Buletin Geoinformasi, Jld 1, No. 3, ms. 112 – 118, Disember 1997

IMPROVING GEOIDAL HEIGHT ESTIMATES FROM GLOBAL GEOPOTENTIAL MODEL USING REGRESSION MODEL AND GPS DATA

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Abstract

Conventionally, for most application, position of a point is often referred to the geoid as the reference surface. Thus there is an important need for the knowledge of the geoid undulation in the area where positioning tasks is performed. This requirement is made more apparent with the advent of high precision using GPS where the resulting ellipsoid height must be converted to orthometric height. An ideal solution is to use a precise gravimetric solution where the geoidal height at each GPS point is computed and applied. Unfortunately, at the moment there is no such solution available in Malaysia. However, efforts are currently being made to develop a precise gravimetric geoid. For the time being, an alternative method would have to be use and the global geopotential model is one of them. In order to increase the accuracy of computed geoid height from the geopotential model, a regression model is used in conjunction with the GPS data. The resulting accuracy estimates of the geoid height determination increases from around 60 cm to about 10 cm level.

1.0 INTRODUCTION

The advent of high precision relative positioning by use of the Global Positioning System (GPS) has opened up an alternative to the classical method of height determination. Using GPS, it is now possible to determine routinely, very accurate three-dimensional Cartesian coordinate differences (or *baseline* vectors) between observation points. This three-dimensional capability therefore provides the possibility of using GPS for height determination as well as for (horizontal) positioning.

Conventionally, topographic maps, engineering design and construction project plans, usually depict relief by means of orthometric height, which differs from the GPS derived ellipsoidal height. Thus, the application of GPS will be further extended if accurate transformations between GPS ellipsoidal height and the orthometric height can be realized. This can be accomplished on the condition that we know the geoid height, which relates the orthometric height to the GPS ellipsoidal height and this requires the knowledge of the geoid.

The geoid information can be obtained by the use of one of the following geoid solutions, namely; using a plane surface model, a gravimetric geoid model, and a global geoid model. For small area of less than 10 km, which is not too hilly and the gravity field is smooth, a plane surface can be used to approximate the geoid using control points with known orthometric heights (Hajela, 1990; and Khairul, 1993). For a more accurate estimates, a precise gravimetric solution is required but unfortunately it is still not yet available in Malaysia and work is currently undertaken to rectify this situation. An alternative approach would be to use a global geopotential model to estimate the geoidal height. Khairul (1993) have shown that the accuracy of the height determined by using a global geoid model in Malaysia is about 0.5 to 1.0m. In order to improve the height estimates, a regression model employing four parameters were tested using a GPS network at UTM. This paper attempts to report on the results obtained from the study.

2.0 ORTHOMETRIC HEIGHT ESTIMATION

2.1 GPS Heighting

The position of points derived from GPS measurements are usually computed in a three-dimensional Cartesian coordinate system. The basic results of the precise differential GPS survey of a baseline are the Cartesian coordinate differences ΔX, ΔY and ΔZ. Baselines connecting the observed GPS points are then put through a network adjustment such as L3D-HEIGHT (Khairul, 1994) or GEOLAB (Bitwise,1991). The resulting X, Y, and Z coordinates of the GPS points are then transformed, using a reference ellipsoid, into geodetic coordinates in terms of latitude (ϕ), longitude (λ) and ellipsoidal height (h). The following relation relates the orthometric height (H) to the ellipsoidal height (h) :

$$H = h - N \qquad \text{or,} \tag{1}$$

$$H = h_{GPS} - N_{MODEL} \tag{2}$$

where,

h_{GPS} is the GPS derived ellipsoidal height, and,

N_{MODEL} is the geoidal height derived from a geoid model.

or by the relative approach, the orthometric height difference between two GPS points may be deduced from:

$$\Delta H = \Delta h_{GPS} - \Delta N_{MODEL} \tag{3}$$

From the expressions above, the errors in H in eqn. [2] will depend upon the accuracy of the parameters used in its evaluation. It is generally known that, the differences in h between two points measured simultaneously by GPS are much more precise than h at either of the points. This is because of the presence of systematic errors which, being significantly the same at the two points, cancels in the difference. Similarly, ΔN_{MODEL}, is much more precise than the geoid height at either points. This means that for determination requiring highest precision, the approach implied in eqn. [3] is preferred to that in eqn. [2].

