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Ballistic mobility and saturation velocity in low-dimensional nanostructures

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ABSTRACT

Ohm's law, a linear drift velocity response to the applied electric field, has been and continues to be the basis for characterizing, evaluating performance, and designing integrated circuits, but is shown not to hold its supremacy as channel lengths are being scaled down. In the high electric field, the collision-free ballistic transport is predicted, while in low electric field the transport remains predominantly scattering-limited in a long-channel. In a micro/nano-circuit, even a low logic voltage of 1 V gives an electric field that is above its critical value ε_c ($\varepsilon \gg \varepsilon_c$) triggering non-ohmic behavior that results in ballistic velocity saturation. The saturation velocity is an appropriate thermal velocity for a non-degenerate and Fermi velocity for a degenerate system with given dimensionality. A quantum emission may lower this ballistic velocity. The collision-free ballistic mobility in the ohmic domain arises when the channel length is smaller than the mean free path. The results presented will have a profound influence in interpreting the data on a variety of low-dimensional nanostructures.

1. Introduction

Ballistic (B) transport is the name given to the charge flow where the current and related drift velocity do not depend on the scattering interactions. Ohm's law $I = V/R_o$ with linear response of the current to the applied voltage in a circuit depends on the scattering-limited ohmic resistance R_o which in turn depends on the ohmic mobility μ_o. However, this law breaks down when an applied electric field exceeds its critical value $\varepsilon_{co} = V_t/\ell_o$ for the onset of non-ohmic behavior. For a typical mean free path of 0.1 μm, $\varepsilon_{co} = 0.0259\,\text{V}/0.1\,\mu\text{m} = 2.59\,\text{kV/cm}$. As an applied electric field, $\varepsilon = V/L$ for a macro-device (say $L = 1$ cm) has been below this critical value, ohmic behavior was never under scrutiny. As devices scale below 1 μm, $V_{co} = \varepsilon_{co}L = 0.259$ V or lower. For a typical logic voltage of 1.8 V or above, we are clearly in the non-ohmic domain. Thornber [1] determined that saturation velocity is invariant under scaling of the magnitude of the scattering rates, which alters mobility, while mobility is invariant under scaling of the magnitude of the momentum, which alters saturation velocity. The saturation velocity obtained from Arora's distribution function [2] correctly confirmed Thornber's findings.

As channel length goes below the mean free path, there is a talk of mobility being ballistic as well. The ballistic (B) mobility arises due to the fact that the probability of a B electron going without collision is almost unity. In that case, even the ohmic mobility is ballistic, the collision mean free path being replaced by the channel length ($\ell_o \cong L$). These are distinct features of both the mobility and saturation velocity being B in a modern device [3]. This may transform the ways we characterize, evaluate performance, and make predictions in simulations programs.

2. Ballistic intrinsic velocity

The energy spectrum in a semiconductor with varying low-dimensionality configuration is given by

$$E_k = E_{c0} + \frac{\hbar^2}{2m^*}(k_x^2 + k_y^2 + k_z^2) \quad (3D), \tag{2.1}$$

$$E_k = E_{c2} + \frac{\hbar^2}{2m^*}(k_x^2 + k_y^2) \quad (2D), \tag{2.2}$$

$$E_k = E_{c3} + \frac{\hbar^2 k_x^2}{2m^*} \quad (1D), \tag{2.3}$$

with

$$E_{cd} = E_{co} + \varepsilon_{0d} \quad \text{(Quantum limit)}. \tag{2.4}$$

Here, $\varepsilon_{03} = 0$ and ε_{0d} ($d = 2,1$) is the zero-point quantum energy that increases the effective bandgap for a low-dimensional nanostructure. This has no bearing on a single band transport as

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the Fermi energy is always relative to the conduction band edge in the quantum limit. In a high electric field, the difference in the quantum energy of the first excited state and the zero-point energy of the ground state may trigger a quantum emission that limits the saturation velocity below the intrinsic velocity.

The velocity response to the electric field in a high electric field results in velocity saturation. The current in a resistive (channel) is limited by this saturation value $I_{sat3} = nqv_{sat}A$ (3D), $I_{sat2} = n_s q v_{sat} W$ (2D), $I_{sat1} = n_\ell q v_{sat}$ (1D) which in turn depends on the doping concentration n_d ($d = 1, 2, 3$). In the presence of a very high electric field [4], all electrons are streamlined opposite to the direction of the applied electric field. The ultimate unidirectional drift velocity is the saturation velocity that is the average of its absolute value $|v| = \sqrt{2E_k/m^*}$, where E_k is the kinetic energy for a given dimensionality and m^* is the carrier effective mass. When this averaging is taken by including the Fermi–Dirac distribution and density of states as a weight for a given dimensionality, the intrinsic B velocity for a semiconductor is obtained as

