Dividend or Reinvestment whenTaxes are Considered?

Tor Brunzell, Sören Holm and Bengt Jonsson

$$q(s) = \begin{cases} 1 & \text{if } \exp\{\int_s^t (g(u)-r(u))du\} < \dfrac{(1-\tau_1(s))(1-\tau_2(s))}{(1-\tau_3(s))(1-\tau_4(s))} \\[3mm] 0 & \text{if } \exp\{\int_s^t (g(u)-r(u))du\} > \dfrac{(1-\tau_1(s))(1-\tau_2(s))}{(1-\tau_3(s))(1-\tau_4(s))} \end{cases}$$

Dividend or Reinvestment when Taxes are Considered?

Tor Brunzell, Sören Holm and Bengt Jonsson

Contents

Dividend or Reinvestment when Taxes are Considered?

TOR BRUNZELL[1], SÖREN HOLM[2] AND BENGT JONSSON[2]

1) Stockholm University, School of Business, SE-106 91 Stockholm, Sweden
E-mail: tb@fek.su.se
2) Swedish University of Agricultural Sciences, Dept. of Forest Resource
Management, SE-901 83 Umeå, Sweden
E-mail: Soren.Holm@slu.se; Bengt.Jonsson@slu.se

Abstract The article analyzes theoretically how a firm maximizes the value of shareholder's wealth with its policy for dividend/reinvestment of the profit from operations, when taxes are considered. In the study, the net present value, i.e. the present value after deduction for taxes, is used as objective function. Four different taxes are considered.
The analysis shows the terms on which it is profitable to receive dividend payout or to reinvest at an arbitrary time. Under the assumption of a unique maximum net present value, the terms at the time for the maximum net present value are also presented.
The article can be applied in forest economy.

Key words Financing, Management.

Jel Code C61, E62, G31, G35

Dividend or Reinvestment when Taxes are Considered?

1 Introduction

This study is intended to be an examination of the relationship between dividend, investment policies and taxes. The aim is to analyze how a firm maximizes the value of shareholder's wealth with its policy for dividend/reinvestment of the profit from operations, when taxes are considered. This is equivalent to maximizing the market value of the firm. Consequently, the net present value, i.e. the present value after deduction for taxes, is used as objective function in the study. Four different taxes are considered:
(a) corporate tax when dividend payout is received; (b) shareholder's personal income tax on dividends; (c) corporate tax when reinvestment is made and (d) shareholder's capital gains tax.

We assume an ideal economy characterized by rational behavior and perfect certainty (deterministic approach) and to some extent under risk (stochastic perspective). A limitation in the analysis is that the sum of funds used for dividends and investments is equal to the profits from operations. Thus, neither external financing nor repurchasing of shares are regarded.

This issue is earlier treated by many economic scientists e.g. Lintner (1956), Gordon (1959, 1962, 1994), Miller *et al.* (1961), Farrar *et al.* (1967), Fama (1974), Brennan (1970), Miller *et al.* (1978) and Elston (1996).

In principle, the article has an application in forestry: To cut (dividend) or not to cut (reinvestment), when taxes are considered?

2 Model

2.1 Concepts and notations

In this study, the net present value, i.e. the present value after deduction for taxes, is used as objective function. In accordance with this, total net present value over time t, $PV(t)$, is the sum of the net present values of dividends $PV_D(t)$ and gains of reinvestments $PV_R(t)$. Thus, $PV(t) = PV_D(t) + PV_R(t)$.

In the study, the following concepts and denotations are used.

'Gross' means 'with no deduction for taxes'; 'net' means 'after deduction for taxes'.

t	denotes	an arbitrary terminal time point,
s	"	time point ending period s in the discrete model; time in the continuous model where $0 \le s \le t$,
$G(s)$	"	the shareholder's gross profit from operations in period s; it is assumed to be non-negative,
$q(s)$	"	dividend ratio, i.e., relative dividend fraction of the firm's gross profit at time point s where $0 \le q(s) \le 1$; q is decision function,
$1-q(s)$	"	retention ratio, i.e., relative reinvestment fraction of the firm's gross profit at time point s where $0 \le q(s) \le 1$,
$r(s)$	"	market-required rate of return in period s (interest rate); same notation in both the discrete and continuous model, but for consistency with slightly different values,
$\delta(s)$	"	discount factor in period s in the discrete model, i.e., $\delta(s) = (1 + r(s))^{-1}$,
$g(s)$	"	rate of return on reinvestment in period s (growth rate); same notation in the discrete and continuous model, but for consistency with slightly different values,
$\gamma(s)$	"	growth factor in period s, the discrete model, $\gamma(s) = 1 + g(s)$.

