Collective behavior of El Farol attendees

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Abstract

Arthur's paradigm of the El Farol bar for modeling bounded rationality and inductive behavior is undertaken. The memory horizon available to the agents and the selection criteria they utilize for the prediction algorithm are the two essential variables identified to represent the heterogeneity of agent strategies. The latter is enriched by including various rewarding schemes during decision making. Though the external input of comfort level is not explicitly coded in the algorithm pool, it contributes to each agent's decision process. Playing with the essential variables, one can maneuver the overall outcome between the comfort level and the endogenously identified limiting state. The distribution of algorithm clusters significantly varies for shorter agent memories. This in turn affects the long-term aggregated dynamics of attendances. We observe that a transition occurs in the attendance distribution at the critical memory horizon where the correlations of the attendance deviations take longer time to decay to zero. A larger part of the crowd becomes more comfortable while the rest of the bar-goers still feel the congestion for long memories. Agents' confidence on their algorithms and the delayed feedback of attendance data increase the overall collectivity of the system behavior.

Introduction

Agent-based modeling allows agents to construct their own decision making mechanisms from information allocated to all in common. Simple prediction rules and independent decision making processes lead to a coordinated equilibrium that is an emergent property of the system (1, 2).

Arthur's El Farol bar problem brings up a paradigm (3). A total number of N players must decide independently whether to attend the bar or not. If a player forecasts that the total attendance will exceed the comfort level, L, she will not show up, otherwise she will go. We are interested in the way the players predict the total attendance and its long-term characteristics. The problem is based on the agents' knowledge of the overall attendance history, but the individual actions are not known. Yet, a collective behavior emerges from the system, whereby the average attendance is bounded on two sides by the threshold imposed from the outside, and the randomness that would take over the system in the absence of the threshold.

The problem has been investigated from many perspectives (4-7). In particular, Johnson and coworkers have explored the variation of volatility with the pool sizes available to the agents, as well as the effect of various selection schemes on attendance (8). Challet et al. have proposed a statistical mechanical model on the problem that becomes exact in the limit of randomness (9). It has also been analytically shown that the problem is mean reverting in nature; therefore, the average attendance will remain within the region $[N/2, L]$ (10). Yet, a controlled study that explores the effect of the parameters that may influence the output of the system has not been performed. Amongst these are (i) the different types of algorithms utilized by the agents, (ii) the strategy employed to select algorithms from this pool, and (iii) the memory horizon for which attendance data is available to the agents. In this study, we systematically study the effect of each of these parameters on the average attendance recorded. Probability distributions and the time correlations of the attendances are determined. We find that the emergent behavior in these systems is not only limited by an upper and lower bound, but also that it follows predictable patterns within these limitations once a critical value of short memory is surpassed. Furthermore, long-lived correlations in the time evolution of the attendances may be induced by introducing internal and external constraints.

Agent-based modeling of the El Farol bar

We use a system of N potential attendants. Each week we seek to find the number of people who will actually attend the bar, a_o. The attendance data of the last m weeks [a_m, a_{m-1}, …, a_1] are available to the agents. Each attendant, j, takes a decision to attend or not by making a prediction, q_j, on the possible value of a_o, using one of the many available algorithms. The algorithm pool here consists of (i) point-wise hypothesis – the agent uses the attendance data of the k^{th} previous week ($1 \leq k \leq m$) as his prediction, $q_j = a_k$; (ii) arithmetic average – the agent uses the average of the last k ($1 < k \leq m$) weeks as his prediction, $q_j = \frac{1}{k}\sum_{i=1,k} a_i$; (iii) weighted-average – the agent uses a weighted average of the last k ($1 < k \leq m$) weeks, where the more recent a week's data is, the larger weight it has, $q_j = \sum_{i=1,k} \frac{2ia_i}{k(k+1)}$; (iv) trend – the agent makes a least squares fit to the last k weeks' data ($1 < k \leq m$), and uses its extrapolation to the following week as his prediction, bounded by [0,100]. There are, therefore, a total of $4m - 3$ algorithms available, where m

is the memory of the system. Once the agent makes his prediction q_j, he will not attend if the prediction is larger than a previously set comfort level, L.

