Vehicle Equipment Selection Model for Preliminary Design Using Fuzzy Multi-Criteria Decision Making Method

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Abstract: Equipments selection process is one of the key planning and design activities of marine, land and air vehicles. Equipments are often selected from a set of possible alternatives. More than one decision makers are usually involved and selection of one particular choice is made against a set of strategic selection criteria. Approximate mode of decisionmaking is normally employed and hence selection criteria are subjectively specified. This paper presents the fuzzy approach to the subjective representation of design data and the multi-criteria decision making method to the treatment of subjective criteria rating and ranking of equipment to serve as collective decision making tool.

1.0 Introduction

To reduce cost some systems and equipment fitted to newly designed vehicles are of-the-shelf types. Hence designing vehicles involves the selection of systems and equipment such as engine monitoring and control system for high performance car, propulsion plants for ships and radio and telecommunication systems for aircrafts. They are selected from a wide range of choices and selection is made against a set of selection criteria determined by the designers. It is a multi-criteria decision making process and hence a few important notes worth mentioning.

Firstly, the selection criteria normally cover the three different aspects; technical, economic and environmental. Technical aspect includes elements such as technology level, technical specification, technical complexity and user friendliness. Economic aspect includes elements such as equipment cost, maintenance cost and maintenance level. Environmental aspect includes elements such as safety standard, risk level and environmental friendliness. Therefore, it is obvious that some criteria are subjective in nature while some are of crisp quantifiable values.

Secondly, therefore, and especially at the early stage of the design spiral, decision making in the selection process can be done on an approximation basis. At that stage it is quite acceptable to accept an equipment base on its approximate compliance to the selection criteria determined by the designers. In other words, the equipment complies with the selection criteria to a certain degree only. The designers define the range of specification that represents compliance and hence they are considered as having the expert knowledge on the matter. It is also common for the designers to represent each compliance range linguistically such as 'safe', 'moderately safe', 'very safe' and so on.

Thirdly, a decision is made in a mutually exclusive manner. A decision made on one selection criteria will not effect the decision to be on the next. Neither has it been affected by any decision made earlier. This further reflects the ambiguous nature of the decision making process.

Fuzzy sets theory is a convenient and flexible mathematical tool for dealing with approximation using linguistic description (Avineri, 2000). It has been earlier applied, for example, to the selection of material handling equipment for the mining industry (Deb et al., 2002) and to the selection of advanced manufacturing system for investment evaluation purposes (Karsak, 2001). It employs approximate, rather than exact, modes of reasoning, and therefore incorporates imprecise, linguistic and subjective values. Fuzzy method simply mimics human way of expressing opinions in the simplest way.

The paper applies fuzzy concepts to equipment selection in the preliminary design of marine, land and air vehicles and has been organized in the following manner. Section 2.0 summarises the theoretical foundation upon which the decision making model, methodology and the analytical tools have been based. Section 3.0 describes the proposed model and indicates the fuzzification, fuzzy operations and the algorithm behind the mean operator method. Section 4.0

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discusses strength of the fuzzy multi-criteria decision making concept being applied, the versatility of the model developed and the accuracy of the results and highlights some problems and possible improvement.

2.0 THEORETICAL BACKGROUND

2.1 Fuzzification and Defuzzification Method

A fuzzy set is defined by a function $\mu_A(x)$: $X - [0,1]$ and often denoted by $A = \{(x, \mu(x)) | x \in X\}$. μ_A is a generalised characteristic function (the membership function of the fuzzy set *A*), *x* is one particular element that belongs to *A*, *X* is the universe of discourse. The conditions are $\mu_A(x) = 1$ if *x* is totally in *A*, $\mu_A(x) = 0$, if *x* is totally out of *A* and $0 < \mu_A(x) < 1$ if *x* is partly in *A*.

 A set whose membership function is piecewise continuous is called fuzzy number. A fuzzy number according to the concept of fuzzy set can be represented in a triangular form as in Figure 2 (other forms are trapezoidal and S-shaped). A triangular fuzzy number with a centre *a* may be seen as a fuzzy quantity "*x* is approximately equal to *a*". 'A linguistic variable can be defined as a variable whose values are not numbers, but words or sentences in natural or artificial language' (Karsak, 2001). Linguistic variable such as 'large' or 'small' is taken as a representation of phenomenon too complex to be described using the conventional quantitative terms.

