Estimation Based PID controller-Sensorless Wave Based Technique ⋆

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Abstract: This paper presents a sensorless estimation algorithm for estimating flexible system parameters, dynamics and externally applied forces or torques due to system interaction with the environment. The proposed algorithm makes it possible to design a chain of observers that require measuring actuator's current and velocity along with performing two off-line experiments that do not require any additional measurement from the flexible system. The output of these observers are estimates of the system parameters, estimates of the system dynamics in configuration, motion and acceleration level. Eventually, the estimated positions are used to control the motion and vibration of the flexible lumped system without taking any measurement from the system. Experimental results show the validity of the proposed sensorless estimation algorithm and the possibility of controlling motion and vibration of flexible systems by focusing all the measurements on the actuator side keeping the system free from any attached sensors.

Keywords: Modal analysis, Mechanical waves, Torque observer, Parameter estimation, Motion estimation, Estimation based-PID controllers.

1. INTRODUCTION

Feedback control relays on measurements taken from the system using attached sensors and states estimates obtained by predesigned observers. These observers can be designed to estimate observable system states if the input and output are measured. In other words, measurements from the system are required to design such observers and used as basis of the estimation process. Therefore, measurements from the system are necessary for feedback control. Surprisingly enough that feedback control can be accomplished without taking any measurement from the system. Strictly speaking, the plant can be kept free from any attached sensors and all measurements can be focused on the actuator side. This can be misleading unless the reflected mechanical waves from the system are considered as a natural alternative to the actual feedback.

These mechanical waves possess information about system parameters, dyncmics and external disturbances. Moreover, these waves can be estimated from any of the system's boundries at which an actuator is located. In other words, actuators are usually located at a systems' positions where mechanical waves reconstructievly interfere. Therefore, these waves can be estimated from the actuator side without taking any measuremnt from the system.

The Question that arises is how to decouple each piece of information out of these meahcnical waves in order to be used to accomplish sensorless motion control. This paper proposes a sensolress estimation algorithm that can be used to estimate the system parameters and dynamics, then the estimated positions are used instead of the actual measurements to control the motion and vibration of a lumped flexible system in a sensorless manner.

Actuator is used to launch fourier synthesized or filtered control inputs and receiving reflected mechanical waves from the system that can be estimated through actuator's current and velocity. In Ohnishi (1996), Tsuji (2005) the reflected mechanical waves were considered as disturbance on the actuator that can be estimated by an observer designed in Sugita (2000). Robust motion control is achieved when this disturbance is rejected Murakami (1993) turning the system into acceleration control if the inertia and torque constants are assumed to be unity Katsura and Kouhei (2007). In this paper reflected mechanical waves are decoupled out of the total disturbance and instead of rejecting this disturbance by an additional compensation control input, it is analysed and used to estimate system parameters and dynamics.

Occonar W.J.O'Connar (2007) pointed out that actuators can be used to launch mechanical waves to the system and absorb the reflected waves keeping the system free from any residual vibrations but requires the measurement of the first lumped mass and assuming that system if free from external applied forces. Vibration control can be achieved by a variety of approaches K.Miu (1993), point-to-point vibration control is an affective method to set the position of the last mass at certain position insuring that the system is free from any potential or kinetic energy Bhat and Miu (1990), that implies vibrationless motion control but masses positions have to be measured. Multi-switch Bang-Bang control Bellman and Gross (1980), command shaping Singer and Seering (1990) and laplace domain

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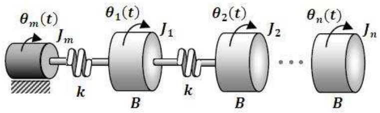
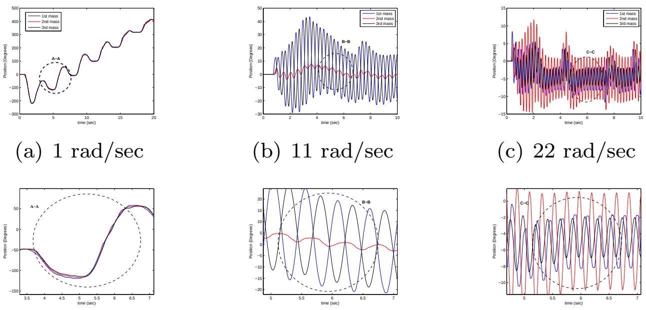