The algorithm pool available to each agent affects the results. In one scenario, all algorithms in the pool may be available to all agents. However, note that alternatively, a sub-pool specific to each agent may be assigned *a priori*, and the agents may only choose from within this subset. Johnson and coworkers have shown that this choice significantly affects the volatility, where a minimum value of the volatility is obtained for a given fraction of the pool size, and it has a larger value for lower or higher fractions; maximum volatility is recorded for a fraction of one (8). The sub-pool may consist of a particular type of strategy (i) – (iv) above, or a mixture of those. The union of all the sub-pools should then define the total of the algorithms. The algorithms are initially assigned to the sub-pools using a uniform distribution. The extent to which the memory of each agent extends into the past is equivalent to m in all these cases. In what follows, one property we investigate in detail is how m affects the outcome. Since the fraction that gives the minimum volatility depends on m, we choose to make all the algorithms available to all the agents; i.e. the sub-pool fraction is one. Volatility is therefore at its maximum for the entire memory horizon.

Another property that significantly influences the results is the way the agents pick future algorithms from their sub-pool. This may be done in one of a multitude of ways: In the simplest of the applications, each agent hangs on to his current algorithm as long as it is successful, and changes it as soon as he fails in his prediction (denoted by ***scheme I*** here). Alternatively, various rewarding schemes that evaluate the success of the algorithms in retrospect may be adopted. In one scheme, the agent re-evaluates the predictions of all of his algorithms, and picks the one that would have been successful *and* provides the closest value to a_o (***scheme II***). In another scheme, the agent keeps a log of the success of his algorithms, by giving a point to all algorithms that would have succeeded at the current step. The agent then picks the one that has the highest cumulative score as his next predictor, or randomly selects between algorithms in case of equivalent scores (***scheme III***).

We have also implemented a modification of ***scheme I***, where agents employ insistence by changing their current algorithms only after a predetermined number of failures, instead of an immediate alteration (***scheme I´***). As another variation of ***scheme I***, we have studied the effect of feeding delayed information to the agents (***scheme I˝***). These latter two approaches uncover the effect of altering the system dynamics internally, by adjusting the response time of the agents (***scheme I´***), or externally, by imposing an outside condition in addition to the comfort level L (***scheme I˝***). As we shall see, such choices significantly affect the fundamental properties of the system.

Effect of memory on mean attendance and distributions of active algorithms

In figure 1 we display the variation of the average attendance with increasing memory for the three schemes discussed above. We observe that there are upper and lower bounds on the attendance: As the amount of information available to the agents increases, the average attendance approaches a value that is equal to or less than the comfort level, L. The ultimate value attained depends on the choice of the algorithm selection scheme, I, II, or III. The lower bound, on the other hand, tends to $N/2$; i.e. when there is very limited amount of information provided to the agents, their predictions become randomized, irrespective of the comfort level. The change from the lower bound,

$N/2$, to the upper bound, occurs in an S-shaped curve; however, for short memories, depending on the selection scheme used, some of the data points deviate from this curve, as shown with the hollow circles in figure 1.

The reason for this deviation becomes clear if we investigate the fraction of algorithms utilized by the agents at different memories. This is shown in figure 2 for the three selection procedures, and the four types of algorithms that may be utilized by the agents. We find that in all cases, the distributions converge once the agents extend their memory horizon past a critical value. That memory value also corresponds to the point where the scatter in the data in figure 1 disappears, and the data track the curves marked by the solid line. For scheme I, since the algorithms are freely and readily changed from the total pool, that position is rapidly reached at a short memory of $m = 4$. For schemes II and III, however, the corresponding position is reached at $m = 9$ and the scatter in the data at lower memory values is reflected into figure 1. We therefore mark the approximate expected trace of the data, had they followed the converged distributions in all memory values, by the dashed line.

Also interesting is the fact that the point-wise hypotheses (cycle detectors) are the most frequently used algorithms in all schemes. In scheme I where the algorithms are changed immediately in case they fail, they appear nearly twice as much as all the other types, indicating that they are the more often winners at instantaneous steps. That they make-up ⅔ of the algorithms utilized in scheme II, where the agents change to the predictor with the least error, points to the fact that they are also more precise predictors. Furthermore, in scheme III which uses cumulative rewarding, the competitive edge of cycle detectors gets compounded, and 90% of the algorithms utilized are composed of these. This over-expression of the point-wise hypotheses hints that there are cycles that occur in the data, an observation that may be corroborated by the correlations in the attendance profiles that emerge (see figure 3 in the next section). The remaining algorithm types behave almost equally well in scheme I, having values reduced slightly below that expected of random choice (23, 19, and 20% respectively for arithmetic average, weighted average, and trend, as opposed to the expected nearly 25% each); the reductions contribute almost equally to the enhancement in the cycle detectors. However, once rewarding is introduced in schemes II and III, weighted average algorithms are used less. In particular, in III their probability of appearance is nearly zero for all memories. Also, trend algorithms behave slightly better than arithmetic averages in II and III, opposite of the observation in scheme I.

