Height Determination Using GPS Data, Local Geoid and Global Geopotential Models

by

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Abstract

Orthometric heights are normally derived using the spirit levelling. This requires the spirit level equipment to be set up from point to point along a levelling line which is a time consuming and tedious task. GPS offers a new alternative in orthometric height determination very accurately over a comparatively short period. The ellipsoidal height derived from GPS technique can be transformed into orthometric height if we know the geoidal height normally derived from a gravimetric geoid of the area. Unfortunately, we have yet to compute an accurate gravimetric geoid for such purpose, largely due to non-existence of gravity data for a larger part of the country. As an alternative, a study was undertaken to look into the feasibility of using a simple local geoid solution and a global geopotential geoid model solution. The data used in this study consist of GPS data and known orthometric height of several GPS points. This paper present some of the results obtained so far in estimating orthometric height from GPS data in local environment.

1.0 INTRODUCTION

The position of points derived from GPS measurements are usually computed in a three-dimensional Cartesian coordinate system, and are then transformed into the more recognisable geodetic latitudes (ϕ), longitudes (λ), ellipsoidal heights (h) or in term of geodetic coordinate differences $\Delta\phi$, $\Delta\lambda$ and Δh. GPS ellipsoidal heights are very useful for deformation and subsidence studies and other applications where the emphasis is not so much in locating a precise point in space as in the relative change of height from one time epoch to another. It is, however, the case that the ellipsoidal heights delivered by GPS are not the same as those historically obtained with spirit levelling (providing *orthometric heights*). Conventionally, topographic maps, engineering design and construction project plans, usually depict relief by means of orthometric height. Thus, the application of GPS will be further extended if accurate transformations between GPS ellipsoidal height differences and the orthometric height differences can be realised. This can be accomplished on the condition that we know the *geoidal height*, or rather, the geoidal height difference which relates the orthometric height difference to the GPS ellipsoidal height difference. Hence today, a great deal of interest is being shown in the development of the geoid models which are important to provide the necessary geoid height to transform GPS ellipsoidal heights to orthometric heights.

2.0 GPS HEIGHTING

The basic results of the precise differential GPS survey of a baseline are the Cartesian coordinate differences ΔX, ΔY and ΔZ. Baselines connecting the observed GPS points are then put through a network adjustment such as L3D-HEIGHT (Khairul, 1994). The resulting X, Y, and Z coordinates of the GPS points are then transformed, using a reference ellipsoid, into geodetic coordinates in terms of latitude (ϕ), longitude (λ) and ellipsoidal height (h). The orthometric height (H) is related to the ellipsoidal height (h) by the following relation:

$$H = h - N \quad \text{or,} \qquad [1]$$

$$H = h_{GPS} - N_{MODEL} \qquad [2]$$

where,

h_{GPS} is the GPS derived ellipsoidal height, and,

N_{MODEL} is the geoidal height derived from a chosen geoid model,

or by the relative approach, the orthometric height difference between two GPS points may be deduced from:

$$\Delta H = \Delta h_{GPS} - \Delta N_{MODEL} \qquad [3]$$

From the expressions above, the errors in H in eqn. [2] will depend upon the accuracy of the parameters used in its evaluation. It is generally known that, the differences in h between two points measured simultaneously by GPS are much more precise than h at either of the points. This is because of the presence of systematic errors which, being significantly the same at the two points, cancels in the difference. Similarly, ΔN_{MODEL}, is much more precise than the geoid height at either points. This means that for determination requiring highest precision, the approach implied in eqn. [3] is preferred to that in eqn. [2].

