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The Forest Management Planning Package

Theory and application

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Abstract

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The Forest Management Planning Package represents a general fundamental structure of a forest management planning system based on two inventory phases. It is an existing calculation system used in practical forestry in Sweden.

The Forest Management Planning Package integrates economic theory, objective inventory measurements and accurate growth forecasts. The core of the system is a chain of models depicting the production possibilities of a forest holding.

Detailed growth forecasts and economic calculations with high resolution (individual trees) permit analysis of various silvicultural treatment options in all types of Swedish stands.

A non-linear objective function and mathematical optimization result in a compromise between maximum net present value and sustained net-revenue profile. Application of the system contributes to a much improved economic result through the removal of uncertainties concerning the real production possibilities and it has significantly altered the management strategies of the forest companies that have implemented it.

Key words: bioeconomics, forest management planning, operative planning, strategic planning, forest inventory, long-term forecasting, timber assessment calculation, optimization, sustained yield, non-linear programming.

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Preface

In this publication we present an economically effective resource management system for practical use that focuses mainly on the forest timber resource. The system is based on forest economic theory going back to the early 1800s. However, as Clark (1989) puts it, interest in renewable-resource economics has increased in recent years. A new phrase, sustainable development, has come into popular use. The phrase clearly indicates a concern with conservation for the long-term benefits of humanity.

Modern resource management is thus concerned with the economics of sustainable use of biological resources. It is important to find practical application for these bioeconomic principles, if we wish to realize the vision of sustainable development.

In accordance with this, we have in our system formulated a workable objective function, which is a compromise between basic economic principles of net present value maximization and sustainable development. In Swedish forestry practice decisions have been taken intuitively in accordance with this objective function since the early 1900s. Our planning system lends support to a selection of management activities that reflects great responsibility for both the present and the future.

The development of the planning system presented here started in the early 1970s. The system comprises the whole forest management planning process, and its core is the integration of forest inventory, forecast and optimization methods. It has been in practical use here for about a decade, and has had quite an effect on Swedish forestry. We continue to develop the system in response to future experiences and needs; emphasis is put on operational planning and on widening the scope of the objective function to include amenity services.

Introduction

Forest management planning in a general planning context

If we wish to extract large quantities of wood from the forest, and at the same time allow large quantities of wood to remain, we have a forest management problem. The need for forest management planning is born out of a conflict between the present and the future, i.e. the problem of sustainable development. In a world with unlimited resources, there are no such conflicts and thus no forest management problems.

Planning, to an great extent, is to shape the future. According to the rational theory of planning (Simon, 1976), a decision-maker must be able to envision the consequences of different options before making decisions. These options can then be ranked with regard to their desirability.

By planning, we imply a process that guides our actions to the results we most desire. Planning comprises both normative as well as technical aspects. We must articulate our values and let them guide our actions in a rational ("goal-oriented") way.

By plan, we mean a representation of a treatment option in which treatments and consequences are described. A treatment option is in this context a sequence of treatments over time, applied to a whole forest or a part of a forest.

In some simple situations, it might be possible to articulate explicitly our "innermost desires", i.e. our values. We call such a representation of values an objective function. This function is defined with the set of possible treatment options as a domain, and rates these options unambiguously with regard to the desirability of their consequences.

A decision-maker cannot normally be expected to articulate his or her values explicitly without particular preparation. A form of planning denoted "strategic planning" may then be employed. This is a search process in which the characteristics of desired options successively are identified. The result of a strategic planning effort is two-fold: a concrete picture of what it is that the decision-maker wishes to accomplish, i.e. the goal, and guidance in the form of rules and other necessary means of reaching the goal.

In a planning process, some form of model is often required (Dykstra, 1984). Planning is largely a matter of construction and analysis of realistic models. To be able to analyse a problem that in reality is very complex, it is necessary to simplify. In a model, only the portions of reality relevant to the problem are described. To make meaningful analysis possible, this model must depict all components of reality that are important to the problem, as well as their interaction. The effect of less important components is depicted in a summary but correct fashion.

After careful consideration, the model-builder must decide which components are important to the problem and determine how they influence it. One often encounters different models built for the same problem. By assigning the components different weights and kinds of interaction, one arrives at models which are different approximations of reality.

Thus, the model is a simplification and approximation, but it can nonetheless be of considerable complexity and size. To facilitate the solution of complex problems, a model can be broken down into smaller parts that function together in a process. The problem of finding the best solutions in time and space for forestry is so complicated that the model must be relatively complex to be useful.

For a long time, forest management planning had to be confined to the use of simple, intuitive and "manual" models. This restricted the analyses, in the main, to the level of annual cut. Such simple models are still being used. The development of the forest, for instance, is depicted by the progression of age classes. Level of annual cut is determined using rotation calculations that form a basis for choosing the area to be cut annually, hence also for the volume of the cut.

However, powerful computers, improved measurement techniques and advanced sampling theory have made it possible to utilize more complex and realistic models.

Forest management implies carrying out treatments in complex production systems. The purpose of forest management planning is to provide a basis for the allocation of these treatments, so that the desired result can be obtained.

To do this, methods are required for (cf. Jonsson, Holm & Kallur, 1992):

1. formulating normative management goals, i.e defining the results which one wishes to achieve (goal formulation);

2. finding effective treatment options which will produce the desired results, i.e. optimizing the choice of management activities (optimization methods);

3. describing the outcome of treatments in the production system, i.e. predicting the result of various management activities (forecasting methods);

4. charting the production system, i.e. surveying the forest to obtain a basis for making predictions (inventory methods).

Together, these parts form a forest management planning system.

The problem of forest management planning can now be formulated in the following way: For a forest holding, a set of possible treatment options (sequences of treatments over time) exists. The problem is to select the treatment option that maximizes utility in some sense.

Arguments for a quantitative approach to forest management planning

The need for goals

In order to act consistently and effectively, it is necessary to define the results which one wishes to achieve – i.e. to formulate goals.

Concrete and operative goals for timber-production are formed at the crossroads between knowledge of the physical results that are possible to achieve, and our assessment today of the market-value of these results. A major part of the results in forest management become visible in the distant future. Long-term forecasts of the outcome of different options of action are therefore a natural component of strategic management planning.

The primary goal of such forecasts is to define the limits of possible yield, not to predict the future. The other natural part of strategic forest management planning is to assess the value in economic terms of those possible outcomes.

These two search processes – definition of the yield potential and assessment of the value of these outcomes – are both necessary to the formulation of specific goals for the guidance of forest management.

Our experience from application of the forest management planning system presented in this paper, is that the decision-maker's consciousness of strategic problems increases through analysis. Insight into relationships and problems is gained that might otherwise have been overlooked. The system is called the Forest Management Planning Package (FMPP).

In forest management, the goal is to achieve the highest possible sustained yield. The way in which this yield is measured determines the real content of this formulation. Over the years, many suggestions for yield measures or formulations of goal in forest management have been made. Our experience indicates that it is worth paying attention to some yield measures of interest (Table 1; cf. Jacobsson & Jonsson, 1991).

Net present value is a general yield measure, which subsumes all the other measures shown in Table 1. The difference between these ways of measuring goal-accomplishment lies in the way in which we express revenues, costs and interest rates in the calculation of net present value.

This is the reason why we persist in advocating the use of present value calculation as a logical form for the choice of forest management

Table 1. *Different yield measures or formulations of goal in forest management*

Goal	Economic production factors considered
I. Highest production of wood – interest rate is zero – all cubic metres have equal value, regardless of whether or not they are utilized – no costs	LAND
II. Highest timber yield – interest rate is zero – all utilized cubic metres have equal value – no costs	LAND
III. Highest yield of value – interest rate is zero – prices and costs are considered	LAND LABOR
IV. Highest net present value – all factors above are considered	LAND LABOR CAPITAL

program. The fact that this invites the use of other values than zero for costs and interest rates is something that we believe is of benefit to forest management.

The challenge of uncertainty

We have often met the objection that all statements about the value of the future yield of timber are meaningless. After all, nothing can be known about the future. This would render economic theory and the present-value calculation useless for solving forest management problems.

If one is not allowed to use economic theory to solve forest management problems, however, one is left helpless. The reason for this is that economic theory is the general tool that humanity has created for husbanding scarce resources.

