BIT-ERROR RATE PERFORMANCE ANALYSIS OF SPECTRUM BASED DETECTOR FOR FSK DIGITAL MODULATION

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Abstract—**FSK is widely used digital modulation technique due to its simplicity in implementation using noncoherent detection. Further availability of digital signal processing algorithms such as the FFT (Fast Fourier Transform) and necessary supporting technology makes it possible to implement spectrum based detector for FSK. This paper formulates the BER (Bit-error rate) performance of the spectrum based detector. Generally, the performance lies between the optimum that is the coherent detector and the suboptimum that is the noncoherent detector. Verification was performed by computer simulation to confirm the results.**

I. INTRODUCTION

Digital modulation using FSK (Frequency Shift-Keying) is one of the basic and widely used method for transmitting binary data over a passband channel such as the PSTN (Public Switched Telephone Network), HF (High Frequency) circuits and UHF (Ultra High Frequency) digital cellular communication systems [1]-[3]. Although not as spectrum efficient as PSK (Phase Shift-Keying), the detection process at the receiver is simplified by noncoherent detector. However, at the expense of suboptimal BER (bit-error rate) performance compared to coherent detector. Due to the availability of algorithms such as FFT (Fast Fourier Transform) and signal processing technology such as the DSP processor, it is cost effective to use spectrum estimation to detect data [4]. Thus, the objective of this paper is to compare the bit-error rate performance for FSK using the spectrum based detector with coherent and noncoherent detector, and derive its theoretical formulation.

II. SIGNAL MODEL

The received signal is within a bit-duration is given as

$$y(t) = x(t) + n(t) \qquad (1)$$

where $x(t)$ is the true signal, and $n(t)$ is the interference due to additive white Gaussian noise with zero mean and power N_0. The true signal $x(t)$ is [4]

$$x(t) = x_1(t) = A\cos 2\pi f_1 t \quad \text{bit '1'}$$
$$x_0(t) = A\cos 2\pi f_0 t \quad \text{bit '0'}$$
$$t_0 \le t \le t_0 + T_b \qquad (2)$$

where A is the signal amplitude, f_1 and f_0 are the frequencies of the signal, t_0 is any arbitrary time instant, and T_b is the bit-duration. For simulation purposes, the subcarrier frequencies used are 1400 and 1800 Hz, bit-rate of 100 bits/sec and sampling frequency of 8000 Hz. The time and frequency domain representation of the signal for transmitting a binary sequence '1110010' is shown in Fig. 1.

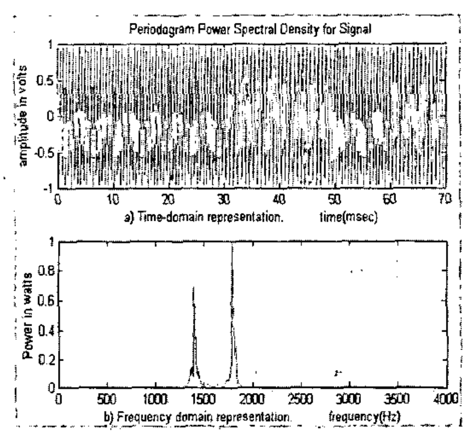

Figure 1. Time and frequency representation of an FSK signal.

III. COHERENT AND NONCOHERENT DETECTOR

There are two ways for detecting data : coherent and noncoherent detector. Coherent detector is more optimum in terms of the bit-error rate (BER) performance but is more complex compared to noncoherent detector due to the need for carrier recovery circuit for phase syncronization.

A. Coherent Detector

Coherent detector of data requires exact synchronization in phase between the received signal and the possible signal form. The output of the receiver is defined as [5]

$$z(t) = \int_{t_0}^{t_0+T_b} y(t)x_1(t)dt - \int_{t_0}^{t_0+T_b} y(t)x_0(t)dt$$

$$t_0 \le t \le t_0 + T_b \tag{3}$$

where $x_0(t)$ and $x_1(t)$ are reference signal used at the receiver. The reference signals are required to be similar in form to the received signal.

By sampling $z(t)$ at intervals nT_b, the detection is performed as follows

$$\begin{aligned} z(n) &= +ve \text{, then binary bit "1" is detected} \\ &= -ve \text{, then binary bit "0" is detected} \end{aligned} \tag{4}$$

The BER for coherent detector is

$$BER = Q\left(\sqrt{\frac{A^2T_b}{4N_0}}\right) \tag{5}$$

where $Q(\,)$ is

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty \exp\left(-\frac{z^2}{2}\right)dz \tag{6}$$

B. Noncoherent Detector

For noncoherent detector, the same methodology for detection is applied except that the output of the receiver is [5]

$$z(t) = \int_{t_0}^{t_0+T_b} |y(t)*h_1(t)|^2\,dt - \int_{t_0}^{t_0+T_b} |y(t)*h_0(t)|^2\,dt \tag{7}$$

where * indicates a convolution operation, and $h_1(t)$ and $h_0(t)$ are the impulse response of bandpass filters centered at f_1 and f_0 respectively with bandwidth $2/T_b$.

