An Analytical Design Method for Milling Cutters With Nonconstant Pitch to Increase Stability, Part I: Theory

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Chatter vibrations result in reduced productivity, poor surface finish and decreased cutting tool life. Milling cutters with nonconstant pitch angles can be very effective in improving stability against chatter. In this paper, an analytical stability model and a design method are presented for nonconstant pitch cutters. An explicit relation is obtained between the stability limit and the pitch variation which leads to a simple equation for determination of optimal pitch angles. A certain pitch variation is effective for limited frequency and speed ranges which are also predicted by the model. The improved stability, productivity and surface finish are demonstrated by several examples in the second part of the paper. [DOI: 10.1115/1.1536655]

1 Introduction

Chatter vibrations develop due to dynamic interactions between the cutting tool and workpiece. Especially for highly flexible machining systems, chatter may develop even at very slow cutting speeds which are commonly used to suppress vibrations in metal cutting. In general, additional operations, mostly manual, are required to remove the chatter marks left on the surface. Thus, chatter vibrations result in reduced productivity, increased cost and inconsistent product quality.

Stable depth of cut in milling operations can be increased by using speeds that correspond to high stability lobes with regular pitch cutters or by using special geometry cutters such as the ones with nonconstant pitch. Fluctuating spindle speed has also stabilizing effect on cutting system as demonstrated in [1–4]. This can be an effective method of chatter supression in milling especially for the cases where part dynamics vary during machining [3], provided that the spindle drive system has the required bandwith to be able to oscillate the speed at required rates [4]. In order to have a significant increase in the chatter free material removal rate using stability lobes, usually high spindle speeds have to be used with regular pitch cutters. This is not possible for many operations, as required speeds may not be available on the machine tool. Higher cutting speeds may also present machinability problems for some materials, such as titanium alloys. Furthermore, for very flexible workpieces such as thin-walled parts, the stable depth of cut is usually very small, even at high stability lobes. The operations which require large stable depth of cuts have to be performed using very slow speeds to increase process damping, but of course by losing stability lobe effect. In some cases, chatter may develop even at slow speeds.

Milling cutters with nonconstant pitch, or variable pitch cutters, may result in significant improvements in the stability when designed properly. Unlike the stability lobe or process damping effects, they can be effective for both high and low speed applications. Particularly for the cases where slow cutting speeds have to be used, very high stability can be achieved due to combined effects of process damping and nonconstant pitch. This requires proper selection of pitch angles which is the topic of this paper. These cutters are effective for a certain speed and chatter frequency ranges which can be predicted, and extended, by the model presented here. Most of the modern cutting tool grinders have the capability to make variable pitch cutters which makes the implementation of them practical in industry.

The first accurate modeling of self-excited vibrations in orthogonal cutting was performed by Tlusty [5] and Tobias [6]. They identified the most powerful source of self-excitation, *regeneration*, which is associated with the structural dynamics of the machine tool and the feedback between the subsequent cuts on the same cutting surface. These and the following other fundamental studies are applicable to orthogonal cutting where the direction of the cutting force, chip thickness and system dynamics do not change with time. On the other hand, the stability analysis of milling is complicated due to the rotating tool, multiple cutting teeth, periodical cutting forces and chip load directions, and multi-degree-of-freedom structural dynamics.

In the early milling stability analysis, Tlusty [7] used his orthogonal cutting model considering an average direction for the cut. Later, however, Tlusty et al. [8] showed that the time domain simulations would be required for accurate stability predictions in milling. Sridhar et al. [9,10] performed a comprehensive analysis of milling stability which involved numerical evaluation of the dynamic milling system's state transition matrix. Minis et al. [11,12] used Floquet's theorem and the Fourier series for the formulation of the milling stability, and numerically solved it using the Nyquist criterion. Budak [13] developed a stability method which leads to analytical determination of stability limits. The method was verified by experimental and numerical results, and demonstrated to be very fast for the generation of stability lobe diagrams [14,15]. This method was also applied to the stability of ball-end milling [16]. In this paper, the application of the model to stability of nonconstant pitch milling cutters is presented. In case of variable pitch cutters, the phase between two waves is not constant for all teeth disturbing the regeneration mechanism. This reduces the modulation in chip thickness and slows down vibrations increasing the stability of cutting. Many different variation patterns can be used, however as it is shown in the paper, the stability limit is maximized by using a certain pitch variation (optimal), which can be determined by the model presented.

