DESIGN OF REINFORCED CONCRETE BEAMS WITH WEB OPENINGS

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ABSTRACT: The provision of transverse openings in floor beams to facilitate the passage of utility pipes and service ducts results not only in a more systematic layout of pipes and ducts, it also translates into substantial economic savings in the construction of a multi-storey building. To investigate the problem of openings in beams, the author initiated a research program in the early 1980s. Since then extensive research has been carried out giving a comprehensive coverage on both circular and large rectangular openings under various combinations of bending, shear and torsion. In this paper, major findings relevant to the analysis and design of such beams under the most commonly encountered loading case of bending and shear are extracted and summarized. An attempt has been made to answer the frequently asked questions related to creating an opening in an already constructed beam and how to deal with multiple openings. It has been shown that the design method for beams with large openings can be further simplified without sacrificing rationality and having unreasonable additional cost.

Keywords: beams (reinforced concrete); opening; serviceability; strength; structural design.

1. INTRODUCTION

In the construction of modern buildings, a network of pipes and ducts is necessary to accommodate essential services like water supply, sewage, air-conditioning, electricity, telephone, and computer network. Usually, these pipes and ducts are placed underneath the beam soffit and, for aesthetic reasons, are covered by a suspended ceiling, thus creating a *dead space*. Passing these ducts through transverse openings in the floor beams leads to a reduction in the dead space and results in a more compact design. For small buildings, the savings thus achieved may not be significant, but for multistory buildings, any saving in story height multiplied by the number of stories can represent a substantial saving in total height, length of air-conditioning and electrical ducts, plumbing risers, walls and partition surfaces, and overall load on the foundation.

It is obvious that inclusion of openings in beams alters the simple beam behavior to a more complex one. Due to abrupt changes in the sectional configuration, opening corners are subject to high stress concentration that may lead to cracking unacceptable from aesthetic and durability viewpoints. The reduced stiffness of the beam may also give rise to excessive deflection under service load and result in a considerable redistribution of internal forces and moments in a continuous beam. Unless special reinforcement is provided in sufficient quantity with proper detailing, the strength and serviceability of such a beam may be seriously affected.

In his extensive experimental study, Prentzas (1968) considered openings of circular, rectangular, diamond, triangular, trapezoidal and even irregular shapes. However, circular and rectangular openings are the most common ones in practice. When the size of opening is concerned, many researchers use the terms *small* and *large* without any definition or clear-cut demarcation line. From a survey of available literature, it has been noted (Mansur and Tan, 1999) that the essence of such classification lies in the structural response of the beam. When the opening is small enough to maintain the beam-type behavior or, in other words, if the usual beam theory applies, then the opening may be termed as small opening. In contrast, large openings are those that prevent beam-type behavior to develop. Thus, beams with small and large openings need separate treatments in design.

In this paper, beams containing small and large openings are treated separately. Based on the research work reported in the literature, an attempt has been made to give a comprehensive treatment of openings under bending and shear, addressing the major issues concerning structural design. It has been shown that the design of beams with large openings can be further simplified by maintaining its rationality and upholding construction economy.

2. BEAMS WITH SMALL OPENINGS

Openings that are circular, square, or nearly square in shape may be considered as small openings provided that the depth (or diameter) of the opening is in a realistic proportion to the beam size, say, about less than 40% of the overall beam depth. In such a case, beam action may be assumed to prevail. Therefore, analysis and design of a beam with small openings may follow the similar course of action as that of a solid beam. The provision of openings, however, produces discontinuities or disturbances in the normal flow of stresses, thus leading to stress concentration and early cracking around the opening region. Similar to any discontinuity, special reinforcement, enclosing the opening close to its periphery, should therefore be provided in sufficient quantity to control crack widths and prevent possible premature failure of the beam.

2.1 Pure Bending

In the case of pure bending, placement of an opening completely within the tension zone does not change the load-carrying mechanism of the beam because concrete there would have cracked anyway in flexure at ultimate. Mansur and Tan, (1999) have illustrated this through worked out examples, supported by test evidence. Thus, the ultimate moment capacity a beam is not affected by the presence of an opening as long as the minimum depth of the compression chord, h_c, is greater than or equal to the depth of ultimate compressive stress block, that is, when

$$h_c \leq \frac{A_s f_y}{0.85 f_c' b} \tag{1}$$

in which A_s = area of tensile reinforcement; f_y = yield strength of tensile reinforcement; f_c' = cylinder compressive strength of concrete; b = width of the compression zone. However, due to reduced moment of inertia at a section through the opening, cracks will initiate at an earlier stage of loading. In spite of this, the effects on maximum crack widths and deflection under service load have been found to be only marginal, and may safely be disregarded in design.

2.2 Combined Bending and Shear

In a beam, shear is always associated with bending moment, except for the section at inflection point. When a small opening is introduced in a region subjected to predominant shear and the opening is enclosed by reinforcement, as shown by solid lines in Fig. 1, test data reported by Hanson (1969), Somes and Corley (1974), Salam (1977), and Weng (1998) indicate that the beam may fail in two distinctly different modes. The first type is typical of the failure commonly observed in solid beams except that the failure plane passes through the center of the opening (Fig. 1a). In the second type, formation of two independent diagonal cracks, one in each member bridging the two solid beam segments, leads to the failure (Fig. 1b). Labeled respectively as *beam-type* failure and *frame-type* failure (Mansur,1998), these modes of failure require separate treatment.

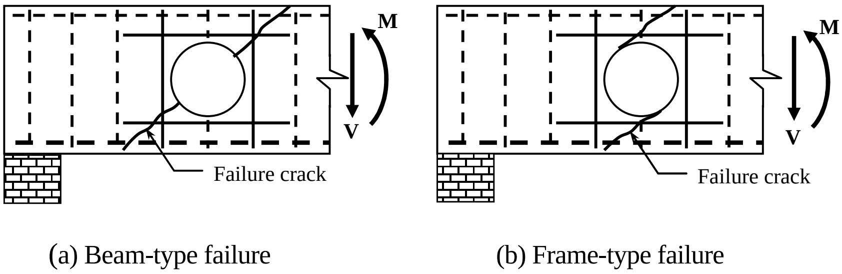


Fig 1. The two modes of shear failure at small openings.