3.0 DERIVATION OF GEOIDAL HEIGHT

The most precise method of obtaining accurate geoidal height is using gravimetric observations. Numerical integration of gravimetric observations using Stokes integral equation provides us with a local gravimetric geoid solution. Unfortunately, we have yet to compute our very own precise gravimetric geoid. The reasons behind it are many but the most significant one is related to the difficulties encountered in observing gravity values in large parts of the country due to the topography. However, plans are underway to utilised modern approach in gravimetric measurement such as using an airborn gravimeter. So when no local gravimetric geoid solution is available, a local geoid surface fitting solution may be employed as an alternative for small area of gentle undulation, but for large area, a global geoid solution should be used instead.

3.1 Local Geoid Surface Fitting Solutions

This solution involves the use of a local geoid surface model using a surface fitting procedure. The fundamental theory of this solution is based on the following three assumptions:

(i) the adjusted GPS observations are of very high quality and considered to be exact,
(ii) the orthometric heights of at least three GPS stations in the network are known and also considered to be exact,
(iii) the area involved is small and that the geoid features does not vary rapidly,

If we have three or more GPS points with known orthometric heights (referred here as Height Control Point, (HCP), then a local geoid surface solution using a surface fitting model can be employed. This is accomplished by taking N_i as a function of the position of each HCP in the network. The surface fitting model would take the following mathematical form:

$$N_i = F(\phi_i, \lambda_i) = a_0 + a_1 x_i + a_2 y_i + a_3 x_i y_i + \dots\dots\dots \qquad [4]$$

where;

a_0, a_1, a_2, a_3 are the unknown model coefficients,

$x_i = (\phi_i - \phi_o)\rho_m/\mu$

$y_i = (\lambda_i - \lambda_o)v_m \cos\phi_m/\mu$

$\phi_m = (\phi_i + \phi_o)/2$

and;

ϕ_i, λ_i are the latitude and longitude of station i,

ϕ_o, λ_o are the latitude and longitude of a point chosen as the origin,

ρ_m is the prime vertical radius of curvature at mid-latitude,

v_m is the meridian radius of curvature at mid-latitude,

ϕ_m is the mid-latitude between ϕ_i and ϕ_o, and,

$\mu = 206264.806''$

When using 3 HCPs, eqn. [4] will represent a simple plane surface passing through all the three points on the surface of the geoid. The east-west and the north-south tilt of the plane surface are designated by the model coefficients a_0, a_1 and a_2. Once these coefficients are computed using the least squares method, the unknown orthometric height of the other GPS points can then be derived. If more than three HCPs are available, a more complex (curve) surface can be modelled in place of the plane surface.

3.2 Global Geoid Solutions

Global geoid solutions are obtained from global geopotential models which are given as a set of coefficients consisting of a series of spherical harmonic functions. The coefficients of the various terms in the series are determined using a combination of satellite orbit analyses (for the long wavelength geoid features), terrestrial gravity (medium to short wavelength features) and geoidal heights measured by satellite altimetry over the ocean (medium to short wavelength features).

Some global geopotential models are derived from satellite observations only and are known as satellite-only solutions. These models such as GEM9(Lerch et al., 1979), GEM-L2(Lerch et al., 1982) and GEM-T2(Marsh et al., 1989) involve only low-order spherical harmonics and thus contain relatively few coefficients. Other global geopotential models which are known as combined solutions are obtained by adding surface gravimetry and altimetry data to the satellite-only solutions and they usually contain more coefficients. For example, the development of the GEM10B(Lerch et al.,1981) model are based on the GEM-9 satellite-only model. OSU86F(Rapp and Cruz, 1986) is based on the GEM-L2 model, while OSU89B(Rapp and Pavlis, 1990) and OSU91A(Rapp et al., 1991) are based on the GEM-T2 model.

The geoidal height N_{GM} from a global geoid solution is computed from a set of normalised geopotential coefficients using the following equation:

$$N_{GM} = \frac{GM}{r\gamma} \sum_{n=2}^{n_{MAX}} \left(\frac{a}{r}\right)^n \sum_{m=0}^{n} \left[\overline{C}^*_{nm} \cos m\lambda + \overline{S}_{nm} \sin m\lambda\right] \overline{P}_{nm}(\sin\phi) \qquad [5]$$

where,

n_{MAX} is the maximum degree at which the coefficients are known.

