

TIDAL TERMS IN UNIVERSAL TIME: EFFECTS OF ZONAL WINDS AND MANTLE Q

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Abstract. The zonal response coefficient κ is studied, and some improvements in its determination are made. First, the reprocessing of the Bureau International de l'Heure data with the 1980 International Astronomical Union nutation series is found to result in improved estimates of κ . Second, a time series of the angular momentum of the atmosphere is found to have power in the tidal bands, and it is demonstrated that removing the atmospheric influence from the rotation data leads to better estimates of κ . The frequency dependence of κ due to finite dissipation in the earth is computed, and the observations are subsequently shown to limit the allowable models of dissipation. If Q varies with frequency σ , as $Q \sim \sigma^\alpha$, then the M_f and M_m terms in UT place an upper bound of about $1/3$ on α . Finally, dynamic ocean tide models are studied, and it is concluded that the rotation data cannot distinguish between these and an equilibrium tide.

Introduction

The short-period terms in the length of day (lod) are the result of the action of the tide raising potentials of the sun and moon on the axial moment of inertia of the earth. Jeffreys [1928] first recognized that these terms should be observable in the rotation rate of the earth, although the pendulum clocks available at that time were barely accurate enough for the task. As timekeeping improved, the possibility of more terms becoming observable led Woolard [1959] to compile a table of the 20 largest terms. The geophysical interest in this problem has recently quickened as the improved rotation data have revealed that corrections to the Jeffreys and Woolard amplitudes are necessary.

A second-degree zonal potential U_2^0 , such as that due to the lunar fortnightly (M_f) or monthly (M_m) tide is symmetric about the polar axis and causes the polar flattening of the earth to vary as the potential increases and decreases. The resulting deformation of the earth is characterized by an induced potential which is proportional to the prescribed potential through the Love number k_2 . MacCullagh's formula then relates the induced potential to the change in polar moment of inertia ΔC :

$$\Delta C = -k_2 \frac{1}{3} \left(\frac{5}{\pi}\right)^{\frac{1}{2}} \frac{r^3}{G} U_2^0$$

where r is the radius of the earth, G is the gravitational constant, and C is the polar moment of inertia of the earth. Making allowance for the

fluidity of the core and the inertia of the ocean tide induced by the prescribed potential, the relative change in rotation rate is then

$$\frac{\Delta\Omega}{\Omega} = \frac{1}{3} \kappa \left(\frac{5}{\pi}\right)^{\frac{1}{2}} \frac{r^3}{GC} U_2^0 \quad (1)$$

where k_2 has been replaced with κ , the zonal response coefficient. In Jeffreys's original work and in Woolard's, this was simply the second-degree Love number $k_2 = 0.300$. The idealized earth model for which this number is appropriate is one in which the response to the tidal potential is purely elastic, with a seismically determined rigidity structure, no oceans, no atmosphere, and a fluid core which is viscous enough to follow any variations in the mantle's rotation rate.

Agnew and Farrell [1978] correctly suggested that the change in the moment of inertia of the oceans had a major influence on κ , and they produced a numerical value based on an equilibrium tide model. Merriam [1980], Wahr et al. [1981] and Yoder et al. [1981] attempted to account for the presumed lack of core-mantle coupling, and predicted amplitudes were brought into agreement with the mean of all observations to within about 2%. Some uncertainties remained, however, which some recent developments offer the possibility of correcting.

1. The Bureau International de L'Heure (BIH) has begun to revise its data using the 1980 International Astronomical Union nutation series in the reduction process, and data are available from 1978 to 1982. Since the older nutation series can reportedly produce errors of up to 1.8 ms in universal time (UT), [Capitaine and Feissel, 1983], or nearly twice the amplitude of the M_f and M_m signals, its effect on the estimates of the amplitudes of the tidal signals may be large.

