A Theoretical Development to Simplify the Analysis of Rectangular Patch Microwave Sensor Built on Multi-layer Substrate

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Abstract - This paper presents a method to simplify the analysis of a square patch microwave sensor built on a multi-layer substrate. The main feature of this method is the simplification of a multi-layer sensor structure into a sensor structure with a single homogenous layer. The equivalent dielectric constant for the new single homogenous layer was determined using the method proposed in this paper. Effectively it eliminates the problem of mode matching between layers and thus reduces the amount of calculation and computing time. The method was compared with the experimental results and was found to give an error less than 4 % in the worse case.

1. Introduction

Microwave sensors can be designed and according to the given specifications can take any form and shapes. In many applications the microwave sensor is in the form of a patch resonator. Some of these resonators were fabricated onto a substrate or onto multi-layer substrates. In order to accurately predict the characteristic of these resonator several method of analyses were employed. In most applications, the analyses require to solve the electromagnetic field in each layer. A considerable number of techniques to analyse the fields have been reported in the published literature [1 –6] and to date there are at least five techniques available with varying degrees of accuracy [6]. In the patch resonator applications, some of the frequently used techniques of analysis include the transmission line theory, modal-expansion cavity model, spectral domain integral equation and finite difference time domain. While rigorous methods are more complete in that they offer accurate radiation models, surface wave effect and the field internal to the patch, they are quite complicated and cannot be readily implemented especially for preliminary design work. It was reported that the accuracy of the rigorous approach may not be much better than that of the simple transmission line or cavity models provided the latter are used within their range of validity [6]. One draw back of these simple models i.e. transmission line and cavity model, is that they are not efficient when applied to a patch resonator built on a multi-layer substrates as shown in figure 1. Each mode in any particular layer has to be solved and matched with an adjacent layer [7 – 8]. This paper presents a theoretical development backed with experimental work in an attempt to simplify the cavity model analysis for microwave sensor built on multi-layer substrate.

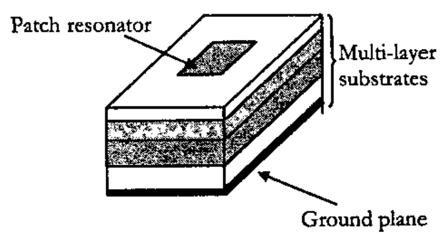

Figure 1 A single square patch microstrip resonator

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2. Equivqlent permittivity analysis

For microwave sensor built as a single patch resonator on a multi-layer structure as shown in figure 1, the cavity under consideration is the space between the patch resonator or the conducting patch and the ground plane. As a result, the usual assumptions of the cavity model, which are applicable for the patch resonator on a single dielectric layer, have to be modified. The modified assumptions are as follows;

1. Due to the close proximity between the conducting patch and the ground plane, only transverse magnetic (TM) modes are assumed to exist. The z component of the electric field is a function of z since the cavity is a multi-layered.
2. The cavity is assumed bounded by perfect electric walls on the top and the bottom and by a perfect magnetic wall along the edge.
3. Across the dielectric-dielectric interfaces, the tangential electric field and the normal electric flux density are continuous.

Based on these assumptions and the resonator as shown in figure 1, the Maxwell's equation in the cavity region take the form

$$\nabla \times \vec{E}_d = -j\omega\mu_{od}\vec{H}_d \quad (1)$$

$$\nabla \times \vec{H}_d = J + j\omega\varepsilon_d\vec{E}_d \quad (2)$$

$$\nabla \cdot \vec{E}_d \equiv 0 \quad (3)$$

$$\nabla \cdot \vec{H}_d \equiv 0 \quad (4)$$

Where subscript, d, indicates the corresponding 'd'th dielectric layer and $\vec{E}$ and $\vec{H}$ are the Electric and Magnetic fields respectively. The current source, J, in equation (2) is assumed to be z independent because of the thinness of the dielectric layers. As a result, the continuity equation gives $\nabla \cdot J = -j\omega\rho \equiv 0$. As a consequence of this, the volume charge density distribution is identically zero in the region, which is the reason for writing $\nabla \cdot \vec{E}_d \equiv 0$ in equation (3). Taking the curl of equation (1) gives,

$$\begin{aligned} \nabla \times \nabla \times \vec{E}_d &= \nabla(\nabla \cdot \vec{E}_d) - \nabla^2 \vec{E}_d \\ &= -j\omega\mu_o \nabla \times \vec{H}_d \\ &\Rightarrow \left(\nabla^2 + k_d^2\right)E_z \equiv j\omega\mu_o J_z \quad (5) \end{aligned}$$