$\overline{C}^{*}_{nm}$ are the $\overline{C}_{nm}$ less the zonal coefficients of the the normal potential of the selected reference ellipsoid.

G is the gravitational constant.
M is the mass of the earth, including the atmosphere.
a is the earth's equatorial radius.
r is the distance from the earth's centre of mass.
ϕ, λ are the geocentric latitude and longitude.
$\overline{P}_{nm}(\sin\phi)$ is the normalised associated Lagendre function.
g is the normal gravity.
n, m are the degree and order respectively.

Generally, the more coefficients there are in a model, the more detailed the model usually is since it contains shorter wavelength information of the earth's gravity field. This means that in general, the best solution to use is one that has determined up to the maximum degree and order of 360. In this study, the global geopotential model adopted is the OSU91A which was developed using 30' by 30' mean gravity anomalies derived from terrestrial and altimetric data (Rapp et al., 1991). From previous study conducted by Ahmad and Kearsley(1994), it was found that this model is best suited for Malaysia. One of the reason for this may be due to the utilization of gravity data observed over various parts of Malaysia in the OSU91A solution. Once the geoidal heights N_{GM} have been derived, the orthometric height of GPS stations in the network can simply be computed using eqn. [1].

4.0 THE EXPERIMENT

4.1 The GPS/Height Network

In order to test and evaluate the proposed method of height determination, a network of points with known heights is clearly needed. As such a network of 10 points with known heights was established within UTM campus for this purpose. The distribution of the points is shown in Figure[1.0]. The heights of the 10 points are derived using the conventional levelling method from two bench marks (i.e., N184 and N185) previously established in the campus and this is shown in Table[2.0]. The GPS observations were made using three Ashtech™ and one Topcon™ receivers. A total of 24 baselines were processed using the GPPS™ post-processing softwares.

Table 1.0 Height of Known Points

STATION	HEIGHT (m)
BM 02	18.478
BM 03	25.897
BM 04	32.139
BM 05	39.412
BM 06	33.958
BM 07	13.813
BM 08	09.315
BM 09	28.893
BM 10	25.688
BM 11	18.969
BC 10	145.190

4.2 The Tests

Two approaches described previously in (3.1) and (3.2) were tested using the GPS network.

Using geoid surface fitting technique

With reference to eqn.[4], a number of surface fitting models can be formed subject to the number of known HCP available. For this study three models were used. The models takes the following form:

$$\text{Model P1 : } N_i = a_0 + a_1 x_i + a_2 y_i \qquad [6]$$

$$\text{Model P2 : } N_i = a_0 + a_1 x_i + a_2 y_i + a_3 x_i y_i \qquad [7]$$

$$\text{Model P3 : } N_i = a_0 + a_1 x_i + a_2 y_i + a_3 \Delta h_i \qquad [8]$$

In the model P3, $\Delta h_i = h_i - h_0$, h_i being the ellipsoidal height of the GPS point and h_0 is the ellipsoidal height at the point selected as the origin. The rational of adding the height term Δh in the model is based on the assumption that the geoid undulation generally follow approximately the topography of the area.

The first model (P1) requires at least 3 HCP to form the surface model. This model is basically a plane surface that passes throught all the three points on the surface of the geoid. It is generally expected that this kind of model is most suitable for small area where the terrain is considered flat in nature. Table [2.0] shows the estimated orthometric height derived using this model. The HCPs used consist of BC10, BM08 and BM11 selected according to their location and forming a triangle enclosing the test area. The r.m.s calculated from the difference between the true and estimated height for six points is found to be 10.8 cm.