The forest management problem does not differ in principle from the general problem of resource management. The production time is longer, but a long production time is no problem in principle, as long as there are expectations of the world continuing to exist.

We know with certainty that our actions today influence the size of the potential future timber harvest, as well as the structure and quality of this harvest. Even if we today do not wish to express or are not able to express, any opinions of the value of the future timber harvest, we shall still indirectly take a standpoint through our current actions.

By attempting to express an opinion about the future in economic terms, and then relying on net present-value calculations to formulate a strategy in accordance with this opinion, we shall be able to act consistently and effectively. Thus, it is important to have a vision of the future. Without an opinion of the value of the future forest, we shall simply be groping in the dark.

Optimization or not?

A timber assessment is a means of linking activities and expected results in forest management. Timber assessments have always been an important aid in considering forest management problems.

In principle, there are two different ways in which a timber assessment can be used in strategic planning. One of the possibilities – consequence assessment – involves an attempt to forecast what will happen if the forest is managed according to certain specified programs. The outcome of the assessment is not primarily used to evaluate whether the specified program is a good solution to the forest management problem. One simply assumes that the program is good.

The alternative to consequence assessment is optimized timber assessment. Such an assessment involves systematically investigating the consequences of a large number of different options of managing a forest, then evaluating the results in reference to a clearly specified evaluation norm. If the set of treatment options becomes very large, one is forced by time constraints to be systematic in generating and evaluating the options. Mathematical optimization methods are an aid in this work.

An important element in all timber assessments – optimized or not – is to ensure that the results of the formulated treatment options of action are feasible, i.e. assessible.

The sustained net-revenue profile approach

If the elements of the goal are periodical net revenues, then the efficient solutions to forest management problems – i.e. the solutions which lie on the outer boundary of what forest management can yield – can be found with the help of simple net present-value calculations. Figure 1 shows how it is possible to use the net present value to separate solutions which lie on the outer boundary of the management possibilities offered by the particular forest.

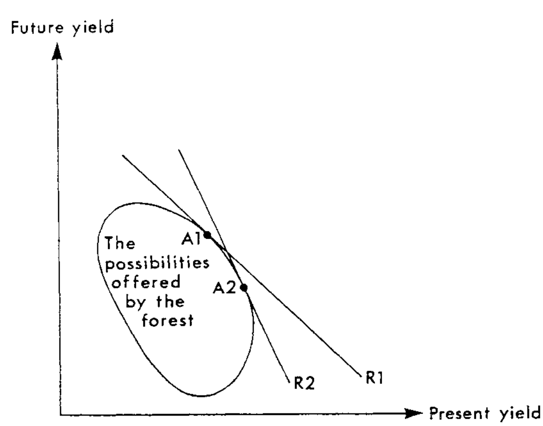

Fig. 1. The possibilities of forest management and the efficient solutions.

The choice of interest rate, which is represented in the figure by lines $R1$ and $R2$, determines the balance between present and future yield – what we call the profile. A low interest rate $R1$ gives more in the future and less today. A high interest rate $R2$ gives more today and less in the future.

If we were to turn this around, and instead first select which of the solutions we wanted – $A1$ or $A2$ – that is, first select the profile, then it is possible to establish afterwards, which rate of interest this solution indirectly builds upon.

In Figure 1, the problem has been simplified to cover only two time periods. The same principles apply to a real problem, however, with many time periods or continuous, unlimited time.

By using the decision-maker's desires concerning the shape of the net-revenue profile as a starting point, it is possible to reduce the problem of choosing the interest rate. We thus consciously lose a minor fraction of the theoretically present value, as calculated with a fixed and known interest rate. However, since the true level of the future interest rate is unknown, the question of whether we lose or gain in the end is not answered.

It is generally much easier to obtain information from the decision-maker concerning the desirable shape of the net-revenue profile than concerning which interest rate to apply. Thus, we have concluded that the profile approach is a useful technique in the context of real world decision-making, where fixed and known interest rates are nonexistent. The approach based on a sustained net-revenue profile makes it possible for decision-makers to formulate goals which are compatible, if not in all aspects, at least in the most important, with sound economic theory.

Modelling forest dynamics

The primary production of forestry, i.e. the production of harvestable trees, has two important dimensions: space and time. The process is regulated with the help of various measures, such as planting, thinning and final fellings. A meaningful regulation of this process is only possible through an overview and understanding.

Objective sampling schemes allow us, at a reasonable cost, to create a realistic state-description of the process in space, i.e. the state of the forest today. To obtain an overview of the primary production process, we must have methods at our disposal for forecasting the size, structure and quality of the growing stock over time. The basis for any meaningful optimization is that the development of the forest, as a result of different treatment options, can be described with a reasonable level of precision. We are not concerned with the development of idealized type-forests here, but rather with the actual forest we possess today. If we cannot manage this, optimization is meaningless at best, and at worst directly harmful.

Analysis or prejudgements

Much would be gained if a clear distinction were made between restrictions caused by external factors and self-imposed "good forestry rules". The restrictions caused by external factors are real, and do determine the boundaries of what is possible. The most important of these external boundaries is set by the growth of the forest. Self-imposed rules, however, must be questioned and re-examined when they place obstacles in the way of good economic results for forest management.

The traditional formulation of forest management programs, in terms of rules and restrictions, must for reasons of simplicity be founded on a number of typical stands and stand developments. The programs are illustrated by one or several norms regarding the appearance of stands at different ages. An attempt is made in the planning process to mold the real-life state of the forest to the norm. The problem is that this may correspond to an economically unde-

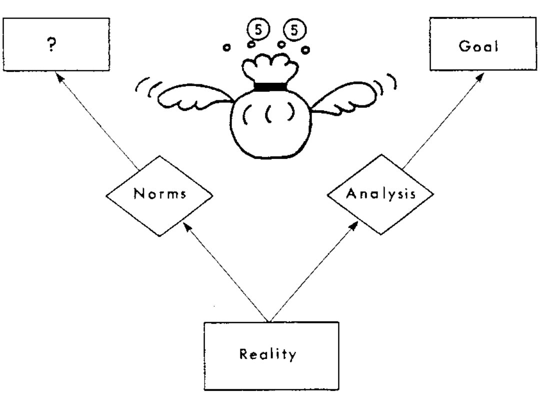

Fig. 2. Norms for how the forest should look may correspond to an economically undefined goal.

fined goal (Figure 2) and will therefore be a very expensive behaviour.

An effective way of reaching the goal is to analyse the actual development potential of the stands in relation to the entire forest holding, with reference to economic conditions. A possible result of this is that we may have to accept a state of the forest which deviates from the norm for a longer time than would otherwise have been necessary. However, the aim of forest management is to fulfill economic goals, not to ensure, at any price, a norm for how a stand ought to appear.

Thus, when we have assumed the economic conditions (prices and interest rate) and formulated an operative, measurable goal, then there exists a theoretically true optimal treatment program for a specific forest area. This means that the compartment structure is determined and that the management plan for the forest within these boundaries has been updated as far as treatment options are concerned.

It is now our task

– to try to discover the optimal treatment program through analysis;

– to avoid forcing our conception of a well-managed stand – based on norms – upon the forest.

Application of the Forest Management Planning Package is an iterative search process, in which the results prompt re-evaluation of the assumptions, which in turn will lead to new results, and so on (see Figure 3). In the end, this search process leads to a treatment option close to the optimal one.

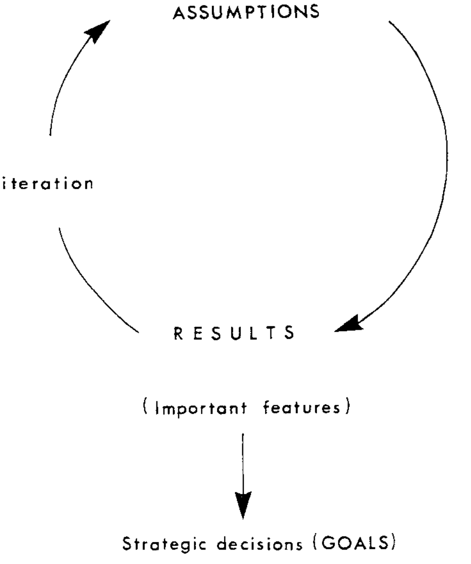

Fig. 3. Strategic planning is an iterative search process.

