Analytical Modeling of Chatter Stability in Turning and Boring Operations: A Multi-Dimensional Approach

E. Budak (2), E. Ozlu
Faculty of Engineering and Natural Sciences, Sabanci University, Istanbul, Turkey

Abstract
In this study, an analytical model for the stability of turning and boring processes is proposed. The proposed model is a step ahead from the previous studies as it includes the dynamics of the system in a multi-dimensional form, uses the true process geometry and models the insert nose radius in a precise manner. Simulations are conducted in order to compare the results with the traditional oriented transfer function stability model, and to show the effects of the insert nose radius on the stability limit. It is shown that very high errors in stability limit predictions can be caused when the true process geometry is not considered in the calculations. The proposed stability model predictions are compared with experimental results and an acceptable agreement is observed.

Keywords:
Chatter, Turning, Boring

1 INTRODUCTION

Being one of the most important problems in machining, chatter vibrations must be avoided as they result in high cutting forces, unreasonable surface finish and reduced productivity. Although chatter is a more common problem in milling, it can be a limiting factor in some turning and boring operations where slender and flexible tools and workpieces are involved. Therefore, prediction of chatter stability can be critical for these operations as well.

The mechanics of instability in cutting processes was first understood by Tlusty [1] and Tobias [2]. They observed that the modulated chip thickness due to vibrations affect cutting forces dynamically, which in return increases vibration amplitudes yielding a process known as regenerative chatter, and the key process parameter in the process stability is the depth of cut. Tlusty [1] proposed an oriented transfer function approach where the dynamic forces and the dynamic displacements are oriented in the resultant force direction. Although this is a valid model for a 1D cutting process, it may yield inaccurate results for the cases where the geometry of the system is more complicated, e.g. milling or turning/boring processes with inclination angles. This has been demonstrated for the milling stability by Minis and Yanushevsky [3] and Budak and Altintas [4], and in the stability analysis of turning by Ozlu and Budak [5]. In an early study, Kaneko et al. [6] modeled the self excited chatter in turning operations using a 2D model where the conclusions were mostly based on the experimental observations. Minis et al. [7] used an oriented approach and failed to integrate the 3D turning geometry into the model. In a later study, Kuster [8] modeled the regenerative effect with a 3D approach for boring operations, but couldn't obtain the stability lobes. Later, Rao et al. [9] used a multi directional approach [4, 10] to model the stability in turning by employing a cross coupling term which complicates the solution. In one interesting study that is performed by Rigal et al. [11] the stability is modeled by FEM, but the experimental verification and applications were not presented. Atabey et al. [12] and Lazoglu et al. [13] proposed an analytical model for the force prediction in boring, and using time domain solutions they predicted workpiece topography as well. In a recent study, Chandiramani et al. [14] employed a multi-dimensional approach to model the turning dynamic system using an oversimplified process geometry. The turning stability studies summarized above solved the stability equations in the time domain using numerical methods.

The most commonly used stability model in turning applications is the one dimensional oriented-transfer function (1DOTF) stability model [1]. On the other hand, the model proposed in this paper is an analytical one for the prediction of the stability limit for multi-dimensional dynamic turning and boring operations [15]. The model is a step ahead from the previous studies as it treats the dynamic system in a multi-dimensional manner, i.e. the tool and workpiece dynamics are not oriented in one direction, but modeled in a multi-directional form. The model also uses the true geometry of the processes, i.e. the tool angles and the nose radius are included in the formulation. The problem is solved in the frequency domain rather than the time domain simulations resulting in analytical equations for the stability limit.

The paper is organized as follows. First, the basic stability model is formulated in section 2 where the solution procedure is presented for turning and boring including the nose radius. Subsequently, in section 3 a comparative simulation is conducted to demonstrate the error between the 1DOTF and the proposed stability model. The experimental verifications of the presented models are presented section 4.

