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## SLIDING MODE CONTROL OF ACTIVE SUSPENSION SYSTEM

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**Abstract.** The purpose of this paper is to present a new approach in controlling an active suspension system. This approach utilized the proportional integral sliding mode control scheme. Using this type of sliding surface, the asymptotic stability of the system during sliding mode is assured compared to the conventional sliding surface. The proposed control scheme is applied in designing an automotive active suspension system for a quarter-car model and its performance is compared with the existing passive suspension system. A simulation study is performed to prove the effectiveness of this control design.

*Key words:* Automotive control, sliding mode control, robust control, mismatched uncertainties.

**Abstrak.** Kertas kerja ini mempersembahkan suatu pendekatan baru dalam mengawal sistem gantungan aktif. Pendekatan yang dicadangkan ialah menggunakan skim kawalan mod gelangar perkadaran-pengkerbebaan. Menggunakan permukaan gelangar jenis ini, kestabilan asimptotik sistem adalah lebih pasti berbanding dengan jenis permukaan gelangar konvensional. Skim kawalan yang dicadangkan diaplikasikan penggunaannya dalam mereka bentuk sistem gantungan aktif automotif model kereta suku. Keupayaan sistem kawalan yang direka bentuk akan dibandingkan dengan sistem gantungan pasif yang sedia ada. Kajian dalam bentuk penyelakuan dijalankan untuk membuktikan keberkesanan rekabentuk sistem kawalan berkenaan.

*Kata kunci:* Kawalan automotif, kawalan mod gelangar, kawalan tegap, ketakpastian tak terpadan.

### 1.0 INTRODUCTION

In the past millennium, there had been widespread interest in using advanced control techniques to improve the performance of vehicle suspension system. Performance of the suspension system has been greatly increased due to increasing vehicle capabilities. Several performance characteristics have to be considered in order to achieve a good suspension system [1]. These characteristics deal with regulation of body movement, regulation of suspension movement and force distribution. Ideally the suspension should isolate the body from road disturbances and inertial disturbances associated with cornering and braking or acceleration. Furthermore, the suspension must be able to minimize the vertical force transmitted to the passengers for passengers comfort. These objectives can be achieved by minimizing the vertical car body acceleration.

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An excessive wheel travel will result in non-optimum attitude of tyre relative to the road that will cause poor handling and adhesion. Furthermore, to maintain good handling characteristic, the optimum tyre-to-road contact must be maintained on four wheels.

An early design for automobile suspension systems focused on unconstrained optimizations for passive suspension system which indicate the desirability of low suspension stiffness, reduced unsprung mass, and an optimum damping ratio for the best controllability [2]. Thus the passive suspension system, which approach optimal characteristics had offered an attractive choice for a vehicle suspension system and had been widely used for passengers. However, the suspension spring and damper do not provide energy to the suspension system and control only the motion of the car body and wheel by limiting the suspension velocity according to the rate determined by the designer. To overcome the above problem, active suspension systems have been proposed by various researchers [3–7]. Active suspension systems dynamically respond to changes in the road profile because of their ability to supply energy that can be used to produce relative motion between the body and wheel. Typically, active suspension systems include sensors to measure suspension variables such as body velocity, suspension displacement, wheel velocity and wheel or body acceleration. An active suspension is one in which the passive components are augmented by actuators that supply additional forces. These additional forces are determined by a feedback control law using data from sensors attached to the vehicle. Various control strategies such as optimal state-feedback [3], backstepping method [4], optimal state-feedback [5], fuzzy control [6] and sliding mode control [7] have been proposed in the past years to control the active suspension system. The sliding mode control has relatively simpler structure and it guarantees the system stability.

