\tau_i)d\tau \tag{3}$$

where $N(\xi,\zeta_s)$ is the geoid height which is estimated from sea surface dynamic topography $\xi$ and the sea-surface height anomaly $\zeta_s$ as described by Imawaki et al [3]. Following, Inglada and Garello (1999), the mathematical expressions for second –order Volterra kernels ($D_{2xx}$ and $D_{2yy}$ of instantaneous surface geostrophic velocity $U_g$ are as follows;

$$D_{2xx}(f_{1x},f_{1y},f_x-f_{1x},f_y-f_{1y})=\left[\xi_x\frac{\partial D_{1x}(f_{1x},f_{1y})}{\partial\zeta}(f-f_1)-\xi_x\bar{f}+\frac{\partial N(\vec{\xi})}{\partial\vec{\xi}}.(\bar{U}_{t_0}-\beta(\frac{\partial\zeta_s}{\partial y}-\frac{\partial\xi}{\partial y})\right]\times \beta\frac{\partial\xi}{\partial y}\bar{f} \tag{4}$$

$$D_{2yy}(f_{1x},f_{1y},f_x-f_{1x},f_y-f_{1y})=\left[\xi_y\frac{\partial D_{1y}(f_{1x},f_{1y})}{\partial\zeta}(f-f_1)-\left[\xi_y\bar{f}+\frac{\partial N(\vec{\xi})}{\partial\vec{\xi}}.(\bar{U}_{t_0}-\beta(\frac{\partial\zeta_s}{\partial x}-\frac{\partial\xi}{\partial x})\right]\right]\times \beta\frac{\partial\xi}{\partial x}\bar{f} \tag{5}$$

where $f_x, f_y, \bar{f}$ are the frequency domain in x and y dimensions while $\bar{f}$ is average frequency domain. Furthermore, $\beta$ is the ratio of gravity acceleration to the Coriolis parameter expressing the effect rotation of the earth. In order to estimate the sea surface current from the altimeter data, we assume that there is a linear relationship between 2-D Fourier transform of mean surface slope variations obtained from JASON-1 satellite altimetry data ($F(f_x, f_y)$) and the second –order Volterra kernels $D_{2xx}$ and $D_{2yy}$ as follows:

$$F(f_x,f_y)=U_x(f_x,f_y).D_{2xx}(f_x,f_y) \tag{6}$$

$$F(f_x,f_y)=U_y(f_x,f_y).D_{2yy}(f_x,f_y) \tag{7}$$

Equations 6 and 5 are used to estimate the current at *x* and y directions where the inverse Volterra model is used to estimate the current flow as described by Inglada and Garello [2] by

$$D(f_x,f_y)=\begin{cases}\dfrac{1}{D_{2xx}(f_{1x},f_{1y},f_x-f_{1x},f_y-f_{1y})}\\[2ex]\dfrac{1}{D_{2yy}(f_{1x},f_{1y},f_x-f_{1x},f_y-f_{1y})}\end{cases} \tag{8}$$

The current direction can be obtained by using the traditional tan formula as follows:

$$\theta = \tan^{-1}(\frac{U_y}{U_x}) \quad (9)$$

### 3.2 Lax-Wendrof Scheme

The second-order accurate dispersive Lax-Wendrof scheme is used for sea surface current flow in JASON-1 altimeter data which can be written in predictor–corrector form in a staggered grid. The staggered grid consists of a primary grid where points are labeled with *(i, j, k)* and a dual grid where points are labeled with *(i + 12,j + 12, k + 12)* . In 2D, the centers of the cell edges in the primary grid are given by *(i + 12, j)*, etc. Following Liska and Wendroff [5], 2D, numerical flow for the two-step forms of Lax-Wendrof scheme is

$$U_{i,j}^{n+1} = U_{i,j}^{n} + \frac{\Delta t}{2\Delta x}(v(U^{n+1/2}{}_{i+1/2,j+1/2}) + v(U^{n+1/2}{}_{i+1/2,j-1/2}) - v(U^{n+1/2}{}_{i-1/2,j+1/2}) - v(U^{n+1/2}{}_{i-1/2,j-1/2})) + \frac{\Delta t}{\Delta y}(G(U^{n+1/2}{}_{i+1/2,j+1/2}) + G(U^{n+1/2}{}_{i-1/2,j+1/2}) - G(U^{n+1/2}{}_{i+1/2,j-1/2}) - G(U^{n+1/2}{}_{i-1/2,j-1/2})) \quad (10)$$

where *v* and *G* are smooth functions as described by Liska and Wendroff [5]. The values at the center of all faces of the primary cell on time level $n + 1/4$ are computed using the analog of 2D predictor by

$$U_{i,j+1/2,k+1/2}{}^{n+1/4} = \frac{1}{4}(U^n{}_{i,j,k} + U^n{}_{i,j+1,k} + U^n{}_{i,j,k+1} + U^n{}_{i,j+1,k+1}) + \frac{\Delta t}{4\Delta y}(G(U^{n+1/6}{}_{i,j,k+1/2})) \quad (11)$$

The finite difference scheme model was applied with traditionally rectangular grids due to most their simplicity. Model numerical solutions was enhanced by using the curvilinear nearly orthogonal, coastline-following grids. This technique was implemented to overcome the problems raised due to complicated SCS topography, and boundary conditions.