2. Rotation data may be able to provide some information on the possible frequency dependence of Q because anelastically corrected rigidities, and hence Love numbers and κ , might be measurably different from those appropriate to seismic frequencies. The dependence of κ on frequency is computed here, and it is found that κ_{M_f} and κ_{M_m} could be as much as 5% larger than seismic rigidities indicate.

3. Lambeck and Cazenave [1974] suggested that zonal winds in the atmosphere might have enough power at monthly periods to corrupt the measurement of κ from the rotation data. An atmospheric angular momentum data set covering 1976-1982, compiled by Rosen and Salstein [1983], is examined, and it is found that while there is little evidence for tidal terms in the wind data, there is sufficient power in the band to influence measurements of κ .

4. A possible weakness of the most recent

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studies of κ is that they use an equilibrium ocean tide model, whereas there is some evidence that at least M_f is not equilibrium. Schwiderski [1982] has now computed dynamic M_f and M_m tides and the second-degree zonal term, computed by C. Goad (personal communication, 1984), is used here to find a dynamic κ . The results, however, are found to support an equilibrium tide model as well as they do the dynamic tide model. This is not surprising because the amplitudes of the second-degree zonal coefficient of both the M_f and M_m tides are well below the claimed accuracy of Schwiderski's tide models, and the deviation of this coefficient from its equilibrium counterpart is only a few millimeters, so that one could argue that it is not meaningful.

A persistent feature of analyses of tidal terms in the lod is that $\kappa_{M_f} > \kappa_{M_m}$, whereas the ratio of 1 for these two parameters is expected on the basis of equilibrium tidal theory. This result is obtained in nearly all studies and is found in the data of independent observing programs. Capitaine [1982] has presented evidence that both parameters vary with epoch and that this is particularly true for κ_{M_f} . One of the principal reasons for undertaking this work was the idea that the winds and the dynamic ocean tide might jointly resolve this problem. In fact, it appears that errors in the nutation series used in the reduction of the rotation data are responsible for most of the anomaly in the ratio $\kappa_{M_f}/\kappa_{M_m}$.

Results Based on the 1980 IAU Nutation

Beginning with its annual report for 1982 the BIH has introduced the 1980 IAU nutation series into its processing of measurements of earth rotation. This series, developed by Wahr [1981] for an elastic earth model with a fluid core, replaces the older series developed by Woolard [1959] that is based on a rigid earth model. The errors in Woolard's theory can reportedly produce errors of up to 1.8 ms in UT, or twice the amplitude of the M_f and M_m terms, so that an improved nutation series could have important consequences for the measurement of κ .

Two BIH ΔUT series, one based on the Woolard nutation series, (BIH annual reports 1978-1981, Table 6), the other based on the IAU nutation series (published in the 1982 BIH annual report, Table 6), of 5-year duration, 1978-1982, were examined to see if the introduction of the 1980 IAU nutation series improved the determination of κ .

To compute κ , a nominal UT time series comprising the 50 largest tidal terms was generated for comparison with the observed series. After removing the drift from both series with a high pass filter designed to have a minimal effect on M_f and M_m an amplitude and a phase were least squares fitted to every tidal constituent which was large enough to be detectable and which could be resolved from other waves in the band. The zonal response coefficient κ was then obtained as the ratio of the amplitude of a wave in the BIH data to the amplitude of the same wave in the nominal series.

A small correction is required to remove the effects of the 5-day averaging of the BIH. Before the daily observations of the BIH are averaged to

produce the 5-day means of Table 6 of the BIH annual report, a nominal tidal series, with $\kappa = 0.312$, is removed. The same nominal series is then added to the results of the 5-day averaging. The effect of this procedure is that the anomalous amplitude of the waves is averaged and the nominal part ($\kappa = 0.312$) is not. Values of κ determined from the BIH data must therefore be corrected for averaging. Averaging over 5 days reduces a wave of period P days by $\sin(5\pi/P)/(5\pi/P)$ which amounts to 0.7931 at M_f and 0.9465 at M_m . If κ_o is the value of κ found from the BIH data, then κ_t is the value corrected for averaging the anomalous part over 5 days:

At M_f

$$\kappa_t = 0.312 + \frac{(\kappa_o - 0.312)}{0.7931}$$

At M_m

$$\kappa_t = 0.312 + \frac{(\kappa_o - 0.312)}{0.9465}$$

The 4 years overlap between the two data sets is not a long enough time span to permit accurate measurements of κ , but at this point the aim is only to see if the introduction of the 1980 IAU nutation series improves the determination at all. In the 4 years of overlap the Woolard series gave $\kappa_{M_f} = 0.225 \pm 0.043$ $\kappa_{M_m} = 0.253 \pm 0.024$, while the 1980 series gave $\kappa_{M_f} = 0.342 \pm 0.023$ and $\kappa_{M_m} = 0.346 \pm 0.020$. The estimated standard errors are smaller when the 1980 series is used, indicating perhaps less variance in the 1980 series, but more importantly, the estimates of κ and the ratio $\kappa_{M_f}/\kappa_{M_m}$ are closer to expected values. Two 3-year subintervals of the 4-year epoch confirm these conclusions and further indicate that the 1980 series permits a more stable estimate of κ suggesting that much of the variation of κ with epoch found by Capitaine [1982] may be attributable to errors in the Woolard series.

Frequency Dependence of κ Due to Anelasticity

In a dispersive medium the rigidity and hence the Love numbers will be frequency dependent. Merriam [1981] attempted to measure this effect by using gravity tide observations (the gravimetric factor $\delta = 1 + h_2 - 3/2 k_2$ being a linear combination of Love numbers) but was unsuccessful because the nature of the calibration of gravimeters is such that one is forced to measure the small change in δ between semidiurnal and diurnal frequencies rather than the much larger change from seismic to tidal frequencies. More accurate earth tide measurements in the semidiurnal-diurnal band will eventually provide some information, but alternatively, one could look at perhaps less accurately measured phenomena of much longer period. Smith and Dahlen [1981] use the observed period of the Chandler wobble (435 days) to constrain models of the dispersive decrease in rigidities. J. B. Merriam (unpublished manuscript, 1984) extends this with observations of the zonal tides at M_f , M_m , Ssa in the rotation data and Lageos measurements of Ssa and the lunar nodal tide.

Any anelastic medium will have a frequency dependent rigidity, but the functional form of the dependence is modified by the behavior of the specific dissipation function Q within the absorption band. If Q is independent of frequency, then the rigidity μ , relative to a reference frequency σ_2 , is

$$\frac{\mu(\sigma_1)}{\mu(\sigma_2)} = 1 - 2(\pi Q)^{-1} \ln(\sigma_2/\sigma_1) \quad (2)$$

[Kanamori and Anderson, 1977]. With a nominal Q of 500 the relation (2) implies a decrease in rigidity of about 2% between a reference period of 1 s and the fortnightly period of the M_f tide. However, M_f and M_m are well outside the seismic band in which most Q measurements are made, and while observations are generally consistent with a constant Q in the seismic band, the uncertainty is such that even the small permissible frequency dependence can become important if it is extrapolated into the earth tides band. Anderson and Minster [1979] have postulated a frequency dependent Q in which $Q \sim \sigma^\alpha$ in an absorption band of unspecified width which includes the seismic band. Physically, this may be interpreted as an increase in the density of relaxation mechanisms through the absorption band as the period increases. In this case the rigidity will vary with frequency in the absorption band as

$$\frac{\mu(\sigma_1)}{\mu(\sigma_2)} = 1 - \cot \frac{\pi\alpha}{2} \left[\left(\frac{\sigma_2}{\sigma_1}\right)^\alpha - 1 \right] \frac{1}{Q_2} \quad (3)$$

provided that the implied $Q \gg 1$. Implicit in (3) is that σ_1 is within the absorption band. At this point the only thing that can be said about the width of the absorption band is that it is at least as wide as the seismic band (1 s $\sim 10^4$ s). While (3) may not be valid at M_m , a measurement of α through the tidal band may help to define the long-period cutoff of the absorption band. For periods of 2 weeks or more the difference between (2) and (3) can be considerable, implying relatively large variations in κ (Figure 1) and leading to the possibility of measuring α if M_f and M_m are indeed within the absorption band.