Variable Structure Control with Sliding Mode Control (SMC) was first proposed and elaborated in early 1950's in the Soviet Union by Emelyanov and several coresearchers such as Utkins and Itkis [8]. During the last two decades, significant interest on VSC and SMC have been generated in the control research community. VSC has been applied into general design method being examined for wide spectrum of system types including nonlinear system, multi-input multi-output (MIMO) systems, discrete-time models, large-scale and infinite-dimension systems, and stochastic systems. The most eminent feature of VSC is it is completely insensitive to parametric uncertainty and external disturbances during sliding mode [9].

Essentially, VSC utilizes a high-speed switching control law to drive the nonlinear plant's state trajectory onto a specified and user-chosen surface in the state space which is called the sliding or switching surface, and to maintain the plant's state trajectory on this surface for all subsequent time. This surface is called the switching surface because if the state trajectory of the plant is "above" the surface, a control path has one gain and a different gain if the trajectory drops "below" the surface. During the process, the control system's structure varies from one to another, thus earning the name variable

structure control (VSC). To emphasize the important role of the sliding mode, the control is also called sliding mode control (SMC).

There are two steps in the SMC design. The first step is designing a sliding surface so that the plant restricted to the sliding surface has a desired system response. This means the state variables of the plant dynamics are constrained to satisfy another set of equations which define the so-called switching surface. The second step is constructing a switched feedback gains necessary to drive the plant's state trajectory to the sliding surface. These constructions are built on the generalized Lyapunov stability theory.

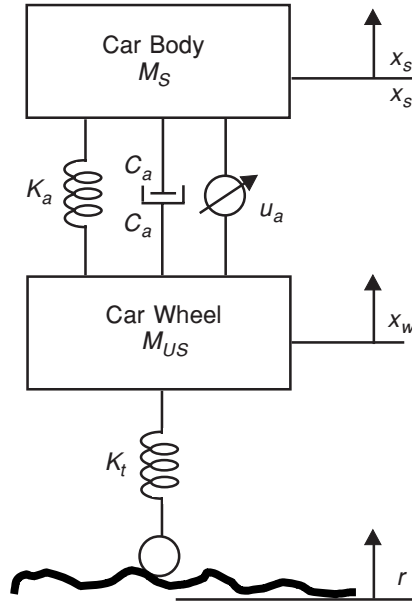
In this paper we will consider a control scheme that can improve further the ride comfort and road handling of the active suspension system. The control of the active suspension system is complicated further due to the inherent mismatched condition present in the system. The mismatched condition exist in the system due to the road profile which acts as the disturbance input. The proposed control scheme differs from the previous sliding mode techniques in the sense that the sliding surface is based on the proportional-integral (PI) sliding mode control. The additional integral in the proposed sliding surface provides one more degree of freedom and also reduce the steady state error. It will be shown that the proposed controller would be able to overcome the mismatched condition provided a certain condition is satisfied. A computer simulation will be performed to demonstrate the effectiveness and robustness of the proposed control scheme.

## 2.0 DYNAMIC MODEL OF THE VEHICLE SUSPENSION

Most of the past active suspension designs were developed based on the quarter-car model as in Figure 1. The following state-space model (equation (1)) can be easily obtained from Figure 1.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -K_a/M_s & -C_a/M_s & 0 & -K_a/M_s \\ 0 & 0 & 0 & 1 \\ K_a/M_{us} & C_a/M_{us} & -K_a/M_{us} & -C_a/M_{us} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/M_s \\ 0 \\ -1/M_s \end{bmatrix} u_a + \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix} \dot{r} \quad (1)$$

where  $M_s$  and  $M_{us}$  are the masses of the car body and wheel respectively,  $x_s$  and  $x_w$  are the displacements of car body and wheel respectively,  $K_a$  and  $K_t$  are the spring coefficients,  $C_a$  is the damper coefficient,  $\dot{r}$  is the road disturbance and  $u_a$  is the control force from the hydraulic actuator and assumed as the control input. The state variables are defined as forms:  $x_1 = x_s - x_w$  for suspension travel,  $x_2 = \dot{x}_s$  for car body velocity,  $x_3 = x_w - r$  for wheel deflection and  $x_4 = \dot{x}_w$  for wheel velocity. Equation (1) can be written in vector-matrix form as



