Window Width Estimation and the Application of the Windowed Wigner-Ville Distribution in the Analysis of Heart Sounds and Murmurs

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Abstract : This paper looks at the analysis of heart sounds and murmur using time-frequency signal analysis. The techniques used are the Wigner-Ville distribution (WVD) and windowed Wigner-Ville distribution (WWVD) that belonged to the bilinear class of time-frequency distribution. These techniques developed to provide high-resolution time-frequency representation for time-varying signals. Due the nonlinear operation involved, interference terms are introduced in the time-frequency representation. The signals of interest are modeled as multicomponent signals and the characteristics of the signal in time-lag plane are observed. From the time-lag plane, the interference components are identified, and the appropriate window width is selected in the WWVD to remove the interference. Analysis results show that WWVD produces more accurate time-frequency representation compared to the WVD and the signal-to-interference is used to quantify the improvement.

I INTRODUCTION

Time-frequency signal analysis techniques are developed to allow the representation of the instantaneous characteristics of time-varying signals jointly in the time-frequency plane. One of the earliest method developed was the spectrogram [1] that is based on the moving time-windowed Fourier transform. However, the choice of the window function results in the compromise in the time and frequency resolution. This is overcome by Wigner-Ville distribution that belongs to the general class of bilinear time-frequency distribution [2][3]. A major problem with this method is the presence of interference due to cross terms in the time-frequency distribution. Other time-frequency distributions such as the exponential kernel distribution (EKD) [4] and reduced interference distribution (RID) [5] are efforts made to overcome this problem.

Two time-frequency distributions presented are the Wigner-Ville distribution (WVD) and the Windowed Wigner-Ville distribution (WWVD). The paper presents the use of both of these time-frequency distributions in the analysis of heart sounds and murmurs. A model for the heart sound is presented and its characteristics are represented in the time-lag plane. The interference problem is identified and with the appropriate window width selection the interference terms are minimized using the WWVD.

II SIGNAL DEFINITION

Heart sounds and murmurs can be measured based on the auscultation using a stethoscope. The signal is periodic and within a period can be generally expressed as

$$z(n) = \sum_{l=0}^{4} z_l(n) \qquad 0<n<N_p \tag{1}$$

where N_p is the period of the signal and $z_l(n)$ is the individual component of the heart sound. The various component of heart sounds that are represented by $z_l(n)$ are described as follows

1) $z_0(n)$-the 4 th heart sound (S4) that has low energy and does occur for normal heart conditions.

2) $z_1(n)$-the 1 st heart sound (S1) that has high energy and represent the sound of a normal heart.

3) $z_2(n)$-a period of lull for normal heart but of significant activity for heart conditions such as valvular regurgigation or stenosis conditions.

4) $z_3(n)$-the 2 nd heart sound (S2) that has high energy and similar to the 1 st heart sound is a feature of a normal heart.

5) $z_4(n)$-the 3 rd heart sound (S3) that could have energy levels approaching either the S1 and S2 heart sounds.

Each of the signal component $z_l(n)$ is a limited duration complex pulse sinusoid centered at time n_l samples and of

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pulse duration N_0 samples. The following conditions ensure that the individual signal components do not overlap in time

$$n_0<n_1<n_2<\ldots\ldots<n_4$$
$$n_1=n_0+N_0,\ n_2=n_1+N_0,\ \ldots\ldots\ N_0,$$
$$n_4=n_3+N_0 \tag{2}$$

Each individual component of $z_l(n)$ can be is expressed as

$$z_l(n)=\prod\nolimits_{N_0}(n-n_l)c_l\exp(j2\pi f_l(n-n_l)-\phi_l) \tag{3}$$

where c_l is the amplitude, f_l is the frequency, n_l is the time delay and ϕ_l is the phase. The term $\prod_{N_0}$ is referred as a box function that is defined as

$$\prod\nolimits_{N_0}(n-n_l)=1 \quad \text{for } n_l-N_0/2<n<n_l+N_0/2$$
$$=0 \quad \text{elsewhere} \tag{4}$$