Fig. 1. Lumped flexible inertial system



(a) 1 rad/sec (b) 11 rad/sec (c) 22 rad/sec

(d) Zoom of-a (e) Zoom of-b (f) Zoom of-c

Fig. 2. Modal matrix experimental interpretation

synthesis Bhat and Miu (1991) are very efficient vibration control techniques that requires some measurement from the system. The paper is organized as follow, in section 2 the mechanical waves are estimated using actuator's parameters, then system parameters are determined. Rigid and flexible motions of the lumped flexible system are estimated in section 3 and used to present estimation based PID controllers. Finally section 4 includes the experimental results and final conclusions.

2. PARAMETER ESTIMATION

2.1 Modal Analysis of Lumped Flexible System

For the inertial lumped flexible system shown in Fig.1, the matrix equation of motion is

$$\mathbf{J}\,\underline{\ddot{\theta}} + \mathbf{B}\,\underline{\dot{\theta}} + \mathbf{K}\,\underline{\theta} = \underline{\mathbf{T}} \qquad (1)$$

where **J**, **B** and **K** are the inertia, damping and stiffness matrices. $\underline{\theta}$ and $\underline{\mathrm{T}}$ are the generalized coordinates and torque inputs vectors. Assuming zero damping and equal inertial masses for a three degree of freedom flexible system and taking laplace transform of (1) it can be represented in the following linear system form

$$\mathbf{A}\,\underline{\theta} = \underline{\mathbf{T}}\ . \qquad (2)$$

Solving the for the eigenvalues of the homogenous version of (1) and finding the corresponding eigenvectors we get the following modal matrix

$$\mathbf{M} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -2 \\ 1 & -1 & 1 \end{bmatrix}\ . \qquad (3)$$

Figure.2 shows the experimental interpretation of the modal matrix, where the first eigenvector represents the rigid body motion of the system as all masses have constant amplitude and in phase. The second eigenvector indicates that the second mass is not moving while the first and third are oscillating with the same amplitude but out of phase. According to the third eigenvector the first and third masses have the same amplitude and in phase while the third mass has twice the amplitude and out of phase. In any event, the point here is to indicate that if the forcing function was filtered or fourier synthesized so that the input has zero energy at the resonances of the system all the lumped masses will be moving with the same amplitude with respect to each other. Therefore, the number of coordinates used to describe the system will be droped from n to a single coordinate. The mechanical dynamics of the actuator is described using the following equatiion of motion

$$J_m\ddot{\theta}_m + B(\dot{\theta}_m - \dot{\theta}_1) + k(\theta_m - \theta_1) = i_a k_t \qquad (4)$$

making the following definition

$$\tau_{\mathbf{ref}} \triangleq B(\dot{\theta}_m - \dot{\theta}_1) + k(\theta_m - \theta_1) \triangleq \sum_{i=1}^{n} J_i\ddot{\theta}_i \qquad (5)$$

where $\tau_{\mathbf{ref}}$ is the reflected torque wave on the actuator that can be estimated using the actuator's current and velocity. i_a and k_t are the actuator's current and torque constant.