The effectiveness of variable pitch cutters in suppressing chatter vibrations in milling was first demonstrated by Slavicek [17]. He assumed a rectilinear tool motion for the cutting teeth, and applied the orthogonal stability theory to irregular tooth pitch. By assuming an alternating pitch variation, he obtained a stability

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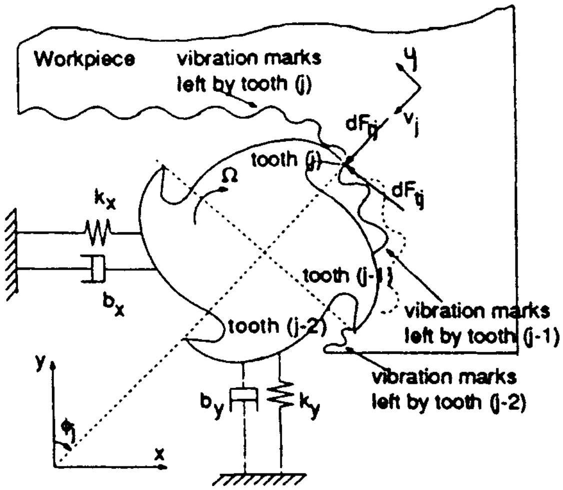

Fig. 1 Chatter model for milling

limit expression as a function of the variation in the pitch. Opitz et al. [18] considered milling tool rotation using average directional factors, however they, too, considered alternating pitch with only two different pitch angles. Their experimental results and predictions showed significant increase in the stability limit using cutters with alternating pitch. Vanherck [19] considered different pitch variation patterns in the analysis by assuming rectilinear tool motion. His computer simulations showed the effect of pitch variation on stability limit. Tlusty et al. [20] analyzed the stability of milling cutters with special geometries such as irregular pitch or serrated edges, using numerical simulations. These studies mainly concentrated on the effect of pitch variation on the stability limit, however they do not address the cutting tool design, i.e., determination of optimal pitch variation. It is quite difficult to determine optimal pitch angles for a given milling system by simulating the stability for different pitch configurations, particularly using numerical time domain simulations. Recently, Altintas et al. [21] adapted the analytical milling stability model to the case of variable pitch cutters which can be used more practically to analyze the stability with variable pitch cutters.

In this paper, a complete analytical model is presented for the design of variable pitch angles for a given milling system. For given chatter frequency, spindle speed and number of cutting teeth, pitch angles can be optimized to maximize stability limits. In the paper, first the analytical stability model for equal pitch cutters is summarized, then the extension of the model to variable pitch cutters is presented. Optimal pitch-angle-prediction procedure is followed by conclusions. The application of the developed method is demonstrated with several examples in the second part of the paper.

2 Stability of Milling With Standard Cutters

2.1 Modeling of Dynamic Milling. Budak and Altintas [13–15] considered both milling cutter and workpiece to have two orthogonal modal directions as shown in Fig. 1. Milling forces excite both cutter and workpiece causing vibrations which are imprinted on the cutting surface. Each vibrating cutting tooth removes the wavy surface left from the previous tooth resulting in modulated chip thickness which can be expressed as follows:

$$h_j(\phi)=s_t\sin\phi_j+(\nu^o_{j_c}-\nu^o_{j_w})-(\nu_{j_c}-\nu_{j_w}) \qquad (1)$$

where the feed per tooth s_t represents the static part of the chip thickness, and $\phi_j=(j-1)\phi_p+\phi$ is the angular immersion of tooth (j) for a cutter with constant pitch angle $\phi_p=2\pi/N$ and N teeth as shown in Fig. 1. $\phi=\Omega.t$ is the angular position of the cutter measured with respect to the first tooth and corresponding to the rotational speed Ω (rad/sec). ν_j and ν_j^0 are the dynamic displacements due to tool and workpiece vibrations for the current and previous tooth passes, for the angular position ϕ_j, and can be expressed in terms of the fixed coordinate system as follows:

$$\nu_{j_p}=-x_p\sin\phi_j-y_p\cos\phi_j \quad (p=c,w) \qquad (2)$$

where c and w indicate cutter and workpiece, respectively. The static part in Eq. (1), ($s_t\sin\phi_j$), is neglected in the stability analysis. It should be noted that even though the static chip thickness varies in time as the milling cutter rotates, it does not contribute to the regeneration and thus can be eliminated in chatter stability analysis. If Eq. (2) is substituted in Eq. (1), the following expression is obtained for the dynamic chip thickness in milling:

$$h_j(\phi)=[\Delta x\sin\phi_j+\Delta y\cos\phi_j] \qquad (3)$$

where

$$\Delta x=(x_c-x_c^o)-(x_w-x_w^o)$$
$$\Delta y=(y_c-y_c^o)-(y_w-y_w^o) \qquad (4)$$

where (x_c,y_c) and (x_w,y_w) are the dynamic displacements of the cutter and the workpiece in (x) and (y) directions, respectively. The dynamic cutting forces on tooth (j) in the tangential and the radial directions can be expressed as follows:

$$F_{t_j}(\phi)=K_t a h_j(\phi);\ \ F_{r_j}(\phi)=K_r F_{t_j}(\phi) \qquad (5)$$

where a is the axial depth of cut, and K_t and K_r are the empirical cutting force coefficients. After substituting h_j from Eq. (1) into (5), and summing up the forces on each tooth ($F=\Sigma F_j$), the dynamic milling forces can be resolved in x and y directions as follows:

$$\begin{Bmatrix}F_x\\F_y\end{Bmatrix}=\frac{1}{2}aK_t\begin{bmatrix}a_{xx} & a_{xy}\\ a_{yx} & a_{yy}\end{bmatrix}\begin{Bmatrix}\Delta x\\ \Delta y\end{Bmatrix} \qquad (6)$$

where the directional coefficients are given as:

$$a_{xx}=-\sum_{j=1}^{N}\ \sin 2\phi_j+K_r(1-\cos 2\phi_j)$$
$$a_{xy}=-\sum_{j=1}^{N}\ (1+\cos 2\phi_j)+K_r\sin 2\phi_j$$
$$a_{yx}=-\sum_{j=1}^{N}\ -(1-\cos 2\phi_j)+K_r\sin 2\phi_j \qquad (7)$$
$$a_{xx}=-\sum_{j=1}^{N}\ -\sin 2\phi_j+K_r(1+\cos 2\phi_j)$$

The directional coefficients depend on the angular position of the cutter which makes Eq. (6) time-varying:

$$\{F(t)\}=\tfrac{1}{2}aK_t[A(t)]\{\Delta(t)\} \qquad (8)$$

$[A(t)]$ is periodic at the tooth passing frequency $\omega=N\Omega$ and with corresponding period of $T=2\pi/\omega$. In general, the Fourier series expansion of the periodic term is used for the solution of the periodic systems [22]. The solution can be numerically obtained by truncating the resulting infinite determinant. However, in chatter stability analysis inclusion of the higher harmonics in the solution may not be required as the response at the chatter limit is usually dominated by a single chatter frequency. Starting from this idea, Budak and Altintas [13–15] have shown that the higher harmonics do not affect the accuracy of the predictions, and it is sufficient to include only the average term in the Fourier series expansion of $[A(t)]$:

$$[A_0]=\frac{1}{T}\int_0^T[A(t)]dt \qquad (9)$$

As all the terms in $[A(t)]$ are valid within the cutting zone between start and exit immersion angles (ϕ_{st},ϕ_{ex}), Eq. (9) reduces to the following form in the angular domain:

$$[A_0]=\frac{1}{\phi_p}\int_{\phi_{st}}^{\phi_{ex}}[A(\phi)]d\phi=\frac{N}{2\pi}\begin{bmatrix}\alpha_{xx} & \alpha_{xy}\\ \alpha_{yx} & \alpha_{yy}\end{bmatrix} \tag{10}$$

where the integrated, or average, directional coefficients are given as:

$$\begin{aligned}\alpha_{xx}&=\tfrac{1}{2}[\cos 2\phi-2K_r\phi+K_r\sin 2\phi]_{\phi_{st}}^{\phi_{ex}}\\ \alpha_{xy}&=\tfrac{1}{2}[-\sin 2\phi-2\phi+K_r\cos 2\phi]_{\phi_{st}}^{\phi_{ex}}\\ \alpha_{yx}&=\tfrac{1}{2}[-\sin 2\phi+2\phi+K_r\cos 2\phi]_{\phi_{st}}^{\phi_{ex}}\\ \alpha_{yy}&=\tfrac{1}{2}[-\cos 2\phi-2K_r\phi-K_r\sin 2\phi]_{\phi_{st}}^{\phi_{ex}}\end{aligned} \tag{11}$$

Substituting Eq. (11), Eq. (8) reduces to the following form:

$$\{F(t)\}=\tfrac{1}{2}aK_t[A_0]\{\Delta(t)\} \tag{12}$$

2.2 Chatter Stability Limit. The dynamic displacement vector in Eq. (12) can be described as:

$$\{\Delta(t)\}=(\{r_c\}-\{r_c^0\})-(\{r_w\}-\{r_w^0\}) \tag{13}$$

where

$$\{r_p\}=[\{x_p\}\{y_p\}]^T \quad (p=c,w) \tag{14}$$

The response of the both structures at the chatter frequency can be expressed as follows:

$$\{r_p(i\omega_c)\}=[G_p(i\omega_c)]\{\mathsf{F}\}e^{i\omega_c t} \quad (p=c,w) \tag{15}$$

where $\{\mathsf{F}\}$ represents the amplitude of the dynamic milling force $\{F(t)\}$, and the transfer function matrix is given as:

$$[G_p]=\begin{bmatrix}G_{p_{xx}} & G_{p_{xy}}\\ G_{p_{yx}} & G_{p_{yy}}\end{bmatrix} \quad (p=c,w) \tag{16}$$