In sum, for a given scheme, beyond a critical size of memory, the attendances fall onto an S-shaped curve. For shorter memories, attendances show large deviations from this curve. The critical memory where the average attendance begins to follow a predictable pattern corresponds to the converged distributions of the algorithm types used from the algorithm pool. The question remains, however, as to the origin of the dependence between the attendance and agent memory.

Effect of memory on attendance distributions and deviation clustering

In this section, we confine our attention to scheme I, and the threshold value $L = 60$. The probability distribution of the attendance for this set is displayed in figure 3. In the case of low memories, the probability distribution of attendance is widely spread over the attendance range exhibiting almost a double hump (blue line). One hump is centered on 70 with low variance and the second one is furnished with very high variance around

40. As the memory takes higher values, the lower end of the distribution moves towards the center and the hump at the lower end becomes more and more pronounced, while the higher end also shifts slightly towards the tail. When the memory reaches 60 (green line) the bump heights are equalized. Hereafter, with the higher memory values (dashed black line and black line are associated with the memory 90 and 150, respectively), the higher end distribution gets shallower loosing its maximum. Finally, the attendance distribution converges when the memory is around 200. The red line in figure 3(a) is obtained for the case where the memory is 210. The converged distribution is left only with one peak whose maximum is very close to the threshold value with a very thick tail of high attendances.

We note that the "transition" occurring in the attendance distribution resembles the phase transition phenomena frequently observed in nature. The point where the transition occurs in figure 3(a) (where the double humps are equalized) is also the memory level at which the correlations of the attendance deviations take longer time to decay to zero. In figure 3(b), we present the correlations of the attendance deviations and the correlations of the absolute attendance deviations (inset). The most dominant correlations occur in consecutive weeks, a negative value which prevails for all memory levels, representing the immediate pull back to the mean. The correlation between the attendances of every other week, on the other hand, tends to be maximum at the point where the two humps in the distributions are equalized (i.e. the transition point, $m = 60$, in figure 3a) and then disappears for very long memories. A positive correlation between the attendances of every third week, in contrast, exist even at $m = 210$.

Exhibited in the inset of figure 3(b) are the correlations of the absolute attendance. While an examination of the correlations lends one to assume that correlations die, the correlations of the absolute attendance display decay in the first two weeks and then remains positive and significantly high. This is a manifestation of the intuition that the attendances tend to revert the mean. Yet, the inset of figure3(b) signifies that “large changes tend to be followed by large changes, of either sign, and small changes tend to be followed by small changes.” The structural issues pertinent to deviation clustering have been recently reviewed (11). Note that the non-diminishing correlations between the data are manifested in the success of the point-wise algorithms (figure 2).

Effect of agents’ rigidity and delayed information allocation to cooperative behavior

The outcome of the simulations based upon ***scheme I´*** is displayed in figure 4(a) and (b). The attendance distributions at the long memory horizon are not altered significantly if the agents insist on keeping their current algorithms. Yet the attendance distributions at the memory level where the transition takes place are not distorted qualitatively, only the lower attendance hills shift slightly to the lower attendance regime. These observations are independent of the rigidity degree of the agents in response to the number of the failures that their algorithms experience, as shown in figure 4(a) in black, blue, and red curves for one, ten, and 100 failures, respectively. The correlations of the attendances, on the other hand, do not remain unresponsive to the agents’ reckless attitude to failures as shown in figure 4(b). Sticking to failed algorithms amplifies the collectivity of the system as the correlation time increases significantly. As expected, the most significant increase in the collective memory is obtained at the transition.