Similar to the traditional shear design approach, it may be assumed in both the cases that the nominal shear resistance, V_n is provided partly by the concrete, V_c, and partly by the shear reinforcement crossing the failure plane, V_s. That is,

$$V_n = V_c + V_s \tag{2}$$

Design for bending may be carried out independently in the usual manner and combined with shear design solutions.

2.2.1. Beam-type failure

In designing for beam-type failure, a 45° inclined failure plane, similar to a solid beam may be assumed, the plane being traversed through the center of the opening (Fig. 2). Following the simplified approach of the ACI Code (1995), the shear resistance V_c provided by the concrete may be estimated (Mansur, 1998) by considering the net concrete area available as

$$V_c = \frac{1}{6}\sqrt{f_c'}\, b_w (d - d_o) \tag{3}$$

in which b_w = web width; d = effective depth; and d_o = diameter of opening.

For the contribution of the shear reinforcement, V_s, reference may be made to Fig. 2. It may be seen that the stirrups available to resist shear across the failure plane are those by the sides of the opening within a distance $(d_v - d_o)$, where d_v is the distance between the top and bottom longitudinal rebars, and d_o is the diameter (or depth) of opening, as shown. Thus,

$$V_s = \frac{A_v f_{yv}}{s}\left(d_v - d_o\right) \tag{4}$$

in which A_v = area of vertical legs of stirrups per spacing s; f_{yv} = yield strength of stirrups.

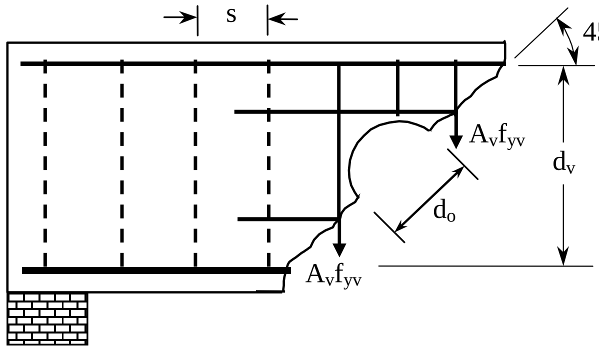


Fig 2. Shear resistance, V_s, provided by shear reinforcement at an opening.

Knowing the values of V_c and V_s, the required amount of web reinforcement to carry the factored shear through the center of the opening may be calculated in the usual way. This amount should be contained within a distance $(d_v - d_o)/2$, or preferably be lumped together on either side of the opening. Other restrictions applicable to the usual shear design procedure of solid beams must also be strictly adhered to.

2.2.2 Frame-type failure

Frame-type failure occurs due to the formation of two independent diagonal cracks, one on each of the chord members above and below the opening, as shown in Fig. 1(b). It appears that each member behaves independently similar to the members in a framed structure. Therefore, each chord member requires independent treatment, as suggested by Mansur (1998).

In order to design reinforcement for this mode of failure, let us consider the free-body diagram at beam opening, as shown in Fig. 3. Clearly, the applied factored moment, M_u, at the center of the opening from the global action is resisted by the usual bending mechanism, that is, by the couple formed by the compressive and tensile stress resultants, N_u, in the members above and below the opening. These stress resultants may be obtained by

$$(N_u)_t = \frac{M_u}{\left(d - \frac{a}{2}\right)} = -(N_u)_b \qquad (5)$$

subject to the restrictions imposed by Eq. (1). In this equation, d = the effective depth of the beam, a = depth of equivalent rectangular stress block, and the subscripts t and b denote the top and bottom cross members of the opening, respectively.

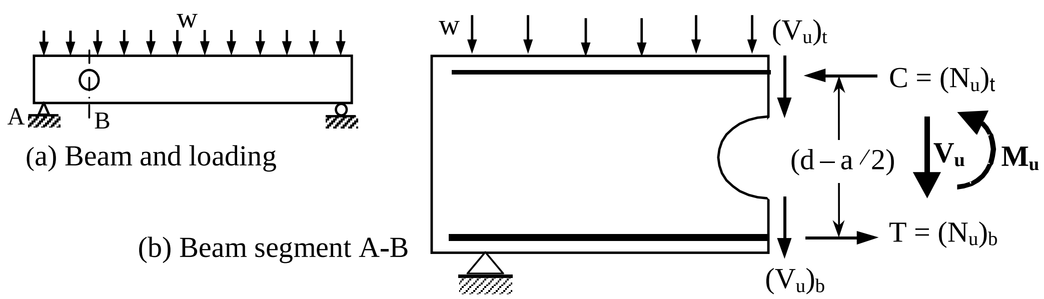


Fig 3. Free-body diagram at beam opening.

The applied shear, V_u, may be distributed between the two members in proportion to their cross-sectional areas (Nasser et al., 1967). Thus,

$$(V_u)_t = V_u \left[\frac{A_t}{A_t + A_b} \right] \qquad (6a)$$

and

$$(V_u)_b = V_u - (V_u)_t \qquad (6b)$$

Knowing the factored shear and axial forces, each member can be independently designed for shear by following the same procedure as for conventional solid beams with axial compression for the top chord and axial tension for the bottom chord.

2.2.3 Reinforcement details

Consideration of beam-type failure will require long stirrups to be placed on either side of the opening, while that of the frame-type failure will need short stirrups above and below the opening. For anchorage of short stirrups, nominal bars must be placed at each corner, if none is available from the design of solid segments. This will ensure adequate strength. For effective crack control, nominal bars should also be placed diagonally on either side. The resulting arrangement of reinforcement around the opening is shown in Fig. 4.