Our goal and model

Utility, net present value, and sustained net-revenue profile

In the literature of forest management planning, utility is mainly connected with present value (e.g. Faustmann, 1849; Dykstra, 1984; Johansson & Löfgren, 1985; Holten-Andersen, 1991) and sustained yield (e.g. Faustmann 1849; Samuelson, 1976; Johnson & Scheurmann, 1977; Johnson, Jones & Kent, 1980; Haig & Krutilla, 1985).

We have concluded that a treatment option which generates a high degree of utility is a compromise between:

– *high net present value, i.e. high net value of present-time equivalents of revenues and costs in a wide sense; and*

– *a reasonable distribution of net revenue over time (sustained net-revenue profile).*

Reasons for desiring a sustained net-revenue profile might include:

1. An assumption that forestry, being a basis for many activities in society as well as for the forest owner, best serves its purpose by means of a continuous and sustained (relatively speaking) yield of value (Brundtland, 1987). A sustained net-revenue flow from forestry would in other words have a value of its own, derived from its stabilizing effect. We may call this the "sustained-yield argument".

2. An assumption that a number of factors, represented either schematically or not at all in the model, would change the solution towards greater smoothness if they were included in the model. Important such factors include prices and interest rates, whose magnitude in reality may depend on the magnitude of the entity to which they are applied (Walker, 1971; Johnson & Scheurmann, 1977; Lappi & Siitonen, 1985). Even if the model were refined, inadequately depicted factors will always remain. We may call this the "crude model argument".

The theory of forest economics is largely concerned with the treatment of an idealized normal or steady-state forest, which among other things is characterized by an even distribution of area over age-classes. In managing an optimal normal forest, there is no conflict between high net present value and sustained net revenue. In reality, however, one seldom or never encounters normal forests. A useful model for strategic planning must permit the relationship between net present value and sustained net-revenue profile to be studied. One way of allowing the aspect of sustained net-revenue profile to be accounted for in a model, is not to discount net revenue directly, but rather to discount a function of net revenue. A distinction is made between net revenues and utility in a single time-period. In the following, the rationale behind this approach will be discussed.

If a decision-maker prefers a smooth net-revenue profile over time to an uneven profile, he will gain if he transfers net revenue from periods with high net revenue to periods with low net revenue. This, in turn, indicates that the marginal utility of the net revenue decreases (Figure 4).

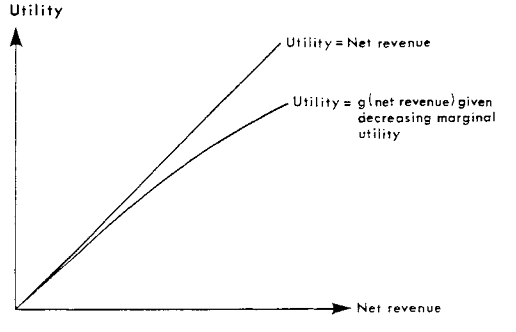

Fig. 4. Schematic relationship between utility and net revenue per annuma

A reasonable assumption regarding the relationship between utility and net revenue is that it is a homogeneous function; in other words, an increase in net revenue of x per cent will increase utility by y per cent, regardless of the magnitude of the net revenue. It is also assumed that the relationship between utility and net revenue can be depicted as a continuous concave function.

A simple function that fulfills these assumptions is

$$u(N) = c \cdot N^b,$$

where

c is a constant and $c > 0$

b is a constant in the interval $0 < b \leq 1$

N is the net revenue.

The function $u(N)$ is furthermore the solution to the differential equation

$$u'(N) = b \cdot \frac{u(N)}{N}.$$

This means that the marginal utility of the net revenues, calculated in this way, is proportional to the mean utility of the net revenues.

If the magnitude of b is less than 1, marginal utility is strictly decreasing. If the function stated above is used in a present-value calculation (and b is less than 1), the value of options with an even flow of net revenue over time will be positively affected. The smaller the magnitude of b, the more pronounced this effect will be.

The decision-maker can study the balance between the goals of smoothness and high net present value by changing the magnitude of b.

Every treatment option $H \in A$ can be assigned the net revenue flow it creates $N(t)$, i.e. net revenue flows over time t; "A" denotes the set of possible treatment options. Our task is to find H so that utility is maximized. Thus, the problem of forest management planning can be given the following mathematical formulation:

$\max_{H \in A} U$ where

$$U = \int_0^\infty e^{-rt} \cdot u(N_H(t))\, dt = \int_0^\infty e^{-rt} \cdot c \cdot [N_H(t)]^b dt; \qquad (1)$$

U denotes utility

u denotes utility function

t denotes point in time

r denotes rate of interest

$N_H(t)$ denotes net revenue of treatment option H at time t.

We shall return to these questions when we build our model.

Our model

We shall be using a deterministic model, in which the state and the development of the forest (the production system) at a point s in space and at time t are determined solely by the initial state $I(s,0)$ and the treatment option $H(s)$. The treatment option $H(s)$ is a sequence of treatments over time that is to be determined in advance and applied to a point s.

The state of the forest at time T can be written

$$I(s,T) = I(s,0) + \int_0^T g(H(s), I(s,t))\, dt$$

where g is a function describing the development of the forest state.

A general model is:

$$U = \int_0^\infty e^{-rt} \cdot c \cdot (N_H(t))^b\, dt = \int_0^\infty e^{-rt} \cdot c \cdot \left\{ \int_S \int_X a(x,t) \cdot f[x,t, H(s), I(s,t)]\, dx\, ds \right\}^b dt, \qquad (2)$$

where

S denotes area of the forest holding

x denotes type of product (output) or means of production (input)

$a(x,t)$ denotes function yielding the price of x at time t

f denotes function of treatment option and state of the forest; the function gives the yield of output x or the consumption of input x at time t;

$H(s)$ denotes treatment option applied to the point s over time

$I(s,t)$ denotes state of the forest (production system) at point s and time t.

This general model will be developed into a model applicable in practice. It constitutes the core of a system for planning primary forest production (timber production), the so-called Forest Management Planning Package (Jonsson, 1978, 1982).

The first step in this development is that the continuous time-scale is approximated with a discrete scale using 5-year intervals. Furthermore, we introduce discrete x-values. All treatments and results in the form of products – which in reality occur at any time during a 5-year period – are now assumed to be concentrated to the middle of the period (t_p).

The model now takes the form

$$U = \sum_{p=1}^{\infty} e^{-rt_p} \cdot c \cdot \left\{ \int_S \sum_x a_{xp} \cdot f[x, t_p, H(s), I(s, t_p)] \, ds \right\}^b, \quad (3)$$

where

p denotes a 5-year period; $p = 1, 2, \ldots$

t_p denotes $t_0 - 2.5 + 5\,p$

The yield of net revenues from the forest holding during a period p is represented by the integral in this expression.

We can imagine that the holding is divided into a large but finite number of small area elements (plots). The yield of timber from these plots is the sum of the yield of all trees that are or will be established on them. The size of the plot is chosen to reflect the fact that trees grow individually under the influence of their immediate surroundings. The effect of external factors that affect growth is approximately the same for all trees on a plot, if the plot is sufficiently small. Such factors are, for instance, competition between trees and site quality.

For reasons of operational economics, a small plot should be treated at the same time as other, adjacent plots. For this reason, we introduce an additional spatial concept: the compartment, which thus is a treatment unit (see section "Phase 1", p. 26). It is constituted by a number of adjacent plots.

In the procedure for estimating the state and the future development of the forest, we use a sample of compartments from the forest holding and a sample of plots within the sampled compartments.

Our task is to find H so that U is maximized, where

$$U = \sum_{p=1}^{\infty} e^{-rt_p} \cdot c \cdot \left\{ \sum_{i=1}^{M} \sum_{j=1}^{N_i} \sum_{x=1}^{k} a_{xp} \cdot f(x, t_p, H_i, I_{ijt_p}) \right\}^b; \quad (4)$$

i denotes compartment

M denotes number of all compartments

j denotes plot

N_i denotes number of all plots within compartment i

H_i denotes treatment option for compartment i; H for the whole forest is built up by the combination of the single H_i

I_{ij} denotes state of the forest within the plot j in compartment i.

