

**TOWARDS A REDEFINITION OF THE NATIONAL GEODETIC
VERTICAL DATUM BY THE INTEGRATED GEODESY
ADJUSTMENT - A VIEWPOINT**

by

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Abstract

Determination of Mean Sea Level (MSL) as a vertical datum for orthometric heights is a vital requirement for the readjustment of the National Geodetic Levelling Network. The vertical datum is defined to be the geoid whereby its practical realization was brought by tide gauge observations at very few selected points e.g. ports. Geodetic levelling observation could use these tide gauges as fixed points. Unfortunately, the so-called determined MSL differs significantly from the geoid due to local phenomena, such as sea surface topography, ocean currents, tides, etc. The solution leads to distortion in the levelling network or other height information.

Using integration approach of the Global Positioning System (GPS) data connected between tide gauges, satellite altimetry data on sea near the tide gauges, derived potential differences (from levelling nets and gravity observations), gravity anomalies and the deflection of the vertical surrounding the tide gauges, the required reference surface could be precisely determined. From geodetic point of view, these data contribute to the vertical datum problem and is briefly presented in this paper by the expression of the associated parameters of the Integrated Geodesy Adjustment's Concept.

1.0 Introduction

Height of points are defined with respect to the geoid. Historically, the conceptual role in the vertical datum definition is an equipotential surface of the earth's gravity field that most closely coincide with the MSL. However, the vertical network used as a vertical control for mapping and surveying activities such as earth deformation studies, engineering projects, marine projects, etc. suffer irregularities in this form of vertical datum. Vertical datum is defined as a point of zero height which can relate the physical height information of the whole earth to the same geoid. Conventionally, this surface is represented in the form of tide gauges data determined by the periodic tidal observations.

Since the present vertical datum which was known as a local MSL at various selected tide gauge stations held fixed (assumed to be zero elevation) be considered to be on the same geoid, errors introduced by this approach were quite significant. It is now well recognised that due to the effect of sea surface topography, the vertical datums of many countries were not precisely determined (Rizos, 1980). The sea surface topography is the time-variable equipotential surface

which causes the ocean current, water density variation, salinity, air pressure, wind stress, etc. In addition, close to the shore the sea-bed topography and river discharge may also play a significant contribution. The relationship between sea surface topography and the geoid which defined the tide gauge is shown in figure 1.0. The principle purpose of the tide gauge is to measure differences in height of the instantaneous sea level and the land.