Under usual circumstances, introduction of a small opening with proper detailing of reinforcement does not seriously affect the service load deflection. However, in case any doubt one can follow the procedure described in Art. 3.2.3 for beams with large openings to calculate the service load deflections and check them against the permissible values.

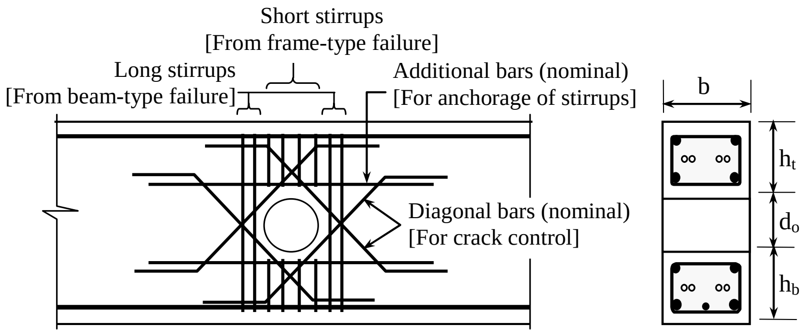


Fig 4. Reinforcement details around a small opening.

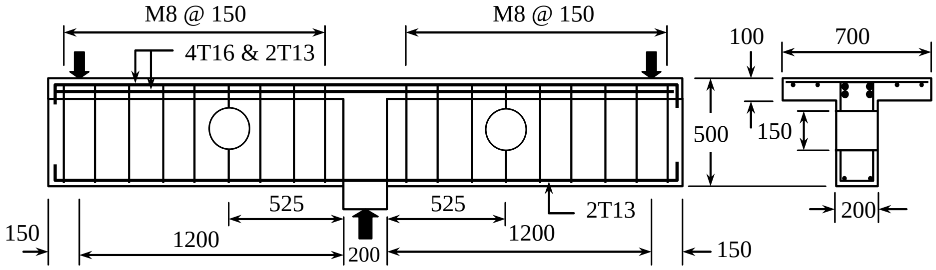
2.3 Effects of Creating Openings in Existing Beams

It is obvious that transverse openings through beams are a source of potential weakness. When the service systems are preplanned and the sizes and locations of openings required to achieve the necessary layout of pipes and ducts are decided upon well in advance, adequate strength and serviceability may be ensured by following the method described in the preceding section.

However, this is not always the case. While laying the ducts in a newly constructed building, the M&E contractor frequently comes up with the request to drill an opening for the sake of simplifying the arrangement of pipes. When such a request comes, the structural designer finds it difficult to give a decision. Of course, from the owner’s viewpoint, creating an opening may represent some financial savings, but the structural engineer would have to take the risk of jeopardizing the safety and serviceability of the structure.

Another situation arises in an old building where concrete cores are taken for structural assessment of the building. In this case, however, the holes are generally filled in by non-shrink grout. If the structure is to stay, then the question is whether or not such repair is adequate to restore the original level of safety and serviceability of the structure. In a recent study (Mansur et al., 1999), an attempt was made to answer some of these frequently asked questions of the effect of drilling holes in an existing beam.

As part of the study, four prototype T-beams simulating the conditions that exist in the negative moment region of a continuous beam were tested. All beams were 2.9 m long and contained a central stub to represent the continuous support. The cross section consisted of a 400-mm-deep and 200-mm-wide web and, a 100-mm-thick and 700-mm-wide flange. For symmetry, one opening, 150 mm in diameter, was created on each side of the central stub at a distance 525 mm from the face of the central stub, and all beams contained the same amount and arrangement of reinforcement as can be seen in Fig. 5.



NOTE: All dimensions are in mm.

Fig 5. Reinforcement details of beams tested by Mansur et al. (1999).

The beam, designated *S* in Table 1, contained no openings. It served as a reference to assess the performance of the remaining beams with openings. Beam *O* was intended to gage the effects of creating an opening. The openings in beam *O-G* were filled in with non-shrink grout to simulate field conditions, while beam *O-FRP* was strengthened by externally bonded carbon fiber reinforced polymer plates in an attempt to restore the original response.

Table 1. Principal results of beam tested by Mansur et al. (1999).

Beam	Cylinder compressive strength, f_c' (MPa)	Load at shear cracking (kN)	At service load*		$\frac{P_u}{P_u \text{ of beam S}}$ (kN)
			Maximum crack width (mm)	Maxumum deflection (mm)	
S	30.8	200	0.27	4.38	1.000
O	29.7	134	>1.00	5.57	0.713
O-G	37.9	195	0.98	4.91	0.820
O-FRP	37.1	215	0.25	3.57	1.003

* Assumed service load = Ultimate load of beam S / 1.7

The cracking patterns of the beams after failure are presented in Fig. 6. It may be seen that beam *O*, which contained an opening exhibited a cracking pattern remarkably similar to that of the solid beam *S*. The major diagonal crack, which led to the failure traversed through the center of the opening. Beam *O-G*, in which the openings were filled with non-shrink grout, behaved in a similar manner except that failure crack bypassed the center of the opening and progressed along the opening periphery, as can be seen in Fig. 6. In contrast, beam *O-FRP*, which was strengthened by FRP plates, had almost the same behavior as the solid beam except that it had less number of narrower web cracks.