Where $k_d = \omega\sqrt{\mu_o\varepsilon_d}$ the wave number in the dth dielectric layer. Solving the inhomogeneous wave equation (5) is done by firstly determining the eigenfuction φ_{dl}, which satisfy the following homogeneous wave equation

$$\left(\nabla^2 + k_{dl}^2\right)\varphi_{dl} = 0 \quad (6)$$

Where the second letter in the subscript, l, is the mode number. Without lost of generality and to simplify the analysis, let consider a microstrip patch resonator with circular shape constructed on two layers substrate as shown in figure 2.

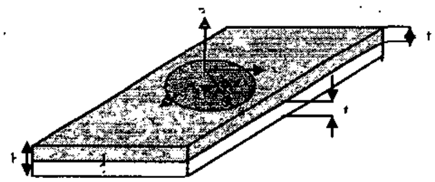


Figure 2. A microstrip patch resonator with circular shape constructed on twolayers substrate.

The field solution to equation (6) at a point (ρ, φ, z) would be as follows,

In dielectric 1;

$$E_{z1} = C_1 J_n(k_1\rho)\cosh\varphi\cos(\beta_1(h-z)) \quad (7)$$

$$E_{\rho 1} = C_1\beta_1 k_1^{-1} J_n'(k_1\rho)\cosh\varphi\sin(\beta_1(h-z)) \quad (8)$$

$$E_{\varphi 1} = -C_1 n\beta_1\rho^{-1}k_1^{-2}J_n(k_1\rho)\sinh\varphi\sin(\beta_1(h-z)) \quad (9)$$

$$H_{\rho 1} = -C_1 j\omega\varepsilon_1 n\rho^{-1}k_1^{-2}J_n(k_1\rho)\sinh\varphi\cos(\beta_1(h-z)) \quad (10)$$

$$H_{\varphi 1} = -C_1 j\omega\varepsilon_1 k_1^{-1}J_n'(k_1\rho)\cosh\varphi\cos(\beta_1(h-z)) \quad (11)$$

where

$$k_1^2 = \omega^2\mu_o\varepsilon_1 - \beta_1^2 \quad (12)$$

In dielectric 2

$$E_{z2} = C_2 J_n(k_2\rho)\cosh\varphi\cos\beta_2 z \quad (13)$$

$$E_{\rho 2} = -C_2\beta_2 k_2^{-1} J_n'(k_2\rho)\cosh\varphi\sin\beta_2 z \quad (14)$$

$$E_{\varphi 2} = C_2 n\beta_2\rho^{-1}k_2^{-2}J_n(k_2\rho)\sinh\varphi\sin\beta_2 z \quad (15)$$

$$H_{\rho 2} = -C_2 j\omega\varepsilon_2 n\rho^{-1}k_2^{-2}J_n(k_2\rho)\sinh\varphi\cos\beta_2 z \quad (16)$$

$$H_{\varphi 2} = -C_2 j\omega\varepsilon_2 k_2^{-1}J_n'(k_2\rho)\cosh\varphi\cos\beta_2 z \quad (17)$$

where

$$k_2^2 = \omega^2\mu_o\varepsilon_2 - \beta_2^2 \quad (18)$$

On applying the magnetic wall condition at $\rho = a$ to equations (8) and (14) gives

$$C_1\beta_1 k_1^{-1}J_n'(k_1 a)\cosh\varphi\sin(\beta_1(h-z)) = 0$$

$$-C_2\beta_2 k_2^{-1}J_n'(k_2 a)\cosh\varphi\sin\beta_2 z = 0$$

$$\Rightarrow k_1 = k_2 = k_{nm} = \frac{x_{nm}}{a} \quad (19)$$

Where x_{nm} is the mth root of J_n' which is the derivative of the Bessel function of the first kind of order n. Applying the condition that the tangential E and normal D must be continuous across $z = t_2$, gives