 Therefore within a universe of discourse a linguistic variable represents a range of values that make up a fuzzy set. The universe of discourse can be partitioned into as many linguistic variables as deemed necessary and partitions can overlap as shown in Figure 3. The linguistic variables are usually defined as fuzzy sets with appropriate membership functions (Hong and Lee, 1996). H is a linguistic variable representing a partition that describes a certain phenomenon with a characteristic 'high' in the universe of discourse. In fuzzy set theory membership is a matter of degree. In the above expression $\mu(A)$ is defining the degree of relevant of x to the set A. Membership of x to A is imprecise or vague and $\mu(A)$ is its measure of uncertainty. The fuzzy proposition is true to the degree to which *x* belongs to the fuzzy set.

A symmetric triangular fuzzy number with centre *a* and width $\alpha > 0$ has a membership function of the following form

$$
A(x) = \begin{cases} 1 - \frac{|a - x|}{\alpha} & \text{if } |a - x| \le \alpha \\ 0 & \text{otherwise} \end{cases}
$$
. The notation use is A=(a, \alpha)

 The process of assigning membership functions to fuzzy variables is either intuitive or based on some algorithmic or logical operations (Karsak, 2001). Intuition is simply derived from the capacity of the experts to develop membership functions through their own intelligence, experience and judgement (Hong and Lee, 1996; Karsak, 2001). Triangular membership functions are chosen for application considering their intuitive representation and ease of computation (Karsak, 2001). A fuzzy number can be defuzzified using the centre of gravity method. Figure 4 illustrates the operation of defuzzifying using such method.

2.2 Fuzzy Aggregation and Ranking

 The straight forward method for aggregating fuzzy sets in the context of decision-making uses the aggregating procedures frequently used in utility theory or multi-criteria decision theory (Raj, 1999; Avineri, 2000; Karsak, 2001; Deb et al., 2002]. The method considers a mean value between varieties of goal using average between the optimistic lower bound (minimum degree of membership) and the pessimistic upper bound (the maximum degree of membership). Hence the method is called mean operator method. This method is normally used for analysis involving empirical data and the continuous use of this method shows that it is an adequate model for human aggregation procedures in decision environments.

 The procedure for the mean operator method is as below. Let there be a selection situations where: (i) there is a designer *j* from a group of designers $j = 1$ to *m*, (ii) there is a set of alternative equipment *i* for $i = 1$ to *n* for the vehicle, (iii) the equipment is to be selected using a '2-level selection approach' against; first, putting weightage on *k* strategic subjective selection criteria, and second, putting rating on each alternative for each k for $k = 1$ to p (iv) the designers will use linguistic weighing $W = (VP, P, F, G, VG)$ for the strategic subjective criteria where $VP =$ very poor, $P =$ poor, $F =$ fair, $G =$ good and VG = very good and linguistic rating $R = (VL, L, M, H, VH)$ where VL = very low, $L =$ low, $M =$ medium, $H = high$ and $VH = very high$.

 If two designers are involved in weighing and rating four different equipment the rating and ranking can be tabulated in Table 2. R_{ijk} and W_{kj} are all fuzzy numbers $(R_{ijk}^a, R_{ijk}^b, R_{ijk}^c)$ and $(W_{kj}^a, W_{kj}^b, W_{kj}^c)$ respectively. Aggregating within one particular *k* gives $W = \frac{1}{m} \otimes [W_{j=1} \oplus W_{j=2} \oplus W_{j=3} \oplus ... \oplus W_{j=m}]$ (*W* in (row 3, column 4) of Table 2)

and $R_i = \frac{1}{m} \otimes [R_{i,j=1} \oplus R_{i,j=2} \oplus R_{i,j=3} \oplus ... \oplus R_{i,j=m}]$ (R_i in (row 4, column 4) of Table 2). Aggregating W and R_i for one particular *k* gives $F_i = (W \otimes R_i)$ where F_i is called the fuzzy suitability index. Aggregating across all *k* gives $Fi =$

 $\frac{1}{\sigma} \otimes \left[(R_{i1} \otimes W_1) \oplus (R_{i2} \otimes W_2 \oplus ... \oplus (R_{ip} \otimes W_p) \right]$ *p* \otimes \mid $(R_{i1} \otimes W_1) \oplus (R_{i2} \otimes W_2 \oplus ... \oplus (R_{ip} \otimes$

