# ENHANCEMENT OF HEIGHT SYSTEM FOR MALAYSIA USING SPACE TECHNOLOGY : <br> THE STUDY OF THE DATUM BIAS INCONSISTENCIES IN PENINSULAR MALAYSIA 

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#### Abstract

The algorithm for orthometric height transfer using GPS has been widely presented. Its practical limitations are mostly due to datum bias inconsistencies and lack of precise geoid. In most applications, datum biases are assumed to be systematic over short baselines and therefore could be eliminated by differential heighting techniques. In this study, optimal algorithms were investigated to model biases between local vertical datum in Peninsular Malaysia and the datums implied by by EGM96, OSU91A and the regional Gravimetric Geoid in South_East Asia.

The study has indicated that local vertical datum is not physically parallel to the datums implied by the above geoids. The shift parameters between the datums implied by the GPS/leveling data, and the EGM96, OSU91A and the gravimetric datums are about $41 \mathrm{~cm},-54 \mathrm{~cm}$ and -8 cm respectively. Also the maximum tilts of the planes fitting the residual geoids above these datums relative to GPS/Leveling datum are of the order of 36, 51 and 33 centimeters per degree. It is therefore necessary to take into account the effect of inconsistent datum bias particularly for baseline height transfer.

The level of accuracy achieved by the bias corrected relative orthometric height differences of the EGM96, OSU91A and the gravimetric geoid models combined with GPS/leveling data for baseline lengths up to 36 km , is sufficient to replace the conventional tedious, time consuming ordinary leveling technique for rapid height transfer for land surveying and engineering applications.


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### 1.0 INTRODUCTION

### 1.1 Background

The need to have a unified vertical reference frame becomes so apparent in the era of information technology. The reliability of information requiring geographical position needs the points on the earth to be defined uniquely and accurately. It is the role of geodesist to provide the reference system which is capable to support the need for present and future advancements in mapping science and information technology.

Unification of vertical reference systems requires both geometrical and physical connections in the study area. Precise geometrical connection is provided from the regional GPS network while the physical relationship is to be provided from the regional or global precise geoid covering the whole of study area.

The algorithm for orthometric height transfer using GPS has been widely presented and used. Its practical limitations are mostly due to datum bias inconsistencies and lack of precise geoid.

For the past 100 years or so, traditional spirit leveling has been a technique of choice in the determination of orthometric heights. It is simple, the operation is effective, the method has remained basically unchanged, and yet it can achieve a remarkable precision. However, the observational time is too lengthy, making it a slow, labour-intensive, painstaking and costly operation. It is also a line operation whereby points whose heights are required need to be interconnected by a series of leveling lines. This makes levelling operation prone to many systematic errors which are difficult to detect and eliminated. Thus in recent times, many efforts were made to develop alternative techniques and technologies to levelling to suit current needs.

Nowadays, the many benefits offered by space based measurements systems such as the Global Positioning System (GPS) have made it a suitable alternative over traditional levelling. Orthometric height determination using GPS is one of the possible applications that is gaining popularity. In recent times, the use of GPS for surveying, engineering and mapping applications is expanding at an astounding rate. This has prompted many countries, including Malaysia to not only upgrade their existing reference systems to be GPS-compatible, but also to seriously look into the potential use of GPS in heighting.

However, there are several problems associated with the use of GPS in vertical positioning. First GPS gives elevations above a reference ellipsoid, World Geodetic System 1984 (WGS84). It gives heights that cannot be used directly with traditional orthometric height datums especially in determining the directions of water flow. Neither could the ellipsoidal heights be directly incorporated into the gravity based height systems.

Therefore, GPS-based levelling technique needs to use ellipsoidal height differences as well as the geoidal height differences in order to obtain orthometric height differences. In order to derive GPS height with respect to this vertical datum, the geoid-ellipsoid separation needs to be deducted. As such, orthometric heights can only be obtained from the knowledge of ellipsoidal height from GPS and geoidal heights from a geoid model.

As most engineering and mapping activities are referenced to an orthometric height surface, GPS users requiring orthometric heights need to perform geoid modeling.

This research explores the potential use of GPS in height surveys. First it outlines the principles and applications of modelling the geoid over Peninsular Malaysia. Particular attention is directed towards the use of EGM96 global geopotential model and gravity computation using the fast Fourier transform (FFT) technique where an analysis of the geoids in the region is performed. Following this, both geoid heights are compared with the GPS-levelling geoid. The second part of the research describes the about the detailed
study on the datum bias. The test area is Perak. The third part of the research describes a GPS survey carried out on bench marks in a test area. The aim is to investigate the achievable accuracy of GPS heighting in Johor. This is to enable a comparison to be made between GPS-derived orthometric heights and spirit levelled heights of bench marks. This work will demonstrate GPS as a viable technique of transforming GPS ellipsoidal heights to orthometric heights using a local geoid model. Graphical and statistical comparisons are also made with GPS-derived geoid model to determine the effectiveness of GPS height survey.

### 1.2 Objectives

Main objectives of this research are :

- To model vertical datum bias
- To evaluate accuracy of long baseline height determination
- To evaluate status of NGVD (initial study)
- To investigate the application of GPS Heighting


### 2.0 RESEARCH METHODOLOGY

### 2.1 Background Theory

### 2.1.1 GPS Heighting

Due to the fact that ellipsoidal heights are geometric values and orthometric heights are physical values reflecting local variations in gravity as well as changes in topography, the conversion from ellipsoidal to orthometric height requires a geoid height model. The process of converting ellipsoidal heights to orthometric heights or ellipsoidal height differences to orthometric height differences from a known geoid model is known as GPS heighting. In this study, four different geoid models were considered for this investigation. These are the EGM96, OSU91A , gravimetric and the GPS/leveling geoids.

### 2.1.1.1 Absolute GPS Heighting without Datum Bias Term

When dealing with absolute GPS heighting without datum bias term, we are assuming that, the local vertical datum is coincident with the geoid as shown in Fig. 1.


Figure 1 Absolute GPS heighting without datum bias term

The orthometric heights above EGM96, OSU91A and the gravimetric geoids are computed from:

$$
\begin{equation*}
H_{P}=h_{P}-N_{P} \tag{1}
\end{equation*}
$$

The orthometric height differences $(\delta H)$ of the EGM96, OSU91A and the gravimetric models relative to the local vertical datum are computed from:

$$
\begin{equation*}
\delta H_{P}=H_{P}^{B M}-H_{P}=H_{P}^{B M}-h_{P}+N_{P} \tag{2}
\end{equation*}
$$

Where;
$H^{B M}$ : $\quad$ is the true orthometric height from leveling at the Bench Marks

### 2.1.1.2 Relative GPS Heighting without Datum Bias Term

However due to the fact that in surveying and geodetic applications, differential (relative) GPS is used to provide ellipsoidal height differences with respect to a fixed base station, the absolute orthometric heights of equation (1) may not be relevant. Instead the change in orthometric height over a GPS baseline (A to B) is determined by using the corresponding change in geoid height. Also when dealing with relative GPS heighting without datum bias term, we are assuming that, the local vertical datum is coincident with the geoid.

Relative orthometric, ellipsoidal and geoidal height differences of the EGM96 model, OSU91A model, gravimetric model and the GPS/leveling data relative to the fixed station are computed from:

$$
\begin{equation*}
\Delta H_{A B}=H_{B}-H_{A}=\Delta h_{A B}-\Delta N_{A B} \tag{3}
\end{equation*}
$$

Residual orthometric height differences of the EGM96, OSU91A and the gravimetric model relative to the local vertical datum are computed from:

$$
\begin{equation*}
\delta \Delta H_{A B}=\Delta \stackrel{B M}{H_{A B}}-\Delta H_{A B} \tag{4}
\end{equation*}
$$

with:

$$
\begin{aligned}
{ }^{B M} & { }^{B M}{ }^{B M} \\
\Delta H_{A B}^{B M} & H_{B}
\end{aligned}
$$

These relative orthometric height differences are scaled by their corresponding distances and expressed in parts per million (ppm) as:

$$
\begin{equation*}
\text { Error in ppm }=\delta \Delta H_{A B} / S_{A B} \tag{5}
\end{equation*}
$$

### 2.1.1.3 Absolute GPS Heighting with Datum Bias Term

When dealing with absolute GPS heighting with datum bias term, the assumption is that the local vertical datum does not coincide with and may not be parallel to the geoid as shown in Fig. 2.


Figure 2 Absolute GPS heighting with datum bias term

The bias corrected geoidal heights referred to the EGM96, OSU91A and gravimetric models were computed from:

$$
\begin{equation*}
N_{P}^{*}=N_{P}-\delta N_{P} \tag{6}
\end{equation*}
$$

Also the bias corrected orthometric heights above EGM96, OSU91A and the gravimetric geoid models are computed as follows:
$H_{P}^{*}=h_{P}-N_{P}^{*}=h_{P}-N_{P}+\delta N_{P}$

The bias corrected orthometric height differences of EGM96, OSU91A and the gravimetric model relative to the local vertical datum (GPS/leveling datum) are computed from:

$$
\begin{equation*}
\delta H_{P}^{*}=H_{P}^{B M}-H_{P}^{*}=H_{P}^{B M}-h_{P}+N_{P}-\delta N_{P} \tag{8}
\end{equation*}
$$

### 2.1.1.4 Relative GPS Heighting with Datum Bias Term

Also when dealing with relative GPS heighting with datum bias term, the assumption is that, the local vertical datum does not coincide with and may not be parallel to the geoid as shown in Fig. 3.


Figure 3 Relative GPS heighting with datum bias term.

Relative bias corrected orthometric and geoidal height differences of the EGM961, OSU91A, and the gravimetric models relative to the fixed station are computed from:

$$
\begin{equation*}
\Delta H_{A B}^{*}=H_{B}^{*}-H_{A}^{*}=\Delta h_{A B}-\Delta N_{A B}^{*} \tag{9}
\end{equation*}
$$

With:

$$
\Delta N_{A B}^{*}=\Delta N_{A B}-\delta N_{A B}
$$

$$
\Delta \delta N_{A B}=\delta N_{B}-\delta N_{A}
$$

Residual bias corrected orthometric height differences of the EGM96, OSU91A and the gravimetric models relative to the local vertical datum are computed from:

$$
\begin{equation*}
\delta \Delta H_{A B}^{*}=\Delta H_{A B}^{B M}-\Delta H_{A B}^{*} \tag{10}
\end{equation*}
$$

Also these relative orthometric height differences are scaled by their corresponding distances and expressed in parts per million (ppm) as:

$$
\begin{equation*}
\text { Error in ppm }=\delta \Delta H_{A B}^{*} / S_{A B} \tag{11}
\end{equation*}
$$

### 2.1.2 Vertical Datum Bias Inconsistencies

The GPS/leveling- derived geoidal heights should be combined with gravimetric data to determine the best possible geoid. This is simply done by using the GPS/leveling data to shift and tilt the gravimetric geoid to get the best possible match in a region. Mean Sea Level heights at different locations does not lie on the same equipotential surface, vertical datums of the world have inconsistent reference surfaces at the $\pm 2 \mathrm{~m}$ discrepancy level (Rapp and Balasubramania ,1992). The estimation of orthometric heights from the combination of ellipsoidal heights derived from GPS and existing geoid models in absolute or relative mode is subject to the following error sources:

### 2.1.2.1 Errors in GPS Ellipsoidal Heights

The main error source in GPS ellipsoidal heights were found to occur depending on the mode (point positioning, differential, kinematics, etc) in which the GPS survey is conducted, due to the resolution of the GPS observable (C/A, L1, L1\&L2,..tc) used ( Featherstone et al 1998). In addition to the mode of GPS survey and the observables used, other errors inherent to GPS surveying affects the GPS heights. These are :

- Vertical dilution of precision (VDOP).
- Satellite ephemeris and GPS baseline length
- The atmosphere
- Multipath
- Antenna orientation and phase center
- Measurement of antenna height
- Fixing the integer ambiguities


### 2.1.2.2 Errors in Geoid Determination

The determination of geoid undulation indicates four main error sources in undulation estimation.

- Errors associated with the gravity data in the spherical cap surrounding the computation point.
- Errors associated with the gravity anomalies being given at discrete locations (mean values) instead of as a continous function.
- Commision errors associated with the potential coefficients of the geopotential model used in the combination process.
- Ommision errors associated with the neglected potential coefficients above degree $N_{\text {MAX }}$.
- Errors associated with topographical attraction effects on the gravity anomalies
- Errors due to indirect effects on the computed geoid
- Spherical approximation errors


### 2.1.2.3 Errors in Orthometric Heights

For the assessment of the accuracy of a geoid model in a survey area, orthometric heights derived from spirit leveling are needed. These heights are subject to systematic and random errors inherent in most local and regional leveling networks. These errors could be due to the following:

- Instrumental errors
- Atmospheric errors
- Data reduction errors
- Improper leveling network adjustments


### 2.1.2.4 Vertical Datum Errors

- Non-parallelism of equipotential surfaces
- Vertical deformation errors
- Vertical datum definition errors


### 2.1.3 Modeling Vertical Datum Biases

Since a local vertical datum and a given geoid are theoretically parallel surfaces, datum biases (inconsistencies) should, at least form a uniform surface that can be represented by a plane surface. The geoidal height differences computed at the GPS stations from EGM96, OSU91A and the gravimetric geoid models relative to the local vertical datum, are subject to errors and biases due to several factors as mentioned above. These biases can be reduced or absorbed by fitting a plane surface to these residuals as follows:

$$
\begin{aligned}
& \delta N(\phi, \lambda)=c_{1} \rho \Delta \phi+c_{2} v \cos \phi \Delta \lambda+c_{0} \\
& \rho=\frac{a\left(1-e^{2}\right)}{\left(1-e^{2} \sin ^{2} \phi\right)^{\frac{3}{2}}} \\
& v=\frac{a}{\left(1-e^{2} \sin ^{2} \phi\right)^{\frac{1}{2}}} \\
& \Delta \phi=\phi-\phi_{0} \\
& \Delta \lambda=\lambda-\lambda_{0}
\end{aligned}
$$

Where;
$\rho \quad: \quad$ meridian radius of curvature
$v \quad: \quad$ prime vertical radius of curvature
$\phi \quad: \quad$ geodetic latitude of the station
$\lambda \quad: \quad$ geodetic longitude of the station
$\phi_{0} \quad: \quad$ geodetic latitude of the origin
$\lambda_{0} \quad: \quad$ geodetic longitude of the origin
$c_{1} \quad: \quad$ north-south tilt of the plane
$c_{2} \quad: \quad$ east-west tilt of the plane
$c_{0} \quad: \quad$ shift between the local vertical datum and the gravimetric datum
$a \quad: \quad$ semi-major axis of the reference ellipsoid
e : first eccentricity of the ellipsoid

Our interest lies in the maximum tilt of this plane with respect to some horizontal plane on which the residual would be random, as well as the direction of the maximum tilt. The unit of tilt is distance per degree. The maximum tilt ( T ) of the plane fitting the residuals and the azimuth (A) with respect to north at the origin point of the plane are given as:

$$
\begin{align*}
& T=\sqrt{a^{2}+b^{2}}  \tag{15}\\
& A=\arctan (b / a) \tag{16}
\end{align*}
$$

From the data of Table 3, the residual differences between GPS/leveling geoid and the EGM96, OSU91A and the gravimetric geoid are computed. Considering these differences as observations, the datum bias parameters $\mathrm{C}_{0}, \mathrm{C}_{1}$ and $\mathrm{C}_{2}$ are estimated from a least squares. Also the maximum tilt (T) and the direction of maximum tilt (A) are computed from equation (9) and equation (10) respectively. The datum biases between the local vertical datum and EGM96,OSU91A and the gravimetric geoid at the twenty-seven GPS stations computed from a least squares .

### 2.1.4 Geoid Computation Method

As mentioned, the geoid can be defined as the gravity equipotential surface which best approximates mean sea level over the whole earth and as such it also acts as the datum for orthometric height system (Kuang et. al., 1996). Since the geoid reflects the earth's gravity field, its shape is irregular and undulating. So, a reference surface of an ellipsoid, a regular mathematical figure, is selected to approximate the geoid surface. The geoidal height and the deflection of vertical describe the relationship between the two surfaces and they can be determined by various approaches. Many advances in the field of geoid computations, both in terms of theory and algorithm development have occurred in the past decade. These include the gravimetric method, the astro-geodetic method, satellite dynamic solution for potential coefficients and direct determination from 3-D geocentric coordinates and orthometric heights (Vanicek \& Krakiwsky, 1986; Torge, 1980). Each of these approaches has its advantages and disadvantages.

A gravimetric geoid model provides a very high resolution and local accuracies. However, one major drawback that it has is that it is subject to long-wavelength systematic effects. With the recent developments in methodology and techniques, older geoids based on astro-geodetic have now been superseded by global geoids evaluated using gravimetric methods. Nowadays GPS technique allows the determination of geoid heights N of points of which height is known from levelling H . It provides geocentric position of the point with $h$ as the height above a reference ellipsoid such as WGS-84 or GRS80 by the relationship $\mathrm{N}=\mathrm{h}-\mathrm{H}$.

The gravimetric determination of the geoid at any point of geographical coordinates ( $\varphi$, $\lambda$ ) essentially relies on the classical Stokes’ Integral (Heiskanen \& Moritz, 1967). This amounts to evaluating the integral for the sphere as follows:

$$
\begin{equation*}
N=\frac{R}{4 \pi \gamma} \iint_{\sigma} S(\psi) \Delta g d \sigma \tag{17}
\end{equation*}
$$

where R is the mean radius of the Earth, $\gamma$ is the normal gravity on the GRS80 reference
ellipsoid at the geodetic latitude of the computation point, $\Delta g$ is the free-air gravity anomaly, $\mathrm{d} \sigma$ is the element of surface area for integration on the sphere and $\psi$ is the angular distance between the point of computation and $\mathrm{d} \sigma$, and $S(\psi)$ is the Stokes' function of the angular distance $\psi$ given by:

$$
\begin{equation*}
S(\psi)=\frac{1}{\sin \frac{\psi}{2}}-4-6 \sin \frac{\psi}{2}+10 \sin ^{2} \frac{\psi}{2}-\left(3-6 \sin ^{2} \frac{\psi}{2}\right) \ln \left(\sin \frac{\psi}{2}+\sin ^{2} \frac{\psi}{2}\right) \tag{18}
\end{equation*}
$$

The gravimetric geoid separation N can be computed by determining the long- and medium-wavelength contributions $\mathrm{N}_{\mathrm{egm}}$ from the global geopotential model (GGM) coefficients of EGM96, the short- and part of the remaining medium-wavelength effect $\mathrm{N}_{\Delta \mathrm{g}}$ from terrestrial gravity data and associated height information $\mathrm{N}_{\mathrm{h}}$ from a digital elevation model (DEM). This procedure is represented by the following relationship, such that:

$$
\begin{equation*}
N=N_{\text {egm }}+N_{\Delta g}+N_{h} \tag{19}
\end{equation*}
$$

The method used in this study breaks the gravity field into the three component and solve them separately. The principles of the computation procedure are as follows:

- Long wavelength values are determined by global geopotential model;
- Topographical masses are removed and restored mathematically through reductions of anomalies; and
- Short wavelength gravity anomalies are interpolated by fast Fourier transform of the Stoke's function.