2.2 Reflected Torque Estimation

Considering the parameters variation (4) becomes

$$J_{mo}\ddot{\theta}_m = k_{to}i_a - \tau_{\mathbf{ref}} - \triangle J_m\ddot{\theta}_m + \triangle k_t i_a \qquad (6)$$

where J_{mo} and k_{to} are the nominal inertia and torque constant, while $\triangle J_m$ and $\triangle k_t$ are the variation from these nominal values. The last three terms are considered as disturbance d on the actuator that can be estimated through a low pass filter with g_{dist} corner frequency as follows

$$\widehat{d} = \frac{g_{dist}}{s + g_{dist}}[\triangle J_{mo}\ddot{\theta}_m - i_a k_{to}]. \qquad (7)$$

This in turn implies that the reflected torque wave τ_{ref} can be decoupled out of the disturbance d

$$\widehat{\tau}_{ref} = -\widehat{d} - \widehat{\triangle J_m}\ddot{\theta}_m + i_a\widehat{\triangle k_t} \qquad (8)$$

where $\widehat{\triangle k_t}$ and $\widehat{\triangle J_m}$ can be determined by a parameter identification process. Fig.3 shows the block diagram implementation of the reflected torque estimation process where disturbance is estimated then reflected torque is decoupled.

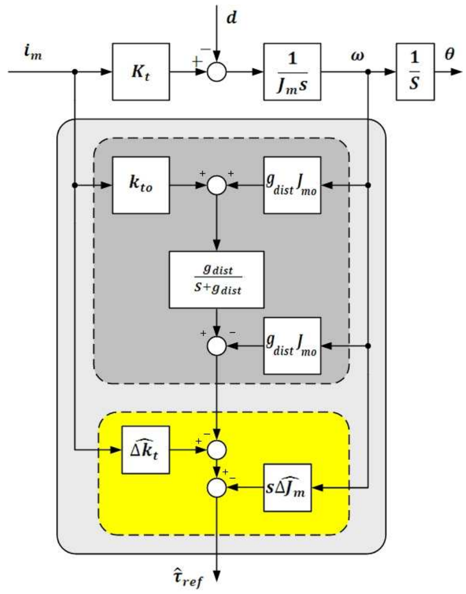


Fig. 3. Torque observer using actuator parameters

2.3 Rigid Body Motion Estimation

Equation (5) can be rewritten using the estimate of the reflected torque as follows

$$\widehat{\tau}_{\mathbf{ref}} \triangleq B(\dot{\theta}_m - \dot{\theta}_1) + k(\theta_m - \theta_1) = \sum_{i=1}^{n} J_i \ddot{\theta}_i \ . \tag{9}$$

Assuming that the input forcing function doesn't contain any energy at the system resonance frequencies. In other words, the forcing function is filtered and/or fourier synthesized such that the system's flexible modes are not excited. In this special case the motion of the system can be descried by a single coordinate as all the masses are moving with the same amplitude with respect to each other. Therefore, the system rigid motion can be estimated as follows

$$\widehat{\theta}(t) = \frac{1}{\sum_{i=1}^{n} J_i} \int_0^t \int_0^t \hat{\tau}_{\mathbf{ref}}(\mathbf{t}) \ d\tau d\tau + c_1 t + c2 \tag{10}$$

where $\widehat{\theta}(t)$ is the estimate of the rigid body motion, while c_1 and c_2 are integration constants

2.4 Parameters Estimation

Equation (9) can be rewritten using the estimate of the rigid body position of the system

$$\widehat{\tau}_{\mathbf{ref}} \triangleq B(\dot{\theta}_m - \dot{\widehat{\theta}}) + k(\theta_m - \widehat{\theta}) \ . \tag{11}$$

Defining

$$\underline{\xi} \triangleq (\theta_m - \widehat{\theta}) \tag{12}$$

$$\underline{\eta} \triangleq (\dot{\theta}_m - \dot{\widehat{\theta}})$$

where $\underline{\xi}$ is a vector of data points representing the difference between the actuator and estimated system rigid position, $\underline{\eta}$ is the time derivative of this signal. B and k are the uniform damping coefficient and the uniform joint stiffness. Rewriting (11)

$$\underline{\widehat{\tau}}_{\mathbf{ref}} = k \ \underline{\xi} + B \ \underline{\eta} \tag{13}$$

putting the previous equation in a matrix form

$$\left[\underline{\xi} \ \underline{\eta} \right] \begin{bmatrix} K \\ B \end{bmatrix} = \underline{\widehat{\tau}}_{\mathbf{ref}} \tag{14}$$