Table 1 A matrix illustrating fuzzy aggregating operation

Criteria		$k=1$			$k=2$			$k=3$			$k=4$		Σ
Designer	j=1	$i=2$	Σ	j=1	$j=2$	Σ	j=1	$j=2$	Σ	$i = r$	$j=2$	Σ	
Weight	W_{1j}	W_{12}	$\mathbf{W_{k}}$	W_{21}	$\mathbf{w}_{\mathbf{z}}$	W_2	$\mathbf{W_{31}}$	$\rm w_{\rm 32}$	W_3	W_{41}	$W_{\mathcal{Q}}$	W_4	
E apt. $_{i=1}$	$\rm R_{\ddot{\mathbf{i}} \mathbf{k}}$	R_{121}	R_{n}	$\rm R_{II2}$	R_{122}	R_{12}	$\rm R_{II3}$	R_{123}	R_{13}	R_{II4}	R_{124}	R_{14}	R_{ik}
Eqpt. $_{\pm 2}$	$\rm R_{211}$	$\rm R_{221}$	R_{21}	R_{212}	$\rm R_{222}$	R_{22}	R_{213}	R_{223}	$\rm R_{23}$	R_{214}	$\rm R_{224}$	R_{24}	R_{2m}
E apt. μ_3	$\rm R_{311}$	R_{321}	$\rm R_{31}$	R_{312}	$\rm R_{322}$	R_{32}	R_{313}	R_{323}	R_{33}	R_{314}	R_{324}	R_{34k}	R_{3m}
E qpt. $_{i=4}$	R_{411}	R_{21}	$\rm R_{41}$	R_{412}	R_{422}	$R_{\mathcal{Q}}$	R_{413}	R_{423}	R_{q3}	R_{414}	R_{24}	R_{44}	R_{4m}

 Ranking of fuzzy sets is based on '…extracting various features from the fuzzy sets such as its centre of gravity. Raj (1999) and Prodanovic and Simonovic (2002) provides good comments on the various methods of ranking fuzzy numbers. Ranking equation by Chen's method (Deb et al., 2002)

is
$$
V(F_i) = \frac{1}{2} \left\{ \frac{(\delta - x_{\min})}{(x_{\max} - x_{\min})} - (\gamma - \delta) + 1 - \frac{(x_{\max} - \alpha)}{(x_{\max} - x_{\min})} + (\beta - \alpha) \right\}
$$
. The maximising set is

 $M = \{(x, \mu M(x)) | x \in R\}$ with membership function. *k* $M(x) = \left| \frac{(x - x_{\min})}{(x_{\max} - x_{\min})} \right|$ ⎦ $\frac{(x-x_{\min})}{(x-x_{\min})}$ լ $\mathsf I$ $f(x) = \begin{cases} \frac{(x - x_{\min})}{(x_{\max} - x_{\min})} \end{cases}$ max $-\,x$ min $\mu M(x) = \left| \frac{(x - x_{\min})}{(x - x_{\min})} \right|$ for $x_{\max} \ge x \ge x_{\min}$ and

 $\mu_M(x) = 0$, otherwise. The minimising set $N = \{(x, \mu N(x)) | x \in R \}$ is *k* $N(x) = \left| \frac{(x - x_{\text{max}})}{(x_{\text{min}} - x_{\text{max}})} \right|$ ⎦ $\frac{(x-x_{\text{max}})}{(x-x_{\text{max}})}$ ⎣ $\mathsf I$ $f(x) = \begin{cases} \frac{x - x_{\text{max}}}{x_{\text{min}} - x_{\text{max}}} \end{cases}$ min $-\chi$ max $\mu N(x) = \left| \frac{(x - x_{\text{max}})}{(x - x_{\text{max}})} \right|$ for

 $x_{\text{max}} \ge x \ge x_{\text{min}}$ and $\mu_v(x) = 0$ otherwise; where *k* represents planner's preference on level of risk, $k = 1$ when designers are conservative or neutral, $k = 0.5$ when designers are optimist or risk taker, $k = 2$ when designers are pessimist or risk averse, x_{min} = inf D (operator 'inf' represents infimum, which is the global minimum [21]), x_{max} = sup D (operator 'sup' represent supremum, which is the global maximum [21], $D = U_{i=1,n}$, (i.e. the set containing the elements), $D_i = \{x | \mu_{F_i}(x) > 0\}$ (the elements with positive membership degree μ_{F_i}).

3.0 The Proposed Model

 T build up the model to demonstrate the application of the fuzzy multi-criteria decision making let there be a case where systems and equipment to be fitted to a vehicle can be selected based on nine selection criteria presented in Table 2 below. In practice it is also quite common to use a nine-digit likert scale of $1 - 9$; 1 being the least preferred to decide on these selection criteria. In cases where the selection criteria are represented by actual technical specification values and figures, these figures can also be partitioned into several ranges. The figures can be split into nine partitioned to match the likert scale. Fuzzification comprises of deciding what are the minimum and maximum of these figures, the number of partitions, the figures to be represented by each partition, and the linguistic naming of each partition.