From the 31 classes of heart sounds, four are chosen for analysis because they represent most of the possible combination of heart sounds. The sounds and their mathematical representation based on Equation (1) are summarized in as follows

Normal heart

$$z(n)=z_1(n)+z_3(n) \tag{5}$$

Aortic stenosis

$$z(n)=z_1(n)+z_2(n)+z_3(n) \tag{6}$$

S4 gallop preceding S1

$$z(n)=z_0(n)+z_1(n)+z_3(n) \tag{7}$$

Quadruple rhythm (S4-S1-S2-S3)

$$z(n)=z_0(n)+z_1(n)+z_3(n)+z_4(n) \tag{8}$$

Each of the individual components of $z_l(n)$ in Equation (5) to (8) are defined in Equation (3). In general, it is found that the frequency f_1 and f_3 for the S1 and S2 heart sounds are identical. Other parameters are not expected to be identical.

III BILINEAR TIME-FREQUENCY DISTRIBUTIONS

The formulation for the bilinear class of discrete time-frequency distribution [1][2] is

$$\rho_z(n,k)=\sum_{m=0}^{N-1}G(n,m)\underset{(n)}{*}\left[z(n+m)z^*(n-m)\right]\exp\left(-j\frac{4\pi km}{N}\right) \tag{9}$$

where $G(n,m)$ is the time-lag kernel function and $z(n)$ is the discrete-time analytical signal. The product $z(n+m)z^*(n-m)$ is often referred as the bilinear product $K_z(n,m)$. Basically, the bilinear product transforms the time domain signal $z(n)$ into the time-lag plane that is described by the function $K_z(n,m)$. Thus, Equation (9) can be written as

$$\rho_z(n,k)=\sum_{m=0}^{N-1}\left[G(n,m)\underset{(n)}{*}K_z(n,m)\right]\exp\left(-j\frac{4\pi km}{N}\right) \tag{10}$$

In the above equation, the term $G(n,m)\underset{(n)}{*}K_z(n,m)$ is referred as the time-varying autocorrelation function and is given by the notation $R_z(n,m)$.

If the time-lag kernel function is defined as $G(n,m)=\delta(n)$, then the formulation in Equation (9) becomes the Wigner-Ville distribution (WVD)

$$W_z(n,k)=\sum_{m=0}^{N-1}z(n+m)z^*(n-m)\exp\left(-j\frac{4\pi km}{N}\right)$$
$$=\sum_{m=0}^{N-1}K_z(n,m)\exp\left(-j\frac{4\pi km}{N}\right) \tag{11}$$

The windowed Wigner-Ville distribution (WWVD) differs from the WVD because the time-lag kernel function is defined as $G(n,m)=\delta(n)g(m)$. When substituted into Equation (9), the time-frequency distribution is

$$W_{z,w}(n,k)=\sum_{m=0}^{N-1}g(m)z(n+m)z^*(n-m)\exp\left(-j\frac{4\pi km}{N}\right) \tag{12}$$

The window function $g(m)$ is defined as

$$g(m)\neq 0 \quad \text{for } -N_g<m<N_g$$
$$=0 \quad \text{elsewhere} \tag{13}$$

where N_g is the window width. Any one of the popular window functions such as the rectangular, Hamming, Hanning or Bartlett window functions can be chosen for the WWVD.

IV CHARACTERISTICS OF THE BILINEAR PRODUCT

In this section, the analytical expressions for the auto bilinear product and cross bilinear product are derived. From these expressions, it is of interest to determine the range of values in time and lag, time deviation frequency and lag frequency for the individual auto bilinear product and cross bilinear product components. Based on these factors, the interference conditions are defined for the time-frequency representations.