2 ANALYTICAL MODEL

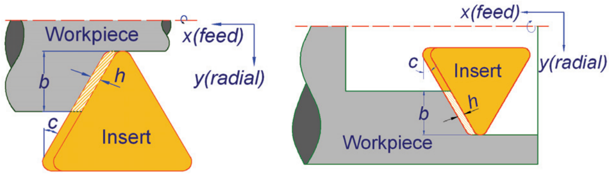
In the proposed stability model, first of all the relationship between the dynamic chip thickness and the cutting forces are modeled. Then, the multi-dimensional dynamic force equation is formulated which is shown to reduce to an eigenvalue problem. In this section, the dynamic force equation is derived neglecting the nose radius. In the later subsections, the insert nose radius is included in the formulations for turning and boring operations. In the mathematical analysis, the global coordinate system (lathe axes; *x*, *y*, and *z*) which can be seen in Figure 1 and 2 is used. The basic parameters that identify the turning process are shown in Figure 1 and 2.a, where α is the normal rake angle, and *i* and *c* are the inclination and side edge cutting angles, respectively, both measured on the rake face. From Figures 1 and 2, one can deduce that the dynamic displacements in the cutting direction (*z*) do not affect the dynamic chip thickness. By this observation, the dynamic problem is reduced to a 2D model. Therefore, the dynamic chip thickness resulting from the vibrations of the tool and the workpiece can be expressed as follows:

$$h(t) = \Delta x \cos c \pm \Delta y \sin c \tag{1}$$

where:

$$\begin{aligned} \Delta x &= x_c(t) - x_w(t) - x_c(t-\tau) + x_w(t-\tau) \\ \Delta y &= y_c(t) - y_w(t) - y_c(t-\tau) + y_w(t-\tau) \end{aligned} \tag{2}$$

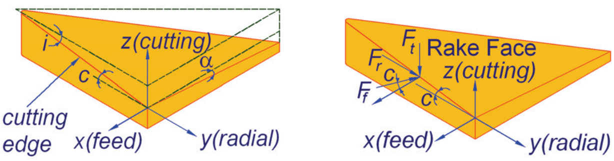
where $x_c(t)$,$x_w(t)$ and $y_c(t)$, $y_w(t)$ are the cutter and workpiece dynamic displacements in the *x* and *y* directions for the current pass. Similarly, $x_c(t-\tau)$,$x_w(t-\tau)$ and $y_c(t-\tau)$, $y_w(t-\tau)$ are the cutter and workpiece dynamic displacements for the previous pass in the *x* and *y* directions, respectively. τ is the delay term which is equal to the one spindle revolution period in seconds. It should be noted here that, since the static chip thickness does not contribute to the regeneration mechanism, it is ignored in the stability analysis.



(a) (b)

Figure 1: Basic parameters and axes in (a) turning operations, (b) boring operations.

Although the dynamic problem can be considered as 2D, the cutting process is 3D in nature, due to the existence of the inclination angle. Then, the forces on the cutting edge need to be modeled by an oblique cutting model [16].The total cutting force acting on the cutting edge is divided into three components as shown in Figure 2.b.



(a) (b)

Figure 2: 3D view of the (a) cutting angles, (b) forces on the insert.

The dynamic cutting forces on the tool in the base coordinate system can be expressed by a transformation and using Equation (1) as follows:

$$\begin{Bmatrix} F_x \\ F_y \end{Bmatrix} = b[A] \begin{Bmatrix} \Delta x \\ \Delta y \end{Bmatrix} \tag{3}$$

where the directional coefficient matrix [*A*] includes the cutting angles and cutting force coefficients [15] and, F_x and F_y are the cutting force components in the *x* and *y* directions, respectively. Note that F_t and consequently F_z is not included in the formulation as it does not take part in the regeneration mechanism. However, it is affected by the regeneration, and if needed it can be determined.

For the stability analysis of the dynamic turning process, a procedure, which is similar to the one used by Budak et al. [4, 10] for the milling stability, is followed. Expressing the dynamic chip thickness in terms of the transfer functions [$G(i\omega_c)$] and the cutting forces, the dynamic force equation can be obtained as follows[15]:

$$\{F\}e^{i\omega_c t} = b\left(1 - e^{-i\omega_c \tau}\right)[A][G(i\omega_c)]\{F\}e^{i\omega_c t} \tag{4}$$

where $\Lambda = b\left(1 - e^{-i\omega_c \tau}\right)$ which reduces the stability solution into an eigenvalue problem. Consequently the stability limit at the chatter frequency can be calculated [4,10,15,17].

