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Engines of Liberation

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Electricity was born at the dawn of the last century. Households were inundated with a flood of new consumer durables. What was the impact of this consumer durable goods revolution? It is argued here that the consumer goods revolution was conducive to liberating women from the home. To analyse this hypothesis, a Beckerian model of household production is developed. Households must decide whether or not to adopt the new technologies, and whether a married woman should work. Can such a model help to explain the rise in married female labour-force participation that occurred in the last century? Yes.

> The housewife of the future will be neither a slave to servants nor herself a drudge. She will give less attention to the home, because the home will need less; she will be rather a domestic engineer than a domestic labourer, with the greatest of all handmaidens, electricity, at her service. This and other mechanical forces will so revolutionize the woman's world that a large portion of the aggregate of woman's energy will be conserved for use in broader, more constructive fields.
> Thomas Alva Edison, as interviewed in *Good Housekeeping Magazine*, LV, no. 4 (October 1912, p. 436)

1. INTRODUCTION

The dawn of the last century ushered in the Second Industrial Revolution: the rise of electricity, the internal combustion engine and the petrochemical industry. As this was happening, another technological revolution was beginning to percolate in the home: the household revolution. This introduced labour-saving consumer durables, such as washing machines and vacuum cleaners. It also saw the introduction of other time-saving products, for instance, frozen foods and ready-made clothes. The impact of the household revolution was no less than the industrial revolution. At the turn of the last century most married women laboured at home. Now, the majority work in the market. It will be argued here that technological progress in the household sector played a major role in liberating women from the home.

1.1. The analysis

To address the question at hand, Becker's (1965) classic notion of household production is introduced into a dynamic general equilibrium model. In particular, household capital and labour can be combined to produce home goods, which yield utility. This isn't the first time that household production theory has been embedded into the neoclassical growth model. Benhabib, Rogerson and Wright (1991) have done so to study the implication of the household sector for business cycle fluctuations. The analysis undertaken here differs significantly, though, from the above work. It assumes that over the last century there has been tremendous investment-specific technological progress in the production of household capital. These new and improved capital goods allow household production to be undertaken using less labour. Additionally, the price of these durables declines over time due to technological advance in the capital goods producing sector, spurring adoption. The formalization of the labour-shedding nature of new technologies is reminiscent of Krusell, Ohanian, Rios-Rull and Violante's (2000) analysis of the impact that biased technological progress had on the postwar skill premium.

Households in the analysis are taken to differ by ability or income. At the heart of the developed framework are two interrelated decisions facing each household. First, they must choose whether or not to adopt the new technology at the going price. This decision is complicated by the fact that prices fall over time: should they buy today or wait for a lower price. Additionally, the decision to adopt is influenced by the household's income level. Second, they must decide whether the woman in the family should work in the market or not.

The question at hand is whether or not such a framework can help to explain the increase in female labour-force participation over the last century. This is a quantitative matter. So to address this question, the developed model is simulated to see if it can match the observed rise in female labour-force participation. Two exogenous time series are inputted into the simulation. First, prices for appliances are assumed to drop at the historically average rate. Second, over the last century the ratio of female to male wages has risen. To control for this, a series for the gender gap is inputted into the model. The simulation results display several features:

(1) female labour-force participation increases over time;
(2) housework declines over time;
(3) the diffusion of new appliances through the economy is gradual with the rich adopting first.

The upshot of the analysis is that technological advance in the household sector may be an important factor in explaining the rise in U.S. female labour-force participation over the last century.

Just how important depends upon the configuration of the model that is simulated. The baseline model with lumpy durables and indivisible labour has little trouble generating an increase in female labour-force participation of the magnitude observed in the data. Technological progress in the household sector *taken alone* can account for 28% points of the 51% point rise in female labour-force participation produced by the model. Interestingly, the narrowing of the gender gap alone accounts for little, only 10% points, of the simulated rise. This speaks to the presence of an interaction effect in the model. Without labour-saving durable goods, the elasticity of female labour supply is low. As household durables are introduced into the economy the responsiveness of female labour supply to a narrowing in the gender gap increases. A version of the model run with divisible household goods and labour suggests that the durable goods revolution induces slightly more than one half of the observed rise in female labour-force participation.

There are of course other explanations for the increase in female labour-force participation. For example, Galor and Weil (1996) argue that economic development led to a change in the nature of jobs (from brawn to brain so to speak) that was favourable to women's participation.[1] A gradual change in social norms may have occurred as more and more women worked; this is stressed by Fernandez, Fogli and Olivetti (2002). There is no quarrel here with either of these two hypotheses. It would be difficult, and even undesirable, however, to incorporate these and other explanations into a single framework.[2] Theory, by essence, is the process of abstraction. Furthermore, given the paucity of historical evidence it may never be possible to gauge with

1. In their model capital and brains are complementary in the market production function. As the capital stock rises so does the demand for brains. Hence, in the Galor and Weil (1996) analysis the rise in female labour-force participation is also due to technological advance, but in the market sector as opposed to the home sector stressed here. The current analysis complements theirs.

2. Costa (2000) presents a recent overview of the various factors behind the rise in married female labour-force participation. The classic source detailing the rise in female labour-force participation is Goldin (1990).

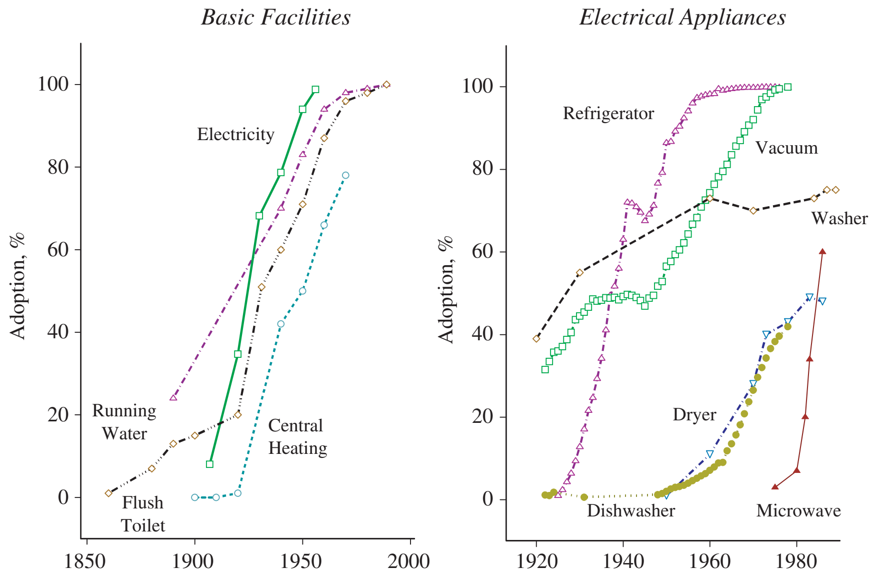

FIGURE 1

The diffusion of basic facilities and electrical appliances through the U.S. economy

much precision the contribution of each force to the rise in female labour supply. Attention will now be directed to the available evidence.

1.2. Historical facts

Durable goods. The household revolution was spawned by massive investment-specific technological progress in the production of household capital. This era saw the rise of central heating, dryers, electric irons, frozen foods, refrigerators, sewing machines, washing machines, vacuum cleaners and other appliances now considered fixtures of everyday life. The spread of electricity, central heating, flush toilets and running water, through the U.S. economy is shown in Figure 1.[3] Likewise, Figure 1 also plots the diffusion of some common electrical appliances through American households. Investment in household appliances as a percentage of GDP more than tripled over the last century. It represented about 0·5% of GDP in 1988, which was about 2·9% of total investment spending. Similarly, the stock of appliances as a percentage of GDP has also risen continuously, roughly doubling in magnitude, as Figure 2 shows.[4] Last, the data suggests that the poorer a family was, the slower they were to purchase durable goods—for instance, see Day (1992, Table 8, p. 319). The relative price of new goods fell rapidly after their introduction. Poor households tended to purchase durable goods at later dates, when their prices were lower, than did rich ones.

Time savings. To understand the impact of the household revolution, try to imagine the tyranny of household chores at the turn of the last century. In 1890 only 24% of houses had

3. Sources: (i) Central heating, Lebergott (1976, p. 100); (ii) Electricity, Vanek (1973, Table 1.1); (iii) Running water and flush toilets, Lebergott (1993, Tables II.14 and II.15); (iv) Dishwashers, refrigerators and vacuum cleaners, *Electrical Merchandising* (1947); (v) Dryers and microwaves, Burwell and Sweezy (1990, Figures 11.8 and 11.10); (vi) Washers, Lebergott (1993, Table II.20).

4. Sources: *Survey of Current Business* and *Fixed Reproducible Tangible Wealth in the United States, 1925–1989*. Washington, D.C., U.S. Department of Commerce.

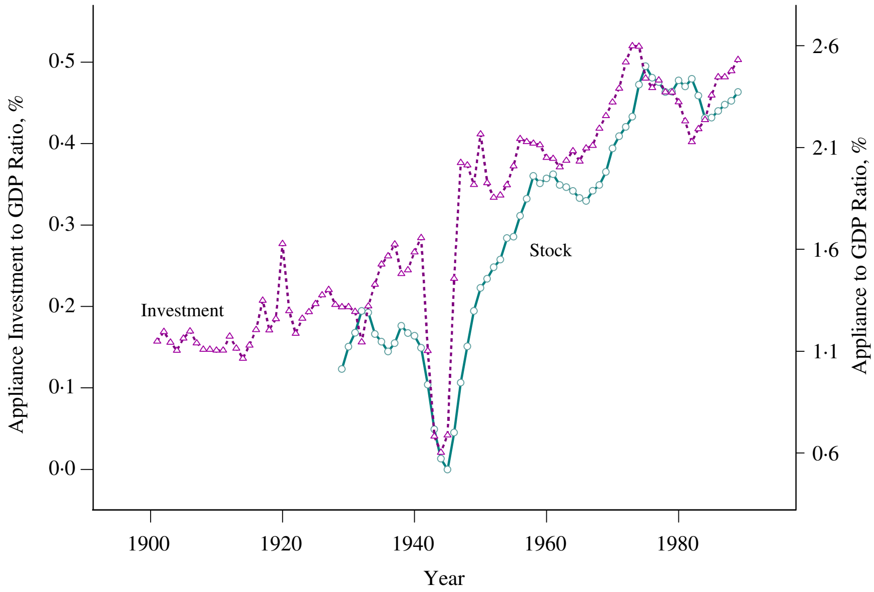

FIGURE 2
Household appliances

running water, none had central heating. So, the average household lugged around the home 7 tons of coal and 9000 gallons of water per year. The simple task of laundry was a major operation in those days. While mechanical washing machines were available as early as 1869, this invention really took off only with the development of the electric motor. Ninety-eight per cent of households used a 12 cent scrubboard to wash their clothes in 1900. Water had to be ported to the stove, where it was heated by burning wood or coal. The clothes were then cleaned via a washboard or mechanical washing machine. They had to be rinsed out after this. The water needed to be wrung out, either by hand or by using a mechanical wringer. After this, the clothes were hung out to dry on a clothes line. Then, the oppressive task of ironing began, using heavy flatirons that had to be heated continuously on the stove.