$\underline{\widehat{\tau}}_{\mathbf{ref}}$ is a vector of reflected torque wave data points, making the following definition

$$\mathbf{A} \triangleq \left[\underline{\xi} \ \underline{\eta} \right]$$

Equation (14) represents an over-determined system where the number of equations are more than the number of unknowns, and the optimum parameters can be determined as follow

$$\begin{bmatrix} \widehat{K} \\ \widehat{B} \end{bmatrix} = \mathbf{A}^{\dagger} \ \underline{\widehat{\tau}}_{ref} \tag{15}$$

where $\mathbf{A}^{\dagger}$ is the pseudo inverse of $\mathbf{A}$, $\widehat{k}$ and $\widehat{B}$ are the joint stiffness and damping coefficients estimates.

3. POSITION ESTIMATION-FLEXIBLE OSCILLATION

3.1 Flexible Motion Estimation

As the control input can contain energy at the system's resonances, system flexible modes can be excited as it is shown in Fig. 2.Therefore, the flexible motion of the system has to be estimated. Recalling (11) and replacing the actual with the estimated parameters, we obtain the following first order differential equation

$$\widehat{B}\dot{\theta}_1 + \widehat{k}\theta_1 = \alpha \tag{16}$$

$$\alpha \triangleq \widehat{B}\dot{\theta}_m + \widehat{k}\theta_m - \widehat{\tau}_{\mathbf{ref}} \ .$$

Solving the previous differential equation we obtain

$$\widehat{\theta}_1(t) = e^{-\frac{\widehat{B}}{k}t} \int_o^t \beta e^{\frac{\widehat{B}}{k}\tau} d\tau + e^{-\frac{\widehat{B}}{k}t} c_1 \tag{17}$$

where $\widehat{\theta}_1(t)$ is the estimate of the first inertial mass position regardless to the frequency of the forcing function. For the first equation of motion of (1) we have

$$\widehat{B} \ \widehat{\dot{\theta}}_2 + \widehat{k} \ \theta_2 = \gamma \tag{18}$$

$$\gamma \triangleq J_1 \widehat{\ddot{\theta}}_1 - \widehat{B}(\dot{\theta}_o - \widehat{\dot{\theta}}_1) - \widehat{k}(\theta_o - \theta_1) + \widehat{B} \ \widehat{\dot{\theta}}_1 + \widehat{k} \ \widehat{\theta}_1$$

$$\zeta \triangleq \frac{\gamma}{\widehat{B}}$$

solving (18) we obtain

$$\widehat{\theta}_2(t) = e^{-\frac{\widehat{B}}{k}t} \int_o^t \zeta e^{\frac{\widehat{B}}{k}\tau} d\tau + e^{-\frac{\widehat{B}}{k}t} c_2 \tag{19}$$

where $\widehat{\theta}_2(t)$ is the estimate of the second lumped inertial mass. And the general position estimate of any of lumped mass of the system is

$$\widehat{\theta}_i(t) = e^{-\frac{\widehat{k}}{B}t} \int_o^t \Omega \ e^{\frac{\widehat{k}}{B}\tau} d\tau + e^{-\frac{\widehat{B}}{k}t} c_i \tag{20}$$

where

$$\Omega \triangleq \frac{\Psi}{\widehat{B}}$$

$$\Psi \triangleq g(J_{i-1}, \widehat{\theta}_{i-1}, \widehat{\dot{\theta}}_{i-1}, \widehat{\ddot{\theta}}_{i-1}, \widehat{k}, \widehat{B})$$

