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DECENTRALIZED PROPORTIONAL-INTEGRAL SLIDING MODE TRACKING CONTROL FOR A CLASS OF NONLINEAR INTERCONNECTED UNCERTAIN SYSTEM

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Abstract. A decentralized sliding mode controller for a class of nonlinear interconnected uncertain systems is presented in this paper. It is assumed that the plant to be controlled is represented by interconnected sub-systems and the local dynamics of each sub-system is represented by its nominal and bounded parametric uncertainties. It is also assumed that the interconnection dynamics is also represented in the same manner and it is further assumed that the matching conditions hold for every sub-system. A robust decentralized sliding mode controller is derived such that for each sub-system, the actual trajectory tracks the desired trajectory using only the local states information. The Proportional-Integral sliding mode is chosen to ensure the stability of the overall dynamics during the entire period i.e. the reaching phase and the sliding phase. Mathematical proof of the proposed controller is presented and the results are verified using a case study.

Key words: Large scale system, Decentralized sliding mode control, Tracking controller, Matched uncertainties

Abstrak. Kertas kerja ini membincangkan pengawal ragam gelangsar ternyahpusat untuk sistem dalam kategori takpasti, tak lelurus tersaling hubung. Andaian yang dipakai dalam kertas kerja ini ialah loji yang hendak dikawal dianggap sebagai suatu sistem yang diwakili oleh subsistem-subsistem yang tersaling hubung di antara satu sama lain dan dinamik setempat untuk setiap subsistem pula diwakili oleh nilai nominalnya di samping ketakpastian berparameter terbatas. Di samping itu, dinamik untuk saling hubungan di antara subsistem juga diandaikan sebagai diwakili dengan cara yang serupa dan syarat padanan (*matching conditions*) benar untuk semua subsistem. Suatu pengawal ragam gelangsar ternyahpusat yang *robust* telah berjaya dihasilkan supaya untuk setiap subsistem, trajektori sebenar sistem akan menjejak trajektori yang dikehendaki menggunakan hanya mklumat dari pemboleh ubah keadaan setempat. Ragam gelangsar berkadaran-kamiran (Pi) sengaja dipilih untuk memastikan kestabilan dinamik keseluruhan sistem terjamin (meliputi fasa menjangkau dan fasa menggelangsar). Pembuktian secara matematik pengawal yang dicadangkan turut di ketengahkan dalam kertas kerja ini dan keputusannya pula diperiksa benar tidaknya melalui satu kajian kes.

Kata kunci: Sistem skala besar, Kawalan ragam gelangsar ternyahpusat, Pengawal menjejak, Ketakpastian terpadan

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1.0 INTRODUCTION

The majority of large-scale uncertain systems consist of several independent dynamic sub-systems interconnected together. Naturally, there exist interactions between each sub-system. Furthermore, input couplings may exist in each sub-system. There are numerous real-world systems that belong to this category such as power systems, communication systems, robotics, social systems and economic systems. The control of such a large-scale system are very complicated due to the high dimensionality and complexity of the systems equations, interactions between the sub-systems, coupling of the inputs and uncertainties resulted from the modeling errors and disturbances. The decentralized control is an effective way of controlling such a systems since each sub-system can be independently controlled. Due to its simplicity, much effort has been concentrated on developing such a controller in the various fields [1-3].

The broad explanations on decentralized control of large-scale uncertain system can be found in [4-9] and their bibliographies. Generally, there are two classes of decentralized control, namely, the *local approach* and the *global approach*. The local approach is based on the state information of each sub-system only while the global approach utilizes the extra feedback information from the states of neighboring sub-systems beside the information from the local sub-system. The research work presented in this paper falls into the former category. This is due to the argument that the local approach reflects a true decentralized control as compared to the global one.

A Variable Structure Control (VSC) system is a system whose structure is intentionally altered to achieve the desired performance. Many attempts have been made by researchers in harnessing the robust property of the variable structure control strategy in controlling large-scale uncertain systems. In [10], the multilevel control concepts based on variable structure theory is used to decompose a large control problem into a two-level algorithm such that each sub-system is stabilized with local discontinuous controllers and higher level corrective control is designed to take into account the effect of interactions among the sub-systems. The application of decentralized nonlinear variable structure control on AC-DC power system has also been explored [11]. The global stability is proved by using variable structure theory. Based on the characteristics of the system and the sufficient condition for the existence of the equivalent control, the decentralized nonlinear sliding surface is chosen so that the reduced order dynamics exhibits a linear behavior in a sliding mode. The results showed that the variable structure approach is effective and the controller posses high property of robustness with respect to the perturbation in system parameters. In robotics, an algorithm for trajectory control of robot manipulators by decentralized feedback is developed in a model following framework [12]. Here, a variable structure control law is used with or without force measurement and the controller proves to be robust with respect to parametric uncertainties and robot's payload variations.

One of the fascinating phenomena of the variable structure control is the discontinuous nature of the control action whose primary function is to switch between two distinctively different system structures such that a new type of system motion called *sliding mode* exists in the manifold. A sliding mode exists, if in the vicinity of the switching surface, the velocity vectors of the state trajectory always point toward the switching surface [13-14]. When a system is in the sliding mode, its dynamics is strictly determined by the dynamics of the sliding surfaces and hence insensitive to parameter variations and system disturbances provided certain matching conditions are satisfied [15]. This condition states that the disturbances and uncertainties present in the system must be in the range space of the systems input matrix [16]. Throughout this research, it is assumed that the matching condition is always satisfied. It should be noted however that prior to the sliding mode i.e. during the *reaching phase*, the system is not secluded against the influences of the parameter variations and system disturbances.

The main contributions of this work can be divided into two parts. First, it is shown that if the matching condition is fulfilled, then the error dynamics of the large-scale systems with bounded uncertainties, interaction between sub-systems and, input couplings is stable during the sliding mode. In fact its closed-loop dynamics can easily be shaped-up using any of the well-known pole placement method. Secondly, a decentralized Proportional-Integral sliding mode control is proposed so that the hitting condition is met. The bounds of interconnections as well as the input couplings between the sub-systems are explicitly taken into account while the neighboring states are not required.

2.0 PROBLEM FORMULATION

Consider an uncertain composite system S defined by an N interconnected sub-systems. Each sub-system can be described by

$$S_i : \dot{x}_i(t) = [A_i + \Delta A_i(t)]x(t) + B_i u_i(t) + \sum_{\substack{j=1 \\ j \neq i}}^{N} [A_{ij} + \Delta A_{ij}(t)]x_j(t) + \sum_{\substack{j=1 \\ j \neq i}}^{N} B_{ij} u_j(t) \qquad (1)$$

where $x_i(t) \in R^{n_i}$, $u_i(t) \in R^{m_i}$ respectively, represent the state and input of sub-system S_i, A_i, B_i, A_{ij} and B_{ij} are constant nominal matrices of appropriate dimensions and i=1,2,...N. $\Delta A_i(t)$ and $\Delta A_{ij}(t)$ are the matrices representing uncertainties present in the system and the interconnection, respectively. The system considered in equation (1) is different from those considered by other researcher such as in [19] in the sense that the input coupling term is not zero.

