AN ANT COLONY OPTIMIZATION APPROACH FOR THE CAPACITATED VEHICLE ROUTING PROBLEM WITH SIMULTANEOUS DELIVERY AND PICK-UP

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Abstract. We propose an Ant Colony Optimization (ACO) algorithm to the NP-hard Vehicle Routing Problem with Simultaneous Delivery and Pick-up (VRPSDP). In VRPSDP, commodities are delivered to customers from a single depot utilizing a fleet of identical vehicles and empty packages are collected from the customers and transported back to the depot. The objective is to minimize the total distance traveled. The algorithm is tested with the well-known benchmark problems from the literature. The experimental study indicates that our approach produces comparable results to those of the benchmark problems in the literature.

1.   Introduction

The Vehicle Routing Problem with Simultaneous Delivery and Pick-up (VRPSDP) is a variant of the VRP where the vehicles are not only required to deliver goods but also to pick up some goods from the customers. Customers receiving goods are called linehauls while customers sending goods are called backhauls. The objective is to minimize the total distance traveled by the vehicles and/or the number of vehicles used subject to maximum distance and maximum capacity constraints on the vehicles. VRPSDP may be classified into three categories: (i) Delivery First, Pick-up Second: the vehicles pick up goods after they have delivered their goods; (ii) Mixed Delivery and Pick-up: linehauls and backhauls can occur in any sequence on a vehicle route; and (iii) Simultaneous Delivery and Pick-up: the vehicles simultaneously deliver and pick-up goods [11]. Delivery First Pick-up Second and Mixed VRPDP problems are jointly referred to as the VRP with backhauling (VRPB). Delivery First Pick-up Second and Mixed VRPDP problems are jointly referred to as the VRP with backhauling (VRPB). Each customer has either a delivery demand or a pick-up to be satisfied. Products to be delivered are loaded at the depot while picked up products are transported back to the depot. A set of vehicle routes has to be designed so that all

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customers are serviced exactly once and no "pick-up customer" is visited before any other "delivery customer" on the same route.

In this paper, we address VRPSDP with a fleet of uniform vehicles each having the same capacity. The delivery and pick- up items are identical in the sense that each unit consumes the same amount of vehicle capacity. Delivery and pick-up locations are unique and the delivery to a customer is only allowed from the depot. The objective is to serve all customers with the minimum total distance.

Although VRP has been intensively studied in the literature research on VRPSDP is scant. The problem is first introduced by Min [10], where book distribution and recollection activity between a central library and 22 remote libraries at a county in Ohio, with limited number of trucks and available capacity, is discussed. Min utilizes a cluster-first route-second approach and solves Traveling Salesman Problems (TSP) to optimality as sub-problems. Halse [9] studies VRPSDP as well as many other special case problems in the VRP literature. Cases with a single depot and multiple vehicles and number of nodes varying between 22 and 150 are addressed. Lagrangean relaxation and column generation approach is utilized. A cluster-first route-second type heuristic is developed in which nodes are first distributed to vehicles and then the problem is solved using 3-opt algorithm.

Angelelli and Mansini [1] study the VRPSDP with time windows constraints. They implement a branch-and-price approach based on a set covering formulation. Gendreau et al. [8] study the VRPSDP for a single vehicle case and develop two heuristics. Casco et al. [3] introduce a load-based insertion procedure for VRPB where the insertion cost for backhaul customers is based on the load still to be delivered. Salhi and Nagy [12] modify this method by allowing backhauls to be inserted in clusters rather than one by one. This procedure is also capable of solving simultaneous problems.

Dethloff [6] presents insertion-based heuristics using four different criteria. He develops 40 instances to test his algorithm. He also compares his results with those of [12] and reports an improvement on Min's [10] problem. In the problem structure in [12], nodes are represented as disjoint delivery or pick-up nodes so repetitive servicing is allowed. Besides, the problem puts a limit on the maximum route length and introduces multiple depots rather than a single depot case.

In this paper, we propose Ant Colony Optimization (ACO) algorithm for the VRPSDP introducing a new visibility function. To our knowledge, no ACO based approach has been previously proposed for this problem. Only Wade and Salhi [25] address the Mixed VRPB and Reimann et al. [20] address the VRPB with time windows using an ACO based approach. The remainder of the paper is organized as follows: In Section 2 description of the problem is provided. Section 3 is devoted to the discussion of the proposed ACO approach. Section 4 discusses computational experiments and numerical results. Finally, concluding remarks and future research issues are presented in Section 5.