3.2 Estimation Based-PID Controller

Since the position of any lumped mass can be estimated using (20), these estimates can be used as a virtual feedback instead of the actual measurements taken using attached sensors to the system. The error is no longer defined as the difference between desired reference and actual measurement taken from the point of interest, it became the difference between the reference and the position estimate obtained using (20)

$$\widehat{e}(t) = \theta_{ref}(t) - \widehat{\theta}_i(t) \tag{21}$$

the control law of the Estimation based PID control is

$$u(t) = k_p \widehat{e}(t) + k_i \int_0^t \widehat{e}(t) dt + k_d \frac{d\widehat{e}(t)}{dt} \ . \tag{22}$$

3.3 Summary of the Estimation Based Control Process

The steps of the entire process are:

(1) Fourier synthesize the input such that it contains zero energy at the system resonance frequencies.
(2) Reflected torque estimation using actuator parameters.
(3) Rigid body motion estimation using (10).
(4) Estimation of the uniform system parameters using (15).
(5) Using the recursive formula (20) to determine the estimate of the i^{th} mass required to be controlled.

(6) Feeding back the position estimate of the i^{th} mass to the controller and changing the controller gain according to the required transient response.

Figure.4 shows the entire estimation and control process that is based on two measurements taken from the actuator. The previous estimation algorithm is based on extracting system parameters from certain frequency range at which system motion is described by single coordinate, then these parameters are used in the flexible motion observer to estimate the position of any lumped mass regardless to the frequency of the forcing function.

4. EXPERIMENTAL RESULTS

The implementation of the pervious algorithm is performed on a three degree of freedom inertial flexible system with the following parameters where g_{lpf} is the velocity

Table 1. Experimental parameters

Parameter	Value	Parameter	Value
J_1	5152.99 gcm^2	g_{dist}	100 rad/sec
J_2	5152.99 gcm^2	g_{lpf}	100 rad/sec
J_3	6192.707 gcm^2	f_{init}	1 rad/sec
J_m	209 gcm^2	k_{act}	1.627 KN/m
k_b	235 rpm/v	k_t	40.6 mNm/A

low pass filter cut off frequency, f_{init} is the forcing function frequency that keeps the flexible system rigid, k_{act} is the theoretical spring constant along the flexible system that is known before hand by the following calculation

$$k_{act} = \frac{Gd}{8c^3 n} = \frac{70 \times 10^9 \times 2}{8 \times (\frac{8}{2})^3 \times 21} = 1.627\ kN/m \qquad (23)$$

where G is the modulus of rigidity, c is the spring ratio, d and n are the coil diameter and the effective number of turns.

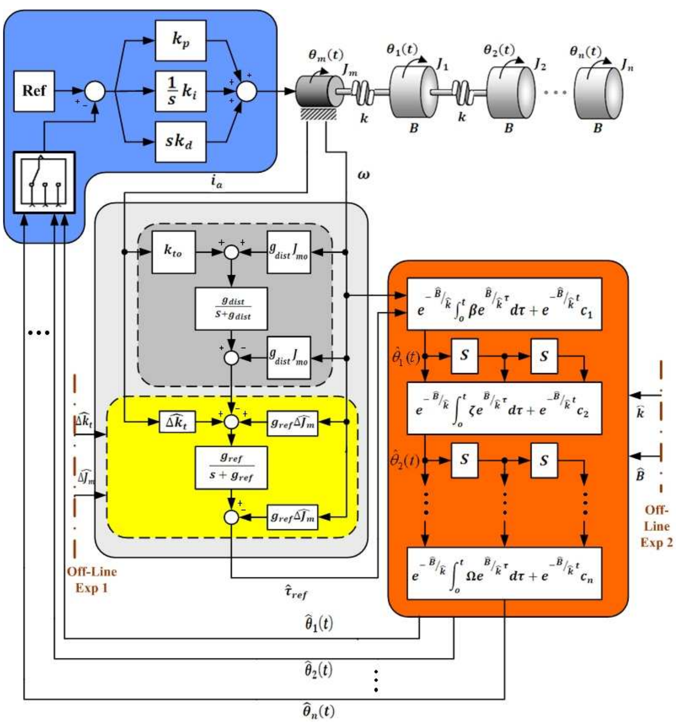
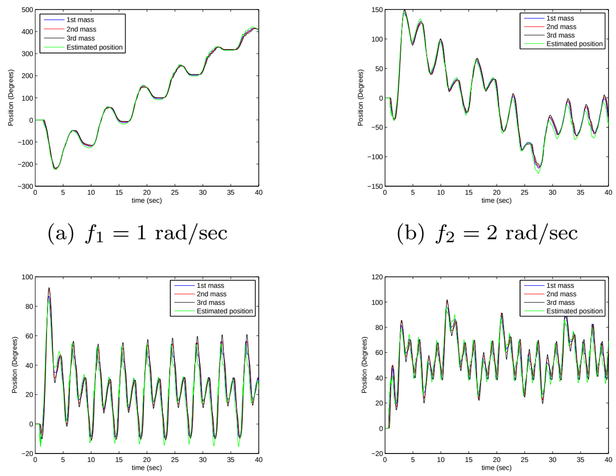


