CURVE FITTING TECHNIQUE FOR OPTIMAL PULSEWIDTH MODULATION (PWM) ONLINE CONTROL OF A VOLTAGE SOURCE INVERTER (VSI)

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Abstract: A curve fitting technique to allow online computation of switching angles based on the optimal PWM switching strategies is presented. Detail description of the curve fitting technique as applied to the optimal PWM switching angles trajectories for a VSI particularly for ac power conditioner or UPS applications is provided in this paper. This includes some analysis on the accuracy of the calculations based on the curve fitting technique compared with the exact solutions. The approach taken to actually implement this technique on a practical system is also highlighted.

Keywords
Optimal PWM, curve fitting technique, VSI.

1. INTRODUCTION

Control of a VSI output from the aspect of modulation can generally be classified into two switching strategies. These are the carrier modulated natural or regular sampled sinusoidal PWM and optimal or programmed PWM switching strategies. It is a well known fact that optimal PWM switching strategies offer several distinct advantages over the carrier modulated PWM type. In particular, a VSI employing optimal PWM switching strategies is found to achieve great reduction in its effective switching frequency. This contributes to reduced switching losses for the same amount of reduction in lower order harmonics when compared to the one using the carrier modulated sinusoidal PWM switching strategies.

Optimal PWM switching strategies are designed to optimise some specific performance criteria such as harmonic voltage elimination, harmonic current minimisation or harmonic loss minimisation. The approach taken is to define a general PWM waveform in terms of a set of switching angles for all the values of the per unit amplitude of the VSI output voltage fundamental. These angles are determined based on a chosen performance criterion using numerical minimisation search techniques applied to a set of nonlinear transcendental equations. Lookup tables of the precalculated optimised switching angles are then programmed into a microprocessor's memory to allow for online generation of the optimised switching angles using interpolation techniques. Such an online implementation can turn out to be tedious and time consuming especially when dealing with a large number of switching angles which increase the memory requirement on the microprocessor. On the other hand, approximation of the actual optimal PWM switching strategies through the development of equations or certain algorithms have allowed online real time computation of near optimal or suboptimal switching angles. Recent studies on optimal PWM switching strategies have shown great effort in developing such techniques.

This paper presents a curve fitting technique to allow on-line computation of the optimal PWM switching angles. It gives better control range and resolution for a VSI, particularly for ac power conditioner or UPS applications where it is crucial to maintain high quality output voltage at a fixed frequency regardless of the loading conditions. In this case, elimination of the lower order harmonics of the VSI output voltage is chosen as the performance criterion. Solutions trajectories of the nonlinear equations designed to meet this criteria while providing output voltage control is first obtained using the usual numerical minimisation search technique. Each of these trajectories is then approximated using MATLAB's curve fitting function The polynomial equations obtained for each switching angle solution trajectory for the amplitude of the VSI output voltage fundamental are then used as the basis for a simple online real time computation of an optimal PWM switching angles for the VSI.

II.OPTIMAL PWM SWITCHING STRATEGIES BASED ON HARMONIC ELIMINATION

Fig. 1 illustrates a generalised quarter-wave symmetric PWM waveform of a single phase full bridge VSI based on the unipolar PWM switching scheme.

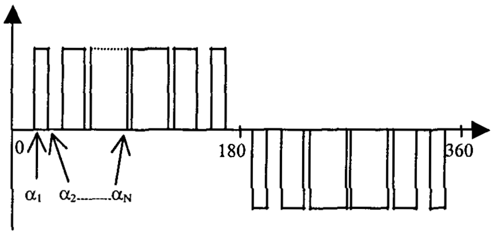


Fig. 1 Generalized quarter wave symmetric PWM waveform

The Fourier coefficients of .the waveform are given by [1]

$$a_n = \frac{4}{n\pi}\left[\sum_{k=1}^{N}(-1)^{k+1}\cos(n\alpha_k)\right] \qquad (1)$$

$$b_n = 0 \qquad (2)$$

where
n = harmonic order
N = number of switching angles per quarter cycle
α_k= kth switching angle.

A set of solutions is obtainable by equating any N-1 harmonics to zero and assigning a specific value to the amplitude of the fundamental of the VSI output voltage (ap1). These equations are nonlinear as well as transcendental in nature. The equations to eliminate N-1 lower order harmonics such as 3,5,7 etc. are in the form of [2]

$$\begin{bmatrix} \cos\alpha_1 & -\cos\alpha_2 & \cdots\cdots\cdots (-1)^{N+1}\cos\alpha_N \\ \cos 3\alpha_1 & -\cos 3\alpha_2 & \cdots\cdots\cdots (-1)^{N+1}\cos 3\alpha_N \\ \vdots & & \\ \cos(x)\alpha_1 & -\cos(x)\alpha_2 & \cdots\cdots\cdots (-1)^{N+1}\cos(x)\alpha_N \end{bmatrix} = \begin{bmatrix} \frac{\pi\, ap1}{4} \\ 0 \\ \vdots \\ 0 \end{bmatrix} \qquad (3)$$

