Modeling Dynamics of Parallel Milling Processes in Time-Domain

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Abstract: The use of parallel milling processes is increasing in various industries due to several advantages of these machine tools. Parallel milling processes are the processes where more than one milling tool simultaneously cut a workpiece. Due to the increased number of cutting tools, they have the potential for considerable increase in productivity as a result of higher material removal rate (MRR). However, dynamic interactions between milling tools may reduce stability limits. Generally, direct dynamic coupling between two milling tools on such a machine is weak since they are located on different spindles. However, there can be a strong dynamic coupling in case of milling a flexible workpiece. In this case, the vibrations caused by one of the tools may have regenerative effects on the other one. In order to address this problem, a stability model that works in time domain has been developed. The model is capable of simulating cases where two flexible milling tools are cutting a flexible workpiece. Several example cases are simulated with the model and results are presented.

Keywords: Stability, simultaneous/parallel milling, chatter vibrations

1. INTRODUCTION

In parallel milling more than one cutting tool cut a workpiece at the same time. Since number of cutting tools are higher with respect to standard milling, they have the potential for increased material removal rate; hence improved productivity. This potential can be used as long as chatter vibrations are eliminated and stable processes are achieved. Chatter vibrations can be avoided using stability diagrams which can be predicted by modeling the dynamics of parallel milling processes. Thus, understanding of parallel milling dynamics is critical for high productivity.

Parallel milling operations can be performed on machining centers with dual or multi spindles. Alternatively, they could be performed on turning centers using live tooling or mill-turn machines. In all cases, cutting tools may be dynamically interdependent or independent depending on the machine configuration and workpiece flexibility. Milling tools are generally on independent spindles or turrets. In case the workpiece is rigid, the dynamic coupling between the tools will not be significant. Hence, the stability of these processes can be analyzed by standard milling stability models, e.g. by the model presented in [Budak, E. and Altintas, Y., 1998]. On the other hand, dynamic cutting forces and displacements on a milling tool may affect the other milling tool(s) if the workpiece is flexible. For these cases, stability models that can simulate the interaction between the milling tools are needed.

There has been considerable number of works in 3-axis milling stability formulation. [Minis et al, 1990] solved the 2-dof milling stability in an iterative manner. Later, [Budak and Altintas, 1998] formulated the milling stability analytically and developed single and multi-frequency methods to obtain stability diagrams. Added lobe phenomenon which is seen in low radial immersion conditions has been presented by several authors ([Davies et al., 2002], [Insperger et al., 2003], [Merdol and Altintas, 2004]). [Campomanes and Altintas, 2003] and [Sims, 2005] are among the authors who developed time-domain models to simulate milling process dynamics. The most notable advantage of time-domain models is that nonlinearities such as loss of tool-material contact can be taken into consideration. Generating stability diagrams using time-domain models, on the other hand, is computationally expensive.

There are very few works on dynamics of parallel machining operations. [Lazoglu et al, 1998] developed a time-domain model for parallel turning operations, and using simulation results they showed that parallel working tools decrease the stability limits of each other. Later, [Ozdoganlar and Endres, 1999] formulated the dynamics of the parallel turning process and presented experimental verification on a modified vertical milling machine. [Olgac and Sipahi, 2005] developed an analytical method for prediction of stability diagrams for simultaneous machining. They basically determine the stability limits by analyzing the characteristic roots of the system.

In the paper, a time-domain model for parallel milling processes with two milling tools cutting a common workpiece is presented. The definitions and formulations for parallel milling process dynamics are given in the next section. Time-domain model and overview of the method used to predict stability diagrams are presented in section 3. Finally, the results of the model are demonstrated on example cases in the last section.

2. DYNAMICS OF PARALLEL MILLING

Definitions of the coordinate systems and process parameters used in the time-domain model are presented in this section. Chip thickness and cutting force formulation are given next. Then, calculation of dynamical response of tools and workpiece to cutting forces is presented.

2.1. Coordinate systems and Process Parameters

An example parallel milling process is illustrated in Figure 1. In this process two flexible milling tools are cutting a flexible workpiece simultaneously. The cutting tool on the upper side is numbered as the first tool and the tool below is named as the second tool. Three coordinate systems are used to represent the parallel milling process. The first coordinate system is the XYZ coordinate system on the workpiece. X, Y and Z axes are aligned with the machine tool axes. The other coordinate systems are the tool coordinate systems. $x_1y_1z_1$ is the coordinate system on the first cutting tool where x_1 represents the feed direction, z_1 is the tool axis direction and y_1, which is the cross-feed direction, is determined according to the right handed coordinate system notation. Similarly, $x_2y_2z_2$ is the coordinate system on the second milling tool.