A DEFINITION OF THE BILINEAR PRODUCT

Based on the signal definition in Equation (1), bilinear product of the signal obtained from Equation (9) is

$$K_z(n,m) = z(n+m)z^*(n-m) = \sum_{i=0}^{N_s-1} z_i(n+m)\sum_{l=0}^{N_s-1} z_l^*(n-m) \quad (14)$$

By putting the terms for $i=l$ together, the bilinear product is now

$$K_z(n,m) = \sum_{l=0}^{N_s-1} K_{zl}(n,m) + \sum_{i=0}^{N_s-1}\sum_{l=0}^{N_s-1} K_{zi,zl}(n,m) \quad (15)$$

where $K_{zl}(n,m)$ is the auto bilinear product of the component signal $z_l(n)$ and $K_{zi,zl}(n,m)$ is the cross bilinear product for the signal component $z_i(n)$ and $z_l(n)$. The auto bilinear product and the cross bilinear product components are defined as

$$K_{zl}(n,m) = z_l(n+m)z_l^*(n-m) \quad (16)$$

$$K_{zi,zl}(n,m) = z_{zi}(n+m)z_{zl}^*(n-m) \quad (17)$$

Based on this definition, the bilinear product for the normal heart sound and quadratic rhythm sound are

$$K_{z,normal}(n,m) = \sum_{l=odd}^{3} K_{zl}(n,m) + \sum_{\substack{i=odd\\ i\neq l}}^{3}\sum_{l=odd}^{3} K_{zi,zl}(n,m) \quad (18)$$

$$K_{z,quad}(n,m) = \sum_{l=0}^{1} K_{zl}(n,m) + \sum_{l=3}^{4} K_{zl}(n,m) + \sum_{i=0}^{1}\sum_{l=3}^{4} K_{zi,zl}(n,m) \quad (19)$$

The terms $K_{zl}(n,m)$ and $K_{zi,zl}(n,m)$ are referred as the auto bilinear and cross bilinear components respectively. Thus, the bilinear product consists of the auto bilinear product and the cross bilinear product components with the auto bilinear product and the cross bilinear product of the signal components.

B AUTO BILINEAR PRODUCT COMPONENTS

Based on the signal and bilinear product definitions in Equation (1) and (9), the auto bilinear product for an arbitrary signal component $z_l(n)$ is

$$K_{zl}(n,m) = \prod_{N_0}(n-n_l+m)\prod_{N_0}(n-n_l-m)c_l^2 \times \exp(j2\pi 2f_l m) \quad (20)$$

The maximum overlap of the box functions in time occurs at lag $m=0$. Thus, Equation (20) when expressed in time at this lag value is

$$K_{zl}(n,0) = \prod_{N_0}(n-n_l)c_l^2 \quad (21)$$

Equation (20) is centered at time instant $n=n_l$, and when define over lag becomes

$$K_{zl}(n_l,m) = \prod_{N_0}(m)c_l^2 \exp(j2\pi 2f_l m) \quad (22)$$

The lag frequency f_m for auto bilinear product is $2f_l$ that is twice the frequency of the true signal and the results in Equation (21) is also the instantaneous energy of the signal component $z_l(n)$.

Based on the results in Equation (21) and (22), the range of values in the time-lag plane, and the lag frequency f_m and time deviation frequency $f_{\delta n}$ for the auto bilinear product are

$$\begin{aligned} &K_{zl}(n,m)\neq 0 && \text{for } n_l-N_0/2<n<n_l+N_0/2, \text{ and } -N_0/2<m<N_0/2\\ &f_m=2f_l,\ f_{\delta n}=0 &&\\ &K_{zl}(n,m)=0 && \text{elsewhere}\\ &f_m=0,\ f_{\delta n}=0 && \end{aligned} \quad (23)$$

C CROSS BILINEAR PRODUCT COMPONENTS

In this section, it is desired to determine the range values in time and lag for the cross bilinear product component. By substituting the signal definition in Equation (1) into (9), the cross bilinear product for the signal component $z_{l+1}(n)$ and $z_l(n)$ is

$$K_{z(l+1),zl}(n,m) = \prod_{N_0}(n-n_l+m)\prod_{N_0}(n-n_l-m)c_l^2 \times \exp(j\phi_{l+1,l})\exp(j2\pi(f_{l+1}-f_l)n)\exp(j2\pi 2(f_{l+1}+f_l)m) \quad (24)$$

where $\phi_{l+1,l}$ is the time-lag phase term

$$\phi_{l+1,l} = -2\pi(f_{l+1}n_{l+1} - f_l n_l) - (\phi_{l+1} - \phi_l) \quad (25)$$

