CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION

Beams composed of two or more elements are commonly used to span large openings. If the elements of such a beam are interconnected, the elements act simultaneously and thus the load carrying capacity of the beam is greater than the sum of the individual's capacities. A steel-concrete composite beam consists of concrete and steel acting as a beam together. Example of composite beams are reinforced concrete beam (RC), conventional composite beam (i.e. composite beam with concrete slab), RC beam stiffened with tension materials such as steel or composite materials and profiled composite beam. The advantages of composite beams are light weight, high strength, improved durability and the capacity to withstand dynamic loads.

1.2 COMPOSITE BEHAVIOUR

Steel-concrete composite structures are defined as structures built up by concrete and steel components connected by shear connectors to form an interacting unit. The behaviour of composite members depends on the degree of shear connection between the components. Rigid shear connectors usually develop full composite action between the individual components whilst, flexible shear connectors generally permit the development of partial composite action. The latter requires consideration of the interlayer slip between the components.

Despite the established studies on the static behaviour, investigations concerning the dynamic responses of composite beams with partial interaction are scarce. A beam is said to be undergoing free vibration when it is disturbed from its static equilibrium position and then allowed to vibrate without any external dynamic excitation. The analytical result describing free vibration provides a basis to determine the natural frequency of a beam. This is important when it comes to forced vibration. The largest response amplitude, defined as resonance occurs when the frequency of the vibration is the same as the natural frequency. As far as composite beams are concerned, investigations are limited only on the dynamic behaviour of beams with full interaction whilst work on partial interaction allowing slip only analytically solved by Wu et.al. (2007), on one hand, and Hamed and Rabinovitch (2005,2007) solved the problems using variational principles, on the other. However, so far there is no finite element formulation for free vibration of composite beams.

1.3 RESEARCH BACKGROUND AND RATIONALE

Existing partial interaction theories of composite beams are based on force or direct equilibrium whereby differential equations are derived and solved. As discussed in the literature review, the difficulty of the problem increases as parameters to be considered increases. Increment in the number of parameters results in higher order differential equation which might as well coupled. Newmark (1951), in allowing the effect of slip, has derived and solved a second order differential equation whilst Adekola (1968) dealt with fourth and second order coupled differential equations due to the allowance of uplift. Mohd Yassin (2007) have extra in terms due to friction in their differential equation as compared to Adekola (1968). In the study of vibration, Wu et.al. (2007) derived a sixth order partial differential equation despite the fact that only slip was taken into account. A direct comparison with Newmark (1951) would reveal that the inclusion of vibration modifies the second order ordinary differential equation into sixth order partial differential equation. Hamed and Rabinovitch (2005,2007) solved vibration of composite beams using Hamilton's variational principle for both free and forced vibration.

Obviously, the problems of partial interaction and vibration of composite beams are complex and difficult. Recent works based on variational principle only applied conventional Ritz method. It is therefore the main interest of this study to derive finite element formulation for the problem.

1.4 PURPOSE AND OBJECTIVES OF STUDY

The purpose of this study is to provide an alternative approach to the Ritz variational approach in treating the partial interaction and vibration of composite beam by formulating finite element approach to the problem. The objectives of the study are as follows:

1) To establish stiffness matrix and mass matrix of composite beams allowing partial interaction and free vibration
2) To obtain the natural frequencies of the composite beam
3) To verify the formulation by comparing the results (i.e static deflection and natural frequency) with those obtains through analytical solutions

1.5 ASSUMPTIONS AND LIMITATIONS

The study is based on a few assumptions and limitations which are:

1) Equal curvature between the two subelements, whereby uplift will not occur.
2) This study is to find the eigenvalues only and not the eigenvectors.
3) Linear and elastic analysis.

1.6 OUTLINE OF THESIS

This chapter introduces the study by stating why it is needed, the problem statements, objectives and what are the results that will be obtained. Chapter 2 elaborates more on the topic in terms of the partial interaction and also vibration. Previous studies are analyzed and problems are outlined in this chapter. Finite element (FE) formulations are treated in Chapter 3. Chapter 4 discusses on the results achieved by the FE formulations and being verified by existing solutions while Chapter 5 deals with the conclusion of the study and recommendations that can be made to further improve the study.

CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION

A composite beam is normally defined as a steel beam being connected to the concrete slab using shear connectors. Composite beams are stronger and lighter compared to conventional beam due to composite action. Composite action can be explained by referring to Figure 2.1.

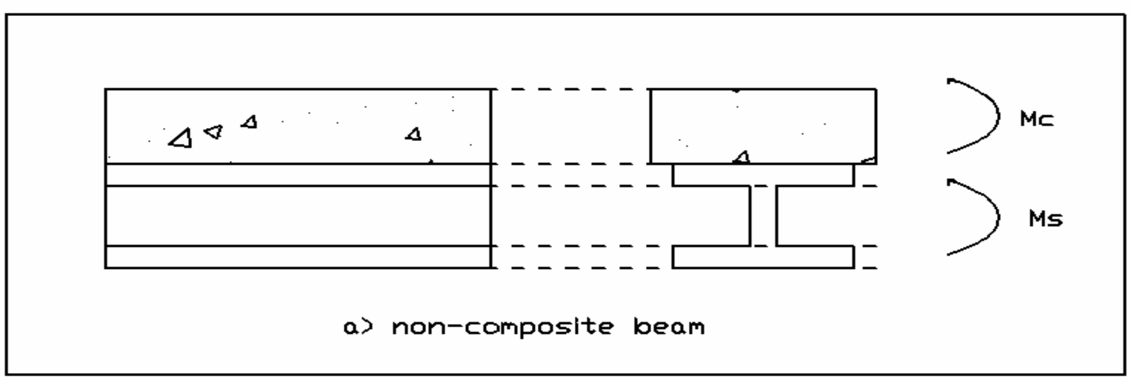

Figure 2.1 : Composite behaviour (continues)

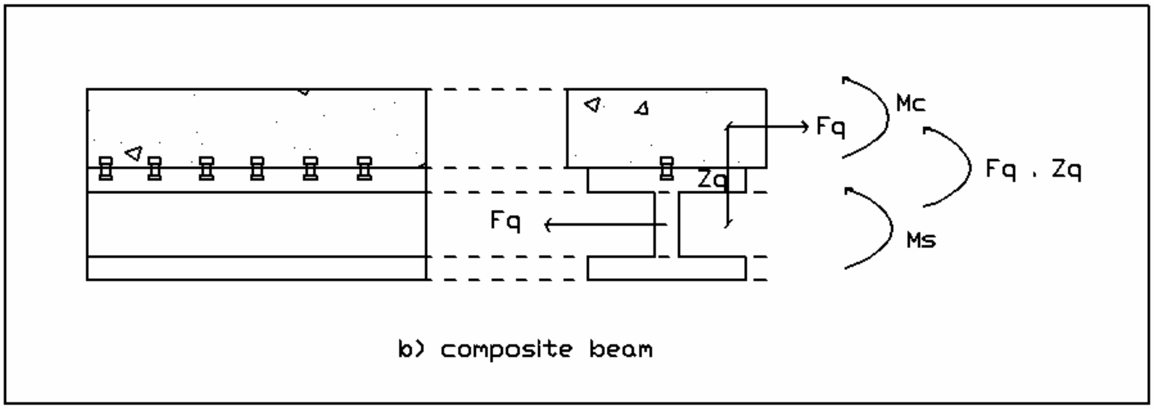

Figure 2.1 : Composite behaviour (continued)

Figure 2.1 shows composite action due to the use of shear connectors. There will exists a resultant force, F_q as shown in Figure 2.1b) which is absent in the non-composite section. This additional resultant force produces an extra moment term that increase the total moment capacity of the composite beam as compared to non-composite beam.

For non-composite beam, the total moment is :

$$M=M_c+M_s \tag{2.1}$$

where M_c and M_s are resisting moments produced by concrete and steel respectively.

Total moment for composite beam is given as :

$$M=M_c+M_s + F_q\, z \tag{2.2}$$

where F_q is the additional resultant force produced by the composite action and z is the distance between the neutral axis of concrete and steel.