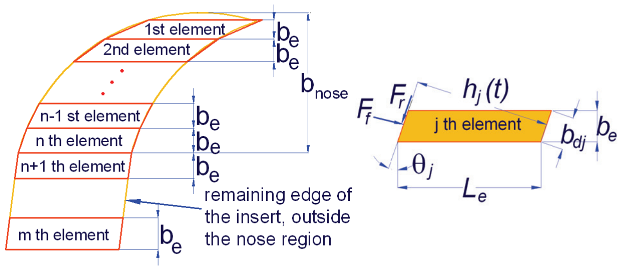


Figure 3: Elements dividing the chip thickness.

2.1 Stability Model for Turning Operations

In this section, the stability model discussed above is implemented for the turning processes with inserts having nose radius. For the stability analysis, when the (nose radius/stable depth of cut) ratio increases, the importance of including the nose radius in the model increases as well. In order to model the insert nose radius, the chip thickness at the nose radius is divided in to many trapezoidal elements as shown in Figure 3. Each element has a different 'side edge cutting angle' which is the main reason behind the strong effect of the nose radius on the stability. By this observation, it can be concluded that the nose radius alters the effects of the tool and workpiece dynamics on the stability limit by changing the contributions of the transfer functions on the process dynamics, similar to the effect of side cutting edge angle, which is also discussed further in the following sections. Hence, each element's contribution to the dynamic system should be included in the analysis. The dynamic force equation (Equation 4) when the *j*th element is in the cut reduces to the following augmented matrix equation [15]:

$$\left\{ \begin{bmatrix} F_{1x} \\ F_{1y} \end{bmatrix} \\ \vdots \\ \begin{bmatrix} F_{jx} \\ F_{jy} \end{bmatrix} \right\} e^{i\omega_c t} = b_e\left(1 - e^{-i\omega_c \tau}\right)[G_0] \left\{ \begin{bmatrix} F_{1x} \\ F_{1y} \end{bmatrix} \\ \vdots \\ \begin{bmatrix} F_{jx} \\ F_{jy} \end{bmatrix} \right\} e^{i\omega_c t} \tag{5}$$

where b_e represents the element height and [G_0] is composed of the transfer functions and directional coefficients including the cutting angles and the force coefficients . Equation (5) also reduces to an eigenvalue problem similar to Equation (4) [15].

2.2 Stability Model for Boring Operations

The insert nose radius has similar effects in boring operations. However, since the stable depths of cut in boring are comparable to the insert nose radius, this effect is more pronounced. The transfer function matrix [$G(i\omega_c)$] is assumed to include only the transfer functions in the *y*-direction as in almost all of the boring operations the tool and the workpiece are much more rigid in the *x*-direction. As a result, the dynamic force equation for the boring processes with inserts having a nose radius reduces to the following when the *j*th element is in the cut [15]:

$$\sum_{p=1}^{j}[F_p]e^{i\omega_c t} = b_e\left(1 - e^{-i\omega_c \tau}\right)\left\{\sum_{p=1}^{j}[A_p]\right\}[G(i\omega_c)]\sum_{p=1}^{j}[F_p]e^{i\omega_c t} \tag{6}$$

Again, Equation (6) reduces to an eigenvalue problem similar to Equation (4), and consequently the stability limit reduces to a 1D equation [15].

2.3 Prediction of Stability Limits and Lobes

In the aforementioned sections, the stability limit solutions have been showed to reduce to an eigenvalue problem. In the cases where the insert is without a nose radius the stability lobe calculations are rather straight forward. The stable depth of cut is calculated by solving the eigenvalue problem [15] and by sweeping the chatter frequency around the flexible modes of the dynamic system. Then, the corresponding spindle speeds are determined as explained in [4,10,17]. However, when the insert has a nose radius, there are two unknowns according to the proposed nose radius model: the height and the number of the meshing elements in the cut. In order to solve the stability limit for those cases a search-based solution procedure is proposed as follows. The number and the height of the meshing elements are selected in the beginning, and the stability limit is calculated by increasing the number of elements in the cut starting with one element. If the determined stability limit is found to be smaller than the height of the elements that are in the cut, the calculation is stopped; otherwise it is continued by adding the following element in the solution. Once the stability limit is obtained, by sweeping the chatter frequency the corresponding spindle speeds are calculated for different lobes. It should be noted here that the stability lobes of turning and boring processes are relatively narrow compared to milling lobes due to the spindle speed limitations and the single cutting tooth. Thus the main objective of the analysis and the experiments in this study is to determine the absolute stability limit.