The state vector of the composite system S is defined as

$$X(t) = \left[x_1^T(t), x_2^T(t), \ldots, x_n^T(t) \right]^T \qquad (2)$$

Let a continuous function be the desired state trajectory, where $X_d(t)$ is defined as:

$$X_d(t) = \left[x_{d1}^T(t), x_{d2}^T(t), \ldots, x_{dn}^T(t) \right]^T \tag{3}$$

$$x_{di}(t) \in R^{n_i}$$

Define the tracking error, $z_i(t)$ as

$$z_i(t) = x_i(t) - x_{di}(t) \tag{4}$$

In designing the controller, the following assumptions are made:

i) The uncertainty matrices $\Delta A_i(t)$ and $\Delta A_{ij}(t)$ are bounded by β_{ii} (> 0) and β_{ij} (> 0), respectively, i.e.,

$$\left\| \Delta A_i(t) \right\| \leq \beta_{ii}, \ \left\| \Delta A_{ij}(t) \right\| \leq \beta_{ij} \tag{5}$$

ii) $\Delta A_i(t)$, A_{ij}, $\Delta A_{ij}(t)$, and B_{ij} belong to the class of matrices which satisfy the perfect matching condition [16]:

$$\text{rank } [\, B_i \mid \Delta A_i \,] = \text{rank } [\, B_i \mid A_{ij} \,] = \text{rank } [\, B_i \mid \Delta A_{ij} \,] = \text{rank } [\, B_i \mid B_{ij}(t) \,]$$

$$= \text{rank } [\, B_i \,] \tag{6}$$

iii) There exist a Lebesgue function $\Omega_i(t) \in R$, which is integrable on bounded interval such that

$$\dot{x}_{di}(t) = A_i x_{di}(t) + B_i \Omega_i(t) \tag{7}$$

where A_i and B_i are the i-th subsystem nominal system and input matrices, respectively. Since A_i, B_i and $x_{di}(t)$ are known in advance, $\Omega_i(t)$ can easily be computed.

iv) The pair (A_i, B_i) is controllable.

Using equation (4) and equation (1), the error dynamics can be written as

$$\begin{aligned} \dot{z}_i(t) = {} & [A_i + A_i(t)]z_i(t) + [A_i + A_i(t)]x_{di}(t) - \dot{x}_{di}(t) \\ & + \sum_{\substack{j=1 \\ j \neq i}}^{N} [A_{ij} + \Delta A_{ij}(t)]x_j(t) + B_i u_i(t) + \sum_{\substack{j=1 \\ j \neq i}}^{N} B_{ij} u_j(t) \end{aligned} \tag{8}$$

Substituting equation (7) into equation (8) gives

$$\dot{z}_i(t) = [A_i + \Delta A_i(t)]z_i(t) + \Delta A_i(t)x_{di}(t) - B_i\Omega_i(t) + \sum_{\substack{j=1 \\ j\neq i}}^{N}[A_{ij} + \Delta A_{ij}(t)]x_j(t) + B_iu_i(t) + \sum_{\substack{j=1 \\ j\neq i}}^{N} B_{ij}u_j(t) \tag{9}$$

Define the local Proportional-Integral (PI) sliding surface for S_i as

$$\sigma_i(t) = C_iz_i(t) - \int_0^t [C_iA_i + C_iB_iK_i]z_i(\tau)d\tau \tag{10}$$

where $C_i \in R^{m_i \times n_i}$ and $K_i \in R^{m_i \times n_i}$ are constant matrices. The matrix K_i satisfies

$$\lambda_{\max}(A_i + B_iK_i) < 0 \tag{11}$$

and C_i is chosen such that C_iB_i is nonsingular. For this class of system, the sliding manifold can be described as

$$\sigma(t) = [\sigma^T{}_1, \sigma^T{}_2, ..., \sigma^T{}_N]^T \tag{12}$$

The control problem is to design a decentralized controller for each sub-system using the Proportional-Integral sliding mode given by equation (10) such that the system state trajectory $X_i(t)$ tracks the desired state trajectory $X_{di}(t)$ as closely as possible for all t in spite of the uncertainties and non-linearities present in the system. The whole task can be divided into two parts; firstly it must be assured that the system error dynamics is asymptotically stable (approaching zero) during the sliding mode, and secondly a decentralized controller must be designed in such a way that whatever error the system has during the initial stage, the system is directed towards the sliding surface (without sacrificing the stability aspect of the controlled system) during the reaching phase.

3.0 SYSTEM DYNAMICS DURING SLIDING MODE

Differentiating equation (10) gives

$$\dot{\sigma}_i(t) = C_i\Delta A_i(t)z_i(t) + C_i\Delta A_i(t)x_{di}(t) - C_iB_i\Omega_i(t) + \sum_{\substack{j=1 \\ j\neq i}}^{N} C_i[A_{ij} + \Delta A_{ij}(t)]x_j(t) + C_iB_iu_i(t) + \sum_{\substack{j=1 \\ j\neq i}}^{N} C_iB_{ij}u_j(t) - C_iB_iK_iz_i(t) \tag{13}$$

Equating equation (13) to zero gives the equivalent control, $u_{eqi}(t)$:

$$u_{eqi}(t) = K_i z_i(t) + \Omega_i(t) - (C_i B_i)^{-1}\{C_i \Delta A_i(t) z_i(t) + C_i \Delta A_i(t) x_{di}(t) + \sum_{\substack{j=1 \\ j\neq i}}^{N} C_i [A_{ij} + \Delta A_{ij}(t)] x_j(t) + \sum_{\substack{j=1 \\ j\neq i}} C_i B_{ij} u_j(t)\} \quad (14)$$

The system dynamics during sliding mode can be found by substituting the equivalent control (14) into the system error dynamics (9):

$$\dot{z}_i(t) = [A_i + B_i K_i] z_i(t) + [I_{n_i} - B_i (C_i B_i)^{-1} C_i]\{\Delta A_i z_i(t) + \Delta A_i x_{di}(t) + \sum_{\substack{j=1 \\ j\neq i}}^{N} [A_{ij} + \Delta A_{ij}(t)] x_j(t) + \sum_{\substack{j=1 \\ j\neq i}}^{N} B_{ij} u_j(t)\} \quad (15)$$

Define

$$P_{si} \triangleq [I_{n_i} - B_i (C_i B_i)^{-1} C_i] \quad (16)$$

P_{si} is a *projection operator* and satisfies the following two equations [17]:

$$C_i P_{si} = 0 \quad \text{and} \quad P_{si} B_i = 0 \quad (17)$$

In view of assumption (ii), then it follows that by the projection property, equation (15) can be reduced as

$$\dot{z}_i(t) = [A_i + B_i K_i] z_i(t) \quad (18)$$

The conditions established by assumption (ii) merely state that for the system given by equation (15) to be invariant with respect to $z_i(t)$, $x_{di}(t)$, $x_j(t)$ and $u_j(t)$ in sliding mode, the columns of matrix $\Delta A_i(t)$, A_{ij}, $\Delta A_{ij}(t)$ and B_{ij} should belong to the range space of B_i [18].

Hence if the matching condition is satisfied, the system error dynamics during sliding mode are independent of the interconnection between the subsystems and couplings between the inputs, and, insensitive to the parameter variations. Equation (18) suggests that the closed-loop error dynamics can be easily shaped up by assigning appropriate gain K_i such that the system of equation (18) is stable.

4.0 DECENTRALIZED SLIDING MODE CONTROLLER DESIGN

The composite manifold (12) is asymptotically stable in the large, if the following hitting condition is held [19]:

$$\sum_{i=1}^{N} \frac{\sigma_i^T(t)}{\|\sigma_i(t)\|} \dot{\sigma}_i(t) < 0 \tag{19}$$

As a proof, let the positive definite Lyapunov function be

$$V(t) = \sum_{i=1}^{N} \|\sigma_i(t)\| \tag{20}$$

Differentiating equation (20) with respect to time, t yields

$$\begin{aligned} \dot{V}(t) &= \sum_{i=1}^{N} \tfrac{1}{2}[\sigma_i^T(t)\sigma_i(t)]\{\sigma_i^T(t)\dot{\sigma}_i(t) + \dot{\sigma}_i^T(t)\sigma_i(t)\} \\ &= \sum_{i=1}^{N} \frac{\sigma_i^T(t)}{\|\sigma_i(t)\|} \dot{\sigma}_i(t) \end{aligned} \tag{21}$$

Following the Lyapunov stability theory, if equation (19) holds, then the sliding manifold $\sigma(t)$ is asymptotically stable in the large.