$$v_{id} = v_{thd} \frac{\Im_{(d-1)/2}(\eta_{Fd})}{\Im_{(d-2)/2}(\eta_{Fd})}, \tag{2.5}$$

with

$$v_{thd} = v_{th} \frac{\Gamma[(d+1)/2]}{\Gamma(d/2)}, \tag{2.6}$$

$$v_{th} = \sqrt{\frac{2k_B T}{m^*}}. \tag{2.7}$$

Fig. 1 gives a plot of intrinsic B velocity normalized to the thermal velocity of Eq. (2.7). In the non-degenerate regime, this ratio is $2/\sqrt{\pi} = 1.128$ for a 3D semiconductor, it drops to $\sqrt{\pi}/2 = 0.886$ for a 2D nanostructure and further drops to $1/\sqrt{\pi} = 0.564$ for a 1D nanostructure. The intrinsic velocity is a strong function of temperature in the non-degenerate regime and is independent of carrier concentration. It is, therefore, convenient for many investigators to take it as an ultimate saturation or ballistic velocity. However, in the degenerate regime, the 1D nanowires have a distinct speed advantage and may be a factor to consider in the design of nanowire transistors. In the strongly degenerate regime, the B velocity is independent of temperature and is a strong function of carrier concentration n_d for a given dimensionality (n_1 per unit length, n_2 per unit area, and n_3 per unit volume). This B velocity is given by

$$v_i = 2\frac{d}{d+1}\frac{\hbar}{m^*}\sqrt{\pi}\left[\Gamma\left(\frac{d+2}{2}\right)\frac{n_d}{2}\right]^{1/d}. \tag{2.8}$$

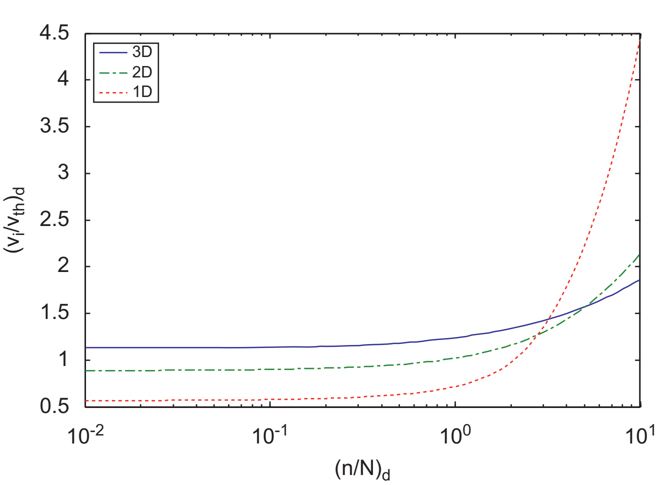

Fig. 1. Relative intrinsic velocity versus normalized carrier density.

3. Ballistic mobility

Ballistic transport, as the name suggests, is the description of a collision-free trajectory of a carrier. In that respect, the trajectory within a free path described by mean scattering length ℓ_o is always B. It is natural for carriers to avoid collision if the length L of the conducting channel is less than ℓ_o. As shown in Fig. 2, the electrons at a point x arrive from left and right a distance $x \pm \ell_o$ away. The net exchange is between carriers going in the two directions (parallel and anti-parallel to the applied electric field in the negative x-direction). The band diagram is tilted. It is interesting to investigate the behavior of a macro-resistor in terms of a series of B resistors as advocated by Büttiker [5]. A drift resistor of macro-length L comprises a series of B resistors. Within Büttiker's approach, carriers are removed from the device and injected into a "virtual" reservoir where these are thermalized and re-injected into the device so that the number of electrons is conserved. Therefore, the internal contacts of a B chain behave as Büttiker probes, and then we expect that a B chain can realistically describe a macro-resistor in any transport regime. In a low electric field, the equilibrium Fermi–Dirac distribution is slightly perturbed [6], with band diagram almost flat. An electron coming from the left at point x has velocity $v_+ = +v_i + (q\varepsilon\tau_c/m^*)$ and that arriving from the right has a velocity $v_- = -v_i + (q\varepsilon\tau_c/m^*)$. Since the number of electrons going either way is equal, the average drift velocity at x is given by

$$v_d = \frac{(v_+ + v_-)}{2} = \frac{q\tau_c}{m^*}\varepsilon = \mu_0 \varepsilon. \tag{3.1}$$

The ohmic mean free path $\ell_o = v_i \tau_c = v_i \mu_o m^*/q$ is then easily obtainable from the experimental mobility.