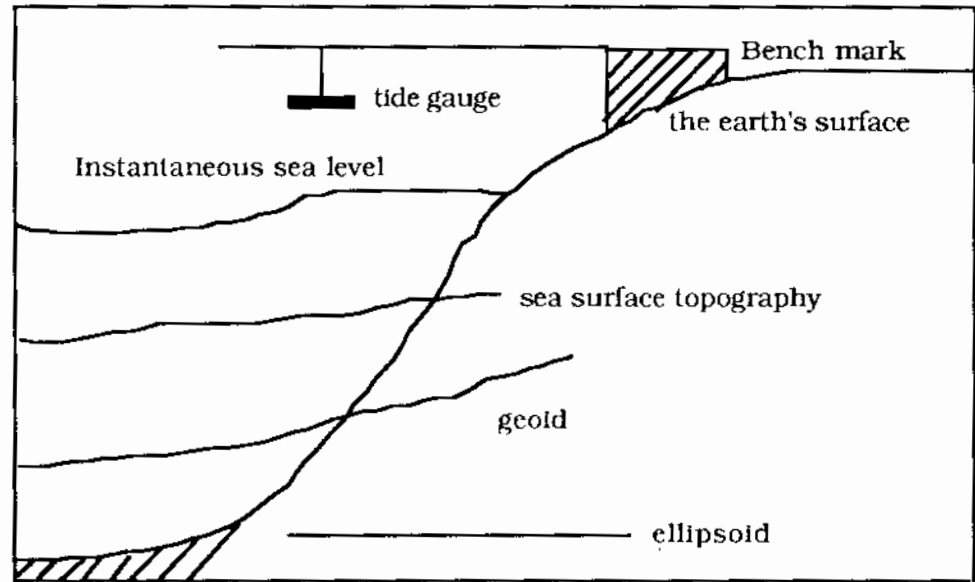


Figure 1.0

Realising those discrepancies occurred in the vertical networks, it is very important to redefine the status of the present National Geodetic Vertical Datum. With the aids of the modern ideas in software engineering, new development in human-computer interfacing and the advancement of satellite and surveying technologies, the determination of the vertical datum will guarantee an optimal solution in terms of accuracy, practicality and reliability. Here, based on terrestrial gravity anomalies, deflection of the vertical, GPS data connected between tide gauges potential differences between tide gauges (as derived from combination of levelling and gravity observations) and Seasat altimetry data, the integrated approach could be applied for the solution of the above mentioned problem.

2.0 Background

In the past, the undisturbed ocean surface had been taken as the geoid which is represented in the form of tidal observations at selected site for several years. It had been the practice in this country to connect the vertical nets to several tide gauges, widely dispersed around the perimeter of nets, eg. Port Kuantan, Port Klang, Cendering, etc. However the sea surface topography which is the departure of the MSL from the geoid at each tide gauge is the different from one tide gauge to another. Therefore, the resulting adjustment of the levelling data to fit the different MSL led to the distortion of the National Geodetic Levelling Network. In other words, the results show discrepancies due to the inconsistencies between different vertical datums. Simulation approach in Table:1.0 shows that the distribution of errors resulted from different fixed tide gauges is quite significant. Here, a certain

question can be implicitly asked: to what extent the sea surface topography in our region will contribute to the definition of vertical datum?

Table. 10

Tide gauge (held fixed)	Distribution errors (cm)
Port Kelang	0 1.8
Kuantan	0 1.5
Cendering	0 1.8
Johor Bahru	0 2.2
Tanjung Keling	0 2.0
Lumut	0 1.7
Penang	0 2.1

(Source Kaldzu, 1989)

With the advancement and revolutionising of the computer and satellite positioning technology in the field of surveying, the problem of modelling the vertical datum could be solved. For example, the GPS derived orthometric heights and positions, a new treatment of satellite altimetry-data and the wealth information of the Earth Model such as GEMOC, OSU86F GPMZ will produce a result of a unique vertical reference that minimises the corresponding errors at the tide gauges whereby the effect of the sea surface topography is filtered out. Conceptually, this approach is called "The Integrated Geodesy Adjustment". The model was originally developed by Egg and Krarup (1973), and is considered to be the best possible approach that combined various type of surveying information in term of land based and satellite based data sets in one integrated model. Hence, it could established a vertical network origin whereby the elevation of this point is 'zero'. After reduction the potential differences, orthometric height (or any height information) and gravity anomalies, are then precisely referred to a uniform equipotential surface.

3.0 Significant of Findings

The significant of the findings will contribute to the followings:

- 1) This is a response to the requirement of precise geodetic information (vertical information) coming from different social sectors related to the planning of social and economical development
- 2) It will be of technical and economical benefits when the new survey is linked to the adjusted network, such as permanent vertical position and correlation with other surveys carried out in the same reference system.
- 3) The result of the redefinition of vertical networks will give an opportunity to have a reliable vertical information with known accuracy compatible with the new procedures and scientific equipments
- 4) It will be appropriate for developing countries like Malaysia to have systematic technical frame work which allow it to take advantage in

the best way of information related to the physical features, natural disaster controls, resources and socio-economical conditions for a better knowledge of the country in complete and global sense.

4.0 Integrated Modelling

Analysis of GPS surveying data has shown that GPS can be used to establish precise relative positions in three-dimensional Earth-centered coordinate system. The availability of precise satellite altimeter data by Seasat or ERS-1 will contribute accurate separation between sea surface topography and the geoid. On the other hand, a new method of computing the potential differences using combination of gravimetric and levelling data together with low to high degree reference gravity field model will ensure the improvement of the geoidal computations. Therefore, if all data as mentioned in section 1.0 is available is considered, the Integrated Geodesy Adjustment to determine the vertical datum could be performed.