The resulting bit-error rate is

$$BER = \frac{1}{2}\exp\left(-\frac{A^2}{2N_p}\right) \tag{8}$$

where σ_p^2 is

$$N_p = \frac{4N_0}{T_b} \tag{9}$$

For both detection schemes, the signal-to-noise ratio (SNR) in decibels is

$$SNR_{dB} = 10\log\left(\frac{A^2T_b}{2N_0}\right) \tag{10}$$

In general, the performance degradation for the noncoherent over the coherent detector is about 6 dB for a constant BER of 10^{-4}.

C. Phase Synchronization Error

Synchronization is critical to ensure that the bit-error rate performance is optimal for coherent detector. Given that a received signal within a bit-duration is

$$y(t) = A\cos(2\pi f_1 t + \varphi) \quad t_0 \le t \le t_0 + T_b \tag{11}$$

where ϕ is the phase present in the signal. If the reference signal is

$$x_1(t) = A\cos(2\pi f_1 t) \quad t_0 \le t \le t_0 + T_b \tag{12}$$

then the output signal is

$$z(t) = \int_{t_0}^{t_0+T_b} y(t)x_1(t)dt = \frac{A^2T_b}{2}\cos(\varphi) \tag{13}$$

The results show how critical the phase shift affects the magnitude of the output signal and subsequently the bit-error rate. This problem is solved by introducing carrier recovery algorithms that is described in [6].

The phase synchronization error for noncoherent dectection can be demonstrated by applying the signal defined in Equation 11 into 7. The output of the noncoherent detector $z(t)$ in Equation 7 compares the signal energy that passes through either one of the filters to estimate the received data. Thus, the spectrum representation for Equation 7 is

$$\begin{aligned} z(t) &= \frac{1}{T_b}\int_{-\infty}^{\infty} |Y(f)H_1(f)|^2\,df \\ &- \frac{1}{T_b}\int_{-\infty}^{\infty} |Y(f)H_0(f)|^2\,df \end{aligned} \tag{14}$$

Since $y(t)$ can only pass through $h_1(t)$, then the output of detector is

$$z(t) = \frac{1}{T_b}\int_{-\infty}^{\infty} |Y(f)H_1(f)|^2\,df \tag{15}$$

The power spectrum of $Y(f)$ obtained from the Fourier transform is

$$\frac{1}{T_b}|Y(f)|^2 = Y(f)Y^*(f)$$

$$= \frac{A^2 T_b}{4}\left[\sin c(\pi(f_1 - f)T_b)^2 + \sin c(\pi(f_1 + f)T_b)^2\right] \tag{16}$$

Since the phase is absent from the power spectrum, then the noncoherent detector is insensitive to phase synchronization error.

IV. SPECTRUM BASED DETECTOR

This section will explain the foundation of the spectrum based detector and the derivation of the theoretical bit-error rate.

A. *Theory and Methodology*

If $x(t)$ is defined within a bit-duration T_b, then its power spectrum $S_{xx}(f)$ can be estimated as [7]

$$S_{xx}(f) = \frac{1}{T_b}|X(f)|^2 = \frac{1}{T_b}\left|\int_{t_0}^{t_0+T_b} x(t)\exp(-j2\pi ft)dt\right|^2 \tag{17}$$

where $X(f)$ is the amplitude spectrum. If the complex exponential is defined in terms of the cosine and sine terms, the power spectrum estimate is

$$S_{xx}(f) = \frac{1}{T_b}\left|\int_{t_0}^{t_0+T_b} x(t)h_I(t,f)dt\right|^2 + \frac{1}{T_b}\left|\int_{t_0}^{t_0+T_b} x(t)h_Q(t,f)dt\right|^2 \tag{18}$$

where

$$h_I(t,f) = \cos(2\pi ft)$$
$$h_Q(t,f) = \sin(2\pi ft) \tag{19}$$

Thus, its equivalent form in terms of the power spectrum is

$$S_{xx}(f) = \frac{1}{T_b}\left[\int_{-\infty}^{\infty} X(v)H_I(f-v)dv\right]^2 + \frac{1}{T_b}\left[\int_{-\infty}^{\infty} X(v)H_Q(f-v)dv\right]^2 \tag{20}$$

The power spectrum is maximized if both $H_I(f)$ and $H_Q(f)$ matched in frequency in $X(f)$.