2. Problem Description

The problem deals with a single depot distribution/collection system servicing a set of customers using a homogeneous fleet of vehicles. Each customer requires two types of service: a delivery and a pick-up. The critical feature of the problem is that both activities have to be carried out simultaneously by the same vehicle. Products to be delivered are

loaded at the depot and products picked up are transported back to the depot. The objective is to find the set of routes servicing all the customers with the minimum total distance [1].

From a practical point of view VRPSDP models situations such as distribution of bottled soft drinks, LPG tanks, laundry service of hotels where the customers are typically visited only once but for a double service and grocery stores where reusable specialized pallets/containers are used for the transportation of merchandise. Also, regulations may force companies to take responsibility for their products throughout their lifetime.

Mathematically, VRPSDP is described by a set of homogenous vehicles V, a set of customers C, and a directed graph G (N, A). $N = \{0,...,n+1\}$ denotes the set of vertices. Each vehicle has capacity Q and each customer i has delivery and pick-up requests d_i and p_i, respectively. The graph consists of $|C|+2$ vertices where the customers are denoted by $1,2,...,n$ and the depot is represented by the vertices 0 and $n+1$. $A = \{(i, j): i?j\}$ denotes the set of arcs that represents connections between the depot and the customers and among the customers. No arc terminates at vertex 0 and no arc originates from vertex $n+1$. A cost/distance c_{ij} is associated with each arc (i, j). Finally, Q, d_i, p_i, c_{ij} are assumed to be non-negative integers.

If P is assumed as an elementary path in G, $P = \{0 = i_0, i_1,..., i_p, i_{p+1} = n + 1\}$, a feasible solution for our problem can be represented by a set of disjoint elementary paths originating from 0 and ending at $n+1$. These paths visit every customer exactly once while satisfying the capacity constraints. Thus, the pick-up demands already collected plus the quantities to be delivered must not exceed the vehicle capacity. The objective is to minimize the total distance traveled by all the vehicles. (Refer to Dethloff [6] for the mathematical model)

Anily [2] proves that the VRPB is *NP*-hard in the following way: If $P_j = 0$ (j ? J) or even $P_j = D$ (j ? J) then the problem reduces to the VRP which is known to be *NP*-hard. Thus, VRPB is also *NP*–hard. VRPB may be considered as a special case of the VRPSDP where either the delivery demand D_j or the pick-up demand P_j of each customer equals zero [12]. Hence, VRPSDP is also *NP*–hard.

3. Description of the ACO based approach

ACO is based on the way real ant colonies behave to find the shortest path between their nest and food sources. While walking ants leave an aromatic essence, called pheromone, on the path they walk. Other ants sense the existence of pheromone and choose their way according to the level of pheromone. Greater level of pheromone on a path will increase the probability of ants following that path. The level of pheromone laid is based on the path length and the quality of the food source. It will increase when the number of ants following that path increases. In time all ants are expected to follow the shortest path.

ACO simulates the described behavior of real ants to solve combinatorial optimization problems with artificial ants. Artificial ants find solutions in parallel processes using a constructive mechanism guided by artificial pheromone and a greedy heuristic known as visibility. The amount of pheromone deposited on arcs is proportional to the quality of the solution generated and increases at run-time during the computation. ACO was first introduced for solving the TSP [5]. Since then many implementations of ACO have been proposed for a variety of combinatorial optimization problems such as quadratic assignment problem, scheduling problem, sequential ordering problem, and vehicle routing

problems. The interested reader is referred to [7] for details of ACO metaheuristic. In what follows is the description of the algorithm.

3.1. Initialization

An initial amount of pheromone t_0 is deposited on each arc. In the literature, it has been observed that $t_0 = 1/nL_{init}$, where L_{init} is the length of an initial feasible route and n is the number of customers, can generate the good routes. The initial route is constructed by starting at the depot and then selecting the not yet visited closest feasible customer as the next customer to be visited. A customer is infeasible if it violates the capacity. If no feasible customer is available then the route is terminated at the depot and a new route is initiated.