$$-C_1 n\beta_1\rho^{-1}k_1^{-2}J_n(k_1\rho)\sinh\varphi\sin(\beta_1 t_1)$$

$$= C_2 n\beta_2\rho^{-1}k_2^{-2}J_n(k_2\rho)\sinh\varphi\sin\beta_2 t_2$$

$$\Rightarrow -C_1\beta_1\sinh(\beta_1 t_1) = C_2\beta_2\sin(\beta_2 t_2) \quad (20)$$

$$\varepsilon_1 C_1 J_n(k_1\rho)\cosh\varphi\cos\beta_1 t_1$$

$$= \varepsilon_2 C_2 J_n(k_2\rho)\cosh\varphi\cos\beta_2 t_2$$

$$\Rightarrow \varepsilon_1 C_1\cos\beta_1 t_1 = \varepsilon_2 C_2\cos\beta_2 t_2 \quad (21)$$

Assuming t_1 and t_2 are to be small compared to the wavelength, gives

$$-C_1\beta_1^2 t_1 \cong C_2\beta_2^2 t_2 \quad (22)$$

$$\varepsilon_1 C_1 \cong \varepsilon_2 C_2 \quad (23)$$

$$\Leftrightarrow \frac{C_1}{C_2} \cong -\left(\frac{\beta_2}{\beta_1}\right)^2\frac{t_2}{t_1} \cong \frac{\varepsilon_2}{\varepsilon_1} \quad (24)$$

Using equations (12), (18), (19), (22) and (23) the expression for resonant frequency is given by,

$$-\frac{\varepsilon_2}{\varepsilon_1}\left(\omega_{nm}^2\mu_o\varepsilon_1 - k_{nm}^2\right)t_1 = \left(\omega_{nm}^2\mu_o\varepsilon_2 - k_{nm}^2\right)t_2$$

$$\omega_{nm} = k_{nm}\sqrt{\frac{\varepsilon_2 t_1 + \varepsilon_1 t_2}{\mu_o\varepsilon_1\varepsilon_2(t_1 + t_2)}} \quad (25)$$

Comparing equation (25) with the resonant frequency formula for a single layer substrate, it can be said that the equivalent permittivity of the two-layers substrate is given by

$$\varepsilon_{eq} = \frac{\varepsilon_1\varepsilon_2(t_2 + t_1)}{(\varepsilon_2 t_1 + \varepsilon_1 t_2)} \quad (26)$$

$$\varepsilon_{eq} = \left(\frac{t_1}{\varepsilon_1} + \frac{t_2}{\varepsilon_2}\right)^{-1}\cdot(t_1 + t_2) \quad (27)$$

For multi-layer substrates, the same technique as above can be applied by considering two layers at each time. In this case equation (27) becomes:

$$\varepsilon_{eq} = \left(\sum_{n=1}^{N}\frac{t_n}{\varepsilon_n}\right)^{-1}\cdot\sum_{n=1}^{N}t_n \quad (28)$$

where t_n = height of the nth substrate layer

ε_n = permittivity of the nth substrate layer.

3. The proposed simplified multi-layer model

Extending the idea of equivalent permittivity to the multi layer substrate and calculating it using equation (28), the author proposed to simplify the resonator structure as shown in figure 1 into a resonator constructed on a single equivalent homogenous substrate as shown in figure 3 where the cavity model is best applied.

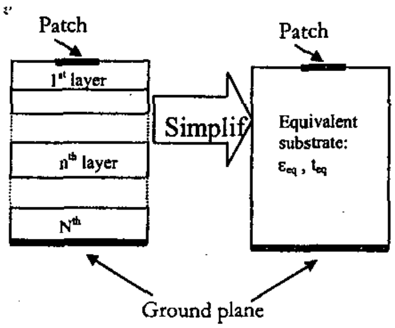


Figure 3. A single patch microstrip resonator constructed on multi-layer substrate is simplified into a resonator on single layer homogenous substrate.

The equivalent substrate is considered lossless and has a permittivity given (28). from the same equation (28) the equivalent dielectric constant is given by

$$\varepsilon_{req} = \left(\sum_{n=1}^{N} \frac{t_n}{\varepsilon_{rn}}\right)^{-1} \cdot \left(\sum_{n=1}^{N} t_n\right) \qquad (29)$$

The height of the equivalent substrate is taken to equal to the total height of the substrates and is given by

$$t_{eq} = \sum_{n=1}^{N} t_n \qquad (30)$$

The rest of the dimensions such as the width and the length of the patch were unchanged.