Technical	Economic	Environmental & others		
Technology level	Equipment cost	Safety standard		
Operator skill requirement	Maintenance cost	Comfort measures		
Maintenance skill requirement	Licence and upgrading cost	Environmental friendliness		

Table 2: Equipment selection criteria during preliminary design

 If each selection criteria are to be split into five equally spaced partitions and linguistically named as VERY LOW (VL), LOW (L), MEDIUM (M), HIGH (H) and VERY HIGH (VH), its equally spaced overlapping membership function will be of the form as in Figure 3 below. Observe that each linguistic term is representing a range of specification values or likert scale values.

Figure 3: A format of membership function for selection criteria

 To be able to implement the mean operator method proposed by Raj (1999), Avineri (2000), Karsak (2001) and Deb(2002) two unknown are required; (i) *woi* which is the weight of importance of the selection criteria (*sc*), and (ii) *er* the equipment rating for each choice . Thus, for a nine-criteria case

$$
woi = woi''_{sc1}, woi''_{sc2}, woi''_{sc3}, woi''_{sc4}, woi''_{sc5}, woi''_{sc6} woi''_{sc7}, woi''_{sc8}, woi''_{sc9}, \text{ and}
$$

\n
$$
er = er''_{sc1}, er''_{sc2}, er''_{sc3}, er''_{sc4}, er''_{sc5}, er''_{sc6}er''_{sc7}, er''_{sc8}, er''_{sc9}
$$

\nwhere, in fuzzy terms $woi''_{sc} = (woi_{sc_a}, wois_{sc_b}, woi_{sc_c})$ and $er''_{sc} = (er_{sc_a}, er_{sc_b}er_{sc_c})$.

 Referring to Figure 3 the fuzzy *woi* and *er* in likert scale correspond to the designers' selection of linguistic names above as in the Table 3 below. The designers decision on *woi* and *er* can be summarized in Table 4.

LINGUISTIC NAME	woi and er
VL.	(1, 1, 3)
	(1, 3, 5)
М	(3, 5, 7)
H	(5, 7, 9)
/H	

 Table 3: Fuzzy *woi* and *er*

Table 4: Summary of designers' choice of *woi* and *er*

 The fuzzy suitability index (*fsi*) for each equipment choice *ch* is then calculated by multiplying the equipment rating (*er*) with *woi*[″]. The product is fsi' ["] which is a fuzzy number (fsi_a, fsi_b, fsi_c) . The net fsi'' for each *ch* is the normalised sum of (er ["]X *woi*") and divided by 9.

$$
fsi''_{ch} = \frac{1}{9} \otimes \begin{bmatrix} (er''_{sc1} \otimes woi''_{sc1}) \oplus (er''_{sc2} \otimes woi''_{sc2}) \oplus \\ (er''_{sc3} \otimes woi'_{sc3}) \oplus (er''_{sc4} \otimes woi''_{sc4}) \oplus \\ (er''_{sc5} \otimes woi''_{sc5}) \oplus (er''_{sc6} \otimes woi''_{sc6}) \oplus \\ (er''_{sc7} \otimes woi''_{sc7}) \oplus (er''_{sc8} \otimes woi''_{sc8}) \oplus \\ (er''_{sc9} \otimes woi''_{sc6}) \end{bmatrix}
$$

where the calculation of normalized fuzzy suitability index for equipment choice-1 ($fsi^{\dagger}ch_1$) is