Table 2.0 Solution using model P1 and three known heights

STATION	GEODETIC COORDINATE (WGS 84)			TRUE ORTH. HEIGHT (m)	ESTIMATED HEIGHT(m)	DIFF. (m)
	ϕ	λ	h			
BM 03	01 33 34.1931	103 38 01.1370	32.0257	25.897	25.906	0.009
BM 04	01 33 54.6876	103 37 43.2444	38.1366	32.139	31.947	0.192
BM 05	01 33 54.0404	103 38 01.8391	45.4659	39.412	39.327	0.085
BM 06	01 33 29.9165	103 38 13.1441	40.4659	33.958	34.037	0.079
BM 07	01 33 22.0442	103 38 26.5890	19.9838	13.813	13.945	0.132
BM 10	01 33 46.2812	103 38 21.2902	31.8193	25.688	25.740	0.052
RMS = 10.8 cm						

The second model P2 was tested to see if it fits more closely with the actual geoid surface of the area. This is made apparent from the use of the fourth term in the model which will represent any curvature of the geoid surface. In this model at least four known heights are needed to fit the geoid surface. The four points chosen are BC10, BM02, BM09 and BM11. The computed orthometric heights are shown in Table[3.0]. The r.m.s computed from this model is 3.5 cm which is an improvement over model P1.

Table 3.0 Solution using model P2 and four known heights

STATION	GEODETIC COORDINATE (WGS 84)			TRUE ORTH. HEIGHT (m)	ESTIMATED HEIGHT(m)	DIFF. (m)
	ϕ	λ	h			
BM 03	01 33 34.1931	103 38 01.1370	32.0257	25.897	25.908	0.011
BM 04	01 33 54.6876	103 37 43.2444	38.1366	32.139	32.187	0.048
BM 05	01 33 54.0404	103 38 01.8391	45.4659	39.412	39.459	0.047
BM 06	01 33 29.9165	103 38 13.1441	40.4659	33.958	33.961	0.003
BM 07	01 33 22.0442	103 38 26.5890	19.9838	13.813	13.773	0.040
BM 10	01 33 46.2812	103 38 21.2902	31.8193	25.688	25.723	0.035
RMS = 3.5 cm						

The third model P3, which is a slight variation from model P2 was also used in this experiment. This model contain the term Δh_i which refers to the difference between the ellipsoidal height of each GPS point and the average ellipsoidal height within the test network.This is to reflect the assumption suggesting the geoid generally follows the terrain. Again similar computation steps as above were taken. Table [4.0] shows the result computed using 4 HCPs consisting of BC10, BM02, BM09 and BM11. The r.m.s computed is found to be 6.6 cm and it is apparent that the use of a height term in the model does not contribute any significant improvement to the result compared to those given by model P2.

Table 4.0 Solution using model P3 and four known heights

STATION	GEODETIC COORDINATE (WGS 84)			TRUE ORTH. HEIGHT (m)	ESTIMATED HEIGHT(m)	DIFF. (m)
	ϕ	λ	h			
BM 03	01 33 34.1931	103 38 01.1370	32.0257	25.897	25.924	0.027
BM 04	01 33 54.6876	103 37 43.2444	38.1366	32.139	32.200	0.061
BM 05	01 33 54.0404	103 38 01.8391	45.4659	39.412	39.499	0.087
BM 06	01 33 29.9165	103 38 13.1441	40.4659	33.958	33.960	0.002
BM 07	01 33 22.0442	103 38 26.5890	19.9838	13.813	13.793	0.020
BM 10	01 33 46.2812	103 38 21.2902	31.8193	25.688	25.806	0.118
RMS = 6.6 cm						

Using Global Geopotential Model

Global geopotential model as discussed previously can be used to derive the geoidal height to correct the ellipsoidal height to give us the orthometric height. The geoidal height is computed using eqn.[5] using a set of coefficients. As discussed previously there are quite a number of global geopotential model available that can be utilised but for this study OSU91A(Rapp et al.) coefficients were used. Table[5.0] shows the orthometric heights derived using the OSU91A coefficients. The r.m.s computed from this model is about 82 cm and this accuracy is not suitable for most engineering application.