Reasonable values of κ for the earth range as high as 1/3, but specific measurements are few. The most reliable estimate is probably that obtained by requiring that the reduced rigidities associated with a frequency dependent Q lengthen the period of the Chandler wobble by the 8 or so days needed to explain the observed period. This method yields $\alpha = 0.15 \pm 0.04$ [Smith and Dahlen, 1981]

Introducing variations in μ such as (2) or (3) into earth tide models leads to the evaluation of frequency dependent Love numbers which can, in turn, be used to define frequency dependent κ .

Merriam [1980] derives an expression for κ in terms of the Love numbers and the equilibrium tide height in the oceans which can be written as

$$\kappa = 0.886k_2 + (1.129 + 0.894k_2')\gamma_2 \quad (4)$$

$$\gamma_2 = \beta \xi_2^0 \frac{g}{U_2^0}, \quad \beta = 3\rho_w/5\rho_0$$

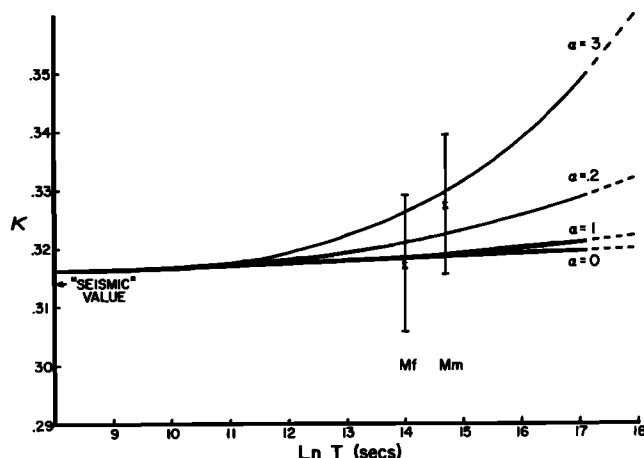


Fig. 1 The expected variation of κ with the natural log of the period T in seconds for several values of the parameter α and the observed values at M_f , M_m of κ . Error bars are the estimated standard errors on the least squares fit of the BIH-ATM series to a nominal series composed of the 50 largest tidal terms. BIH refers to the Bureau International de L'heure ΔUT time series 1978-1982 based on the 1980 IAU nutation theory and ATM refers to the ΔUT series implied by the variations in atmospheric angular momentum.

where k_2 and k_2' are the body force and load Love numbers, respectively; γ_2 represents the influence of the ocean tide; ρ_w is the density of seawater; ρ_0 the average density of the earth; g the acceleration of gravity; and U_2 the prescribed tidal potential.

The numerical factors, which are essentially unaffected by small variations in the rigidity, are the result of the lack of core-mantle coupling and would all be 1 if that coupling was complete (in the sense of the core being solid). The factor γ_2 is itself a function of the Love numbers and unfortunately a rather complicated one. Its strict evaluation would require a solution of the equilibrium tidal equations over a range of frequencies and α . It is possible to avoid a lot of that effort with very little loss in accuracy by considering the following. The second-degree tide height in global oceans is [Merriam, 1973]

$$\xi_2^0 = \frac{\gamma_2}{1 - \beta\gamma_2'} \frac{U_2^0}{g}$$

where $\gamma_2 = 1 + k_2 - h_2$, $\gamma_2' = 1 + k_2' - h_2'$. When the equilibrium tidal equations for nonglobal oceans are solved either in a truncated matrix formulation [Merriam, 1973] or iteratively [Agnew and Farrell, 1978], the second-degree zonal tide height is then numerically