The transformations of displacements or forces among these three coordinate systems are necessary in the model. The transformation of entities from $x_1y_1z_1$ and $x_2y_2z_2$ to XYZ coordinates can be performed by two transformation matrices. T_1 and T_2, which transform from $x_1y_1z_1$ to XYZ and from $x_2y_2z_2$ to XYZ, respectively, are presented in the following equation:

$$T_1 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}, \quad T_2 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \tag{1}$$

In the example process shown in Figure 1, the milling tools are parallel to each other, i.e. z_1 and z_2 are parallel. This is a common configuration seen on parallel machine tools. In this paper, the model is developed for the cases where z_1 and z_2 are parallel but the formulation can be extended to other cases with slight modifications.

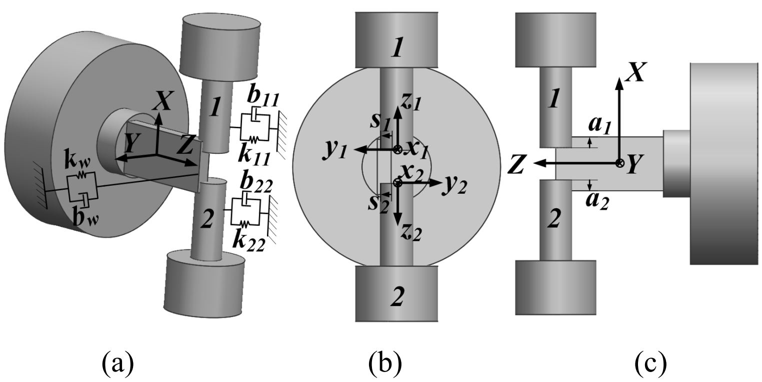


Figure 1; An example parallel milling process (a) 3D view (b) XY view (c) XZ view

Since there are two cutting tools in the parallel milling processes, the number of cutting parameters doubles. The process parameters for the i^{th} milling tool are defined here. Axial and radial depths are represented by a_i and s_i as shown in Figure 1. The spindle speed and feed per tooth are symbolized by rpm_i and f_i, respectively. The clockwise or counter-clockwise rotating tools can be used at the same time. Depending on the type of the cutting tool, and workpiece orientation with respect to the cutting tools, the cutting types can be up-milling or down-milling. For example, if both of the milling tools are rotating in clockwise direction in the example process (Figure 1), the first tool is cutting in up-milling mode while the second tool is cutting in down-milling mode. The immersion angle of the j^{th} cutting flute at the tool tip which is measured from positive y_i direction is represented by φ_{ij}. In general, the cutting tools may not contact the workpiece at the same angular position; hence there will be a lag angle, ψ, between the flutes of milling tools. The lag angle can be controlled if the spindles are vector controlled spindles, otherwise lag angle is not under operator's control.

2.2. Chip Thickness

The chip thickness on the i^{th} cutting tool depends on the dynamic displacement vector $\boldsymbol{d_i}$, the local immersion angle $\varphi_{ij}(z)$ and feed per tooth f_i. The dynamic displacement vector $\boldsymbol{d_i}$ represents the relative displacements of the i^{th} milling tool with respect to the workpiece.

$$\boldsymbol{d_i} = \begin{bmatrix} \Delta x_i \\ \Delta y_i \\ \Delta z_i \end{bmatrix} = \begin{bmatrix} x_{ti}(t) - x_{ti}(t-\tau_i) \\ y_{ti}(t) - y_{ti}(t-\tau_i) \\ z_{ti}(t) - z_{ti}(t-\tau_i) \end{bmatrix} - \begin{bmatrix} x_{wi}(t) - x_{wi}(t-\tau_i) \\ y_{wi}(t) - y_{wi}(t-\tau_i) \\ z_{wi}(t) - z_{wi}(t-\tau_i) \end{bmatrix} \tag{2}$$

where x_{ti}, y_{ti} and z_{ti} represent the present displacements of the i^{th} cutting tool in x_i, y_i and z_i directions. Similarly, x_{wi}, y_{wi} and z_{wi} are the displacements of the workpiece on the region that is in contact with the i^{th} cutting tool. The delayed terms are the corresponding displacements one tooth period τ_i before. τ_i depends on the spindle speed rpm_i and number of flutes n_i on the i^{th} cutting tool.