A flowchart depicting the determination of the three signals and the generation of a gravimetric geoid is given in Figure 4. Figure 5 demonstrates the three contributions of the long, medium and short wavelengths of the Earth's gravity field. $\mathrm{N}_{\text {egm }}$ represents long wavelength geoid features of the EGM96 geopotential model over distances in the order
of 100 km . Due to the scarcity of global gravity data coverage, the long wavelength components are less accurate. This defficiency is compensated by satellite-derived gravity data. Since the strength of the gravity field degrades with distance from the geocentre, only the low frequency (ie. the long wavelength) component $\mathrm{N}_{\text {egm }}$ can be detected. It changes very smoothly with magnitude in metres. On the other hand, terrestrial gravity data gives detailed local information about the short wavelength contribution $\mathrm{N}_{\Delta \mathrm{g}}$ of the Earth's gravity field. $\mathrm{N}_{\Delta \mathrm{g}}$ has the order of magnitude of in decimeters. It represents regional geoid features with wavelengths of 20 to $100 \mathrm{~km} . \mathrm{N}_{\mathrm{h}}$ is introduced by the topography as a contribution of the DEM. It represents the wavelength features below 20 km and changes rapidly with magnitudes in centimeters.


Figure 4 A flowchart to compute a gravimetric geoid via FFT and using the R-C-R approach


Figure 5 Contributions of Global Gopotential Model, terrestrial gravity data and height information to gravimetric geoidal separation (adapted from Sideris, 1993)

From Equation (17), it can be seen that the approach by Stokes in the determination of a geoid model requires a global gravity coverage. Practically, this is impossible to achieve, not only because of many areas of the world still has no gravity data but also due to the fact that many countries still treat the data as highly classified piece of information. These difficulties can be avoided by using the "remove-compute-restore" technique (Schwarz et. al., 1990), which is one of the most commonly routine in computing regional gravimetric geoid nowadays. This technique requires the removal of spherical harmonic model (SHM) gravity anomalies before computing the Stokes’ Integral, and the restoration of SHM geoid undulations following it.

The "remove-compute-restore" approach exploits the linearity of Equation 19 where the computation of a local geoid is split into three distinct parts:
i. First, remove from the terrestrial free-air gravity anomalies $\Delta g_{f a}$, the total effect of the EGM96 global geopotential model (the long wavelength contribution), $\Delta g_{\text {egm }}$ and the terrain (short wavelength information), $\Delta g_{h}$ to derive the residual gravity anomalies, $\Delta g_{\text {rga }}$. This is carried out by first interpolating the anomalies from EGM96 and then subtracting them from the observed gravity anomalies at each gravity observation points, such that:

$$
\begin{equation*}
\Delta g_{r g a}=\Delta g_{f a}-\Delta g_{e g m}-\Delta g_{h} \tag{20}
\end{equation*}
$$

From EGM96, a reference gravity anomaly $\Delta g_{\text {egm }}$ is computed using:
$\Delta g_{e g m}=\frac{G M}{R^{2}} \sum_{n=2}^{360}(n-1) \sum_{m=0}^{n}\left[C_{n m} \cos m \lambda_{p}+S_{n m} \sin m \lambda_{p}\right] \times P_{n m}\left(\sin \phi_{p}\right)$
where $G$ is the Newton's gravitational constant, $C_{n m}$ and $S_{n m}$ are the fully normalised spherical harmonic coefficients obtained from EGM96 geopotential model to degree and order $360, P_{n m}\left(\sin \phi_{p}\right)$ is the fully normalised associated Legendre polynomial, $n$ and $m$ are the degree and order of EGM96 respectively and 360 is the maximum degree of EGM96.
ii. Next, compute the residual geoidal heights, $\mathrm{N}_{\Delta \mathrm{g}}$ via FFTGEOID software ( Li , 1994), based on 1D-FFT technique using the derived residual gravity anomalies, $\Delta g_{\text {rga }}$

By applying the Stoke's integral, it is now possible to compute the gravimetric geoid for Peninsular Malaysia. However, the formula represents the surface integral over the whole area of interest, it will result in constraints in terms of computing time. One way to overcome this problem is to employ the FFT approach.

The 1D-FFT or one-dimensional fast Fourier transform is one of the most recent and popular approach for the evaluation of Stoke's integral. The integration is done in an area of interest, instead of the whole surface of the Earth as required by the Stoke's integral. FFT technique requires fully reduced gridded gravity anomalies. The gridding algorithm employed in this study used a method of Kriging.

FFT technique evaluates the discrete spherical Stokes integral parallel by parallel without any approximation, along each parallel of latitude. In this technique, a rectangular zone of geoidal heights is produced by integrating the Cartesian rectangular zone ( $\mathrm{x}, \mathrm{y}$ ) of gravity anomalies within the following geographical boundaries: $0^{\circ} 00^{\prime} \mathrm{N} \leq \phi \leq 8^{\circ} 00^{\prime} \mathrm{N}$ and $99^{\circ} 00^{\prime} \mathrm{E} \leq \lambda \leq 105^{\circ} 00^{\prime} \mathrm{E}$. FFT technique expedites many of the computations used in modeling the gravity field very efficiently using gridded digital terrain data and mean gravity values. In order to reduce spectral leakage and to eliminate circular convolution effects, a $100 \%$ zero padding is applied on the east and west edges of the gravity anomaly input grid.

The Stoke's equation is written as:

$$
\begin{equation*}
N_{\Delta g}\left(\varphi_{p}, \lambda_{p}\right)=\frac{R \Delta \varphi \Delta \lambda}{4 \pi \gamma} \sum_{i=1}^{M}\left\{\sum_{j=1}^{N} S\left(\varphi_{P}, \varphi_{i}, \lambda-\lambda_{j}\right) \Delta g\left(\varphi_{i}, \lambda_{j}\right) \operatorname{Cos} \varphi_{i}\right\} \tag{22}
\end{equation*}
$$

The terms in the brackets contain 1D discrete convolution with respect to $\lambda$ which can then be evaluated by the 1D FFT technique. Following this, the 1D-FFT expression is as follows:

$$
\begin{equation*}
N_{\Delta g}\left(\varphi_{p}, \lambda_{p}\right)=\frac{R \Delta \varphi \Delta \lambda}{4 \pi \gamma} F^{-1}\left\{\sum_{\varphi_{Q}=\varphi_{1}}^{\varphi_{\text {max }}} F\left[S\left(\psi_{P Q}\right)\right] F\left[\Delta g\left(\varphi_{Q}, \lambda_{Q}\right) \operatorname{Cos} \varphi_{Q}\right]\right\} \tag{23}
\end{equation*}
$$

where the operators F and $\mathrm{F}^{-1}$ denote the direct and inverse 1D discrete Fourier transforms respectively, $\Delta \varphi$ and $\Delta \lambda$ are the used latitudinal and longitudinal grid spacing, and $\varphi_{1}$ and $\varphi_{\max }$ are the southern and northern grid boundaries respectively.

Thus, the geoid undulation for all points on a parallel can be obtained.
iii. And finally, the last step is to restore back the effect of the EGM96 global geopotential model, $\mathrm{N}_{\text {egm }}$ (the long wavelength contribution) to the residual geoidal heights, $N_{\Delta g}$ and add the terrain effect term, $N_{H}$ (computed from digital elevation model) to form the final geoid undulations.

Again from EGM96, the reference geoidal undulation $N_{\text {egm }}$ or the longwavelength geoid component was also made on a 2' x 2' grid within the geographical boundaries specified above. This is also the grid configuration in which the final MYGeoid02 geoid heights are given. $N_{\text {egm }}$ was computed using:

$$
\begin{equation*}
N_{e g m}=R \sum_{n=2}^{360} \sum_{m=0}^{n}\left[C_{n m} \cos m \lambda_{p}+S_{n m} \sin m \lambda_{p}\right] P_{n m} \sin \phi_{p} \tag{24}
\end{equation*}
$$

### 2.2 Research Area

This research has been mainly conducted in three areas which each area has its own objectives. The areas are:

- Peninsular Malaysia GPS Network. This area research aims to model the geoid and datum bias over Peninsular Malaysia.
- The State of Perak. This area research aims to be detailed study of datum bias.
- The State of Johor. This area research aims to investigate the accuracy of GPS heighting.


### 2.2.1 Preliminary Geoid Computation Over Peninsular Malaysia

The geoid is most commonly defined as the hypothetical equipotential surface of the Earth's gravity field, which would closely coincide with undisturbed mean sea level (MSL). Undisturbed MSL would only exist if the oceans were acted on by the Earth's gravitational field whilst, ignoring other forces such as tides, ocean currents and winds. Geoid determination remains as one of the basic task of ongoing researches in physical geodesy. This is because the majority of measurements are referred to Earth's gravity field. The geoid also reflects variations in the gravity field. Traditionally its surface has served as the fundamental reference for orthometric height and its differences, gravity potentials and other vertical heights.

The past decade had seen a renewed interest in the determination of geoid on global, regional and local scales. Great strides have been achieved both in terms of theory, techniques and algorithm development in the quest to achieve precise geoids. There has also been an increase in worldwide coverage of terrestrial gravity collection. In Peninsular Malaysia, the national gravity database is continuously being updated. The geographical distribution of gravity data over the land areas has not only been widespread but also densified substantially over the last decade. The number of gravity points was less than 2000 in 1989 (Kadir et. al., 1989) and by 1995 it had increased to more than

5000 points (Ses \& Majid, 1998). In 2000, the gravity database at DSMM has a total number of over 10000 points. However, up to now, there is no precise geoid available yet for Peninsular Malaysia and its surrounding seas, although attempts have been made in recent years. This is mainly due to the paucity of gravity anomaly data sets and their rather poor distribution in the region.

The importance contributions from the determination of the geoid can be summarised as follows:

- better understanding of the geology and the geophysics through geodetic perspective, since it is possible to study the gravity field features directly from the geoid. Variations in gravity field can be used in geothermal exploration and geophysical prospecting, especially in areas of potential oil and gas bearing structures (Hwang \& Shih, 1998).
- greater use of GPS in height determination with high precision since it has the potential to replace the costly, labourious and time-consuming spirit levelling. It permits the determination of orthometric heights by combining geoid heights computed from a geoid model and ellipsoidal heights obtained from the GPS observations.
- more accurate connection of one local vertical datum to another, especially for the control of levelling networks (Kadir et. al., 1999) and verification of apparent sea slope along the coasts (Ses \& Gilliland, 1996).

In this section, the best available data is used to compute a preliminary national geoid and compare to a global gravity model. Comparisons are also made with the geoid produced involving some 230 GPS points that have precise levelling heights across the nation.


Figure 6 Topographical features of Peninsular Malaysia

### 2.2.1.1 Topographical Setting

Peninsular Malaysia extends from latitude $1^{\circ} 20^{\prime} \mathrm{N}$ to $6^{\circ} 40^{\prime} \mathrm{N}$ and from longitude $99^{\circ}$ $35^{\prime} \mathrm{E}$ to $104^{\circ} 20^{\prime} \mathrm{E}$. The total length of the country is roughly 804 km . Figure 6 illustrates the topographical features of Peninsular Malaysia. The Main Range of mountains trend roughly in a north-south direction with maximum elevations of over 2000 metres. It is long, narrow and rugged and lies nearer to the western side of the country. From the Thailand border, it runs continuously southwards into Negeri Sembilan where its height gradually diminishes and the hills merge with the coastal plain near the coast. The relief becomes less pronounced southwards especially over much of south of Pahang and Johor.

Narrow stretches of lowland lie on either side of the Main Range. On the eastern side of Peninsular Malaysia, another major barrier occurs. The barrier consists of several close parallel ranges with relatively deep valleys between. The ranges extend from the Kelantan coast in the north to the Pahang coast in the south. The heights in general are less than those in the Main Range.

### 2.2.1.2 Data Compilation and Preparation

The different types of data used in the present study are as follows:

The EGM96 Global Geopotential Model: Global geopotential models (GGMs) describe the Earth's gravitational potential in terms of an infinite series of spherical harmonics outside the Earth's attracting masses. They are determined by a combination of satellite and terrestrial observations and used as reference fields in the determination of local and regional geoids. The geopotential is usually given as a truncated set of harmonic coefficients. The maximum complete degree and order of these expansions has now reached 360 (Kirby \& Featherstone, 1997), corresponding to a 111km wavelength at the equator. The smallest wavelength of geoid undulation that can be modeled is $180^{\circ} / 360$ or $0.5^{\circ}$, which is equivalent to about 55 km .

The most recent estimate of the global gravity field that is currently available is the Earth Gravity Model 1996 or EGM96. It is the national Aeronautics and Space Administration (NASA), National Imagery and Mapping Agency (NIMA), and The Ohio State University (OSU) joint global geopotential model in spherical harmonics complete to degree $l$ and order $m 360$ (Lemoine et. al., 1997). It is also considered as the best global geopotential model currently available. This is due to the inclusion of new data, improved computational procedures and the use of geodetic satellites with different inclinations in to improve detection of low frequencies at low and mid latitudes. Globally, EGM96 is estimated to give the geoid undulation with an accuracy of 1 m .

Terrestrial gravity data: Generally, the gravity data over Peninsular Malaysia and the adjacent marine areas is poor. The data used in this study are 10400 point free air gravity anomalies which encompass the Peninsular Malaysia and surrounding marine area: $0^{\circ} 00^{\prime}$ $\mathrm{N} \leq \phi \leq 8^{\circ} 00^{\prime} \mathrm{N}$ and $99^{\circ} 00^{\prime} \mathrm{E} \leq \lambda \leq 105^{\circ} 00^{\prime} \mathrm{E}$. The data belong to the DSMM Gravity Data Bank and are referenced to the International Standardisation Network 1971 (IGSN71).


Figure 7 The geographical distribution of the available gravity measurements

Figure 7 shows the geographical distribution of the available gravity measurements. The distribution varies from a very dense network in the Wilayah Persekutuan Kuala Lumpur, to less dense in the southern and western regions, to a sparse configuration in the central region and along the east coast.

Digital Elevation Model (DEM): DEM is a statistical representation of selected points on a continuous terrain relief in a digital form. It is used to better account for the terrain effects on the gravity field. The DEM for Peninsular Malaysia comes from the 30 " point topography database, GTOPO30 which is distributed by US Geological Survey.

According to Higgins, et. al. (1996), the aims of using DEMs in the computation of geoid are firstly to better account for the terrain effects on the gravity field by terrain related corrections. Secondly, it is used to interpolate the irregularly spaced raw gravity data.

As mentioned, the gravity data over Peninsular Malaysia is generally poor. The middle area especially contains sparse or no gravity data at all. This necessitates some form of fill-in by prediction using DEM. Terrain corrections were computed from the DEM using the 1-D FFT technique. These corrections were then applied to the residual gravity anomalies (after the removal of the long wavelength contributions of EGM96). The resultant gravity data set thus obtained is referred to as terrain corrected data.

Satellite altimetry-derived marine gravity data: The satellite altimetry-derived geoid heights data was extracted from a database of the Kort og Matrikelstyrelsen (KMS) or the Danish National Survey in Denmark. It comes on a 1.5’ by 1.5’ grid, covering both land and marine areas.

Using a small program that reads height data less than or equal to zero, the marine data were selected. A number of erroneous observations was deleted in accordance to the following adopted criteria:

- subsatellite points close to the shore line;
- short arcs;
- error value of more than 0.16 or 0.18 ; and
- high sea-surface heights.

From these selected altimeter data, the long wavelength effects of EGM96 were removed. Figure 8 illustrates a map of free air gravity anomalies contoured at 5 mGals for the marine and land areas around Peninsular Malaysia.


Figure 8 A map of free-air gravity anomalies (contour interval is 15 mgals )


Figure 9 The distribution of 230 GPS points having both the ellipsoidal and orthometric heights to derive $\mathrm{N}_{\text {GPS }}$ and be compared with $\mathrm{N}_{\text {GRAV }}$ and $\mathrm{N}_{\text {EGM96 }}$

GPS and spirit-levelled height data: A total of 230 GPS points that have elevations established by precise levelling were compiled. Figure 9 illustrates the distribution of these points, which varies from a uniform network to very few points in the northern central region. The geometric geoid height at these points will be compared with the gravimetric geoid height derived from EGM96 and local model solutions. This will give some indication of the accuracy of the gravimetric geoid derived.

### 2.2.1.3 Geoid Computation Method

As mentioned, the geoid can be defined as the gravity equipotential surface which best approximates mean sea level over the whole earth and as such it also acts as the datum for orthometric height system (Kuang et. al., 1996). Since the geoid reflects the earth's gravity field, its shape is irregular and undulating. So, a reference surface of an ellipsoid, a regular mathematical figure, is selected to approximate the geoid surface. The geoidal height and the deflection of vertical describe the relationship between the two surfaces and they can be determined by various approaches. Many advances in the field of geoid computations, both in terms of theory and algorithm development have occurred in the past decade. These include the gravimetric method, the astro-geodetic method, satellite dynamic solution for potential coefficients and direct determination from 3-D geocentric coordinates and orthometric heights (Vanicek \& Krakiwsky, 1986; Torge, 1980). Each of these approaches has its advantages and disadvantages.

A gravimetric geoid model provides a very high resolution and local accuracies. However, one major drawback that it has is that it is subject to long-wavelength systematic effects. With the recent developments in methodology and techniques, older geoids based on astro-geodetic have now been superseded by global geoids evaluated using gravimetric methods. Nowadays GPS technique allows the determination of geoid heights N of points of which height is known from levelling H. It provides geocentric position of the point with h as the height above a reference ellipsoid such as WGS-84 or GRS80 by the relationship $\mathrm{N}=\mathrm{h}-\mathrm{H}$.

### 2.2.2 Detailed Study Of Datum Bias In State of Perak

The selected test network consists of a subset of bench marks from the PLN in the northwest of Peninsular Malaysia. It covers the zone with limits $3^{\circ} 30^{\prime} \mathrm{N} \leq \phi \leq 5^{\circ} 30^{\prime} \mathrm{N}$ and $100^{\circ} 00^{\prime} \mathrm{E} \leq \lambda \leq 101^{\circ} 30^{\prime} \mathrm{E}$. This translates to an approximate network


Figure 10 Location of test area for GPS heighting study


Figure 11 Distribution of GPS stations and GPS-occupied bench marks


Figure 12 Distribution of gravity data
coverage area of roughly $220 \mathrm{~km} \times 160 \mathrm{~km}$ as depicted in Figure 10. The nature of the terrain around the test area is relatively undulating. Height variation is from 2.15 m to 88.60m along the GPS profile.

The chosen network consists of 57 stations as depicted in Figure 11, whilst Figure 12 shows the distribution of the gravity data over the area. The aim of selecting bench marks from the PLN was to provide data for constructing and evaluating contours of geoidal separation across the project area. The bench marks were so selected in order to give good point distribution through the network. The inter-station distance between the selected points were between 2 to 28 km with a mean spacing of 14 km . This also ensured inter-station drive times to be about 30 minutes since more than one session in a day was involved in the campaign.

### 2.2.2.1 GPS on Bench Marks: Network and Computation

A total number of 43 pre-selected bench marks and 14 GPS stations have been occupied by GPS. The area of study in the north of Peninsular Malaysia was chosen for this work. Specifically, this area covers most of the state of Perak.