2.2 Derivation of Geoidal Height

One way to derive the geoidal height is by employing a global geoid solution. Global geoid solutions are obtained from global geopotential models, which are given as a set of coefficients consisting of a series of spherical harmonic functions. The coefficients of the various terms in the series are determined using a combination of satellite orbit analyses (for the long wavelength geoid features), terrestrial gravity (medium to short wavelength features) and geoid heights measured by satellite altimetry over the ocean (medium to short wavelength features). The geoidal height from a global geoid model N_{GM} is computed from a set of normalized geopotential coefficients using the following equation:

$$N_{GM} = \frac{GM}{r\gamma} \sum_{n=2}^{n_{MAX}} \left(\frac{a}{r}\right)^{n} \sum_{m=0}^{n} \left[\overline{C}^{*}_{nm} \cos m\lambda + \overline{S}_{nm} \sin m\lambda\right] \overline{P}_{nm}(\sin\phi) \tag{4}$$

where,

n_{MAX} is the maximum degree at which the coefficients are known.

$\overline{C}^{*}_{nm}$ are the $\overline{C}_{nm}$ less the zonal coefficients of the the normal potential of the selected reference ellipsoid.

G is the gravitational constant.

M is the mass of the earth, including the atmosphere.

a is the earth's equatorial radius.

r is the distance from the earth's center of mass.

ϕ, λ are the geocentric latitude and longitude.

$P_{nm}(\sin \phi)$ is the normalized asociated Legendre function

γ is the normal gravity

n, m is the degree and order respectively

Generally, the more coefficients there are in a model, the more detailed the model usually is since it contains shorter wavelength information of the earth's gravity field. This means that in general, the best solution to use is one that has been determined up to the maximum degree and order of 360, which, theoretically at least, can model features in the geoid with half wavelength of 0.5 degrees or 55 km. In this study, the global geopotential model adopted is the OSU91A which was developed using 30' by 30' mean gravity anomalies derived from terrestrial and altimetric data (Rapp et al., 1991). These data are then combined with GEM-T2 (Marsh et al., 1989) to produce the model complete to degree and order of 360. This model was chosen on the basis that it is the most up-to-date global geopotential model made available.

2.3 Improving the Geoid Height Estimates

The geoidal height computed using the solution described above may contain biases due to several factors, such as the problem arising from the differences in the GPS and geoid model datums. This is especially apparent in the case of using a global solution where the biases may consist of long-wavelength errors contributed by geopotential model errors, bad gravity coverage and a bad elevation datum for the gravity observations, since barometric levelling is commonly used. These biases can be reduced or absorbed by implementing some kind of transformation procedure such as that used by Forsberg et al. (1990). The geoid change ($N' - N_{MODEL}$) due to these biases can be expressed in geodetic coordinates in the form of a regression formula (ibid.):

$$N' - N_{MODEL} = a_1 + a_2 \cos\phi \cos\lambda + a_3 \cos\phi \sin\lambda + a_4 \sin\phi \qquad [5]$$

By using at least four known geoidal heights, N', in the above equation, the four coefficients in the regression model can be computed. These coefficients are then used in computing the 'correction' that will be applied to N_{MODEL} in deriving the geoid height and the orthometric heights at the other points.

3.0 THE EXPERIMENT

3.1 The GPS/Height Network

In order to test and evaluate the proposed method of height determination, a network of points with known heights is clearly needed. A network of 10 points with known heights was established within UTM campus for this purpose. The heights of the 10 points are derived using the conventional levelling method. Figure [1.0] shows the distribution of all the 10 points. The GPS observations were made using three Ashtech™ and one Topcon™ receivers. A total of 24 baselines were processed using the GPPS™ post-processing software.