We estimate the optimal H by maximizing

$$U = \sum_{p=1}^{\infty} e^{-rt_p} \cdot c \cdot \left\{ \sum_{i=1}^{m} q_i \sum_{j=1}^{n_i} \sum_{x=1}^{k} a_{xp} \cdot f(x, t_p, H_i, I_{ijt_p}) \right\}^b; \quad (5)$$

where

m denotes number of sampled compartments

n_i denotes number of sample plots within the sampled compartment i

q_i denotes projection factor for sample plots within compartment i (depending on the sampling method)

When the function $u(N)$ is implemented in the model, the constant c, which has no significance in the optimization, can be given the value 1.

Contents of the model

Net revenues

For the time being, the Forest Management Planning Package covers only that part of forestry which focuses on timber production. Other functions of forestry, such as the production of recreation possibilities, environmental protection, berries, wildlife, waterflow, etc., are assumed to be independent of the design of timber production. The products x which are included in the model refer only to input and output of timber production. In principle, however, different levels of amenity services can be analysed by studying the costs in terms of reduced value of timber production.

The timber production process can be divided into two sub-processes:

– The primary production process, having resources for silviculture as input and trees mature for harvesting as output;

– The secondary production process, having mature trees as input, as well as resources for logging, transportation, storage, and sales. The output is timber products at permanent processing facilities.

The Forest Management Planning Package builds on the schematic assumption that the design of the secondary production process is given and fixed when the trees are delivered from the primary production process.

This means that the primary production process is optimized, given an assumed technological solution of the secondary production process.

Thus, given a treatment option *H*, the flow of products and means of production between forestry and its ambience can be divided into:

- Means of production for primary production – silviculture;
- Means of production for secondary production – logging, transportation, storage, and sales;
- Products in the form of timber from trees that have been logged, transported, stored, and sold.

Revenues in forestry are composed of revenues from the harvested trees. Costs consist partly of the costs of the primary production, which depend on the silvicultural program, and partly of the costs of the secondary production, which depend on the size and structure of those trees that are delivered from the primary production process.

Given knowledge of the technology used in the secondary production process, the costs of this process can be estimated. These costs are deducted from the revenues generated by the sales of timber from the harvested trees. This results in a net value (stumpage value) from the harvested trees that is taken as a revenue of the primary production process. Thus, the revenue side of the model consists of stumpage values, while the cost side consists of silvicultural costs. The difference between these revenues and costs during a certain period constitutes the net revenue for that period. This is the sense in which the term *net revenue* is used in this work.

It might be of interest to include logging in the model if different logging methods vary in their effect on primary production (e.g. different methods of thinning). If such is the case, the costs of these particular methods should be taken into account in the calculation of net revenues.

Forest inventory methods

The forest inventory section of the Forest Management Planning Package is founded on both "guesstimation" and measurement.

Matérn (1978) submits the following regarding "guesstimation" and measurement (translated from Swedish): "It may perhaps sound like a contradiction if I now say that there is great room for subjective methods, guesstimation, 'eyeballing' in forest inventory. But observation by 'oculation' alone is not sufficient; it must be complemented with data that allow us to 'translate' them into measurement results."

For reasons of cost, it is not feasible to measure all compartments, nor to measure all plots within chosen compartments. We must resort to approximations, and do so by using a stratified sample of compartments, in which several circular plots are measured (Jonsson, 1978, 1982). The frame from which the stratified sample is chosen is a register of all compartments.

In the context of the Forest Management Planning Package, a compartment register consists of data generated by subjective methods of inventory. These data are translated into measurement values with stated precision, based on objective measurements of a small sample from this register. The sample is selected by means of some probability procedure.

Such a procedure is known in statistics as two-phase sampling or double sampling (Cochran, 1977). It can be extended and applied on several levels; we then speak of multi-phase sampling.

The Forest Management Planning Package uses two inventory phases. The first phase consists of creating a compartment register. The second phase consists of a sample of compartments that are inventoried according to a "basic

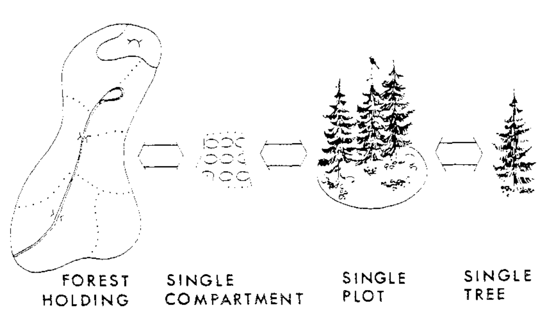

Fig. 5. The representation of the forest in the Forest Management Planning Package.

method for circular-plot inventory". This method is designed as a sample of plots within the sampled compartment, and all individual trees on the sample plots are recorded (Figure 5).

Only the measured sample plots from the sampled compartments are used in our model. These are taken to represent non-measured parts of sampled compartments, as well as compartments not measured at all. If the sample is allocated in an efficient way and is sufficiently large, the resulting picture of the real forest holding is an approximation that is useful for strategic analyses.

A need for more extensive in-place information arises only in planning of a more operative nature. By matching the subjective data in the compartment register with objective data and analysis results from the strategic planning based on the second phase, it is possible to create a basis for drawing general conclusions about the state and appropriate treatment of all compartments.

This two-phase procedure has many advantages in a planning context:

– It is efficient in the sense that it estimates means and totals with a certain precision for the forest holding at lower cost than any other known method (see Li, 1988).

– It uses the compartment as an inventory unit. This unit is of fundamental significance in all forest management planning, and having the inventory unit coincide with the relevant treatment unit is a great advantage.

– Individual values for compartments that have only been estimated subjectively can be calibrated (Jonsson & Lindgren, 1978; Li, 1988). This removes the effect of systematic errors in the subjective estimates.

– It yields detailed information with high resolution about the sampled compartments; this information is necessary for making good forecasts for these compartments, as well as for whole forest holdings. This, in turn, is a necessary condition if optimal solutions to the modelled problem are to resemble optimal solutions to the real problem.

– Treatment decisions founded on very good measured data can be related to the simple data actually available for non-measured compartments. These relations between decisions and compartment data can be established with regard to the weaknesses and strengths inherent in the data material available for non-sampled compartments.

Growth calculations

The need to make forecasts of forest development as far ahead as a hundred years makes the dynamic part of the Forest Management Planning Package, i.e. growth calculation, its most sensitive element. Great care is required, since there is a significant risk of going wrong and thereby producing misleading results.

How should growth forecasts be made? Assume a compartment consisting of a number of plots and a number of trees on each plot. Our compartment can then be described using compartment means for, e.g.,

- site quality
- age
- diameter
- number of stems per ha
- species composition.

The compartment can also be characterised using the variation among plot means concerning

- site quality
- age
- diameter
- number of stems per ha
- species.

Moreover, there is variation among trees within plots concerning

- age
- species
- diameter.

There are several possible methods of making growth predictions:

1. On each plot, site quality and individual trees are observed with regard to diameter, age, and species. In this case, functions that depict the growth of individual trees can be used. All information about variation is then accounted for (Näslund, 1942; Jonsson, 1974*a*, 1980; Söderberg, 1986).

2. Based on the same observations as in case 1, it is also possible to use functions that depict the collective growth of all trees on a plot (Ekö, 1985). In this case, available information about variation among trees within plots is not utilized.

3. Information from the same observations as

in case 1 can be used for calculation of compartment means. Functions depicting collective, plotwise growth are then used as in case 2. In this case, no information about variation within or among plots is utilized. Moreover, the functions are being used outside their domain of validity.

4. Compartment means can be determined using data derived from subjective methods for measurement of basal area (use of a relascope at subjectively chosen points in the stand), height, age, etc., and site quality. Functions depicting collective, plotwise growth are then used. In this case, no information about variation within or among plots is utilized. The functions are used outside their domain of validity on data that contain unknown errors of a more or less systematic nature (Larsson, 1990).