**Figure 1** A quarter-car model

$$\dot{x}(t) = Ax(t) + Bu(t) + f(t) \quad (2)$$

where  $x(t) \in \mathfrak{R}^n$  is the state vector,  $u(t) \in \mathfrak{R}^m$  is the control input, and the continuous function  $f(t)$  represents the uncertainties with the mismatched condition. The following assumptions are taken as standard:

*Assumption i:* There exists a known positive constant such that  $\|f(t)\| \leq \beta$ , where  $\|\bullet\|$  denotes the standard Euclidean norm.

*Assumption ii:* The pair is controllable and the input matrix has full rank.

### 3.0 SWITCHING SURFACE AND CONTROLLER DESIGN

In this study, PI sliding surface define as follows is utilized:

$$\sigma(t) = Cx(t) - \int_0^t (CA + CBK)x(\tau) d\tau \quad (3)$$

where  $C \in \mathfrak{R}^{m \times n}$  and  $K \in \mathfrak{R}^{n \times n}$  are constant matrices. The matrix  $K$  satisfies  $\lambda(A + BK) < 0$  and  $C$  is chosen so that  $CB$  is nonsingular. It is well known that if the

system is able to enter the sliding mode, hence  $\sigma(t) = 0$ . Therefore the equivalent control,  $u_{eq}(t)$  can thus be obtained by letting  $\dot{\sigma}(t) = 0$  [8] i.e.,

$$\dot{\sigma}(t) = C\dot{x}(t) - \{CA + CBK\}x(t) = 0 \quad (4)$$

If the matrix  $C$  is chosen such that  $CB$  is nonsingular, this yields

$$u_{eq}(t) = Kx(t) - (CB)^{-1}Cf(t) \quad (5)$$

Substituting equation (5) into system (2) gives the equivalent dynamic equation of the system in sliding mode as:

$$\dot{x}(t) = (A + BK)x(t) + \{I_n - B(CB)^{-1}C\}f(t) \quad (6)$$

Theorem 1: If

$$\|\tilde{F}(t)\| \leq \beta_1 = \|I_n - B(CB)^{-1}C\|\beta \quad (7)$$

the uncertain system in equation (6) is boundedly stable on the sliding surface  $\sigma(t) = 0$ .

**Proof:** For simplicity, we let

$$\tilde{A} = (A + BK) \quad (8)$$

$$\tilde{F}(t) = \{I_n - B(CB)^{-1}C\}f(t) \quad (9)$$

and rewrite (6) as

$$\dot{x}(t) = \tilde{A}x(t) + \tilde{F}(t) \quad (10)$$

Let the Lyapunov function candidate for the system is chosen as

$$V(t) = x^T(t)Px(t) \quad (11)$$

Taking the derivative of  $V(t)$  and substituting equation (6), gives

$$\begin{aligned} \dot{V}(t) &= x^T(t) [\tilde{A}^T P + P\tilde{A}]x(t) + \tilde{F}^T(t)Px(t) + x^T(t)P\tilde{F}(t) \\ &= -x^T(t)Qx(t) + \tilde{F}^T(t)Px(t) + x^T(t)P\tilde{F}(t) \end{aligned} \quad (12)$$

where  $P$  is the solution of  $\tilde{A}^T P + P\tilde{A} = -Q$  for a given positive definite symmetric matrix  $Q$ . It can be shown that equation (12) can be reduced to:

$$\dot{V}(t) = -\lambda_{\min}(Q)\|x(t)\|^2 + 2\beta_1\|P\|\|x(t)\| \quad (13)$$

Since  $\lambda_{\min}(Q) > 0$ , consequently  $\dot{V}(t) < 0$  for all  $t$  and  $x \in B^c(\eta)$ , where  $B^c(\eta)$

is the complement of the closed ball  $B(\eta)$ , centered at  $x = 0$  with radius  $\eta = \frac{2\beta_1 \|P\|}{\lambda_{\min}(Q)}$ .