The vibrations at the previous tooth period, i.e., at $(t-T)$, can be defined as follows

$$\begin{aligned}\{r_p^o\}&=[\{x_p(t-T)\}\{y_p(t-T)\}]^T\\ \{r_p^o\}&=e^{-i\omega_c T}\{r(i\omega_c)\}\end{aligned} \quad (p=c,w) \tag{17}$$

By substituting Eqs. (13)–(17) into the dynamic milling force expression given by Eq. (12), the following is obtained

$$\{\mathsf{F}\}e^{i\omega_c t}=\tfrac{1}{2}aK_t(1-e^{i\omega_c T})[A_0][G(i\omega_c)]\{\mathsf{F}\}e^{i\omega_c t} \tag{18}$$

where

$$[G(i\omega_c)]=[G_c(i\omega_c)]+[G_w(i\omega_c)] \tag{19}$$

Equation (18) has a non-trivial solution only if its determinant is zero,

$$\det[[I]+\Lambda[G_0(i\omega_c)]]=0 \tag{20}$$

where $[I]$ is the unit matrix, and the oriented transfer function matrix is defined as:

$$[G_0]=[A_0][G] \tag{21}$$

and the eigenvalue (Λ) in Eq. (20) is given as

$$\Lambda=-\frac{N}{4\pi}K_t a(1-e^{-i\omega_c T}) \tag{22}$$

If the eigenvalue Λ is known, the stability limit can be determined from Eq. (22). Λ can easily be computed from Eq. (20) numerically. However, an analytical solution is possible if the cross transfer functions, G_{xy} and G_{yx}, are neglected in Eq. (20):

$$\Lambda=-\frac{1}{2a_0}(a_1\pm\sqrt{a_1^2-4a_0}) \tag{23}$$

where

$$\begin{aligned}a_0&=G_{xx}(i\omega_c)G_{yy}(i\omega_c)(\alpha_{xx}\alpha_{yy}-\alpha_{xy}\alpha_{yx})\\ a_1&=\alpha_{xx}G_{xx}(i\omega_c)+\alpha_{yy}G_{yy}(i\omega_c)\end{aligned} \tag{24}$$

This is a valid assumption for majority of the milling systems, i.e., the cross transfer functions are negligible, such as slender end mills and plate-like workpieces. If the cross transfer functions have to be included in the analysis, the eigenvalue can only be obtained by solving Eq. (20) numerically.

Since the transfer functions are complex, Λ will have complex and real parts. However, the axial depth of cut (a) is a real number. Therefore, when $\Lambda=\Lambda_R+i\Lambda_I$ and $e^{-i\omega cT}=\cos\omega_c T-i\sin\omega_c T$ are substituted in Eq. (22), the complex part of the equation has to vanish yielding

$$\mathrm{K}=\frac{\Lambda_I}{\Lambda_R}=\frac{\sin\omega_c T}{1-\cos\omega_c T} \tag{25}$$

The above can be solved to obtain a relation between the chatter frequency and the spindle speed [14,15]:

$$\begin{aligned}\omega_c T&=\varepsilon+2k\pi\\ \varepsilon&=\pi-2\psi;\quad \psi=\tan^{-1}\kappa\\ n&=\frac{60}{NT}\end{aligned} \tag{26}$$

where ε is the phase difference between the inner and outer modulations, k is an integer corresponding to the number of vibration waves within a tooth period, and n is the spindle speed (rpm). After the imaginary part in Eq. (22) is vanished, the following is obtained for the stability limit [14,15]:

$$a_{\lim}=-\frac{2\pi\Lambda_R}{NK_t}(1+\kappa^2) \tag{27}$$

Therefore, for given cutting geometry, cutting force coefficients, transfer functions, and chatter frequency ω_c, Λ_I and Λ_R can be determined from Eq. (23), and can be used in Eqs. (26) and (27) to determine the corresponding spindle speed and stability limit. When this procedure is repeated for a range of chatter frequencies and number of vibration waves, k, the stability lobe diagram for a milling system is obtained. It has been shown [14,15] that the stability lobes generated by using the analytical model are in very good agreement with the time domain simulations and experimental data.