Composite interactions exist when the two materials are fully or partially connected together by shear connectors. Interaction refers to the transferring of stress from one material to the other and also the relative displacement between them. This study requires the understanding of the following fundamentals :

I. Partial Interaction Theory
II. Vibration
III. Total Potential Energy
IV. Hamilton Principle
V. Finite Element

2.2 PARTIAL INTERACTION THEORY

The behaviour of composite members depends to a large degree on the type of connection between the materials. Rigid shear connectors usually develop full composite action between the individual materials of the member, thus conventional principle of analysis can be applied. Flexible shear connectors, on the other hand, generally permit development of only partial composite action because of strain incompatibility, therefore the analysis procedure requires consideration of the interlayer slip between the materials.

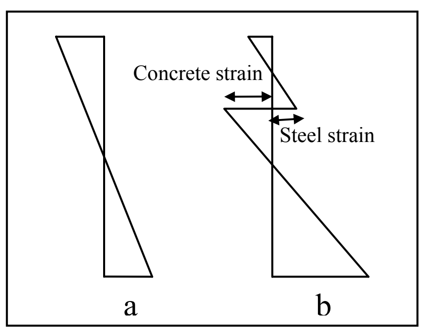

Figure 2.2 : Strain distribution for a) full composite action and b) partial interaction

Slip occurs at the interface of concrete and steel with partial interaction as shown in Figure 2.2b). Both the strains are not equal when they bend together. Because of the flexible shear connectors, the stress from concrete to steel is not fully transferred.

Strain inequality of both concrete and steel will produce slip. The rate of change of slip is equal to the difference between the strain in the concrete and the strain in the steel at the level at which slip occurs.

Partial interaction results from existence of imperfect composite action. Strain incompatibility occurs between the interfaces of the materials. Horizontal shear is transferred from one element to the other through the shear connectors. Newmark (1951) allows for slip only and the differential element is as shown in Figure 2.3. The second order differential equation is as given in Eq. (2.3).

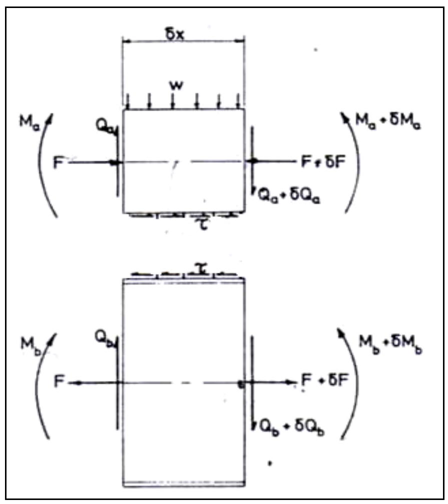

Figure 2.3 : Differential element of Newmark (1951)

$$\frac{d^2F}{dx^2} - \emptyset_1 F = \emptyset_2 M \tag{2.3}$$

where F is force and M is the applied moment, φ_1 and φ_2 are constants.

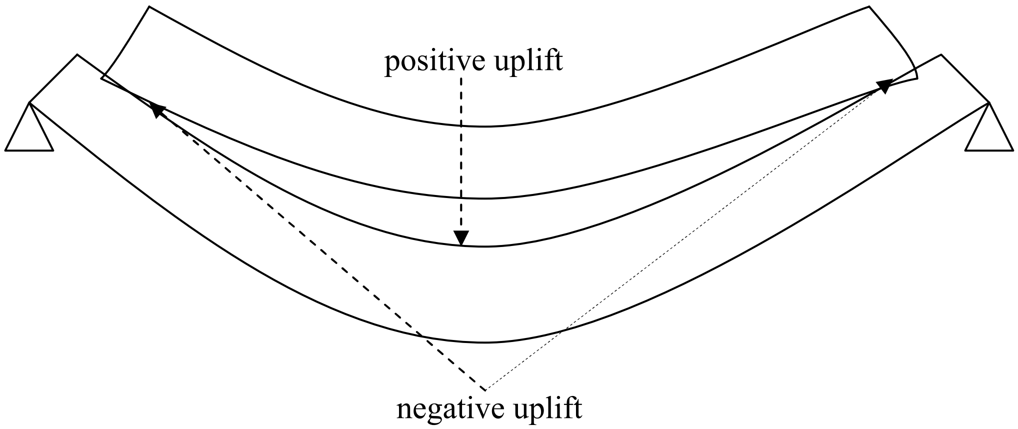

Figure 2.4 : Uplift in composite beam

Figure 2.4 shows uplift deformation in a composite beam because of different curvature between concrete and steel. Negative uplift is the region at the end of the composite where both materials produce normal stresses towards each other while positive uplift occurs at the middle region where there are no contact between concrete and steel.