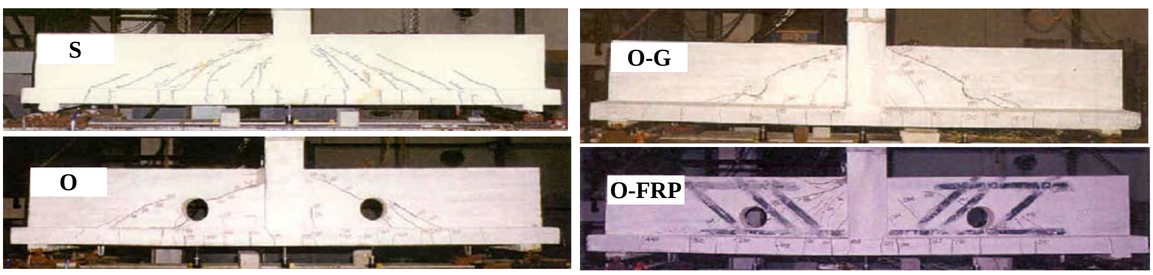


Fig 6. Cracking patterns of beams tested by Mansur et al. (1999).

The maximum width of cracks and midspan deflections of the beams are plotted against the applied load in Figs. 7 and 8, respectively. Table 1 shows the summary of principal test results. Taking the response of the solid beam *S* as the required target, it may be seen that creating an opening in existing beams leads to early cracking (Table 1), wider crack widths at all loading stages (Fig. 7), smaller post-cracking stiffness (Fig. 8), and significantly reduces the load carrying capacity of a beam (Table 1 and Fig. 8). This serves as a warning that drilling an opening in an existing beam might seriously affect the safety and serviceability of the structure.

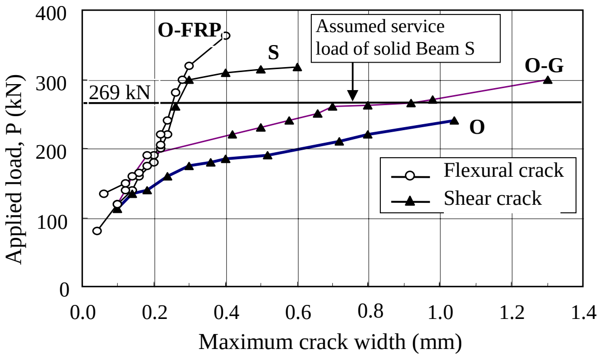


Fig 7. Load vs. maximum crack width curves (Mansur et al., 1999).

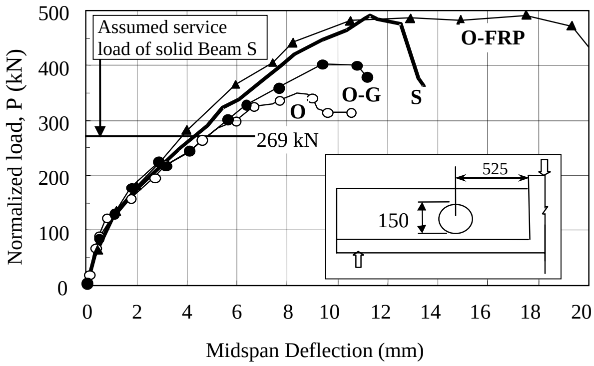


Fig 8. Load vs. midspan deflection curves (Mansur et al., 1999).

Filling the opening with non-shrink grout, as was done in beam *O-G*, results in some improvement over the corresponding beam *O* without any grout in-fill, but the overall performance was far beyond the target performance of the solid beam *S*. However, strengthening by externally bonded FRP plates as used in beam *O-FRP* can completely eliminate the weakness introduced by creating an opening in an already constructed beam.

The results of these beams clearly indicate that drilling an opening near the support region of an existing beam may seriously impair the safety and serviceability of the structure. Also, filling the opening by non-shrink grout is not adequate to restore the original strength and stiffness. The risk may, however, be minimized by limiting the size of opening or drilling the opening without cutting any stirrups. In any case, the designer must carefully analyze and assess the situation. Unless larger than usual factor of safety is incorporated in the original design or suitable measures to strengthen the beam is undertaken, no opening should be created in existing beams.

3. BEAMS WITH LARGE RECTANGULAR OPENING

Similar to a beam with small openings, incorporation of a large opening in the pure bending zone of a beam will not affect its moment capacity provided that the depth of the compression chord is greater than or equal to the depth of ultimate compressive stress block, and that instability failure of the compression chord is prevented by limiting the length of the opening (Mansur and Tan, 1999).

In practice, openings are located near the supports where shear is predominant. In such a case, tests have shown that a beam with insufficient reinforcement and improper detailing around the opening region fails prematurely in a brittle manner (Siao and Yap, 1990). When a suitable scheme consisting of additional longitudinal bars near the top and bottom faces of the bottom and top chords, and short stirrups in both the chords, as shown in Fig. 9, is furnished, then the chord members behave in a manner similar to a Vierendeel panel and failure occurs in a ductile manner. The failure of such a beam is shown in Fig. 10. Clearly, the failure mechanism consists of four hinges, one at each end of the top and bottom chords.

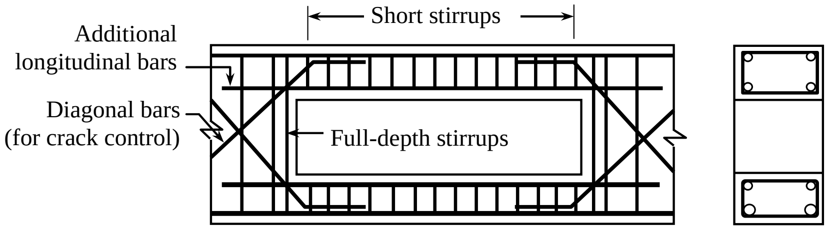


Fig. 9. A suitable reinforcement scheme for the opening region.

Fig 10. Ductile failure of a beam under combined bending and shear.

3.1 Analysis for Ultimate Strength

The experimental observations of the final mode of failure have led to the development of a method of analysis for predicting the ultimate strength of a beam with large rectangular opening (Mansur et al., 1984). It is based on the collapse load analysis in which the basic requirements of equilibrium, yield and mechanism are satisfied simultaneously. The main ingredients of this method, which yields a closed-form solution for the collapse load, are briefly described below for a simply-supported beam subject to a point load, *P*, at a solid section distant, *X*, from its right support, as shown in Fig. 11.