3 Chatter Stability of Milling Cutters With Nonconstant Pitch

3.1 Stability Analysis. The fundamental difference in the stability analysis of milling cutters with nonconstant pitch angle is that the phase delay between the inner and the outer waves, is different for each tooth:

$$\varepsilon_j=\omega_c T_j \quad (j=1,\ldots,N) \tag{28}$$

where T_j is the jth tooth period corresponding to the pitch angle ϕ_{pj}. The dynamic chip thickness and the cutting force relations given for the standard milling cutters apply to the variable pitch cutters, as well. The directional coefficients given in Eq. (10) are evaluated at the average pitch angle to simplify the formulation. Then, the characteristic equation given in Eq. (22) is valid for the variable pitch cutters, however the eigenvalue expression will take the following form due to the varying phase:

$$\Lambda=\frac{a}{4\pi}K_t\sum_{j=1}^{N}(1-e^{i\omega_c T_j}) \tag{29}$$

The stability limit can be obtained from Eq. (29) as:

$$a_{\text{lim}}^{vp} = -\frac{4\pi}{K_t}\frac{\Lambda}{N-C+iS} \tag{30}$$

where

$$C = \sum_{j=1}^{N} \cos \omega_c T_j$$

$$S = \sum_{j=1}^{N} \sin \omega_c T_j \tag{31}$$

Since the eigenvalue is an complex number, if $\Lambda = \Lambda_R + i\Lambda_I$ is substituted in Eq. (30), the following is obtained:

$$a_{\text{lim}}^{vp} = -\frac{4\pi}{K_t}\left[\frac{\Lambda_R(N-C)+\Lambda_I S}{(N-C)^2+S^2} + i\,\frac{\Lambda_I(N-C)-S\Lambda_R}{(N-C)^2+S^2}\right] \tag{32}$$

As a_{lim} is a real number, the imaginary part of Eq. (32) must vanish:

$$N-C = S\frac{\Lambda_R}{\Lambda_I} \tag{33}$$

Substituting into Eq. (32), a_{lim} is obtained as:

$$a_{\text{lim}}^{vp} = -\frac{4\pi}{K_t}\frac{\Lambda_I}{S} \tag{34}$$

It is interesting to note that the stability limit obtained for the equal pitch cutters, Eq. (27), can be put into a similar form by substituting κ from Eq. (25):

$$a_{\text{lim}} = -\frac{4\pi}{K_t}\frac{\Lambda_I}{N \sin \omega_c T} \tag{35}$$

Note that for equal pitch cutters, $S = \Sigma \sin \omega_c T$ in Eq. (34) becomes $N \sin \omega_c T$ in Eq. (35) as the phase ($\omega_c T$) is the same for all teeth.

The stability limit with variable pitch cutters can be determined using Eqs. (33) and (34). Unlike for the equal pitch cutters, in this case the solution has to be determined numerically since an explicit equation for the chatter frequency-spindle speed relation cannot be obtained from Eq. (33). Also, the cutter pitch angles have to be known in advance. However, optimization of pitch angles for a given milling system has more practical importance than the stability analysis of an arbitrary variable pitch cutter. Therefore, the rest of the analysis focuses on the optimization of the pitch angles to maximize the stability against chatter.

Equation (34) indicates that in order to maximize the stability limit, $|S|$ has to be minimized. From Eq. (31), S can be expressed as follows:

$$S = \sin \varepsilon_1 + \sin \varepsilon_2 + \sin \varepsilon_3 + \ldots \tag{36}$$

where $\varepsilon_j = \omega_c T_j$. The phase angle, which is different for every tooth due to the nonconstant pitch, can be expressed as follows:

$$\varepsilon_j = \varepsilon_1 + \Delta\varepsilon_j \quad (j = 2, \ldots, N) \tag{37}$$

where $\Delta\varepsilon_j$ is the phase difference between tooth j and tooth (1) corresponding to the difference in the pitch angles between these teeth. Considering the number of vibration waves in one cutter revolution m can further develop this relation:

$$m = \frac{\omega_c}{\Omega} \tag{38}$$

where Ω is the spindle speed (rad/sec). Note that m is summation of full number of waves and the remaining fraction of a wave, and thus it is, in general, a non-integer number.

If θ is defined as the tooth immersion angle corresponding to one full vibration wave as shown in Fig. 2, it is determined as:

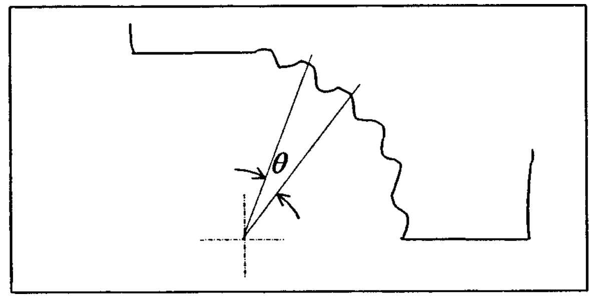

Fig. 2 Tooth immersion angle corresponding to one full vibration wave left on the surface

$$\theta = \frac{2\pi}{m} = \frac{2\pi\Omega}{\omega_c} \tag{39}$$

Therefore, the pitch angle variation ΔP corresponding to $\Delta\varepsilon$ can be determined as:

$$\Delta P = \frac{\Delta\varepsilon}{2\pi}\theta = \frac{\Omega}{\omega_c}\Delta\varepsilon \tag{40}$$

Thus, ΔP and $\Delta\varepsilon$ are linearly proportional.