Allocating delayed information to the agents as abbreviated by ***scheme I″*** modifies the attendance distributions and the correlations significantly. The results are summarized in figure 5(a) and (b). First of all, for the low memory horizon (m = 20), the distributions tend to become shallower, losing the humps at the extremes if the delay is one or two weeks as displayed in figure 5(a). Increasing the delay to five weeks makes the agents more inclined towards the all-or-none behavior, showing peaks at the extremes. The output of very long memory horizon simulations (m = 2500) indicate that the attendance distributions will eventually reach that of the on-time information case (***scheme I***) for small delays (one or two weeks). However, at a moderate delay of five weeks, convergence is still not obtained at these very high memory allocations. The time correlations, on the other hand, display persistent diminishing cyclic patterns that survive for long periods, figure 5(b). For both low and very long memories, strong cooperativity is induced by the delays. This behavior may find its roots in the delayed control of mechanical systems (12).

Concluding Remarks

Whether the average attendance will converge to the externally provided comfort level or not depends on the algorithm selection procedures of the agents. Changing the algorithm used whenever it fails, irrespective of the past success of the algorithm, and picking up another one randomly (scheme I), drives the average attendance to the comfort level, L, as the agents use more information from the past. Taking into account a merit based stickiness to the algorithms employed in the past (schemes II and III), exhibit considerable deviation from the path that carries the average attendance to L. As shown in figure 1, stickiness not only alters the plateau levels, but also yields large fluctuations at the approach-to-plateau pathways, especially for shorter memory allocations (empty circles).

Information carrying capacity modulates the fractional use of the type of algorithms available in the pool. For short time horizons, it is rather difficult to estimate the distinct distributions. The algorithm clusters, namely, cycle detectors, trend followers, average takers, are shared by the agents sporadically, exhibiting a transient regime as observed in figure 2. As more of the past information is made available to all agents, independent of the rewarding scheme, fractional distributions are collectively balanced. Note, however, that reaching the detailed balance in the distributions does not coincide with reaching the plateau value of the attendance in figure 1, but is rather associated with the regions of small variations from the paths described by the S-shaped curves.

The mean-reverting nature of the process is characterized by the correlation of the attendances in figure 3(b). Metaphorically, the energy wells associated with different memory horizons of the process may be obtained by the negative logarithm of the attendance distributions. In figure 3(a), we showed that the attendance distribution experiences a phase transition when the areas of the wells are equalized at a memory of 60. After the transition, a larger crowd feels the comfort, yet, still many attends without joy.

When agents happen to be unresponsive to the number of failures of the algorithms that they select, the attendance distributions are not changed considerably. The correlations, on the other hand, become longer lasting at the memory level where the transition takes place. Also, correlations are significantly higher at short time intervals for the long memories. For the delayed revealing of the attendance data, both the attendance distributions and the correlations are extremely altered. The distributions converge at very high memory allocations and the correlations indicate the sign of high collectivity. Thus, it is possible to manipulate the time evolution of the attendance internally if agents agree to delay algorithm modification upon failure, or externally by providing past data with delay.

REFERENCES

1. *Adaptive Agents, Intelligence, and Emergent Human Organization: Capturing Complexity through Agent-Based Modeling* (2002) in *Proc. Natl. Acad. Sci. USA*, Vol 99 suppl. 3.
2. Tesfatsion L, Judd K, eds. *Handbook of Computational Economics*. Vol. 2. (2006).
3. Arthur WB (1994) *Amer. Econ. Rev.* 84: 406-411.
4. Arthur WB (1999) *Science* 284: 107-109.
5. Arthur WB (2006) in *Handbook of Computational Economics*, eds. L. Tesfatsion and K. Judd (North-Holland), ch. 32.
6. Challet D, Marsili M, Zhang Y-C (2005) *Minority Games: Interacting Agents in Financial Markets* (Oxford University Press).
7. Bell AM, Sethares, WA, Buckley, JA (2003) *IEEE Trans. Signal Process.* 51: 875-885.
8. Johnson NF, Jarvis S, Jonson R, Cheung P, Kwong YR, Hui PM (1998) *Physica A* 258: 230-236.
9. Challet D, Marsili M, Ottino G (2004) *Physica A* 332: 469-482.
10. Lus H, Aydin CO, Keten S, Unsal HI, Atilgan AR (2005) *Physica A* 346: 651-656.
11. Granger CWJ, Machina MJ (2006) *J Econometrics* 135: 15-29.
12. Gu K, Niculescu S-I (2003) *J. Dyn. Syst. Meas. Control-Trans. ASME* 125: 158-165.