3.2 The Tests

Using Global Geopotential Model

Global geopotential model as discussed previously can be used to derive the geoidal height to correct the ellipsoidal height to give us the orthometric height. The geoidal height is computed using eqn.[4.0] using a set of coefficients. As discussed previously there are quite a number of global geopotential model available that can be utilized but for this study OSU91A(Rapp et al.) coefficients were used. Table[1.0] shows the orthometric heights derived using the OSU91A coefficients. The r.m.s computed from this model is about 82 cm and this accuracy is not suitable for most engineering application.

A strategy to improve the orthometric height estimation using the global geopotential model was attempted. This is because the geoidal height computed using the solution described above may contain biases due to several factors, such as the problem arising from the differences in the GPS and geoid model datums. These biases can be reduced or absorbed by implementing some kind of transformation procedure such as that used by Forsberg et al. (1990) as described in eqn.[5.0]. By using at least four known geoidal heights, N', the four coefficients in the regression model can be computed. These coefficients are then used in computing the 'correction' that will be applied to N_{MODEL} in deriving the geoid height and the orthometric heights at the other points. Table[2.0] shows the orthometric heights of GPS points computed in this manner. The resulting r.m.s of 10.4 cm signifies a significant improvement in the height estimation.

STATION	GEODETIC COORDINATE (WGS 84)			TRUE ORTH. HEIGHT	ESTD. HEIGHT	DIFF.
	ϕ	λ	h (m)	(m)	(m)	(m)
BM 03	01 33 4.1931	103 38 1.1370	32.0257	25.897	25.109	0.788
BM 04	01 33 4.6876	103 37 3.2444	38.1366	32.139	31.242	0.897
BM 05	01 33 4.0404	103 38 1.8391	45.4659	39.412	38.550	0.862
BM 06	01 33 9.9165	103 38 3.1441	40.4659	33.958	33.188	0.770
BM 07	01 33 2.0442	103 38 6.5890	19.9838	13.813	13.036	0.777
BM 10	01 33 6.2812	103 38 1.2902	31.8193	25.688	24.880	0.808
RMS = 81.8 cm						

Table 1.0 Solution using global geopotential model (OSU91A) only

STATION	GEODETIC COORDINATE (WGS 84)			TRUE ORTH. HEIGHT	ESTD. HEIGHT	DIFF.
	ϕ	λ	h	(m)	(m)	(m)
BM 03	01 33 34.1931	103 38 1.1370	32.0257	25.897	25.895	0.002
BM 04	01 33 54.6876	103 37 3.2444	38.1366	32.139	32.048	0.091
BM 05	01 33 54.0404	103 38 1.8391	45.4659	39.412	39.237	0.175
BM 06	01 33 29.9165	103 38 3.1441	40.4659	33.958	33.949	0.009
BM 07	01 33 22.0442	103 38 6.5890	19.9838	13.813	13.831	0.018
BM 10	01 33 46.2812	103 38 1.2902	31.8193	25.688	25.525	0.163
RMS = 10.4 cm						

Table 2.0 Solution using OSU91A and REGRESSION MODEL

4.0 CONCLUSIONS

Based on the tests described above, the following conclusions can be made:

- Using geoidal height from a global geopotential model may give an accuracy of about 80-cm to 1 metre to the height determination. This level of accuracy is below the requirement of many engineering applications.

- Employing a linear regression to overcome biases that may arise from using a global geopotential model does contribute a significant improvement on the height estimation.

- For some engineering applications, the use of GPS data in conjunction with additional known height of several points has the potential of replacing the conventional spirit levelling for height determination.