## 4 Results and Discussion

The geostrophic current patterns have been modeled by Volterra- Lax-Wendrof Scheme are shown in Figs 3, 4 and 5. Fig. 3 shows the typical water circulation during the inter-monsoon period of October 2003 of the South China Sea. In October 2003, non fixed current patterns basically was dominated at the northern and center parts of the South China Sea. The maximum current magnitude was 1.0 m/s was found in the northern and eastern coast of Malaysia. The minim current velocity was 0.4 m/s and observed in the Gulf of Thailand. It is interested to find that the stream current pattern was a dominant feature in Malacca Straits with current magnitude of 0.8 m/s. The east coast of Malaysia was dominated by southwards current flow which moving parallel to coastline. Part of these currents was turned south-east and moving parallel to Borneo coastline with current velocity of 1.0 m/s.

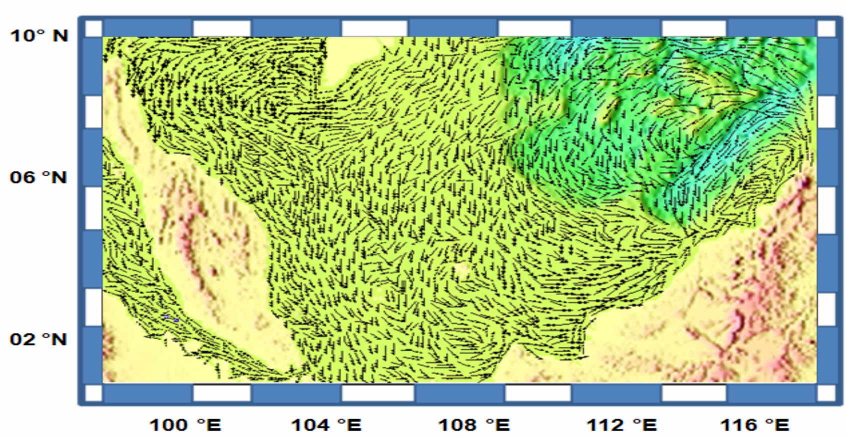


**Fig. 3.** Sea Surface Current Simulated from Volterra-Lax- Wendrof Scheme During October 2003

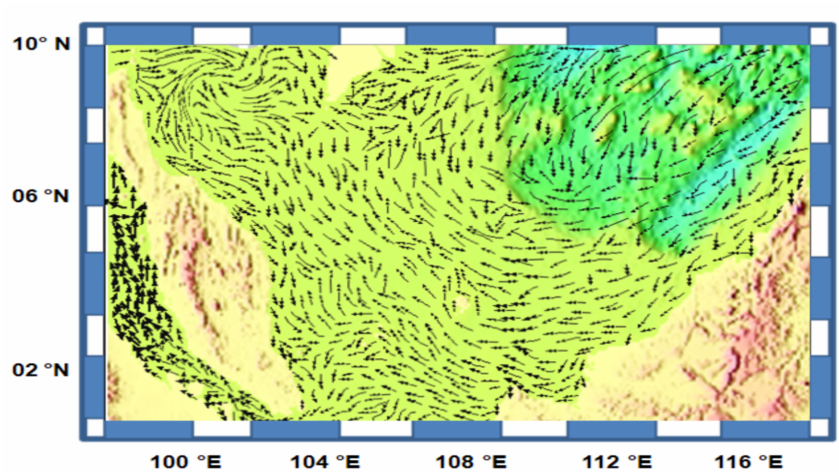


**Fig. 4.** Sea Surface Current Simulated from Volterra-Lax- Wendrof Scheme During April 2004

In April 2004, there was weak current flow along the east coast of Malaysia with magnitude 0.3 m/s. This current was moved towards southward direction. In Gulf of Thailand there were two eddies were moved opposite to each other with maximum velocity of 0.3 m/s. The southern eddy was moved anticlockwise while the north eddy was moved clockwise. It is obvious that there was southward current flow along the Borneo coastline with current velocity of 0.5 m/s. This current was turned to north and formed small scale eddy in the southern part of Vietnam coastline. The results confirm the study of Wryki [9], Maged [6] and Alejandro [1] . In March 2005, the current magnitude is increased and generate an eddy in the center of the South China sea. The maximum current magnitude was 1.2 m/s. the dominate current direction was northward. The Gulf of Thailand dominated by anticlockwise eddy with maximum velocity of 1.0 m/s.