$$\tau_i = \frac{rpm_i}{60 n_i} \tag{3}$$

The displacements of the tools in the axial direction do not result in regenerative effect. Thus, the dynamic chip thickness is calculated using the following formula:

$$h_i = (f_i + \Delta x_i)\sin\varphi_{ij}(z_i) + \Delta y_i \cos\varphi_{ij}(z_i) \tag{4}$$

The local immersion angle $\varphi_{ij}(z)$ varies along the tool axis depending on the following equation:

$$\varphi_{ij}(z_i) = \varphi_{ij} - \frac{\tan(\beta_i)}{R_i} z_i \tag{5}$$

where β_i and R_i are the helix angle and the radius of the i^{th} milling tool, respectively; z_i represents the axial position on the milling tool.

2.3. Dynamic Cutting Forces

Using the linear-edge force model [Budak, et al., 1996], differential cutting forces in radial, tangential and axial directions on the i^{th} cutting tool's j^{th} flute can be written as follows:

$$\begin{aligned} dFr_{ij}(\varphi_{ij}) &= (K_{re} + K_{rc} h_i) dz_i \\ dFt_{ij}(\varphi_{ij}) &= (K_{te} + K_{tc} h_i) dz_i \\ dFa_{ij}(\varphi_{ij}) &= (K_{ae} + K_{ac} h_i) dz_i \end{aligned} \tag{6}$$

where K_{rei}, K_{tei}, K_{aei} and K_{rci}, K_{tci}, K_{aci} are radial, tangential, axial edge and cutting force coefficients on the i^{th} tool, respectively. dz_i is the height of the axial differential element. In previous works by [Budak, E., 2006] and [Altintas, 2000], the calculation of static cutting forces was presented. In this paper, in order to calculate dynamic cutting forces, the static force formulation in [Budak, E., 2006] is modified by using the dynamic chip thickness formulation presented in Eq.(4). Finally, dynamic cutting forces in x_i, y_i and z_i directions are determined for given immersion angle of φ_i as follows:

$$
\begin{aligned}
Fx_i(\varphi_i) &= \sum_{j=1}^{n_i} \int_{z_{\lim 1}}^{z_{\lim 2}} dFx_{ij}(\varphi_{ij}) \\
Fy_i(\varphi_i) &= \sum_{j=1}^{n_i} \int_{z_{\lim 1}}^{z_{\lim 2}} dFy_{ij}(\varphi_{ij}) \\
Fz_i(\varphi_i) &= \sum_{j=1}^{n_i} \int_{z_{\lim 1}}^{z_{\lim 2}} dFz_{ij}(\varphi_{ij})
\end{aligned}
\tag{7}
$$

$z_{\lim 1}$ and $z_{\lim 2}$, are the integration limits that are also used for modeling of standard 3-axis milling processes [Altintas, 2000; Budak, 2006].

2.4. Tool and Workpiece Dynamics

Tool and workpiece dynamics can be represented by transfer functions, or frequency response functions, which are measured by impact hammer tests. The response of the i^{th} tool at its tip to the dynamic cutting forces can be obtained using the following relation:

$$
\begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} = \begin{bmatrix} G_{x_i x_i} & G_{x_i y_i} & G_{x_i z_i} \\ G_{y_i x_i} & G_{y_i y_i} & G_{y_i z_i} \\ G_{z_i x_i} & G_{z_i y_i} & G_{z_i z_i} \end{bmatrix} \begin{bmatrix} Fx_i \\ Fy_i \\ Fz_i \end{bmatrix} \tag{8}
$$

The cross-transfer functions, e.g. G_{xizi}, G_{yizi} etc., are neglected since their magnitudes with respect to the direct ones are considerably low. Moreover, the direct transfer functions in tool axis direction, i.e. G_{zizi}, are also neglected since milling tools are relatively rigid in this direction. So, $G_{x_i x_i}$ and $G_{y_i y_i}$ are the only transfer functions required in the formulation. In the hammer tests, excitation is given from the tool tip with a hammer and response of the tool is measured by an accelerometer at the tool tip. Since the only response at the tool tip is of interest, one transfer function measurement is adequate although the tool tip response can include multi-dof behavior. The modal data is fit to measured transfer functions using Cutpro® software. $G_{x_i x_i}$ and $G_{y_i y_i}$ can be calculated using the following relation:

$$
G_{p_i p_i}(j\omega) = \sum_{r=1}^{q} \frac{\frac{1}{m_r}}{(j\omega)^2 + 2\xi_r \omega_{n,r}(j\omega) + \omega_{n,r}{}^2} \quad , p = x,y \tag{9}
$$

where q represents number of modes determined from the transfer function measurement at the tool tip and ω is the frequency variable. m_r, ζ_r and $\omega_{n,r}$ are modal mass, modal damping ratio and modal natural frequency corresponding to the r^{th} mode.