The project utilised six Trimble 4000 SSE GPS receivers and Trimble Compact L1/L2 with ground plane antennas. The Trimble SSE is a dual-frequency, full wavelength GPS receiver. In order to achieve a relatively high degree of accuracy, most stations were occupied twice. As many as four receivers were moved at a time in a leapfrog fashion, reobserving two stations previously occupied. Each GPS loop was arranged so that it contained observations from at least two different sessions. The survey was designed in such a way so that there would be redundant information to detect data outliers.

There were a total of 22 sessions in the GPS campaign with two observation sessions per day. The antennae were orientated to the North throughout the campaign. GPS data was collected above a 15-degree elevation mask. The baselines were observed for a period of
between 2 to 3 hours at a 15 -second epoch rate. Six or more satellites were continuously visible at all times. The survey was conducted during periods having Vertical Dilution of Precision (VDOP) less than four.

Primary network connections were also made to fourteen GPS stations of the Peninsular Malaysia Scientific and Geodetic Network (PMSGN) and GPS densification network. As a matter of providing initial quality control of the project, a preliminary daily baseline solution was performed each evening. Figure 13 shows the network configuration of the ‘GPS on BMs' project.

Trimble's proprietary processing package GPSurvey version 2.30 software was used to process the non-trivial vectors at a 30 -second epoch and 20 degrees cutoff angle, whilst Geolab v2.4C was used in the network adjustment. Double difference carrier-phase observations were used in a least-squares adjustment to determine the fifty-six baselines. Three stations from the Peninsular Malaysia Scientific and Geodetic Network (PMSGN) were held fixed in the adjustment. Broadcast ephemeredes were used in all reductions. This resulted in forty-three ellipsoidal heights referred to the WGS-84 ellipsoid. Table 1 gives a description of the occupied points involved in the campaign.

Table 1 Network point type summary

| Station <br> Description | Number of <br> Stations | Symbol | Remarks |
| :--- | :---: | :---: | :--- |
| Bench Mark | 36 | 0 | Intermediate Bench Mark <br> Standard Bench Mark |
| GPS Points | 3 | 0 | $\triangle$ |



Figure 13 Network configuration of 'GPS on BMs' Project in Perak

### 2.2.3 Investigation On Accuracy of GPS Heighting In The State of Johor

The Department of Survey and Mapping Malaysia kindly made available twenty-seven GPS derived co-ordinates in Johore State co-located with leveling for this study. The GPS derived co-ordinates on WGS84 datum associated with leveling tabulated in Table 2 were given in terms of geodetic latitude ( $\phi$ ), geodetic longitude ( $\lambda$ ), ellipsoidal height (h) and orthometric height (H). The distribution of this data set is shown in Fig. 14.

| ST. NO | NAME | $\phi$ | $\lambda$ | h | H |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | DOP2 | 1.380 | 103.610 | 90.910 | 83.800 |
| 2 | GP15 | 2.060 | 102.560 | 4.002 | 1.960 |
| 3 | GP16 | 2.130 | 102.730 | 36.230 | 33.470 |
| 4 | GP43 | 2.600 | 103.780 | 11.180 | 4.990 |
| 5 | GP44 | 2.470 | 103.070 | 55.650 | 51.870 |
| 6 | GP47 | 2.390 | 102.930 | 51.180 | 47.860 |
| 7 | GP48 | 1.980 | 102.930 | 164.750 | 160.990 |
| 8 | GP49 | 1.630 | 103.200 | 7.360 | 1.950 |
| 9 | GP50 | 1.550 | 103.400 | 10.120 | 3.810 |
| 10 | GP51 | 1.630 | 103.670 | 42.710 | 35.520 |
| 11 | GP52 | 1.370 | 104.270 | 67.810 | 58.720 |
| 12 | GP53 | 1.800 | 103.900 | 31.820 | 24.000 |
| 13 | GP54 | 1.930 | 104.090 | 12.880 | 4.660 |
| 14 | GP55 | 2.080 | 103.890 | 36.740 | 29.330 |
| 15 | GP56 | 2.390 | 103.870 | 11.570 | 4.680 |
| 16 | GP58 | 2.120 | 103.430 | 77.620 | 72.100 |
| 17 | GP59 | 1.970 | 103.230 | 45.180 | 40.150 |
| 18 | GP61 | 2.190 | 103.190 | 41.830 | 37.320 |
| 19 | GP84 | 1.860 | 102.940 | 7.130 | 3.150 |
| 20 | GP85 | 1.910 | 102.740 | 4.930 | 1.880 |
| 21 | GP90 | 1.930 | 104.110 | 10.780 | 2.470 |
| 22 | GP91 | 1.880 | 103.690 | 26.310 | 19.550 |
| 23 | TG05 | 2.650 | 102.920 | 193.730 | 190.800 |
| 24 | TG07 | 1.680 | 104.070 | 194.270 | 186.200 |
| 25 | TG09 | 2.460 | 103.640 | 223.000 | 216.700 |
| 26 | TG10 | 1.460 | 104.070 | 193.250 | 183.500 |
| 27 | TG19 | 1.470 | 103.260 | 136.070 | 130.400 |

Table 2 Johor GPS data associated with leveling.


Fig. 14. The distribution of the GPS data in Johor State

### 2.2.3.1 Computations

There are several approaches with which to transform GPS ellipsoidal heights to orthometric heights using either the point mode equation or the relative mode equation . The ellipsoidal height $h$ derived from GPS measurements is given above a geocentric ellipsoid (which is WGS84 for GPS). Also the geoid undulations are required above the same ellipsoid to determine orthometric heights of interest. While ellipsoidal height $h$ might be very accurate to the several millimeter level, the accuracy of the orthometric height, which is the height of interest is limited by the uncertainity of the geoid undulation $N$. Some of These methods used to evaluate $N$ in this study are reviewed below.

### 2.2.3.1.1 Gravimetric Geoid

A regional gravimetric geoid model over the area of South-East Asia was computed from $1^{0} \times 1^{0}, 30^{\prime} \times 30^{\prime}$ mean surface gravity anomaly data combined with a truncated EGM96 $(70,70)$ spherical harmonics potential coefficient set ( Kadir et al.,1998 ). To compute the gravimetric geoid in South-East Asia, the Molodensky modified Stokes's kernel was used instead of the original ellipsoidal Stokes's kernel. This reduces truncation errors as the former tapers of more rapidly than the latter (the influence of distant gravity anomalies on local geoid heights is, thus reduced). The reduction is proportional to the degree $L$ of the satellite model being used to recover the long wavelength component of the geoid. The total gravimetric geoid was obtained from the summation of the long wavelength and the short wavelength components. This solution was referred to the World Geodetic System 1984 (WGS-84). The gravimetric geoid heights ( $N_{\text {GRAV }}$ ) at the twenty-seven GPS stations in Johore State were estimated from the above geoid model.

### 2.2.3.1.2 Geopotential Geoid

One of the methods to estimate global geoid undulations is by using geopotential models. These are mathematical models in which gravitational potential of the Earth can be
expanded into a series of spherical harmonics. In this study, the EGM96 geoid heights ( $N_{\text {EGM }}$ ) and the OSU91A geoid heights ( $N_{\text {OSU }}$ ) on GRS80 were estimated from EGM96 and OSU91A model coefficient sets together with the parameters of the GRS80 ellipsoid.

### 2.2.3.1.3 Geometric Geoid

The geometric geoid could be derived from a combination of GPS measurements associated with spirit leveling. The end product of GPS measurements are either the ellipsoidal co-ordinates ( $\phi, \lambda, h$ ) or Cartesian co-ordinates ( $X, Y, Z$ ) on the WGS84 system. The spirit leveling results in heights referenced to the geoid. These heights are termed orthometric heights ( $H$ ). The geoid height ( $N$ ) at any control station with ellipsoidal co-ordinates ( $\phi, \lambda, h$ ) and known orthometric height $(H)$ is calculated in a point mode using equation (22) or in a relative mode from equation (23)

$$
\begin{align*}
& N_{P}=h_{P}-H_{P}  \tag{25}\\
& N_{B}=N_{A}+\Delta h_{A B}-\Delta H_{A B} \tag{26}
\end{align*}
$$

If the co-ordinates of the control station were given in rectangular form ( $X, Y, Z$ ) then they have to be converted into ellipsoidal form to deduce the ellipsoidal height ( $h$ ) before applying the above equations for geoid height recovery. The ellipsoidal coordinates ( $\phi, \lambda, h$ ) could be obtained from the Cartesian co-ordinates ( $X, Y, Z$ ) associated with the parameters of the WGS84 datum using the non-iterative reverse conversion formulae given in Bowring (1985) as:

$$
\begin{align*}
& \phi=\arctan \left[\frac{Z+\varepsilon^{2} b \sin ^{3} u}{P-e^{2} a \cos ^{3} u}\right]  \tag{27}\\
& \lambda=\arctan \left[\frac{Y}{X}\right] \tag{28}
\end{align*}
$$

$h=P \sec \phi-v$
with:
$u=\arctan \left[\frac{a Z}{b P}\right]$
$P=\sqrt{X^{2}+Y^{2}}$
$\varepsilon=\frac{e^{2}}{1-e^{2}}$
where;
u : the parametric latitude;
a : the semi-major axis of the reference ellipsoid;
b : the semi-minor axis of the reference ellipsoid;
$e \quad: \quad$ the first eccentricity of the reference ellipsoid.
$\varepsilon \quad: \quad$ the second eccentricity of the reference ellipsoid.

Using the data of Table 2, the geometric geoid ( $N_{G P S}$ ) on GRS80 ellipsoid at the twentyseven GPS stations is computed from equation (25).

### 3.0 RESULTS

### 3.1 Results of The Vertical Datum Bias For Peninsular Malaysia and Comparisons with GPS/Levelling Heights

The result from the geoid computation takes the form usually indicated by a contour plot whereby it shows the position of the geoid with respect to the selected ellipsoid. Figures 15 and 16 illustrate the contours of the EGM96 and preliminary national geoid model for Peninsular Malaysia (referred to here forth as MYGeoid02) respectively. It should be noted that MYGeoid02 is considered preliminary because of the lack of gravity data in the central region as mentioned before. However, based on the dense coverage of the gravity points, it can be considered as being more reliable in the western coastal regions and in the southern part of Peninsular Malaysia.

Both geoids, EGM96 and MYGeoid02, are rising from north-west to south-east. They coincide with the reference ellipsoid GRS80 roughly in the central region, from $102^{\circ} \mathrm{E}$ meridian to $103.5^{\circ} \mathrm{E}$ meridian and from $2^{\circ} \mathrm{N}$ to $5^{\circ} \mathrm{N}$. The contours descend from 10 m in the south to -15 m in the north. Results also indicate that the maximum and minimum values of the MYGeoid02 are 9.45 m and -14.66 m respectively. Whilst for the EGM96 geoid, the figures are 9.53 m and -14.47 m . MYGeoid02 and EGM96 have a mean undulation value of 9.45 m and 9.53 m respectively.


Figure 15 EGM96 geoid model for Peninsular Malaysia at 5-min grid (contour interval is 2 m )


Figure 16 Preliminary geoid model for Peninsular Malaysia, MYGeoid02 at 2-min grid (contour interval is 2 m )

GPS/Levelling geoid heights for 230 points in Peninsular Malaysia are also computed by subtracting adjusted orthometric heights from the corresponding GPS ellipsoidal heights. Comparisons are made between the offset of EGM96 and MYGeoid02 models with these GPS-derived geoid height at each of those points. The available N-values from GPS are compared to get a qualitative estimate of the fit of the geoids, GPS and the orthometric heights. Table 3 shows descriptive statistics of the absolute fit of MYGeoid02 and EGM96 to the 230 GPS/Levelling control data. The removal of suspected data outliers (2.6\%) in the geometric geoid improved the standard deviations considerably as shown in Table 4.

Table 3. Comparison of geoid models to GPS-derived geoid heights before removal of suspected data outliers (values in metres)

| Geoid Model | $\mathbf{N}_{\text {Model }}-\mathbf{N}_{\text {GPs }}$ |  |  |  |
| :---: | :--- | :---: | :---: | :---: |
|  | Minimum | Maximum | Mean | Std. <br> Dev. |
|  | -2.653 | 1.785 | 0.100 | 0.471 |
| EGM96 | -2.710 | 1.687 | -0.051 | 0.498 |

Table 4. Comparison of geoid models to GPS-derived geoid heights after removal of suspected data outliers (values in metres)

| Geoid Model | $\mathbf{N}_{\text {Model }}-\mathbf{N}_{\text {GPs }}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Minimum | Maximum | Mean | Std. <br> Dev. |
|  | -0.943 | 0.688 | -0.065 | 0.389 |
| EGM96 | -0.760 | 0.827 | 0.052 | 0.402 |

The two geoid models are found to have approximately the same relative accuracy. Results show that the standard deviations of the differences are $\pm 40 \mathrm{~cm}$ for the EGM96 solution and $\pm 39 \mathrm{~cm}$ for MYGeoid02 solution. The 40 cm magnitude of the fit can be considered as satisfactory. From the inspection of the standard deviations of both geoids, it can be seen that MYGeoid02 represents only a slight improvement over EGM96. Some possible explanations for this include:

- increased resolution since MYGeoid02 is computed on a 1-min grid compared to the 5-min resolution of EGM96;
- additional new gravity data; and
- use of DEM data since MYGeoid02 features in the central mountain range of Peninsular Malaysia strongly correlates with the terrain.


Figure 17 Comparison between gravimetric geoids with geometric geoid

Figure 17 illustrates the results obtained from the comparison between the two gravimetric geoids and the geometric geoid. From the examination of the Figure, it may
be noted that both EGM96 and MYGeoid02 geoids exhibit similar trends. Overall, it can be concluded that, with the limited amount of gravity points available in Peninsular Malaysia, the use of DEM, the inclusion of satellite-altimeter-derived gravity data and the adoption of suitable computational techniques, MYGeoid02 is the more suitable geoid model for the transformation of GPS-derived ellipsoidal height differences to orthometric height differences.

Figure 18 and Figure 19 show the vertical datum bias for Peninsular Malaysia.


Figure 18 :
Vertical Datum Bias For EGM’96


Figure 19 :
Vertical Datum Bias For Gravimetric Geoid

Leveling and GPS data have been used to compute one flat surface using the equations as follows:
$\delta \mathrm{N}(\phi, \lambda)=\mathrm{a} \Delta \phi+\mathrm{b} \Delta \lambda \cos (\phi)+\mathrm{c}$
where :
b = East-west direction tilt.
a $\quad=$ North-south direction tilt.

| c | $=$ distance between two geoid models. |  |
| :--- | :--- | :--- |
| Tilt | $=$ maximum surface tilt | $=\left(a^{2}+\mathrm{b}^{2}\right)^{1 / 2}$ |
| Azimuth | $=$ tilt direction | $=\tan ^{-1}(\mathrm{~b} / \mathrm{a})$. |

The variables $\mathrm{a}, \mathrm{b}$ and c are solved using least square adjustment. The result is shown in the following:

|  | EGM96 | Gravimetrik |
| :--- | :---: | :---: |
| Number of data used | 233 | 135 |
| a (sec) | 0.1642 | 0.4529 |
| b (sec) | 0.6085 | 0.2161 |
| c (m) | 0.1142 | 0.1759 |
| Tilt (sec) | 0.6303 | 0.5018 |
| Ppm | 3.1 | 2.4 |
| Azimut (deg) | 331.4929 | 21.6530 |

Table 5: Datum Bias Parameter

For Local gravimetric model, the area consists of latitude $1.45^{\circ} \mathrm{N}-3.5^{\circ} \mathrm{N}$ and longitude $100.5^{\circ} \mathrm{E}-104.5^{\circ} \mathrm{E}$.

To ensure that the data fit the formed model, the residual must be small and has a normal distribution. This residual show the random error of data. In this study, the resulting residual shows that there is still blunder where in fig. 20 and 21 , the residual close to 1 m. However, they still show the characteristic of a normal distribution.


Figure 20 : Residual Histogram From Model Geoid EGM96 and $\mathrm{N}_{\text {MSL }}$.


Figure 21 : Residual Histogram From Model Geoid Local Gravimetric and $\mathrm{N}_{\text {MSL }}$

Using the model explained above, the GPS heighting is conducted to obtain the orthometric height. The orthometric heights obtained here are close to the Mean Sea Level Height . Figure 22 show the result :


Figure 22 :Bar Chart Showing The Comparison Between The Orthometric Height With or Without §N ( EGM96)

### 3.2 Results of The Detailed Study of Datum Bias In The State of Perak

Since GPS ellipsoidal heights and orthometric heights are available for the forty-three bench marks in the test network, a product of GPS-derived geoid heights was obtained. As the objective of this investigation is to confirm the potential of GPS approach in determining orthometric heights, a set of geoidal heights was also compiled from a local geoid. The extent of agreement to which GPS-derived geoid heights match gravimetric geoidal heights is used as an indicator of the ability of GPS to replicate spirit levelling.

The analysis of the local geoid model to geoid heights determined from the 'GPS on BMs' project in the test area involves only thirty-seven bench marks; the remaining five bench marks are used as a check. The analysis is carried in two parts (Veronneau, 1993). The first part of analysis is to allow an investigation be made on the overall accuracy of the model in the test area. It involves the estimation of geoid heights at every station using the local geoid model (Figure 23). These geoid heights are interpolated using WINTER (Windows Interpolation) software which can be downloaded without charge from the AUSLIG website (http://www.auslig.gov.au/ geodesy/ausgeoid/geoid.htm). The software can interpolate in a batch mode a file of positions using either bilinear or bicubic interpolation. This is followed by a direct comparison to the respective geoid heights as derived from GPS observations combined with levelling data using the following expression:

$$
\delta N_{1}=\left(h_{G P S}-H_{\text {Levelling }}\right)-N_{\text {Model }}
$$

where $h_{G P S}$ is the GPS derived ellipsoidal height, $H_{\text {Levelling }}$ is the orthometric height and $N_{\text {Model }}$ is the computed geoidal height.

The results of the comparison is summarised in Table 5. The standard deviation of the discrepancies of the GPS-derived geoid heights from the local geoid model on thirtyseven bench marks is 15 cm . The value of $\delta N_{1}$ ranges from 11.8 cm to 51.2 cm .


Figure 23 Local geoid model for the State of Perak

The second part of the analysis concerns with the evaluation of the accuracy in relation to the baseline distance between stations. The computed geoidal height differences, $\Delta N_{\text {Model }}$ along forty baselines in the network are compared with the corresponding results obtained from GPS derived ellipsoidal height differences, $\Delta h_{G P S}$ and orthometric height differences, $\Delta H_{\text {Levelling. }}$. The comparison can be expressed as:

$$
\delta N_{2}=\left(\Delta h_{G P S}-\Delta H_{\text {Levelling }}\right)-\Delta N_{\text {Model }}
$$

This type of analysis is especially useful in identifying problematic stations when associated repeatedly to baselines having largest errors in ppm. The results of the comparison are shown in Table 6. The value of $\delta N_{2}$ ranges from -35.3 cm to 28.7 cm . It can be seen that larger discrepancies are obtained when the baselines cross the geoid contour lines (Figure 23) and made the geoid height more significant.