The amount of time freed by modern appliances is somewhat speculative. Controlled engineering studies documenting the time saved on some specific task by the use of a particular machine would be ideal. Unfortunately, these studies seem hard to come by. The Rural Electrification Authority supervised one such study based on 12 farm wives during 1945–1946. They compared the time spent doing laundry by hand to that spent using electrical equipment. The women also wore a pedometer. One subject, Mrs. Verett, was reported on in detail.[5] Without electrification, she did the laundry in the manner described above.[6] After electrification Mrs. Verett had an electric washer, dryer and iron. A water system was also installed with a water heater. They estimated that it took her about 4 h to do a 38 lb load of laundry by hand, and then about 4·5 h to iron it using old-fashioned irons. By comparison it took 41 min to do a load of the laundry using electrical appliances and 1·75 h to iron it. The woman walked 3181 feet to do the laundry by hand, and only 332 feet with electrical equipment. She walked 3122 feet when ironing the old way, and 333 the new way.

5. This study is reported in *Electrical Merchandising* (1947), 1 March, pp. 38–39.
6. She actually used a gas-powered washing machine instead of a scrubboard.

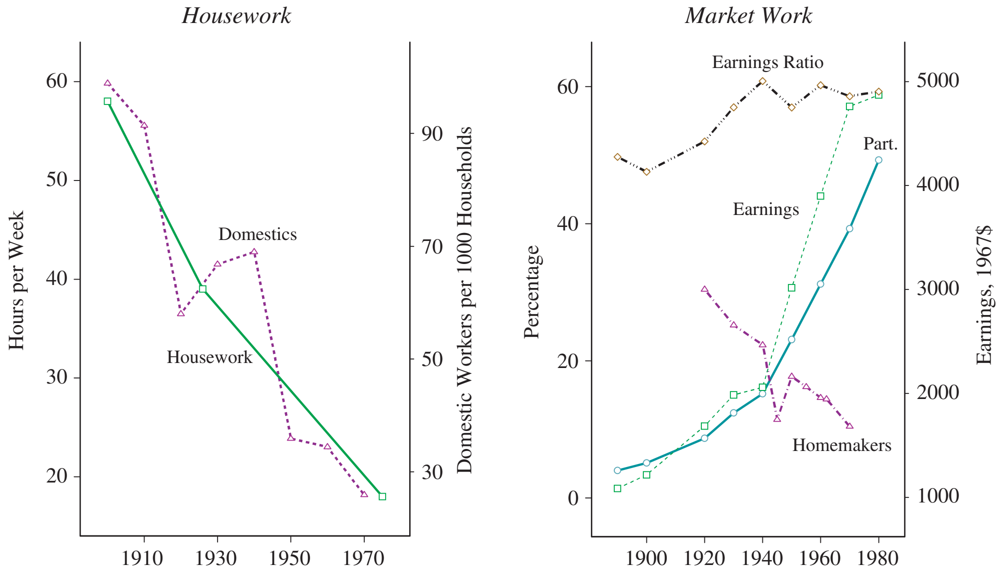

FIGURE 3
Housework and market work

In 1900 the average household spent 58 h a week on housework—meal preparation, laundry and cleaning. This compares with just 18 in 1975.[7] At the same time the number of paid domestic workers declined, presumably in part due to the labour-saving nature of household appliances. Hence, the time spent on the more onerous household chores, such as those associated with cooking, cleaning, doing laundry, etc., declined considerably in the last century—see Figure 3.[8]

Female labour-force participation. What was the effect of this massive technological advance in the production of household capital on labour-force participation? A case can be made that it helped to liberate women from the home. As can be seen from Figure 3, female labour-force participation rose steadily since 1890. At the same time the number of homemakers continuously declined. Real income per full-time female worker grew fivefold over this period. In 1890 a female worker earned about 50% of what a male did, and by 1970 this number had risen to only 60%. It seems unlikely that the small rise in the relative earnings of a female worker could explain on its own the dramatic rise in labour-force participation, unless the elasticity of female labour supply is very large.

2. THE ECONOMIC ENVIRONMENT

The world is made up of overlapping generations. Each generation lives J periods. Hence, in any period there are J generations around.

7. Interestingly, Roberts and Rupert (1995) report, using data from the Panel Study on Income Dynamics, that between 1976 and 1988 the time spent on housework by a working wife fell significantly from 20·2 h per week to 15·9. The time spent by a non-working wife dropped very slightly from 34·0 to 32·2 h per week.

8. Sources: (i) Housework, Lebergott (1993, Table 8.1); (ii) Domestics, Oppenheimer (1970, Table 2.5); (iii) Ratio of female to male earnings, and participation, Goldin (1990, Table 5.1); (iv) Homemakers, Vanek (1973, Table 1.22).

Tastes. Let tastes for an age-j household be given by

$$\sum\nolimits_{i=j}^{J} \beta^{i-j}[\mu \ln m^i + \nu \ln n^i + (1-\mu-\nu)\ln l^i], \tag{1}$$

where m^i and n^i are the consumptions of market and non-market produced goods, and l^i is the household's leisure.

Income. A household is made up of a male and a female. They are endowed with two units of time, which they split up between market work, home work and leisure. Work in the market is indivisible. Set market time at ω. To some this may be a liability, but note that the historical data shown in Figure 3 measures female labour-force participation, an *extensive* margin concept and *not* hours worked, which could be modelled along the intensive margin. It will be assumed that males always work in the market. The household can choose whether or not the female will work in the market. Each household is indexed by an ability level, λ, shared by both members. This is drawn at the beginning of their lives. They make all decisions knowing the value of λ. Let ability λ be drawn from a lognormal distribution. Normalize the mean of λ at unity. Therefore, assume that $\ln\lambda \sim N(-\sigma^2/2, \sigma^2)$. Denote the ability distribution function by $L(\lambda)$. The market wage for an efficiency unit of male labour is given by w. A woman earns the fraction ϕ of what a man does. Hence, in a given period, a family of efficiency level λ will earn the amount $w\lambda\omega$ if the female stays at home and the amount $w\lambda\omega + w\phi\lambda\omega$ if she works. The family may also have assets. Denote these by a and let the gross interest rate be r.

Household production. Home goods are produced according to the following Leontief production function.[9] Specifically,

$$n = \min\{d, \zeta h\}, \tag{2}$$

where d represents the stock of household durables and h proxies for the time spent on housework. The variable ζ represents labour-augmenting technological progress in the household sector. Durable goods are assumed to be lumpy. This assumption is needed to capture the notion of technology adoption and diffusion. Technology adoption as measured in Figure 1 is an extensive margin concept—at any point in time some fraction of the population may not own the new technology. Historically, the adoption of appliances was decreasing in income—again, Day (1992) presents some evidence. With standard tastes and technology if durable goods were divisible then all households would instantly adopt them at any price, albeit in perhaps miniscule quantities. Last, all housework is done by women.

The durable goods revolution, a preview. A household technology is defined by the triplet (d, h, ζ). Recall that household capital, d, is lumpy, and assume that housework, h, is indivisible—these twin assumptions are dropped in Section 6. Let the time price of the technology be q—this is set in terms of hours of work (at the mean skill level). The cost of the technology is equal to the price of the durable goods, d, needed to operate it. Before the arrival of electricity suppose that $d = \delta$, $h = \rho\eta$ and $\zeta = \delta/(\rho\eta)$, where $0 < \rho\eta < 1-\omega$ and $\rho > 1$. Using this old technology, $n = \min\{d, \zeta h\} = \delta$ units of non-market goods can be produced. The price of the old household technology will be set to zero. Now, imagine that electricity comes along together with a new set of durable goods. Define this new technology by the triplet (d', h', ζ'). Here $d' = \kappa\delta$, $h' = \eta$ and $\zeta' = \kappa\delta/\eta$, where $\kappa > 1$. Note that $\zeta' = \kappa\rho\zeta$, so that technological progress can be broken down into the additional amount of capital services

9. Given the adopted set-up, this choice for the household production function amounts to an innocuous normalization. Footnote 22 will make this clear.

provided and the amount of household labour freed up. That is, with the new technology capital services rise by a factor of κ. The old technology requires more labour, a factor ρ more.[10] The new technology produces $n' = \min\{d', \zeta' h'\} = \kappa\delta > \delta$ units of non-market goods. Should a household adopt the new technology? This will depend on its price, q', of course.

Market production. Market production is undertaken according to the standard neoclassical production function

$$\mathbf{y} = \xi \mathbf{k}^{\alpha}(z\mathbf{l})^{1-\alpha}, \tag{3}$$

where $\mathbf{y}$ is output, $\mathbf{k}$ represents the aggregate business capital stock and $\mathbf{l}$ is aggregate labour input. Labour-augmenting technological progress is captured by the variable z. Market output can be used for the consumption of market goods, $\mathbf{m}$, gross investment in business capital, $\mathbf{i}$, and for gross investment in household capital, $\mathbf{d}$. Hence

$$\mathbf{m} + \mathbf{i} + \mathbf{d} = \mathbf{y}. \tag{4}$$

The law of motion for business capital is

$$\mathbf{k}' = \chi \mathbf{k} + \mathbf{i}, \tag{5}$$

where χ factors in physical depreciation.

3. THE HOUSEHOLD'S DECISION PROBLEM

Asset accumulation and labour-force participation decisions. Consider the dynamic programming problem facing an age-j household. Suppose that the household has already made its decision about whether or not to adopt the new technology for the current period. Then the household's state of the world is summarized by the triplet (a, τ, λ). Here $\tau \in \{0, 1, 2\}$ is an indicator function giving the state of the household's technology. When $\tau = 0$ the household does not use the new technology in the current period. When $\tau = 1$ the household purchases and uses the new technology in the current period. Last, $\tau = 2$ denotes the case where the household has adopted previously. The lifetime utility for an age-j household with assets, a, state of technology τ, and ability level λ is represented by $V^j(a, \tau, \lambda)$. It is easy to see that the decisions regarding female labour-force participation and asset accumulation are summarized by

$$\begin{aligned} V^j(a, 0, \lambda) = \max\{&\max_{a'}\{\mu \ln(w\lambda\omega + \phi w\lambda\omega + ra - a') + \nu \ln(\delta) \\ &+ (1 - \mu - \nu)\ln(2 - 2\omega - \rho\eta) \\ &+ \beta \max[V^{j+1}(a', 0, \lambda), V^{j+1}(a', 1, \lambda)]\}, \\ &\max_{a'}\{\mu \ln(w\lambda\omega + ra - a') + \nu \ln(\delta) \\ &+ (1 - \mu - \nu)\ln(2 - \omega - \rho\eta) \\ &+ \beta \max[V^{j+1}(a', 0, \lambda), V^{j+1}(a', 1, \lambda)]\}\}, \end{aligned} \tag{P1}$$

$$\begin{aligned} V^j(a, 1, \lambda) = \max\{&\max_{a'}\{\mu \ln(w\lambda\omega + \phi w\lambda\omega + ra - a' - wq) + \nu \ln(\kappa\delta) \\ &+ (1 - \mu - \nu)\ln(2 - 2\omega - \eta) + \beta V^{j+1}(a', 2, \lambda)\}, \\ &\max_{a'}\{\mu \ln(w\lambda\omega + ra - a' - wq) \\ &+ \nu \ln(\kappa\delta) + (1 - \mu - \nu)\ln(2 - \omega - \eta) \\ &+ \beta V^{j+1}(a', 2, \lambda)\}\}, \end{aligned} \tag{P2}$$