4. Experimental work

Rectangular patch resonators were constructed on several multi-layer substrates to test the proposed method. In all resonator structures, the total substrate heights were several time smaller than the operating wavelength in order to satisfy the assumptions of the proposed method. For t_{eq} much less than the operating wavelength, the cavity model solution for the z-component electric field underneath the square patch is given by [9]

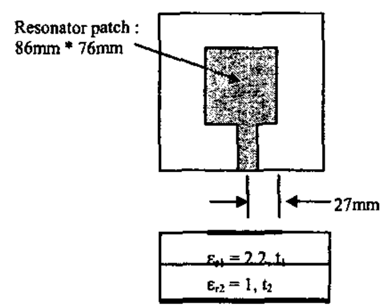
$$E_z(x,y) = jI_0\sqrt{\frac{\mu_0}{\varepsilon_{eq}}}\,k \sum_{m=0}\sum_{n=0} \frac{\psi_{mn}(x,y)\psi_{mn}(x_0,y_0)}{k^2-k_{mn}^2} G_{mn} \qquad (31)$$

For simplicity in fabricating the coupling structure of the resonators, direct conductor connections were chosen in this experimental work. This simply means that either direct microstrip line or direct probe connection to the patch.

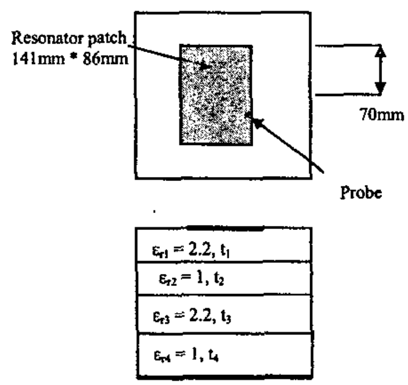
From equation (31) the input impedance is given by

$$Z_{in} = \frac{V_f}{I_0} = -j\omega\mu_0 h \sum_{m}^{\infty}\sum_{n}^{\infty} \frac{\psi_{mn}^2(x_0,y_0)}{k^{\cdot} - k_{mn}^2} G_{mn}$$

Several resonators were fabricated to test the simplified model and figure 4(a) and 4(b) shows two of them.



4(a)



4(b)

Figure 4 Microstrip patch resonators used in this investigation

5. Experimental and Calculated Results

A network analyser was employed to measure the return loss versus frequency characteristic of these resonators. Table 1 and table 2 summarised the result obtained either from calculation or experiment. These comparison results show a close agreement between calculated and measured data. The worst case shows a discrepancy of less than 4% for mode resonant frequencies TM_{10}, TM_{01}, TM_{02} TM_{20} and TM_{11}.

6. *Conclusion*

It can be concluded that using the simplified multi-layer model the calculation of equivalent permittivity is easily done and this value can be used to analyse a simple resonator structure on multi-layer substrate. The problem of solving the modes in each layer of substrate is simplified to solving just the modes in a single layer. The problem with mode matching is eliminated completely which in turn reduces the amount of calculation and computing time. Using one substrate of high value dielectric constant, ε_{rh} say, this investigation shows that other substrates having dielectric constants less than ε_{rh} can be simulated by combining it with an appropriate layer of air dielectric having height calculated using equation (29).

Table 1 A Summary of the results obtained for resonator 4(a)

	Mode frequencies (GHz)					
	Calculated values			Measured values		
	TM_{10}	TM_{01}	TM_{11}	TM_{10}	TM_{01}	TM_{11}
Resonator 4(a)						
1) t_2 = 1.56 mm	1.518	1.350	2.032	1.509	1.359	2.033
2) t_2 = 2.90 mm	1.532	1.374	2.057	1.533	1.334	2,083

For resonator 4(a) t_1 is equal to 1.59 mm

For resonator 4(b) t_1 and t_4 are equal to 1.59 mm and 0.25 mm repectively

Table 2 Summary of the results obtained for resonator 4(b)

	Mode frequencies (GHz)					
	Calculated values			Measured values		
	TM_{10}	TM_{01}	TM_{11}	TM_{10}	TM_{01}	TM_{11}
Resonator 4(b) t_2 = 0.02 mm & t_3 = 0.635 mm	1.347	1.083	2.166	1.326	1.094	2.125
Resonator 4(b) t_2 = 0.04 mm & t_3 = 1.27 mm	1.244	0.995	1.991	1.236	1.012	1.961

7. References

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