where
$$x = 2N - 1 \qquad (4)$$

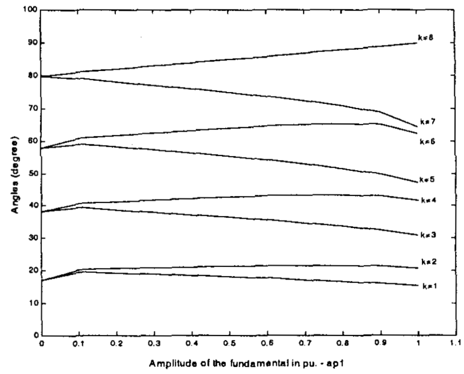
The VSI switching frequency f_{sw} is defined as [2]

$$f_{sw} = (N + 1)\,f \qquad (5)$$

where f is the operating frequency of the VSI. The order of the first and second significant harmonic component of the VSI output voltage is given by [2]

$$D_1 = 2N + 1 \qquad (6)$$

$$D_2 = 2N + 3 \qquad (7)$$



k: kth switching angle

Fig. 2 Switching angles solutions trajectories for N=8

In this work, N= 8 is chosen to eliminate 7 lower order harmonics which are 3,5,7,9,11,13 and 15. Fig. 2 illustrates the switching angles solutions trajectories for N=8. These optimal PWM switching angles solutions are found by solving the nonlinear equations concerned using a numerical method subroutine available from MATLAB's NAG Foundation Toolbox. This subroutine is capable of finding the solution of system of nonlinear equations using function values. With proper set up of the nonlinear equations and initial guessing values of the switching angles, a set of solutions is obtained at each assigned ap1 satisfying the criterion

$$\alpha_1 < \alpha_2 < \alpha_3 < \text{------} \alpha_N < \pi/2 \qquad (8)$$

III. CURVE FITTING TECHNIQUE

The approach taken is to represent each of the switching angles solutions trajectory with an equation in terms of ap1. MATLAB's curve fitting function based on polynomial regression is found to be adequate as a tool to best approximate these trajectories. In this case, each trajectory is modeled by a polynomial equation in the form of

$$\alpha_k = a_o + a_1(ap1) + a_2(ap1^2) + \ldots..a_m(ap1^m) \qquad (9)$$

where m is the polynomial order. The unknown coefficients (a_o ,a_1 ,a_2 ..... a_m) are computed by doing a least square fit which minimises the sum of squares of the deviations of the data which in this case is the values of α_k corresponding to each ap1, from the equation. To further improve the accuracy of the equation, the switching angles solutions trajectories are divided into two segments. The first segments of the trajectories lie between ap1 = 0 to ap1 = 0.5 while the second segments are between ap1 = 0.5 to ap1 = 1. This means each solutions trajectory is represented by two equations but only one is utilised at a time depending on ap1. Consequently, the number of orders of the polynomial equations is also reduced.

A.Determining the Order of the Polynomial Equations

A measure of goodness of fit in the curve fitting technique is the residual which is defined as the difference between the observed and the predicted data.[3]. The order of the polynomial equations representing each segment of each solutions trajectory is determined by comparing the residuals for the various fits Analysis made based on this concluded that m = 5 for the polynomial equations is passable in representing the switching angles solutions trajectories. For comparison purposes, the residuals for segment 2 of α_5's solutions trajectory for m = 3 and m = 5 are shown in Fig. 3.

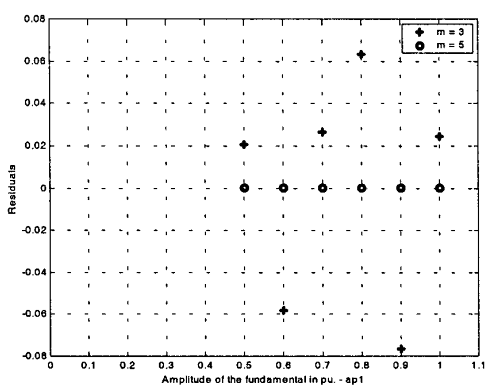
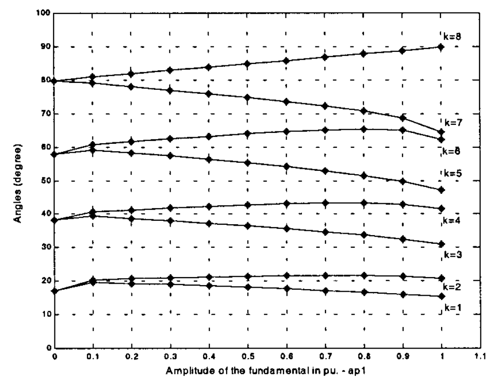


Fig. 3 Residuals for segment 2 solution trajectory of α_5

B. Accuracy of the equations

The accuracy of the solutions calculated using the polynomial equations for each trajectory is shown in Fig. 4. By substituting any value of ap1 between 0 to 1 into the polynomial equations, the exact solutions of the optimised switching angles are obtained.