Table 5.0 Solution using global geopotential model (OSU91A) only

STATION	GEODETIC COORDINATE (WGS 84) ϕ	λ	h	TRUE ORTH. HEIGHT (m)	ESTIMATED HEIGHT(m)	DIFF. (m)
BM 03	01 33 34.1931	103 38 01.1370	32.0257	25.897	25.109	0.788
BM 04	01 33 54.6876	103 37 43.2444	38.1366	32.139	31.242	0.897
BM 05	01 33 54.0404	103 38 01.8391	45.4659	39.412	38.550	0.862
BM 06	01 33 29.9165	103 38 13.1441	40.4659	33.958	33.188	0.770
BM 07	01 33 22.0442	103 38 26.5890	19.9838	13.813	13.036	0.777
BM 10	01 33 46.2812	103 38 21.2902	31.8193	25.688	24.880	0.808
RMS = 81.8 cm						

A strategy to improve the orthometric height estimation using the global geopotential model was attempted. This is because the geoidal height computed using the solution described above may contain biases due to several factors, such as the problem arising from the differences in the GPS and geoid model datums. These biases can be reduced or absorbed by implementing some kind of transformation procedure such as that used by Forsberg et al. (1990). The geoid change ($N' - N_{MODEL}$) due to these biases can be expressed in geodetic coordinates in the form of a regression formula (ibid.):

$$N' - N_{MODEL} = a_1 + a_2 \cos\phi \cos\lambda + a_3 \cos\phi \sin\lambda + a_4 \sin\phi \qquad [9]$$

By using at least four known geoidal heights, N', in the above equation, the four coefficients in the regression model can be computed. These coefficients are then used in computing the 'correction' that will be applied to N_{MODEL} in deriving the geoid height and the orthometric heights at the other points. Table[6.0] shows the orthometric heights of GPS points computed in this manner using 4 HCPs(BC10, BM02, BM09 and BM11). The resulting r.m.s of 10.4 cm signifies a significant improvement in the height estimation.

Table 6.0 Solution using OSU91A and four known heights

STATION	GEODETIC COORDINATE (WGS 84) ϕ	λ	h	TRUE ORTH. HEIGHT (m)	ESTIMATED HEIGHT(m)	DIFF. (m)
BM 03	01 33 34.1931	103 38 01.1370	32.0257	25.897	25.895	0.002
BM 04	01 33 54.6876	103 37 43.2444	38.1366	32.139	32.048	0.091
BM 05	01 33 54.0404	103 38 01.8391	45.4659	39.412	39.237	0.175
BM 06	01 33 29.9165	103 38 13.1441	40.4659	33.958	33.949	0.009
BM 07	01 33 22.0442	103 38 26.5890	19.9838	13.813	13.831	0.018
BM 10	01 33 46.2812	103 38 21.2902	31.8193	25.688	25.525	0.163
RMS = 10.4 cm						