5. Compartment means are estimated as in case 4, and the results are aggregated to larger prediction units, such as age classes. Functions depicting collective, plotwise growth are used. In this case, more information remains unused than in case 4.

Several increasingly schematic cases could be outlined that would distance us even further from the simple fact that trees grow individually while interacting with each other, not as a more or less unstructured collective.

As a consequence of this, the Forest Management Planning Package has been designed to predict growth by utilizing information with the highest degree of resolution, such as in case 1 above (Figure 6). The development of efficient measuring instruments, e.g. the electronic data caliper (Jonsson, 1981, 1991), has made this approach practically feasible.

The individual-tree concept allows for computations to be traced and assessed for feasibility at all steps (Figure 6). Computer printouts can be requested showing growth during the forecasting period for:

- single trees
- single plots
- single compartments
- forest holdings.

Prices

A weakness in all long-range planning is the uncertainty pertaining to future prices of products and means of production. This difficulty notwithstanding, it is necessary to specify reasonable values for these prices. Not to do so would be tantamount to either assuming goals independent of prices or to abstaining from planning.

Uncertainty may be explicitly handled in a forest management planning system if future prices follow a known distribution (Lohmander, 1987; Gong, 1991). However, the complexity of such modelling and uncertainty concerning the distributions describing the uncertainty, has made us choose a deterministic approach to future prices.

Deterministic price changes, i.e. different values of a_{xp} between periods, can be handled

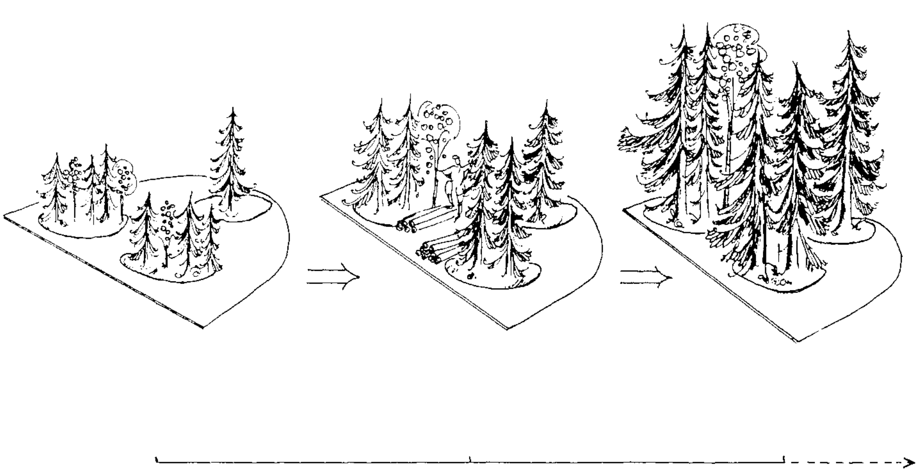

Fig. 6. Illustration of the structure and the resolution of the timber assessment calculation in FMPP. Individual tree growth on sample plots within a compartment.

by our system, but the basis for their estimation is most often weak.

Real rate of interest and real prices

When using the Forest Management Planning Package for analysing treament programs, it is recommendable to employ real rates of interest and real prices. This is in accordance with the following statement by Samuelson (Samuelson, 1976, p. 475): "This means that essentially all we need in order to discuss forest economics correctly is to concentrate on (1) the real rate of interest (i.e., the actual interest rate on money minus the presumed known rate of overall price inflation), and (2) the real price of lumber outputs and inputs (i.e., the percentage real rate of rise for $P_{lumber}/P_{general}$)".

We have followed this recommendation in this paper.

Principles of optimization

Optimization consists of achieving a desired profile of net revenues over time, and of positioning this profile as high as possible. As has been observed earlier, this profile is a compromise between high net present value and sustained net-revenue profile. The decision-maker can select the desired profile by varying r and b in the model.

An algorithm has been developed which is capable of giving solutions to the model, i.e. finding the combination of treatment options for the compartments that maximizes the objective function for the entire forest holding and thus positions a desired profile at the highest level. One limitation, however, is that all treatment options cannot be studied, since their number in principle is unlimited. For that reason, the algorithm is applied to a large but limited set of treatment options that has been formulated by the decision-maker, and which hopefully includes an option close to that which is theoretically the best. A great deal of knowledge about forest management is required to formulate these options.

The production possibilities

Products, means of production, and their prices

The backbone of our model is made up of estimations of potential timber cut. A potential timber cut calculation produces an estimate of the flow from and to the forest of products and means of production in all periods, given an initial state I and an option of treatment H. The forest is represented in model (5) by small, sample plots within sampled compartments.

The estimation of potential timber cut is essentially based on methods developed by Jonsson (1974*a*, 1974*b*, 1980) and implemented by Söderberg (1986).

Products and their prices

The product of the primary production process is trees which are ready for harvesting. Thus, in this context x refers to a number of tree characteristics. The price a_{xp} for a tree delivered from the primary production process is equal to the stumpage value. This value can be calculated as the difference between the combined revenue of all components of the tree having passed through the secondary process and the costs which can be attributed to the tree in this production process (logging costs, etc.).

Given a set of product prices, this difference, i.e. the price a_{xp} for a tree depends on

- tree species
- tree size
- tree quality
- tree concentration
- location
- other circumstances that affect costs in the secondary production process.

Thus, to estimate the stumpage value, we must first predict species, tree size, and tree quality at different points in time. This prediction is based upon initial state, the means of prediction, and the treatment option. The forecast of tree size is clearly the most important.

The revenue from every individual tree is estimated with the help of model trees. Prior to predicting growth, the values of a number of model trees are calculated by cross-cutting and using a price-list containing the prices of different types and sizes of log. The model trees cover three dimensions: tree species, tree size, and tree quality. The valuation of a single tree is made by linear interpolation between the values of model trees of the same species and tree quality (Figure 7).

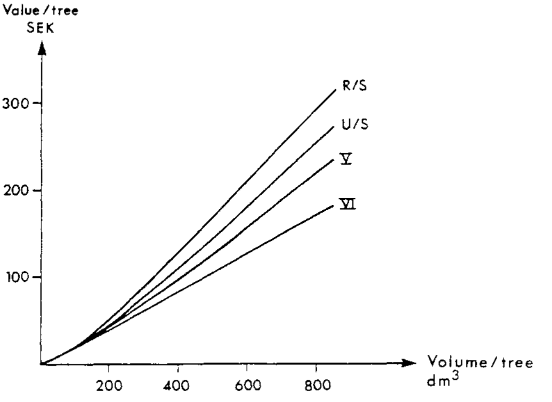

Fig. 7. The revenues from a single pine tree. R/S, U/S, V, and VI denote different timber-quality classes.

To calculate harvesting costs, it is necessary, as mentioned above, to know how densely the trees grow, the geographic location of the compartment, and other circumstances that affect these costs.

The harvesting costs are estimated using locally calibrated cost functions. The functions could e.g. be of the type

$$HC = \beta_0 + \beta_1 \cdot Vol + \beta_2 \cdot \frac{1}{St},$$

where

HC	denotes harvesting cost for a tree
Vol	denotes tree volume
St	denotes tree concentration in stems/ha
β_0, β_1, β_2	denote parameters locally calibrated for mean terrain conditions, etc.

The difference between revenues and harvesting costs gives a_{xp}, i.e. the stumpage value in period p for an individual tree.

Means of production and their costs

As a rule, a treatment option includes inputs of means of production in the form of silvicultural resources; however, it is possible to conceive of an option in which no silvicultural resources are expended. The amount of silvicultural resources, therefore also the magnitude of costs, is strongly related to type of treatment, area treated, and soil and ground characteristics. The predictions of silvicultural resource expenditure can consequently be produced by simple functions of treatment type, soil and ground type, and amount of area treated.

Price developments

The price a_{xp} for a product or means of production x in period p is calculated in two steps. In the first step, which is taken in conjunction with the calculation of the growth and yield forecasts, the level of resolution is that of the single tree. Two independent revenues of each tree are calculated according to two product price lists. The cost of final felling, thinning, fertilizing, and other silvicultural activities is calculated at the same time.

In the second step, the revenue is calculated as a weighted mean of the revenues according to the two independent product price lists. The compartment results are multiplied by price-development factors, which are differentiated with regard to time period and the type of revenue/cost.