3 SIMULATION RESULTS

In this section, a comparative analysis is conducted in order to compare the effects of the insert nose radius and the inclination angle on the stability limit using both the 1DOTF approach and the proposed model. The simulation parameters are as follows: The rake angle is 5º, the side edge cutting angle is 30º, the natural frequency of the tool and the workpiece is 1000 Hz, the stiffness of the tool is $3x10^7$ N/m, the stiffness of the workpiece is $30x10^7$ N/m and the damping ratio is 0.01. The absolute stability limits were determined using both models, and the resulting errors of the 1DOTF approach are shown in Figure 4 as a function of the insert nose radius.

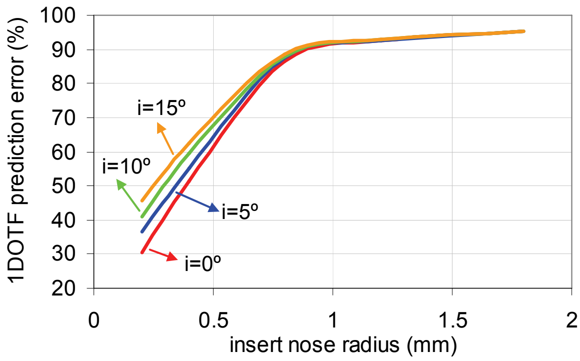


Figure 4: Error in stability limit prediction by the 1DOTF model. (*i*: inclination angle)

Firstly, it can be deduced from Figure 4 that the error is mainly due to the insert nose radius rather than the inclination angle, and it approaches to 100% for higher values of the nose radius. This error is expected since the 1DOTF stability model does not include the insert nose radius effect in the formulation. The effect of the insert nose radius on the process dynamics is the same as the side edge cutting angle. In a dynamic turning process without an insert nose radius, or a side edge cutting angle, the dynamics of the system are only affected by the dynamics in the chip thickness direction, i.e. the *x*-axis as shown Figure 1. The effect of the dynamics in the *y* direction contributes to the dynamic system if there is a side edge cutting angle, or an insert nose radius. An increase in insert nose radius, or side edge cutting angle, increases the effect of the dynamics in the *y* direction on the system stability. Since the 1DOTF stability model cannot handle this effect, as the insert nose radius increases the difference between the predictions of two methods increases, as well.

4 EXPERIMENTAL RESULTS

Chatter experiments were conducted in order to verify the proposed stability model. The workpiece material was AISI 1040 steel and the cutting tool used was a coated carbide insert (TPGN type). The cutting angles on the insert were controlled by the ground insert seats. The transfer functions of the tool and the workpiece were measured by the modal test setup, and sound measurements were used to determine the chatter frequencies for the unstable cuts. Three experiment cases are presented in this section. In the first case, a turning chatter experiment is performed where the tool is more flexible than the workpiece, and has an insert having 0.4 mm nose radius. The parameters used during experiments and analysis can be found in Table 1 (Case 1), and the results along with a chatter sound spectrum, and an example surface finish can be seen in Figure 5. As can be observed from the figure, a close agreement between the experiments and the analytical solution is obtained.