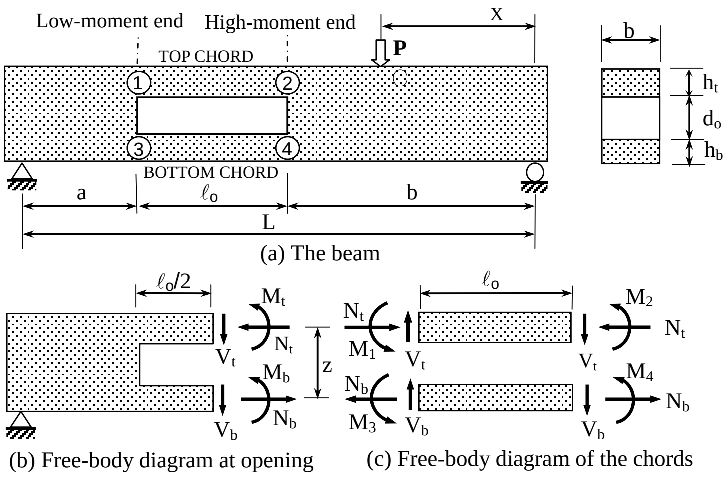


Fig 11. Beam with a large opening under bending and shear.

3.1.1. Equilibrium

For the beam in Fig. 11, the free-body diagram through the opening center and those of the chord members above and below it may be represented by Figs. 11(b) and (c), respectively. It may be noted that the unknown actions at the center of the opening are the axial forces (N_t and N_b), the bending moments (M_t and M_b), and the shear forces (V_t and V_b) in the chord members. Three equilibrium equations relate these six unknowns. These are:

$$M_t + M_b + N z = M_m \tag{7}$$

$$N_t + N_b = 0 \tag{8}$$

$$V_t + V_b = V_m \tag{9}$$

in which M_m and V_m are the applied moment and shear force, respectively, at the center of the opening. Thus, the beam is statically indeterminate to the third degree.

3.1.2. Yield Condition

As shown in Figs. 11(b) and (c), the chord members are subject to combined bending moment, *M*, shear force, *V*, and axial force, *N*. Therefore, an interaction curve (to be used as yield surface) among these stress resultants is required for the analysis. If it is assumed that the chord members are adequately reinforced in shear, then the interaction diagram between *M* and *N*, may be adopted as the required yield surface. One such interaction diagram is shown in Fig. 12.

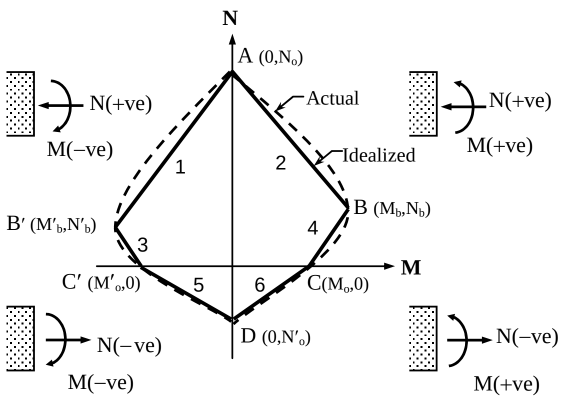


Fig 12. Piecewise linear approximation of yield surface.

For a beam, the top and bottom chord members can be considered as eccentrically loaded compression and tension members, respectively, and the interaction diagram for these members may be obtained by the general method of equilibrium and strain compatibility. For simplicity, the non-linear interaction curve may be approximated by the piecewise linear surface *ABCDC′B′A*, as shown by the solid lines in Fig. 12.

3.1.3. Collapse Mechanism

Consistent with experimental observations, the assumed mechanism consists of four hinges in the chord members, with one at each corner of the opening, as shown in Fig. 13.

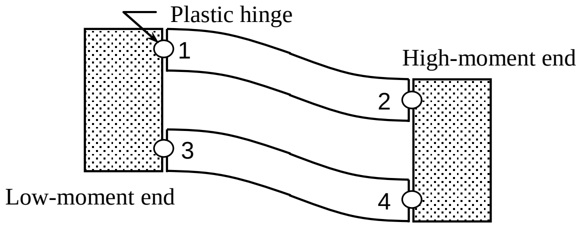


Fig 13. Assumed collapse mechanism for a beam with large openings.

3.1.4. Solution

It is assumed that the beam segments beyond the opening region are rigid, and collapse occurs by the formation of the above mechanism (Fig. 13). Hinges in the top chord member are denoted by 1 and 2 and in the bottom chord member by 3 and 4.

When the beam is subject to a positive bending moment, the axial stress resultant in the top chord member is a compressive force. The axial stress resultant in the bottom chord member would be a tensile force equal in magnitude to the thrust in the top chord. Consequently, the portion of the yield surface above the M-axis (Fig. 12) will be applicable to the top chord and that below the same axis will be applicable to the bottom chord.

In general, the chord members are not identically reinforced. Therefore, each member will have different yield surface. The respective yield surface will be unsymmetrical, that is, $M_b \neq M'_b$, $N_b \neq N'_b$, and $M_o \neq M'_o$ (Fig. 12). Thus, there are four possible combinations of the yield planes *AB′*, *B′C′*, *AB*, and *BC* on which hinges at locations 1 and 2 may form. Designated by Case 1, Case 2(a), Case 2(b), and Case 3, these combinations are clearly shown in Fig. 14. In all the cases, hinge at 3 forms on *C′D* and that at 4 on *CD*. It may be noted that if the top chord is symmetrically reinforced, only Cases 1 and 3 need to be considered.