Theorem: The global hitting condition (19) of the composite manifold (12) is satisfied if every local control $u_i(t)$ of system (9) is given by:

$$\begin{aligned} u_i(t) = -(C_iB_i)^{-1}&[\gamma_{i1}\|z_i(t)\| + \gamma_{i2}\|x_i(t)\| + \gamma_{i3}\|x_{di}(t)\| + \gamma_{i4}\|\Omega_i(t)\|] \\ &SGN(\sigma_i(t)) + \Omega_i(t) \end{aligned} \tag{22}$$

where

$$\gamma_{i1} > \frac{\beta_{ii}\|C_i\| + \|C_iB_iK_i\|}{1 + \sum_{\substack{j=1 \\ j\neq i}}^{N} C_jB_{ji}(C_iB_i)^{-1}} \tag{23}$$

$$\gamma_{i2} > \frac{\sum_{\substack{j=1 \\ j\neq i}}^{N} [\|C_jA_{ji}\| + \beta_{ji}\|C_j\|]}{1 + \sum_{\substack{j=1 \\ j\neq i}}^{N} C_jB_{ji}(C_iB_i)^{-1}} \tag{24}$$

$$\gamma_{i3} > \frac{\beta_{ii}\|C_i\|}{1+\sum_{\substack{j=1\\ j\neq i}}^{N} C_j B_{ji}(C_i B_i)^{-1}} \tag{25}$$

$$\gamma_{i4} > \frac{\|C_j B_{ji}\|}{1+\sum_{\substack{j=1\\ j\neq i}}^{N} C_j B_{ji}(C_i B_i)^{-1}} \tag{26}$$

Proof:

See the Appendix.

It should be noted that during the reaching phase, the decentralized control given by equation (22) guarantees that not only the reaching condition is met, it also assures that based on the Lyapunov theory, the system trajectory during this reaching phase is stable in the large. The block diagram of the proposed controller is as shown in Figure 1.

5.0 AN ILLUSTRATIVE EXAMPLE

The nonlinear interconnected plant to be controlled is of the form

$$S_1 : \quad \frac{d}{dt}\begin{bmatrix}\theta_1(t)\\ \dot{\theta}_1(t)\end{bmatrix} = \begin{bmatrix}0 & 1\\ 2\cos\theta_1(t) & \dot{\theta}_1(t)\end{bmatrix}\begin{bmatrix}\theta_1(t)\\ \dot{\theta}_1(t)\end{bmatrix} + \begin{bmatrix}0\\ 3\end{bmatrix}u_1(t)$$
$$+\begin{bmatrix}0 & 0\\ \sin\theta_1(t) & \sin\theta_1(t)\sin\theta_2(t)\end{bmatrix}\begin{bmatrix}\theta_2(t)\\ \dot{\theta}_2(t)\end{bmatrix} + \begin{bmatrix}0\\ 2\end{bmatrix}u_2(t)$$

$$S_2 : \quad \frac{d}{dt}\begin{bmatrix}\theta_2(t)\\ \dot{\theta}_2(t)\end{bmatrix} = \begin{bmatrix}0 & 0\\ \cos\theta_1(t)\sin\theta_2(t) & \sin\theta_2(t)\end{bmatrix}\begin{bmatrix}\theta_1(t)\\ \dot{\theta}_1(t)\end{bmatrix} + \begin{bmatrix}0\\ 2\end{bmatrix}u_1(t)$$
$$+\begin{bmatrix}0 & 1\\ \dot{\theta}_2(t) & \cos\theta_2(t)\end{bmatrix}\begin{bmatrix}\theta_2(t)\\ \dot{\theta}_2(t)\end{bmatrix} + \begin{bmatrix}0\\ 4\end{bmatrix}u_2(t)$$

This downscaled interconnected large scales system is basically nonlinear and the input of each sub-system is accompanied with couplings from other sub-system input. Define the state vector as

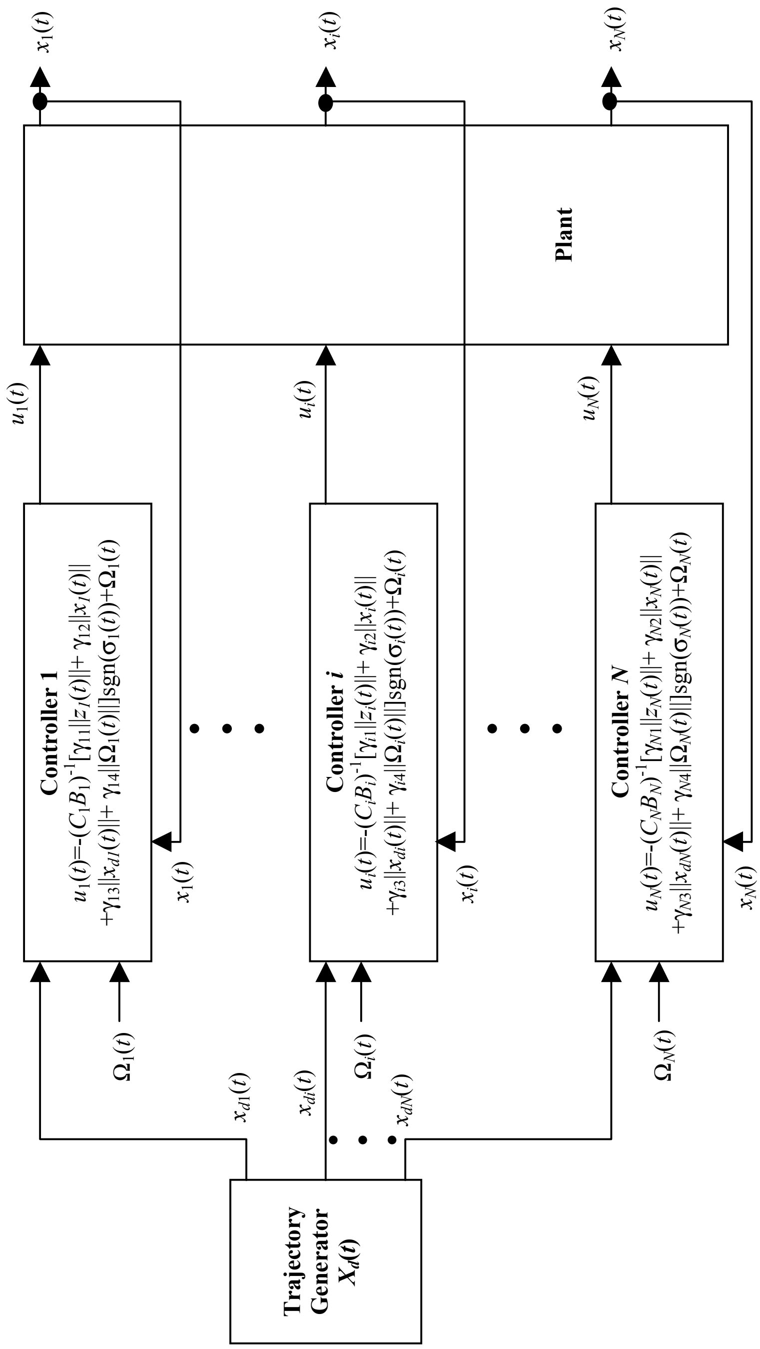

Figure 1 Block Diagram of the Proposed Decentralized Proportional-Integral Sliding Mode Controller

$$[x_1(t) \mid x_2(t)]^T \triangleq [x_{11}(t) \quad x_{12}(t) \mid x_{21}(t) \quad x_{22}(t)]^T = \left[\theta_1(t) \quad \dot{\theta}_1(t) \mid \theta_2(t) \quad \dot{\theta}_2(t)\right]^T$$

and assume that the bounds $\theta_i(t)$ and $\dot{\theta}_i(t)$ are given as follows:

$$-270^\circ \le \theta_1 \le 270^\circ, \quad 0^\circ s^{-1} \le \dot{\theta}_1 \le 50^\circ s^{-1}, \quad -25^\circ \le \theta_2 \le 75^\circ, \quad 0^\circ s^{-1} \le \dot{\theta}_2 \le 30^\circ s^{-1}$$