Adekola (1968) dealt with fourth and second order coupled differential equations due to the allowance of both slip and uplift as shown in Eq. (2.4). A new term *T*, normal stress at the interface of the elements is introduced compared to Newmark (1951). Figure 2.5 shows the differential element.

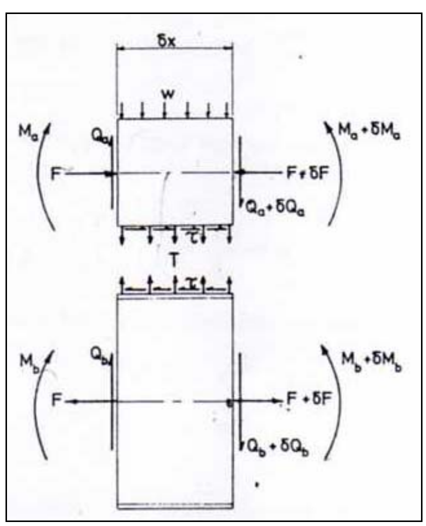

Figure 2.5 : Differential element of Adekola (1968)

$$\frac{d^4T}{dx^4} + \chi_1 T - \chi_2 \frac{d^2F}{dx^2} = \chi_3 \tag{2.4a}$$

$$\frac{d^2F}{dx^2} - \varnothing_1 F - \varnothing_3 \frac{d^2T}{dx^2} - \mu \frac{dT}{dx} = \varnothing_2 M \tag{2.4b}$$

where μ is the coefficient of friction, T is the normal stress at the interface and φ_1, φ_2, φ_3, χ_1, χ_2 and χ_3 are constants.

Comparing Adekola's (1968) with Newmark's (1951), which allow only slip, the second order differential equation is now a fourth and second order coupled differential equations and also the introduction of a new term, T due to the allowance of both slip and uplift. However, Adekola (1968) has shown that the magnitude and the effect of uplift is small compared to slip and it is usually ignored.

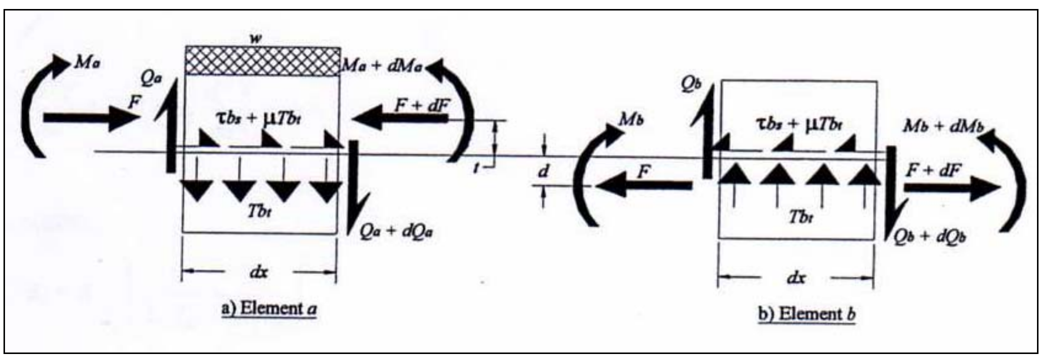

Figure 2.6 : Differential element of Mohd Yassin (2007)

Mohd Yassin (2007) have extra in terms due to friction in their differential equation as compared to Adekola (1968). Adekola (1968) states that the normal stresses produce frictional stresses in the negative uplift region where it is neglected because the region of negative uplift is very small, as shown in Figure 2.4.