The relation given by Eq. (3.1) between the average (drift) velocity v_d and electric field ε is linear with the coefficient μ_o, the scattering-limited mobility. In a nanoscale resistor of length L, the

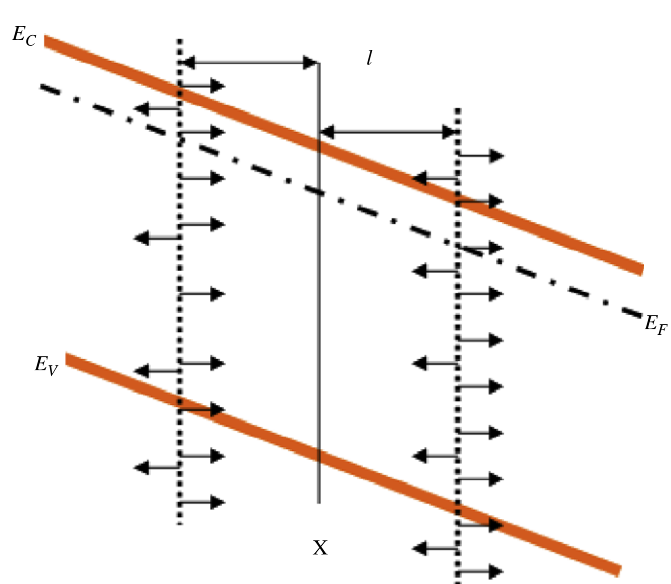

Fig. 2. Partial streamlining of random motion of the drifting electrons on a tilted energy band diagram in an electric field.

mean free length ℓ_o is limited by the length of the resistor L. Therefore, the effective mean free path in this case is [7]

$$\ell_L = \ell_o(1 - e^{-(L/\ell_0)}), \tag{3.2}$$

where the factor multiplying ℓ_o is the probability of a collision on electron completing a distance L [7]. The mobility is then described by

$$\mu_L = \mu_o(1 - e^{-(L/\ell_0)}), \tag{3.3}$$

$$\mu_o = \frac{q\ell_o v_{id}}{dk_B T} = \frac{2q\ell_o v_{id}}{m^* dv_{th}^2}, \tag{3.4}$$

As expected for a macro-resistor ($L \gg \ell_o$), the scattering-limited mobility is $\mu_L = \mu_o = q\ell_o/m^* v_{Bd}$ with $v_{Bd} = v_{th}^2/2v_{id}$. However, when $L \ll \ell_o$ for a submicro-resistor, the mobility becomes B with mean free path $\ell_o \cong L$ resulting in $\mu_B = qL/m^* v_{Bd}$. Equation (3.3) is consistent with the conclusion arrived at by Wang and Lundstrom [8] and Shur [9] except for the minor differences. Wang and Lundstrom consider the ballistic velocity as appropriate for 3D and unidirectional, only half Maxwellian directed toward the channel. Shur considers this as a full Maxwellian velocity. Our results are consistent with that of Shur in the non-degenerate regime only. The results reported in this paper go well beyond Shur's predictions as these are not only valid for degenerate statistics, but also valid for all three dimensionalities ($d = 1, 2, 3$).

In a high electric field, there is a possibility of B transport interrupted by the onset of quantum emission which is the coherence length $\ell_Q = E_Q/qE$ during which an electron gains enough energy to emit a quantum of average energy E_Q. In the presence of quantum emission, the mean free path is further limited by

$$\ell = \ell_L(1 - e^{-(\ell_Q/\ell_L)}). \tag{3.5}$$

Even if no change in the distribution is considered, this transformation of electric field-dependent mean free path gives the velocity saturation $v_{sat} = E_Q/m^* v_i$. However, the change in the distribution of electrons transforms the nature of this saturation [4]. Here, $E_Q = (N_o + 1)\hbar\omega_o$ is the average energy of a quantum with $N_o = 1/[\exp(E_Q/k_B T) - 1]$, the Bose–Einstein distribution, giving the probability of emission. The energy $\hbar\omega_o$ is not limited to the optical phonon emission, but can be energy of any quantum that includes energy spacing between the two lowest quantized energy levels or presence of an electromagnetic quantum. Perhaps, future experiment can assess the nature and energy of this quantum that limits the saturation velocity, but has no impact on ballistic mobility.

4. Conclusion

To conclude, we have indicated that the saturation velocity limiting the drift of carriers is the intrinsic velocity that depends on carrier concentration and the ambient temperature. On the other hand, in low electric field, even the low-field mobility may become ballistic as the carrier mean free path is replaced by the channel length of a device. In ultra-small devices, both mobility and saturation velocity may become ballistic. In a high electric field, the saturation velocity may become smaller than the intrinsic velocity because of the onset of a quantum emission. Greenberg and de Alamo [10] have indeed provided a direct evidence of saturation-limited transport in a 5-μm channel of InGaAs. In fact, they have specifically shown that the ohmic parasitic resistances in the contact regions may blow-up when the current is closer to its saturation limit.

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