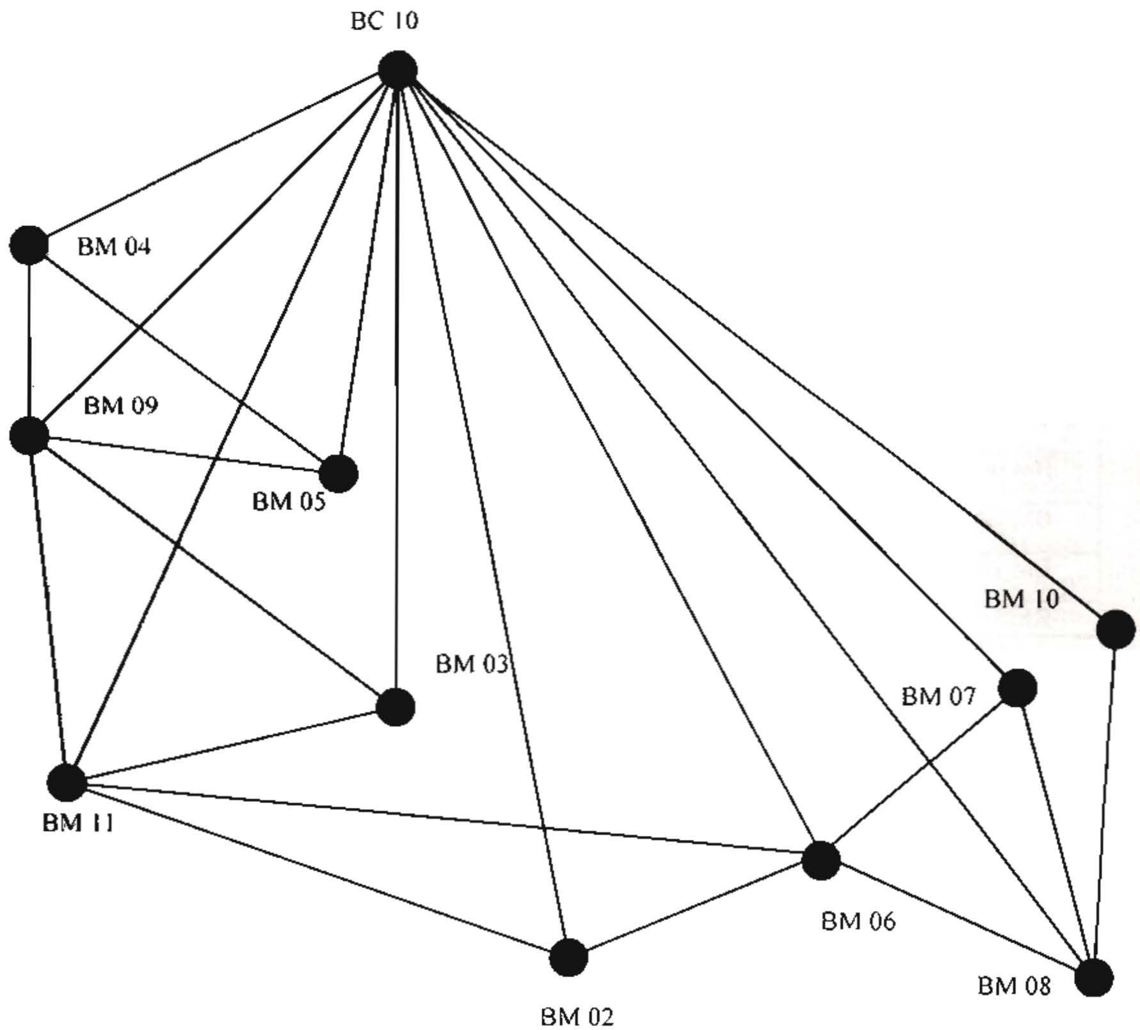

Figure 1.0 The Test GPS Network

Acknowledgements
Assistance received from a number of people in implementing this study is greatly appreciated. Dr. R. H. Rapp for providing the OSU91A geopotential coefficients; and the Research and Development Unit, Universiti Teknologi Malaysia for funding this work.

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