10. Since $\zeta'/\zeta = \kappa\rho > 1$, the technology is labour saving in the sense that if d and h could be freely chosen it must transpire that $d'/h' > d/h$—given the Leontief assumption. Furthermore, it is easy to see that if $\zeta'/\zeta > d'/d$ then $h' < h$.

and

$$\begin{aligned}V^j(a,2,\lambda) = \max\{&\max_{a'}\{\mu\ln(w\lambda\omega+\phi w\lambda\omega+ra-a')+\nu\ln(\kappa\delta)\\&+(1-\mu-\nu)\ln(2-2\omega-\eta)+\beta V^{j+1}(a',2,\lambda)\},\\&\max_{a'}\{\mu\ln(w\lambda\omega+ra-a')\\&+\nu\ln(\kappa\delta)+(1-\mu-\nu)\ln(2-\omega-\eta)\\&+\beta V^{j+1}(a',2,\lambda)\}\}.\end{aligned} \tag{P3}$$

Denote the female labour-force participation decision that arises from these problems by the indicator function $p = P^j(a,\tau,\lambda)$. Here $p = 1$ denotes the event where the woman works. Likewise, the household's asset decision is represented by $a' = A^j(a,\tau,\lambda)$. (Note that prices, w, r and q, are suppressed from the value functions and decision rules when it is convenient.)

The adoption decision. Now, suppose that a household currently does not own the new technology. The household faces a choice about whether to adopt the new technology in the current period or not. The decision problem facing an age-j household is

$$\max_{\tau\in\{0,1\}} V^j(a,\tau,\lambda). \tag{P4}$$

Let $T^j(a,\lambda)$ represent the indicator function that summarizes the decision to adopt ($\tau = 1$) the new technology or not ($\tau = 0$). The solution to this problem is simple:

$$T^j(a,\lambda) = \begin{cases}1, & \text{if } V^j(a,1,\lambda) > V^j(a,0,\lambda),\\ 0, & \text{if } V^j(a,1,\lambda) \le V^j(a,0,\lambda).\end{cases}$$

It only applies to those agents who have not adopted previously. The law of motion for technology must specify that $\tau^{j+1} = 2$ if either $\tau^j = 1$ or $\tau^j = 2$.

Decision rules. Consider generation j. Denote an age-j household's current asset holdings by a^j and its state of technology by τ^j. Now, note that for the first generation $a^1 = 0$. This implies that a^{j+1} and τ^{j+1} can be represented by $a^{j+1} = \mathbf{A}^j(\lambda)$ and $\tau^j = \mathbf{T}^j(\lambda)$. To see that this is so, suppose that $a^j = \mathbf{A}^{j-1}(\lambda)$ and $\tau^{j-1} = \mathbf{T}^{j-1}(\lambda)$. First, note that if $\tau^{j-1} = 1$ or 2 then $\tau^j = 2$. Therefore, in this case, $\tau^j = \mathbf{T}^{j-1}(\lambda) + 1$ or $\tau^j = \mathbf{T}^{j-1}(\lambda)$, respectively. If $\tau^{j-1} = 0$ then $\tau^j = T^j(\mathbf{A}^{j-1}(\lambda),\lambda)$. Hence, write $\tau^j = \mathbf{T}^j(\lambda)$. Second, observe that $a^{j+1} = A^j(\mathbf{A}^{j-1}(\lambda),\mathbf{T}^j(\lambda),\lambda) \equiv \mathbf{A}^j(\lambda)$. To start the induction off, let $\tau^0 = 0 \equiv \mathbf{T}^0(\lambda)$ and $a^1 = 0 \equiv \mathbf{A}^0(\lambda)$. Similarly, an age-$j$ household's participation decision can be written as $\mathbf{P}^j(\lambda)$.

4. COMPETITIVE EQUILIBRIUM

Market-clearing conditions. At each point in time all factor markets must clear. This implies that the market demand for labour must equal the market supply of labour. Therefore,

$$\mathbf{l} = J\omega\int \lambda L(d\lambda) + \phi\omega\sum_{j=1}^{J}\int \lambda\mathbf{P}^j(\lambda)L(d\lambda). \tag{6}$$

The market supply of labour is obtained by summing males' and females' labour supplies across ability levels and generations. Likewise, the next period's business capital stock must equal today's purchases of assets so that

$$\mathbf{k}' = \sum_{j=1}^{J}\int \mathbf{A}^j(\lambda)L(d\lambda). \tag{7}$$

Since the market sector is competitive, factor prices are given by marginal products. Hence,

$$w = (1-\alpha)z\xi(z\mathbf{l}/\mathbf{k})^{-\alpha}, \tag{8}$$

and

$$r' = \alpha\xi(z'\mathbf{l}'/\mathbf{k}')^{1-\alpha} + \chi. \tag{9}$$

It is time to define the competitive equilibrium under study.

Definition. A stationary competitive equilibrium consists of a set of allocation rules $\mathbf{A}^j(\lambda)$, $\mathbf{P}^j(\lambda)$ and $\mathbf{T}^j(\lambda)$, for $j = 1, \ldots, J$, and a set of wage and rental rates, w and r, such that

(1) The allocation rules $\mathbf{A}^j(\lambda)$ and $\mathbf{P}^j(\lambda)$ solve problems (P1) to (P3), given w, r and q.
(2) The allocation rule $\mathbf{T}^j(\lambda)$ solves problems (P1) to (P4), given w, r and q.
(3) Factor prices clear all markets, implying that (6) to (9) hold.

Balanced growth. Represent the pace of labour-augmenting technological progress by γ so that $\gamma = z'/z$. Let $z_0 = 1$ so that $z_t = \gamma^t$. Conjecture that **y**, **m**, **i**, **d** and **k** all grow at this rate too. Also, posit that along a balanced-growth path the aggregate stock of labour, **l**, is constant. This conjecture is consistent with the forms of (3) to (5). This implies from (8) and (9) that r is constant over time, while w grows at rate γ. It remains to be shown that $\mathbf{A}^j_{t+1}(\lambda) = \gamma\mathbf{A}^j_t(\lambda)$, $\mathbf{P}^j_{t+1}(\lambda) = \mathbf{P}^j_t(\lambda)$ and $\mathbf{T}^j_{t+1}(\lambda) = \mathbf{T}^j_t(\lambda)$. Observe that this solution will be consistent with the factor market-clearing conditions (6) and (7).

Lemma 1. *Along a balanced-growth path,* $\mathbf{A}^j_{t+1}(\lambda) = \gamma\mathbf{A}^j_t(\lambda)$, $\mathbf{P}^j_{t+1}(\lambda) = \mathbf{P}^j_t(\lambda)$ *and* $\mathbf{T}^j_{t+1}(\lambda) = \mathbf{T}^j_t(\lambda)$.

Proof. Suppose that along a balanced-growth path $V^{j+1}(\gamma a, \tau, \lambda; \gamma w) = V^{j+1}(a, \tau, \lambda; w) + [(1 - \beta^{J-j})/(1 - \beta)]\mu \ln \gamma$.[11] Now, by eyeballing problems (P1) to (P3) it is easy to see that if $A^j(a, \tau, \lambda; w)$ and $P^j(a, \tau, \lambda; w)$ are the solutions to these problems when the state of the world is $(a, \tau, \lambda; w)$, then $A^j(\gamma a, \tau, \lambda; \gamma w) = \gamma A^j(a, \tau, \lambda; w)$ and $P^j(\gamma a, \tau, \lambda; \gamma w) = P^j(a, \tau, \lambda; w)$ are the solutions when the state of the world is given by $(\gamma a, \tau, \lambda; \gamma w)$. It then follows that $V^j(\gamma a, \tau, \lambda; \gamma w) = V^j(a, \tau, \lambda; w) + [(1 - \beta^{J-j+1})/(1 - \beta)]\mu \ln \gamma$. Finally, note from problem (P4) that $T^j(a, \lambda; w) = T^j(\gamma a, \lambda; \gamma w)$. Therefore, if $A^j(a, \tau, \lambda; w)$, $P^j(a, \tau, \lambda; w)$ and $T^j(a, \lambda; w)$ solve problems (P1) to (P3) today—when the state is $(a, \tau, \lambda; w)$—then $\gamma A^j(a, \tau, \lambda; w)$, $P^j(a, \tau, \lambda; w)$ and $T^j(a, \lambda; w)$ will solve them tomorrow—when the state will be $(\gamma a, \tau, \lambda; \gamma w)$. ‖

Remark. Technological advance in the market sector (which leads to higher earnings for both men and women) has no effect on female labour-force participation. This is also true in the standard home production model, *à la* Benhabib *et al.* (1991). To see this, let tastes remain the same as (1) but rewrite (2) as $n = d^{\varepsilon}(\zeta h)^{1-\varepsilon}$. Drop all indivisibilities. Furthermore, let $\zeta_t = \gamma_\zeta^t < \gamma^t$, so that productivity in the marketplace rises faster than at home. Along a balanced-growth path **y**, **m**, **i**, **d**, **k** and w will all grow at rate γ. Aggregate household production, **n**, will grow at the rate $\gamma^{\varepsilon}\gamma_\zeta^{1-\varepsilon}$. It is easy to check that aggregate market hours, **l**, remains constant. This transpires because the implicit relative price of home goods for an age-j, type-λ household, $(\nu/\mu)m^j(\lambda)/n^j(\lambda)$, rises at rate $(\gamma/\gamma_\zeta)^{1-\varepsilon} > 1$. This exactly offsets the lower increase in the marginal productivity of labour for this type of household at home, $(1 - \varepsilon)\{d^j(\lambda)/[\zeta h^j(\lambda)]\}^{\varepsilon}\zeta$, vis-à-vis at work, w. Therefore, a rise in *both* male and female wages will have *no* effect on

11. Note that the household's problem is a function of w, r and q. Hence, these factor prices should be entered into the value functions. Since r is constant along a balanced-growth path it has been suppressed in the value function—same with q.

female labour-force participation. Hence, the standard model cannot account for the rise in female labour-force participation, at least without modification in a non-neutral direction.