Unless the workpiece is flexible in Y and/or Z directions, the dynamics of the tools in the considered case are independent of each other. In such a case the dynamics and stability of the tools can be analyzed separately. However, the workpiece in the considered case is flexible, and thus it is a dynamically parallel process. The flexibility of the workpiece in X and Z directions can be neglected since the workpiece is noticeably rigid in these directions with respect to Y direction. The frequency response functions at two different points -one on the upper side and one on the lower side of the workpiece- where the first and second tool is in contact with the workpiece are measured (*Figure 2*). Since the feed in both of the cutting tools is in –Z direction, the workpiece dynamics is variable during the process. The stability analysis is performed for the beginning of the process; hence the measurement points are selected close to the beginning of the process as shown in *Figure 2*. However, the stability analysis can be repeated at different machining stages by using the updated part frequency responses for the relevant points [Alan, S., et al., 2010]. The response of the workpiece at two points to the cutting forces on the workpiece can be determined using the following equation.

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \qquad (10)$$

where Y_i is the displacement of the workpiece and F_i is the cutting force at the i^{th} point. The transfer functions, G_{ik} can be defined using the following equation [W. de Silva, C., 2007]:

$$G_{ik}(j\omega) = \sum_{r=1}^{q_w} \frac{\frac{U_{ir}U_{kr}}{m_r}}{(j\omega)^2 + 2\xi_r\omega_{n,r}(j\omega) + \omega_{n,r}^2} \quad , i, \mathrm{k} = 1,2 \qquad (11)$$

U_{ir} and U_{kr} are the elements of the modal shape matrix which are obtained by Cutpro® modal analysis module and q_w is the number of modes used in the analysis. The size of the mode shape matrix is 2xq_w. Modal data and mode shape matrix are obtained by modal analysis of two measured transfer functions which are G_{11} and G_{12}. The displacements of the workpiece in Eq.(10) can be transformed to tool coordinate systems using the inverse of transformations presented in Eq. (1).

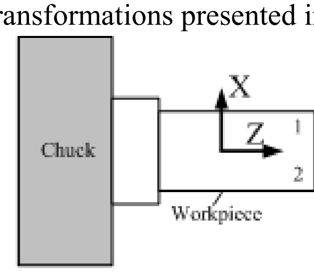


Figure 2; Measurement points on the workpiece

3. TIME-DOMAIN MODEL

The time-domain model that simulates the dynamic behavior of parallel milling operations needs all the process parameters which are stated in the previous sections. Moreover, dynamic chip thickness, dynamic cutting forces, tool and workpiece dynamics should be written in terms of process parameters. In the model, the parallel milling process is simulated at discrete time intervals in Simulink® environment. Each discrete time corresponds to an immersion angle on each tool. Dynamic displacements of the cutting tools and workpiece are calculated using the measured transfer functions and modal shape matrices by Eq. (8) and Eq. (10), respectively. The relative displacements of the tools with respect to the workpiece at the present time and one tooth period before are used to form the dynamic displacement vector $\boldsymbol{d_i}$ by Eq.(2) that is responsible for the regeneration effect. Since the displacements in the z_i directions do not affect the regeneration mechanism for the presented parallel milling process, the first two terms which include displacements in x_i and y_i directions are used to calculate the dynamic chip thickness using Eq.(4). Finally, cutting forces corresponding to the calculated chip thickness values and given process parameters are calculated for the present immersion angle. This calculation steps are continued with the next discrete simulation time. The block diagram notation of the presented time domain model is given in Figure 3. Depending on the variation of dynamic cutting forces, displacements and/or frequency spectrum of these variations, processes can be classified as stable, marginal or unstable.