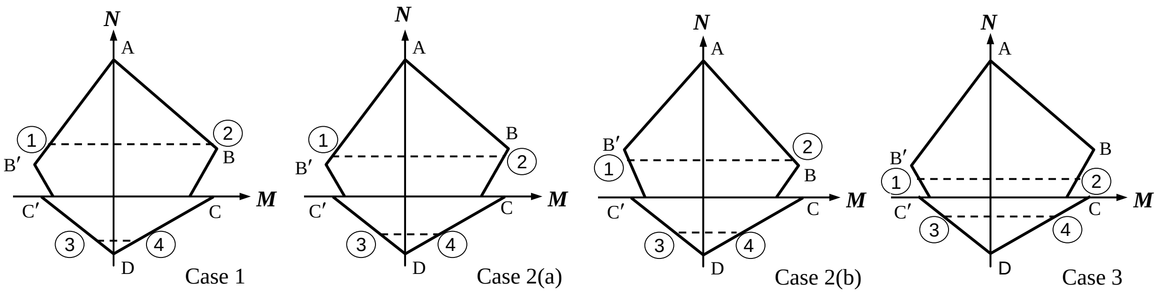


Fig 14. Possible combinations of yield planes for the formation of a mechanism.

Using the above basic information, a closed-form solution giving the exact collapse load may be obtained. The full details of this analysis may be found elsewhere (Mansur et al., 1984).

When the chord members are provided with adequate reinforcement, the beam may fail at a solid section beyond the opening. Therefore, the collapse load, P_u, as obtained from the foregoing analyses, must not be greater than the strength of the solid section. Otherwise, the later case will govern.

3.2. Structural Design

Structural design of beams with openings entails the check for ultimate strength and serviceability with respect to cracking and deflection. Although other methods like *plasticity truss* method or *strut-and-tie* method (Mansur and Tan, 1999) are available, the limit analysis method outlined above may be used for checking the strength of a beam with openings at collapse. Crack control is ensured through proper detailing of reinforcement at the corners of openings. In general, deflections have to be calculated and compared to allowable values.

3.2.1. Design for Ultimate Strength

Design is however, a reverse process to analysis, where the amount and disposition of reinforcement are generally sought. Although, the foregoing limit analysis method can be recast in a graphical form suitable to obtain direct design solutions (Mansur et al., 1985), the method is relatively complex and it requires development of new sets of design charts. The complexity arises mainly due to the consideration of the generalized arrangement of reinforcement in the chords. However, the design solution may be considerably simplified if the chord members are reinforced symmetrically. This is a feasible option because opening length represents only a small fraction of the total span of the beam.

In a general situation, the problem is statically indeterminate to the third degree, as mentioned in Art. 3.1. Equilibrium provides only three equations. Therefore, three additional equations need to be formulated to solve for the three unknown actions. This may be accomplished as outlined below.

When the chord members are symmetrically reinforced then the moments at two ends of each chord member (potential hinge location) are numerically the same at plastic collapse. That is, $M_1 = M_2$ and $M_3 = M_4$. From the free-body diagram of the chord members (Fig. 11c), it may be readily shown that the contraflexure points occur at midpoint of the chord members. This means that $M_t = 0$ and $M_b = 0$. Eq. (7) then reduces to

$$N z = M_m \tag{10}$$

If the total shear, V_m, through the center of the opening due to global action is suitably apportioned between the chord members, that is, if

$$V_t = k_v V_m \tag{11}$$

in which k_v is a known value, then the problem reduces to a statically determinate one and the critical sections at the ends of the chord members that are subject to combined bending, shear and axial force can be designed in the usual way following the provisions of any current building codes.

There are, however, three schools of thought regarding the distribution of applied shear between the chord members at an opening. The first, as proposed by Lorensten (1962), assumes that the compression chord carries the total shear and the tension chord merely acts as a link carrying no shear. This is probably true when the opening is near the bottom. The second proposal (Nasser et al., 1967 and Reagan and Warwaruk, 1967), distributes the total shear between the chord members in proportion to their cross-sectional areas. And the third, suggested by Barney et al., (1977), distributes the shear force in proportion to the flexural stiffness of the chord members. Accordingly,

Lorensten (1962): $$k = 0 \tag{12}$$

Nasser et al. (1967): $$k_v = A_t / (A_t + A_b) \tag{13}$$

Barney et al. (1977): $$k_v = I_t / (I_t + I_b) \tag{14}$$

Clearly, the three proposals would lead to widely varying amounts of shear to be assigned to each chord. However, such an assumption is not necessary if the chord members are symmetrically reinforced. The salient points in such a design process are described in the flowing steps:

Step 1: *Determine longitudinal reinforcement for the compression chord.*

First, assuming that the beam contains no openings, design the longitudinal reinforcement. If the beam is subject to a sagging moment, the main reinforcement, A_s, will be at the bottom. The top reinforcement will be lighter than the bottom reinforcement and the same amount is usually continued throughout the length of the beam, including the opening region. Thus, the top reinforcement in the compression chord is known. Use the same amount and arrangement at its bottom as additional reinforcement required to restore the strength and avoid brittle failure of the beam due to the provision of openings.

Step 2: *Determine the shear force carried by the compression chord.*
Since the amount and arrangement of longitudinal reinforcement in the compression chord are known, and as the axial force acting on it is given by Eq. (10), the moment capacity of the section may be estimated in the usual manner. Because of symmetry, the capacity in positive and negative bending will be numerically the same. Therefore, from the free-body diagram of Fig. 11(c), the amount of shear force that can be transmitted through the compression chord at ultimate may be obtained as

$$(V_u)_t = 2(M_u)_{t,2} / \ell_o \tag{15}$$

where ℓ_o is the length of opening.