5. FINDINGS

5.1. Some preliminary analysis

Calibration. Take the model period to be 5 years. First, there are four parameters governing market production: α, χ, ξ and z. Set labour's share of income at 70% and assume that the annual rate of depreciation on the market capital stock is 10%. Thus, $\alpha = 0{\cdot}70$ and $\chi = (1 - 0{\cdot}10)^5$. Normalize the coefficient on the market production function to be one so that $\xi = 1{\cdot}0$. Suppose that z grows at some constant rate, γ. Given the isoelastic structure of the model, there is no need to incorporate this technological progress explicitly into the framework.[12] Therefore, assume $z = 1$. Second, there are four parameters controlling household production, namely ω, η, ρ and κ. Now, ω, η and ρ can be pinned down from time-use data. In a week there are 112 non-sleeping hours available per adult. If full-time work involves a 40 h workweek, then $\omega = 0{\cdot}36$. In 1900 about 58 h a week were spent on housework, while 18 were in 1975. So, set $\eta = 0{\cdot}16$ and $\rho\eta = 0{\cdot}52$. In 1925 the per-capita stock of appliances was \$66 (in 1982\$) while in 1980 it was \$528.[13] Hence, $\kappa = 8{\cdot}00$. Third, there are several household taste and type parameters that need to be picked. To this end, let $J = 10$ so that a household has a working life of 50 years. Set the annualized discount factor at 0·96, implying that $\beta = 0{\cdot}96^5$. The lognormal distribution for household skill type λ will be discretized so that $\lambda \in \Lambda \equiv \{\lambda_1, \ldots, \lambda_{100}\}$. The skill distribution is parameterized by setting $\sigma = 0{\cdot}70$, in line with findings on empirical income distributions contained in Knowles (1999). Just two utility parameters μ and ν remain. The determination of these two parameters will now be discussed.

The household sector, circa 1980. The time is 1980. Fifty per cent of married women work now—Goldin (1990, Table 5.1). They earn 59% of what a man does—again, Goldin (1990, Table 5.1). Almost everybody owns electrical appliances. Appliance investment amounts to 0·45% of GDP. The model's steady state can be calibrated to match this situation by choosing the utility parameters μ and ν, and the time price of durables q appropriately. This transpires when $\mu = 0{\cdot}33$, $\nu = 0{\cdot}20$ and $q = 0{\cdot}03$—after setting $\phi = 0{\cdot}59$. With these parameter values female labour-force participation is 51% and the appliance investment to GDP ratio is 0·0045. If a period is 5 years then there are about 8800 working hours (5 years $\times$ 11 months $\times$ 4 weeks $\times$ 40 h) per adult. Hence, the mean male would only need to work about 264 h to purchase modern appliances. The steady-state interest rate is 4·75%, which lies slightly above the rate of time preference. At this interest rate, the investment-to-output ratio is 0·18. This is not that far off its 1980 value of 0·14. The standard deviation for (the ln of) household income is 0·72, close to the value of 0·69 estimated by Knowles (1999, Table 1.4).

Female labour-force participation is a decreasing function (actually a non-increasing one) of household income, as Figure 4 illustrates. Women from very poor households retire later. (Just multiply the participation rate by 10 to get the period a woman retires in.) Why is labour participation a decreasing function of λ, when a household purchases appliances? This is due to the lumpy nature of durables. The fixed cost of appliances becomes less significant for a household as λ rises. Hence, the household cuts back on market work. The fixed cost is very small for most households (as measured as a percentage of lifetime income) in the economy when

12. That is, there exists a simple one-to-one mapping between the model with growth and the model without growth. This mapping is discussed in footnote 21 and is presented in the Appendix.

13. Source: *Fixed Reproducible Tangible Wealth in the United States, 1925–1989* (Table A.17). The series was deflated by the 14 + population.

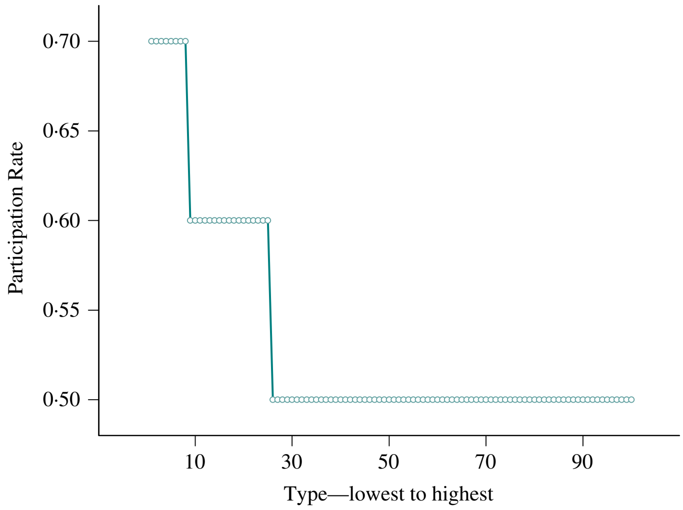

FIGURE 4
Female labour-force participation

$q = 0{\cdot}03$. It becomes more burdensome as the lower end of the type distribution is approached; *i.e.* for $\lambda \leq \lambda_{25}$, which represents the lowest 6% of the population.

The household sector, circa 1900. Now, move back in time to 1900. No one owns an electrical appliance. At this time in history, almost all married women stayed at home. In 1900 only 5% of all married women worked, reports Goldin (1990, Table 5.1). The gender gap is 0·48—once again, Goldin (1990, Table 5.1). The model predicts that no one will work when the new technology is not around (and $\phi = 0{\cdot}48$). The annual interest rate for the model is 5·9%. Once again it lies above the rate of time preference. The standard deviation (for the ln) of household income in the model is 0·70.

Surprisingly, female labour-force participation is *not* a function of income when nobody adopts the new technology. This transpires because the income and substitution effects from a change in λ exactly cancel out given the assumed form for tastes. The fact that female labour-force participation is (i) not a function of λ when nobody adopts the new technology, but (ii) is a decreasing function of λ when everyone adopts the new technology is a general result as the next two lemmas establish.[14]

Lemma 2. *If* $\mathbf{T}^j(\lambda) = \mathbf{0}$ *for all* $j = 1, \ldots, J$ *and* $\lambda \in \Lambda$, *then* $\mathbf{P}^j(\lambda) = \pi^j$ *for all* j *and* λ.

Proof. Take a household of type λ and let $p^j = \mathbf{P}^j(\lambda)$. Its market consumption decision must satisfy the Euler equation

$$\frac{1}{m^j(\lambda)} = \beta r \frac{1}{m^{j+1}(\lambda)}. \tag{10}$$

14. Lemma 2 implies that for the 1900 steady-state female labour-force participation must lie in the 11-point set $\{0, 0{\cdot}1, 0{\cdot}2, \ldots, 1{\cdot}0\}$. Given the discrete nature for aggregate labour-force participation, the algorithm for computing the model's equilibrium is a little temperamental when no one adopts appliances. The equilibrium reported is actually for $\phi = 0{\cdot}47$ and not $\phi = 0{\cdot}48$, because of this.

Using the household budget constraint, this implies that

$$m^j(\lambda) = (\beta r)^{j-1}\frac{1-\beta}{1-\beta^J}\Omega(\lambda),$$

where $\Omega(\lambda)$ is the present value of the household's income—at age 1—net of the cost of purchasing consumer durables. Since the household does not adopt, $\Omega(\lambda)$ is given by

$$\Omega(\lambda) = \sum\nolimits_{j=1}^{J}\frac{w\lambda\omega + \phi w\lambda\omega p^j}{(r)^{j-1}} = w\lambda\omega\left[\frac{1-1/r^J}{1-1/r} + \phi\sum\nolimits_{j=1}^{J}\frac{p^j}{(r)^{j-1}}\right].$$

It is then easy to calculate that lifetime utility is given by

$$\begin{aligned} V^1(0,0,\lambda) = \mu&\left\{\frac{1-\beta^J}{1-\beta}\left[\ln\left(\frac{1-\beta}{1-\beta^J}\right) + \ln\Omega(\lambda)\right] + \sum\nolimits_{j=1}^{J}(j-1)\beta^{j-1}\ln(\beta r)\right\} \\ &+ \nu\frac{1-\beta^J}{1-\beta}\ln\delta + (1-\mu-\nu)\sum\nolimits_{j=1}^{J}\beta^{j-1}\{p^j\ln(2-2\omega-\rho\eta) \\ &+ [1-p^j]\ln(2-\omega-\rho\eta)\}. \end{aligned} \tag{11}$$

Now, there are $2^J - 1$ other possible work combinations. Let p^{*j} denote some other arbitrary work profile and $V^{*1}(0,0,\lambda)$ represent the lifetime utility associated with this particular participation sequence. To obtain $V^{*1}(0,0,\lambda)$ replace p^j with p^{*j} in (11). For p^j to be optimal it must happen that $V^1(0,0,\lambda) \geq V^{*1}(0,0,\lambda)$. Observe that $V^1(0,0,\lambda) - V^{*1}(0,0,\lambda)$ is *not* a function of λ, however—note that $\ln\Omega(\lambda) = \ln(w\lambda\omega) + \ln[(1-1/r^J)/(1-1/r) + \phi\sum_{j=1}^{J}p^j/r^{j-1}]$. Hence, p^j cannot be a function of λ. ‖

Lemma 3. *The present value of female labour-force participation,* $\sum_{j=1}^{J}p^j/r^{j-1}$, *is non-increasing in type,* λ, *holding fixed the date of adoption,* ς. *Similarly,* $\sum_{j=1}^{J}p^j/r^{j-1}$ *is non-increasing in* ς, *holding fixed* λ.

Proof (*by contradiction*). Consider two types of households, λ^* and λ^{**}, with $\lambda^* < \lambda^{**}$. Let p^{*j} denote the optimal participation policy associated with λ^*, and p^{**j} represent the corresponding policy linked with λ^{**}. Analogously, let $B^{*1}(\lambda)$ and $B^{**1}(\lambda)$ be the period-1 L.H.S. of the Bellman equations connected with the policies. These can be obtained by replacing $p^j = \mathbf{P}^j(\lambda)$ in (11) with p^{*j} and p^{**j}, respectively, and adding $[(\beta^{\varsigma-1}-\beta^J)/(1-\beta)]\nu\ln\kappa$.