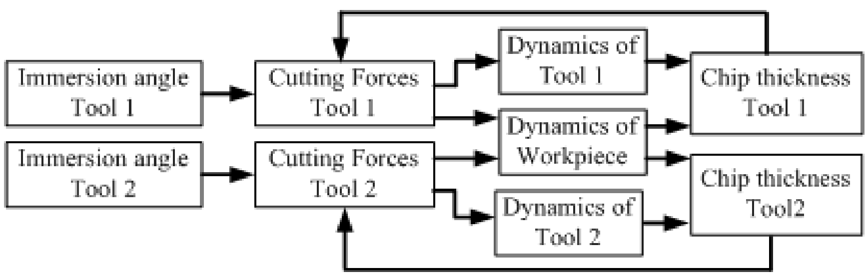


Figure 3; Block diagram notation of the time domain model

The stability diagrams are used to determine stable process parameters to avoid chatter vibrations and the presented time-domain model can be used to predict stability diagrams for a parallel milling process. There are two cutting tools in the presented parallel milling process but the stability diagram for each tool cannot be obtained independently since there is dynamic coupling between two tools through the flexible workpiece. Stability diagram for only one of the tools can be predicted after the process parameters of the other cutting tool are all set. With that purpose, a spindle speed range of interest is selected. For each spindle speed, the time domain model is simulated starting from low to higher axial depth of cuts until the process becomes unstable.

The spindle speeds and number of flutes of the cutting tools can be different in parallel milling which results in different tooth periods, i.e. different delay terms in Eq.(2). Furthermore, if the feed velocities are also different, there will be a relative translational motion between the two tools. This makes determination of the interaction between the cutting tools difficult; hence feed per tooth value of the second tool is selected according to the following equation in order to have the same feed:

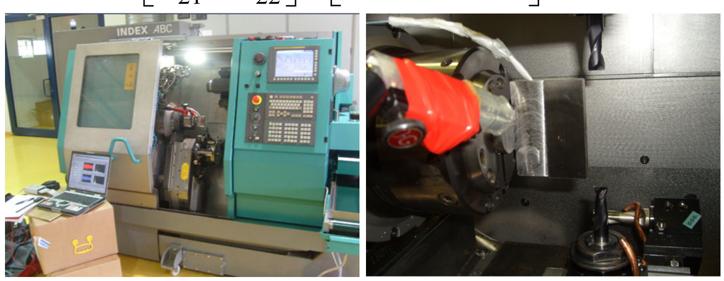
$$f_2 = f_1 \frac{rpm_1 * n_1}{rpm_2 * n_2} \qquad (12)$$

4. SIMULATION RESULTS

The presented time-domain model is simulated on several example cases. Although the experimental verification is not presented here, the measurements for the example cases are performed on an Index ABC parallel machining centre (Figure 4(a)). The workpiece and two milling tools are also shown in (Figure 4(b)). The workpiece material is 1050 steel. The cutting tools are clock-wise rotating, 12 mm diameter and 2 flute flat-end mills with 30 deg. helix angles. The overhang lengths of the upper tool and the lower tool are 40.5 mm and 47.8 mm, respectively.

The measured modal data of the first tool and second tool, which include natural frequencies (f_n) damping ratios (ζ) and stiffness values (k), are presented in Table 1. The modal data of the workpiece is tabulated in Table 2, and the corresponding modal shape matrix U is given in Eq.(13). Note that the first mode is a bending mode whereas the second mode is a torsional one.

$$\boldsymbol{U} = \begin{bmatrix} U_{11} & U_{21} \\ U_{21} & U_{22} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0.930 & -0.844 \end{bmatrix} \qquad (13)$$



(a) (b)

Figure 4; (a) Parallel machining centre (b) Workpiece and two milling tools in the parallel milling process

Using the stability model presented in [Budak, E. and Altintas, Y., 1998], the stability diagram of each tool working in single mode can be determined. The flexibility of the workpiece is also included in the calculations. The first tool's absolute stability limit is determined as 0.5 mm with chatter frequency of 1550Hz when it's working in up milling mode (Figure 5(a)). The absolute stability of the second tool in down milling is calculated as 0.3mm at chatter frequency of 760 Hz (Figure 5(b)). When the second tool's cutting type is changed to up milling, the absolute stability increases to 0.9 mm at 734 Hz chatter frequency. Comparing the calculated chatter frequencies with the natural frequencies of the system, it can be concluded that the workpiece flexibility is dominant for the stability of this case.