Table 6 Discrepancies between the local geoid model and geoid heights derived from GPS observations for 37 bench marks in Perak

| Station | Longitude $\lambda\left({ }^{\circ}\right.$ ) | Latitude $\phi\left({ }^{\circ}\right.$ ) | $\mathbf{h}_{\text {GPS }}-\mathbf{H}_{\text {Levelling }}(\mathbf{m})$ | $\mathbf{N}_{\text {Model }}(\mathrm{m})$ | $\delta \mathbf{N}(\mathrm{m})$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| A0085 | 101.2814 | 4.11628 | -5.696 | -5.578 | -0.118 |
| A0089 | 101.2607 | 4.20096 | -5.940 | -5.968 | 0.028 |
| A0092 | 101.2179 | 4.23137 | -6.224 | -6.171 | -0.053 |
| A0123 | 100.9473 | 4.36234 | -7.535 | -7.712 | 0.177 |
| A0152 | 101.1357 | 4.51839 | -7.035 | -7.014 | -0.021 |
| A0363 | 100.6897 | 4.51937 | -9.078 | -9.067 | -0.011 |
| A0424 | 100.9588 | 5.05571 | -8.406 | -8.622 | 0.216 |
| A0500 | 100.6478 | 4.39344 | -9.034 | -9.058 | 0.024 |
| A0585 | 100.6946 | 4.38580 | -8.832 | -8.868 | 0.036 |
| A0600 | 100.7688 | 4.32683 | -8.316 | -8.564 | 0.248 |
| A0635 | 101.0108 | 4.49590 | -7.598 | -7.702 | 0.104 |
| A0701 | 101.1621 | 4.29531 | -6.449 | -6.643 | 0.194 |
| A0726 | 100.6425 | 4.22529 | -8.712 | -8.880 | 0.168 |
| A0832 | 100.8545 | 4.52206 | -8.176 | -8.382 | 0.206 |
| A0840 | 100.7847 | 4.39410 | -8.382 | -8.516 | 0.134 |
| A0974 | 101.0345 | 4.25155 | -6.942 | -7.177 | 0.235 |
| A0979 | 101.0592 | 4.32813 | -6.940 | -7.192 | 0.252 |
| A0983 | 101.0511 | 4.39260 | -7.093 | -7.268 | 0.175 |
| A1285 | 100.8766 | 4.05006 | -7.128 | -7.533 | 0.405 |
| A1381 | 100.5459 | 5.00448 | -10.293 | -10.647 | 0.354 |
| A1396 | 100.7165 | 4.15890 | -8.202 | -8.370 | 0.168 |
| A1555 | 100.8010 | 4.05374 | -7.527 | -7.890 | 0.363 |
| A1597 | 100.8980 | 4.25431 | -7.517 | -7.867 | 0.350 |
| A1601 | 100.9294 | 4.16724 | -7.160 | -7.572 | 0.412 |
| A1606 | 100.9372 | 4.07804 | -6.968 | -7.333 | 0.365 |
| A1622 | 101.0355 | 4.48418 | -7.349 | -7.504 | 0.155 |
| A1802 | 100.7162 | 4.67351 | -9.112 | -9.151 | 0.039 |
| A1831 | 100.6757 | 4.88922 | -9.656 | -9.827 | 0.171 |
| A1839 | 100.6150 | 4.96478 | -10.003 | -10.124 | 0.121 |
| S0091 | 101.0785 | 4.81665 | -7.715 | -7.633 | -0.082 |
| S0294 | 100.9676 | 5.10751 | -8.559 | -8.704 | 0.145 |
| S0376 | 100.6309 | 5.13059 | -10.356 | -10.482 | 0.126 |
| S0379 | 100.7725 | 5.11354 | -9.644 | -9.723 | 0.079 |
| S0411 | 100.7842 | 4.50004 | -8.579 | -8.649 | 0.070 |
| A0933 | 100.9695 | 3.97195 | -6.478 | -6.990 | 0.512 |
| S0461 | 100.9268 | 4.97805 | -8.437 | -8.633 | 0.196 |
| S0462 | 100.9227 | 4.93639 | -8.462 | -8.586 | 0.124 |
| No. of points |  |  | 37 |  |  |
| Minimum ( $\delta \mathrm{N}_{1}$ ) |  |  | -0.118 m |  |  |
| $\text { Maximum }\left(\delta \mathrm{N}_{1}\right)$ |  |  | 0.512 m |  |  |
| Mean ( $\delta \mathrm{N}_{1}$ ) |  |  | 0.164 m |  |  |
| Std. Dev. $\left(\delta \mathrm{N}_{1}\right)$ |  |  | $\pm 0.146 \mathrm{~m}$ |  |  |
| RMS ( $\delta \mathrm{N}_{1}$ ) |  |  | $\pm 0.218 \mathrm{~m}$ |  |  |

Table 7 Comparison of geoidal height differences, i.e. observed $(\Delta h-\Delta H)$ and computed $\Delta N$


Statistical analyses of the fitness of the two data sets are also carried out. First, the RMS value as defined as follows:

$$
R M S= \pm \sqrt{\frac{\sum_{i=1}^{n}\left(\Delta h_{\text {GPS }}-\Delta H_{\text {Leveling }}-\Delta N_{\text {Model }}\right)_{i}^{2}}{n}}
$$

where $n$ is the number of baselines and $\Delta N$ value was interpolated from the respective geoids. The second statistic computed is the RMS/s where $s$ is the mean length of all baselines.


Figure 24 The values of $\delta N_{2}$ as a function of baseline length

The values of $\delta \mathrm{N}_{2}$ from Table 7 are also plotted against the length of the baselines as shown in Figure 24. It demonstrates that the accuracy of the baseline degrades as the baseline length increases. Fotopoulos et al., (2002) have conducted several tests using various baseline lengths. It was also found that the longer the length, the poorer the achievable $\Delta H$ accuracy from GPS/geoid levelling. Among the reasons given for this result are partly due to the spatial decorrelation of GPS errors and the contribution of the
corrector surface model eror.

The third value is the mean relative accuracy $\bar{a}$ or the mean ppm value,

$$
\bar{a}=\frac{1}{n} \sum_{i=1}^{n}\left|\frac{\Delta h_{\text {GPS }}-\Delta H_{\text {Levelling }}-\Delta N_{\text {Model }}}{s_{i}}\right|
$$

where $s_{i}$ is the individual baseline length. The results from the three statistical tests are also given in Table 7. It is shown that the RMS of the discrepancies between the the computed geoidal heights and the GPS-derived geoidal heights was 8.0 to 9.9 ppm of the distance separating the points.

The permissible discrepancy for leveling in Peninsular Malaysia is $3 \mathrm{~mm} \sqrt{\mathrm{~km}}$ for $1^{\text {st }}$ order 2-way levelling and $12 \mathrm{~mm} \sqrt{ } \mathrm{~km}$ for $2^{\text {nd }}$ order levelling. Thus, the accuracy of spirit levelling is a function of the square root of the levelling distance. On the other hand, GPS accuracy depends on the interdistance between points. This is clearly demonstrated in Figure 9.15 whereby the 8.0 ppm line is plotted together with the levelling permissible discrepancy curves. The results demonstrate that the accuracy achievable by GPS heighting do not meet the requirements of the first order spirit levelling in this part of the country. It appears that at best GPS heighting technique can provide $2^{\text {nd }}$ order levelling standard only for distances of up to 2.25 km , at a precision that equates to 18 mm .


Figure 25 Comparison of accuracies of orthometric height differences from GPS (8.0ppm) and from spirit levelling

The accuracy and density of gravity data is paramount in any gravimetric determination. The test area is located close to the coast. This means that the altimeter data are affected by wave action and also by loss of signal lock through back scattering from the coastal land. To the north and south-east of the test area, there are disparities of the coverage of gravity data. These are probably among the reasons for the relatively less satisfactory performance of the gravimetric geoid in the test area.

Checks were also carried out with points not included in the analyses. Table 8 presents the results at five check points within the test area. The average difference between GPSderived and local geoid heights is -0.0412 m , with values ranging from -0.224 m to 0.079 m . The standard deviation of the differences is 0.1148 m .

In practice, the accuracy of the relative orthometric height $\Delta H$ is dependent on the accuracy of the geoidal height, $\Delta N$, and of the relative ellipsoidal height, $\Delta h$, in the relation:

$$
\sigma_{\Delta H}^{2}=\sigma_{\Delta N}^{2}+\sigma_{\Delta h}^{2}
$$

Table 8 Geoidal height differences of 5 check points

| No. | Longitude <br> $\lambda\left({ }^{\circ}\right)$ | Latitude <br> $\phi\left({ }^{\circ}\right)$ | $\mathbf{N}_{\text {GPS }}$ <br> $(\mathbf{m})$ | $\mathbf{N}_{\text {Model }}$ <br> $(\mathbf{m})$ | Difference <br> $(\mathbf{m})$ |
| :---: | :---: | :---: | :---: | :---: | :--- |
| 1 | 101.0108 | 4.49590 | -7.598 | -7.374 | -0.224 |
| 2 | 101.2607 | 4.20096 | -5.940 | -5.923 | -0.017 |
| 3 | 100.8545 | 4.52206 | -8.176 | -8.255 | 0.079 |
| 4 | 100.9294 | 4.16724 | -7.160 | -7.180 | 0.020 |
| 5 | 100.7162 | 4.67351 | -9.112 | -9.048 | -0.064 |
| Mean |  |  |  |  |  |
| Standard Deviation | -0.0412 m |  |  |  |  |
| $\pm 0.1148 \mathrm{~m}$ |  |  |  |  |  |

If the relative accuracy of $\Delta h$ is taken as 2 ppm over a line of 20 km , the $\sigma_{\Delta h}$ inferred is 40mm. Similarly, if $\sigma_{\Delta N}$ is 15 mm as a result of a 'hypothetical' improvement to MYGeoid02, the relative accuracy required for $H$ is roughly 43 mm . This equates to 2.2ppm.

Figure 23 shows a plot of the 2.2 ppm line with the $1^{\text {st }}$ and $2^{\text {nd }}$ order levelling permissible discrepancy curves. It can be suggested that, with cm-accuracy geoid, the use of GPS in height surveys is comparable to $1^{\text {st }}$ order levelling in not more than 2 km . On the other hand, the technique exceeds $2^{\text {nd }}$ order levelling for distances up to 30 km .


Figure 26 Comparison of accuracies of orthometric height differences from GPS (2.2ppm) and from spirit levelling

### 3.2.1 Modelling Vertical Datum Bias

As can be seen from Table 6 the direct comparison between the local geoidal heights $N$ and height differences between GPS heights ( $h$ ) and levelling heights ( $H$ ) of some 37 points results in discrepancies as denoted by $\delta N_{1}$. Figure 27 presents a contour plot of $\delta N_{1}$ values, together with the data points which were used. The influence of the Main Range mountain chain can be clearly seen in the southern part of the test area where the largest values of $\delta N_{1}$ are found. This is expected, since the topography features in the area are undulating and rough. This made both geoid modelling and spirit levelling less accurate than in flat areas.


Figure 27 Contour Plot of Discrepancies $\left(\delta N_{1}\right)$ between the local geoid model and geoid heights derived from GPS observations at 37 common points of GPS and levelling in Perak (shown as solid circles; contour interval is 0.05 m )

There are a number of factors that contribute to the computed values of $\delta N_{1}$. These values, as in Table 6, reflect the vertical datum inconsistencies between the available height data, long wavelength geoid errors, and GPS and levelling errors included in the ellipsoidal and orthometric heights (Vergos \& Sideris, 2002). In Peninsular Malaysia, biases are known to exist in the computed local geoids, with values reported as -8 cm , 41cm and -54cm (Fashir et al. 2000a; Fashir et al. 2000b; Kadir et al., 2000). These inconsistencies could be improved by fitting a bias and tilt to the discrepancies. In order to reduce the amount of the deviations to a minimum, a three-parameter model or a planar surface function is applied (modified from Fashir et al. 2000a) with pointwise derivation given in Section 5.9 of Heiskanen \& Moritz (1967):

$$
\delta N=(h-H)-N=\sum_{i=0}^{n} \sum_{j=0}^{n} a_{i j} x^{i} y^{j}+v
$$

or

$$
\delta N=a_{00}+a_{01} y+a_{10 X}+v
$$

where $a_{00}$ is the shift parameter between the vertical datum implied by the GPS/levelling data and the gravimetric datum, $a_{01}$ and $a_{10}$ are the north-south and eastwest tilts of the plane surface, $(i, j)$ denotes the horizontal location of the points, $x$ and $y$ are the horizontal coordinate components of the regression surface and $v_{i}$ are the residuals to be minimised.

A four-parameter transformation model with one additional parameter added is represented by the following formula:

$$
\delta N=a_{00}+a_{01} y+a_{10} x+a_{11} x y+v
$$

where now $a_{01}, a_{10}$, and $a_{11}$ are the translation parameters in $\mathrm{x}, \mathrm{y}$ and z axes between the coordinate system implied by the GPS data and the one implied by the gravimetric data.

It should be noted that the choice of the parametric form of the corrector surface model, which can vary from a simple plane to more complicated higher-order polynomial and trigonometric corrector surfaces, is not a trivial task (Kotsakis et al., 2001; Featherstone, 2000). For the test carried out in the present study, a four-transformation model is used. The polynomial coefficients $a_{00}, a_{01}, a_{10}$, and $a_{11}$ in the four-parameter transformation model can be estimated through a least squares fit and tested. Once these have been solved for, the gravimetric geoid can be corrected into a geoid compatible with the GPS-derived geoid. Thus, the resulting geoid will be associated with a local vertical datum, similar to the levelled heights given for the GPS-occupied stations.

An in-house software was used to perform the least squares estimation and the result gives the following values: $a_{00}=0.1502 \mathrm{~m}, a_{01}=-13.3918, a_{10}=47.3674$ and $a_{11}=$ -9.3516. This indicates that the local vertical datum is not physically parallel to the datum implied by the local geoid model by a shift of 15 cm . It also shows that the shift and translation parameters are statistically significant for the area under study. Having applied the transformation, the statistics of the results is presented in Table 9.

From the Table , after the fit, using the four-parameter transformation model, the RMS of the residuals is at $\pm 12.5 \mathrm{~cm}$ and a standard deviation of $\pm 13.4 \mathrm{~cm}$. The RMS difference before the fit, as indicated in Table 6, is $\pm 21.8 \mathrm{~cm}$ with a standard deviation of $\pm 14.6 \mathrm{~cm}$. The contour plot of the residuals is illustrated in Figure 29. The significant improvement in the results after the application of bias and tilt indicate that the four-parameter model can be considered as adequate for the reduction of $\delta N_{i}$ values.

However, results also indicate that at the current level of limitations of the geoid in particular, it cannot reach the millimeter level requirements of a first-order levelling operation. On a positive note though, the data set of GPS-derived geoidal heights can be a valuable resource for evaluating and testing the geoid model and/or vice-versa. This could also be another way of checking and identifying stations that may have Errors in their heights.

Table 9 Results of 4-parameter transformation for 37 common points

| Station | Longitude <br> $\lambda\left({ }^{\circ}\right)$ | Latitude <br> $\boldsymbol{\phi}\left(^{\circ}\right)$ | Residuals $\left(\boldsymbol{v}_{\boldsymbol{i}}\right)$ after transformation |
| :--- | ---: | ---: | :--- |
| $(\mathbf{m})$ |  |  |  |



Figure 28 Contour Plot of Residuals after bias and tilt fit with the four-parameter transformation model for 37 common points of GPS and levelling in Perak (shown as solid circles; contour interval is 0.05 m )

### 3.3 Results of Investigation On Accuracy of GPS Heighting In The State of Johor

Figure 29 shows a plot of absolute orthometric height differences derived from the EGM96, OSU91A and the gravimetric geoid model relative to the local vertical datum. Table 11 lists these differences. The results of Table 10 indicates that, the mean of these differences has values $-36 \mathrm{~cm},-48 \mathrm{~cm}$ and -3 cm with uncertainties of $30 \mathrm{~cm}, 37 \mathrm{~cm}$ and 31 cm respectively. Figure 30 shows that there are large differences at stations 11, 24, 25 and 26. More investigations are required as to the causes of the large differences at these stations; the leveling and the GPS results should be checked carefully. These four stations are considered as outliers. These outliers may be due to Benchmark movement, ellipsoidal height error, and / or inaccuracies in the geoidal height model.


Fig. 29. Absolute orthometric height differences of EGM96, OSU91A and Gravimetric geoid models relative to local vertical datum

|  | $\mathrm{H}_{\text {GPS }}-\mathrm{H}_{\text {EGM }}$ | $\mathrm{H}_{\text {GPS }}-\mathrm{H}_{\text {OSU }}$ | $\mathrm{H}_{\text {GPS }}-\mathrm{H}_{\text {GRAV }}$ |
| :---: | :---: | :---: | :---: |
| MAXIMUM | 0.350 | 0.500 | 0.715 |
| MINIMUM | -1.180 | -1.020 | -0.713 |
| MEAN | $\mathbf{- 0 . 3 5 9}$ | $\mathbf{- 0 . 4 7 9}$ | $\mathbf{- 0 . 0 3 1}$ |
| RMS | 0.303 | 0.368 | 0.305 |
| STD | 0.297 | 0.361 | 0.299 |

Table 10 Statistics of absolute orthometric height differences of EGM96, OSU91A and the gravimetric model relative to the local vertical datum.

|  | $H_{\text {GPS }}-H_{\text {EGM }}^{*}$ | $H_{\text {GPS }}-H_{\text {OSU }}^{*}$ | $H_{\text {GPS }}-H_{\text {GRAV }}^{*}$ |
| :--- | :--- | :--- | :--- |
| MAXIMUM | 0.526 | 0.566 | 0.512 |
| MINIMUM | -0.983 | -0.903 | -0.932 |
| Mean | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ |
| RMS | 0.274 | 0.266 | 0.263 |
| STD. $D$ | 0.269 | 0.261 | 0.258 |

Table 11 Statistics of bias corrected absolute orthometric height differences of EGM96, OSU91A and gravimetric models relative to the local vertical datum

Figure 30 shows a plot of absolute orthometric height differences of the EGM96, OSU91A and the gravimetric geoid model relative to the local vertical datum after rejecting stations 11, 24, 25 and 26 from the data set. These differences are listed in Table 12. The overall accuracies of these differences improved significantly, i.e., the uncertainties of the differences are $15 \mathrm{~cm}, 23 \mathrm{~cm}$ and 16 cm for the EGM96, OSU91A and the gravimetric geoid model relative to the local vertical datum respectively.


Fig. 30. Absolute bias corrected orthometric height differences of EGM96, OSU91A and Gravimetric geoid models relative to local vertical datum after rejecting outliers.

|  | $H_{\text {GPS }}-H_{\text {EGM }}$ | $H_{\text {GPS }}-H_{\text {OSU }}$ | $H_{\text {GPS }}-H_{\text {GRAV }}$ |
| :--- | :--- | :--- | :--- |
| MAXIMUM | -0.180 | -0.080 | 0.245 |
| MINIMUM | -0.650 | -0.940 | -0.290 |
| MEAN | $\mathbf{- 0 . 3 5 5}$ | $\mathbf{- 0 . 5 1 5}$ | $\mathbf{- 0 . 0 4 2}$ |
| RMS | 0.149 | 0.233 | 0.159 |
| STD. $D$ | 0.145 | 0.228 | 0.156 |

Table 12 Statistics of absolute orthometric height differences of EGM96, OSU91A and the gravimetric model relative to the local vertical datum after rejection of outliers

Figure 31 shows a plot of bias corrected absolute orthometric height differences derived from the EGM96, OSU91A and the gravimetric geoid model relative to the local vertical datum. Table 12 lists these differences. Results of Table 11 suggests that, after a threeparameter trend of equation (6) was removed, the differences assumed zero means with uncertainties of $27 \mathrm{~cm}, 27 \mathrm{~cm}$ and 26 cm respectively.