With these bounds, the given plant can be written as an interconnected uncertain system as in equation (1) with the nominal matrices calculated as follows:

$$A_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0.4363 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1 \\ 0.2168 & 0.6294 \end{bmatrix}$$

$$A_{12} = \begin{bmatrix} 0 & 0 \\ 0 & 0.3706 \end{bmatrix}, \quad A_{21} = \begin{bmatrix} 0 & 0 \\ 0.6943 & 0.2717 \end{bmatrix}$$

The bound on the system matrices A_i and interconnection matrices A_{ij} can be obtained as follows:

$$\beta_{11} = 2.0470, \ \beta_{12} = 1.1816, \ \beta_{22} = 0.4538, \ \beta_{21} = 0.7455$$

It is assumed that each sub-system is required to track a pre-specified cycloidal function of the form:

$$\theta_{di}(t) = \begin{cases} \theta_i(0) + \dfrac{\Delta_i}{2\pi}\left[\dfrac{2\pi t}{\tau} - \sin\left(\dfrac{2\pi t}{\tau}\right)\right], & 0 \le t \le \tau \\ \theta_i(\tau), & \tau \le t \end{cases}$$

where $\Delta_i = \theta_i(\tau) - \theta_i(0)$, $i = 1,2$. In this example, the final time, τ is set at 10 s.

In this study, the gains are chosen as follows:

K_1 = [6.667 3.1454] such that $\lambda(A_1 + B_1K_1) = \{-4,-5\}$;
K_2 = [1.5655 1.4074] such that $\lambda(A_2 + B_2K_2) = \{-2,-3\}$;
C_1 = [2 1] and C_2 = [3 1].

These gains can be chosen arbitrarily but in this work the above-mentioned values of K_i's are intentionally selected to represent the case of over-damped response while the values of C_i's here are selected such that the matrix C_iB_i is nonsingular.

Therefore, from equation (23)–(26):

$$\lambda_{11} > 13.5583; \ \lambda_{12} > 1.8618; \ \lambda_{13} > 0.4833; \ \lambda_{14} > 2;$$
$$\lambda_{21} > 6.5704; \ \lambda_{22} > 2.0085; \ \lambda_{23} > 1.4351; \ \lambda_{24} > 2.$$

For simulation purposes, two sets of controller parameters are chosen:

Case 1: $\lambda_{11} = 10$; $\lambda_{12} = 1$; $\lambda_{13} = 0.1$; $\lambda_{14} = 1$; $\lambda_{21} = 6$; $\lambda_{22} = 1.5$; $\lambda_{23} = 1$; $\lambda_{24} = 1$

Case 2: $\lambda_{11} = 14$; $\lambda_{12} = 2$; $\lambda_{13} = 1$; $\lambda_{14} = 3$; $\lambda_{21} = 7$; $\lambda_{22} = 3$; $\lambda_{23} = 2$; $\lambda_{24} = 3$

In case 1, the controller parameter is selected to study the performance of the system if the gain conditions of equations (23)–(26) are not met; while in case 2 the controller parameters is selected to represent a situation where the conditions imposed on the controller are met. The simulation study was extensively conducted using MATLAB and SIMULINK as a platform.

The simulation results for case 1 are shown in Figure 2a-h. Figure 2a and Figure 2e illustrate the position tracking error for sub-system 1 and sub-system 2, respectively. From the plots, it can be observed that although sub-system 2 tracking error is considerably good, the tracking performance for sub-system 2 is far beyond satisfactory. This observation can be made easier by comparing the tracking errors of each subsystem as indicated in Figure 2b and Figure 2f. The sliding phenomenon for subsystem 1 and sub-system 2 is shown in Figure 2c and Figure 2g, respectively. While the dynamics of sub-system 1 fails to slide, it can be clearly seen that the dynamics of sub-system 2 is eventually slides and hence insensitive to the uncertainties present in the system. The control input generated for both sub-systems are shown in Figure 2d and Figure 2h.

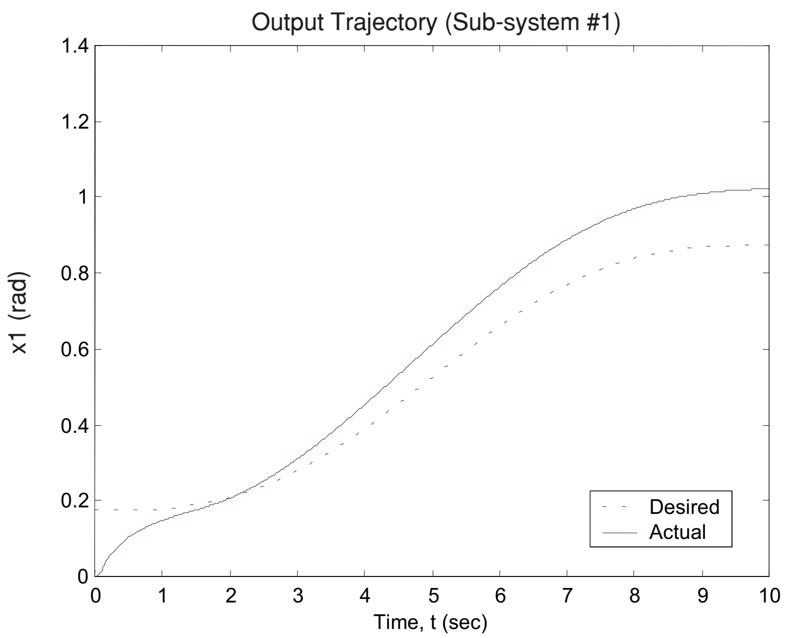

Figure 2(a) The Output and Desired Trajectories of Subsystem 1 for γ_{11}=10, γ_{12}=1, γ_{13}=0.1, γ_{14}=1, γ_{21}=6, γ_{22}=1.5, γ_{23}=1, and γ_{24}=1

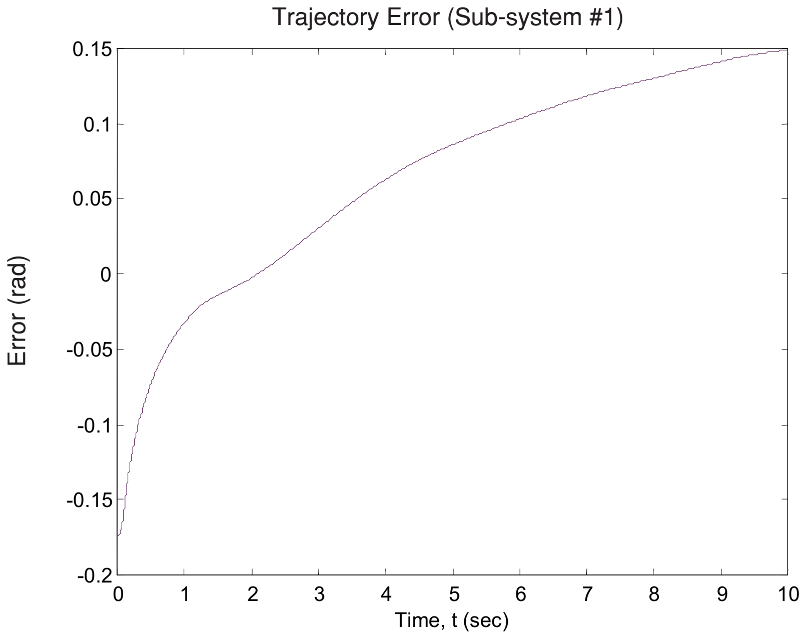

Figure 2(b) The Tracking Error of Subsystem 1 for γ_{11}=10, γ_{12}=1, γ_{13}=0.1, γ_{14}=1, γ_{21}=6, γ_{22}=1.5, γ_{23}=1, and γ_{24}=1