However, Mohd Yassin (2007) needs to consider friction because of the continuous rib connections in beams, where normal stresses and hence frictional stresses exist along the span of the beam. Figure 2.6 shows the differential element which a new term, μ is introduced to allow the effect of friction. This resulted in a differential equation as shown in Eq. (2.5) which have the absolute value of T.

$$\frac{d^2F}{dx^2} - \phi_4 F = -\phi_5 M + \phi_6 \frac{d^2T}{dx^2} - \frac{\mu b_t d|T|}{dx} \tag{2.5a}$$

$$\frac{\partial^4 T}{\partial x^4} + \emptyset_1 T = \emptyset_2 \frac{\partial^2 F}{\partial x^2} - \emptyset_3 w \tag{2.5b}$$

Obviously, when the parameters to be considered in the analysis increases, the difficulty also increases resulting in higher order differential equations coupled differential equations and larger number of terms in the differential equations. The difficulties are in getting the roots of characteristic equations where higher order differential equations will result in higher order characteristic equation and also the number of boundary conditions needed will be higher, where it is impossible to obtain.

2.3 VIBRATION

A beam is said to be undergoing free vibration when it is disturbed from its static equilibrium position and then allowed to vibrate without any external dynamic excitation. The analytical result describing free vibration provides a basis to determine the natural frequency of a beam. This is important when it comes to forced vibration. The largest response amplitude, defined as resonance occurs when the frequency of the vibration is the same as the natural frequency.

As far as composite beams are concerned, investigations are limited only on the dynamic behaviour of beams with full interaction whilst work on partial interaction allowing slip only recently by Wu et.al. (2007)

Study on the vibration of composite beam is limited. Wu (2007) has derived the governing partial differential equations of motion but allowing slip only. This study derived a sixth order partial differential equation, given as ;

$$EI\frac{\partial^6 w}{\partial x^6} - (EI\alpha^2 + \beta^2 H)\frac{\partial^4 w}{\partial x^4} + \beta^2 m\frac{\partial^4 w}{\partial x^2 \partial t^2} + \alpha^2 H\frac{\partial^2 w}{\partial x^2} - \alpha^2 m\frac{\partial^2 w}{\partial x^2} = -\alpha^2 q + \beta^2\frac{\partial^2 q}{\partial x^2} \tag{2.6}$$

where H is the external axial force, w is the bending deformation, q is the intensity of loading and α^2 and β^2 are constants.

Wu et.al.(2007) solved the problem by using non-dimensionless quantities and the general solutions of the homogeneous differential equation are based on the root characteristic of its eigen equation. Hamed and Rabinovitch (2005,2007) solved vibration of composite beams using Hamilton's variational principle. The function is given in the next section. Newton time integration approach has been directly applied to the analytical model resulting in a set of ordinary differential equations that is solved at every time step. The results are presented in terms of displacements, stress resultants and stresses in the beam, fiber reinforced polymer strip and adhesive layer.

2.4 TOTAL POTENTIAL ENERGY

Total Potential Energy is a type of functional where it can be derived from the Principle of Virtual Work. The total potential energy is given as :

$$\Pi = U + V \tag{2.7}$$

where U and V are the strain energy and load potential respectively.

2.4.1 FUNDAMENTALS

Total Potential Energy is a type of functional where it is an integral of a function which contains *x, y*, their derivatives and other functions.

$$I = \int_{x_1}^{x_2} F\left(x, u, v, u', v', u'', v'' \ldots\ldots, u^n, v^n\right) dx \tag{2.8}$$

where F is a function of variables x, u, v, u' and etc.

Extremizing a functional means that satisfying $\delta I = 0$, where any value of the functions will not change I. The general form of Euler –Lagrange equation is given in Eq. 2.9 by satisfying $\delta I = 0$.

$$\sum_{0}^{n} (-1)^n \frac{\partial^n}{\partial x^n}\left(\frac{\partial F}{\partial u^n}\right) = 0 \tag{2.9}$$

Total Potential Energy is a type of a functional and its stationary principle is actually a functional extremization. The total potential energy of a beam with distributed load (*q*) is derived through stationary principle of Total Potential Energy and can be given as :

$$I = \int_0^L \left(\frac{EI}{2} \left(\frac{d^2 y}{dx^2} \right)^2 - qy \right) dx \tag{2.10}$$

2.5 HAMILTON PRINCIPLE / DYNAMIC ANALYSIS

Free vibrations lead to the notions of natural frequency and also the time-varying displacement. The time-varying displacement *y(x,t)* of any cross section along a beam that is vibrating with a natural frequency ω_n can be expressed as $y(x,t) = f(x)\sin(\omega_n t)$. The maximum kinetic energy of a uniform beam of mass γ per unit length is :

$$T_{max} = \frac{\gamma}{2} \int_0^L (y\omega_n)^2 \, dx \tag{2.11}$$

where ω is the natural frequency.