Figure Captions:

Figure 1. Average attendance versus memory of agents for N = 100 agents, and a comfort level of L = 60. Each data point is the result of 50 runs of 2000 weeks. The first 100 weeks of attendance data are not included in the averages to remove transient effects. Error bars are smaller than the data points. Dashed lines approximate the paths that would have been followed at short memories, had the algorithm types followed the converged distribution profiles (see figure 2 and the text for details). Solid lines at longer memories are drawn through the points to guide the eye.

Figure 2. The distributions reached by each of the algorithm types (point-wise, arithmetic average, weighted average, and trend) at different memory values. Note that there are $4m – 3$ algorithms available to the agents at each memory value, m. The sum of the values at each m is equal to one.

Figure 3. Attendance profiles of agents acting according to ***scheme I*** for different memory horizons. (a) Probability distributions of the attendance; note the transition observed at the memory, $m = 60$ (green line). The fat tail does not disappear even in the longer memory horizons, indicating a significant amount of uncomfortable bar-goers. (b) Correlations of the attendance deviations from the average. The absolute attendance deviations are given in the inset, where the correlations decay fast in the first two weeks and stay significant at later times.

Figure 4. Effect of the resistance of agents to change their algorithms, ***scheme I´***, on (a) the probability distributions of the attendance, and (b) the time correlations, during transition (m = 70, top, dashed curves, values on right y-axis) and at convergence (m = 210, bottom, solid curves). Sticking with a failing algorithm for one (black), 10 (blue) and 100 weeks (red) are shown.

Figure 5. Effect of feeding delayed information to the agents, ***scheme I˝***, on (a) the probability distributions of the attendance, and (b) the time correlations, at short memory (m = 20, top, dashed curves, values on right y-axis) and at very long memory (m = 2500, bottom, solid curves). Delaying attendance information for one (black), two (blue) and five weeks (red) are shown. Note that these simulations were carried out for 10000 weeks.