To detect a signal $x(t)$ with frequency f_1 from the power spectrum $S_{xx}(f)$, both $H_I(f)$ and $H_Q(f)$ are evaluated at $f=f_1$ where $f_1>0$ that are defined as

$$H_{I,1}(f) = \frac{AT_b}{2}\sin c(\pi(f_1 - f)T_b) + \frac{AT_b}{2}\sin c(\pi(f_1 + f)T_b)$$

$$H_{Q,1}(f) = -j\frac{AT_b}{2}\sin c(\pi(f_1 - f)T_b) + j\frac{AT_b}{2}\sin c(\pi(f_1 + f)T_b) \tag{21}$$

Similar this is also defined for $f=f_0$ where $f_0>0$. Thus, the binary data can be detected from the power spectrum evaluated at f_0 and f_1 are

$$S_{xx,1}(f) = \frac{1}{T_b}[X(f)H_{I,1}(f)]^2 + \frac{1}{T_b}[X(f)H_{Q,1}(f)]^2$$
$$S_{xx,0}(f) = \frac{1}{T_b}[X(f)H_{I,0}(f)]^2 + \frac{1}{T_b}[X(f)H_{Q,0}(f)]^2 \tag{22}$$

For $f=f_1$ and $f=f_0$, the term $H_I(f)$ and $H_Q(f)$ described the spectrum of bandpass filters and $S_{xx}(f)$ is the spectrum of the signal that passed through the bandpass filters. The signal powers derived from Equation 22 are

$$P_{x,1} = \int_{-\infty}^{\infty} S_{xx,1}(f)df$$
$$P_{x,0} = \int_{-\infty}^{\infty} S_{xx,0}(f)df \tag{23}$$

From an implementation perspective, the receiver can be implemented based on Equation 18 and the power spectrum is evaluated for $f=f_1$ and $f=f_0$.

B. *Bit-Error Rate Performance*

The binary data in FSK is detected based on the peak of the power spectrum. For a given time instant, the detection performed within a bit-duration is as follows

$P_{x,1} - P_{x,0} > 0$ binary bit '1' is detected

$P_{x,1} - P_{x,0} < 0$ binary bit '0' is detected (24)

An error occurs when a binary '0' is detected when the actual data transmitted is '1' or vice versa. The bit-error rate can be calculated as

$$P_e = P_{e,1}P_1 + P_{e,0}P_0 \tag{25}$$

where P_1 and P_0 are the priori probability that is ½. The probability $P_{e,1}$ and $P_{e,0}$ are defined as

$P_{e,1} = P\{P_{x,1} - P_{x,0} < 0\}$ when actual data is '0'

$P_{e,0} = P\{P_{x,1} - P_{x,0} > 0\}$ when actual data is '1' (26)

Since $P_{e,1}$ and $P_{e,0}$ are equal, then the bit-error rate in Equation 25 simplifies as

$$P_e = P_{e,1} \tag{27}$$

The actual bit-error rate can be derived further based on the following example.

An FSK signal within a bit-duration is

$$x(t) = \cos(2\pi f_0 t) + n(t) = x_0(t) + n(t) \qquad (28)$$

where $x_0(t)$ is the actual signal representing the binary bit '0' and $n(t)$ is the interference due to zero mean additive white Gaussian noise with variance N_0. The spectrum of the signal $x_0(t)$ and the power spectrum of noise is

$$X_0(f) = \frac{AT_b}{2}\operatorname{sinc}(\pi(f_0 - f)T_b) + \frac{AT_b}{2}\operatorname{sinc}(\pi(f_0 + f)T_b) \qquad (29)$$

$$S_{nn}(f) = [N(f)]^2 = N_0 \quad -\infty \le f \le \infty \qquad (30)$$

Based on Equation 22, the power spectrum estimates are

$$S_{xx,1}(f) = \frac{1}{T_b}[N(f)H_{I,1}(f)]^2 + \frac{1}{T_b}[N(f)H_{Q,1}(f)]^2$$

$$S_{xx,0}(f) = \frac{1}{T_b}[\{X_0(f) + N(f)\}H_{I,0}(f)]^2 + \frac{1}{T_b}[N(f)H_{Q,0}(f)]^2 \qquad (31)$$

The power spectrum estimate $S_{xx,1}(f)$ is a Rayleigh random variable since it is the magnitude of two narrowband Gaussian random process centered at f_1. For $S_{xx,0}(f)$, the estimate is the magnitude of two narrowband Gaussian random process with a deterministic sinusoid with frequency f_0. The Rician random variable is applicable. Based on this assumption, the bit-error rate in Equation 31 can be calculated as [5]

$$P_e = \frac{1}{2}\exp\left(-\frac{P_{sig}}{2N_p}\right) \qquad (32)$$

where P_{sig} is the signal power and N_p is the narrow band noise power.