Step 3: *Determine the moments and forces at critical sections and design the tension chord.*
The shear force carried by the tension chord will be the difference between the applied shear and that carried by the compression chord in accordance with Eq. (9). Due to reinforcement symmetry, contraflexure point will be at midspan. The moment at the critical end section is then given by

$$(M_u)_b = (V_u)_b \frac{\ell_o}{2} \tag{16}$$

With the axial tension given by Eq.(10), the required amount of longitudinal reinforcement can be obtained by following the standard design procedure. Reinforcement that has already been determined from the global action can now be taken into account to obtain the desired symmetrical arrangement of reinforcement in the tension chord. Design for shear is identical to a solid beam.

Use of design charts, similar to a column, may expedite the design process. A typical design chart (using the capacity reduction factor ϕ = 0.9) for symmetrical arrangement of reinforcement, approximated by straight lines, is shown in Fig. 15, where $\mu = f_y / 0.85 f_c'$ and $\rho_g = 2A_s / bh$. The simple design steps, as outlined above, are shown in this figure by the direction of arrows with blank circles as the targets.

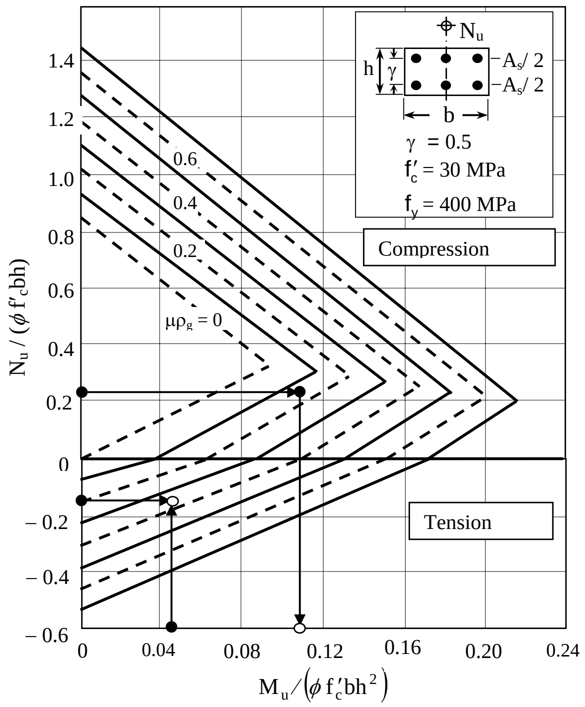


Fig 15. Typical design chart.

3.2.2. Design for Crack Control

There has been a common consensus that adequate control of crack widths under service load may be achieved by using full-depth stirrups adjacent to each side of the opening and/or diagonal bars placed at each corner as shown in Fig. 9. However, differences in opinion exist regarding the amount and the type of reinforcement, diagonal bars or full-depth stirrups, to be provided. According to Nasser et al. (1967), provide diagonal reinforcement to control crack widths. Including the capacity reduction factor, the amount to be provided at each corner of the opening is given by

$$A_d = \frac{\eta V_m}{\phi} \frac{1}{f_{yd} \sin \theta} \tag{17}$$

in which η = stress concentration factor, taken as 2, ϕ = capacity reduction factor, f_{yd} = yield strength of diagonal bars, and θ = inclinational of diagonal bars with the longitudinal axis of the beam.

According to Barney et al. (1977), use of full-depth stirrups alone on either side of the opening without considering any stress concentration would be adequate for crack control. Thus, the amount to be provided on each side is given by

$$A_v = \frac{V_m}{\phi} \frac{1}{f_{yv}} \tag{18}$$

It may be clearly seen that the above two proposals represent two extreme situations. Research conducted at the National University of Singapore (Mansur et al., 1984) suggests that a

50-50 combination of full-depth stirrups and diagonal bars with a stress concentration factor of 2 is preferable. Thus, the amounts of diagonal and full-depth stirrups are respectively given by

$$A_d = \frac{V_m}{\phi} \frac{1}{f_{yd} \sin\theta} \tag{19}$$

and

$$A_v = \frac{V_m}{\phi} \frac{1}{f_{yv}} \tag{20}$$

3.2.3. Calculation of Deflection

It is difficult to set limiting span-effective depth ratios for beams with openings to indirectly satisfy the serviceability limit state of deflection. Therefore, deflections of beams with large openings should be calculated and checked against permissible values. A simple method to estimate maximum service load deflection for simply-supported beams with openings is as follows (Barney et al., 1977).

The model shown in Fig. 16 considers that the chord members act as struts framing into rigid abutments on each side of the opening. The effective length, ℓ_e, of the struts is conservatively taken as the distance between the full-depth stirrups on each side of the opening. To reflect the Vierendeel truss action observed in the tests (see Fig. 10), points of contraflexure are assumed at the mid-length of each strut. Thus each half of the chords bends as a cantilever, as shown. Denoting the moments of inertia for the top and bottom struts as I_t and I_b, respectively, the relative displacement of one end of the opening with respect to the other under the action of V may be obtained as

$$\delta_v = \frac{V\ell_e^3}{12E_c\left(I_t + I_b\right)} \tag{21}$$

where E_c is the modulus of elasticity of concrete. Under service load, I_t may be based on gross concrete section while I_b can be conservatively based on a fully cracked section.

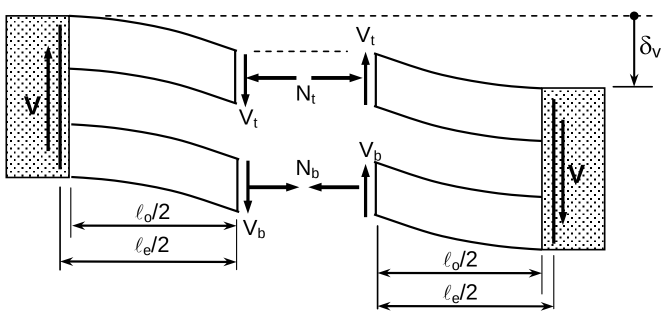


Fig 16. Model for the estimation of deflection at opening (Barney et al., 1977).