$$\xi_2^0 = 0.602 \frac{\gamma_2}{1 - \beta\gamma_2'} \frac{U_2^0}{g}$$

and all the complexities of self-gravitation in a nonglobal ocean have been condensed into the factor 0.602. The usefulness of this result arises from the fact that the factor 0.602 is essentially determined by the geometry of the oceans (i.e., by the ocean function) and is

virtually independent of elastic effects so that the effects of varying the rigidity are conveniently contained in γ_2 and γ_2' . Equation (4) then becomes

$$\kappa = 0.886k_2 + (1.129 + 0.894k_2') \frac{\beta \gamma_2}{1 - \gamma_2'} 0.602 \quad (5)$$

The frequency dependence of κ can then be easily obtained from the frequency dependent Love numbers. The Love numbers themselves have been computed by integrating the elastic equations of motion with the preliminary reference earth model (PREM) of Dziewonski and Anderson [1978]. The PREM model is distinguished because dispersion effects have been modeled as part of the solution and observed seismic velocities and rigidities reduced to a nominal period of 1 s with equation (2). As such, it is not strictly compatible with (3), but in a practical sense this may not be important because the differences between (2) and (3) in the seismic band (where (2) was used in the dispersion correction to construct the model) are quite small. Therefore when (3) is used with this earth model, rigidities are first adjusted to a reference period of 200 s by using (2) (this has very little effect on κ ; (3) might be used with very little difference), and then for longer periods, rigidities are computed using these values and a period of 200 s as reference values in (3). The reference period of 200 s is appropriate because the data used to construct the Q model in PREM were mostly free oscillations.

The variation of κ with frequency for several models of frequency dependence is shown in Figure 1. Values of α in excess of 1/3 can be ruled out at the 1σ level by the observed amplitudes of the M_m and M_f tides. A nominal value of α of about 0.2 leads to values for the zonal coupling coefficient κ about 5% larger than seismic rigidities suggest and requires that the monthly tides be about 1% larger than the semimonthly tides, which is below the level of accuracy of the rotation data.

Measurements of α from long-period oscillations are few but are for the most part in agreement with the range $\alpha < 1/3$. Smith and Dahlen [1981] argue that the 8 day difference between the observed Chandler period and a theoretical period based on a seismic rigidity structure can be explained by a frequency dependent Q with $\alpha = 0.15 \pm 0.043$. Lambeck and Nakiboglu [1983] use an estimate of the amplitude of the 18.6-year tide in the earth's potential from Lageos tracking data [Rubincam, 1984]. Their results require $\alpha = 0.35 \pm 0.05$; however, J. B. Merriam (unpublished manuscript, 1984) points out an error and revises this upward. Further measurements of α at 1/2 year and 18.6 year are reported by J. B. Merriam (unpublished manuscript, 1984).

Zonal Winds and LOD at M_f , M_m

Suspensions that most of the irregular rotation signal at periods less than a year or so is due to variations in the global wind pattern have been convincingly confirmed by a series of papers in the last few years [Hide et al., 1980; Langley et al., 1980; Rosen and Salstein, 1983]. That the atmosphere may have winds at M_f and M_m , or

sufficient power near these tidal lines to influence the lod and hence κ was suggested by Lambeck and Cazenave [1974], but the observations were not sufficiently detailed either temporally or spatially to test this hypothesis. Rosen and Salstein [1983] have recently produced a time series of the angular momentum of the atmosphere consisting of twice daily values (0000 and 1200 UT) from January 1, 1976, to December 1982 that meet the above restrictions and now permits the suggestion of Lambeck and Cazenave to be studied.

The accuracy of the atmospheric angular momentum time series is difficult to assess because of the many factors which must be considered, but Rosen and Salstein suggest that for periods of more than a year the angular momentum may be systematically underestimated due to the failure of the data to sample the upper 10% of the atmosphere. Shorter-period variations are thought to be in error by about 5%. Approximately 10% of the time series is missing, and these missing values were interpolated by a cubic polynomial. Since most of the gaps consist of a half day or a single day and none is more than a fraction of the M_f period, it is unlikely that the interpolated time series contains errors that seriously affect the conclusion.