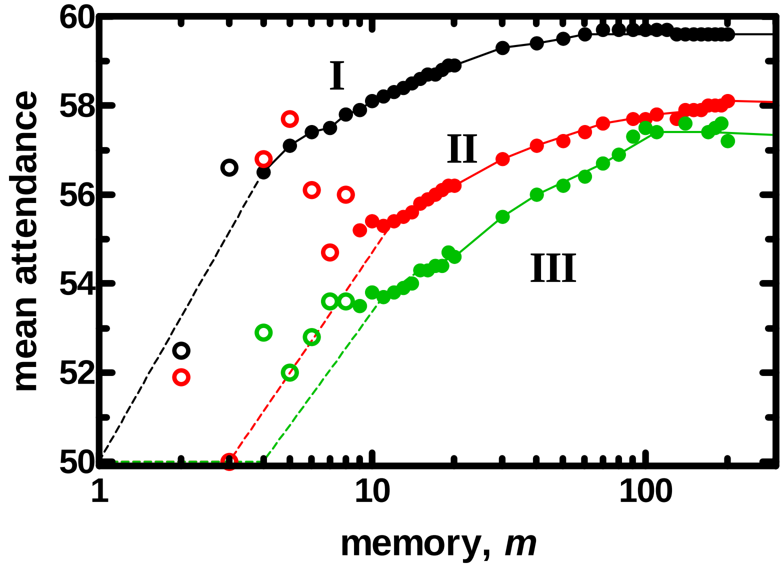


Figure 1

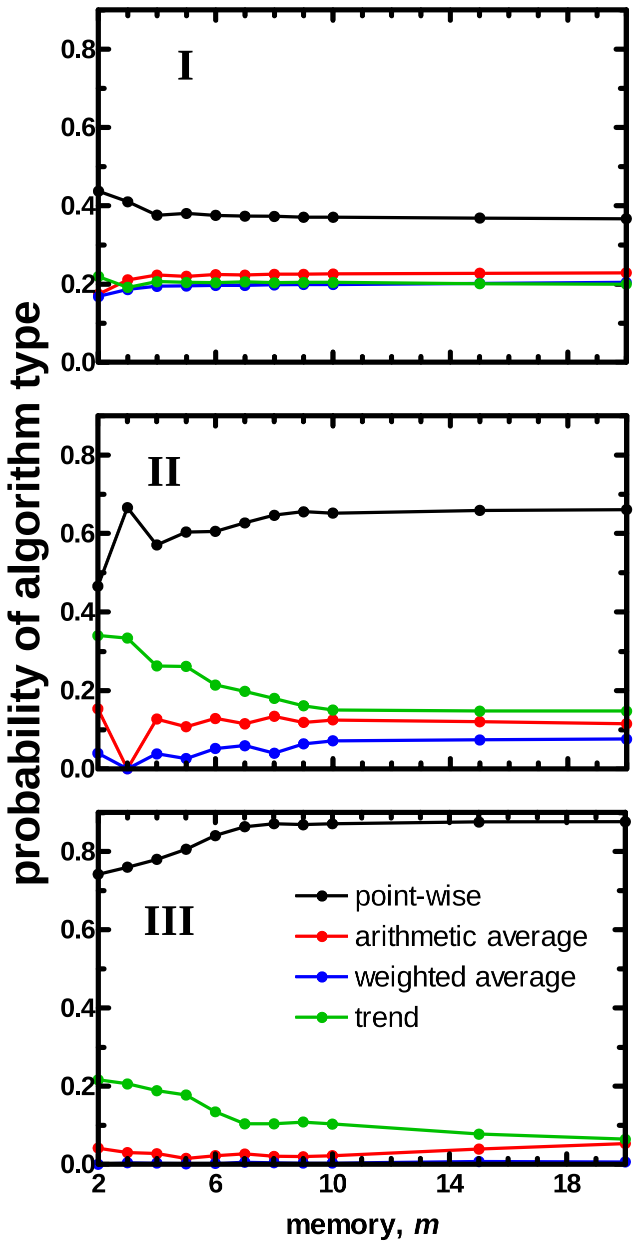