Fig. 4. Estimation Based-Control of Flexible system



(a) $f_1 = 1$ rad/sec (b) $f_2 = 2$ rad/sec

(c) $f_3 = 3$ rad/sec (d) $f_4 = 4$ rad/sec

Fig. 5. Rigid motion estimation experimental results

4.1 Rigid Body Motion Estimation-Experimental results

The uniform parameters are extracted from the system low frequency range, the idea behind this process is to drop as many unknown coordinates as possible reducing the number of coordinates from n to only single unknown which is the position of the entire rigid system that can be estimated using (10) after estimating the reflected torque wave using(8). Therefore, the initial input forcing function is filtered so that its energy content at the system resonances is zero. Fig.5 shows the rigid motion of three masses and their position estimate for different frequencies. It turns out that, below 4 rad/sec the estimation of the rigid motion is following the actual system position. In other words (10) is valid in the low frequency range below 4 rad/sec. Therefore, the parameter estimation experiment has to be performed in this frequency range.

4.2 Uniform Parameters Estimation-Experimental Results

The parameter estimation experiment requires the estimate of the reflected torque along with the difference between the actuator position and the rigid body position estimate data point vector and its derivative, using (15) the system parameters are determined and Table.2 shows the obtained experimental results for both joint stiffness and damping coefficient that are assumed to be uniform along the flexible system. Using the obtained average estimated parameters, the reflected torque wave is reconstructed and compared with the estimated one as shown in Fig.6. the magnified plot of Fig.6-a indicates a high level of noise amplification due to the direct differentiation.

Table 2. Parameters estimation results

Par	1st Exp	2nd	3rd	4th	5th
$\widehat{k}$ KN/m	1.579	1.533	1.645	1.511	1.562
$\widehat{B}$ Nsec/m	0.088	0.087	0.088	0.089	0.089

$$\widehat{k}_{avg} = \frac{\sum_{i=1}^{n} k_i}{n} = \frac{\sum_{i=1}^{5} k_i}{5} = 1.566\ kN/m \qquad (24)$$

$$\widehat{B}_{avg} = \frac{\sum_{i=1}^{n} B_i}{n} = \frac{\sum_{i=1}^{5} B_i}{5} = 0.0882\ Nsec/m \qquad (25)$$

comparing the average estimated stiffness with the theoretical value that is known before hand (23), we conclude that the difference is less than 5 percent and these parameters can be used in the flexible motion recursive observer (20).

4.3 Position Estimation-Experimental Results

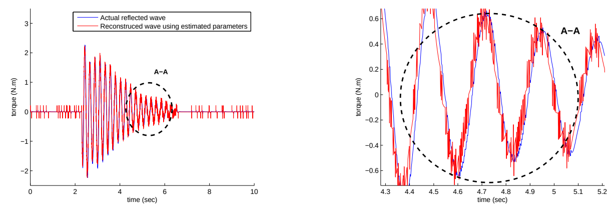
Lumped masses positions can be estimated using (20), where the position estimate of the i^{th} mass requires the determination of all the previous masses position estimates. Fig.7-a shows the flexible system oscillation when an arbitrary forcing function contains some energy at the system resonances While the other figures show the difference between the actual and estimated positions using (20). The estimated position seems to be identical to the actual mass position that makes it possible to use these estimate as feedback to the controller instead of the actual measurements.