Conserving angular momentum between the atmosphere and the mantle, the changes in the length of day can be obtained from

$$\frac{\Delta \text{lod}}{\text{lod}} = \frac{\Delta M}{\Omega C_m}$$

where ΔM is the change in the atmosphere's angular momentum, C_m is the moment of inertia of the mantle, and Ω the mean rotation rate. The mantle's moment of inertia is used rather than the whole earth's on the assumption that the coupling of the core to the mantle is too weak for angular momentum to be transferred from the mantle to the core. This is probably true for periods less than a year, and it seems unlikely that any mechanism (laminar viscous, turbulent, topographic, or electromagnetic) can spin couple the core to the mantle at periods less than 5-25 years [Yoder et al., 1981]. The Δlod time series was then integrated to produce a UT time series, ATM, and subsequently averaged over 5-day intervals centered on the same days as the 5-day means of the BIH data.

The angular momentum of the atmosphere is available from 1976 to 1982, and while a longer time series is more useful, only the 1978 to 1982 data have been used here to match the epoch of the new BIH data. This is important because the atmospheric time series is so variable that its characteristics in the tidal band depend on epoch. Two more years of BIH data with the old nutation series could have been used, but since the errors in the old nutation series and the winds seem to affect the determination of κ at about the same level, it was deemed more desirable to use a homogeneous treatment.

A critical question for this study is the noise level in the atmospheric data and the cutoff period in the ATM series at which the signal due to real variations in the atmospheric angular momentum drops below the noise level. Because atmospheric processes have a large random

TABLE 1. The Zonal Response Coefficient Before and After Removing the Wind Signal

	BIH		BIH-Wind	
	κ	Phase days	κ	Phase days
M_f	0.331 ± 0.023	+0.15	0.317 ± 0.013	+0.22
M_m	0.364 ± 0.016	-0.09	0.327 ± 0.011	-0.13

The BIH series is in both cases the result of processing with the 1980 IAU nutation theory 1978-1982. A positive phase means the observed signal leads the nominal tidal signal.

component, statistical determinations of the noise level in the angular momentum time series may be meaningless in the sense that even a perfectly accurate time series may look like noise. Langley et al. [1980] and Hide et al. [1980] have demonstrated correlation between rotation and atmospheric angular momentum fluctuations to periods of at least 7 weeks, while Barnes et al. [1983] maintain that the correlations exist down to periods of a few weeks. According to Rosen and Salstein [1983] the error in the meteorological data set is equivalent to about 0.2 ms, which is comparable to the noise in the BIH data, so that while it is conceivable that some part of the BIH "noise" might in fact be due to the zonal winds, removal of the wind signal from the BIH data may not reduce the noise level. The only way to test this properly is to remove the wind signal from the UT data and see if the noise level is thereby reduced.

To test the hypothesis that the atmosphere is responsible for at least some of the variance in the BIH data near the tidal frequencies, the coherence between the BIH ΔUT series and a nominal ΔUT series consisting of the 50 largest tidal terms was computed. Removing the atmospheric time series ATM from the BIH series was found to improve the coherence between the BIH series and the nominal tidal series by about 10% near M_m and 2% near M_f .

The results of the least squares solution for from 1978 to 1982 are shown in Table 1. Column 2 contains the results from the BIH data processed with the 1980 IAU nutation series, and column 4 contains the results of removing ATM from the same BIH series before solving for κ . The estimated standard errors are smaller for both waves and the largest effect on the amplitude is at M_m , which is 10% smaller when the wind signal is removed. On the basis of this it is concluded that winds in the atmosphere do disturb measurements of κ from the BIH data and that the BIH data should have the atmospheric signal removed before κ is estimated.