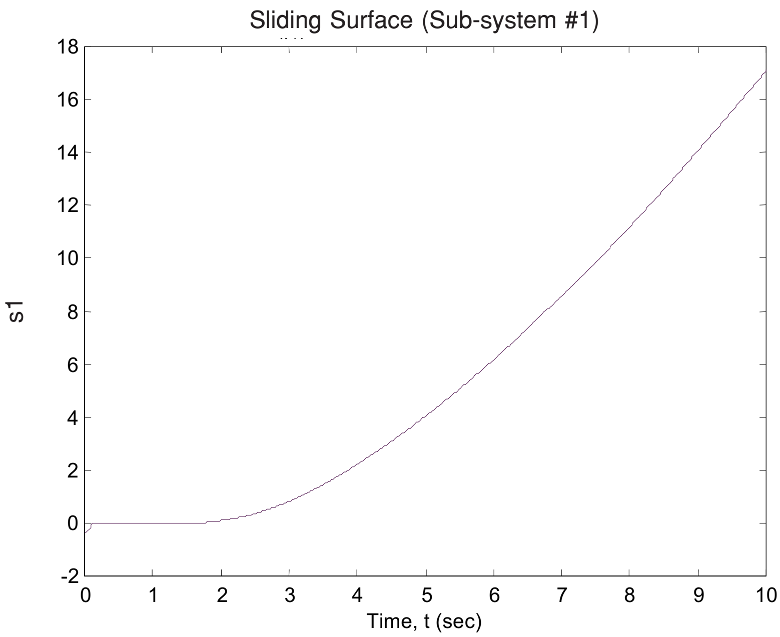

Figure 2(c) The Sliding Surface of Subsystem 1 for γ_{11}=10, γ_{12}=1, γ_{13}=0.1, γ_{14}=1, γ_{21}=6, γ_{22}=1.5, γ_{23}=1, and γ_{24}=1

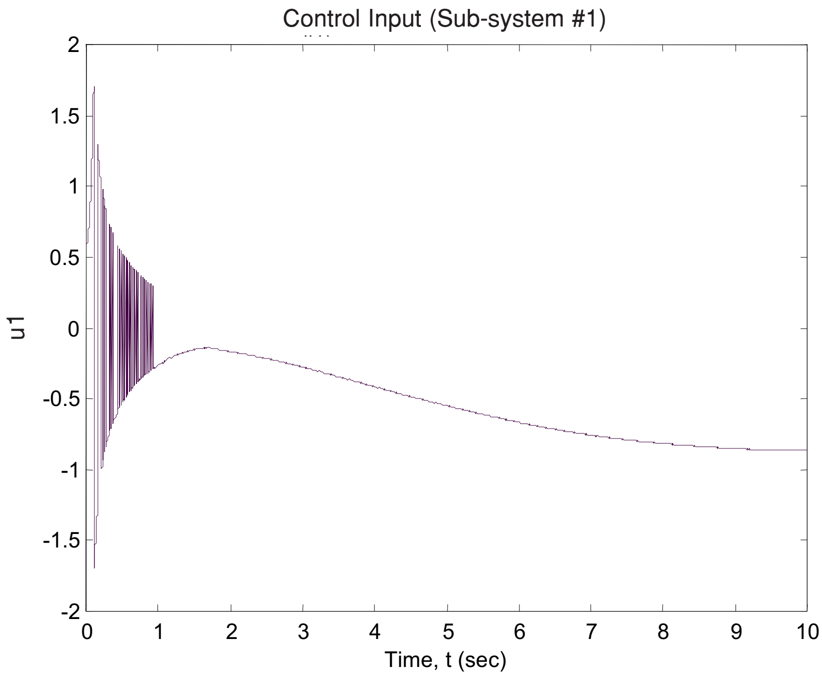

Figure 2(d) The Control Input of Subsystem 1 for γ_{11}=10, γ_{12}=1, γ_{13}=0.1, γ_{14}=1, γ_{21}=6, γ_{22}=1.5, γ_{23}=1, and γ_{24}=1

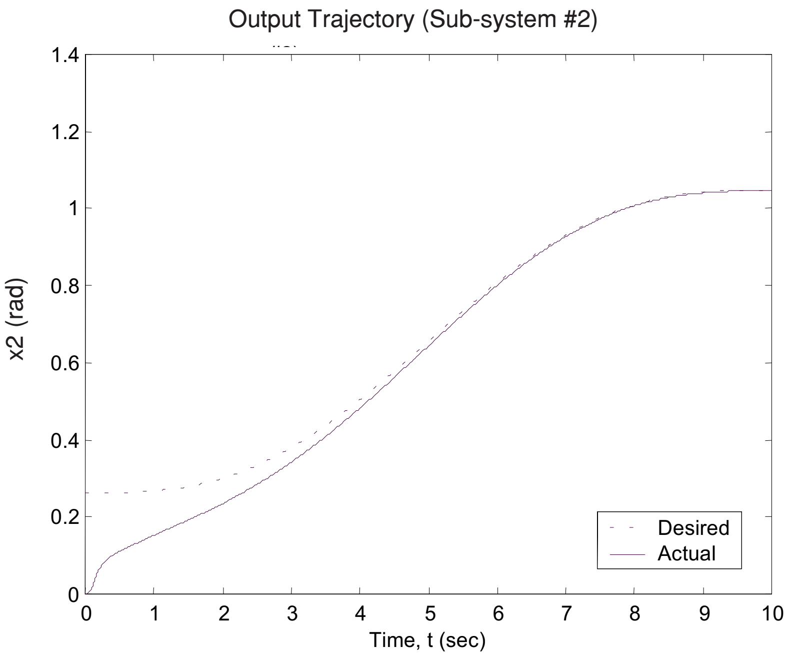

Figure 2(e) The Output and Desired Trajectories of Subsystem 2 for γ_{11}=10, γ_{12}=1, γ_{13}=0.1, γ_{14}=1, γ_{21}=6, γ_{22}=1.5, γ_{23}=1, and γ_{24}=1

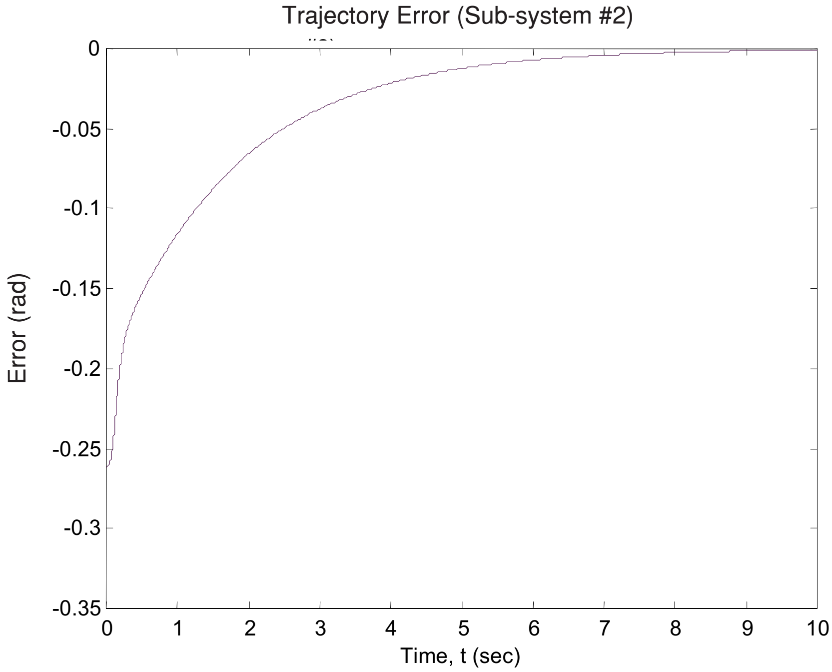

Figure 2(f) The Tracking Error of Subsystem 2 for γ_{11}=10, γ_{12}=1, γ_{13}=0.1, γ_{14}=1, γ_{21}=6, γ_{22}=1.5, γ_{23}=1, and γ_{24}=1