Suppose that the hypothesis is not true. Then, there exists a λ^* and λ^{**} such that $B^{*1}(\lambda^*) > B^{**1}(\lambda^*)$, $B^{*1}(\lambda^{**}) < B^{**1}(\lambda^{**})$ and $\sum_{j=1}^{J}p^{*j}/r^{j-1} < \sum_{j=1}^{J}p^{**j}/r^{j-1}$. Now, observe from the analogue to (11) that, $B^{*1}(\lambda) - B^{**1}(\lambda) = \mu[(1-\beta^J)/(1-\beta)]\{\ln[\Omega^*(\lambda)/\Omega^{**}(\lambda)]\} +$ *constant*. Here $\Omega^*(\lambda)$ and $\Omega^{**}(\lambda)$ are the levels of permanent income (net of adoption cost) associated with the p^{*j} and p^{**j} policies. Now,

$$\begin{aligned} \frac{d[B^{*1}(\lambda) - B^{**1}(\lambda)]}{d\lambda} &= \mu\frac{1-\beta^J}{1-\beta}\frac{q/r^{\varsigma-1}\phi\omega\sum_{j=1}^{J}[p^{**j}-p^{*j}]/(r)^{j-1}}{\{\sum_{j=1}^{J}[\lambda\omega + \phi\lambda\omega p^{**j}/(r)^{j-1}] - q/r^{\varsigma-1}\}^2}\frac{\Omega^{**}(\lambda)}{\Omega^*(\lambda)} \\ &> 0. \end{aligned}$$

Consequently, if $B^{*1}(\lambda^*) > B^{**1}(\lambda^*)$, then $B^{*1}(\lambda^{**}) > B^{**1}(\lambda^{**})$. The desired contradiction obtains. The proof of the second part of the hypothesis parallels the first, *mutatis mutandis*. ‖

Welfare. The lot of families in the artificial economy can be examined by comparing the 1900 and 1980 steady states. As a result of new, more productive household capital, GDP rises by 30% (in continuously compounded terms). It may be tempting to conclude that the gain

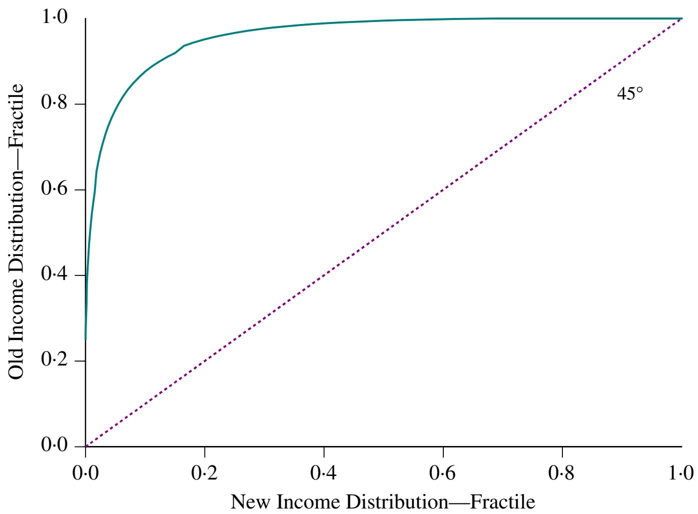

FIGURE 5
An utilitarian view of economic development

in welfare must be less than this. After all, the increase in GDP occurs because more women are working. In fact, welfare increases by 173% (when computed as a compensating variation measured in terms of market consumption). The new technology, together with a narrowing of the gender gap, leads to a 28% increase in market consumption, a 208% increase in non-market consumption (largely due to the fact that household capital rose by eightfold), and a 14% increase in leisure.[15]

Now, take a family living in 1980. They will reside at some percentile in the income distribution and have an associated level of utility. At what spot in the 1900 income distribution would a family have to be located in order to realize this same level of utility? Figure 5 gives the answer. A poor family at the 10-th percentile in 1980 is as well off as someone living in the 90-th percentile in 1900, for instance—just due to the advent of modern appliances.

The effect of declining prices (partial equilibrium). Between 1900 and 1980 the prices for household appliances dropped dramatically; this will be discussed in Section 5.2. The time path of prices has a big impact on the adoption and participation decisions. To see this, consider the following partial equilibrium consumer experiment. For age-1 households, hold the interest rate fixed at 4·7% and imagine that prices fall at 8·3% a year over the course of their lifetimes starting from an (arbitrary) initial value of 22·6. What will be the impact on age-1 households of various types? At this stage, view this consumer experiment as merely illustrating some of the theoretical mechanisms at work in the model.

Figure 6 tells the story. Nobody adopts immediately. Households prefer to wait until prices have dropped to more reasonable levels. Wealthier (or high-type) households adopt first. In line with Lemma 3, adoption goes hand in hand with increased labour effort. Specifically, observe that whenever there is a jump down in the period of adoption there is an immediate leap up in

15. Parente, Rogerson and Wright (2000) show that when household production is incorporated into the standard neoclassical growth model, cross-country differences in welfare are smaller than cross-country differences in GDP. In their framework cross-country income differentials are due to policy distortions. A tax on market activity reduces GDP. Welfare drops by less than GDP, though, because there is an increase in non-market activity. In the current paper, technological progress leads to a rise in all items in the utility function.

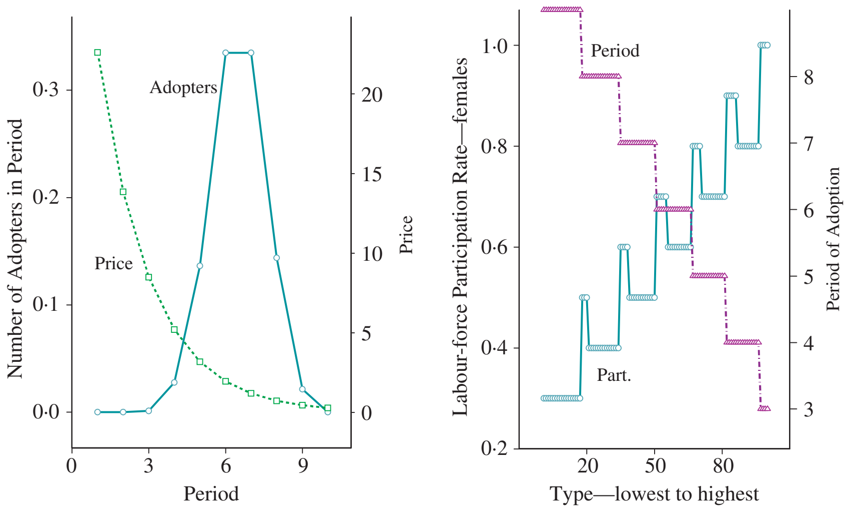

FIGURE 6

The effect of prices on adoption and participation

female labour-force participation.[16] While the profile for labour effort rises over the domain for type it displays a sawtooth pattern. This is consistent with Lemma 3, however, which proved that female labour-force participation is non-increasing in type holding fixed the adoption date.

It may seem that theoretically the date of adoption should be a non-increasing function of λ. This is difficult to establish, though, given the lumpy nature of the adopt and work decisions. The lumpy nature of these decisions can be partially smoothed out by increasing the number of periods that a household lives while holding fixed its lifespan; *i.e.* by shortening the length of a period.

Lemma 4. *Along a balanced-growth path the date of adoption is a non-increasing function in type, at least when the length of a period is sufficiently short.*

Proof. See the Appendix. ‖

5.2. *The durable goods revolution*

The computational experiment (general equilibrium). The time is 1900. The age of electricity has just dawned. This era ushered in many new household goods: dryers, frozen foods, hot water, refrigerators, washing machines, etc. What will be the effect in the artificial economy? To answer this, the transition path from the 1900 steady state to the one for 1980 will be analysed.

To do this, a time path for durable goods prices must be inputted into the model. Hard numbers are hard to come by, but Figure 7 plots quality-adjusted time price series for several appliances.[17] The figure also shows a quality-adjusted price index for eight appliances,

16. Take the case where the type distribution is continuous. Consider some threshold value of λ and a local neighbourhood around it. Suppose that above this value of λ the household adopts at some date ς, while below it they adopt at some later date, say $\varsigma + j$ where the integer $j \geq 1$. As the threshold is crossed the adoption date jumps forward, but λ remains more or less fixed. Hence, the lemma applies.

17. These are based on series contained in Gordon (1990, Tables 7.4, 7.12, 7.15, and 7.22 and 7.23). Each series was deflated by the GDP deflator to get a relative price. The resulting series was then divided by a measure of real wages to convert the data into a time price.

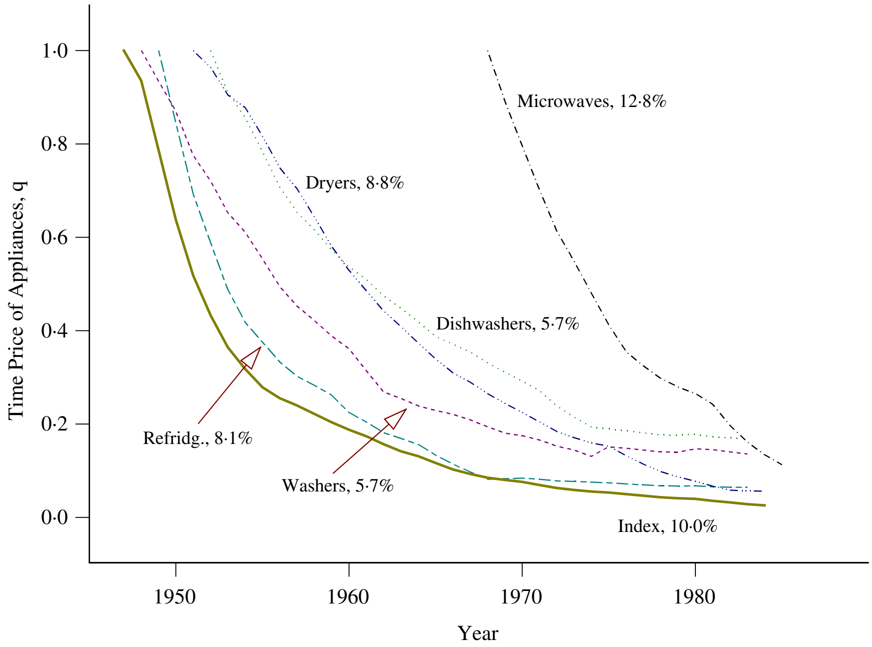

FIGURE 7

Time prices for appliances

namely, refrigerators, air conditioners, washing machines, clothes dryers, TV sets, dishwashers, microwaves and VCRs. This series drops at 10% a year. Assume, then, that time prices decline on average at (a more modest) 8·3% a year for the first 80 years. Thus, $q_{1905} = q_{1985} \times \exp(0{\cdot}083 \times 80)$, where (roughly) in line with the earlier calibration $q_{1985} = 0{\cdot}03$.[18] Likewise, a time path for the gender gap is needed for the simulation. For this a stylized version of Goldin's (1990, Table 5.1) series will be used.[19] The analysis also presumes that agents have perfect foresight; a heroic assumption, for sure.

The rise in female labour-force participation. The upshot of the analysis is shown in Figure 8. The series inputted into the simulation for prices and the gender gap are shown. As can be seen, the model has little problem generating a rise in female labour-force participation. In fact, if anything the underlying forces operating on female labour-force participation are too strong.

To gauge separately the strength of the impact of the durable goods revolution and the narrowing of the gender gap, consider performing two controlled experiments. First, examine the consequence of the narrowing gender gap holding fixed the state of household technology at its 1900 level. Second, study the effect of the durable goods revolution while keeping the gender gap at its 1900 value. Figure 9 shows the results of these two experiments. As can be seen,

18. From the form of the dynamic programming problems (P1) to (P4) it should be apparent that it has been implicitly assumed that once the household has purchased their durables they keep them for the rest of their life. This rules out the *substantial* complications of a second-hand durable goods market. This could create a huge disincentive for older agents to purchase durables. To control for the lack of a resale market, it is assumed that if an age-j agent purchases a durable he then pays $[(J-(j-1)/J)]q$. This amounts to saying that household capital comes in different levels of durability; the more durable the good is, the more you pay.