The maximum deflection of the beam can be calculated as

$$\delta = \delta_w + \delta_v \tag{22}$$

where δ_w is the maximum deflection in the absence of opening.

A more rigorous method to calculate deflections that entails an elastic analysis is also available (Mansur et al., 1992). In the method, the beam is treated as a structural member with several segments constituting the portions with solid beam sections and those with sections traversed by the opening. An equivalent stiffness is adopted for the latter segments and the beam can be analyzed using methods such as the Direct Stiffness Method to obtain the maximum beam deflection under service load. The general guidelines for the selection of the size and location of openings in a beam is available elsewhere (Mansur and Tan, 1999).

3.3 Multiple Openings and Design of Posts

When two or more openings are placed close to each other in a beam, the design of each opening may be carried out in a manner illustrated as above. However, in this case, design and detailing of the post, that is, the element between two adjacent openings, need special attention.

Tests carried out at the laboratory of Portland Cement Association (ACI-ASCE, 1973) have indicated that closely spaced multiple openings can be placed in a beam if each opening has adequate side reinforcement. Fig. 17 shows an inverted T-beam with multiple rectangular openings separated by adequately reinforced posts after it has been tested to failure (Tan et al., 1996).

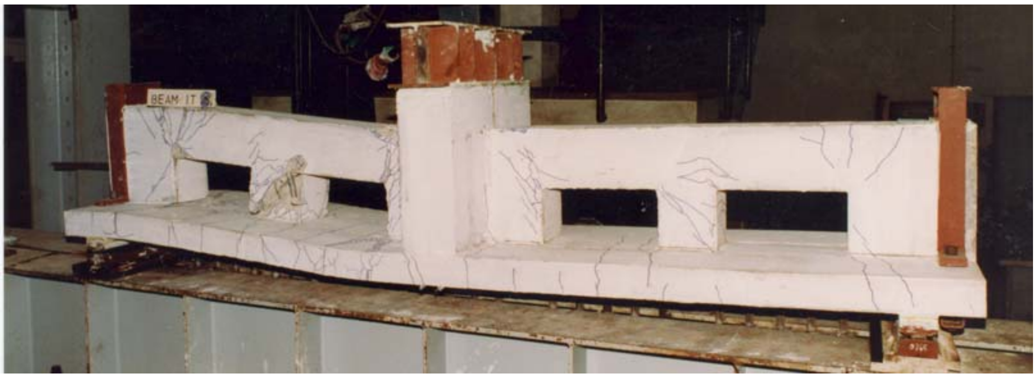


Fig 17. Failure of a beam with multiple rectangular openings.

To ensure that the posts behave rigidly, Barney et al. (1977) recommended that adjacent openings should be separated by posts having overall width-to-height ratios of at least 2.0 where the width of the posts is the distance between adjacent stirrups. It was also suggested that the nominal design shear stress for the posts be limited to $0.17\sqrt{f_c'}$ (MPa).

When two openings are placed close to each other, it is evident from the free-body diagram shown in Fig. 18 that a horizontal shear, V_p, compression force, N_p, and bending moment, M_p, act on the post between the openings.

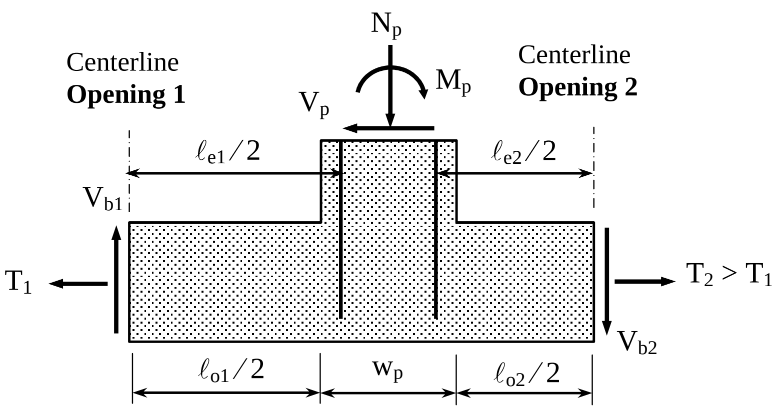


Fig 18. Forces acting on post between adjacent openings.

Assuming that points of contraflexure occur at the mid-length of the chord members in each opening, equilibrium of forces gives

$$V_p = T_2 - T_1 \tag{23}$$

$$N_p = V_{b1} - V_{b2} \tag{24}$$

$$M_p = (T_2 - T_1)\left(d_o + \frac{d_b}{2}\right) - V_{b1}\left(\frac{\ell_{e1} + w_p}{2}\right) - V_{b2}\left(\frac{\ell_{e2} + w_p}{2}\right) \tag{25}$$

where w_p = width of post, taken as the distance between vertical stirrups in the post adjacent to the sides of the two openings; T = tensile force acting on the bottom chord member of opening; V_b = vertical shear force acting on the bottom chord member of opening; e = eccentricity of opening; d_b = depth of bottom chord member; ℓ_o = length of opening, taken as the distance between the vertical stirrups adjacent to the two sides of the opening; d_o = depth of opening; and subscripts 1 and 2 refer to the openings to the left and right of the post, respectively. Knowing the values of V_p, N_p, and M_p, the required reinforcement can be obtained by designing the post as a short, braced column.

4. CONCLUDING REMARKS

This paper gives a brief but comprehensive treatment of the analysis and design of reinforced concrete beams that contain transverse openings through the web and are subjected to combined bending and shear. Recognizing the differences in beam behavior, circular and large rectangular openings are treated separately. Practical situations of drilling an opening in existing beams and special design considerations for beams with multiple openings are also briefly discussed. It has been shown that the design method for large rectangular openings may be considerably simplified if it is decided to use symmetrical arrangement of reinforcement in the chord members. Further details are available in the only book (Mansur and Tan, 1999) available to date on openings through concrete beams.

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