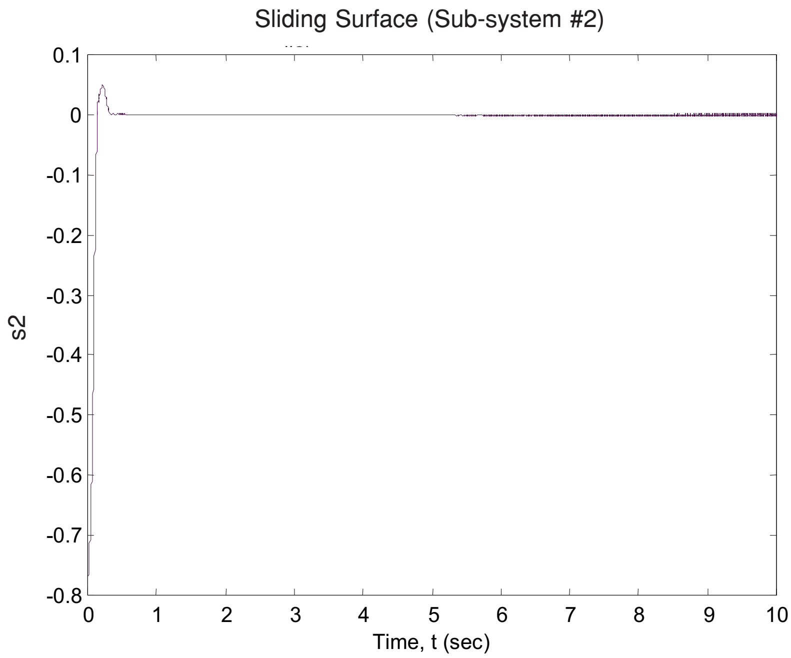

Figure 2(g) The Sliding Surface of Subsystem 2 for γ_{11}=10, γ_{12}=1, γ_{13}=0.1, γ_{14}=1, γ_{21}=6, γ_{22}=1.5, γ_{23}=1, and γ_{24}=1

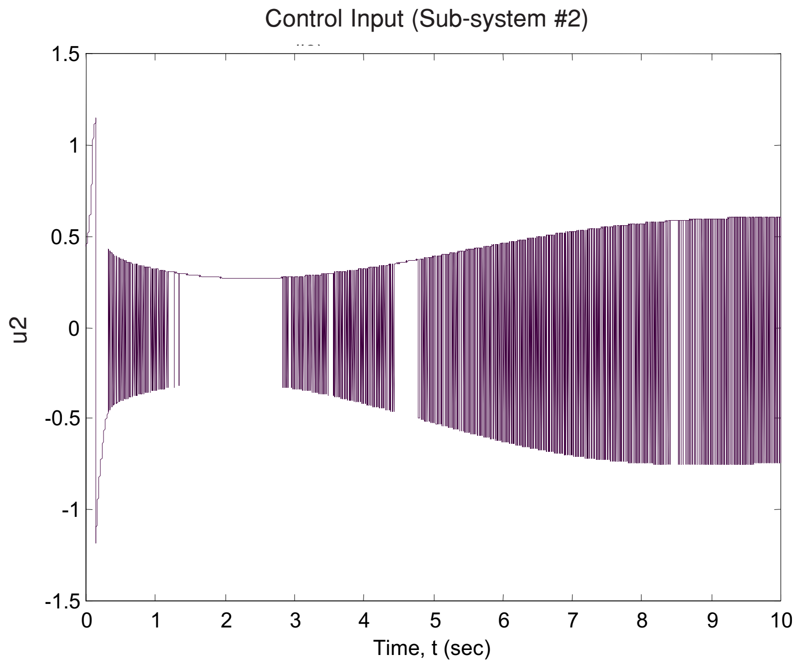

Figure 2(h) The Control Input of Subsystem 2 for γ_{11}=10, γ_{12}=1, γ_{13}=0.1, γ_{14}=1, γ_{21}=6, γ_{22}=1.5, γ_{23}=1, and γ_{24}=1

As for case 2, the results for sub-system 1 and sub-system 2 are shown in Figure 3a-d and Figure 3e-h, respectively. For both sub-systems, the tracking performance is good (Figure 3a and Figure 3e), resulting in the convergence of the tracking error to zero (Figure 3b and Figure 3f). The sub-systems dynamics slide as smoothly as predicted (Figure 3c and Figure 3g) and the control inputs switch accordingly indicating that the hitting condition really takes place (Figure 3d and Figure 3h). In general, it can be concluded that the actual states track the desired states satisfactorily for all of the sub-systems if the conditions of equations (23)–(26) are satisfied.

6.0 CONCLUSIONS

In this paper, a Decentralized Proportional-Integral Sliding Mode controller is proposed for a class of large-scale uncertain system which is not in the input decentralized form. It is shown mathematically that the error dynamics during sliding mode is stable and can easily be shaped-up using the conventional pole-placement technique. The controller utilizes only the information available on the local states and the system stability is guaranteed during the reaching phase. Computer simulation of a hypothetical interconnected plant with input couplings shows that the proposed controller renders the uncertainties system tracks the desired trajectory in spite of the uncertainties, non-linearities and coupling inherent in the system.

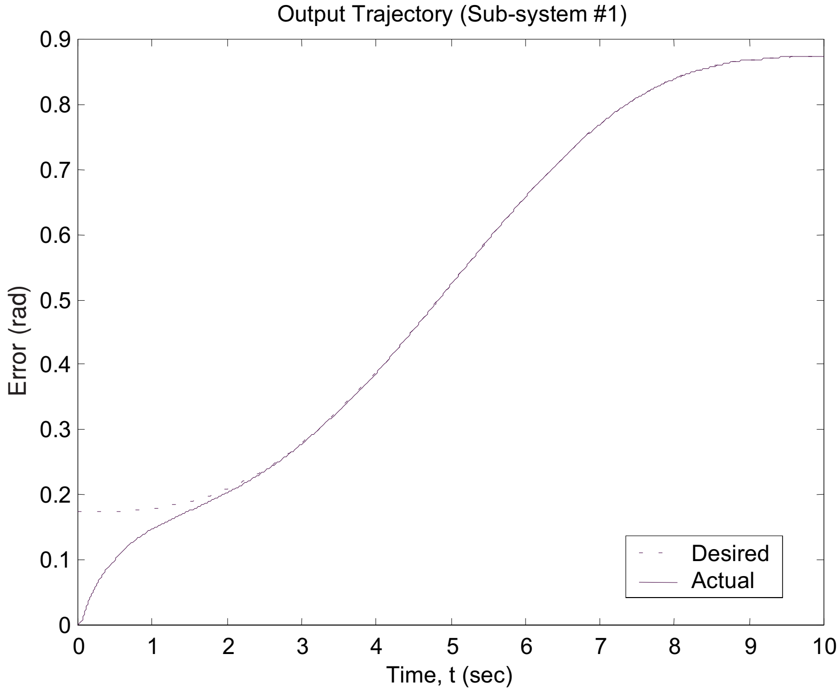

Figure 3(a) The Output and Desired Trajectories of Subsystem 1 for γ_{11}=14, γ_{12}=2, γ_{13}=1, γ_{14}=3, γ_{21}=7, γ_{22}=3, γ_{23}=2, and γ_{24}=3