Tool 1	x_1 direction			y_1 direction		
Mode#	f_n (*Hz*)	ζ	k(*N/m*)	f_n(*Hz*)	ζ	k(*N/m*)
1	2127.2	$5.279*10^{-2}$	$9.107*10^{6}$	2275.2	$6.234*10^{-2}$	$9.459*10^{6}$
Tool 2	x_2 direction			y_2 direction		
Mode#	f_n (*Hz*)	ζ	k(*N/m*)	f_n(*Hz*)	ζ	k(*N/m*)
1	1788	$1.048*10^{-1}$	$2.403*10^{7}$	1731.1	$1.379*10^{-2}$	$1.777*10^{8}$
2	2036.2	$8.883*10^{-2}$	$4.276*10^{7}$	1909.9	$3.288*10^{-2}$	$1.400*10^{7}$
3	-	-	-	2101.6	$2.977*10^{-2}$	$4.910*10^{7}$

Table 1 ; Modal data for the milling tool

	Y direction		
Mode#	f_n (*Hz*)	ζ	k(*N/m*)
1	746.5	$1.691*10^{-2}$	$4.997*10^{6}$
2	1550.1	$2.165*10^{-3}$	$1.160*10^{7}$

Table 2 ; Modal data for the workpiece

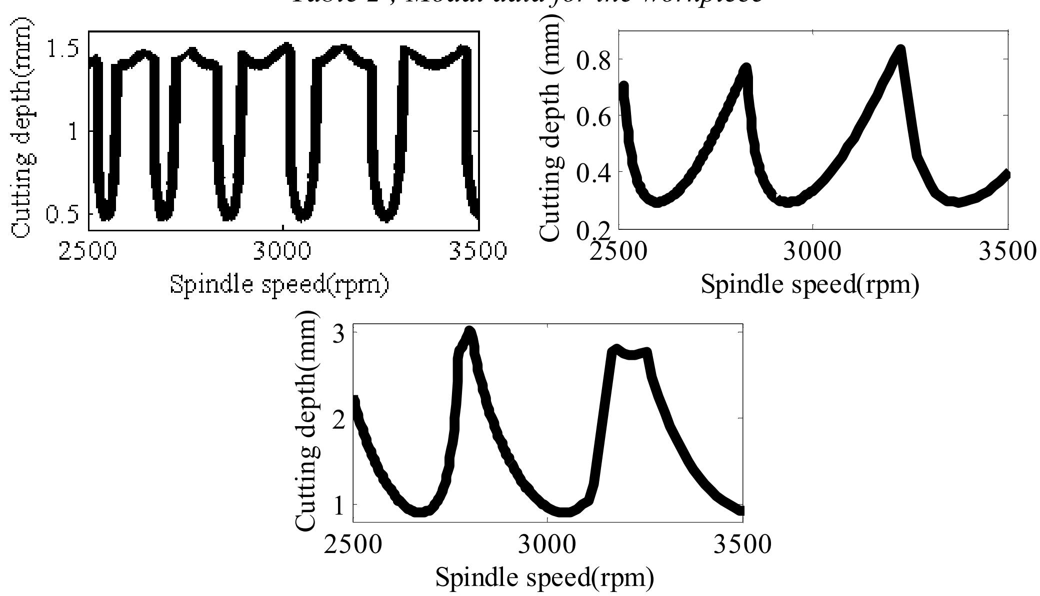


Figure 5; Stability limit diagrams of the tools working in single mode (a)Tool 1(up-milling) (b) Tool 2(down milling) (c) Tool 2(up milling)

The effect of parallel milling on stability limits is analyzed on an example case. The cutting parameters for the first cutting tool are tabulated in Table 3. The second tool is also performing a half immersion operation. The specific cutting force coefficients for the second tool are taken as equal to the ones for the first tool. Since the edge forces do not affect the regeneration mechanism, they are taken as zero. The stability limits for the second tool are predicted for both up milling and down milling modes at several spindle speeds in the range of 2950-3200 rpm by the presented time domain model. These limits are presented in Figure 6 for up milling and down milling operations, separately. In both of the cases the absolute stability is predicted to be 0.35mm. However, the maximum stability limits at the presented lobes are considerably different. The maximum stability limit of the second tool is 1.5 mm for up milling and becomes 3 mm when the cutting type is changed to down milling.

It is of interest to compare the stability diagrams for single mode operations in Figure 5(a) and (b) with the stability diagrams of the parallel milling process in Figure 6. When the second tool is in down milling mode, absolute stability is slightly increased to 0.35mm from 0.3 mm due to the effect of the first tool. Moreover, the maximum stability at the presented lobes is increased to 1.5 mm from 0.8 mm. However, the absolute stability limit is decreased to 0.35 mm from 0.9 mm under the effect of the first tool when the mode of the second tool is up milling. But the maximum stability at the presented lobes is left unchanged around 3 mm. As a result, depending on the milling modes, and whether absolute or maximum stability limits are of interest, the parallel milling may offer certain advantages and disadvantages. However, the additional material removed by the other tool, i.e. the total stable material removal rate, should be taken into account in such comparisons.