The maximum overlap in time for the box functions in Equation (24) occurs at $n=(n_{l+1}+n_l)/2$. When expressed in lag, Equation (24) at this time instant becomes

$$K_{z(l+1),zl}\left(\frac{n_{l+1}+n_l}{2},m\right)=\prod\nolimits_{N_0}\left(\frac{n_{l+1}+n_l}{2}-m\right)c_{l+1}c_l\exp(j\phi_{l+1,l})$$
$$\times\exp\left(j2\pi(f_{l+1}-f_l)\left(\frac{n_{l+1}+n_l}{2}\right)\right)\exp(j2\pi 2(f_{l+1}+f_l)m) \tag{26}$$

From the above equation, the lag frequency f_m is $f_{l+1}+f_l$. Results in Equation (26) shows that the function in lag is centered at $m=(n_{l+1}-n_l)/2$. When expressed over time at this lag instant is

$$K_{z(l+1),zl}\left(n,\frac{n_{l+1}-n_l}{2}\right)=\prod\nolimits_{N_0}\left(n-\frac{n_{l+1}+n_l}{2}\right)c_{l+1}c_l\exp(j\phi_{l+1,l})$$
$$\times\exp\left(j2\pi 2(f_{l+1}+f_l)\left(\frac{n_{l+1}-n_l}{2}\right)\right)\exp(j2\pi(f_{l+1}-f_l)n) \tag{27}$$

Based on Equation (26) and (27), the range of values in time-lag, plane lag frequency f_m and time deviation frequency $f_{\delta,n}$ for the cross bilinear product component are

$K_{z(l+1),zl}(n,m)\neq 0$ for $[(n_{l+1}+n_l)-N_0]/2<n<[(n_{l+1}+n_l)+N_0]/2$, and $[(n_{l+1}-n_l)-N_0]/2<m<[(n_{l+1}-n_l)+N_0]/2$

$f_m=f_{l+1}+f_l$, $f_{\delta,n}=f_{l+1}-f_l$

$K_{z(l+1),zl}(n,m)=0$ elsewhere (28)

$f_m=0$, $f_{\delta,n}=0$

D BILINEAR PRODUCT FOR NORMAL HEART SOUND

The signal definition and the bilinear product for the normal heart sound is defined in Equation (5) and (16). Based on the procedure in Section 4.1 and 4.2, the individual components of the bilinear product are calculated. Figure (1) and (2) shows the bilinear product components and the lag-frequency distribution respectively.

The WVD considers the bilinear product of the signal over the time-lag plane. At time instant such as at n_2, the instantaneous energy of the signal is zero because the bilinear product evaluated at lag zero lag has zero magnitude. Thus, there should not be any signal components in the time-frequency representation at this time instant. However, the Fourier transform in lag at this time instant is not zero since the cross bilinear products $K_{z3,z1}(n,m)$ is present. Thus, $K_{z3,z1}(n,m)$ contributes as interference in the time-frequency representation of the signal.

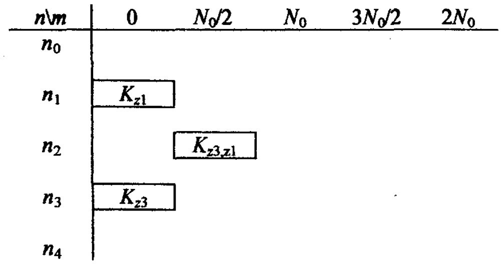

Fig 1 The positions of the auto bilinear product and cross bilinear product of the signal components in the time-lag plane. [The magnitude of all the bilinear product components is at unity. Note : K_{z1} is $K_{z1}(n,m)$ and $K_{z3,z1}$ is $K_{z3,z1}(n,m)$]

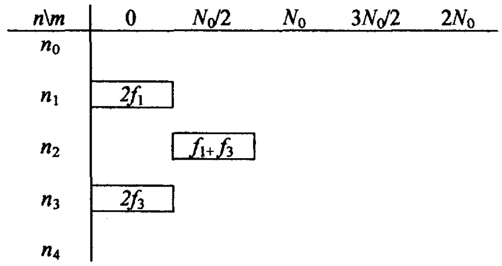

Fig 2 The lag-frequency distribution in the time-lag plane.