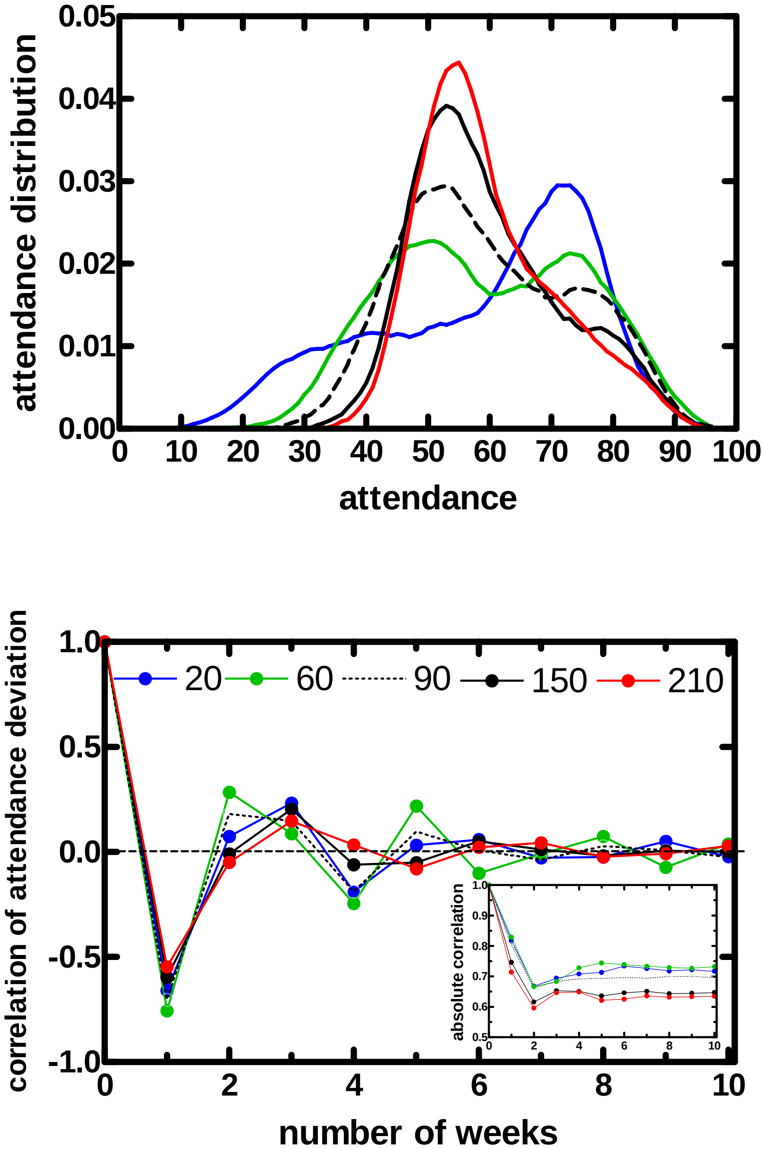
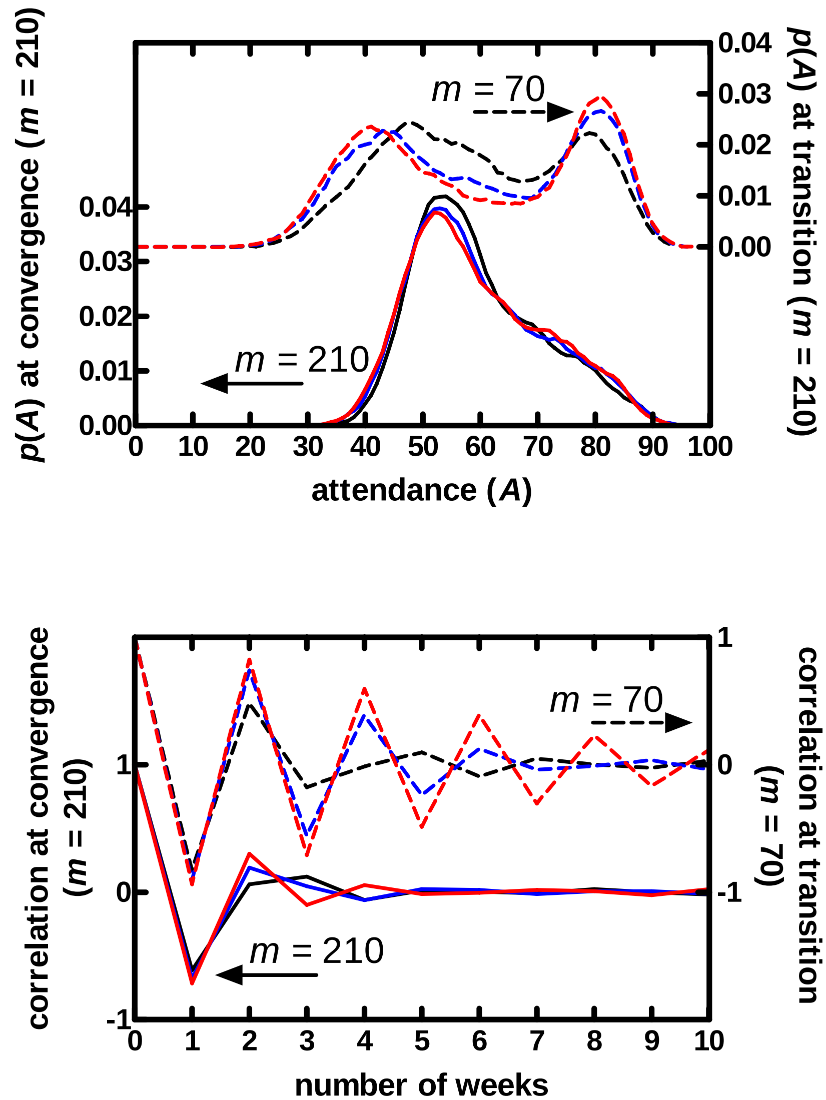