19. A nice feature of the Galor and Weil (1996) analysis is that the gender gap is endogenous. Here it is exogenous.

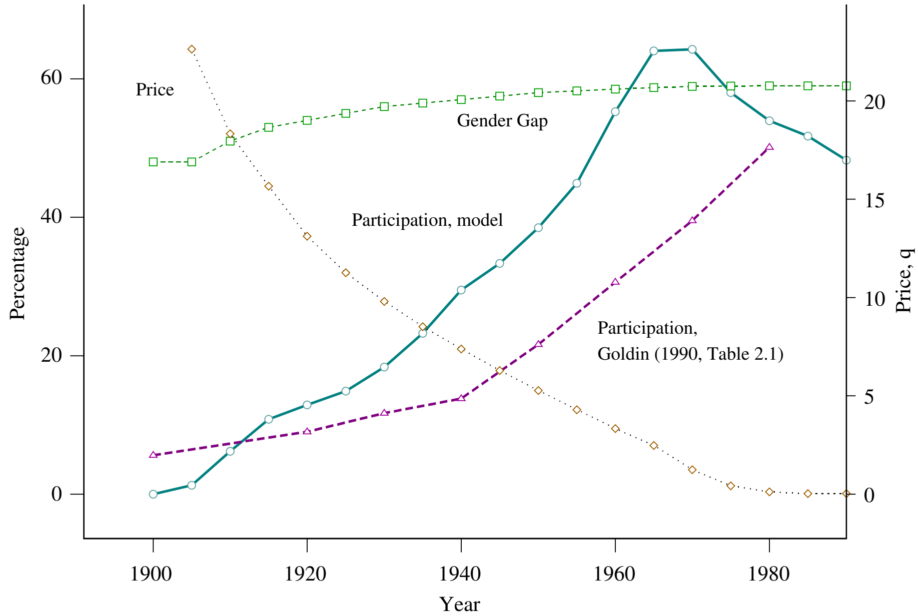

FIGURE 8
Transitional dynamics—the rise in female labour-force participation

a narrowing in the gender gap has only a modest effect on female labour-force participation, if there is no durable goods revolution. Only 10% of women work in 1980. When women are putting in long hours at home, it is hard to entice them into the labour force. The durable goods revolution has a much bigger impact on female labour-force participation. Note that when the gender gap is held fixed labour-force participation is 40% in 1980, converging to a steady-state value of 28%. In the baseline simulation female labour-force participation in 1980 is 54% moving asymptotically to a steady-state value of 51%. The comparison of the steady-state values, speaks to the presence of an interaction effect between the two forces. A narrowing of the gender gap has a bigger influence on female labour-force participation when labour-saving durable goods are available at a reasonable price.

Thus, in the model, the presence of labour-saving durable goods increases the elasticity of female labour supply. Interestingly, the responsiveness of female labour-force participation to changes in wages appears to have increased over time. In particular, labour supply elasticities (both compensated and uncompensated) are larger today than 100 years ago—see Goldin (1990, Table 5.2). The model is consistent, at least qualitatively, with this fact. To see this, consider the responsiveness of steady-state female labour-force participation to an increase in their wages, both with and without new and improved durable goods. Table 1 reports the results of this experiment.

Without modern appliances female labour-force participation rises from 0 to 20% as the gender gap narrows by 20% points. (As a point in fact, one could just as easily say it rises from 0 to 20% as the gender gap narrows by 65% points.) With modern appliances the impact of a change in wages on female labour-force participation is bigger, other things being equal. That is, when the burden of housework is high, the response of female labour supply to changes in wages is muted.

The new appliances catch on slowly at first, disappointingly so. This can be seen from Figure 10, which plots an aggregate diffusion curve. Only wealthy households—high types—

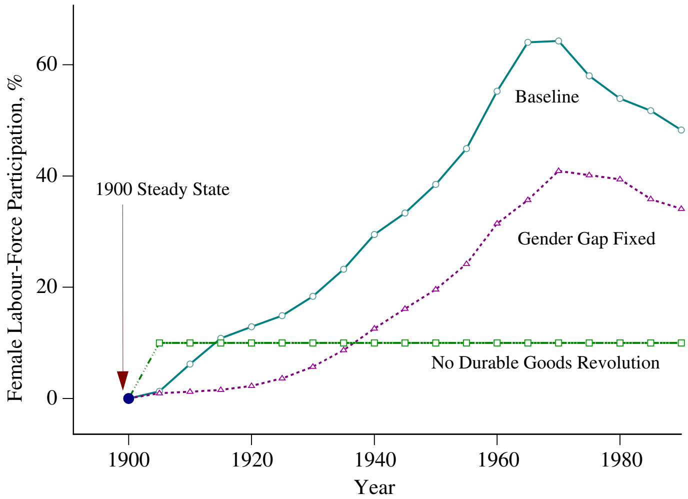

FIGURE 9

Durable goods revolution and gender gap experiments

TABLE 1

Female wages and steady-state labour-force participation

Gender gap, ϕ	Female labour-force participation	
	Without durable goods revol.	With durable goods revol.
0·45	0·00	0·22
0·50	0·00	0·31
0·55	0·10	0·42
0·60	0·10	0·52
0·65	0·20	0·61

can afford to buy when prices are high. The diffusion curves for three age-averaged types of households, are also graphed, namely the poorest families in the economy (λ_1), middle income ones (λ_{50}) and the richest (λ_{100}). The slow nature of diffusion is not surprising, given the model's stark set-up. Households are confronted with an all or nothing decision about whether to buy modern appliances or not. It is easy to imagine a more realistic framework where a continual stream of new, labour-saving durables are introduced into the market. A household would have to decide if and when to buy each of the new appliances. Such an extension would likely possess more appealing diffusion properties.

Welfare, again. The increase in GDP due to the durable goods revolution, together with the narrowing of the gender gap, is shown in the left panel of Figure 11.[20] This rise occurs solely because of the rise in female labour-force participation. Other than the durable goods revolution there is no technological progress. The associated gain in welfare—for the flow of new households into the economy—is also plotted in the left panel of this figure. Again, the improvement in welfare is greater than the increase in GDP.

20. The left panel factors out the effects of growth due to technological progress in the market sector, or to increases in z. This is done by studying the growth-transformed version of the model outlined in the Appendix.

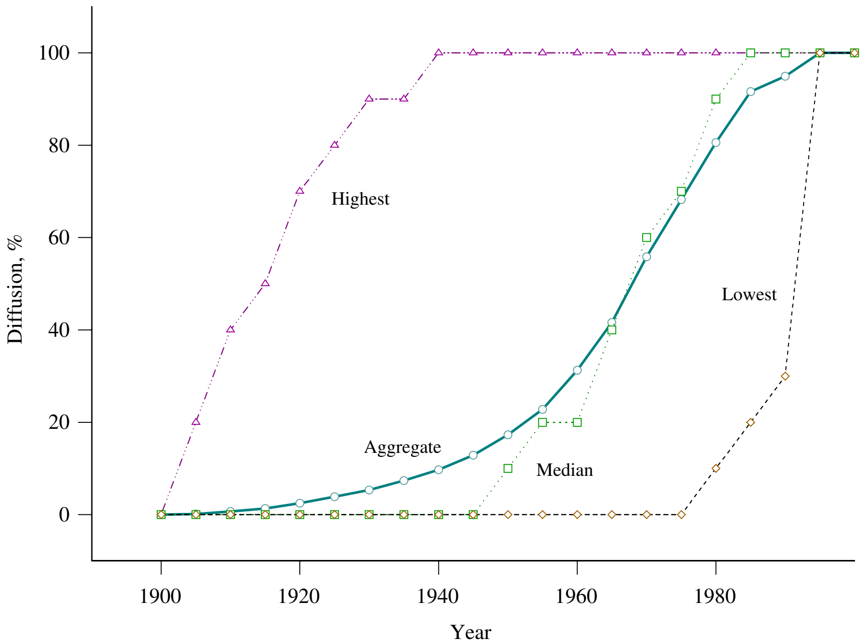

FIGURE 10
Diffusion by type of household

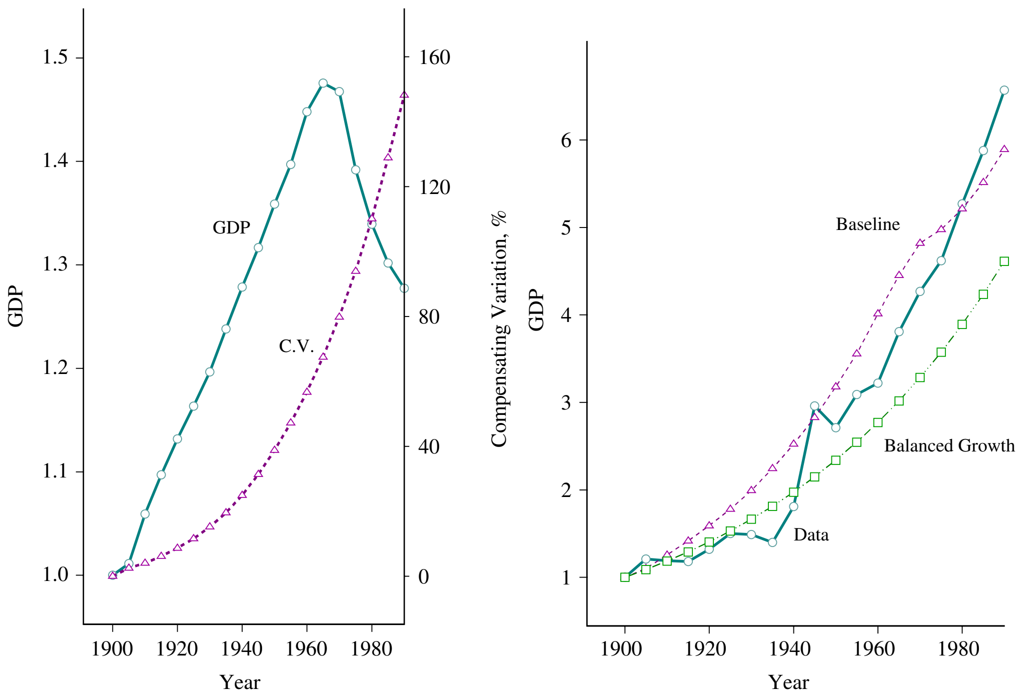

FIGURE 11
The gain in GDP and welfare

Between 1900 and 1980 per-capita real GDP grew at 2·1% per annum. Take this as reflective of the average rate of growth over the last century. In the model over an 80-year period GDP grows by about 0·4% per annum. Therefore, according to the model, the rise in female labour-force participation can be thought of as accounting for about 19% of growth, say, between 1900 and 1980. The right panel of Figure 11 illustrates the idea. Suppose that the rate of labour-augmenting technological advance is constant. In order for the model to match the growth observed between 1900 and 1980 labour-augmenting technological progress must have been 1·7% per year.[21] Without the durable goods revolution or the narrowing of the gender gap the model economy would have simply followed its balanced-growth trajectory. The difference between the balanced-growth path and the path for the baseline simulation is due to the rise in female labour-force participation.