a_1	0.5 *mm*
$nrpm_1$	3000 rpm
f_1	0.05*mm*
s_1	Half immersion up milling
K_{rc1}, K_{tc1} K_{ac1}	484, 1597,517 MPa

Table 3 ; Cutting parameters of the first tool in the example

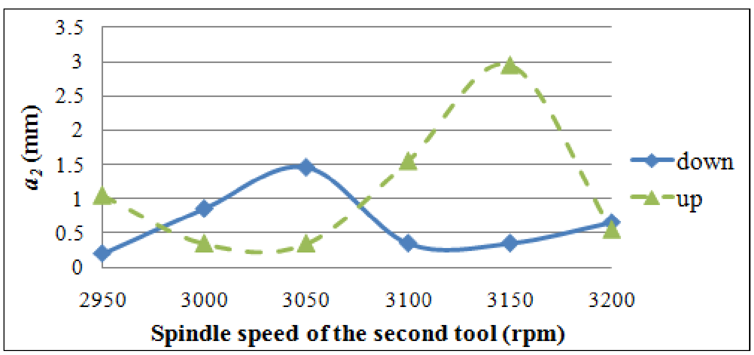


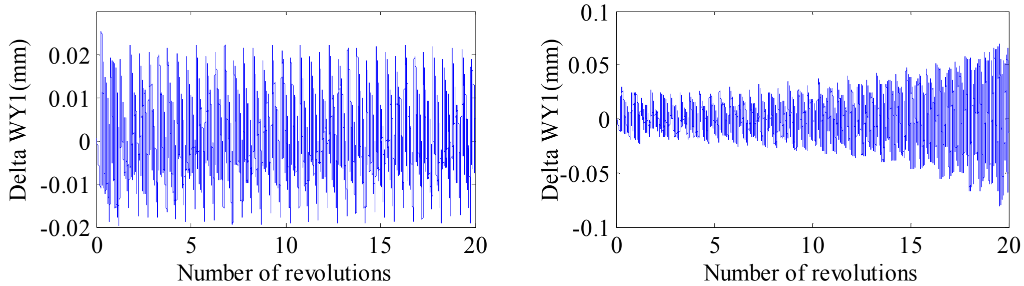
Figure 6; Stability limit diagrams of the second tool for half immersion up and down milling cases(The parameters of the first tool are tabulated in Table 3)

In order to illustrate the results of the time domain model used to determine the stability limits given in Figure 6, the variation of workpiece displacements in Y direction at node 1 (Figure 2) is presented for a stable and unstable case in

(a) *(b)*

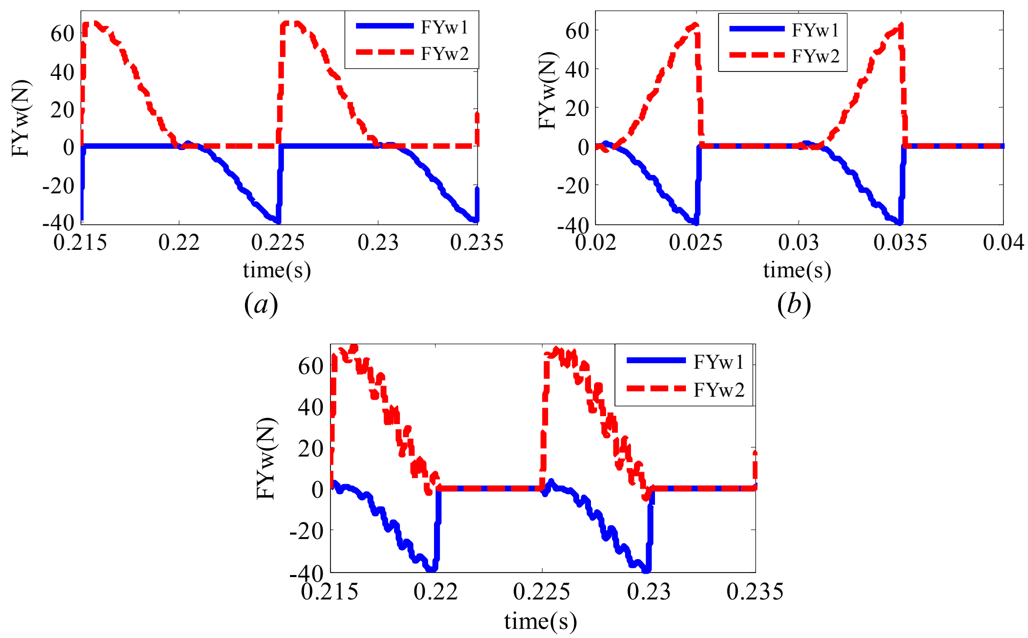
Figure 7. The spindle speed of the second tool is 3000 rpm, cutting mode is down milling and the cutting depths of the second tool are 0.8 mm and 1 mm. Since the stability limit at 3000 rpm for down milling case is determined as 0.85 mm in Figure 6,