4.4 Sensorless Position Control-Experimental Results

Figure.4 shows the entire sensorless estimation and control process, the system is kept free from any attached sensors and only actuator parameters are measured then used by the previous chain of observers to determine the position estimate of any point of interest in the system.

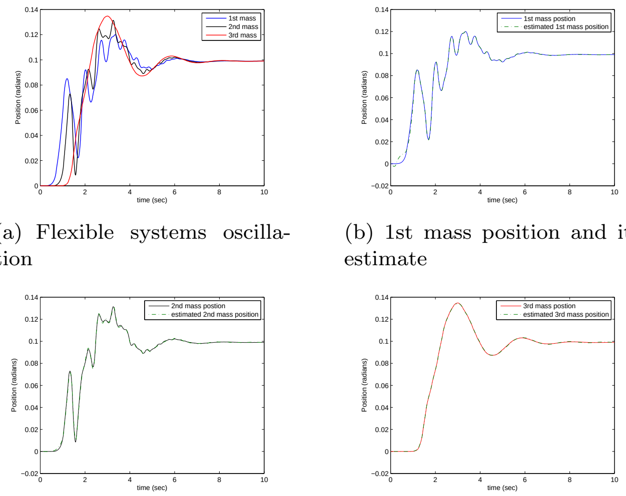
Set-point tracking experiment-1st mass Using the position estimates of the lumped inertial masses as a feedback instead of the actual measurement allows switching the estimates easily to the controller. All the estimates are available, therefore the global behavior of the system can be monitored to ensure that system is controlled and kept free from any residual vibrations. Fig.8a-b shows the sensorless control process of the first mass, where its' position estimate is fed back to the controller, the magnified plot shows 0.1 degrees steady state error in the final response. Fig.8c-d show the response of the other two masses. The objective here is to control the position of any particular mass along with active vibration damping of the other masses keeping the system free from any kinetic and potential to achieve minimum residual vibrations.

Set-point tracking experiment-2nd mass In order to control the second mass position with minimum residual vibration of the system, the estimate of the second mass has to be switched to the controller instead of the first mass. Fig.9-a shows the response of the second mass when its' estimate is fed back to the controller, its' magnified



(a) reflected and reconstructed torque wave (b) Magnified plot

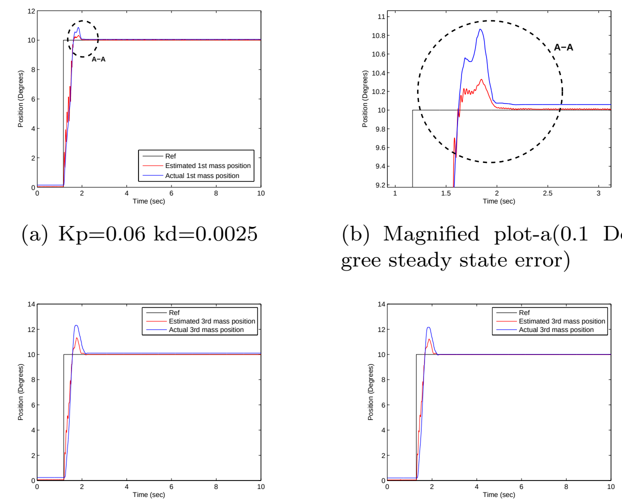
Fig. 6. Reconstructed torque wave using estimated parameters



(a) Flexible systems oscillation (b) 1st mass position and its estimate

(c) 2nd mass position and its estimate (d) 3rd mass position and its estimate

Fig. 7. Flexible motion estimation experimental results



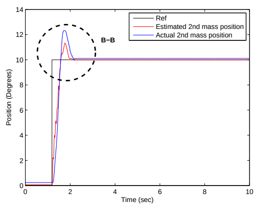
(a) Kp=0.06 kd=0.0025 (b) Magnified plot-a(0.1 Degree steady state error)