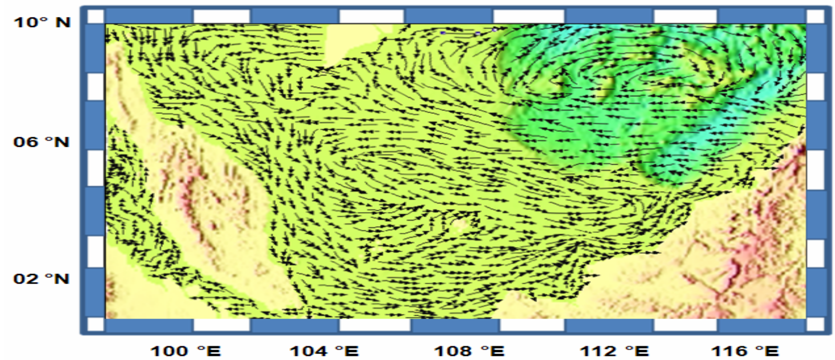


**Fig. 5.** Sea Surface Current Simulated from Volterra-Lax- Wendrof Scheme During March 2005

Fig. 5 shows the regression relationship between average in situ current measurements and the simulated current flow from Volterra- Lax-Wendrof Scheme. The scatter points are more closed to the regression line with $R^2$ value of 0.91 and RMSE of ±0.4 m/s which are indicated high correlation between in situ measured current and simulated one from Volterra-Lax-Wendrof Scheme (Table 1).

**Table 1.** Satistical Summary

| **Statistical Parameters** | **Values** |
|---|---|
| $r^2$ | 0.91 |
| $p$ | 0.002 |
| RMSE | ± 0.4 m |

The Volterra algorithm was shown an overcome of the Coriolis force problem. It does mean that the Volterra algorithm has provided solution for low geostrophic current velocity always appears along Malaysian coastal water due to weakness of Coriolis parameter involves in continuity equation. In addition, Lax-Wendrof scheme can handle mixed sub and super-critical current flows directly, with no regard for directional nature of the computation (left to right or vice versa) [5].

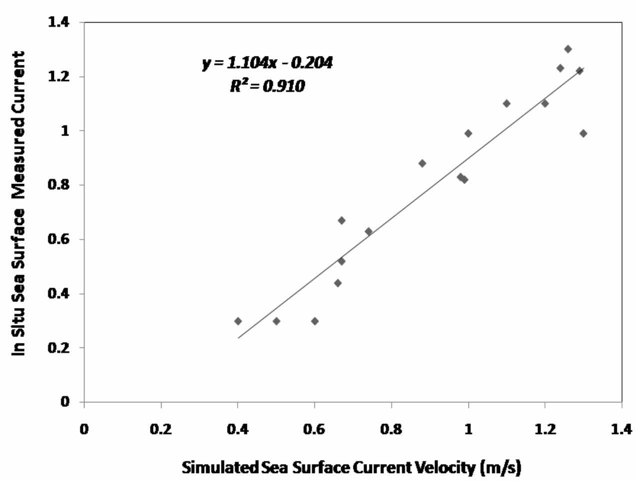


**Fig. 6.** Regression of Simulated Averaged Current and Measured In situ Current

# 5 Conclusions

This work has demonstrated the two-dimensional sea surface current flow modelling from JASON satellite altimetry data in the South China. The new technique was used to reduce the impact of Coriolis parameter in the continuity equation of geostrophic current. Two procedures were involved: second order Volterra Kernel and Lax-Wendrof Scheme. It is interested to name this algorithm as Volterra-Lax-Wendrof scheme. The results showed good correlation between in situ current measured and Volterra-Lax-Wendrof scheme with high $r^2$ of 0.91 and RMSE of ±0.4 m/s. It can be concluded that Volterra-Lax-Wendrof scheme could be used as good model to acquire fine resolution of sea surface current in such sea basin of the South China Sea.

# Acknowledgements

We would like to express our appreciation to Universiti Teknologi Malaysia Research Management Center (RMC) for a great cooperation. In fact, RMC is providing excellent advice, plans and management for UTM researchers. We also acknowledge the financial e-science grants have received from the Malaysian Ministry of Science and Technology via its Intensified Research for Priority Areas programme.

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