6. ROBUSTNESS OF SPECIFICATION: AN EXAMPLE WITH DIVISIBLE EFFORT AND DURABLES

6.1. Theoretical analysis

Set-up. Does the analysis hinge upon the joint assumptions of indivisible effort and lumpy durables? The answer is no. To see this, imagine a (more or less standard) representative household with preferences given by

$$\sum\nolimits_{j=1}^{\infty} \beta^{j-1}[\mu \ln c^j + \nu \ln n^j + (1-\mu-\nu)\ln l^j].$$

These tastes are identical in form to (1), except that now j denotes time and l^j will refer to the female's leisure in period j. Time also goes on forever. Once again assume that a male works a fixed workweek, ω, and does no housework. In each period j the female is free to divide her time between market work, $1 - h^j - l^j$, housework, h^j, and leisure, l^j. Additionally, in any given period j the household can *rent* its desired stock of consumer durables, d^j. Let the period-j *rental* price for durables be denoted by q^j. Last, assume that home goods are produced in line with the CES production function[22]

$$n^j = [\theta(d^j)^\vartheta + (1-\theta)(h^j)^\vartheta]^{1/\vartheta}, \qquad \text{with } \vartheta < 1. \tag{12}$$

The parameter ϑ governs the degree of substitutability between durables and labour in home production, and it plays a crucial role in the subsequent analysis.

Results. The maximization problem facing the representative household can be formulated as

$$\max_{\{c^j,h^j,d^j,l^j\}_{j=1}^{\infty}} \sum\nolimits_{j=1}^{\infty} \beta^{j-1}[\mu \ln c^j + \nu \ln n^j + (1-\mu-\nu)\ln l^j], \tag{P5}$$

21. Let $\{\widehat{y}_t\}_{t=1900}^{1990}$ denote the sequence of GDP that is portrayed in the left panel of Figure 11. Here $\{\widehat{y}_t\}_{t=1900}^{1990}$ is the solution to the growth-transformed version of the model. The sequence of GDP that occurs with technological progress in the market sector, $\{y_t\}_{t=1900}^{1990}$, is simply given by $\{y_t\}_{t=1900}^{1990} = \{\gamma^{t-1900}\widehat{y}_t\}_{t=1900}^{1990}$, where γ is the (assumed constant) rate of labour-augmenting technological progress. The sequence $\{y_t\}_{t=1900}^{1990}$ is shown in the right panel by the series labelled "baseline". See the Appendix for the argument.

22. Recall that the previous analysis used a Leontief production function for the household sector. When durables are lumpy and housework indivisible, this really amounts to an innocuous normalization. To see this, once again let $n = \min\{d, \zeta h\}$. Now, for the old technology set $d = \delta$, $h = \rho\eta$ and $\zeta = \delta/\rho\eta$. For this input bundle the level of home production is δ. But, given the above parameterization (which implies that $d = \zeta h$) one gets the exact same level of output for the given input bundle when $n = [\theta(d)^\vartheta + (1-\theta)(\zeta h)^\vartheta]^{1/\vartheta}$. Observe that things work for any value of ϑ! Likewise, for the new technology set $d' = \kappa\delta$, $h' = \eta$ and $\zeta' = \kappa\delta/\eta$. For this input bundle the level of home production is $\kappa\delta$. Again, the CES production function delivers the same level of household production for the given input bundle (since once again $d' = \zeta' h'$).

subject to the household production function (12) and the intertemporal budget constraint

$$\sum_{j=1}^{\infty} p^j c^j = a^0 + \sum_{j=1}^{\infty} p^j w^j \omega + \sum_{j=1}^{\infty} p^j w^j [\phi^j (1 - h^j - l^j) - q^j d^j]. \tag{13}$$

The variable a^0 in the intertemporal budget constraint (13) denotes the household's initial level of assets. The variable p^j refers (in this section only) to the period-j present-value price of market consumption and is determined by the recursion $p^j = p^{j-1}/r^j$ with $p^1 = 1$.

The upshot of the above maximization problem (after some tedious grinding) are the following solutions for h^j and l^j:

$$h^j = \nu(1-\beta)\frac{\beta^{j-1}}{p^j w^j [\phi^j + q^j Q^j]}\left[a^0 + \sum_{i=1}^{\infty} p^i w^i (\omega + \phi^i)\right], \tag{14}$$

and

$$l^j = (1 - \mu - \nu)(1-\beta)\frac{\beta^{j-1}}{p^j w^j \phi^j}\left[a^0 + \sum_{i=1}^{\infty} p^i w^i (\omega + \phi^i)\right], \tag{15}$$

where the function $Q^j(q^j, \phi^j)$ is defined by

$$Q^j \equiv \{[(1-\theta)/\theta]q^j/\phi^j\}^{1/(\vartheta-1)}.$$

The two equations form the basis for the next two lemmas.

Lemma 5. *A fall in the period-j rental price of durables, q^j, will cause,* ceteris paribus,

(i) *period-j housework, h^j, to drop and period-j market work, $1 - h^j - l^j$, to increase when durables and housework are Edgeworth–Pareto substitutes in utility ($\vartheta > 0$),*
(ii) *h^j to increase and $1 - h^j - l^j$ to decrease when durables and housework are Edgeworth–Pareto complements in utility ($\vartheta < 0$), and*
(iii) *will have no effect on h^l and $1 - h^l - l^l$ for $l \neq j$.*

Proof. Observe that $q^j Q^j$ is decreasing (increasing) in q^j when $\vartheta > 0$ ($\vartheta < 0$). The rest of the proof is immediate from (14) and (15). ‖

Lemma 6. *When durables and housework are Edgeworth–Pareto substitutes ($\vartheta > 0$), an increase in ϕ^j (or a reduction in the gender gap) will cause,* ceteris paribus

(i) *h^j to decrease and $1 - h^j - l^j$ to increase, and*
(ii) *will cause h^l to increase and $1 - h^l - l^l$ to decrease for $l \neq j$.*

Proof. Observe that

$$\begin{aligned}
&\frac{d\{[a^0 + \sum_{i=1}^{\infty} p^i w^i(\omega + \phi^i)]/[p^j w^j(\phi^j + q^j Q^j)]\}}{d\phi^j} \\
&= \frac{p^j w^j}{[p^j w^j(\phi^j + q^j Q^j)]} - \frac{[a^0 + \sum_{i=1}^{\infty} p^i w^i(\omega + \phi^i)]p^j w^j(1 + q^j Q_2^j)}{[p^j w^j(\phi^j + q^j Q^j)]^2} \\
&= -\frac{[a^0 + \sum_{i=1}^{\infty} p^i w^i(\omega + \phi^i)]p^j w^j(1 + q^j Q_2^j) - (p^j w^j)^2(\phi^j + q^j Q^j)}{[p^j w^j(\phi^j + q^j Q^j)]^2} \\
&< 0,
\end{aligned}$$

since $\phi^j Q_2^j > Q^j$ when $\vartheta > 0$. Hence, $dh^j/d\phi^j < 0$ from (14). Likewise, it is trivial to see from (15) that $dl^j/d\phi^j < 0$. Part (i) of the lemma then follows. Part (ii) follows because $dh^l/d\phi^j > 0$ and $dl^l/d\phi^j > 0$ from (14) and (15). ‖

The intuition underlying Lemma 5 is straightforward to understand. If the rental price of durables drops, then the household will demand more of them. When durables and housework are substitutes in the Edgeworth–Pareto sense, an increase in durables decreases the marginal product of housework denominated in utility terms (*i.e.* the marginal product of housework multiplied by marginal utility of home goods). Hence, housework falls. Thus, market work increases. Now, consider a reduction in the gender gap. Females now earn more. Some of this extra income can be used to purchase more durables. When durables and housework are substitutes (complements) in production, this secondary effect will lead to a reduction (increase) in housework. Hence, when $\vartheta > 0$ there will be a unambiguous rise in female labour effort. Otherwise, things are ambiguous. This explains Lemma 6.

6.2. *Quantitative analysis*

Can the above framework generate a rising time path for female labour-force participation similar to that observed in the data? To complete the framework, assume that aggregate output is produced in line with (3) and that the economy must satisfy the resource constraint (4). Let business capital again follow the law of motion (5). For simplicity, suppose that the d^j's take the form of a non-durable intermediate good. This assumption really does no violence to the analysis (it does not affect the theory) and serves to highlight the fact that labour-saving household products and services—such as frozen or take-out foods and ready-made clothes—may operate to increase female labour supply. Last, the analysis in this subsection is intended merely to show that a model with divisible effort and durables has the potential to account for the rise in female labour supply over the last century.

The model's parameter values are chosen to hit two targets. First, recall that in 1900 about 5% of women worked. If they laboured a 40 h week then

$$1 - h^{1900} - l^{1900} = 0{\cdot}05 \times \frac{40}{112} = 0{\cdot}02.$$

The initial steady state for the model should yield this time allocation for a gender gap of 0·48, the value in 1900. Second, in 1980 about 50% of women worked. This implies that

$$1 - h^{1980} - l^{1980} = 0{\cdot}50 \times \frac{40}{112} = 0{\cdot}18.$$

In 1980 the gender gap was 0·59. Additionally, the price of durable goods would have risen by a factor of $\exp(-0{\cdot}083 \times 80)$ between 1900 and 1980, assuming once again an 8·3% annual price decline. Hence, the terminal steady state should give the 1980 level for market work, given $\phi^{1980} = 0{\cdot}59$ and $q^{1980} = q^{1900}\exp(-0{\cdot}083 \times 80)$. The set of parameter values presented below does the trick.

(i) Tastes: $\beta = 0{\cdot}96^5$, $\mu = 0{\cdot}47$, $\nu = 0{\cdot}26$,
(ii) Home production technology, $\theta = 0{\cdot}3$, $\vartheta = 0{\cdot}35$,
(iii) Market production technology, $\alpha = 0{\cdot}30$, $z = 1{\cdot}0$, $\chi = (1{\cdot}0 - 0{\cdot}10)^5$, $q^{1900} = 540$, $\xi = 1{\cdot}0$.

What do the transitional dynamics for the model look like? Figure 12 plots the transitional dynamics for two scenarios. In the first scenario the time path for the gender gap observed in the U.S. data is fed into the model and the price for intermediate goods is assumed to fall at a constant rate of 8·3%.[23] In the second scenario the price for intermediate goods is kept constant

23. The analysis works well with a lower price decline, but a higher value of ϑ is needed. The available evidence suggests that the elasticity of substitution between durables and housework in production is above 1. For instance, using

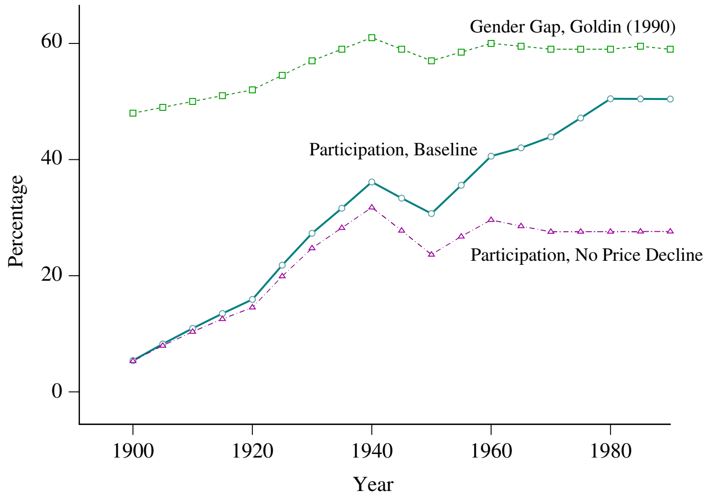

FIGURE 12

The rise in female labour-force participation with divisible effort and non-durable household products and services

(at the level presumed for the 1900 steady state). As can be seen, the introduction of labour-saving goods accounts for about half of the rise in female labour-force participation.[24] Hence, again technological advance in household sector is an important factor in explaining the rise in U.S. female labour-force participation between 1900 and 1980.