cutting depth of 0.8 mm results in a stable process while cutting depth of 1 mm provides a unstable operation which can be seen in Figure 7.



(a) *(b)*

Figure 7; Variation of displacements of the workpiece in Y direction at node 1 (a)a_2=0.8mm (b) a_2=1mm



(*c*)

Figure 8; Variations of cutting forces, FY_{w1} and FY_{w2} (a_2=0.8mm) (a) down milling (b)up milling (c) down milling, lag angle=90 deg

It should be noted that the milling mode, i.e. up milling or down milling, changes the form of cutting force variation. Hence, the interaction of two cutting tools is affected by selection of the milling mode. This effect is shown for a point on Figure 6 where a_1 is 0.5 mm, a_2 is 0.8 mm, and rpm_1 and rpm_2 are both 3000 rpm. The variations of cutting forces, FY_{w1} and FY_{w2}, are presented for one tool rotation in Figure 8(a) and (b). Full lines and dotted lines represent FY_{w1} and FY_{w2}, respectively. FY_{w1} is the cutting force on the workpiece in Y direction due to the first cutting tool whereas FY_{w2} is the cutting force on the workpiece in the same direction due to the second tool. The first tool is in up milling mode while the second tool is in down milling mode in Figure 8(a). In this case, there is a phase difference between forces and there is no interaction between them. On the other hand, in Figure 8(b), both of the cutting tools are in up milling mode and it's seen that they are in phase with each other. For that reason, there is more interaction between forces in this case. This behavior also affects the stability of the system, i.e., the first case is stable while the second case is unstable as presented in Figure 6.

As presented in the section 2.1, there can be a lag angle, ψ, between the first tool and second tool. Its effect on the variation of forces is presented on Figure 8(c). In this case, the lag angle of 90 deg. is applied on the case presented in Figure 8(a). It is seen that there is a time shift between the forces FY_{w1} and FY_{w2} when there is lag angle. Due to its effect on the form of variation of cutting forces, lag angle may affect the stability limits. This effect was only seen in the regions close to the stability limits. For example, when lag angle of 90 deg is applied on the case presented in Figure 7(a) where a_2 is 0.8 mm while stability limit is 0.85 mm, originally stable process becomes unstable as shown in Figure 9. On the other hand, it was seen that the stability is not influenced by the lag angle in the regions away from the stability limits although these results are not presented in the paper.

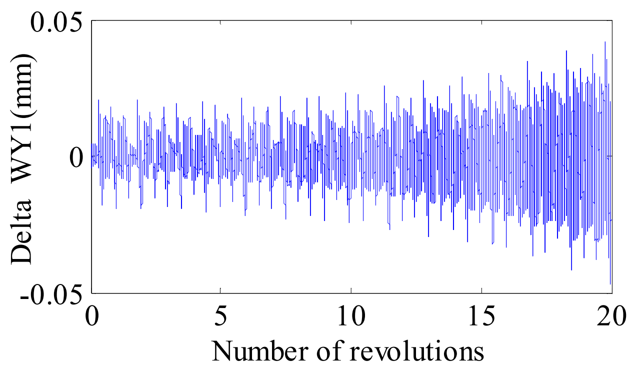


Figure 9; Effect of lag angle of 90 deg. on stability (a_2=0.8mm)

5. CONCLUSION

A time-domain model that can simulate parallel milling processes is presented in this paper. The model is able to include the dynamic interactions of two flexible cutting tools and a flexible workpiece. If the workpiece is rigid, the dynamics of two processes can be analyzed separately using a standard milling stability formulation since there is no presence of dynamic coupling. Otherwise, the presented model is needed to incorporate the dynamic interaction between the cutting tools. The simulation results showed that the process stability strongly depends on the milling mode, part flexibility and other process parameters. It was observed that the total stable material removal rate by two parallel working milling tools may be higher than a single milling tool increasing productivity. The presented model can be a useful tool to select process parameters that results in stable parallel milling processes.

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