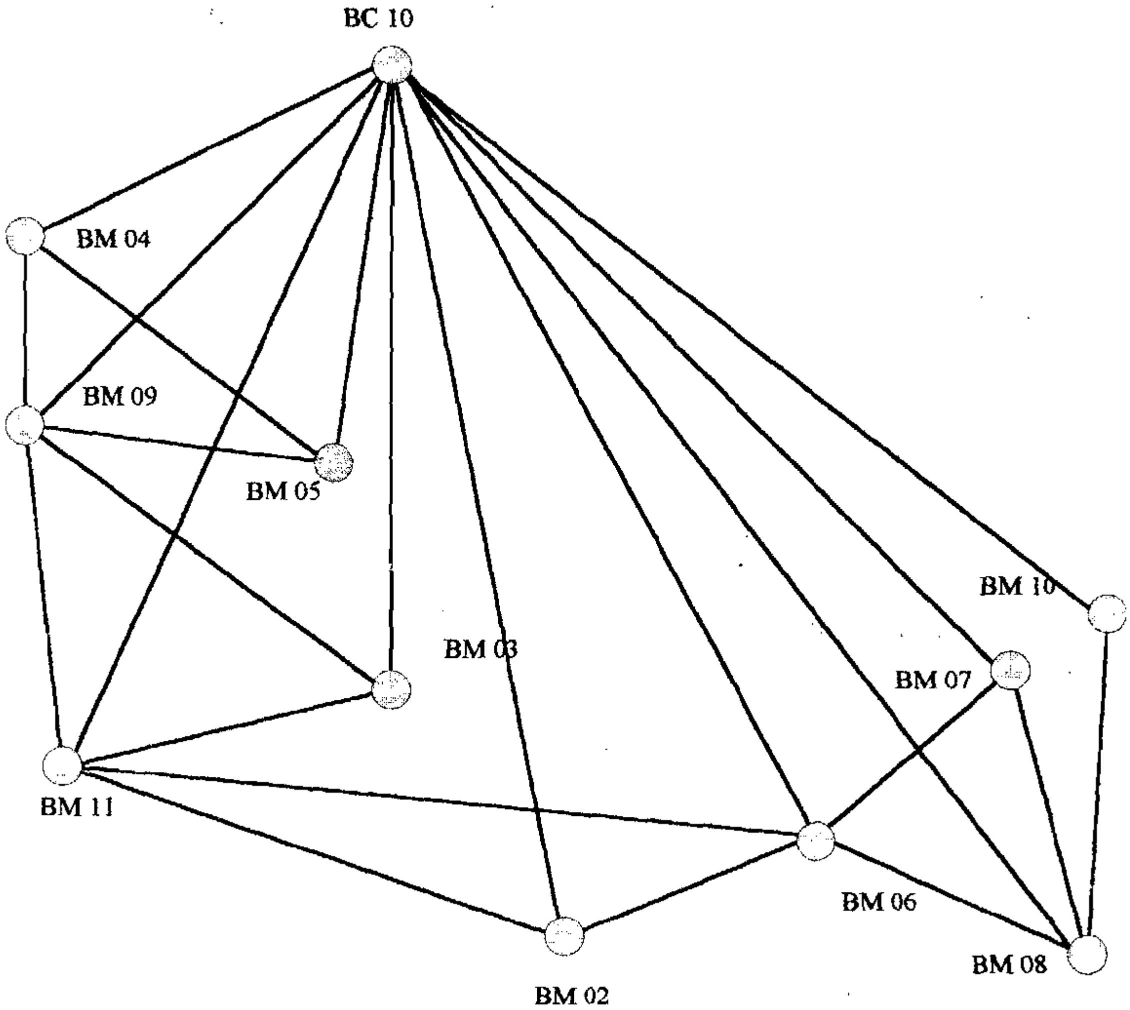

Figure 1.0 The Test GPS Network

5.0 CONCLUSIONS

Based on the tests described above, the following conclusions can be made:

- For small area such as that used in this study, the use of a simple polynomial model (such as model P2) to fit the local geoid surface is adequate in providing estimated orthometric height. The expected accuracy in height determination using this approach is about 10 cm or better. This level of accuracy is more than adequate for most engineering applications.
- For small area, using geoidal height from a global geopotential model may give an accuracy of about 80 cm to 1 metre to the height determination. This level of accuracy is below the requirement of many engineering applications.
- Using a linear regression to overcome biases that may arise from using a global geopotentail model does contribute a significant improvement on the height estimation, but the accuracy level gained is almost on par with that using a simple plane model depicted by model P1.
- For some engineering applications, the use of GPS data in conjunction with additional known height of several points has the potential of replacing the conventional spirit levelling for height determination.

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