Dynamic Ocean Tides at M_m M_f

There has been much speculation on the question of whether or not the M_m , M_f ocean tides are equilibrium. If they are not, can their departure from equilibrium to be detected in the rotation data. Since the oceans affect κ at about the 12% level, a 10% deviation in the amplitude of the

tide from its equilibrium value will only affect at about the 1% level, or below the sensitivity of the UT data.

Schwiderski [1982] has recently computed the M_m , M_f constituents from a hydrodynamical interpolation of tide measurements. The harmonic components of the tide amplitude which influence the length of day are the C_2^0 coefficients which (Table 2) have been supplied by C. Goad (personal communication, 1983). Table 2 also shows the equilibrium coefficients, computed for an earth model with $\alpha = 0.2$. The M_f tide is slightly smaller than its equilibrium value, a result which is expected because a departure from equilibrium for a zonal tide should be manifested as an equatorial deflection of the water mass by the coriolis force. The M_m tide on the other hand has a C_2^0 coefficient which is larger than its equilibrium value by 18%, while its phase is almost exactly equilibrium. The equilibrium tide has a C_2^0 coefficient of about 1 cm so that departure of the dynamic tide from equilibrium is only a couple of millimeters at most. Since the intended accuracy of Schwiderski's tide is a relative error of 5 cm, it is unlikely that 2 mm can be considered real. It seems therefore that at least for the requirements of this work, where only the C_2^0 coefficient is important, the equilibrium tide is as likely to be correct as is the dynamic tide. Furthermore the rotation data cannot be used to distinguish between these two models, and it is doubtful whether this conclusion could be changed in the near future.

Carton [1983] has also computed dynamic tide models of M_f and M_m and his results indicate that the second-degree zonal harmonics of both constituents are smaller the equilibrium amplitudes by slightly less than 10% (Table 2). In terms of κ he finds that nonequilibrium effects of the sea surface elevation lower κ by about 1.6% at M_f and 1.2% at M_m . Zonal currents associated with the nonequilibrium tides compensate for this somewhat so that overall the nonequilibrium κ are smaller than the equilibrium values by 1.3% (M_f) and 1% (M_m).

Summary

The zonal response coefficient has been shown to be dependent on the parameter α which characterizes the frequency dependence of the Q of the mantle. It has been demonstrated that the rotation data on the M_f , M_m waves support an upper limit of about 1/3 on α with the possibility of improving on this slightly as more reprocessed BIH data become available. This result is consistent with measurements of α from Lageos tracking data (J. B. Merriam, unpublished manuscript, 1984) and the observed period of the Chandler wobble [Smith and Dahlen, 1981].

The zonal winds in the atmosphere are found to have sufficient power in the tidal bands to disrupt the measurement of κ , and more reliable measurements of κ can be obtained by removing the influence of variations in the angular momentum of the atmosphere from the rotation data.

The reprocessing by the BIH of its observations with the new IAU nutation series produces substantial improvements in the determination of κ , both in the sense of better agreement with

TABLE 2. The Second Degree Zonal Coefficient of Schwiderski's Dynamic Tide, Carton's Dynamic Tide, and an Equilibrium Tide

	M_f		M_m	
	Amp	Phase	Amp	Phase
	cm	deg	cm	deg
Equilibrium	2.11	+270	1.12	+270
Schwiderski [1982]	1.91	+251	1.34	+270
Carton [1983]	1.91	+263	1.04	+266

The functional form is $\text{amp} \times 1/2 (3 \cos^2 \theta - 1) \cos(\text{ARG} - \text{Phase})$, where θ is the colatitude and ARG is the astronomical argument of the constituent.

expected values and in improved statistical estimates of the uncertainty. While only a few years (1978-1982) of BIH data have been reprocessed with the 1980 IAU nutation series, the tentative conclusion is that many of the anomalous results on the amplitude of the M_f , M_m waves reported in the literature can be attributed to errors in the Woolard nutation theory. It is apparent that the rotation data are not sufficiently accurate to distinguish between an equilibrium tide model for M_f and M_m and the dynamic tides of Schwiderski [1982] and Carton [1983].

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