Fig. 31. Absolute orthometric height differences of EGM96, OSU91A and Gravimetric geoid models relative to local vertical datum after rejecting outliers.

After rejection of outliers, the bias corrected absolute orthometric height differences derived from the EGM96, OSU91A and the gravimetric geoid model relative to the local vertical datum are depicted in Figure 32. These differences are listed in Table 13. The accuracy of these differences improved significantly, i.e., the uncertainties of the differences are $11 \mathrm{~cm}, 12 \mathrm{~cm}$ and 10 cm for the EGM96, OSU91A and the gravimetric geoid model relative to the local vertical datum respectively.


Fig. 32. Relative orthometric height differences derived from EGM96, OSU91A and Gravimetric model relative to GPS/Levelling datum (Station GP58 held fixed).

|  | $H_{G P S}-H_{\text {EGM }}^{*}$ | $H_{\text {GPS }}-H_{\text {OSU }}^{*}$ | $H_{\text {GPS }}-H_{\text {GRAV }}^{*}$ |
| :--- | :--- | :--- | :--- |
| MAXIMUM | 0.166 | 0.186 | 0.195 |
| MINIMUM | -0.225 | -0.240 | -0.191 |
| MEAN | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ |
| RMS | 0.110 | 0.115 | 0.104 |
| STD. $\boldsymbol{D}$ | 0.108 | 0.113 | 0.101 |

Table 13 Statistics of bias corrected absolute orthometric height differences of EGM96, OSU91A and the gravimetric model relative to the local vertical datum after rejection of Outliers.

Fixing station GP58, relative orthometric height differences derived from the EGM96, OSU91A and the gravimetric geoid model relative to the local vertical datum are depicted in Figure 33.These differences expressed in terms of part per million (ppm) are listed in Table 14. As seen in Table 14, the relative agreement derived from the EGM96, OSU91A and the gravimetric geoid model relative to the local vertical datum is about 14
to $0.2 \mathrm{ppm}, 11$ to 0.2 ppm and 14 to 0.2 ppm for baselines of 28 to 125 km respectively. The average agreement in terms of ppm of the above three models relative to GPS/leveling datum is of the order of 3.1, 3.9 and 3.5 for the above baseline lenghts respectively.


Fig. 33. Bias corrected relative orthometric height differences derived from EGM96, OSU91A and Gravimetric model relative to local vertical datum (Station GP58 held fixed).

| ID | STN | $\Delta H_{\text {GPS }}-\Delta H_{\text {EGM }}$ | ppm | $\Delta H_{\text {GPS }}-\Delta H_{\text {OSU }}$ | ppm | $\Delta H_{\text {GPP }}-\Delta H_{\text {GRAV }}$ | ppm | Distance <br> $(\mathrm{km})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | GP58 | fixed |  | fixed |  | fixed |  |  |
| 2 | GP15 | -0.098 | 1.012 | 0.082 | 0.847 | 0.073 | 0.758 | 96.8 |
| 3 | GP16 | 0.000 | 0.000 | 0.120 | 1.544 | 0.151 | 1.944 | 77.7 |
| 4 | GP43 | 0.270 | 4.095 | -0.040 | 0.607 | 0.276 | 4.184 | 65.9 |
| 5 | GP44 | 0.300 | 5.383 | 0.320 | 5.742 | 0.283 | 5.076 | 55.7 |
| 6 | GP47 | 0.220 | 3.488 | 0.290 | 4.598 | 0.250 | 3.968 | 63.1 |
| 7 | GP48 | -0.110 | 1.909 | -0.040 | 0.694 | 0.116 | 2.007 | 57.6 |
| 8 | GP49 | -0.030 | 0.499 | 0.010 | 0.166 | 0.204 | 3.387 | 60.1 |
| 9 | GP50 | 0.100 | 1.578 | 0.010 | 0.158 | 0.144 | 2.276 | 63.4 |
| 10 | GP51 | 0.040 | 0.660 | -0.250 | 4.128 | -0.040 | 0.652 | 60.6 |
| 11 | GP52 | -0.680 | 5.440 | -1.080 | 8.640 | -0.685 | 5.483 | 125.0 |
| 12 | GP53 | 0.010 | 0.158 | -0.330 | 5.229 | -0.101 | 1.600 | 63.1 |
| 13 | GP54 | -0.130 | 1.705 | -0.500 | 6.559 | -0.215 | 2.822 | 76.2 |
| 14 | GP55 | 0.090 | 1.756 | -0.190 | 3.707 | 0.007 | 0.146 | 51.3 |
| 15 | GP56 | -0.230 | 4.014 | -0.130 | 2.269 | 0.189 | 3.293 | 57.3 |
| 16 | DOP2 | 0.430 | 5.087 | -0.300 | 3.549 | -0.126 | 1.488 | 84.5 |
| 17 | GP59 | -0.010 | 0.360 | 0.040 | 1.441 | 0.103 | 3.697 | 27.8 |
| 18 | GP61 | 0.030 | 1.081 | 0.090 | 3.243 | 0.076 | 2.724 | 27.8 |
| 19 | GP84 | -0.150 | 2.436 | -0.090 | 1.462 | 0.152 | 2.475 | 61.6 |
| 20 | GP85 | -0.140 | 1.749 | -0.050 | 0.625 | 0.142 | 1.779 | 80.1 |
| 21 | GP90 | -0.120 | 1.531 | -0.500 | 6.380 | -0.200 | 2.551 | 78.4 |
| 22 | GP91 | -0.130 | 3.310 | -0.330 | 8.402 | -0.200 | 5.095 | 39.3 |
| 23 | TG05 | 0.320 | 3.919 | 0.360 | 4.409 | 0.320 | 3.918 | 81.6 |
| 24 | TG07 | -0.580 | 6.728 | -0.990 | 11.484 | -0.680 | 7.882 | 86.2 |
| 25 | TG09 | 0.620 | 13.977 | 0.440 | 9.919 | 0.600 | 13.531 | 44.4 |
| 26 | TG10 | 0.850 | 8.329 | 0.400 | 3.920 | 0.743 | 7.283 | 102.0 |
| 27 | TG19 | -0.090 | 1.207 | -0.070 | 0.939 | 0.053 | 0.715 | 74.6 |
|  | Mean | -0.030 | 3.131 | 0.105 | 3.872 | -0.063 | 3.490 |  |
|  | Sd | 0.321 | 3.091 | 0.374 | 3.189 | 0.311 | 2.817 |  |

Table 14 Relative orthometric height differences of EGM96, OSU91A and the Gravimetric model relative to the local vertical datum expressed in part per million (Station GP58 held fixed)

After rejection of outliers and fixing station GP58, relative orthometric height differences derived from the EGM96, OSU91A and the gravimetric geoid model relative to the local vertical datum are depicted in Figure 34.These differences expressed in terms of ppm are listed in Table 15. As seen in Table 15, the relative agreement of the EGM96, OSU91A
and the gravimetric geoid model relative to the local vertical datum is about 5 to 0.2 ppm , 8 to 0.2 ppm and 5 to 0.2 ppm for baselines of 28 to 125 km respectively. The average agreement in terms of ppm of the above three models relative to GPS/leveling datum is of the order of 2.1, 3.0 and 2.6 for the above baseline lenghts respectively.


Fig. 34. Bias corrected relative orthometric height differences of EGM96, Osu91A and Gravimetric model after rejecting outliers.

| ID | STN | $\Delta H_{\text {GPS }}-\Delta H_{\text {EGM }}$ | ppm | $\Delta H_{\text {GPS }}-\Delta H_{\text {OSU }}$ | ppm | $\Delta H_{\text {GPS }}-\Delta H_{\text {GRAV }}$ | ppm |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | GP58 | fixed |  | fixed |  | fixed |  |
| 2 | GP15 | -0.098 | 1.012 | 0.082 | 0.847 | 0.073 | 0.758 |
| 3 | GP16 | 0.000 | 0.000 | 0.120 | 1.544 | 0.151 | 1.944 |
| 4 | GP43 | 0.270 | 4.095 | -0.040 | 0.607 | 0.276 | 4.184 |
| 5 | GP44 | 0.300 | 5.383 | 0.320 | 5.742 | 0.283 | 5.076 |
| 6 | GP47 | 0.220 | 3.488 | 0.290 | 4.598 | 0.250 | 3.968 |
| 7 | GP48 | -0.110 | 1.909 | -0.040 | 0.694 | 0.116 | 2.007 |
| 8 | GP49 | -0.030 | 0.499 | 0.010 | 0.166 | 0.204 | 3.387 |
| 9 | GP50 | 0.100 | 1.578 | 0.010 | 0.158 | 0.144 | 2.276 |
| 10 | GP51 | 0.040 | 0.660 | -0.250 | 4.128 | -0.040 | 0.652 |
| 12 | GP53 | 0.010 | 0.158 | -0.330 | 5.229 | -0.101 | 1.600 |
| 13 | GP54 | -0.130 | 1.705 | -0.500 | 6.559 | -0.215 | 2.822 |
| 14 | GP55 | 0.090 | 1.756 | -0.190 | 3.707 | 0.007 | 0.146 |
| 15 | GP56 | -0.230 | 4.014 | -0.130 | 2.269 | 0.189 | 3.293 |
| 16 | DOP2 | 0.430 | 5.087 | -0.300 | 3.549 | -0.126 | 1.488 |
| 17 | GP59 | -0.010 | 0.360 | 0.040 | 1.441 | 0.103 | 3.697 |
| 18 | GP61 | 0.030 | 1.081 | 0.090 | 3.243 | 0.076 | 2.724 |
| 19 | GP84 | -0.150 | 2.436 | -0.090 | 1.462 | 0.152 | 2.475 |
| 20 | GP85 | -0.140 | 1.749 | -0.050 | 0.625 | 0.142 | 1.779 |
| 21 | GP90 | -0.120 | 1.531 | -0.500 | 6.380 | -0.200 | 2.551 |
| 22 | GP91 | -0.130 | 3.310 | -0.330 | 8.402 | -0.200 | 5.095 |
| 23 | TG05 | 0.320 | 3.919 | 0.360 | 4.409 | 0.320 | 3.918 |
| 27 | TG19 | -0.090 | 1.207 | -0.070 | 0.939 | 0.053 | 0.715 |
|  |  | 0.026 | $\mathbf{2 . 1 3 4}$ | -0.068 | $\mathbf{3 . 0 3 2}$ | 0.075 | $\mathbf{2 . 5 7 1}$ |
|  |  | 0.180 | $\mathbf{1 . 6 0 4}$ | 0.238 | $\mathbf{2 . 4 0 6}$ | 0.162 | $\mathbf{1 . 4 0 4}$ |

Table 15 Relative orthometric height differences of EGM96, OSU91A and the gravimetric model relative to the local vertical datum expressed in parts per million after rejecting outliers (Station GP58 held fixed)

Fixing station GP58, bias corrected relative orthometric height differences derived from the EGM96, OSU91A and the gravimetric geoid model relative to the local vertical datum are depicted in figure 35.These differences expressed in terms of part per million (ppm) are listed in Table 16. As seen in Table 17, the relative agreement of the EGM96, OSU91A and the gravimetric geoid model relative to the local vertical datum is about 11 to $0.3 \mathrm{ppm}, 9$ to 0.2 ppm and 12 to 0.3 ppm for baselines of 28 to 125 km respectively.


Fig. 35. Relative orthometric height of EGM96, OSU91A and Gravimetric model relative differences after rejecting outliers.

| ID | STN | $\Delta H_{\text {GPS }}-\Delta H_{\text {EGM }}^{*}$ | ppm | $\Delta H_{\text {GPS }}-\Delta H_{\text {OSU }}^{*}$ | ppm | $\Delta H_{\text {GPS }}-\Delta H_{\text {GRAV }}^{*}$ | ppm | Distanc <br> e <br> $(\mathrm{km})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | GP58 | fixed |  | fixed |  | fixed |  |  |
| 2 | GP15 | -0.044 | 0.455 | -0.210 | 2.169 | -0.065 | 0.667 | 96.8 |
| 3 | GP16 | 0.022 | 0.283 | -0.136 | 1.750 | 0.024 | 0.310 | 77.7 |
| 4 | GP43 | 0.085 | 1.289 | -0.086 | 1.304 | 0.205 | 3.107 | 65.9 |
| 5 | GP44 | 0.188 | 3.373 | 0.065 | 1.166 | 0.122 | 2.187 | 55.7 |
| 6 | GP47 | 0.142 | 2.251 | 0.013 | 0.206 | 0.086 | 1.368 | 63.1 |
| 7 | GP48 | -0.041 | 0.711 | -0.170 | 2.950 | 0.066 | 1.140 | 57.6 |
| 8 | GP49 | 0.154 | 2.563 | 0.102 | 1.698 | 0.298 | 4.968 | 60.1 |
| 9 | GP50 | 0.305 | 4.814 | 0.203 | 3.204 | 0.297 | 4.691 | 63.4 |
| 10 | GP51 | 0.207 | 3.418 | 0.011 | 0.182 | 0.139 | 2.287 | 60.6 |
| 11 | GP52 | -0.443 | 3.544 | -0.510 | 4.080 | -0.328 | 2.627 | 125 |
| 12 | GP53 | 0.107 | 1.695 | -0.047 | 0.745 | 0.071 | 1.125 | 63.1 |
| 13 | GP54 | -0.086 | 1.128 | -0.195 | 2.558 | -0.045 | 0.592 | 76.2 |
| 14 | GP55 | 0.087 | 1.697 | -0.011 | 0.215 | 0.101 | 1.961 | 51.3 |
| 15 | GP56 | 0.097 | 1.693 | -0.069 | 1.204 | 0.193 | 3.363 | 57.3 |
| 16 | DOP2 | 0.248 | 2.934 | 0.029 | 0.343 | 0.111 | 1.315 | 84.5 |
| 17 | GP59 | 0.051 | 1.838 | 0.021 | 0.757 | 0.109 | 3.914 | 27.8 |
| 18 | GP61 | 0.014 | 0.505 | -0.022 | 0.793 | 0.014 | 0.490 | 27.8 |
| 19 | GP84 | -0.039 | 0.633 | -0.173 | 2.810 | 0.137 | 2.232 | 61.6 |
| 20 | GP85 | -0.039 | 0.487 | -0.223 | 2.785 | 0.077 | 0.967 | 80.1 |
| 21 | GP90 | -0.077 | 0.983 | -0.188 | 2.399 | -0.026 | 0.330 | 78.4 |
| 22 | GP91 | -0.054 | 1.375 | -0.151 | 3.845 | -0.087 | 2.218 | 39.3 |
| 23 | TG05 | 0.149 | 1.825 | -0.013 | 0.159 | 0.082 | 1.003 | 81.6 |
| 24 | TG07 | -0.446 | 5.173 | -0.603 | 6.995 | -0.444 | 5.144 | 86.2 |
| 25 | TG09 | 0.491 | 11.069 | 0.394 | 8.882 | 0.543 | 12.246 | 44.4 |
| 26 | TG10 | 1.063 | 10.417 | 0.866 | 8.486 | 1.040 | 10.193 | 102 |
| 27 | TG19 | 0.149 | 1.998 | 0.101 | 1.354 | 0.203 | 2.726 | 74.6 |
|  |  | -0.088 | 2.621 | 0.039 | $\mathbf{2 . 4 2 5}$ | -0.112 | 2.814 |  |
|  | Sd | 0.279 | 2.720 | 0.271 | $\mathbf{2 . 4 0 3}$ | 0.267 | 2.862 |  |

Table 16. Bias corrected relative orthometric height differences of EGM96, OSU91A and the gravimetric model relative to the local vertical datum expressed in parts per million (Station GP58 held fixed)

| ID | STN | $h$ | $\Delta h$ | $H^{*}$ | $\Delta H^{*}$ | $N^{*}$ | $\Delta N^{*}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | GP58 | 77.620 | fixed | 71.993 | fixed | 5.628 | fixed |
| 2 | GP15 | 4.002 | 73.618 | 1.787 | 70.206 | 2.215 | 3.413 |
| 3 | GP16 | 36.230 | 41.390 | 33.386 | 38.607 | 2.844 | 2.784 |
| 4 | GP43 | 11.180 | 66.440 | 5.087 | 66.906 | 6.093 | -0.465 |
| 5 | GP44 | 55.650 | 21.970 | 51.884 | 20.109 | 3.766 | 1.862 |
| 6 | GP47 | 51.180 | 26.440 | 47.838 | 24.155 | 3.342 | 2.286 |
| 7 | GP48 | 164.750 | -87.130 | 160.948 | -88.955 | 3.802 | 1.826 |
| 8 | GP49 | 7.360 | 70.260 | 2.141 | 69.852 | 5.220 | 0.409 |
| 9 | GP50 | 10.120 | 67.500 | 3.999 | 67.994 | 6.121 | -0.493 |
| 10 | GP51 | 42.710 | 34.910 | 35.551 | 36.442 | 7.160 | -1.532 |
| 11 | GP52 | 67.810 | 9.810 | 58.284 | 13.709 | 9.526 | -3.898 |
| 12 | GP53 | 31.820 | 45.800 | 23.963 | 48.030 | 7.857 | -2.229 |
| 13 | GP54 | 12.880 | 64.740 | 4.507 | 67.486 | 8.373 | -2.745 |
| 14 | GP55 | 36.740 | 40.880 | 29.323 | 42.670 | 7.418 | -1.790 |
| 15 | GP56 | 11.570 | 66.050 | 4.765 | 67.228 | 6.805 | -1.177 |
| 16 | DOP2 | 90.910 | -13.290 | 83.803 | -11.810 | 7.107 | -1.479 |
| 17 | GP59 | 45.180 | 32.440 | 40.151 | 31.842 | 5.029 | 0.599 |
| 18 | GP61 | 41.830 | 35.790 | 37.226 | 34.767 | 4.604 | 1.024 |
| 19 | GP84 | 7.130 | 70.490 | 3.179 | 68.814 | 3.951 | 1.677 |
| 20 | GP85 | 4.930 | 72.690 | 1.849 | 70.144 | 3.081 | 2.547 |
| 21 | GP90 | 10.780 | 66.840 | 2.336 | 69.657 | 8.444 | -2.816 |
| 22 | GP91 | 26.310 | 51.310 | 19.355 | 52.638 | 6.955 | -1.327 |
| 23 | TG05 | 193.730 | -116.110 | 190.774 | -118.781 | 2.956 | 2.672 |
| 24 | TG07 | 194.270 | -116.650 | 185.649 | -113.656 | 8.622 | -2.994 |
| 25 | TG09 | 223.000 | -145.380 | 217.135 | -145.142 | 5.865 | -0.237 |
| 26 | TG10 | 193.250 | -115.630 | 184.432 | -112.439 | 8.818 | -3.190 |
| 27 | TG19 | 136.070 | -58.450 | 130.495 | -58.502 | 5.575 | 0.053 |

Table 17 Bias corrected Relative differences derived from gravimetric model and GPS/leveling data (Station GP58 held fixed)

The average agreement in terms of ppm of the above three models relative to GPS/leveling datum is of the order of 2.6, 2.4 and 2.8 for the above baseline lenghts respectively.