(c) 2nd mass position and its estimate responses (d) 3rd mass position and its estimate responses

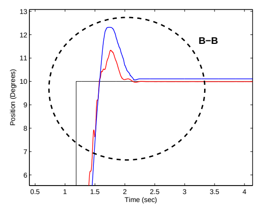
Fig. 8. Sensorless motion control of the 1st mass-Based on the first mass position estimate feedback

plot shows 0.15 degrees steady state error in the final response.

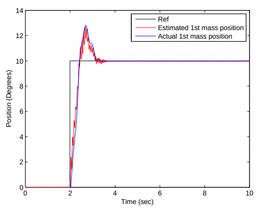
Arbitrary trajectory tracking experiment Figure.10(a-b) show two different trajectories that have to be followed by the point of interest. In this experiment the third mass is considered as the point of interest. For a time varying trajectory tracking the control law has to include a feed forward term, that in turn implies the necessity of obtaining systems dynamics. Surprisingly enough that the proposed algorithm enables determination of system dynamics and parameters that are required to design the feed forward control term. Therefore, the proposed algorithm makes it possible to track any time varying trajectory without attaching any sensor to the system.



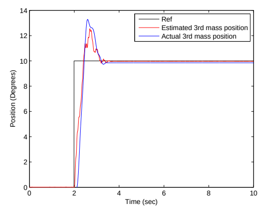
(a) Kp=0.02



(b) Magnified plot-a(0.15 Degree steady state error)

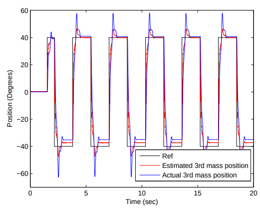


(c) 1st mass position and its estimate responses

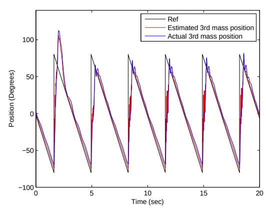


(d) 3rd mass position and its estimate responses

Fig. 9. Sensorless motion control of the 2nd mass-Based on the second mass position estimate feedback



(a) kp=0.08 kd=0.0025



(b) kp=0.06 kd=0.0025

Fig. 10. Sensorless motion control of the 3rd mass-Based on the third mass position estimate feedback

5. CONCLUSION

Uniform parameters and dynamics of any lumped flexible system can be obtained from the reflected mechanical waves on the actuator by extracting system parameters from the system's low frequency range, where the number of generalized coordinates describing the systems position is dropped from n to one coordinate. This simple procedure allows the estimation of the damping coefficient and the joint stiffness. Using this information along with the actuator parameters the flexible motion of the system can be estimated by a chain of observers along with recursive computations.

The experiments show promising results, but the main drawback of the proposed algorithm is the steady state error in the final response. This steady state error is a result of the optimization operations used to estimate the system parameters and also to estimate the actuator parameters variation disturbance. Moreover, the rigid body motion observer contains double integrators that can magnify any tiny initial error. Therefore, due to these reasons an estimation error is expected between the actual and estimated variables which in turn implies that the control action will not be able to bring the actual position to the desired reference keeping the system with a steady state error in the final response. The amount of steady state error depends on the accuracy of the estimation processes. However, reducing this steady state error relays on more accurate estimators designs and careful off line experiments implementation. On the other hand, the proposed algorithms makes it possible to determine system parameters, dynamics and possibly externally applied forces or torques when system interacts with the environment by focusing all the measurements on the actuator side without taking any measurement from the system considering the reflected mechanical waves as a natural feedback from system. Eventually, the experimental results shows the validity of the proposed algorithm and the possibility of accomplishing motion, vibration and force control of flexible systems using actuator as a single platform for measurements keeping the plant free from attached sensors.

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