Hamilton Principle states that, $\pi = T - U - V$. For beam assuming full interactions :

$$\pi = \frac{\gamma}{2} \int_0^L (y\omega_n)^2 \, dx - \int_0^L \left(\frac{EI}{2} \left(\frac{d^2 y}{dx^2} \right)^2 - qy \right) dx \tag{2.12}$$

where *E* is the modulus of elasticity and *I* is the second moment area.

For beams with two material having full interaction, the potential for the Hamilton principle can be given as :

$$\pi = \frac{\rho_1 A_1 + \rho_2 A_2}{2} \int_0^L (y\omega_n)^2 \, dx - \int_0^L \left(\frac{E_1 I_1 + E_2 I_2}{2} \left(\frac{d^2 y}{dx^2} \right)^2 - qy \right) dx \tag{2.13}$$

where ρ_1 and ρ_2 are the densities of the two materials and A_1 and A_2 are the cross-sectional area of the two materials.

Hamed and Rabinovitch (2005,2007) solved vibration of composite beams using Hamilton's variational principle. Hamilton's variational principle is used for the derivation of the equations of motion and the dynamic boundary and continuity conditions. The functional is given as :

$$U = \int_0^L \frac{E_c I_c + E_s I_s}{2} \left(\frac{\partial^2 w}{\partial x^2} \right)^2 dx + \int_0^L \frac{E_c A_c}{2} \left(\frac{\partial u_c}{\partial x} \right)^2 dx + \int_0^L \frac{E_s A_s}{2} \left(\frac{\partial u_s}{\partial x} \right)^2 dx + \int_0^L \frac{k}{2} \left(u_s - u_c + z \frac{\partial w}{\partial x} \right)^2 dx \tag{2.14}$$

where E is the modulus of elasticity, I is the second moment are, A is the cross sectional area of the material, z is the distance between neutral axis of the two materials, k is the modulus of shear connectors, w is the vertical displacement and u is the horizontal displacements where the subscript c and s denotes concrete and steel respectively.

Despite the above works, there are still no FE of composite beams for free vibration allowing slip.

2.6 FINITE ELEMENT

In Finite Element Method (FEM), the basic concept of FEM is the formulation of the following equilibrium equation:

$$[K]\{D\}=\{R\} \tag{2.15}$$

where [K] is the stiffness matrix, {D} is the vector of the degree of freedom (dof) and {R} is the load vector. Solving Eq. 2.15 for {D} is the goal of FEM. In a vibrational analysis, mass matrix, [M] is introduced so that the dynamic stiffness matrix can be assembled. For undamped free vibrations, the dynamic stiffness matrix is given as:

$$[M]\ \{\ddot{U}\}+[K]\ \{U\}= 0 \tag{2.16}$$

The above equation represents an eigenvalue problem which solutions are the natural frequencies of the system.

$$([K]-\lambda[M])\ \{D\}= 0 \tag{2.17}$$

where $\lambda = \omega^2$, ω is the natural frequency.

A non trivial solution for Eq.. 2.17 is by setting the determinant ,$|\ [K]-\lambda[M]\ |= 0$ where the characteristic equation is produced. The roots of the characteristic equation is the value of λ, hence the eigenvalue. The eigenvectors, {D} can be solved by inserting the eigenvalues into Eq. 2.17, but the interest is only on the eigenvalues.

2.7 CONCLUDING REMARKS

From this literature review, it can be seen that when the parameters to be considered in the analysis increases, the difficulty also increases resulting in higher order differential equations, coupled differential equations and larger number of terms in the differential equations. Since there are no finite element formulations yet for free vibrations of composite beams allowing for slip, this study is needed so that the problem can be solved. The understanding of partial interaction, vibration, Total Potential Energy, Hamilton Principle and the finite element itself is necessary in order to succeed in doing this study.