After rejection of outliers and fixing station GP58, bias corrected relative orthometric height differences derived from the EGM96, OSU91A and the gravimetric geoid model relative to the local vertical datum are depicted in figure 33. These differences expressed
in terms of ppm are listed in Table 18. As seen in Table 33, the relative agreement of the EGM96, OSU91A and the gravimetric geoid model relative to the local vertical datum is about 5 to $0.3 \mathrm{ppm}, 4$ to 0.2 ppm and 5 to 0.3 ppm for baselines of 28 to 125 km respectively. The average agreement in terms of ppm of the above three models relative to GPS/leveling datum is of the order of 1.7, 1.6 and 2.0 for the above baseline lenghts respectively

| ID | STN | $\Delta H_{\text {GPS }}-\Delta H_{\text {EGM }}^{*}$ | ppm | $\Delta H_{\text {GPS }}-\Delta H_{\text {OSU }}^{*}$ | ppm | $\Delta H_{\text {GPS }}-\Delta H_{\text {GRAV }}^{*}$ | ppm |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | GP58 | fixed |  | Fixed |  | fixed |  |
| 2 | GP15 | -0.044 | 0.455 | -0.210 | 2.169 | -0.065 | 0.667 |
| 3 | GP16 | 0.022 | 0.283 | -0.136 | 1.750 | 0.024 | 0.310 |
| 4 | GP43 | 0.085 | 1.289 | -0.086 | 1.304 | 0.205 | 3.107 |
| 5 | GP44 | 0.188 | 3.373 | 0.065 | 1.166 | 0.122 | 2.187 |
| 6 | GP47 | 0.142 | 2.251 | 0.013 | 0.206 | 0.086 | 1.368 |
| 7 | GP48 | -0.041 | 0.711 | -0.170 | 2.950 | 0.066 | 1.140 |
| 8 | GP49 | 0.154 | 2.563 | 0.102 | 1.698 | 0.299 | 4.968 |
| 9 | GP50 | 0.305 | 4.814 | 0.203 | 3.204 | 0.297 | 4.691 |
| 10 | GP51 | 0.207 | 3.418 | 0.011 | 0.182 | 0.139 | 2.287 |
| 12 | GP53 | 0.107 | 1.695 | -0.047 | 0.745 | 0.071 | 1.125 |
| 13 | GP54 | -0.086 | 1.128 | -0.195 | 2.558 | -0.045 | 0.592 |
| 14 | GP55 | 0.087 | 1.697 | -0.011 | 0.215 | 0.101 | 1.961 |
| 15 | GP56 | 0.097 | 1.693 | -0.069 | 1.204 | 0.193 | 3.363 |
| 16 | DOP2 | 0.248 | 2.934 | 0.029 | 0.343 | 0.111 | 1.315 |
| 17 | GP59 | 0.051 | 1.838 | 0.021 | 0.757 | 0.109 | 3.914 |
| 18 | GP61 | 0.014 | 0.505 | -0.022 | 0.793 | 0.014 | 0.490 |
| 19 | GP84 | -0.039 | 0.633 | -0.173 | 2.810 | 0.137 | 2.232 |
| 20 | GP85 | -0.039 | 0.487 | -0.223 | 2.785 | 0.077 | 0.967 |
| 21 | GP90 | -0.077 | 0.983 | -0.188 | 2.399 | -0.026 | 0.330 |
| 22 | GP91 | -0.054 | 1.375 | -0.151 | 3.845 | -0.087 | 2.218 |
| 23 | TG05 | 0.149 | 1.825 | -0.013 | 0.159 | 0.082 | 1.003 |
| 27 | TG19 | 0.149 | 1.998 | 0.101 | 1.354 | 0.203 | 2.726 |
|  |  | 0.074 | 1.725 | -0.052 | 1.573 | 0.096 | 1.953 |
|  |  | 0.112 | 1.151 | 0.118 | 1.119 | 0.104 | 1.373 |

Table 18 Bias corrected relative orthometric height differences of EGM96, OSU91A and the gravimetric model relative to the local vertical datum expressed in parts per million after rejecting outliers (Station GP58 held fixed).

Results of Table 19 indicates that local vertical datum is not physically parallel with the EGM96, OSU91A and the gravimetric geoid. The shift parameters between the vertical datum implied by the GPS/leveling datum, and the EGM96, OSU91A and the gravimetric datums are about $-41 \mathrm{~cm},-54 \mathrm{~cm}$ and -8 cm respectively. Also the maximum tilts of the planes fitting the residual geoids above these datums relative to GPS/leveling datum are of the order of 36,51 and 33 centimeters per degree. It is therefore necessary to take into consideration the effect of inconsistent datum bias particularly for long baseline height transfer.

| Residual geoid <br> Cm | $C_{1}$ <br> $\mathrm{~cm} /$ degree | $C_{2}$ <br> $\mathrm{~cm} /$ degree | $c_{0}$ <br> cm | Tilt <br> $\mathrm{cm} /$ degree | Azimuth <br> degrees |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $N_{\text {GPS }}-N_{\text {EGM96 }}$ | -35.8 | -3.7 | -41.0 | 36.0 | 5.90 |
| $N_{\text {GPS }}-N_{\text {OSU91A }}$ | -35.8 | 35.9 | -54.0 | 50.7 | 134.93 |
| $N_{\text {GPS }}-N_{\text {GRAV }}$ | -27.7 | 17.8 | -7.8 | 32.9 | 147.28 |

Table 19 Datum shift parameters of EGM96, OSU91A and gravimetric geoids relative to the local vertical datum (GPS/leveling datum)

When analyzing these absolute errors and the relative errors that follow, it should be kept in mind that these are not purely leveling errors, but contain gravimetric geoid determination errors and GPS position errors as well. Consequently, they could also be used to identify stations that have errors in their ellipsoidal or geoidal heights.

A comparison of different geoid models, as seen from all the tables and figures, indicates that the OSU91A model had the best agreement with the GPS/leveling data when comparing orthometric height differences in an absolute mode. While the EGM96 geoid had the best agreement with the GPS/leveling data when comparing orthometric height differences in a relative mode. The regional gravimetric geiod had the least agreement
with GPS/leveling data. This may be due to poor gravity coverage in the test area. Also in our regional geoid solution, the terrain effect on the anomalies as well as their by product indirect effect on the computed geoid was not considered.

Using bias corrected relative orthometric height differences derived from the EGM96, OSU91A and the regional gravimetric geoid models combined with GPS/leveling data, we can achieve relative accuracy of the order of 1.7, 1.6 and 2 ppm for baseline lengths of 28-125 km respectively. For a baseline length of 36 km , the mean accuracy of these bias corrected relative orthometric height differences derived from the EGM96, OSU91A and the regional gravimetric geoid models are 61,58 and 72 mm respectively. The precision of third order leveling based on the Malaysian height datum for the same baseline length is: $12 \sqrt{K}=12 \sqrt{36}=72 \mathrm{~mm}$. This means, the level of accuracy achieved by the bias corrected relative orthometric height differences of the above three models for baseline lengths up to 36 km , is sufficient to replace the conventional tedious, time consuming ordinary leveling technique for rapid height transfer for land surveying and engineering applications. The other tables and figures relate to the result of this research area can also be read in the appendix.

### 4.0 CONCLUSIONS

### 4.1 Conclusion For Peninsular Malaysia Research Area

Significant progress has been made over the last decade since the initiation of the geoid project in Peninsular Malaysia. This progress can be seen in terms of the gravity data collection (from 2000 points in 1989 to over 10000 points in 2000), the computation algorithm (Stokes integral, LSC, FFT) and the evaluation of results (use of GPS data, satellite altimetry).

A preliminary gravimetric geoid model, MYGeoid02, of the Peninsular Malaysia and its surrounding marine area, covering an area $0^{\circ} 00^{\prime} \mathrm{N} \leq \phi \leq 8^{\circ} 00^{\prime} \mathrm{N}$ and $99^{\circ} 00^{\prime} \mathrm{E} \leq \lambda \leq$ $105^{\circ} 00^{\prime} \mathrm{E}$ was computed and presented. MYGeoid02 was computed using the following data: (1) a global geopotential model, EGM96 spherical harmonic coefficients set complete to degree and order 360, (2) a set of 10400 point free-air gravity anomalies belonging to the DSMM Gravity Data Bank and referred to the GRS80, (3) satellite altimetry-derived geoid heights on a $1.5^{\prime}$ by $1.5^{\prime}$ grid, extracted from a database of the Danish National Survey and (4) a digital terrain data set from the 30 " point topography database, GTOPO. The method applied to obtain the geoid was the Stokes integral in convolution form using gridded gravity anomalies as the input data. The geoid was computed by means of a 1-D FFT convolution about EGM96 spherical harmonic model of the geopotential and based on the "remove-compute-restore" technique. This technique was used, as it is conceptually simple and easy to implement. A $100 \%$ zero padding was appended around the signal matrix to avoid circular convolution effects and terrain correction was also applied to the data.

Both geoids over Peninsular Malaysia and the adjacent marine areas, EGM96 and MYGeoid02, are rising from north-west to south-east. They coincide with the reference ellipsoid GRS80 roughly in the central region, from $102^{\circ}$ E meridian to $103.5^{\circ}$ E meridian
and from $2^{\circ} \mathrm{N}$ to $5^{\circ} \mathrm{N}$. Estimates of the geoidal heights range from 10 m in the south to 15 m in the north. Results also indicate that the maximum and minimum values of the MYGeoid02 are 9.45 m and -14.66 m respectively. Whilst for the EGM96 geoid, the figures are 9.53 m and -14.47 m . MYGeoid02 and EGM96 have a mean undulation value of 9.45 m and 9.53 m respectively.

The computed geoid, MYGeoid02 and also EGM96 geoid for Peninsular Malaysia were compared with GPS/levelling derived geoid. MYGeoid02 seems to show a slight improvement in precision. The results are presented in the form of contour maps. Similar comparisons were made with EGM96 global geoid. Comparison with geoid undulations obtained by GPS/MSL shows the accuracies of MYGeoid02 and EGM96 are 0.39m and 0.40 m respectively. Thus it can be concluded that in general MYGeoid02 is slightly the better model for the transformation of GPS-derived ellipsoidal heights to orthometric heights as compared to EGM96. However, it is anticipated that the accuracy will improve further with the continuing efforts by the DSMM in collecting more gravity points in its monthly routine. Also in the pipeline is the use of airborne gravimetry project, which will begin in 2002.

The advent of GPS has increased the need for an accurate knowledge of geoidal heights. However, the application of the GPS technique for practical levelling is an attractive option provided that the geoid is known precisely enough. With the full constellation of 24 satellites operational that enables excellent geometry and the removal of selective availability, the need to use GPS for levelling can be realised.

### 4.2 Conclusion of The Detailed Study of Datum Bias In The State of Perak

Operational methodologies and accuracy of GPS levelling as demonstrated by the case study in Perak reveal the potential of GPS in height surveys as encouraging. Differences between the computed local geoid and the observed levelled heights were determined for the test area. These differences are found to be quite inhomogeneous, even for stations that are close to each other. However using the derived local geoid, the differences are decreased and tend to be more homogeneous. It can also be noticed that the differences increase in the North-south direction. This is due to the sparse gravity data coverage in the south and may also due to the gravity measurement reduction errors. Comparison between the computed geoidal height differences and the GPS-derived values reveals accuracies of 8.0 to 9.9 ppm can be obtained for average baseline length of 13 km . GPS heighting is generally more cost-effective than the standard levelling techniques. However the accuracy, as the case study has demonstrated, appears to be at best of the order of 8 ppm . As such, GPS heighting can provide $2^{\text {nd }}$ order levelling standard $(12 \mathrm{~mm} \sqrt{ } \mathrm{~K})$ only for distances of less than 2.5 km . For $1^{\text {st }}$ order levelling where the required accuracies are at sub-cm level, the present method is not a feasible alternative to traditional levelling. However, it can be suggested that, with a 'hypothetical' cmaccuracy geoid, the use of GPS in height surveys is comparable to $1^{\text {st }}$ order levelling over lines not more than 2 km and to $2^{\text {nd }}$ order levelling for distances up to 30 km .

Thus, depending on the intended application, GPS technology when used together with a geoid height model can function as a levelling system. Although the traditional spirit levelling has been a technique of choice in the determination of orthometric heights, its main deficiencies are its labour-intensive and time-consuming operation. On the other hand, the local accuracy of a geoid model used in GPS heighting is dependant on the accuracy on the gravity measurements. The coverage or the density of gravity data used in gravimetric geoid determination is paramount. Of equal importance too is the DEM that is used to produce the mean gravity grid.

The present method of GPS heighting, as the research in Perak area has indicated, is not a feasible alternative to the first-order spirit levelling. The presence of systematic errors in GPS heighting produces tilts and biases to the geoid model, whilst gross errors give rise to punctual deformation to the geoid model. The minimisation of the residuals can be carried out quite adequately using a four-parameter transformation model in order to absorb the vertical datum inconsistencies. Results from the Perak test area indicate that a bias of 15 cm could exist in the GPS geoid model and that once this bias is removed, the RMS fit of the residuals is 12.5 cm . Considering the RMS difference between the GPS and gravimetric geoids before the fit was 21.8 cm , the overall improvement can be regarded as significant.

The data set of GPS-derived geoidal heights can be a valuable resource for evaluating and testing the geoid model in any area of interest and vice-versa. This in turn could help in checking and identifying stations that may have errors in their heights. Apart from the need to have a more precise geoid, the accuracy of the ellipsoidal heights from GPS could be improved by using precise ephemerides, especially for longer baselines. It is only then that the accuracy of GPS height surveys be increased once these error contributions are lowered. This would finally pave the way to successfully combine the measurements of horizontal position and height into one complete technique, which presently in Peninsular Malaysia and in most parts of the world still remain as two separate entities

### 4.3 Conclusion of The Investigation On Accuracy of GPS Heighting In The State of Johor

An important aspect of any geodetic positioning technique is to ensure that all outliers have been removed from the data. GPS heighting is a viable alternative for checking and validating ellipsoidal heights and orthometric heights of known Benchmarks. The accuracy of the GPS-derived orthomertic height differences depends on the accuracy of the GPS-derived ellipsoidal height differences and the accuracy of the geoid model considered in the investigation. Efforts to improve the accuracies of geoid height differences will depends on overall national accuracy needs for GPS-derived orthometric height differences and on the cost of differential leveling versus GPS and gravity survey methods.

GPS measurements combined with accurate geoid models can be used for successful rapid height transfer. The investigation of quality of the GPS-derived bias corrected relative orthometric heights is vital for the community of surveyors and oceanographers. Regional geoid models accurate to a sub-decimeter level would be widely used, e.g. by surveyors to rigorously transform the results of GPS positioning to a local datum.

The relative agreement of the EGM96, OSU91A and the gravimetric geoid models relative to the local vertical datum is about 5 to $0.3,4$ to 0.2 and 5 to 0.3 ppm for baselines length of 28 to 125 km respectively. The average agreement in terms of ppm of the above three models relative to GPS/leveling datum is of the order of 1.7, 1.6 and 2 ppm respectively.

Within third order leveling specifications based on the Malaysian height datum, the level of accuracy achieved by the bias corrected relative orthometric height differences of the EGM96, OSU91A and the gravimetric geoid models combined with GPS/leveling data for baseline lengths up to 36 km , is sufficient to replace the conventional tedious, time consuming ordinary leveling technique for rapid height transfer for land surveying and engineering applications.

### 4.4 Overall Conclusions

- Local Vertical Datum is not consistent w.r.t Geoid Models. It means that GPS derived Orthometric Height does not fit Local MSL Height.
- Inconsistencies due to: ellipsoidal or GPS height determination, Geoid Model, and Errors in BM.
- More studies to be done : Using newly derived Gravimetric Geoid and Monitoring the status of BM.


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## APPENDIX

| ST. NO | NAME | $\phi$ | $\lambda$ | $H$ | $H$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | DOP2 | 1.380 | 103.610 | 90.910 | 83.800 |
| 2 | GP15 | 2.060 | 102.560 | 4.002 | 1.960 |
| 3 | GP16 | 2.130 | 102.730 | 36.230 | 33.470 |
| 4 | GP43 | 2.600 | 103.780 | 11.180 | 4.990 |
| 5 | GP44 | 2.470 | 103.070 | 55.650 | 51.870 |
| 6 | GP47 | 2.390 | 102.930 | 51.180 | 47.860 |
| 7 | GP48 | 1.980 | 102.930 | 164.750 | 160.990 |
| 8 | GP49 | 1.630 | 103.200 | 7.360 | 1.950 |
| 9 | GP50 | 1.550 | 103.400 | 10.120 | 3.810 |
| 10 | GP51 | 1.630 | 103.670 | 42.710 | 35.520 |
| 11 | GP52 | 1.370 | 104.270 | 67.810 | 58.720 |
| 12 | GP53 | 1.800 | 103.900 | 31.820 | 24.000 |
| 13 | GP54 | 1.930 | 104.090 | 12.880 | 4.660 |
| 14 | GP55 | 2.080 | 103.890 | 36.740 | 29.330 |
| 15 | GP56 | 2.390 | 103.870 | 11.570 | 4.680 |
| 16 | GP58 | 2.120 | 103.430 | 77.620 | 72.100 |
| 17 | GP59 | 1.970 | 103.230 | 45.180 | 40.150 |
| 18 | GP61 | 2.190 | 103.190 | 41.830 | 37.320 |
| 19 | GP84 | 1.860 | 102.940 | 7.130 | 3.150 |
| 20 | GP85 | 1.910 | 102.740 | 4.930 | 1.880 |
| 21 | GP90 | 1.930 | 104.110 | 10.780 | 2.470 |
| 22 | GP91 | 1.880 | 103.690 | 26.310 | 19.550 |
| 23 | TG05 | 2.650 | 102.920 | 193.730 | 190.800 |
| 24 | TG07 | 1.680 | 104.070 | 194.270 | 186.200 |
| 25 | TG09 | 2.460 | 103.640 | 223.000 | 216.700 |
| 26 | TG10 | 1.460 | 104.070 | 193.250 | 183.500 |
| 27 | TG19 | 1.470 | 103.260 | 136.070 | 130.400 |

Table 20 Johor GPS data associated with leveling.