The window function used in the WWVD can reduce the interference present in the time-frequency representation. From Figure (2), the cross bilinear product that are considered as interference are removed if a window function is defined as

$g(m)=1$ $\quad -N_0/2<m<N_0/2$
$\quad\;\; =0$ $\quad$ elsewhere (27)

As a result, the cross bilinear products component $K_{z3,z1}(n,m)$ is removed from the time-lag plane resulting in a more accurate time-frequency representation.

E BILINEAR PRODUCT FOR QUADRATIC RHYTHM

Equation (8) and (19) defined the quadratic rhythm signal and its bilinear product. Based on the procedure in Section 4.1 and 4.2, the individual components of the bilinear product are calculated. Figure (3) and (4) show the bilinear product and the lag-frequency distribution.

Similarly, the instantaneous energy can be used to indicate if the cross bilinear product contribute as interference in time-frequency representation. Within instant n_2, there are three cross bilinear product components $K_{z4,z0}(n,m)$, $K_{z3,z0}(n,m)$, and $K_{z4,z1}(n,m)$. Since the instantaneous energy within this time instants is zero, then components will appear as interference in the time-frequency representation similar to the normal heart sound described in the previous section.

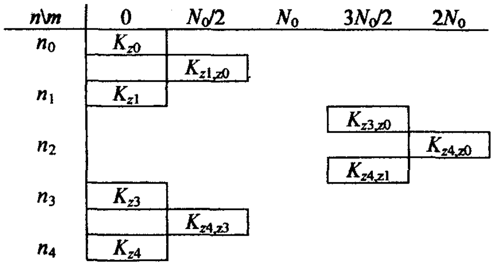

Fig 3 The positions of the auto bilinear product and cross bilinear product of the signal components in the time-lag plane. [Note : K_{z1} is $K_{z1}(n,m)$ and $K_{z4,z3}$ is $K_{z4,z3}(n,m)$

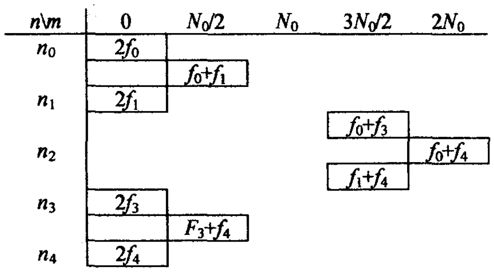

Fig 4 The lag-frequency distribution in the time-lag plane.

From position of the bilinear product components, the components that contributes as interference can be minimized if a window function is defined as

$$g(m)=1 \qquad -N_0/2<m<N_0/2$$
$$=0 \qquad \text{elsewhere} \tag{28}$$

The chosen window function will remove completely all the cross bilinear products that lies at lag greater than $N_0/2$. Thus, the window width derived in this section remove the cross bilinear product components is similar to the one obtained in the previous section.

F CONDITIONS FOR INTERFERENCE IN THE WVD AND WWVD

Based on the examples for the heart sounds, several conclusions can be derived about the auto bilinear and cross bilinear products and how they contribute to the interference present in the time-frequency representation. The results are

1) The auto bilinear products does not contribute to interference and must be must be preserved. The magnitude at lag m=0 is the instantaneous energy of the signal and the lag-frequency is the proportional to the frequency of the true signal.

2) For a given time instant, a cross bilinear product is considered as interference if the lag-frequency is not equal to the lag-frequency of the auto bilinear product. This is not observed in the heart sounds but was demonstrated for communication signals such as frequency shift-keying [6].

3) If the magnitude of the auto bilinear product is zero at a given time instant, all the cross bilinear products that exist are considered as interference terms.

4) The analysis results show that the window width that is equivalent to the pulse duration can remove the interference components and is the compromise between the interference level and smearing in the time-frequency representation.