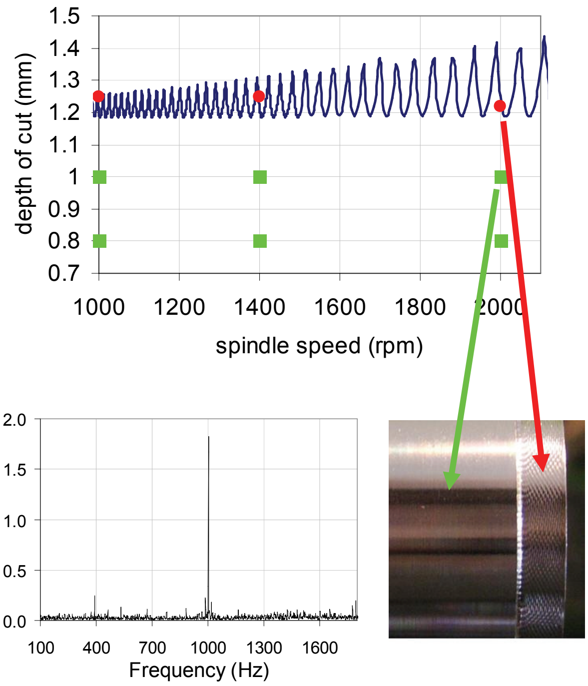


Figure 5: Verification tests for turning with a flexible tool, an example of the chatter sound and a finish surface. (—: analytical, ■: stable cut, ●: chatter)

The aim of the second and third experimental cases is to verify the effect of the insert nose radius on the stability limit. In the second case, a turning chatter experiment is conducted where the workpiece is clamped in such a way that it is more flexible than the tool. The parameters for this case can be found in Table 1 (Case 2), and the test results together with the analytical solution can be seen in Figure 6.a. In the third case, boring chatter experiments are conducted. The parameters for this case can be found in Table 1 (Case 3), and the results can be seen in Figure 6.b.

Case#		1	2	3
Side edge cutting ang.		10°	25°	0°
Rake angle		5°	5°	0°
Inclination angle		5°	5°	0°
Spindle Speed – rpm		-	1400	1400
Cutting force coefficients -MPa	K_f	1180	720	700
	K_r	44	44	-
Nat. freq. of the comp.		1050 Hz	707 Hz	3690 Hz
Stiff. of the comp.(N/m)		$1.2x10^7$	$6.5x10^6$	$2.3x10^7$
Damping ratio		0.015	0.023	0.012

Table 1: Parameters used in the chatter experiments.

It can be observed from Figure 6 that the insert nose radius has a very strong effect on the stability limit due to the increased effects of the dynamics in the *y* direction. Since the workpiece is more flexible in this case, the stability limit is decreased as the insert radius is increased. Similarly in the boring chatter test results shown in Figure 6.b, an increase in the insert nose radius reduces the stability limit due to the increased effects of the tool's flexibility on the dynamic system. It should be noted here that during the chatter tests, high depths of cut were avoided in order to limit cutting forces and boring bar deformations. Overall, the experimental results are in a close agreement with the analytical solutions.

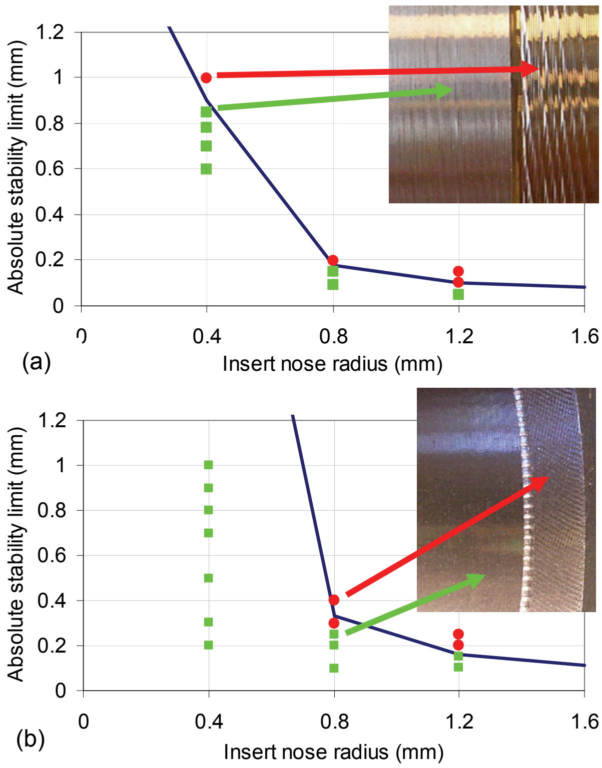


Figure 6: Experimental results and the analytical solutions for (a) turning, and (b) boring experiments. (: analytical, : stable cut, : chatter)