Equation (36) can be expanded as follows by using Eq. (37):

$$S = \sin\varepsilon_1 + \sin\varepsilon_1 \cos\Delta\varepsilon_2 + \sin\Delta\varepsilon_2 \cos\varepsilon_1 + \sin\varepsilon_1\cos\Delta\varepsilon_3 + \sin\Delta\varepsilon_3\cos\varepsilon_1 + \ldots \tag{41}$$

There are many solutions to the minimization of $|S|$, i.e., ($S = 0$). For example, for even number of teeth, $S = 0$ when $\Delta\varepsilon_j = j\pi$. This can easily be achieved by using linear or alternating pitch variation:

$$\text{Linear}: \; P_0, P_0+\Delta P, P_0+2\Delta P, P_0+3\Delta P$$

$$\text{Alternating}: \; P_0, P_0+\Delta P, P_0, P_0+\Delta P, \ldots \tag{42}$$

A more general solution can be obtained by substituting a specific pitch variation pattern into S. Several pitch variation patters such as linear, nonlinear, sinosoidal and random have been tried numerically to see their effect on S. Some of the results will be discussed later in this section. For the linear pitch variation S takes the following form:

$$S = \sin\varepsilon_1(1 + \cos\Delta\varepsilon + \cos 2\Delta\varepsilon + \ldots) + \cos\varepsilon_1(\sin\Delta\varepsilon + \sin 2\Delta\varepsilon + \ldots) \tag{43}$$

It can be found out by intuition that in Eq. (43), $S = 0$ for the following conditions

$$\Delta\varepsilon = k\frac{2\pi}{N} \quad (k = 1, 2, \ldots, N-1) \tag{44}$$

The corresponding ΔP can be determined using Eq. (40).

In the foregoing analysis, the main approach was to increase the stability limit by minimizing S through proper selection of $\Delta\varepsilon$. $\Delta\varepsilon$, on the other hand, also affects the chatter frequency, ω_c, and thus Λ_I, which appears in the numerator of the stability limit expression as given by Eq. (34). Therefore, by changing $\Delta\varepsilon$, both numerator and denominator in Eq. (34) are affected which in general would not guarantee a maximum value. However, the rate of change of the numerator is very small as the affect of $\Delta\varepsilon$ on Λ_I is not significant. S, on the other hand, heavily depends on $\Delta\varepsilon$. Hence, although this approach is not a complete optimization procedure, it can provide acceptable solutions for practical purposes as it will be demonstrated by examples in the second part of the paper.

The increase of the stability with variable pitch cutters over the standard end mills can be determined by considering the ratio of stability limits. For simplicity, the absolute or critical stability

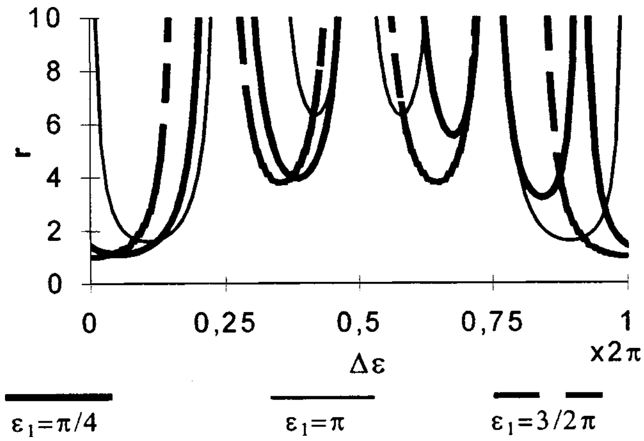

Fig. 3 Effect of $\Delta\varepsilon$ on stability gain for a 4-fluted end mill with linear pitch variation