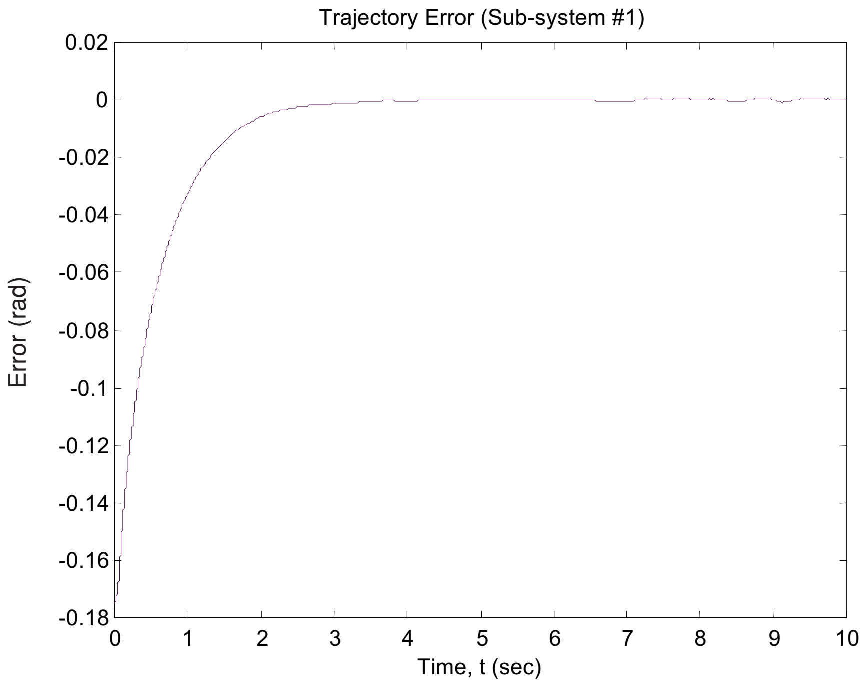

Figure 3(b) The Tracking Error of Subsystem 1 for γ_{11}=14, γ_{12}=2, γ_{13}=1, γ_{14}=3, γ_{21}=7, γ_{22}=3, γ_{23}=2, and γ_{24}=3

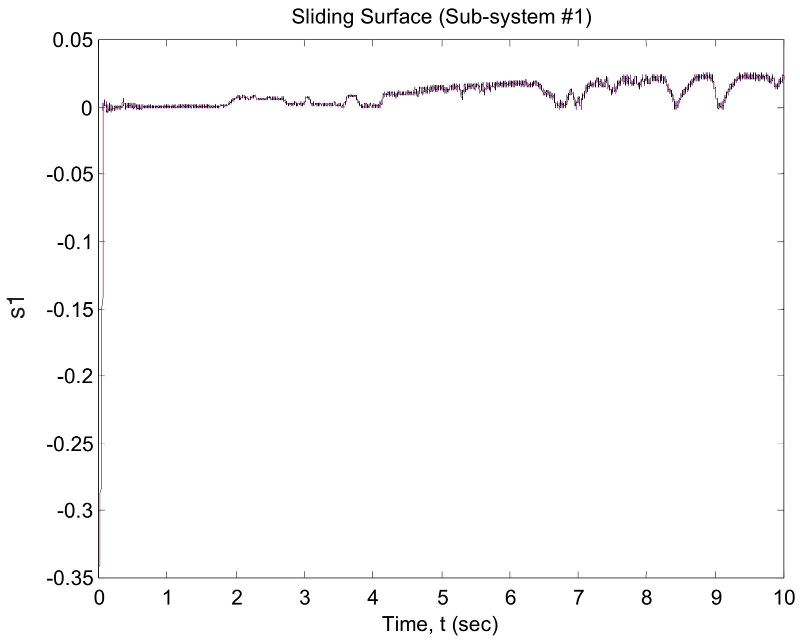

Figure 3(c) The Sliding Surface of Subsystem 1 for γ_{11}=14, γ_{12}=2, γ_{13}=1, γ_{14}=3, γ_{21}=7, γ_{22}=3, γ_{23}=2, and γ_{24}=3

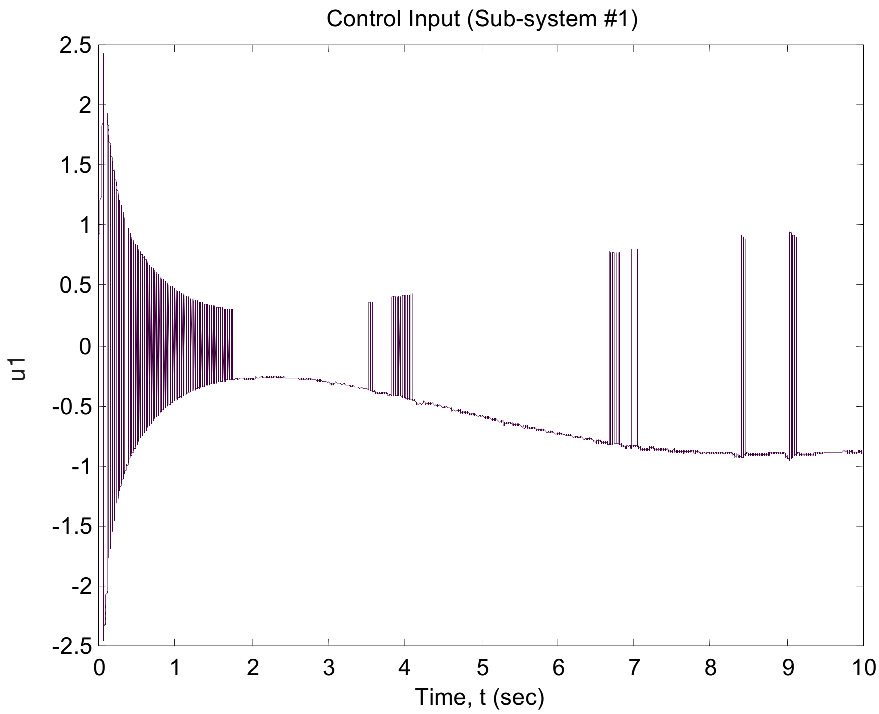

Figure 3(d) The Control Input of Subsystem 1 for γ_{11}=14, γ_{12}=2, γ_{13}=1, γ_{14}=3, γ_{21}=7, γ_{22}=3, γ_{23}=2, and γ_{24}=3

APPENDIX

Using equations (22) and (13), equation (21) can be written as

$$\dot{V}(t) = \sum_{i=1}^{N} \frac{\sigma_i^T(t)}{\|\sigma_i(t)\|}\{[C_i \Delta A_i - C_i B_i K_i] z_i(t) - \gamma_{i1}\|z_i(t)\| SGN(\sigma_i(t))$$

$$+ \sum_{\substack{j=1 \\ j \neq i}}^{N} C_i [A_{ij} + \Delta A_{ij}] x_j(t) - \gamma_{i2}\|x_i(t)\| SGN(\sigma_i(t)) + C_i \Delta A_i x_{di}(t) \tag{A.1}$$

$$-\gamma_{i3}\|x_{di}(t)\| SGN(\sigma_i(t)) - \gamma_{i4}\|\Omega_i(t)\| SGN(\sigma_i(t)) + \sum_{\substack{j=1 \\ j \neq i}}^{N} C_i B_{ij} u_j(t)\}$$