The principle of the Integrated Geodesy Adjustment is fully discussed in Egg and Krarup (1973), Moritz (1980), Landau (1985) and Heins (1986). In the following, the principle of Integrated Geodesy Adjustment will be describe briefly with specific reference is given to the associated parameters of the abovementioned problem. In this case, the Integrated Geodesy Adjustment can be considered as a discrete solution of a free geodetic boundary value problem, determining simultaneously both the coordinates and the functions of the disturbing potential.

Briefly, every geodetic measurement can be expressed as a non-linear function with respect to spaced-position $X(x,y,z)$ and on the earth's gravity field (gravity potential), so,

$$l = F(X,W) \quad \dots(1)$$

where W is the gravity potential given by,

$$W = V + w^2(X^2 + Y^2)/2 \quad \dots(2)$$

V is the potential for the gravitational force, w is the angular velocity of the earth's rotation and (x,y,z) are Cartesian Coordinates in a geocentric reference frame. Z -axis coincides with the rotation of the earth.

By linearising the function in the usual adjustment practise, we have,

$$X = X^0 + dX \quad \dots(3)$$

and

$$W(X) = U(X) + T(X) \quad \dots(4)$$

where,

U is a normal potential associated with the adopted reference system of the International Association of Geodesy. T is a disturbing potential X^0 is an approximate position vector.

By expanding in a Taylor series at point, say $P(X^0, U)$ and putting it in matrix notation, we get a linear system of equations of type,

$$dl = A^T dX + R t + n \quad \dots(5)$$

where,

$$A_i = F_{X_i}(X^0, U) = \frac{\partial F}{\partial X_i}(X^0, U) \quad \dots(6)$$

$$dl = l - F(X^0, U) \quad \dots(7)$$

A and R are known coefficient matrices of the deterministic unknown vectors. R is given by Elssfeller and Heins (1985) as,

$$R(t) = N(t_0) P^T N^T(t) R_3(-\theta(t)) S(t) \quad \dots(8)$$

where N is the nutation matrix, P is the precession matrix, R is the earth-rotation matrix, S is the matrix of the pole coordinate, θ is the sidereal time of Greenwich, t is the time variable and t_0 is the initial time of orbit integration.

In equation (4), n is the noise of the observable data and t contains the disturbing potential (stochastic vector) or signal. By considering equation (4) as a general least square collocation model defined by Moritz(1980), the solution of dX and t is obtained, that is,

$$n^T C_{nn}^{-1} n + t^T K_{tt}^{-1} t = \min \quad \dots(9)$$

Here, C_{nn} and K_{tt} are the appropriate covariance matrices of n and t, respectively. The estimators for dX and t is given by,

$$dX = (A^T D^{-1} A)^{-1} A^T D^{-1} dl \quad \dots(10)$$

$$t = K_{tt}^{-1} R^T D^{-1} (dl - AdX) \quad \dots(11)$$

where element D is given by,

$$D = C_{nn} + RK_{tt}R^T \quad \dots(12)$$

By using equation (9), the residual or disturbing potential and its functionals at tide gauges could be solved. The error estimates for dX and t is given by,

$$E_{xx} = (A^T D^{-1} A)^{-1} \quad \dots(13)$$

$$E_{tt} = K_{tt} - K_{tt} R^T D^{-1} (I - A E_{xx} A^T D^{-1}) R K_{tt} \quad \dots(14)$$

A system of equations could be expressed by using Stepwise Collocation procedure. Based on the basic equation as in equation (4), we have all linear observation equations of the quoted problem, containing of,

$$\begin{aligned} dl &= [dP(t_i, t_j), dW_{ij}, \delta g_i, d\phi_i, d\lambda_i, dH_i]^T \\ X &= [dp^T, d\tau(t_i), \delta z_i, \dots] \\ t &= [T_i, \partial T / \partial r_i, \dots] \end{aligned} \quad \dots(15)$$

where,

- $dP(t_i, t_j)$ = variation of potential difference between two tide gauges i, j at time t_1, t_2 corrected to geopotential unit as observation (altimeter data).
- dW_{ij} = variation of potential difference (in vectorial form) at two tide gauges.
- $d\phi_i, d\lambda_i$ = ellipsoidal latitude and ellipsoidal longitude, respectively (GPS data)
- dH_i = ellipsoidal height differences.
- δg_i = gravity anomalies
- dp = dynamic parameter of the satellite altimetry motion
- $\delta T(t_i)$ = Sea surface topography
- δz_i = height offset of the tide gauges from the geoid
- $\partial T / \partial r$ = variation of disturbing potential with respect to T_i (spherical coordinate of radius r)