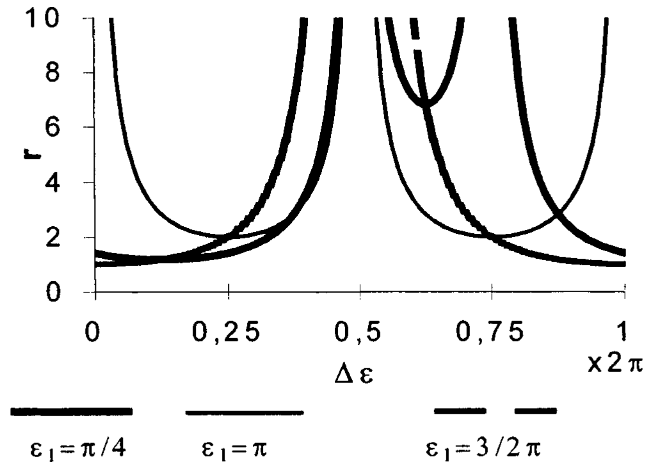

Fig. 4 Effect of $\Delta\varepsilon$ on stability gain for a 4-fluted end mill with alternating pitch variation

limit for equal pitch cutters are considered. The absolute stability limit is the minimum stable depth of cut without the effect of lobing which can be expressed as follows from Eq. (35):

$$a_{cr}=-\frac{4\pi\Lambda_I}{NK_t} \tag{45}$$

Then the stability gain can be expressed as

$$r=\frac{a_{\lim}^{vp}}{a_{cr}}=\frac{N}{S} \tag{46}$$

r is plotted as a function of $\Delta\varepsilon$ in Fig. 3 for a 4-tooth milling cutter with linear pitch variation. The phase ε depends on the chatter frequency, spindle speed and the eigenvalue of the characteristic equation, and therefore the stability analysis have to be performed for the given conditions. However, this can only be done for a given cutting tool geometry (pitch variation pattern). In this study, the intent is to determine the optimal pitch variation for a given milling system. Therefore, three different curves corresponding to different ε_1 values are shown in Fig. 3 to demonstrate the effect of phase variation on r. As expected ε_1 has a strong effect on r, and $3\pi/2$ results in the lowest stability gain. Also, as predicted by Eq. (44), r is maximized for integer multiples of $2\pi/N$, i.e., for $(1/4,1/2,3/4)\times 2\pi$. $\Delta\varepsilon+k2\pi$ $(k=1,2,3,\ldots)$ are also optimal solutions. However, they result in higher pitch variations which is not desired since it increases the chip thickness variation from tooth to tooth, and may result in difficulties in grinding the flutes as some of them may become very close to each other. It is important to note that r cannot be optimized in a straightforward manner by just tuning the spindle speed to achieve $\Delta\varepsilon=2\pi k/N$, as this may change the chatter frequency ω_c and thus $\Delta\varepsilon$. Therefore, the optimal pitch variation can be determined better if the chatter frequency and the spindle speed are known before the cutter is designed. This can be done by simple acoustic measurements using an equal-pitch cutting tool to determine the chatter frequency. However, the chatter frequency may vary in the production environment with the introduction of the variable pitch cutter, or due to the changes in the machine condition, part clamping and workpiece dynamics. Also, there are usually more than a single mode which can cause chatter especially for highly unstable milling systems, and chatter may develop at another vibration mode for which the pitch variation may not be optimal. Modal analysis of the part-tool-spindle system is very useful to determine the other important modes.

As it can be seen from Fig. 3 for a 4-tooth end mill with linear pitch variation, a minimum of $r=4$ gain is obtained for $0.5\pi<\Delta\varepsilon<1.5\pi$. Thus, the target for $\Delta\varepsilon$ should be π, which is one of the optimal solutions for the cutters with even number of flutes, but it is also in the middle of the high stability area. Other variation types were also tried, however they resulted in smaller high-stability gain area than linear variation. For example, Fig. 4 shows the variation of the stability gain with $\Delta\varepsilon$ for a 4-tooth end mill with alternating pitch variation. In this case the high stability gain area is much smaller than the one with the linear variation, and thus the cutting stability is very sensitive to the chatter frequency variations. Figure 5 shows stability gain of linear pitch cutters with different number of teeth. As it can be seen from the figure,