| ST. NO | NAME | $\phi$ | $\lambda$ | $N_{\text {GPS }}$ | $N_{\text {EGM }}$ | $N_{\text {OSU }}$ | $N_{\text {GRAV }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | DOP2 | 1.380 | 103.610 | 7.1100 | 6.7900 | 6.8300 | 7.2658 |
| 2 | GP15 | 2.060 | 102.560 | 2.0420 | 1.8100 | 1.3800 | 1.9986 |
| 3 | GP16 | 2.130 | 102.730 | 2.7600 | 2.4300 | 2.0600 | 2.6389 |
| 4 | GP43 | 2.600 | 103.780 | 6.1900 | 5.5900 | 5.6500 | 5.9441 |
| 5 | GP44 | 2.470 | 103.070 | 3.7800 | 3.1500 | 2.8800 | 3.5271 |
| 6 | GP47 | 2.390 | 102.930 | 3.3200 | 2.7700 | 2.4500 | 3.0997 |
| 7 | GP48 | 1.980 | 102.930 | 3.7600 | 3.5400 | 3.2200 | 3.6743 |
| 8 | GP49 | 1.630 | 103.200 | 5.4100 | 5.1100 | 4.8200 | 5.2365 |
| 9 | GP50 | 1.550 | 103.400 | 6.3100 | 5.8800 | 5.7200 | 6.1958 |
| 10 | GP51 | 1.630 | 103.670 | 7.1900 | 6.8200 | 6.8600 | 7.2595 |
| 11 | GP52 | 1.370 | 104.270 | 9.0900 | 9.4400 | 9.5900 | 9.8054 |
| 12 | GP53 | 1.800 | 103.900 | 7.8200 | 7.4800 | 7.5700 | 7.9510 |
| 13 | GP54 | 1.930 | 104.090 | 8.2200 | 8.0200 | 8.1400 | 8.4651 |
| 14 | GP55 | 2.080 | 103.890 | 7.4100 | 6.9900 | 7.0200 | 7.4325 |
| 15 | GP56 | 2.390 | 103.870 | 6.8900 | 6.3500 | 6.4400 | 6.7313 |
| 16 | GP58 | 2.120 | 103.430 | 5.5200 | 5.1900 | 4.9400 | 5.5495 |
| 17 | GP59 | 1.970 | 103.230 | 5.0300 | 4.7100 | 4.4100 | 4.9574 |
| 18 | GP61 | 2.190 | 103.190 | 4.5100 | 4.1500 | 3.8400 | 4.4644 |
| 19 | GP84 | 1.860 | 102.940 | 3.9800 | 3.8000 | 3.4900 | 3.8576 |
| 20 | GP85 | 1.910 | 102.740 | 3.0500 | 2.8600 | 2.5200 | 2.9376 |
| 21 | GP90 | 1.930 | 104.110 | 8.3100 | 8.1000 | 8.2300 | 8.5399 |
| 22 | GP91 | 1.880 | 103.690 | 6.7600 | 6.5600 | 6.5100 | 6.9901 |
| 23 | TG05 | 2.650 | 102.920 | 2.9300 | 2.2800 | 1.9900 | 2.6401 |
| 24 | TG07 | 1.680 | 104.070 | 8.0700 | 8.3200 | 8.4800 | 8.7795 |
| 25 | TG09 | 2.460 | 103.640 | 6.3000 | 5.3500 | 5.2800 | 5.7298 |
| 26 | TG10 | 1.460 | 104.070 | 9.7500 | 8.5700 | 8.7700 | 9.0368 |
| 27 | TG19 | 1.470 | 103.260 | 5.6700 | 5.4300 | 5.1600 | 5.6467 |

Table 21 Geoid heights from EGM96, OSU91A , Gravimetric and GPS/leveling data

| ST. <br> NO | NAME | $\phi$ | $\lambda$ | $H_{\text {GPS }}$ | $H_{\text {EGM }}$ | $H_{\text {OSU }}$ | $H_{\text {GRAV }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | DOP2 | 1.380 | 103.610 | 83.800 | 84.120 | 84.080 | 83.644 |
| 2 | GP15 | 2.060 | 102.560 | 1.960 | 2.192 | 2.622 | 2.003 |
| 3 | GP16 | 2.130 | 102.730 | 33.470 | 33.800 | 34.170 | 33.591 |
| 4 | GP43 | 2.600 | 103.780 | 4.990 | 5.590 | 5.530 | 5.236 |
| 5 | GP44 | 2.470 | 103.070 | 51.870 | 52.500 | 52.770 | 52.123 |
| 6 | GP47 | 2.390 | 102.930 | 47.860 | 48.410 | 48.730 | 48.080 |
| 7 | GP48 | 1.980 | 102.930 | 160.990 | 161.210 | 161.530 | 161.076 |
| 8 | GP49 | 1.630 | 103.200 | 1.950 | 2.250 | 2.540 | 2.124 |
| 9 | GP50 | 1.550 | 103.400 | 3.810 | 4.240 | 4.400 | 3.924 |
| 10 | GP51 | 1.630 | 103.670 | 35.520 | 35.890 | 35.850 | 35.451 |
| 11 | GP52 | 1.370 | 104.270 | 58.720 | 58.370 | 58.220 | 58.005 |
| 12 | GP53 | 1.800 | 103.900 | 24.000 | 24.340 | 24.250 | 23.869 |
| 13 | GP54 | 1.930 | 104.090 | 4.660 | 4.860 | 4.740 | 4.415 |
| 14 | GP55 | 2.080 | 103.890 | 29.330 | 29.750 | 29.720 | 29.308 |
| 15 | GP56 | 2.390 | 103.870 | 4.680 | 5.220 | 5.130 | 4.839 |
| 16 | GP58 | 2.120 | 103.430 | 72.100 | 72.430 | 72.680 | 72.071 |
| 17 | GP59 | 1.970 | 103.230 | 40.150 | 40.470 | 40.770 | 40.223 |
| 18 | GP61 | 2.190 | 103.190 | 37.320 | 37.680 | 37.990 | 37.366 |
| 19 | GP84 | 1.860 | 102.940 | 3.150 | 3.330 | 3.640 | 3.272 |
| 20 | GP85 | 1.910 | 102.740 | 1.880 | 2.070 | 2.410 | 1.992 |
| 21 | GP90 | 1.930 | 104.110 | 2.470 | 2.680 | 2.550 | 2.240 |
| 22 | GP91 | 1.880 | 103.690 | 19.550 | 19.750 | 19.800 | 19.320 |
| 23 | TG05 | 2.650 | 102.920 | 190.800 | 191.450 | 191.740 | 191.090 |
| 24 | TG07 | 1.680 | 104.070 | 186.200 | 185.950 | 185.790 | 185.491 |
| 25 | TG09 | 2.460 | 103.640 | 216.700 | 217.650 | 217.720 | 217.270 |
| 26 | TG10 | 1.460 | 104.070 | 183.500 | 184.680 | 184.480 | 184.213 |
| 27 | TG19 | 1.470 | 103.260 | 130.400 | 130.640 | 130.910 | 130.423 |

Table 22 Orthometric heights from EGM96, OSU91A, Gravimetric and GPS/leveling data.

| ST. NO | NAME | $\phi$ | $\lambda$ | $H_{\text {GPS }}-\mathrm{H}_{\text {EGM }}$ | $H_{\text {GPS }}-\mathrm{H}_{\text {OSU }}$ | $H_{\text {GPS }}-\mathrm{H}_{\text {GRAV }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | DOP2 | 1.380 | 103.610 | -0.320 | -0.280 | 0.156 |
| 2 | GP15 | 2.060 | 102.560 | -0.232 | -0.662 | -0.043 |
| 3 | GP16 | 2.130 | 102.730 | -0.330 | -0.700 | -0.121 |
| 4 | GP43 | 2.600 | 103.780 | -0.600 | -0.540 | -0.246 |
| 5 | GP44 | 2.470 | 103.070 | -0.630 | -0.900 | -0.253 |
| 6 | GP47 | 2.390 | 102.930 | -0.550 | -0.870 | -0.220 |
| 7 | GP48 | 1.980 | 102.930 | -0.220 | -0.540 | -0.086 |
| 8 | GP49 | 1.630 | 103.200 | -0.300 | -0.590 | -0.174 |
| 9 | GP50 | 1.550 | 103.400 | -0.430 | -0.590 | -0.114 |
| 10 | GP51 | 1.630 | 103.670 | -0.370 | -0.330 | 0.070 |
| 11 | GP52 | 1.370 | 104.270 | 0.350 | 0.500 | 0.715 |
| 12 | GP53 | 1.800 | 103.900 | -0.340 | -0.250 | 0.131 |
| 13 | GP54 | 1.930 | 104.090 | -0.200 | -0.080 | 0.245 |
| 14 | GP55 | 2.080 | 103.890 | -0.420 | -0.390 | 0.022 |
| 15 | GP56 | 2.390 | 103.870 | -0.540 | -0.450 | -0.159 |
| 16 | GP58 | 2.120 | 103.430 | -0.330 | -0.580 | 0.029 |
| 17 | GP59 | 1.970 | 103.230 | -0.320 | -0.620 | -0.073 |
| 18 | GP61 | 2.190 | 103.190 | -0.360 | -0.670 | -0.046 |
| 19 | GP84 | 1.860 | 102.940 | -0.180 | -0.490 | -0.122 |
| 20 | GP85 | 1.910 | 102.740 | -0.190 | -0.530 | -0.112 |
| 21 | GP90 | 1.930 | 104.110 | -0.210 | -0.080 | 0.230 |
| 22 | GP91 | 1.880 | 103.690 | -0.200 | -0.250 | 0.230 |
| 23 | TG05 | 2.650 | 102.920 | -0.650 | -0.940 | -0.290 |
| 24 | TG07 | 1.680 | 104.070 | 0.250 | 0.410 | 0.709 |
| 25 | TG09 | 2.460 | 103.640 | -0.950 | -1.020 | -0.570 |
| 26 | TG10 | 1.460 | 104.070 | -1.180 | -0.980 | -0.713 |
| 27 | TG19 | 1.470 | 103.260 | -0.240 | -0.510 | -0.023 |

Table 23 Absolute orthometric height differences of EGM96, OSU91A and the gravimetric model relative to the local vertical datum.

| ID | STN | $h$ | $\Delta h$ | $H$ | $\Delta H$ | $N$ | $\Delta N$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | GP58 | 77.620 | fixed | 72.100 | fixed | 5.520 | fixed |
| 2 | GP15 | 4.002 | 73.618 | 1.960 | 70.140 | 2.042 | 3.478 |
| 3 | GP16 | 36.230 | 41.390 | 33.470 | 38.630 | 2.760 | 2.760 |
| 4 | GP43 | 11.180 | 66.440 | 4.990 | 67.110 | 6.190 | -0.670 |
| 5 | GP44 | 55.650 | 21.970 | 51.870 | 20.230 | 3.780 | 1.740 |
| 6 | GP47 | 51.180 | 26.440 | 47.860 | 24.240 | 3.320 | 2.200 |
| 7 | GP48 | 164.750 | -87.130 | 160.990 | -88.890 | 3.760 | 1.760 |
| 8 | GP49 | 7.360 | 70.260 | 1.950 | 70.150 | 5.410 | 0.110 |
| 9 | GP50 | 10.120 | 67.500 | 3.810 | 68.290 | 6.310 | -0.790 |
| 10 | GP51 | 42.710 | 34.910 | 35.520 | 36.580 | 7.190 | -1.670 |
| 11 | GP52 | 67.810 | 9.810 | 58.720 | 13.380 | 9.090 | -3.570 |
| 12 | GP53 | 31.820 | 45.800 | 24.000 | 48.100 | 7.820 | -2.300 |
| 13 | GP54 | 12.880 | 64.740 | 4.660 | 67.440 | 8.220 | -2.700 |
| 14 | GP55 | 36.740 | 40.880 | 29.330 | 42.770 | 7.410 | -1.890 |
| 15 | GP56 | 11.570 | 66.050 | 4.680 | 67.420 | 6.890 | -1.370 |
| 16 | DOP2 | 90.910 | -13.290 | 83.800 | -11.700 | 7.110 | -1.590 |
| 17 | GP59 | 45.180 | 32.440 | 40.150 | 31.950 | 5.030 | 0.490 |
| 18 | GP61 | 41.830 | 35.790 | 37.320 | 34.780 | 4.510 | 1.010 |
| 19 | GP84 | 7.130 | 70.490 | 3.150 | 68.950 | 3.980 | 1.540 |
| 20 | GP85 | 4.930 | 72.690 | 1.880 | 70.220 | 3.050 | 2.470 |
| 21 | GP90 | 10.780 | 66.840 | 2.470 | 69.630 | 8.310 | -2.790 |
| 22 | GP91 | 26.310 | 51.310 | 19.550 | 52.550 | 6.760 | -1.240 |
| 23 | TG05 | 193.730 | -116.110 | 190.800 | -118.700 | 2.930 | 2.590 |
| 24 | TG07 | 194.270 | -116.650 | 186.200 | -114.100 | 8.070 | -2.550 |
| 25 | TG09 | 223.000 | -145.380 | 216.700 | -144.600 | 6.300 | -0.780 |
| 26 | TG10 | 193.250 | -115.630 | 183.500 | -111.400 | 9.750 | -4.230 |
| 27 | TG19 | 136.070 | -58.450 | 130.400 | -58.300 | 5.670 | -0.150 |

Table 24 Relative differences derived from GPS/ leveling data (Station GP58 held fixed)

| ID | STN | $h$ | $\Delta h$ | $H$ | $\Delta H$ | $N$ | $\Delta N$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | GP58 | 77.620 | fixed | 72.430 | fixed | 5.190 | fixed |
| 2 | GP15 | 4.002 | 73.618 | 2.192 | 70.238 | 1.810 | 3.380 |
| 3 | GP16 | 36.230 | 41.390 | 33.800 | 38.630 | 2.430 | 2.760 |
| 4 | GP43 | 11.180 | 66.440 | 5.590 | 66.840 | 5.590 | -0.400 |
| 5 | GP44 | 55.650 | 21.970 | 52.500 | 19.930 | 3.150 | 2.040 |
| 6 | GP47 | 51.180 | 26.440 | 48.410 | 24.020 | 2.770 | 2.420 |
| 7 | GP48 | 164.750 | -87.130 | 161.210 | -88.780 | 3.540 | 1.650 |
| 8 | GP49 | 7.360 | 70.260 | 2.250 | 70.180 | 5.110 | 0.080 |
| 9 | GP50 | 10.120 | 67.500 | 4.240 | 68.190 | 5.880 | -0.690 |
| 10 | GP51 | 42.710 | 34.910 | 35.890 | 36.540 | 6.820 | -1.630 |
| 11 | GP52 | 67.810 | 9.810 | 58.370 | 14.060 | 9.440 | -4.250 |
| 12 | GP53 | 31.820 | 45.800 | 24.340 | 48.090 | 7.480 | -2.290 |
| 13 | GP54 | 12.880 | 64.740 | 4.860 | 67.570 | 8.020 | -2.830 |
| 14 | GP55 | 36.740 | 40.880 | 29.750 | 42.680 | 6.990 | -1.800 |
| 15 | GP56 | 11.570 | 66.050 | 5.220 | 67.210 | 6.790 | -1.600 |
| 16 | DOP2 | 90.910 | -13.290 | 84.120 | 0.000 | 6.350 | -1.160 |
| 17 | GP59 | 45.180 | 32.440 | 40.470 | 31.960 | 4.710 | 0.480 |
| 18 | GP61 | 41.830 | 35.790 | 37.680 | 34.750 | 4.150 | 1.040 |
| 19 | GP84 | 7.130 | 70.490 | 3.330 | 69.100 | 3.800 | 1.390 |
| 20 | GP85 | 4.930 | 72.690 | 2.070 | 70.360 | 2.860 | 2.330 |
| 21 | GP90 | 10.780 | 66.840 | 2.680 | 69.750 | 8.100 | -2.910 |
| 22 | GP91 | 26.310 | 51.310 | 19.750 | 52.680 | 6.560 | -1.370 |
| 23 | TG05 | 193.730 | -116.110 | 191.450 | -119.020 | 2.280 | 2.910 |
| 24 | TG07 | 194.270 | -116.650 | 185.950 | -113.520 | 8.320 | -3.130 |
| 25 | TG09 | 223.000 | -145.380 | 217.650 | -145.220 | 5.350 | -0.160 |
| 26 | TG10 | 193.250 | -115.630 | 184.680 | -112.250 | 8.570 | -3.380 |
| 27 | TG19 | 136.070 | -58.450 | 130.640 | -58.210 | 5.430 | -0.240 |

Table 25 Relative differences derived from EGM96 model and GPS/leveling data (Station GP58 held fixed)

| ID | STN | $h$ | $\Delta h$ | $H$ | $\Delta H$ | $N$ | $\Delta N$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | GP58 | 77.620 | fixed | 72.680 | fixed | 4.94 | fixed |
| 2 | GP15 | 4.002 | 73.618 | 2.622 | 70.058 | 1.38 | 3.56 |
| 3 | GP16 | 36.230 | 41.390 | 34.170 | 38.510 | 2.06 | 2.88 |
| 4 | GP43 | 11.180 | 66.440 | 5.530 | 67.150 | 5.65 | -0.71 |
| 5 | GP44 | 55.650 | 21.970 | 52.770 | 19.910 | 2.88 | 2.06 |
| 6 | GP47 | 51.180 | 26.440 | 48.730 | 23.950 | 2.45 | 2.49 |
| 7 | GP48 | 164.750 | -87.130 | 161.530 | -88.850 | 3.22 | 1.72 |
| 8 | GP49 | 7.360 | 70.260 | 2.540 | 70.140 | 4.82 | 0.12 |
| 9 | GP50 | 10.120 | 67.500 | 4.400 | 68.280 | 5.72 | -0.78 |
| 10 | GP51 | 42.710 | 34.910 | 35.850 | 36.830 | 6.86 | -1.92 |
| 11 | GP52 | 67.810 | 9.810 | 58.220 | 14.460 | 9.59 | -4.65 |
| 12 | GP53 | 31.820 | 45.800 | 24.250 | 48.430 | 7.57 | -2.63 |
| 13 | GP54 | 12.880 | 64.740 | 4.740 | 67.940 | 8.14 | -3.2 |
| 14 | GP55 | 36.740 | 40.880 | 29.720 | 42.960 | 7.02 | -2.08 |
| 15 | GP56 | 11.570 | 66.050 | 5.130 | 67.550 | 6.44 | -1.5 |
| 16 | DOP2 | 90.910 | -13.290 | 84.080 | -11.400 | 6.83 | -1.89 |
| 17 | GP59 | 45.180 | 32.440 | 40.770 | 31.910 | 4.41 | 0.53 |
| 18 | GP61 | 41.830 | 35.790 | 37.990 | 34.690 | 3.84 | 1.1 |
| 19 | GP84 | 7.130 | 70.490 | 3.640 | 69.040 | 3.49 | 1.45 |
| 20 | GP85 | 4.930 | 72.690 | 2.410 | 70.270 | 2.52 | 2.42 |
| 21 | GP90 | 10.780 | 66.840 | 2.550 | 70.130 | 8.23 | -3.29 |
| 22 | GP91 | 26.310 | 51.310 | 19.800 | 52.880 | 6.51 | -1.57 |
| 23 | TG05 | 193.730 | -116.110 | 191.740 | -119.060 | 1.99 | 2.95 |
| 24 | TG07 | 194.270 | -116.650 | 185.790 | -113.110 | 8.48 | -3.54 |
| 25 | TG09 | 223.000 | -145.380 | 217.720 | -145.040 | 5.28 | -0.34 |
| 26 | TG10 | 193.250 | -115.630 | 184.480 | -111.800 | 8.77 | -3.83 |
| 27 | TG19 | 136.070 | -58.450 | 130.910 | -58.230 | 5.16 | -0.22 |