7. CONCLUSIONS

Did technological progress unlock the manacles chaining women to the home? That is the question posed here. To address this question, a Beckerian model of household production is embedded into a dynamic general equilibrium framework. Labour-saving technological progress in the household sector is embodied in the form of new consumer durables. The adoption of these technologies frees up the amount of time devoted to housework. The price of these durables falls over time. Households make a decision about when to purchase new durables. Each period they also decide whether or not the woman in the family should work in the market sector. In the baseline model, durables were lumpy and labour indivisible. The first assumption was made to capture the fact that all households do not adopt new technologies at the same time. The second assumption was motivated by the fact that in the historical data female labour-force participation is measured as an extensive margin concept. It was found that the introduction of new and improved household technologies could explain more than half of the observed rise in

dynamic general equilibrium models, McGrattan, Rogerson and Wright (1997) estimate this elasticity to be 1·23, while Chang and Schorfheide (2003) obtain an estimate of 2·45. A midrange value of 1·53 is used here. A word of caution though. The above estimates are based on business cycle data. That is, the trends in female labour-force participation, housework and the decline in the relative price of durables were ignored. Ideally, one would like to estimate the model developed in this section using low-frequency data.

24. As can be seen from Figure 3, there is little narrowing in the gender gap between 1940 and 1980. Data in Blau and Kahn (2000) also suggest that between 1955 and 1980 the gap remained remarkably constant. Over this period however, there was a large increase in female labour-force participation—again see Figure 3. The model predicts this. The set-up with divisible labour and durables attributes a larger fraction of the increase in female labour-force participation to the decreasing gender gap. It has two drawbacks, though: first, as with most macroeconomic models of this form the implied elasticity of (female) labour supply is probably too high, especially for the early part of the last century (see Section 5.2); second, it predicts too much investment in appliances in response to the decline in prices.

female labour-force participation. The rest of the rise was accounted for by the narrowing of the gender gap. Interestingly, the narrowing of the gender gap, alone, could explain only a small fraction of the increase in labour-force participation. When the burden of housework is great it simply is not feasible for women to enter the labour market. Last, it was shown that the gist of the analysis still goes through even when durables are not lumpy and labour is divisible.

Popular wisdom states that the increase in female labour-force participation was due to a narrowing of the gender gap or a change in social norms, spawned by the women's liberation movement. This may well be true, but without the labour-saving household capital ushered in by the Second Industrial Revolution it would not have been feasible for women to spend more time outside of the home, notwithstanding any shift in societal attitudes. While sociology may have provided fuel for the movement, the spark that ignited it came from economics.

APPENDIX

A.1. *Growth transformation*

Consider the consumption decision for an age-1 household. It must satisfy the Euler equation

$$\frac{1}{m^j} = \beta r \frac{1}{m^{j+1}}, \tag{A.1}$$

where m^j is the household's consumption at age j. The household's budget constraint will read

$$m^1 + \frac{m^2}{r} + \frac{m^3}{r^2} + \cdots + \frac{m^J}{r^{J-1}} = \Omega,$$

where Ω is the household's permanent income, net of the cost of purchasing consumer durables. The Euler equation (A.1) implies that $m^{j+1} = \beta r m^j = (\beta r)^j m^1$. Therefore,

$$m^1 = \frac{1-\beta}{1-\beta^J}\Omega.$$

Adding growth would not seem to change this equation much. All variables that grow along a balanced-growth path should be transformed to obtain a stationary representation. Let γ denote the assumed constant pace of labour-augmenting technological progress; *i.e.* $z_{t+1}/z_t = \gamma$ for all t with $z_0 = 1$. Define $\hat{a}^j_{t+1} = a^j_{t+1}/\gamma^t$, $\widehat{\mathbf{k}}_{t+1} = \mathbf{k}_{t+1}/\gamma^t$, $\widehat{m}^j_t = m^j_t/\gamma^t$, $\widehat{w}_t = w_t/\gamma^t$ and $\widehat{\Omega}_t = \Omega_t/\gamma^t$. Then, the Euler equation would appear as

$$\frac{1}{\widehat{m}^j_t} = \beta(r/\gamma)\frac{1}{\widehat{m}^{j+1}_{t+1}}. \tag{A.2}$$

The household's budget constraint now reads

$$\widehat{m}^1_t + \frac{\widehat{m}^2_{t+1}}{(r/\gamma)} + \frac{\widehat{m}^3_{t+2}}{(r/\gamma)^2} + \cdots + \frac{\widehat{m}^J_{t+J-1}}{(r/\gamma)^{J-1}} = \widehat{\Omega}_t.$$

Therefore,

$$\widehat{m}^1 = \frac{1-\beta}{1-\beta^J}\widehat{\Omega}.$$

Here

$$\widehat{\Omega} = \sum\nolimits_{j=1}^{J} \frac{\widehat{w}\lambda\omega + \phi\widehat{w}\lambda\omega P^j(\lambda) - \widehat{w}qI(T^j(\lambda))}{(r/\gamma)^{j-1}}, \tag{A.3}$$

where

$$\widehat{w} = (1-\alpha)(\xi/\gamma^\alpha)\left[\tfrac{r/\gamma - \chi/\gamma}{\alpha(\xi/\gamma^\alpha)}\right]^{\alpha/(\alpha-1)}, \tag{A.4}$$

$$r/\gamma = \alpha(\xi/\gamma^\alpha)(\mathbf{l}/\widehat{\mathbf{k}})^{1-\alpha} + \chi/\gamma, \tag{A.5}$$

and $I(x) = 1$ if $x = 1$ and $I(x) = 0$ if $x \neq 1$. Last, the market-clearing condition for capital would appear as

$$\widehat{\mathbf{k}}' = \sum\nolimits_{j=1}^{J} \int \widehat{\mathbf{A}}^j(\lambda)L(d\lambda).$$

Now, consider the solution to the transformed model with a growth rate of γ. Is there a version of the model without growth that gives the transformed solution? The answer is yes. Let variables in the no-growth economy be indexed by a "$\tilde{}$". The no-growth economy must have a gross interest rate, $\tilde{r}$, equal to r/γ, a fact readily deduced from (A.2) and (A.3). From (A.5) this will transpire if $\tilde{\xi} = \xi/\gamma^{\alpha}$ and $\tilde{\chi} = \chi/\gamma$. This implies that there is no need to solve the model with growth since there always exists a no-growth model that gives the identical solution to the transformed model with growth, a point made in Christiano and Eichenbaum (1992). In the numerical analysis $\tilde{\xi} = 1{\cdot}0$ and $\tilde{\chi} = (1{\cdot}0 - 0{\cdot}10)^5$. The first parameter value amounts to an innocuous normalization of the production function so that $\xi = \gamma^{\alpha} = 1{\cdot}017^{0{\cdot}3\times 5} = 1{\cdot}026$. The second sets $\chi = 1{\cdot}017^5 \times (1{\cdot}0 - 0{\cdot}10)^5$, which implies that the annual depreciation rate in the model with growth is 8·5%.

A.2. *Proof of Lemma* 4

Proof. Consider the continuous-time analogue to the adopt/work problem framed by (P1) to (P4). Let the date of adoption chosen by the household be represented by α. The household will choose an interval $[\sigma, \varepsilon] \subseteq [0, J]$ over which to work. Here σ denotes the start date for working and ε denotes the end date. As an example of how things work, take the case where $\sigma = 0 < \alpha < \varepsilon < J$. Here the woman in a household starts working immediately, builds up some resources to purchase durables at age α, and then retires at ε. After solving out for consumption, a type-λ household's decision problem is

$$\begin{aligned}\max_{\alpha,\varepsilon} \Bigg\{ & \mu \frac{1-e^{-\beta J}}{\beta}\left[\ln\left(\frac{\beta}{1-e^{-\beta J}}\right) + \ln\Omega(\lambda)\right] + \mu\int_0^J (r-\beta) j e^{-\beta j} dj \\ & + \nu\frac{1-e^{-\beta J}}{\beta}\ln\delta + \nu\ln\kappa\int_\alpha^J e^{-\beta j}dj + (1-\mu-\nu)\Bigg[\ln(2-2\omega-\rho\eta)\int_0^\alpha e^{-\beta j}dj \\ & + \ln(2-2\omega-\eta)\int_\alpha^\varepsilon e^{-\beta j}dj + \ln(2-\omega-\eta)\int_\varepsilon^J e^{-\beta j}dj\Bigg]\Bigg\},\end{aligned}$$

subject to

$$\Omega(\lambda) = w\lambda\omega\frac{1-e^{-rJ}}{r} + \phi w\lambda\omega\int_0^\varepsilon e^{-rj}dj - qwe^{-r\alpha}.$$

Now, r represents the *net* interest rate and β is the *rate of time preference*.

The first-order conditions to this problem are

$$\mu\frac{1-e^{-\beta J}}{\beta}wqre^{-(r-\beta)\alpha} = \left[(1-\mu-\nu)\ln\left(\frac{2-2\omega-\eta}{2-2\omega-\rho\eta}\right) + \nu\ln\kappa\right]\Omega(\lambda),$$

and

$$\mu\frac{1-e^{-\beta J}}{\beta}\phi w\lambda\omega e^{-(r-\beta)\varepsilon} = (1-\mu-\nu)\ln\left(\frac{2-\omega-\eta}{2-2\omega-\eta}\right)\Omega(\lambda).$$

Undertaking the requisite comparative statics exercise gives

$$\begin{aligned}\frac{d\alpha}{d\lambda} = & -\{[1-e^{-rJ} + \phi(1-e^{-\varepsilon r})](r-\beta)/r + \phi e^{-r\varepsilon}\} \\ & \times\frac{\left[(1-\mu-\nu)\ln\left(\frac{2-2\omega-\eta}{2-2\omega-\rho\eta}\right) + \nu\ln\kappa\right]\mu(1-e^{-\beta J})e^{-(r-\beta)\varepsilon}\phi(w\omega)^2\lambda}{\beta\det(H)} < 0,\end{aligned}$$

where H is the 2×2 Hessian associated with the maximization problem. To sign the above expression, note that the second-order conditions for a maximum necessitate that the matrix H is negative semidefinite. Necessary conditions for this to transpire are that $\det(H) \geq 0$ and $H_{11}, H_{22} \leq 0$, where H_{11} and H_{22} are the entries in upper left and lower right-hand corner of H. When $r > \beta$ it is easy to see that $d\alpha/d\lambda < 0$. When $r < \beta$ it can be shown that a maximum cannot obtain. It then turns out that $H_{11}, H_{22} < 0$ imply $\det(H) < 0$. There are many other cases to consider, but they all proceed in the same manner. (Basically, the rest of the proof is a boring taxonomy.) ‖

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