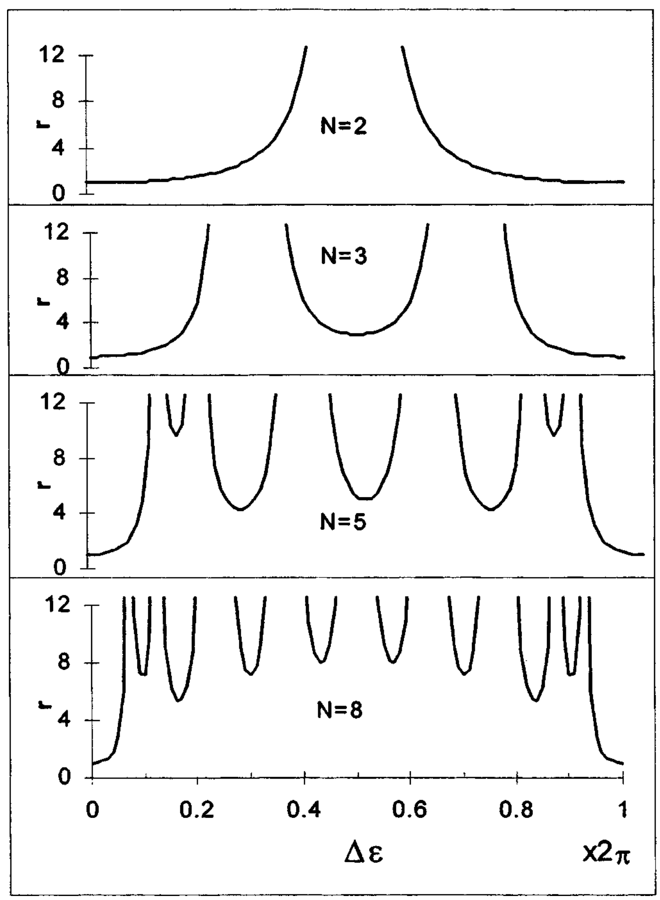

Fig. 5 Effect of number of teeth on stability gain for milling cutters with linear pitch variation

the high stability area increases with the number of teeth. It should be noted, however, increase of stability gain r with number of teeth does not mean that the stable depth of cut increases as the absolute stability limit of the regular pitch cutters reduces with number of teeth as given in Eq. (27).

3.2 Design of Milling Cutters With Linear Pitch Variation. As it was discussed in the previous section, the linear pitch variation gives higher stability gain. The maximum gains are obtained for

$$\Delta\varepsilon = k\frac{2\pi}{N} \quad (k=1,2,\ldots,N-1) \tag{47}$$

Considering possible frequency variations, it is better to keep $\Delta\varepsilon$ close to π, i.e. $k=N/2$ for even number of teeth, $k=(N\pm1)/2$ for odd number of teeth. Then, the relation between the pitch angle variation and the phase given in Eq. (40) takes the following form

$$\Delta P = \pi\frac{\Omega}{\omega_c} \quad \text{for even } N$$

$$\Delta P = \pi\frac{\Omega}{\omega_c}\frac{(N\pm1)}{N} \quad \text{for odd } N \tag{48}$$

where n is the spindle speed in (rpm), $\Delta\varepsilon$ is the pitch variation (rad) and N is the number of teeth. The pitch angles have to satisfy the following relation:

$$P_0+(P_0+\Delta P)+(P_0+2\Delta P)+\ldots+[P_0+(N-1)\Delta P]=2\pi \tag{49}$$

P_0 can be determined from equation (49) as follows:

$$P_0=\frac{2\pi}{N}-\frac{(N-1)\Delta P}{2} \tag{50}$$

Therefore, for given chatter frequency and spindle speed, the optimal pitch variation can be determined from Eqs. (48) and (50). This variation may result in very high stability gain, however due to possible variations in the chatter frequency only $r=N$ can be guaranteed provided that $0.5\pi<\Delta\varepsilon<1.5\pi$ condition is satisfied. The chatter frequency range covered by a particular pitch variation, i.e. with a minimum of $r=N$, can be obtained by substituting for $\Delta\varepsilon$ from Eq. (40):

$$0.5\pi<\Delta\varepsilon<1.5\pi$$

$$\frac{\Omega}{2\Delta P}\pi<\omega_c<\frac{3\Omega}{2\Delta P}\pi \tag{51}$$

The above equation defines the range of chatter frequency for which ΔP linear pitch variation will result in minimum of N times stability increase over the equal pitch cutter, for a defined spindle speed n (rpm). Therefore, an important step in the design is to chose ΔP in such a way that the most flexible natural modes of the milling system are in the range defined above. However, the optimal ΔP for which the stability gain is maximized ($r \gg N$) is given by Eq. (48).

4 Conclusions

Milling cutters with non-constant pitch can be very effective in increasing the chatter free material removal rate. The productivity and surface finish improvements are very significant for particularly low cutting speeds where there are no high-stability-lobes for equal pitch cutters. At low speeds, very high chatter free depth of cuts can be achieved through combination of increased process damping and variable pitch cutters. In this paper, the analytical stability model developed previously for the equal pitch milling cutters is modified to include non-constant pitch effect. This results in an analytical expression for the optimal pitch variation which theoretically may lead to very high increases in the stability limit compared to the absolute (minimum) stability limit of the equal pitch cutters. It is also shown that the optimal pitch variation is very sensitive to the chatter frequency which may vary. However, the proposed design of the pitch angles maximizes the chatter frequency range in which there is large improvement in the stability limit.

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