Since the distance of customers to the depot is an essential characteristic of the tour length we incorporate this in calculating the visibility considering the savings of serving customer i and j on the same route instead of serving them on different tours. The distance between two customers is introduced into the visibility function through the use of Clarke and Wright savings measure [4]:

$$s_{ij} = d_{i0} + d_{0j} - d_{ij} \tag{1}$$

d_{ij} (d_{i0}) denotes the distance between customers i and j (the depot). The higher savings value favors visiting customer j after customer i while the longer distance value prevents it. Thus, the savings per unit distance traveled between customers measures the attractiveness of visiting customer j after customer i. The visibility of selecting customer j after customer i is then computed as follows:

$$h_{ij} = \begin{cases} s_{ij}/d_{ij} & , \ \text{if } s_{ij} \geq 1 \\ 1/d_{ij} & , \ \text{otherwise} \end{cases} \tag{2}$$

Since a high value of $?_{ij}$ indicates that visiting customer j after customer i is a desired choice the tour length is expected to be shorter if the probability of moving from customer i to customer j increases with $?_{ij}$. Furthermore, a candidate list is used to reduce the computation time. The candidate list of each customer is formed as follows: in ACO, visiting customer j after the current customer i is based on the amount of both the pheromone trails t_{ij} and the visibility $?_{ij}$ on arc (i,j). Therefore, at each customer i candidate set $O(i)$ is formed by taking k feasible customers with the largest attractiveness j_{ij}.

3.2. Route construction process

An ant is positioned at each customer and each ant constructs its own tour by successively selecting a not yet visited feasible customer. The choice of the next customer is based on its attractiveness values, which is a function of the pheromone trails and the visibility:

$$j_{ij} = [t_{ij}]^a [h_{ij}]^b \tag{3}$$

a and β are parameters to control the relative weight of trail intensity t_{ij} and visibility $?_{ij}$.

Using the following equations (4) and (5) each ant may either visit the most favorable customer or randomly select a customer to visit based on a probability distribution $p(i,j)$.

$$p(i,j) = \begin{cases} arg \ max_{j \in \Omega(i)} j_{ij} & , \quad \text{if } q \leq q_0 \\ P(i,j) & , \quad \text{otherwise} \end{cases} \tag{4}$$

$$P(i,j) = \begin{cases} \bm{j}_{ij} \Big/ \sum_{h \in \Omega(i)} \bm{j}_{ih} \quad , & \text{if } j \in \Omega(i) \\ 0 \qquad\qquad , & \text{otherwise} \end{cases}$$ (5)

where q_0 ($0 = q_0 = 1$) is a parameter to control exploitation versus exploration.

3.3. Local and global updates

In order to reduce the probability of repeatedly selecting a customer, the amount of the pheromone on the arc is reduced through evaporation. The reduction is made by applying a local updating rule given in equation (6):

$$\bm{t}_{ij} = (1 - \bm{r})\bm{t}_{ij} + \bm{r}\bm{t}_0 ?$$ (6)

where $?$ ($0 = ? = 1$) is the trail persistence parameter. If no feasible customer is available due to the vehicle capacity constraint then the depot is chosen and a new route is started. This process is executed until all customers have been visited.

Once all ants construct their tours, the best $?$ tours are chosen and the global updating rule is applied as follows:

$$\bm{t}_{ij} = (1 - \bm{r}).\bm{t}_{ij} + \sum_{r=1}^{l-1} \Delta \bm{t}_{ij}^r + \bm{l}.\Delta \bm{t}_{ij}^*$$ (7)

If an arc is used by the rth best ant, the pheromone value on arc (i, j) will be increased by $\Delta \bm{t}_{ij}^r = (\bm{l} - r)/L_r$, where L_r is the tour length of rth best ant. The pheromone level on the arcs of the best solution is also increased by $\Delta \bm{t}_{ij}^* = 1/L^*$, where L^* is length of the best tour. The steps of the algorithm are summarized in Figure 1.

```
compute visibility
initialize pheromone
while (max number of iterations is not reached)
    for each ant i
        select the next customer to visit from candidate list O(i)
        update vehicle capacity and candidate list
        perform local pheromone trail update
    end for
    perform global pheromone trail update
    save the best route
end while
```

Figure 1. Pseudo-code of the algorithm.

4. Experimental study

In this section, the proposed ant colony system based approach for VRPSDP is tested on the benchmark problem instance(s) of Min [10] and Dethloff [6]. The algorithm is coded using C++. The parameters were set according to initial experimental runs to different problems: k = (No of Customers)/2, $q_0 = 0.80$, $a = 0.75$, $\beta = 4$, and $? = 15$. The number of iterations is set to 5000 and $\bm{r} = 0.15$.