Table 26 Relative differences derived from OSU91A model and GPS/leveling data (Station GP58 held fixed)

| ID | STN | $h$ | $\Delta h$ | $H$ | $\Delta H$ | $N$ | $\Delta N$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | GP58 | 77.620 | fixed | 72.071 | fixed | 5.550 | fixed |
| 2 | GP15 | 4.002 | 73.618 | 2.003 | 70.068 | 1.999 | 3.551 |
| 3 | GP16 | 36.230 | 41.390 | 33.591 | 38.480 | 2.639 | 2.911 |
| 4 | GP43 | 11.180 | 66.440 | 5.236 | 66.835 | 5.944 | -0.394 |
| 5 | GP44 | 55.650 | 21.970 | 52.123 | 19.948 | 3.527 | 2.023 |
| 6 | GP47 | 51.180 | 26.440 | 48.080 | 23.991 | 3.100 | 2.450 |
| 7 | GP48 | 164.750 | -87.130 | 161.076 | -89.005 | 3.674 | 1.876 |
| 8 | GP49 | 7.360 | 70.260 | 2.124 | 69.947 | 5.237 | 0.313 |
| 9 | GP50 | 10.120 | 67.500 | 3.924 | 68.147 | 6.196 | -0.646 |
| 10 | GP51 | 42.710 | 34.910 | 35.451 | 36.620 | 7.260 | -1.710 |
| 11 | GP52 | 67.810 | 9.810 | 58.005 | 14.066 | 9.805 | -4.255 |
| 12 | GP53 | 31.820 | 45.800 | 23.869 | 48.202 | 7.951 | -2.401 |
| 13 | GP54 | 12.880 | 64.740 | 4.415 | 67.656 | 8.465 | -2.915 |
| 14 | GP55 | 36.740 | 40.880 | 29.308 | 42.763 | 7.433 | -1.883 |
| 15 | GP56 | 11.570 | 66.050 | 4.839 | 67.232 | 6.731 | -1.181 |
| 16 | DOP2 | 90.910 | -13.290 | 83.644 | -11.573 | 7.266 | -1.716 |
| 17 | GP59 | 45.180 | 32.440 | 40.223 | 31.848 | 4.957 | 0.593 |
| 18 | GP61 | 41.830 | 35.790 | 37.366 | 34.705 | 4.464 | 1.086 |
| 19 | GP84 | 7.130 | 70.490 | 3.272 | 68.799 | 3.858 | 1.692 |
| 20 | GP85 | 4.930 | 72.690 | 1.992 | 70.079 | 2.938 | 2.612 |
| 21 | GP90 | 10.780 | 66.840 | 2.240 | 69.831 | 8.540 | -2.990 |
| 22 | GP91 | 26.310 | 51.310 | 19.320 | 52.751 | 6.990 | -1.440 |
| 23 | TG05 | 193.730 | -116.110 | 191.090 | -119.019 | 2.640 | 2.910 |
| 24 | TG07 | 194.270 | -116.650 | 185.491 | -113.420 | 8.780 | -3.230 |
| 25 | TG09 | 223.000 | -145.380 | 217.270 | -145.199 | 5.730 | -0.180 |
| 26 | TG10 | 193.250 | -115.630 | 184.213 | -112.142 | 9.037 | -3.487 |
| 27 | TG19 | 136.070 | -58.450 | 130.423 | -58.352 | 5.647 | -0.097 |

Table 27 Relative differences derived from gravimetric model and GPS/leveling data (Station GP58 held fixed)

| ID | STN | $\delta N_{\text {EGM }}$ | $\delta N_{\text {EGM }}$ | $\delta N_{\text {EGM }}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | DOP2 | -0.152 | -0.214 | 0.159 |
| 2 | GP15 | -0.356 | -0.835 | -0.216 |
| 3 | GP16 | -0.388 | -0.799 | -0.205 |
| 4 | GP43 | -0.595 | -0.589 | -0.149 |
| 5 | GP44 | -0.522 | -0.798 | -0.239 |
| 6 | GP47 | -0.488 | -0.820 | -0.242 |
| 7 | GP48 | -0.341 | -0.673 | -0.128 |
| 8 | GP49 | -0.226 | -0.451 | 0.017 |
| 9 | GP50 | -0.205 | -0.350 | 0.075 |
| 10 | GP51 | -0.243 | -0.282 | 0.100 |
| 11 | GP52 | -0.173 | 0.027 | 0.279 |
| 12 | GP53 | -0.313 | -0.260 | 0.094 |
| 13 | GP54 | -0.366 | -0.238 | 0.092 |
| 14 | GP55 | -0.413 | -0.364 | 0.015 |
| 15 | GP56 | -0.523 | -0.482 | -0.074 |
| 16 | GP58 | -0.410 | -0.543 | -0.078 |
| 17 | GP59 | -0.349 | -0.562 | -0.072 |
| 18 | GP61 | -0.426 | -0.655 | -0.140 |
| 19 | GP84 | -0.299 | -0.626 | -0.093 |
| 20 | GP85 | -0.309 | -0.716 | -0.143 |
| 21 | GP90 | -0.367 | -0.231 | 0.096 |
| 22 | GP91 | -0.334 | -0.364 | 0.035 |
| 23 | TG05 | -0.581 | -0.916 | -0.316 |
| 24 | TG07 | -0.276 | -0.156 | 0.158 |
| 25 | TG09 | -0.539 | -0.589 | -0.135 |
| 26 | TG10 | -0.197 | -0.077 | 0.219 |
| 27 | TG19 | -0.171 | -0.372 | 0.072 |

Table 28 Datum biases of EGM96, OSU91A and gravimetric datums relative to the local

| ID | STN | $N_{\text {GPS }}$ | $N_{\text {EGM }}^{*}$ | $N_{\text {OSU }}^{*}$ | $N_{\text {GRAV }}^{*}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | DOP2 | 7.110 | 6.942 | 7.044 | 7.107 |
| 2 | GP15 | 2.042 | 2.166 | 2.215 | 2.215 |
| 3 | GP16 | 2.760 | 2.818 | 2.859 | 2.844 |
| 4 | GP43 | 6.190 | 6.185 | 6.239 | 6.093 |
| 5 | GP44 | 3.780 | 3.672 | 3.678 | 3.766 |
| 6 | GP47 | 3.320 | 3.258 | 3.27 | 3.342 |
| 7 | GP48 | 3.760 | 3.881 | 3.893 | 3.802 |
| 8 | GP49 | 5.410 | 5.336 | 5.271 | 5.220 |
| 9 | GP50 | 6.310 | 6.085 | 6.07 | 6.121 |
| 10 | GP51 | 7.190 | 7.063 | 7.142 | 7.160 |
| 11 | GP52 | 9.090 | 9.613 | 9.563 | 9.526 |
| 12 | GP53 | 7.820 | 7.793 | 7.83 | 7.857 |
| 13 | GP54 | 8.220 | 8.386 | 8.378 | 8.373 |
| 14 | GP55 | 7.410 | 7.403 | 7.384 | 7.418 |
| 15 | GP56 | 6.890 | 6.873 | 6.922 | 6.805 |
| 16 | GP58 | 5.520 | 5.6 | 5.483 | 5.628 |
| 17 | GP59 | 5.030 | 5.059 | 4.972 | 5.029 |
| 18 | GP61 | 4.510 | 4.576 | 4.495 | 4.604 |
| 19 | GP84 | 3.980 | 4.099 | 4.116 | 3.951 |
| 20 | GP85 | 3.050 | 3.169 | 3.236 | 3.081 |
| 21 | GP90 | 8.310 | 8.467 | 8.461 | 8.444 |
| 22 | GP91 | 6.760 | 6.894 | 6.874 | 6.955 |
| 23 | TG05 | 2.930 | 2.861 | 2.906 | 2.956 |
| 24 | TG07 | 8.070 | 8.596 | 8.636 | 8.622 |
| 25 | TG09 | 6.300 | 5.889 | 5.869 | 5.865 |
| 26 | TG10 | 9.750 | 8.767 | 8.847 | 8.818 |
| 27 | TG19 | 5.670 | 5.601 | 5.532 | 5.575 |

Table 29 Bias corrected geoid heights from EGM96, OSU91A and gravimetric model with GPS/leveling geoid.

| ID | STN | $H_{\text {GPS }}$ | $H_{\text {EGM }}^{*}$ | $H_{\text {OSU }}^{*}$ | $H_{\text {GRAV }}^{*}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | DOP2 | 83.800 | 83.968 | 83.866 | 83.803 |
| 2 | GP15 | 1.960 | 1.836 | 1.787 | 1.787 |
| 3 | GP16 | 33.470 | 33.412 | 33.371 | 33.386 |
| 4 | GP43 | 4.990 | 4.995 | 4.941 | 5.087 |
| 5 | GP44 | 51.870 | 51.978 | 51.972 | 51.884 |
| 6 | GP47 | 47.860 | 47.922 | 47.910 | 47.838 |
| 7 | GP48 | 160.990 | 160.869 | 160.857 | 160.948 |
| 8 | GP49 | 1.950 | 2.024 | 2.089 | 2.141 |
| 9 | GP50 | 3.810 | 4.035 | 4.050 | 3.999 |
| 10 | GP51 | 35.520 | 35.647 | 35.568 | 35.551 |
| 11 | GP52 | 58.720 | 58.197 | 58.247 | 58.284 |
| 12 | GP53 | 24.000 | 24.027 | 23.990 | 23.963 |
| 13 | GP54 | 4.660 | 4.494 | 4.502 | 4.507 |
| 14 | GP55 | 29.330 | 29.337 | 29.356 | 29.323 |
| 15 | GP56 | 4.680 | 4.697 | 4.648 | 4.765 |
| 16 | GP58 | 72.100 | 72.020 | 72.137 | 71.993 |
| 17 | GP59 | 40.150 | 40.121 | 40.208 | 40.151 |
| 18 | GP61 | 37.320 | 37.254 | 37.335 | 37.226 |
| 19 | GP84 | 3.150 | 3.031 | 3.014 | 3.179 |
| 20 | GP85 | 1.880 | 1.761 | 1.694 | 1.849 |
| 21 | GP90 | 2.470 | 2.313 | 2.319 | 2.336 |
| 22 | GP91 | 19.550 | 19.416 | 19.436 | 19.355 |
| 23 | TG05 | 190.800 | 190.869 | 190.824 | 190.774 |
| 24 | TG07 | 186.200 | 185.674 | 185.634 | 185.649 |
| 25 | TG09 | 216.700 | 217.111 | 217.131 | 217.135 |
| 26 | TG10 | 183.500 | 184.483 | 184.403 | 184.432 |
| 27 | TG19 | 130.400 | 130.469 | 130.538 | 130.495 |

Table 30 Bias corrected orthometric heights from EGM96, OSU91A and gravimetric model with true orthometric height from leveling.

| ID | STN | $H_{\text {GPS }}-H_{\text {EGM }}^{*}$ | $H_{\text {GPS }}-H_{\text {OSU }}^{*}$ | $H_{\text {GPS }}-H_{\text {GRAV }}^{*}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | DOP2 | -0.168 | -0.066 | -0.003 |
| 2 | GP15 | 0.124 | 0.173 | 0.173 |
| 3 | GP16 | 0.058 | 0.099 | 0.084 |
| 4 | GP43 | -0.005 | 0.049 | -0.097 |
| 5 | GP44 | -0.108 | -0.102 | -0.014 |
| 6 | GP47 | -0.062 | -0.050 | 0.022 |
| 7 | GP48 | 0.121 | 0.133 | 0.042 |
| 8 | GP49 | -0.074 | -0.139 | -0.191 |
| 9 | GP50 | -0.225 | -0.240 | -0.189 |
| 10 | GP51 | -0.127 | -0.048 | -0.030 |
| 11 | GP52 | 0.523 | 0.473 | 0.436 |
| 12 | GP53 | -0.027 | 0.010 | 0.037 |
| 13 | GP54 | 0.166 | 0.158 | 0.153 |
| 14 | GP55 | -0.007 | -0.026 | 0.007 |
| 15 | GP56 | -0.017 | 0.032 | -0.085 |
| 16 | GP58 | 0.080 | -0.037 | 0.107 |
| 17 | GP59 | 0.029 | -0.058 | -0.001 |
| 18 | GP61 | 0.066 | -0.015 | 0.094 |
| 19 | GP84 | 0.119 | 0.136 | -0.029 |
| 20 | GP85 | 0.119 | 0.186 | 0.031 |
| 21 | GP90 | 0.157 | 0.151 | 0.134 |
| 22 | GP91 | 0.134 | 0.114 | 0.195 |
| 23 | TG05 | -0.069 | -0.024 | 0.026 |
| 24 | TG07 | 0.526 | 0.566 | 0.551 |
| 25 | TG09 | -0.411 | -0.431 | -0.435 |
| 26 | TG10 | -0.983 | -0.903 | -0.932 |
| 27 | TG19 | -0.069 | -0.138 | -0.095 |

Table 31 Bias corrected orthometric height differences of EGM96, OSU91A, and gravimetric models relative to the local vertical datum.

| ID | STN | $h$ | $\Delta h$ | $H^{*}$ | $\Delta H^{*}$ | $N^{*}$ | $\Delta N^{*}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | GP58 | 77.620 | fixed | 72.020 | fixed | 5.600 | fixed |
| 2 | GP15 | 4.002 | 73.618 | 1.836 | 70.184 | 2.166 | 3.434 |
| 3 | GP16 | 36.230 | 41.390 | 33.412 | 38.608 | 2.818 | 2.782 |
| 4 | GP43 | 11.180 | 66.440 | 4.995 | 67.025 | 6.185 | -0.585 |
| 5 | GP44 | 55.650 | 21.970 | 51.978 | 20.042 | 3.672 | 1.928 |
| 6 | GP47 | 51.180 | 26.440 | 47.922 | 24.098 | 3.258 | 2.342 |
| 7 | GP48 | 164.750 | -87.130 | 160.869 | -88.849 | 3.881 | 1.719 |
| 8 | GP49 | 7.360 | 70.260 | 2.024 | 69.996 | 5.336 | 0.264 |
| 9 | GP50 | 10.120 | 67.500 | 4.035 | 67.985 | 6.085 | -0.485 |
| 10 | GP51 | 42.710 | 34.910 | 35.647 | 36.373 | 7.063 | -1.463 |
| 11 | GP52 | 67.810 | 9.810 | 58.197 | 13.823 | 9.613 | -4.013 |
| 12 | GP53 | 31.820 | 45.800 | 24.027 | 47.993 | 7.793 | -2.193 |
| 13 | GP54 | 12.880 | 64.740 | 4.494 | 67.526 | 8.386 | -2.786 |
| 14 | GP55 | 36.740 | 40.880 | 29.337 | 42.683 | 7.403 | -1.803 |
| 15 | GP56 | 11.570 | 66.050 | 4.697 | 67.323 | 6.873 | -1.273 |
| 16 | DOP2 | 90.910 | -13.290 | 83.968 | -11.948 | 6.942 | -1.342 |
| 17 | GP59 | 45.180 | 32.440 | 40.121 | 31.899 | 5.059 | 0.541 |
| 18 | GP61 | 41.830 | 35.790 | 37.254 | 34.766 | 4.576 | 1.024 |
| 19 | GP84 | 7.130 | 70.490 | 3.031 | 68.989 | 4.099 | 1.501 |
| 20 | GP85 | 4.930 | 72.690 | 1.761 | 70.259 | 3.169 | 2.431 |
| 21 | GP90 | 10.780 | 66.840 | 2.313 | 69.707 | 8.467 | -2.867 |
| 22 | GP91 | 26.310 | 51.310 | 19.416 | 52.604 | 6.894 | -1.294 |
| 23 | TG05 | 193.730 | -116.110 | 190.869 | -118.849 | 2.861 | 2.739 |
| 24 | TG07 | 194.270 | -116.650 | 185.674 | -113.654 | 8.596 | -2.996 |
| 25 | TG09 | 223.000 | -145.380 | 217.111 | -145.091 | 5.889 | -0.289 |
| 26 | TG10 | 193.250 | -115.630 | 184.483 | -112.463 | 8.767 | -3.167 |
| 27 | TG19 | 136.070 | -58.450 | 130.469 | -58.449 | 5.601 | -0.001 |

Table 32 Bias corrected Relative differences derived from EGM96 model and GPS/leveling data (Station GP58 held fixed)

| ID | STN | $h$ | $\Delta h$ | $H^{*}$ | $\Delta H^{*}$ | $N^{*}$ | $\Delta N^{*}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | GP58 | 77.620 | fixed | 72.137 | fixed | 5.483 | fixed |
| 2 | GP15 | 4.002 | 73.618 | 1.787 | 70.350 | 2.215 | 3.268 |
| 3 | GP16 | 36.230 | 41.390 | 33.371 | 38.766 | 2.859 | 2.624 |
| 4 | GP43 | 11.180 | 66.440 | 4.941 | 67.196 | 6.239 | -0.756 |
| 5 | GP44 | 55.650 | 21.970 | 51.972 | 20.165 | 3.678 | 1.805 |
| 6 | GP47 | 51.180 | 26.440 | 47.910 | 24.227 | 3.270 | 2.213 |
| 7 | GP48 | 164.750 | -87.130 | 160.857 | -88.720 | 3.893 | 1.590 |
| 8 | GP49 | 7.360 | 70.260 | 2.089 | 70.048 | 5.271 | 0.212 |
| 9 | GP50 | 10.120 | 67.500 | 4.050 | 68.087 | 6.070 | -0.587 |
| 10 | GP51 | 42.710 | 34.910 | 35.568 | 36.569 | 7.142 | -1.659 |
| 11 | GP52 | 67.810 | 9.810 | 58.247 | 13.890 | 9.563 | -4.080 |
| 12 | GP53 | 31.820 | 45.800 | 23.990 | 48.147 | 7.830 | -2.347 |
| 13 | GP54 | 12.880 | 64.740 | 4.502 | 67.635 | 8.378 | -2.895 |
| 14 | GP55 | 36.740 | 40.880 | 29.356 | 42.781 | 7.384 | -1.901 |
| 15 | GP56 | 11.570 | 66.050 | 4.648 | 67.489 | 6.922 | -1.439 |
| 16 | DOP2 | 90.910 | -13.290 | 83.866 | -11.729 | 7.044 | -1.561 |
| 17 | GP59 | 45.180 | 32.440 | 40.208 | 31.929 | 4.972 | 0.511 |
| 18 | GP61 | 41.830 | 35.790 | 37.335 | 34.802 | 4.495 | 0.988 |
| 19 | GP84 | 7.130 | 70.490 | 3.014 | 69.123 | 4.116 | 1.367 |
| 20 | GP85 | 4.930 | 72.690 | 1.694 | 70.443 | 3.236 | 2.247 |
| 21 | GP90 | 10.780 | 66.840 | 2.319 | 69.818 | 8.461 | -2.978 |
| 22 | GP91 | 26.310 | 51.310 | 19.436 | 52.701 | 6.874 | -1.391 |
| 23 | TG05 | 193.730 | -116.110 | 190.824 | -118.687 | 2.906 | 2.577 |
| 24 | TG07 | 194.270 | -116.650 | 185.634 | -113.497 | 8.636 | -3.153 |
| 25 | TG09 | 223.000 | -145.380 | 217.131 | -144.994 | 5.869 | -0.386 |
| 26 | TG10 | 193.250 | -115.630 | 184.403 | -112.266 | 8.847 | -3.364 |
| 27 | TG19 | 136.070 | -58.450 | 130.538 | -58.401 | 5.532 | -0.049 |

Table 33 Bias corrected Relative differences derived from OSU91A model and GPS/leveling data (Station GP58 held fixed)


Fig. 36. Absolute bias corrected orthometric height differences of EGM96, OSU91A and Gravimetric geoid model relative to local vertical datum