Hence, the system is boundedly stable.

Theorem 1 shows that if the condition (7) is satisfied, the system is boundedly stable during sliding mode even in the present of the mismatched condition.

*Remark:* For the system with uncertainties satisfy the matching condition, i.e., then equation (6) can be reduced to [10]. Thus asymptotic stability of the system during sliding mode is assured.

The next step is to design the control scheme that drives the state trajectories of the system in equation (2) onto the sliding surface and the system remains in it thereafter.

For the uncertain system in equation (2) satisfying assumptions (i) and (ii), the following control law is proposed:

$$u(t) = -(CB)^{-1} [CAx(t) + \phi\sigma(t)] - k(CB)^{-1} \frac{\sigma(t)}{\|\sigma(t)\| + \delta} \quad (14)$$

where  $\phi \in \mathfrak{R}^{m \times n}$  is a positive symmetric design matrix, and are the positive constants.

*Theorem 2:* The hitting condition of the sliding surface (3) is satisfied if

$$\|A + BK\| \|x(t)\| \geq \|f(t)\| \quad (15)$$

**Proof:** In the hitting phase  $\sigma^T(t)\dot{\sigma}(t) > 0$ ; using the Lyapunov function candidate

$$V(t) = \frac{1}{2} \sigma^T(t)\sigma(t), \text{ we obtain}$$

$$\begin{aligned} \dot{V}(t) &= \sigma^T(t)\dot{\sigma}(t) \\ &= \sigma^T(t) \left[ -(CA + CBK)x(t) - \phi\sigma(t) - \frac{k\sigma(t)}{\|\sigma(t)\| + \delta} + Cf(t) \right] \\ &\leq - \left[ \left\{ \|\phi\| + \left\| \frac{k}{\|\sigma(t)\| + \delta} \right\| \right\} \|\sigma(t)\|^2 + \{ \|C\| \|A + BK\| \|x(t)\| - \|C\| \|f(t)\| \} \|\sigma(t)\| \right] \end{aligned} \quad (16)$$

It follows that  $\dot{V}(t) < 0$  if condition (15) is satisfied. Thus, the hitting condition is satisfied.

#### 4.0 SIMULATION AND DISCUSSION

The mathematical model of the system as defined in equation (2) and the proposed PI sliding mode controller in equation (14) were simulated on computer by using the MATLAB and SIMULINK software package. Numerical values for the model parameters are taken from [3], and are as follows:

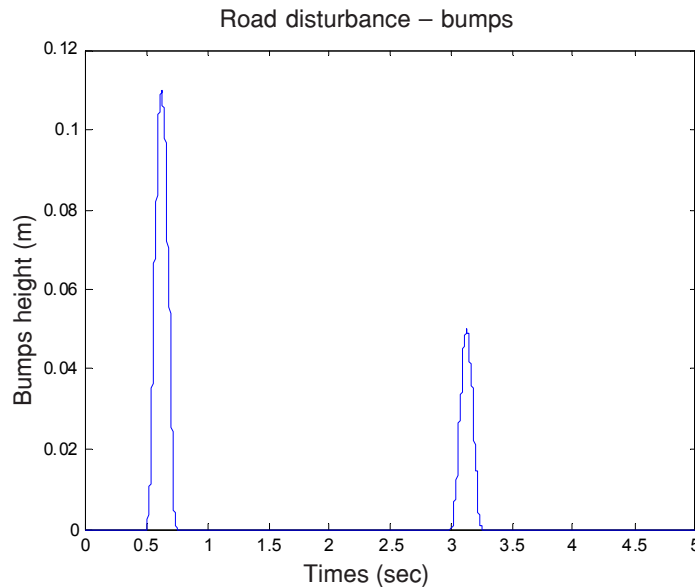
$$M_s = 290\text{kg}, M_{us} = 59 \text{ kg}, K_a = 16812 \text{ N/m}, K_t = 190000 \text{ N/m}, C_a = 1000 \text{ N/(m/sec)}$$