This procedure has been chosen because it makes possible analysis of several different price-development options, without having to repeat the cumbersome individual tree growth calculations in the computer.

At the same time, this procedure precludes the detailed specification of price developments for different products, other than by letting the prices develop continuously in the interval between the two price lists.

Treatment options

The treatment option is identical for all plots within a compartment, and is a sequence of treatments over time. The most important treatment is final felling, which breaks the growth process of a stand generation and makes room for a new one. The macro-structure of a treatment option is thus provided by the shifts between growth generations.

Timber assessment calculations are performed for every stand generation individually, under the assumption that treatments made during one generation do not influence following generations in any other way than by determining the times for shifts in growth generation.

The model is constructed to depict the following types of treatment:
- regeneration
- precommercial thinning (cleaning)
- thinning
- fertilization
- final felling.

The treatment option for a compartment determines when treatments are introduced on the plots within that compartment. Treatments can be modulated with regard to the state of the forest on individual plots. How a treatment is to be performed is thus partly decided at plot level. This is an important feature of the model, particularly with regard to thinnings, where dense plots may be more heavily thinned.

For each compartment, timber assessment calculations are performed for an arbitrarily large number of treatment options. First, however, these options are to be specified by the user.

Principles for forecasts of certain tree characteristics

Forecasts for certain tree characteristics are derived from modelling the size, quality, and mortality of individual trees at different treatment options. We make the simple assumption that trees grow individually but under the influence of each other and of the site conditions. Sudden changes in the close environment of the individual tree, such as a thinning or fertilization, result in growth effects whose correct depiction is important.

On the basis of long experience – not least from Näslund (1935, 1942) – we have attempted to solve the problem of estimating the dimensional growth of individual trees.

According to Jonsson (1974*a*, 1974*b*, 1980), tree growth is the result of a complex process, in which many factors contribute. It is easy, in principle, to determine the influence of one or several factors on something if an experimental approach is applied. This requires, however, that experiments can be laid out and that there is enough time to await the results.

Another mode of investigation (surveying) is based on events that have occurred under natural circumstances prior to the beginning of the investigation. Surveys are thus of a non-experimental nature – something that entails certain problems. It is not a matter of varying one or two factors and keeping the rest under statistical control. Instead, the factors determining growth vary in an uncontrolled way, and it may be difficult to distinguish the effect of one or several factors from that of other factors.

In both cases, available knowledge is used to construct schematic models of the growth process, which are then used in the analysis. A material of empirical data is subsequently used to quantify the role of important factors in the process, or, in other words, to estimate certain parameters in a growth model. The models must be realistic, and at the same time useful with regard to their objective, to data constraints, and so on.

The objective of the models is to produce good estimates of the parameters in functions needed to forecast size, quality, and mortality of individual trees at different treatment options. The main advantage of using a survey material is to achieve representativeness at a low cost. It enables us to make realistic forecasts for a broad spectrum of forest types. The main advantage of using an experimental material is the possibility of isolating the effects of individual growth factors – in this case mainly the effects of treatments. Since the purpose of using the model to a large extent is to provide treatment guidelines, it is essential to separate the effects of treatments from the effects of other growth factors such as site fertility. In our case we use:

– data from the Swedish National Forest Survey for estimating growth (Söderberg, 1986) in the case of no future actions. The same applies to natural mortality;

– data from long-term experimental trials for estimating effects of treatments such as thinning (Jonsson, 1974*a*) and fertilizing (Rosvall, 1980). The same material applies to mortality in extremely dense stands (unthinned stands) (Söderberg, 1986);

– data from the sample tree material, collected in connection with our inventory (FMPP) for estimating tree quality and bark volume;

– data from the "HUGIN'-survey for estimating regeneration results and plant growth (Elfving, 1981).

The estimates are expressed in the form of regression functions for these components.

In our applications, a combination of these functions from all data sets is used to project the future size, quality and mortality of single trees in all types of Swedish forests which are subject to regeneration, precommercial thinning, thinning, and fertilization.

Estimated growth is primarily based on single tree growth undisturbed by future treatments. We call this growth the reference growth. The effects, e.g. of thinning, are expressed as a multiplier affecting this reference growth (see section "Thinning response of single trees" on p. 22 and Figure 8).

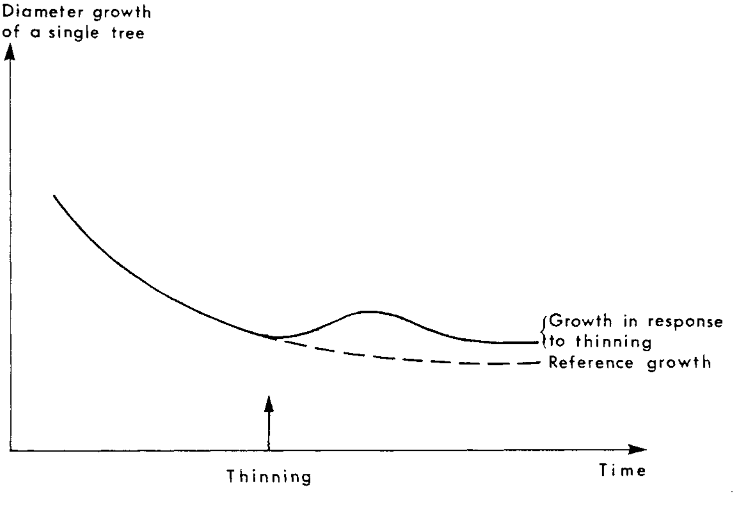

Fig. 8. The reference growth and the thinning response.

Growth functions

A growth function is constructed for our purpose. The approach and its motivation are taken from Jonsson (1969).

The growth of a tree depends both on a number of internal characteristics of the tree and forces and objects in the environment of the tree which release, constrain, favour, inhibit, or block growth.

For the period t_1 to t_2, growth $i(t_1, t_2)$ can be written

$$i(t_1, t_2) = \int_{t_1}^{t_2} f\,[I(t), J(t), K(t)]\; dt,$$

where

- i denotes growth (increment);
- I, J, K denote vectors with a large number of elements; and
- I denotes internal, growth-affecting factors;
- J denotes external, non-climatic growth-affecting factors;
- K denotes external, climatic growth-affecting factors.

Jonsson (1969, 1980) describes this model in detail. He utilizes knowledge about the biological process, as well as schematizations (i.e. annual-ring index) to develop the model into a form in which its parameters can be estimated using regression analysis of an empirical data set. The growth model is primarily a single-tree distance-independent model of basal area growth. Volume growth is obtained secondarily, mainly by means of static form-height functions based on diameter, age, and site index.

The variables in the growth functions should describe properties of the growth-affecting factors as accurately as possible. The choice of these variables is limited both by the observations in the data set at hand, and by constraints in the use of the regression functions, mainly caused by the necessary use of a simple method for collection of field data.

Analytical expressions of the variables should be chosen that adhere to available knowledge, or in case available knowledge is inadequate, such that they are flexible and fully or in part governed by the data. A too rigid expression may introduce systematic errors, while a too flexible expression may reflect particularities in data that are coincidental, rather than of general nature.

The choice of analytical expressions is an important step, which involves more or less subjective decisions.

Söderberg (1986) uses an empirical data set, consisting of sample trees and plot data from the Swedish National Forest Survey, to estimate the reference growth functions used in our system. The sample plots are systematically allocated over the forests of Sweden and are circular, with a 10 metre radius. This material is not primarily produced for yield studies. It is representative, but we cannot expect all information needed for such studies be included. For instance, there is incomplete information about earlier cuts in the sample plot stands.

The following variables have been used in the reference growth functions to describe the factors determining growth:

External, climatic factors as described by
- climatic zone
- latitude
- altitude;

External, non-climatic factors are described by
- soil type
- site index
- thinning history
- basal area per hectare (competition)
- diameter quotient (social position)
- species mixture;

Internal factors (single tree) are described by
- species
- diameter at breast height
- age at breast height.

When implementing a growth function, it is important that the survey measures and describes the forest in the same way as did the survey that yielded the empirical data behind the growth functions. Thus, in the Forest Management Planning Package, the system for site index determination, the sample plot radius, the minimum diameter qualifying a tree to be measured, etc., are all adapted to the norms used by the Swedish National Forest Survey.