5 CONCLUSIONS

In this study, an analytical model for the prediction of the stability limit in turning and boring operations is presented. The model provides a multi-directional approach to the dynamic system by solving the stability limit in a matrix form. In addition the true geometry of processes, i.e. the important cutting angles and the insert node radius, are included in the model. It is presented that the stability limit solution for boring processes reduces to a 1D equation even with the nose radius model. The effect of important process parameters on the stability is demonstrated, and a comparative analysis is presented with the 1DOTF stability model. The model predictions are verified with chatter experiments and overall a close agreement is observed.

REFERENCES

[1] Tobias, S.A. and Fishwick, W., 1958, The Chatter of Lathe Tools Under Orthogonal Cutting Conditions, Transactions of ASME, 80:1079-1088.

[2] Tlusty, J., Polacek, M., 1963, The Stability of Machine Tools Against Self Excited Vibrations in Machining, Int. Res. in Prod. Eng., ASME,465-474.

[3] Minis, I., and Yanushevsky, T., 1993, A New Theoretical Approach for the prediction of the Machine Tool Chatter in Milling, ASME J. Eng. Incl., 115:1-8.

[4] Budak, E., and Altintas, Y., 1998, Analytical Prediction of Chatter Stability in Milling-Part I: General Formulation, Part II: Application to Common Milling Systems, ASME J. Dyn. Sys. Meas. Control, 120:22–36.

[5] Ozlu, E., Budak, E., Comparison of One Dimensional Oriented Transfer Function Stability Model vs. Multi Dimensional Stability Model in Turning Operations, Int. J. of Mach. Tools and Manufacture (submitted for review).

[6] Kaneko, T., Sato, H., Tani, Y., 1984, O-hori, M., Self-Excited Chatter and its Marks in Turning, Transactions of ASME, 222:106-228.

[7] Minis, I. E., Magrab, E. B., Pandelidis, I. O., 1990, Improved Methods for the Prediction of Chatter in Turning Part 3: A Generalized Linear Theory, Transactions of ASME, 112:12-20.

[8] Kuster, F., 1990, "Cutting Dynamics and Stability of Boring Bars", Annals of the CIRP, 39/1:361-366.

[9] Rao, C. B., Shin, Y. C., 1999, A Comprehensive Dynamic Cutting Force Model for Chatter Prediction in Turning, Int. J. of Mach. Tools and Manufacture, 39:1631-1654.

[10] Altintas, Y. and Budak E., 1995, Analytical prediction of stability lobes in milling, Annals of the CIRP, 44:357-362.

[11] Rigal, J., Pupaza, C.,Bedrin, C., 1998, "A model for simulation of Vibrations During Boring Operations of Complex Surfaces", Annals of the CIRP, 47/1:51-54.

[12] Atabey, F., Lazoglu, I., Altintas, Y., 2003, "Mechanics of Boring Processes – Part I", Int. J. of Mach. Tools and Manufacture, 43:463-476.

[13] Lazoglu, I., Atabey, F., Altintas, Y., 2002, "Dynamic of Boring Processes: Part III – Time Domain", Int. J. of Mach. Tools and Manufacture, 42:1567-1576.

[14] Chandiramani, N.K., Pothala, 2006, T., "Dynamics of 2-dof regenerative chatter during turning", Journal of Sound and Vibration, 290:448-464.

[15] Ozlu, E., Budak, E., Analytical Modeling of Chatter Stability in Turning and Boring Operations Part I: Model Development, Part II: Experimental Verification, ASME Journal of Manufacturing Science and Engineering June 2006 (under review)

[16] Armarego, E.J.A. and Whitfield, R.C., 1985, "Computer based modeling of popular machining operations for force and power predictions", Annals of the CIRP, 34:65-69.

[17] Altintas, Y, Weck, M., 2004, "Chatter Stability of Metal Cutting and Grinding", Annals of the CIRP, 53/2:619-642.