From the solution of normal equations we could get the height offset of the tide gauges from the geoid, the sea surface topography, the orbit of the satellite altimeter and also the residual (or disturbing potential and it functionals) at the tide gauges. It should be mentioned that the number of the unknowns could be minimised by expanding the dT_i in a series with a few coefficients as functions of horizontal position, especially when satellite altimeter data is only available near the tide gauges.

The face of the Integrated Geodesy Adjustment is given in figure 2.0. Figure 3.0 shows the distribution of the available data (or to be made available) that contribute to the vertical datum problem in the country.

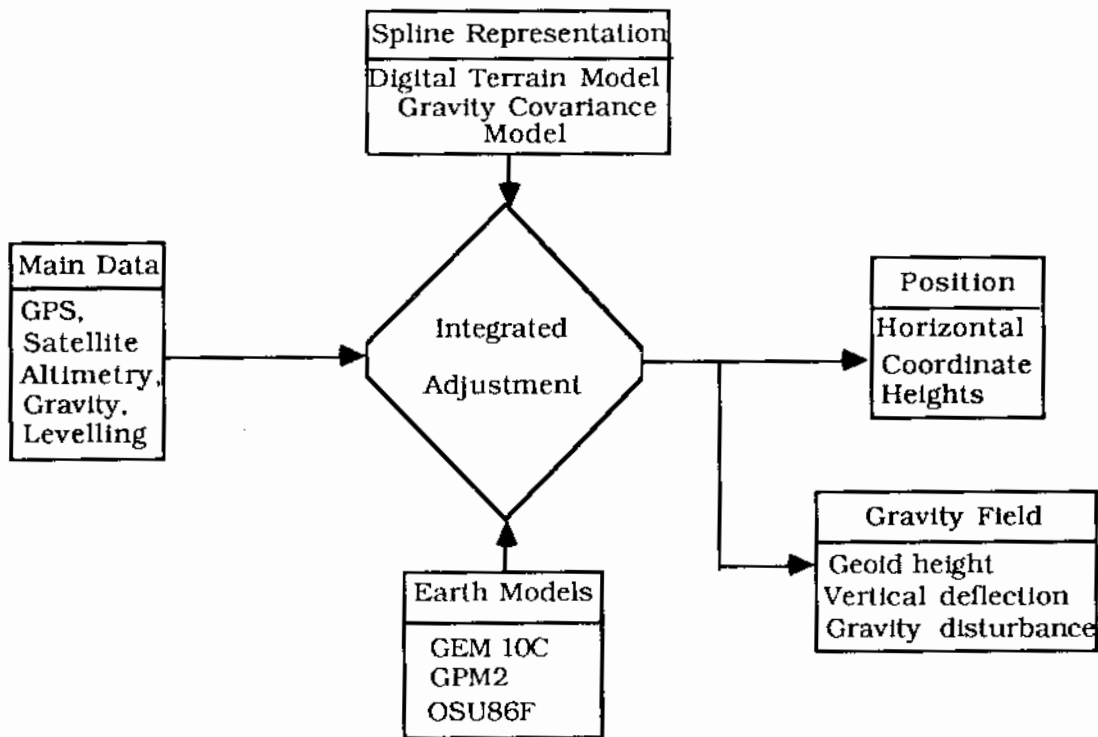


Figure : 2.0
Integrated Solution

5.0 Conclusion

With the advancement of GPS technology, a new treatment of the satellite altimetry, the use of wealth information of the earth model based on gravity data, digital terrain model and a high-degree and order spherical harmonics, the vertical datum problem could be solved. Here, the derived Integrated Geodesy Adjustment model offers a way of computing vertical reference whereby the consideration of noise in the observable data and the resulting error statistics provide a tool for a real assessment of the computed quantities. This is an important requirement for the absolute definition of heights over the country.

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