Problem	Dethloff's Best	Avg Soln	Best Soln	% Imp
Min	89	89,8	88	**1,14%**
SCA3-0	689	705,9	697,0	-1,15%
SCA3-1	765,6	781,5	772,4	-0,88%
SCA3-2	742,8	744,0	742,6	**0,02%**
SCA3-3	737,2	731,0	727,9	**1,27%**
SCA3-4	747,1	766,6	758,9	-1,55%
SCA3-5	784,4	803,8	799,8	-1,92%
SCA3-6	720,4	725,5	719,2	**0,17%**
SCA3-7	707,9	742,3	731,5	-3,23%
SCA3-8	807,2	820,9	799,7	**0,93%**
SCA3-9	764,1	763,4	746,2	**2,40%**
SCA8-0	1132,9	1102,8	1087,5	**4,17%**
SCA8-1	1150,9	1232,2	1230,0	-6,43%
SCA8-2	1100,8	1164,7	1143,1	-3,70%
SCA8-3	1115,6	1264,9	1243,6	-10,29%
SCA8-4	1235,4	1331,6	1286,9	-4,00%
SCA8-5	1231,6	1272,4	1266,2	-2,73%
SCA8-6	1062,5	1181,9	1177,5	-9,77%
SCA8-7	1217,4	1304,2	1293,0	-5,85%
SCA8-8	1231,6	1350,7	1326,6	-7,16%
SCA8-9	1185,6	1230,7	1189,4	-0,32%
CON3-0	672,4	668,6	662,6	**1,48%**
CON3-1	570,6	606,4	600,5	-4,97%
CON3-2	534,8	549,5	538,3	-0,66%
CON3-3	656,9	644,4	636,5	**3,20%**
CON3-4	640,2	630,9	628,7	**1,84%**
CON3-5	604,7	618,9	610,7	-0,99%
CON3-6	521,3	549,3	546,3	-4,58%
CON3-7	602,8	634,8	625,1	-3,57%
CON3-8	556,2	581,6	578,0	-3,77%
CON3-9	612,8	615,4	615,3	-0,40%
CON8-0	967,3	1011,1	1000,0	-3,27%
CON8-1	828,7	845,7	840,5	-1,40%
CON8-2	770,2	837,4	820,0	-6,07%
CON8-3	906,7	970,6	960,0	-5,56%
CON8-4	876,8	909,2	896,3	-2,17%
CON8-5	866,9	946,6	923,8	-6,16%
CON8-6	749,1	827,3	799,2	-6,27%
CON8-7	929,8	959,1	934,2	-0,47%
CON8-8	833,1	887,7	876,5	-4,96%
CON8-9	877,3	914,8	898,2	-2,33%

Table 1. Average improvements with ACO compared to Dethloff's results.

Min [10] reported the objective function value for his problem as 94. Dethloff [6] reported his best solution for Min's problem as 91 with a computation time of 0.27 seconds. Dethloff also reported the best known solution as 89 after 100 hours of computing time on a Pentium III 500 Mhz processor using XPRESS-MP. Our proposed algorithm obtained has been able to improve the solution to 88 with a computation time of 50 seconds on a Pentium IV 2.6 Ghz processor.

Dethloff performed 10 experiments for each data and published the average travel distances. We performed 5 runs for each instance, compare the averages to those of Dethloff's, and compute the gap as (Dethloff Avg/ACO Avg) – 1. We observe that ACO algorithm outperforms in 9 out of 40 problem instances. The average computation time for all problem instances is about 15 minutes. Table 1 reports Dethloff's best solution in comparison with our average and best solutions as well as the gap for all problem instances. We observe that our algorithm performs better for the SCA instances where the customers are scattered. It is also worth noting here that these results are directly obtained from the ACO algorithm and they may be further improved by using a local search heuristic.

5.   Conclusion and future research directions

We address the VRPSDP which has a growing practical relevance in the reverse logistics literature. The computational complexity of the problem necessitates good heuristic solution procedures. We tackle the problem using an ACO based solution approach equipped with a new visibility function. The experimental analysis reveals some improvements to the previously published results in the literature. In addition, our algorithm has provided the best solution in a well-known problem instance. On the other hand, the computation times are larger compared to the other heuristics whose progresses have been declared to be in seconds.

Future work in this area may be dedicated to apply a local search heuristic to further improve the solutions obtained through ACO algorithm. The author is currently investigating 2-opt and 3-opt algorithms. Another future direction may involve attempting to further reduce the number of user controlled parameters and to improve the speed of the procedure. Parallel computing techniques may be utilized to reduce the computational efforts. The approach may be extended to apply to other types of VRP as well.

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