Different tax rates, $\tau_i(s)$, all $0 \le \tau_i(s) < 1$:

$\tau_1(s)$	denotes	corporate tax rate on gross profit in period s, when dividend is received,
$\tau_2(s)$	"	personal income tax rate on gross dividend payout in period s,
$\tau_3(s)$	"	corporate tax rate on gross profit in period s, when reinvestment is made,
$\tau_4(s)$	"	capital gains tax rate, i. e. personal income tax rate in period s on the capitalized reinvestments after a deduction for corporate tax; $\tau_4(s) = \tau_4(t)$ for all $s \le t$ where t is sale date,
$PV(t)$	"	total net present value over t periods,
$PV_D(t)$	"	net present value of dividends over t periods,
$PV_R(t)$	"	net present value of capital gains of reinvestments over t periods.

2.2 Net present value of dividends

The net dividend in period s equals $G(s) \cdot q(s) \cdot (1 - \tau_1(s))(1 - \tau_2(s))$ for $s = 0, 1, \ldots$ and is assumed to be issued in the beginning of the period. By discounting and summing over s we get

$$PV_D(t) = G(0)q(0)(1 - \tau_1(0))(1 - \tau_2(0)) + G(1)q(1)(1 - \tau_1(1))(1 - \tau_2(1))\delta(1) +$$

$$+ G(2)q(2)(1 - \tau_1(2))(1 - \tau_2(2))\delta(1)\delta(2) + \ldots + G(t)q(t)(1 - \tau_1(t))(1 - \tau_2(t))\delta(1) \cdot \ldots \cdot \delta(t) =$$

$$= \sum_{s=0}^{t} G(s)q(s)(1 - \tau_1(s))(1 - \tau_2(s)) \prod_{u=0}^{s} \delta(u) \tag{1}$$

where $\delta(0) = 1$.

For convenience, we now make the discrete expression (1) continuous in time; thus, a change occurs from discontinuous to continuous time. Then $\delta(u)$ is replaced by the continuous version $\exp(-r(u))$. We get

$$PV_D(t) = \int_0^t G(s)q(s)(1 - \tau_1(s))(1 - \tau_2(s)) \cdot \exp\{-\int_0^s r(u)du\}\,ds \tag{2}$$

2.3 Net present value of reinvestments

The net reinvestment gain at time point s equals $G(s) \cdot (1 - q(s)) \cdot (1 - \tau_3(s))$. This is increased by future growth up to time t, where it is taxed by the ratio $\tau_4(t)$ and discounted back to time $s = 0$. With the convention $\tau_4(s) = \tau_4(t)$ for $0 \le s < t$ we get

$$PV_R(t) = G(0)(1 - q(0))(1 - \tau_3(0))(1 - \tau_4(0)) \cdot \prod_{u=1}^{t} \gamma(u) \cdot \prod_{u=1}^{t} \delta(u) +$$

$$+ G(1)(1 - q(1))(1 - \tau_3(1))(1 - \tau_4(1)) \cdot \prod_{u=2}^{t} \gamma(u) \cdot \prod_{u=1}^{t} \delta(u) + \ldots +$$

$$+ G(t)(1 - q(t))(1 - \tau_3(t))(1 - \tau_4(t)) \cdot \prod_{u=1}^{t} \delta(u) =$$

$$= \prod_{u=1}^{t} \delta(u) \cdot \sum_{s=0}^{t} G(s)(1 - q(s))(1 - \tau_3(s))(1 - \tau_4(s)) \cdot \prod_{u=s+1}^{t} \gamma(u) \tag{3}$$

where $\prod_{u=t+1}^{t} \gamma(u)$ is defined as 1.