$$
fsi''_{ch_1} = \frac{1}{9} \otimes \begin{bmatrix} ([e_{r_{sc1_a}}, e_{r_{sc1_b}}, e_{r_{sc1_c}}) \otimes (woi_{sc1_a}, woi_{sc1_b}, woi_{sc1_c})] \oplus \\ ([e_{r_{sc2_a}}, e_{r_{sc2_b}}, e_{r_{sc3_c}}) \otimes (woi_{sc2_a}, woi_{sc2_b}, woi_{sc2_c})] \oplus \\ ([e_{r_{sc3_a}}, e_{r_{sc3_b}}, e_{r_{sc3_c}}) \otimes (woi_{sc3_a}, woi_{sc3_b}, woi_{sc3_c})] \oplus \\ ([e_{r_{sc4_a}}, e_{r_{sc4_b}}, e_{r_{sc4_c}}) \otimes (woi_{sc4_a}, woi_{sc4_b}, woi_{sc4_c})] \oplus \\ ([e_{r_{sc5_a}}, e_{r_{sc5_b}}, e_{r_{sc5_c}}) \otimes (woi_{sc5_a}, woi_{sc5_b}, woi_{sc5_c})] \oplus \\ ([e_{r_{sc6_a}}, e_{r_{sc6_b}}, e_{r_{sc6_c}}) \otimes (woi_{sc6_a}, woi_{sc6_b}, woi_{sc6_c})] \oplus \\ ([e_{r_{sc8_a}}, e_{r_{sc8_b}}, e_{r_{sc8_c}}) \otimes (woi_{sc8_a}, woi_{sc8_b}, woi_{sc8_c})] \oplus \\ ([e_{r_{sc9_a}}, e_{r_{sc9_b}}, e_{r_{sc9_c}}) \otimes (woi_{sc8_a}, woi_{sc8_b}, woi_{sc8_c})] \oplus \\ ([e_{r_{sc9_a}}, e_{r_{sc9_b}}, e_{r_{sc9_c}}) \otimes (woi_{sc9_a}, woi_{sc9_b}, woi_{sc9_c})]) \end{bmatrix}
$$

 The output will be in a triangular fuzzy form (a, b, c). The calculation is repeated for all the equipment under comparison. The fuzzy suitability indexes are then ranked using Chen's method (Deb et al., 2002) as below

 \mathbf{r}

$$
choice_{ranked} = \frac{1}{2} \left\{ \frac{(c_i - x_{\min})}{(x_{\max} - x_{\min}) - (b_i - c_i)} + 1 - \frac{(x_{\max} - a_i)}{(x_{\max} - x_{\min}) + (b_i - a_i)} \right\}
$$

 \mathbf{r}

where

 $x_{\text{max}} =$ maximum *fsi*_c value from all *ch* x_{\min} = minimum *fsi*_{*a*} value from all *ch*

The output will be in crisp form such that $0 < choice_{ranked} < 1.0$. The best choice would be the equipment with the largest Chen's (Deb et al., 2002) ranking value.

4.0 Discussion and Closing

 The fuzzy multi-criteria decision making concept is an established concept. It has been applied to many areas of application. It is versatile and can be applied at different level of decision making. From the model illustrated above it can be seen that the technique assist decision making in many ways. Firstly, it can serve as a quantitative method to decide on qualitative variables. Secondly, representing a range of values under one linguistic term and processing it in a fuzzy way in actual fact is a mean of taking care of uncertainty in decision making. Thus, a separate exercise of estimating the uncertainty I the decision making process is not required. It is already inbuilt within. Thirdly, and the most important is it adopts the human modes of reasoning that uses approximate, imprecise, linguistic, and subjective values. This makes decision making friendlier.

 The above model can easily be modified to suit a particular situation. The types of selection criteria can be adjusted and so as the number of criteria to be used. The designers would only need to decide on the weight of importance of each selection criteria and the equipment rating to each selection criteria decided upon. The ratings are expert knowledge and hence the designers are always considered as the expert when published data are not available. The repetitive number crunching task can be made simpler using spreadsheet of computer program. This could make data entry and other modifications much faster.

 The final results after being ranked using Chen's method (Deb et al., 2002) is normally arranged in a descending order. The highest in the order is the best equipment being considered. However, if the equipment, for some reasons, has to be foregone the next equipment below should be the next choice. The opportunity forgone of not acquiring the earlier can be estimated as the percentage different of the two Chen's value. Thus, if the first ranked equipment is of Chen's value 0.8964 and the second equipment in the order is of 0.7745 Chen's value, the opportunity foregone of taking the second instead of the first is ((0.8964-0.7745) /0.7745)(100); i.e. 15.74%. the opportunity foregone would be in term of cost saving or ease of maintenance or passenger comfort. These can be deduced from the ratings and weight of importance put in place during the decision making.

 Results accuracy is not normally a focus when this concept is applied. Otherwise it contradicts the intention of adopting approximation, vagueness and ambiguity modes of human reasoning. It is suffice to mention here that the degree of relevant of the whole decision making process depends very much on the weight of importance and the equipment ratings used. It can easily be a rubbish-in-rubbish-out exercise depending on the commitment put by those involved in the decision making. However, technique wise, there are several improvement possible such as using types of membership function more accurate then the triangular type, using more partitions and using adjusted non-equal overlapping membership function where smaller partition are use around data presumed to be more important so that those range of data are not lost.

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