If the power spectrum is considered over all frequency range, the signal power and noise power calculated from Equation 31 is

$$P_{sig} = 2\int_{-\infty}^{\infty}\frac{1}{T_b}[\{X_0(f)\}H_{I,0}(f)]^2\,df = \frac{1}{16}A^4T_b^2 \qquad (33)$$

$$N_p = 2\int_{-\infty}^{\infty}\left[\frac{1}{T_b}[N(f)H_{I,1}(f)]^2 + \frac{1}{T_b}[N(f)H_{Q,1}(f)]^2\right]df = \frac{1}{4}A^2T_bN_0 \qquad (35)$$

By substituting Equation 34 and 35 into 32, the bit-error rate is

$$P_e = \frac{1}{2}\exp\left(-\frac{A^2T_b}{8N_0}\right) = \frac{1}{2}\exp\left(-\frac{A^2T_b}{8N_0}\right) \qquad (36)$$

If the detection is performed from the peaks of the power spectrum evaluated at $f=f_1$ and $f=f_0$, the power spectrum is estimated within the frequency

$$f_1 \pm \frac{1}{2}f_{BW},\ f_0 \pm \frac{1}{2}f_{BW},\ f_{BW} = \frac{1}{4T_b} \qquad (37)$$

This is because the signal has finite duration T_b, and the power spectrum mainlobe width is significant.

Similar to Equation 33 and 35, the signal power and noise power calculated from Equation 31 is

$$P_{sig} = 2\int_{f_0-\frac{1}{2}f_{BW}}^{f_0+\frac{1}{2}f_{BW}}\frac{1}{T_b}[\{X_0(f)\}H_{I,0}(f)]^2\,df = \frac{1}{16}A^4T_b^2f_{BW}\left[1+\frac{64}{\pi^4}\right] \qquad (38)$$

$$N_p = 2\int_{f_0-\frac{1}{2}f_{BW}}^{f_0+\frac{1}{2}f_{BW}}\left[\frac{1}{T_b}[N(f)H_{I,1}(f)]^2 + \frac{1}{T_b}[N(f)H_{Q,1}(f)]^2\right]df = \frac{1}{4}A^2T_bN_0f_{BW}\left[1+\frac{8}{\pi^2}\right] \qquad (39)$$

The bit-error rate obtained by substituting Equation 38 and 39 into 32 is

$$P_e = \frac{1}{2}\exp\left(-\frac{\left[1+\frac{16}{\pi 4}\right]}{\left[1+\frac{8}{\pi 2}\right]}\frac{A^2T_b}{8N_0}\right) = \frac{1}{2}\exp\left(-\frac{0.82A^2T_b}{8N_0}\right) \qquad (40)$$

V. RESULTS

The spectrum based detector was tested by computer simulation in the presence of additive white Gaussian noise for signal-to-noise ratio from 5 to 15 dB. In addition, a random delay is included for every packet of received data at the spectrum detector to simulate the effect of synchronization error. Simulation results in Fig. 2 shows that the performance of the spectrum based detetor lies between the proposed theoretical limit that is between the coherent and noncoherent detector. For a constant BER of 10^{-3}, the SNR is 15 dB that is higher by about 2 dB compared to coherent detector. Eventhough the BER is suboptimum compared coherent detector, the spectrum based detector does not required additional circuitary for phase synchronization and insensitive to phase synchronization error.

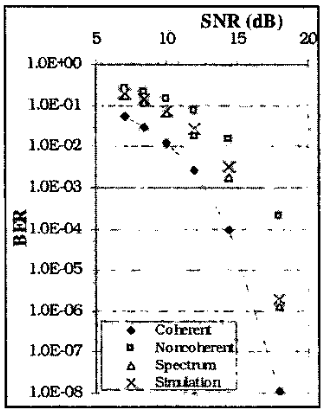

Figure 2. BER simulation results for spectrum based detector.

VI. CONCLUSIONS

The use of spectrum based detector for FSK is an attractive alternative to coherent and noncoherent detector. The proposed spectrum based detector is insensitive to phase synchronization error and its bit-error rate performance is a compromise between coherent and noncoherent detector. A proposed theoretical bit-error formulation is presented and verified by simulation.

VII. REFERENCES

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