7

As above, a change is made from discrete to continuous time, and simultaneously $\delta(u)$ and $\gamma(u)$ are replaced by their continuous analogues r and g. We obtain

$$PV_R(t) = \exp\{-\int_0^t r(u)du\} \cdot \int_0^t G(s)(1 - q(s))(1 - \tau_3(s))(1 - \tau_4(s)) \cdot \exp\{\int_s^t g(u)du\} ds \qquad (4)$$

2.4 Net present value of both dividends and reinvestments

Adding formulas (2) and (4) (cf Jonsson *et al.*, 1992) results in

$$PV(t) = \int_0^t G(s)q(s)(1 - \tau_1(s))(1 - \tau_2(s)) \cdot \exp\{-\int_0^s r(u)du\} ds +$$

$$+ \exp\{-\int_0^t r(u)du\} \cdot \int_0^t G(s)(1 - q(s))(1 - \tau_3(s))(1 - \tau_4(s)) \cdot \exp\{\int_s^t g(u)du\} ds \qquad (5)$$

3 Analysis

3.1 Rearranging (5)

Rearranging (5) by assembling all terms that contain the decision function q we get

$$PV(t) = \exp\{-\int_0^t r(u)du\} \cdot \int_0^t G(s)(1 - \tau_3(s))(1 - \tau_4(s)) \cdot \exp\{\int_s^t g(u)du\} ds +$$

$$+ \int_0^t G(s)q(s)(1 - \tau_1(s))(1 - \tau_2(s)) \cdot \exp\{-\int_0^s r(u)du\} ds -$$

$$- \exp\{-\int_0^t r(u)du\} \cdot \int_0^t G(s)q(s)(1 - \tau_3(s))(1 - \tau_4(s)) \cdot \exp\{\int_s^t g(u)du\} ds =$$

$$= \exp\{-\int_0^t r(u)du\} \cdot \int_0^t G(s)(1 - \tau_3(s))(1 - \tau_4(s)) \cdot \exp\{\int_s^t g(u)du\} ds + \qquad (6a)$$

$$+ \int_0^t G(s)q(s)h(s) \cdot \exp\{-\int_0^s r(u)du\} ds \qquad (6b)$$

where $h(s) = (1 - \tau_1(s))(1 - \tau_2(s)) - (1 - \tau_3(s))(1 - \tau_4(s)) \cdot \exp\{\int_s^t (g(u) - r(u))du\}$.

8

3.2 Result

Consider $PV(t)$ as a functional of $q(\cdot)$, all other functions given. The term (6a) is independent of q. Since G(s) and $\exp\{-\int_0^s r(u)du\}$ are non-negative, it is found that $PV(t)$ is maximized by choosing $q(s) = 1$ whenever $h(s) > 0$, and $q(s) = 0$ whenever $h(s) < 0$. Since all τ_i:s fulfill $0 \le \tau_i < 1$, this entails that optimum for an arbitrary time s is obtained by choosing

$$q(s) = \begin{cases} 1 & \text{if } \exp\{\int_s^t (g(u) - r(u))du\} < \dfrac{(1-\tau_1(s))(1-\tau_2(s))}{(1-\tau_3(s))(1-\tau_4(s))} \\[4mm] 0 & \text{if } \exp\{\int_s^t (g(u) - r(u))du\} > \dfrac{(1-\tau_1(s))(1-\tau_2(s))}{(1-\tau_3(s))(1-\tau_4(s))} \end{cases} \tag{7}$$

We have $\exp\int_s^t (g(u) - r(u))du = \dfrac{1 + g_T(s,t)}{1 + r_T(s,t)}$, where $g_T(s,t)$ is the total rate of return on reinvestment during the period s to t and $r_T(s,t)$ is the corresponding total market-required rate of return; taxes not included. The time length t can be chosen arbitrarily but fixed.