Note that

$$\sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} C_i B_{ij} u_j(t) = \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} C_j B_{ji} u_i(t) \tag{A.2}$$

$$\sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} C_i [A_{ij} + \Delta A_{ij}(t)] x_j(t) = \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} C_j [A_{ji} + \Delta A_{ji}(t)] x_i(t) \tag{A.3}$$

Then equation (A.1) can be written as

$$\dot{V}(t) = \sum_{i=1}^{N} \frac{\sigma_i^T(t)}{\|\sigma_i(t)\|}\{[C_i \Delta A_i - C_i B_i K_i] z_i(t) - [1 + \sum_{\substack{j=1 \\ j \neq i}}^{N} C_j B_{ji} (C_i B_i)^{-1}]\gamma_{i1}\|z_i(t)\| SGN(\sigma_i(t))\}$$

$$+ \sum_{i=1}^{N} \frac{\sigma_i^T(t)}{\|\sigma_i(t)\|}\{\sum_{\substack{j=1 \\ j \neq i}}^{N} C_j [A_{ji} + \Delta A_{ji}] x_i(t) - [1 + \sum_{\substack{j=1 \\ j \neq i}}^{N} C_j B_{ji} (C_i B_i)^{-1}]\gamma_{i2}\|x_i(t)\| SGN(\sigma_i(t))\}$$

$$+ \sum_{i=1}^{N} \frac{\sigma_i^T(t)}{\|\sigma_i(t)\|}\{C_i \Delta A_i x_{di}(t) - [1 + \sum_{\substack{j=1 \\ j \neq i}}^{N} C_j B_{ji} (C_i B_i)^{-1}]\gamma_{i3}\|x_{di}(t)\| SGN(\sigma_i(t))\}$$

$$+ \sum_{i=1}^{N} \frac{\sigma_i^T(t)}{\|\sigma_i(t)\|}\{\sum_{\substack{j=1 \\ j \neq i}}^{N} C_j B_{ji} \Omega_i(t) - [1 + \sum_{\substack{j=1 \\ j \neq i}}^{N} C_j B_{ji} (C_i B_i)^{-1}]\gamma_{i4}\|\Omega_i(t)\| SGN(\sigma_i(t))\} \tag{A.4}$$

Noting that [19]

$$\left(\frac{\sigma_i^T(t)}{\|\sigma_i(t)\|}\right) SGN(\sigma_i(t)) = \sum_{k=1}^{m_i} \frac{|\sigma_{ik}|}{\|\sigma_i(t)\|} \geq 1 \tag{A.5}$$

then each term of equation (A.4) can be simplified individually as follows:

First term of equation (A.4):

$$\sum_{i=1}^{N} \frac{\sigma_i^T(t)}{\|\sigma_i(t)\|} \{[C_i \Delta A_i - C_i B_i K_i] z_i(t) - [1 + \sum_{\substack{j=1 \\ j \neq i}}^{N} C_j B_{ji} (C_i B_i)^{-1}] \gamma_{i1} \|z_i(t)\| SGN(\sigma_i(t))\}$$
$$\leq \sum_{i=1}^{N} -\{[1 + \sum_{\substack{j=1 \\ j \neq i}}^{N} C_j B_{ji} (C_i B_i)^{-1}] \gamma_{i1} - [\beta_{ii} \|C_i\| + \|C_i B_i K_i\|]\} \|z_i(t)\| \tag{A.6}$$

Second term of equation (A.4):

$$\sum_{i=1}^{N} \frac{\sigma_i^T(t)}{\|\sigma_i(t)\|} \{\sum_{\substack{j=1 \\ j \neq i}}^{N} C_i [A_{ij} + \Delta A_{ij}] x_j(t) - [1 + \sum_{\substack{j=1 \\ j \neq i}}^{N} C_j B_{ji} (C_i B_i)^{-1}] \gamma_{i2} \|x_i(t)\| SGN(\sigma_i(t))\}$$
$$\leq \sum_{i=1}^{N} -\{[1 + \sum_{\substack{j=1 \\ j \neq i}}^{N} C_j B_{ji} (C_i B_i)^{-1}] \gamma_{i2} - \sum_{\substack{j=1 \\ j \neq i}}^{N} [\|C_j A_{ji}\| + \beta_{ji} \|C_j\|]\} \|x_i(t)\| \tag{A.7}$$

Third term of equation (A.4):

$$\sum_{i=1}^{N} \frac{\sigma_i^T(t)}{\|\sigma_i(t)\|} \{C_i \Delta A_i x_{di}(t) - [1 + \sum_{\substack{j=1 \\ j \neq i}}^{N} C_j B_{ji} (C_i B_i)^{-1}] \gamma_{i3} \|x_{di}(t)\| SGN(\sigma_i(t))\}$$
$$\leq \sum_{i=1}^{N} -\{[1 + \sum_{\substack{j=1 \\ j \neq i}}^{N} C_j B_{ji} (C_i B_i)^{-1}] \gamma_{i3} - \beta_{ii} \|C_i\|\} \|x_{di}(t)\| \tag{A.8}$$

Last term of equation (A.4):

$$\sum_{i=1}^{N}\frac{\sigma_i^T(t)}{\|\sigma_i(t)\|}\{\sum_{\substack{j=1\\j\neq i}}^{N}C_jB_{ji}\Omega_i(t)-[1+\sum_{\substack{j=1\\j\neq i}}^{N}C_jB_{ji}(C_iB_i)^{-1}]\gamma_{i4}\|\Omega_i(t)\|SGN(\sigma_i(t))\}$$

$$\leq\sum_{i=1}^{N}-\{[1+\sum_{\substack{j=1\\j\neq i}}^{N}C_jB_{ji}(C_iB_i)^{-1}]\gamma_{i4}-\sum_{\substack{j=1\\j\neq i}}^{N}\|C_jB_{ji}\|\}\|\Omega_i(t)\| \tag{A.9}$$

In view of equations (A.6),(A.7),(A.8), and (A.9), equation (A.4) can be written as

$$\begin{aligned}\sum_{i=1}^{N}\frac{\sigma_i^T(t)}{\|\sigma_i(t)\|}\dot{\sigma}_i^T(t) \leq &\sum_{i=1}^{N}-\{[1+\sum_{\substack{j=1\\j\neq i}}^{N}C_jB_{ji}(C_iB_i)^{-1}]\gamma_{i1}-[\beta_{ii}\|C_i\|+\|C_iB_iK_i\|]\}\|z_i(t)\|\\ &+\sum_{i=1}^{N}-\{[1+\sum_{\substack{j=1\\j\neq i}}^{N}C_jB_{ji}(C_iB_i)^{-1}]\gamma_{i2}-\sum_{\substack{j=1\\j\neq i}}^{N}[\|C_jA_{ji}\|+\beta_{ji}\|C_j\|]\}\|x_i(t)\|\\ &+\sum_{i=1}^{N}-\{[1+\sum_{\substack{j=1\\j\neq i}}^{N}C_jB_{ji}(C_iB_i)^{-1}]\gamma_{i3}-\beta_{ii}\|C_i\|\}\|x_{di}(t)\|\\ &+\sum_{i=1}^{N}-\{[1+\sum_{\substack{j=1\\j\neq i}}^{N}C_jB_{ji}(C_iB_i)^{-1}]\gamma_{i4}-\sum_{\substack{j=1\\j\neq i}}^{N}\|C_jB_{ji}\|\}\|\Omega_i(t)\|\end{aligned} \tag{A.10}$$

Let equations (23), (24), (25) and (26) hold, then the global hitting condition (19) is satisfied.

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