probability of algorithm type
I
II
III
0.0
0.2
0.4
0.6
0.8
2
6
10
14
18
memory, m
point-wise
arithmetic average
weighted average
trend

Figure 2



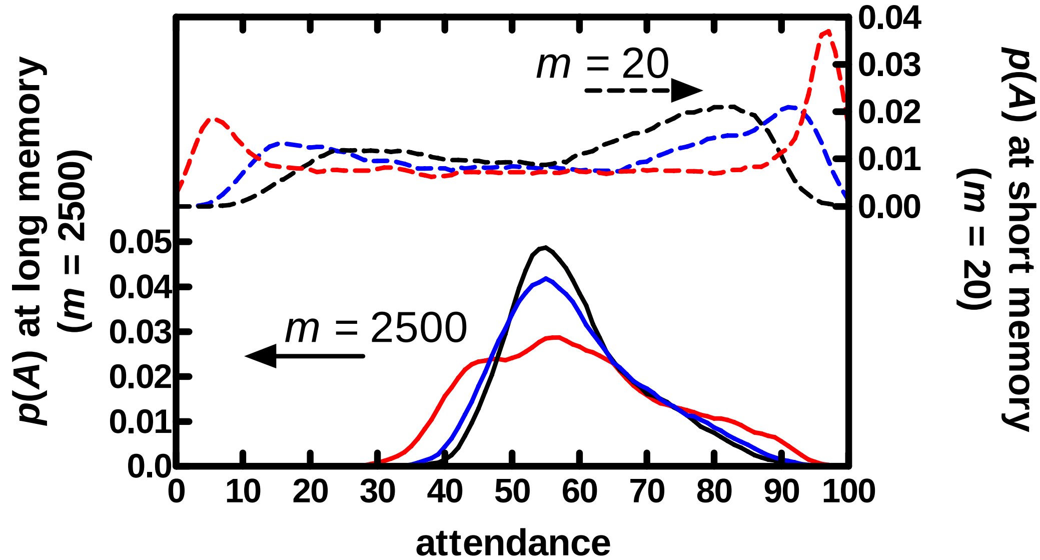
attendance distribution
0.05
0.04
0.03
0.02
0.01
0.00
0
10
20
30
40
50
60
70
80
90
100
attendance
correlation of attendance deviation
1.0
0.5
0.0
-0.5
-1.0
20
60
90
150
210
0
2
4
6
8
10
number of weeks
absolute correlation

Figure 3

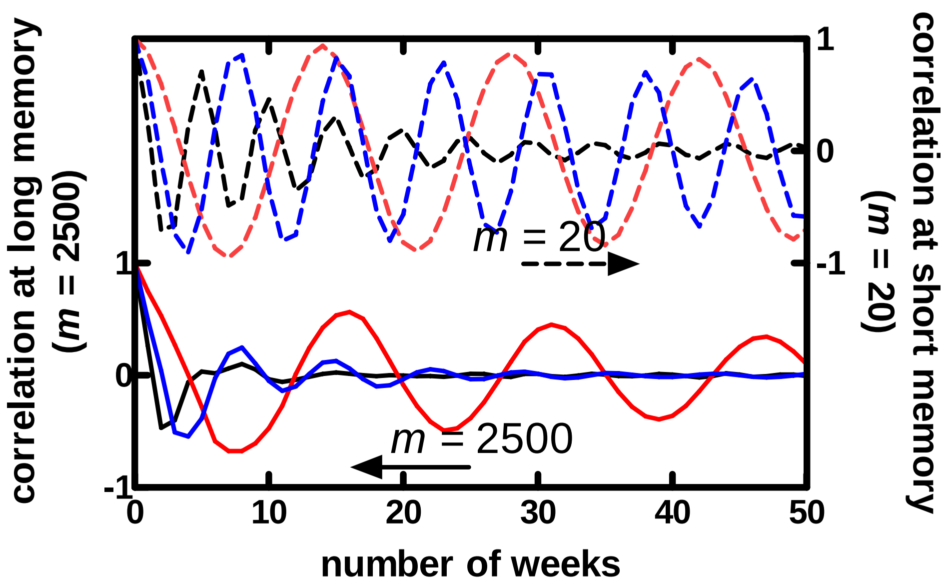


p(A) at convergence (m = 210)
0.04
0.03
0.02
0.01
0.00
p(A) at transition (m = 210)
0.04
0.03
0.02
0.01
0.00
m = 70
m = 210
0 10 20 30 40 50 60 70 80 90 100
attendance (A)
correlation at convergence (m = 210)
1
0
-1
correlation at transition (m = 70)
1
0
-1
m = 70
m = 210
0 1 2 3 4 5 6 7 8 9 10
number of weeks

Figure 4



p(A) at long memory
(m = 2500)
p(A) at short memory
(m = 20)
m = 20
m = 2500
0.04
0.03
0.02
0.01
0.00
0.05
0.04
0.03
0.02
0.01
0.0
0
10
20
30
40
50
60
70
80
90
100
attendance



correlation at long memory
(m = 2500)
correlation at short memory
(m = 20)
m = 20
m = 2500
1
0
-1
0
10
20
30
40
50
number of weeks

Figure 5