3.3 Derivative

Taking the derivative of $PV(t)$ with respect to t (in points of differentiability) we get

$$PV'(t) = \exp\{-\int_0^t r(u)du\} \cdot G(t) \cdot \left[q(t)(1-\tau_1(t))(1-\tau_2(t)) + (1-q(t))(1-\tau_3(t))(1-\tau_4(t)) \right] +$$
$$+ (g(t) - r(t)) \cdot PV_R(t) \tag{8}$$

The first term is the present value of the sum of net dividend and net reinvestment at t. $PV_R(t)$ in the second term is the present value of the accumulated, capitalized net reinvestments up to t (see (4)), $g(t) \cdot PV_R(t)$ is the present value of the revenue of this reinvested capital and $r(t) \cdot PV_R(t)$ is the present value of the cost to keep this capital. At optimum $q(t)$ is either 0 or 1 and the first term is then the present value of either the net dividend ($q(t) = 1$) or the net investment ($q(t) = 0$), in both cases after taxes.

4 Extended model

Now we extend our model to comprise the assumption that the rate of return on reinvestment in period s depends on the size of the reinvestment. To do this $G(s)$ is divided into monetary units ordered from 0 and upwards. The rate of return at time t of the *vth* unit reinvested at time s is denoted $g(s,t,v)$ and the dividend ratio of the *vth* unit at time s is denoted $q(s,v)$.

Then (6) is extended as follows

$$PV(t) = \exp\{-\int_0^t r(u)du\} \cdot \int_0^t (1-\tau_3(s))(1-\tau_4(s)) \int_0^{G(s)} \exp\{\int_s^t g(s,u,v)du\}dvds +$$

$$+ \int_0^t \int_0^{G(s)} q(s,v)h(s,v) \cdot \exp\{-\int_0^s r(u)du\}dvds$$

where $h(s,v) = (1-\tau_1(s))(1-\tau_2(s)) - (1-\tau_3(s))(1-\tau_4(s)) \cdot \exp\{\int_s^t (g(s,u,v)-r(u))du\}$.

In the same way as above it is seen that $PV(t)$ is maximized by choosing

$$q(s,v) = \begin{cases} 1 \text{ if } \exp\{\int_s^t (g(s,u,v)-r(u))du\} < \dfrac{(1-\tau_1(s))(1-\tau_2(s))}{(1-\tau_3(s))(1-\tau_4(s))} \\[4mm] 0 \text{ if } \exp\{\int_s^t (g(s,u,v)-r(u))du\} > \dfrac{(1-\tau_1(s))(1-\tau_2(s))}{(1-\tau_3(s))(1-\tau_4(s))} \end{cases} \qquad (9)$$

If g and r are continuous functions and g non-increasing in v the solution (9) gives an upper limit $v^* = v^*(s,t)$ for optimal reinvestment.

10

5 Random rates of growth and interest

So far all functions are considered as known. Let this assumption be relaxed by assuming the rates of growth and interest to be random variables and that we want to maximize the expected value of the present value $PV(t)$. The same arguments as above lead to the same decision rules (7) and (9), but with the left hand side replaced by its expected value. Since by Jensen's inequality (Feller, 1966) the expected value $E(\exp\{U\}) \geq \exp\{E(U)\}$ for any random variable U it is seen that $q(s) = 0$ more often than expressed by inserting the expected rates in the rules (7) and (9). The expectation is then taken conditionally on the r and g processes up to time s. Thus random rates of growth and interest should increase the retention rate compared to known rates if the expected rates are the same.

6 An example of a practical case

Above we have shown theoretically the "equilibrium" terms on which it is profitable to change from receiving dividend to reinvestment or vice versa, when four different taxes are considered.

In the simple example below, we will calculate the average, annual rate of return on reinvestment – under the above-mentioned terms – during a ten-year period under the following hypothetical terms
 - corporate tax rate on gross profit (τ_1) is 28 %, when dividend is received,
 - personal income tax rate on gross dividend payout (τ_2) is 30 %,
 - corporate tax rate on gross profit (τ_3) is 20 %, when reinvestment is made,
 - capital gains tax rate (τ_4) is 20 %, and
 - annual, market-required rate of return is 5 % during the period,
 - annual rate of return on reinvestment is g % during the period.
According to the discrete version of the solution (7), we get the breakpoint between $q = 0$

and $q = 1$ when $\dfrac{1.0g^{10}}{1.05^{10}} = \dfrac{(1-0.28)\cdot(1-0.30)}{(1-0.20)\cdot(1-0.20)}$, which gives $g = 2.521...$

In this case, the average, annual rate of return on reinvestment is 2.5 per cent during the period, thus equivalent to a market-required annual rate of 5 per cent.