Let the set of typical road disturbance be of the form

$$r(t) = \begin{cases} a(-\cos(8\pi t)/2) & \text{if } 0.50 \leq t \leq 0.75 \text{ and } 3.00 \leq t \leq 3.25 \\ 0 & \text{otherwise} \end{cases}$$

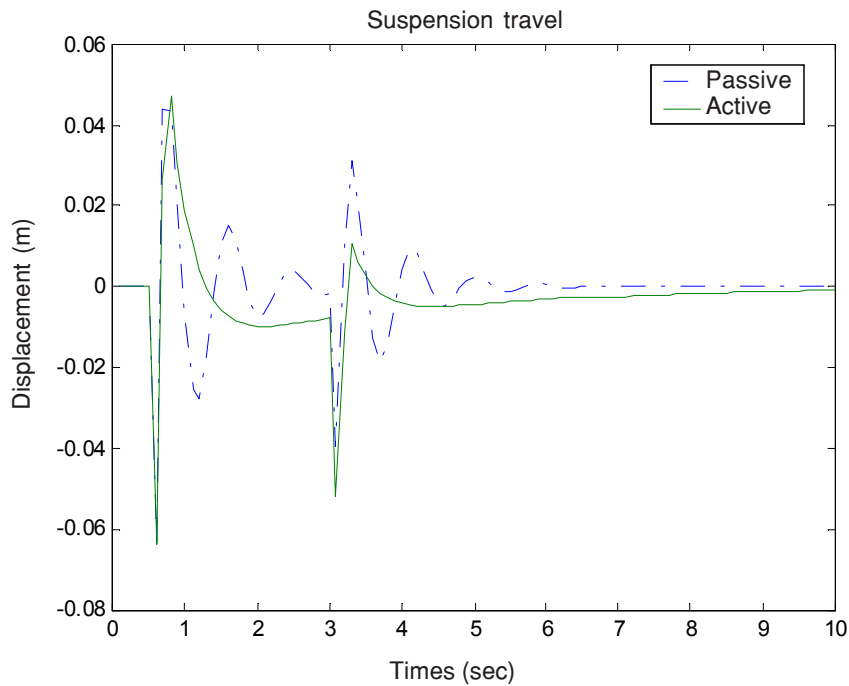
where  $a$  denotes the bump amplitude (see Figure 2). This type of road disturbance has been used by [4,11] in their studies.

Furthermore, the maximum travel distance of suspension travel as suggested by [4] is  $\pm 8$  has been used. The controller parameters has been chosen as follows for the simulation:  $K = [16754 \ 939.14 \ -1.818e5 \ 172.67]$ ,  $C = [200 \ 50 \ 20 \ 10]$ ,  $\phi = 1000$ ,  $k = 1$  and  $\delta = 0.001$ .



**Figure 2** Typical road disturbance – 11 cm and 5cm bumps

In order to fulfill the objective of designing an active suspension system i.e. to increase the ride comfort and road handling, there are two parameters to be observed in the simulations. The two parameters are the car body acceleration and wheel deflection. Figure 3a shows the suspension travel of both the active and passive suspension systems for comparison purposes. The result shows that the suspension travel within the travel limit. Figures 3b and 3c illustrate clearly how the active suspension can effectively absorb the vehicle vibration in comparison to the passive system. The body acceleration in the active suspension system is reduced significantly, which guarantee better ride comfort. Moreover the wheel deflection is also smaller in the active suspension system. Therefore it is concluded that the active suspension system improves the ride comfort while retaining the road handling characteristics, compared to the passive suspension system.