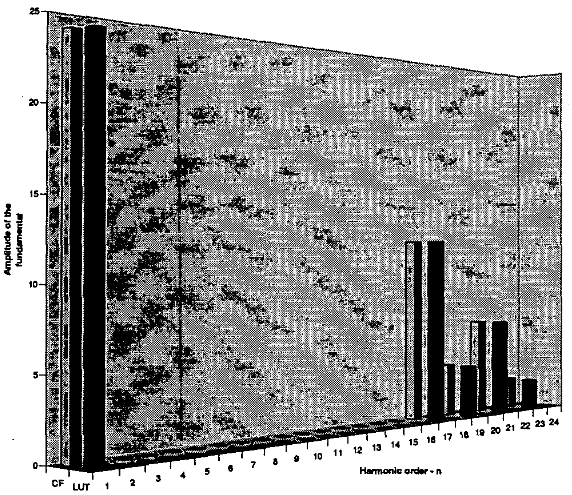
—— Exact solutions
● Equation based solutions
k kth switching angle

Fig.4 Comparison between exact and equation based solutions

IV. IMPLEMENTATION OF THE CURVE FITTING TECHNIQUE

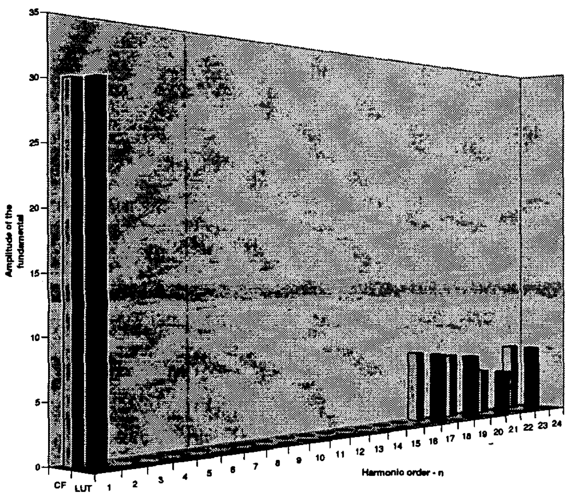
A. Results of the simulation study

To show the feasibility of the proposed curve fitting technique that allows equation based representation of the optimal PWM switching angles solutions trajectories, operation of the single phase bridge VSI is simulated using MATLAB's Simulink. Control of the VSI output voltage with simultaneous lower order harmonic elimination is handled by a block developed based on the polynomial equations. For comparison purposes, another control block is also developed based on lookup tables of the precalculated optimised switching angles. Fig. 5 and Fig. 6 shows the harmonic spectrum of theVSI output voltage utilising both control blocks for ap1 = 0.8 and ap1 = 1. It is evident from these figures that the harmonics in terms of amplitude and placement produced are the same. This confirms the ability of the equations obtained from the curve fitting technique to exactly calculate the optimal PWM switching angles at any given ap1. Such feature is very useful particularly when practically implementing the optimal PWM switching strategies on the VSI.



LUT = Control based on lookup tables
CF = Control based on curve fitting technique

Fig. 5 Harmonic spectrum of the VSI output voltage for ap1 = 0.8



LUT = Control based on lookup tables
CF = Control based on curve fitting technique

Fig. 6 Harmonic spectrum of the VSI output voltage for ap1 = 1

B. Proposed Practical Implementation

A typical practical implementation of the optimal PWM switching strategies on control of a VSI output is by storing the precalculated set of switching angles in a microprocessor's ROM in the form of lookup tables. These lookup tables are subsequently used to generate the PWM waveforms on line in real time. Whenever necessary, the microprocessor uses these lookup tables to calculate the approximate switching angles using simple linear interpolation.

This work proposes incorporation of the equations representing the switching angles solutions trajectories of the optimal PWM switching strategies using the curve fitting technique with the well known capability of the digital signal processors (DSPs) to perform very fast calculations online in real time. With this technique, at a particular ap1, the DSP has to calculate ap1 up the order of 5 once, which involves 4 multiplications. Both segments of the optimal PWM solutions trajectories are represented by equation (5) of the same order. Thus, regardless of ap1, only 5 addition and 5 multiplication operations are involved in the calculation of each optimised switching angle. The overall operations depending on the value of N can easily be accomplished by a DSP within at most a sampling time of 100μ seconds. The advantage of using this technique can be seen from the aspect of better VSI control range and resolution. In addition this technique too can be used as a basis for real time optimal PWM control of VSI output voltages particularly for ac power conditioner or UPS applications, that has the ability to dynamically adapt to changing load conditions.

V. CONCLUSIONS

A curve fitting technique that represents the optimal PWM switching angles solutions trajectories by polynomial equations has been presented. This allows direct online calculations of the optimal PWM switching angles that eliminates lower order harmonics in a VSI output voltage using the high speed calculation capabilities of a DSP. With the equations obtained from this technique, the exact values of the optimal PWM switching angles as the ones initially calculated offline using numerical analysis are computed for every increment of ap1.

VI. REFERENCES

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