Mortality functions

Some 4–8 per cent of the growth in Sweden's forests is lost due to natural mortality. Unlike growth, mortality occurs irregularly in both time and space. A major portion of the mortality cannot be attributed to simple variables of soil, climate, or stand. The data collected by the Swedish National Forest Survey allow the estimation of mean values for natural mortality. These are differentiated with regard to geographical region and species (Bengtsson, unpubl.). Mortality is expressed as a yearly percentage of the basal area. When the Forest Management Planning Package is applied, the level of natural mortality is varied among different plots around the mean value.

When long-term forecasts of forest development without thinnings are made, the means from the Swedish National Forest Survey will underestimate mortality. A special procedure (Söderberg, 1986) is used to estimate mortality in such cases. This procedure is based on two functions:

A. A function that determines the critical basal area which qualifies a sample plot as "dense" and makes it subject to special treatment;

B. A function that estimates mortality in such "dense", unthinned plots.

Both of these functions have been constructed on the basis of an analysis of data from unthinned stands in the permanent forest-development trials of the Faculty of Forestry. If the predicted basal area of an unthinned plot exceeds the critical value determined by function A, then function B is used to estimate its mortality, instead of the means from the Swedish National Forest Survey material.

Thinning response of single trees

The basis for the estimation of growth when a tree has been affected by silvicultural measures, is the growth pattern of an unaffected tree.

The total thinning response of single trees (see Jonsson, 1974*a*), i.e. the expected value of the quotient between the annual-ring widths of a single tree affected and unaffected by thinnings, can be estimated using data from statistically designed thinning experiments. The model used has the following form:

$$\ln Th_p = \beta_1 \sum_{i=1}^{p} g_i + \beta_2 \sum_{i=1}^{p-1} g_i + \beta_3 \sum_{i=1}^{p-2} g_i + \ldots,$$

where

Th_p denotes total thinning response of a single tree in 5-year period p due to all earlier thinnings

g_i denotes thinning intensity, expressed as the percentage of stand basal area that is removed at the beginning of period i

$\beta_1, \beta_2, \ldots, \beta_n$ denote parameters in the model.

Figure 9 shows the total thinning response for a single tree when 35 per cent of the stand basal area is thinned at the beginning of 5-year period 1. In period 5, for instance, the diameter growth in the thinned case is about 10 per cent greater than in the unthinned alternative on site quality class I + II.

Application of the functions for thinning response builds on the assumption that all trees on a sample plot will have the same relative thinning response. The results of studies made

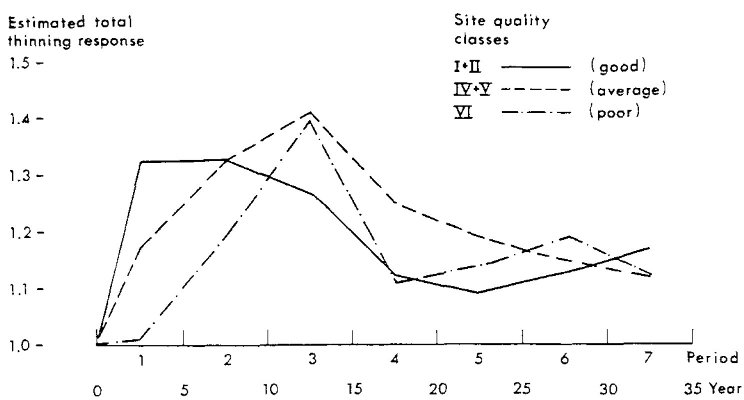

Fig. 9. The total thinning response for a single tree when 35 per cent of the stand basal area is thinned at the beginning of 5-year period 1. The thinning response is the quotient between the mean annual-ring widths of a single tree affected and unaffected by thinning.

by Näslund (1942), Jonsson (1974*a*), and Möller and Pettersson (1979) seem to warrant this assumption.

Available experimental data did not allow for the thinning response to be estimated for more than seven 5-year periods. In longer-term forecasts we use another technique, in which the growth functions are applied to the forest state affected by thinnings, rather than to the unaffected state. The thinning response is then assumed to be adequately captured by the variables in the growth functions – particularly the diameter of the tree and the basal area per hectare of the sample plot, which both are affected by the earlier thinnings.

Response to fertilization

Functions describing the growth response caused by fertilization have been developed by Rosvall (1980). However, Rosvall only studied the response during a 5–10 year period, and after single applications. To deal with long-term effects and with the effect of repeated treatments, we have been forced to make the assumption that the effect of a treatment with fertilizer can be expressed as a shortening of the time required for a forest stand to attain a certain level of development (volume).

Our reference is the forecast of volume growth for the unfertilized compartment. We denote it I_n, where I denotes volume in cubic metres (m^3sk), i.e. the whole trunk including bark and n stands for any 10-year period.

The total effect on volume growth of a single treatment with fertilizer at the beginning of a 10-year period is estimated using the functions developed by Rosvall. We denote this effect by E_n. The shortening of the production time caused by fertilization is then calculated using the expression

$$dt_n = 10 \cdot \frac{E_n}{E_n + I_n}.$$

In the case of repeated treatments, the time-shortening effects are summed. An example will serve to illustrate this.

Without fertilization, the accumulated growth after p 10-year periods will be

$$\sum_{n=1}^{p} I_n.$$

With repeated fertilizations, the same amount of growth will be reached after

$$\sum_{n=1}^{p} (10\text{-}dt_n) \text{ year.}$$

This means that the growth response to fertilization is assumed to last for 10 minus dt_n years, following which either a final felling or a repeated treatment with fertilizer takes place. The effect of repeated treatments is further assumed not to be affected by any possible lasting effects from earlier treatments. This assumption is supported by results from refertilization trials (Kukkola & Saramäki, 1983). Further, the interaction between thinning and fertilization is assumed to be of a multiplicative nature, since the accumulated shortening of production time, calculated with the growth forecast without thinning as a base, is also used in conjunction with thinned options.

This approach – depicting the effect of fertilization as a shortening in production time – has great advantages from the viewpoint of computation economy.

Growth forecasts in young forests

The forecasting method described above cannot be applied to forests in the seedling or sapling stage. The growth functions are intended to be applied, in the main, to trees with a diameter at breast height larger than 50 mm, although they can be used for trees slightly smaller than that. To predict growth on sample plots where the majority of the trees are smaller than 50 mm in diameter, functions depicting height development are used (Elfving, 1981). Diameter and volume are estimated indirectly, using functions correlating them mainly with tree height and number of stems per hectare.

Growth forecasts in newly-established forests

Following final felling, a new generation of forest is established. The development of such new forests is described using components from the HUGIN system (Bengtsson, Holmlund, Lundström & Sandewall, 1989). The growth estimation consists of the following steps:

1. A set of sample plots (normally 30 per site-index class) is selected from the data base of the Swedish National Forest Survey. The selection is made in such a way that the set represents

the region to be analysed with regard to geography and site-index distribution.

2. A regeneration result is estimated for each individual sample plot, given a previously specified treatment program. The estimation procedure produces variation in the regeneration result among sample plots with regard to their site characteristics and to random variation in the data base.

3. The growth of the stands on the sample plots is forecast using functions that depict the growth of young forests (see above) and functions that depict the growth of individual trees (see above).

4. The volumes cut in thinnings are calculated on the base of a previously specified thinning program with set times for thinning of each site-index class.

5. The development at the different sample plots is summed for each site-index class, and a mean development per class is calculated. However, the forecasts are made at the sample-plot level, as described above.

6. Finally, the result is stored in the form of yield tables – one for each combination of region, regeneration program, and site-index class. Normally, five different regeneration programs are represented:
- plantation of Scots pine (Pinus sylvestris L.)
- plantation of Lodgepole pine (Pinus contorta var. latifolia Loud.)
- plantation of Norway spruce (Picea abies (L.) Karst.)
- natural regeneration of softwoods
- natural regeneration of hardwoods.

Timber quality: initial state and development over time

During the survey, all sample trees of Scots pine and Norway spruce > 15 cm at breast height are assigned a timber quality index, ranging from 1 to 4, depending on the quality of the butt log.