**Figure 3(a)** Suspension travel



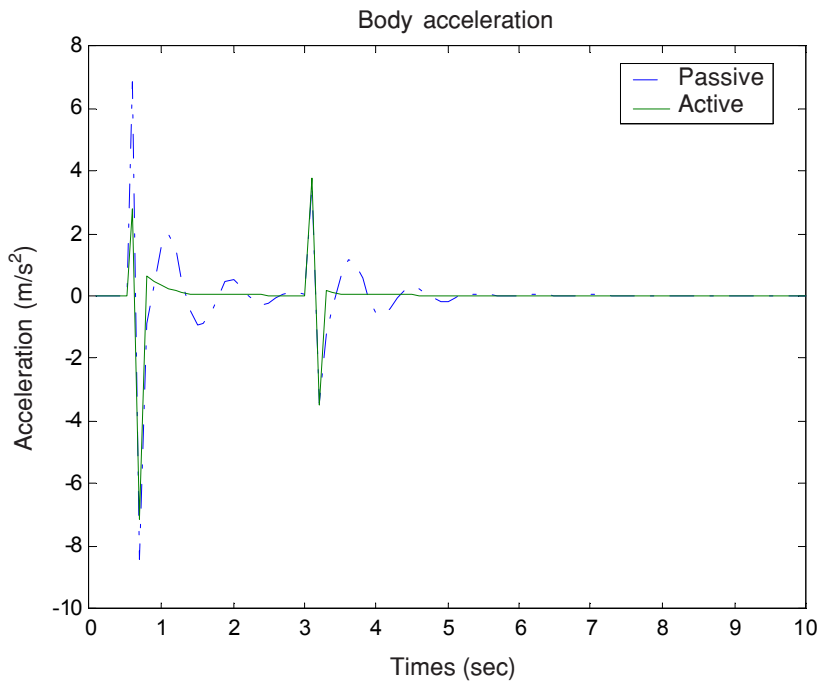


Figure 3(b) Body acceleration

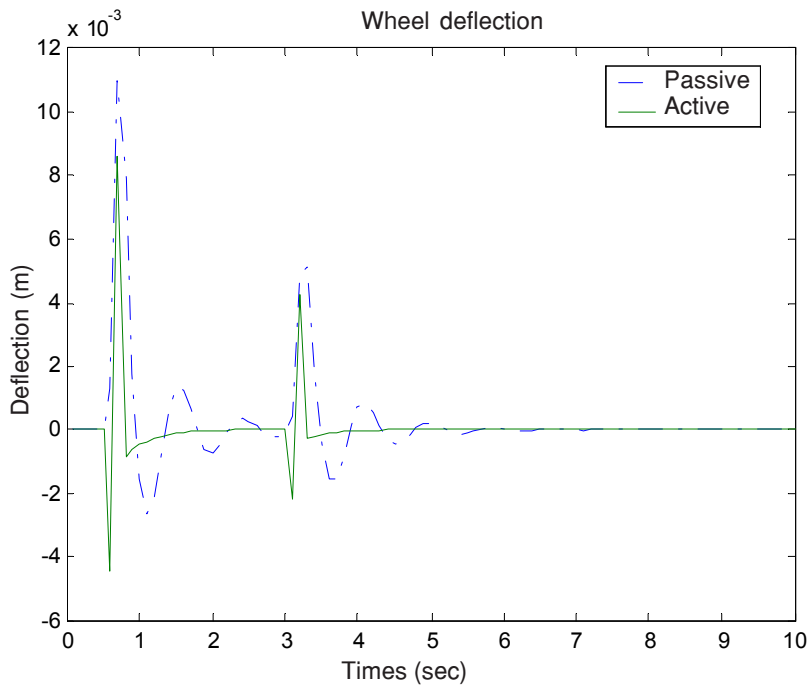


Figure 3(c) Wheel deflection

## 5.0 CONCLUSION

In this paper, the PI sliding mode control technique is proposed for controlling an active suspension system. It has been shown mathematically and through computer simulations that the proposed control scheme is capable of improving the ride comfort and road handling characteristics of the active suspension system.

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