V RESULTS

The time-frequency representations for the good heart sound and quadratic rhythm based on the WVD and WWVD are shown in Figure (5) to (6). In general, the WWVD produces a more accurate time-frequency representation compared to the WVD because the presence of interference is minimized. More specific comparison is also made in terms of the signal-to-interference ratio as shown in Table (1). Thus, the use of the window function in the WWVD has successfully reduced the interference present in the time-frequency representations.

Signal	WVD		WWVD 25		WWVD 50	
	SIR	SIR dB	SIR	SIR dB	SIR	SIR dB
Good heart sound	0.75	-1.3	374	25.7	138	21.4
Quadratic rhythm	0.32	-5	105	20.2	87.4	19.4

Table 1 Comparison between the various time-frequency representations for the good heart sound and quadratic rhythm based on the signal-to-interference ratio (SIR).

VI CONCLUSIONS

Time-frequency distributions such as the WVD and WWVD are developed as a tool to analyze time-varying signals. The instantaneous characteristics for these signals are represented jointly in the time-frequency plane. Due to the nonlinearity of the operations involved, interference terms are introduced in the time-frequency representation. Analysis performed on the bilinear product of the signal in the time-lag plane show that the desired signal characteristics is present in the auto bilinear product components that occur at zero lag values and parallel to the time axis. Thus, the removal of the cross bilinear product terms is necessary to obtained an accurate time-frequency representation of the signal. This is only possible if the WWVD is used because the window function can be used to control the amount of interference present in the time-frequency representation.

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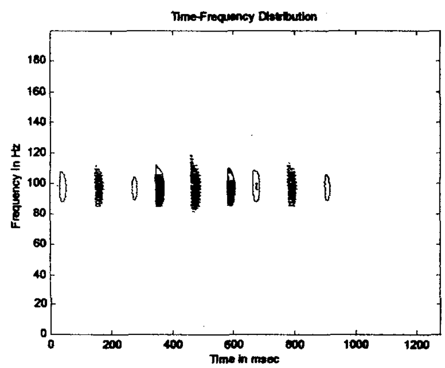
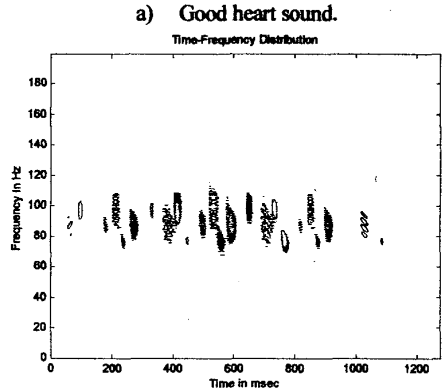
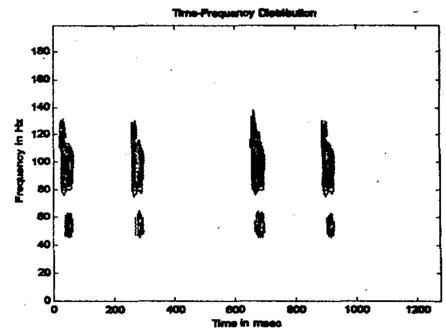


a) Good heart sound.

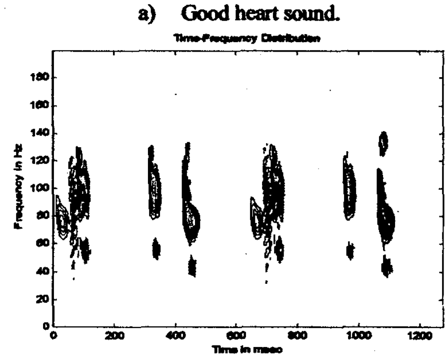


b) Quadratic rhythm.

Fig 5 Time-frequency representation for the good heart sound and quadratic rhythm using the WVD.



a) Good heart sound.



b) Quadratic rhythm.

Fig 6 Time-frequency representation for the good heart sound and quadratic rhythm using the WWVD window width 50 msecs.