7 Conclusion

Under the assumptions that the sum of funds used for dividends and reinvestments is equal to the profits from operations, we find from (7), that the value of the shareholder's wealth (i.e. equivalent to the value of the firm) is affected by dividend policy in a world both with and without taxes. Dividend/reinvestment policy is irrelevant in this respect only if the ratio $\frac{1+g_T(s,t)}{1+r_T(s,t)}$ is equal to the following ratio of tax rates $\frac{(1-\tau_1(s))(1-\tau_2(s))}{(1-\tau_3(s))(1-\tau_4(s))}$; $g_T(s,t)$ is the total rate of return on reinvestment during the period s to t and $r_T(s,t)$ is the corresponding total market-required rate of return.

The dividend policy theorem by Miller *et al.* (1961) states that, in perfectly competitive capital markets, the value of a firm is independent of its payout choice between dividends and retained earnings. Findings by Elston (1996) are broadly consistent with them and with Fama (1974), which supports the perfect capital markets view by finding insufficient evidence to reject the notion that a firm's investment and dividend policies are independent.

Gordon (1994), however, emphasizes that in the real world dividend policy is investment policy and one of them cannot be varied independently of the other.

We find from (7) that it is advantageous for shareholders to receive dividend payout at time s if and only if the ratio $\frac{1+g_T(s,t)}{1+r_T(s,t)}$ is less than the ratio $\frac{(1-\tau_1(s))(1-\tau_2(s))}{(1-\tau_3(s))(1-\tau_4(s))}$ of tax rates. Otherwise, it is better to reinvest.

However, in the case of random rates of growth and interest, the retention rate should be increased compared to known rates of return, if the expected rates are the same.

By equating the derivative PV(t) (see (8)) to zero the relationship between the included components at the time of maximum net present value is determined. Here we assume that the present value, as a function of time, has a unique maximum. Then, the time for maximum net present value occurs when the sum of the net dividend and the net reinvestment is equal to the difference between the cost to keep the accumulated net reinvested capital and the revenue of the same capital.

References

Brennan, M. (1970). Taxes, Market Valuation and Corporate Financial Policy. *National Tax Journal* 23, 417-427.

Elston, J. A. (1996). Dividend and Policy and Investment: Theory and Evidence from US Panel Data. *Managerial and Decision Economics 17*, 267-275.

Fama, E. (1974). The empirical relationship between the dividend and investment decisions of the firm. *American Economic Review* 63, 304-318.

Farrar, D. and Selwyn, L. (1967). Taxes, Corporate Financial Policy and Return to Investors. *National Tax Journal* 20, 444-454.

Feller, W. (1966). An Introduction to Probability Theory and Its Applications. Volume II. John Wiley & Sons, New York. 626 pp.

Gordon, M. (1959). Dividends, Earnings, and Stock Prices. *Review of Economics and Statistics* 41, 99-105.

Gordon, M. (1962). The Savings, Investment and Valuation of a Corporation. *Review of Economics and Statistics* 44, 37-51.

Gordon, M. (1994). *Finance, investment and macroeconomics: The neoclassical and post Keynesian solution.* Aldershot, U.K.: Elgar; distributed in the U.S. by Ashgate, Brookfield, Vt. 1994, xiv, 213 pp.

Jonsson, B. and Jonsson, T. (1992). Optimal Harvesting Time in Single- and Multiple-purpose Forest Management. *Scand. J. For. Res.* 7, 571-574.

Lintner, J. (1956). Distribution of Incomes of Corporations among Dividends, Retained Earnings and Taxes. *American Economic Review* 46, 97-113.

Miller, M. and Modigliani, F. (1961). Dividend Policy, Growth and the Valuation of Shares. *Journal of Business* 34, 411-433.

Miller, M. and Scholes, M. (1978). Dividends and Taxes. *Journal of Financial Economics* 6, 333-364.