The timber quality indices are as follows:
1. denotes timber quality "special"
2. denotes timber quality "unsorted" (qualities I–IV)
3. denotes timber quality "fifth" (V)
4. denotes timber quality "reject" (VI), or pulpwood.

On the basis of these observations, a local timber quality index function is estimated independently for pine and spruce, using the following model

$$Q = \beta_0 + \beta_1 \cdot D + \beta_2 \cdot A,$$

where

Q denotes timber quality index

D denotes diameter at breast height, mm

A denotes age at breast height, years

$\beta_0,\ldots$ denotes parameters.

As an example, a timber quality index function is graphed in Figure 10.

The timber quality index is calculated for an individual tree over time in two steps:

1. For calipered trees > 15 cm at breast height, an initial timber quality index is directly assigned to each tree through a special imputation procedure (cf. section "Age imputation", p. 34). For calipered trees ≤ 15 cm at breast height, each tree is assigned an initial timber quality index by means of the above-mentioned function; a random component is also added to this value.

2. The static quality function is then used in the forecasts to capture the changes in initial timber quality index that occur as a result of increases in diameter and age.

According to this, we obtain the timber qual-

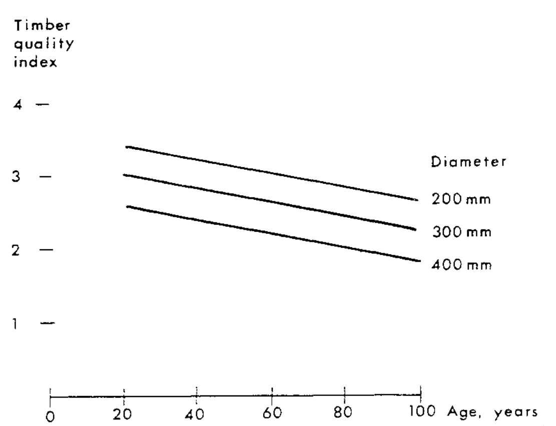

Fig. 10. Illustration of a timber quality index function graphed for individual trees.

ity index over time in the following way for individual trees:

$$Q_t = Q_0 + \beta_1 \cdot (D_t - D_0) + \beta_2 \cdot (A_t - A_0),$$

where index t and 0 denote points in time.

Thus, timber quality index functions have been produced for pine and spruce. They are used to estimate both the changes in the timber quality index and the initial quality index of trees that could not be imputed such an index from the survey data. They are also used to estimate the quality of small trees and saplings. New forests are also treated in this way.

Only pine and spruce have specified quality in our analyses.

The forest survey

The purpose of the forest survey is to yield support for decisions of strategic as well as operative character. Operative decisions concern every single compartment, which calls for comprehensive information about all compartments. The kind of information needed is mainly determined by the requirements of the operative decision-making process. These requirements vary depending on the development stage of the forest, as different decisions are made at different development stages. Strategic decisions usually concern the forest holding in its entirety, which opens up the possibility of relying on information derived from objective samples. Strategic decisions call for supporting data that, above all, must give a correct picture of the possibilities offered by the holding as a whole.

Characteristics of supporting information from timber surveys:

Operative decisions
- information about all compartments,
- low cost per compartment surveyed,
- random errors are usually larger than bias components,
- emphasis on reducing random errors,

Strategic decisions
- information from a sample of compartments sufficient,
- low total cost for the forest holding, but not necessarily for the compartments in the sample,
- random errors may be reduced by large sample size,
- emphasis on reducing potential sources of bias.

The different requirements in the supporting data for operative and strategic decisions can be met by a survey procedure divided into two phases.

Phase 1: A total, fast, and inexpensive description of the entire holding, divided into appropriate compartments;

Phase 2: A detailed, objective measurement of a sample of compartments.

Phase 1

The first survey phase consists of two main parts:

1. Partitioning of the forest holding into compartments and the formalization of the compartment structure on a map.

2. Description of the forest in every compartment and the formalization of this description in a compartment register.

The bridge between these two parts is the identity of the compartment.

Let us here devote some words to the compartment concept. A compartment is a part of a forest holding. According to Model (4) the guidance for the partitioning of a forest holding into compartments should be whether the stand within the entire compartment will be simultaneously given the same silvicultural treatment.

Thus, compartments can be regarded as units which constitute a basis for the treatment of the forest, to achieve the management objective. A compartment structure is imposed on the forest in connection with "area planning". The purpose here is to distinguish and determine the boundaries of different land classes and to divide the forest in the productive category into units, i.e. compartments, with regard to both biological and operational conditions; each compartment should be uniform in some sense.

If we were to distinguish compartments on edaphic and biological grounds (biotopes) only, these would be small and would reflect the variation in the forest more or less in detail. Operational considerations, however, often lead to larger compartments, which will contain edaphic and biological variations within certain acceptable limits. Thus, biotopes are generally too small to be used as operational treatment units. The economies of scale suggest that larger treatment units, consisting of several adjacent biotopes, be used.

There is an optimal partitioning of the forest into compartments in terms of treatment units; this partitioning is difficult to determine in practice. It is decided, among other things, by present and future prices of both products and means of production. It should therefore not be regarded as stable, but rather as varying over time with changes in these conditions. Changing technology, changing prices, and the development of the biotopes themselves, point toward a dynamic division of the forest into compartments in terms of treatment units.

In a given planning situation, we must base our compartment structure on assumptions about these future conditions. Such assumptions concerning prices, etc., are speculative. This means that the spatial structure of compartments is also speculative, and changes with varying assumptions.

Fixed partitioning into compartments is more or less intuitive and leads to some rigidity in our analyses. It will restrict opportunities for obtaining optimal solutions. For the time being, we are confined to working with a fixed compartment structure in the analysis of our treatment programs.

In this primitive situation, the holding should in principle be divided into compartments in such a way that no future treatments will call for the further division of a compartment. As mentioned above, this tends to result in a large number of small compartments. At the same time, administration and other costs tend to favor fewer treatment units. In an essay, Håkansson (1987) studied the economic effects of two different degrees of resolution in the compartment structure of a holding in southernmost Sweden. In this case, a more modulated management approach, with many small compartments, yielded no gains when the increased costs for administration and movement of operational resources had been deducted.

The forest within compartments should be described in order to support future decisions concerning the treatment of the compartment (see section "Priority functions", p. 44).

Besides the factors discussed above, the design of the total survey of the holding in phase 1 ought also to be influenced by the need to support the stratification in phase 2 (see below).

Systematic errors in the phase-1 survey can be scrutinised and corrected with the help of calibration techniques (see section "Calibration methods", p. 36). This allows for the use of subjective measurement techniques and other methods that are fast but not unbiased. All in all, the Forest Management Planning Package allows for a high degree of freedom in the choice of methods for the phase-1 survey.

Phase 2

The purpose of the forest survey in the second phase is to describe the forest in such a way that we can estimate the amount of products (output) and means of production (input) that are produced and consumed, respectively, in the process of forestry.

In Model (5), these quantities are represented by the double summation

$$\sum_{i=1}^{m} \sum_{j=1}^{n_i} q_i f(x, t_p, H_i, I_{ijt_p}).$$

As a basis for the estimation, a stratification and a two-stage sampling procedure are used.

Stage 1: Stratified PPS-sampling of compartments for circular-plot survey;

Stage 2: Systematic sampling of circular plots within each sampled compartment.

This sampling procedure results in an image of the forest holding, based on a number of sampled compartments and on the circular plots surveyed within them. Our Model (5) is subsequently applied to this image. Every sample plot is then taken to represent a larger area than it has in reality. The size of this area is determined by the sampling procedure.

The estimates produced from the sample of compartments are also used to calibrate the phase-1 survey (see section "Calibration methods", p. 36) and to derive operative decision rules (see section "Operative planning", p. 49).

Selection of compartments

The selection of sample plots is done as a stratified PPS-sampling (PPS = Probability Proportional to Size; Cochran, 1977). The aim of stratification is to create strata homogeneous in aspects which are important to the subsequent analyses. For estimation of yield potential, the present value of the forest holding is important (Jacobsson, 1986). For estimation of the product yield from thinnings and from final fellings, age and density are important. A stratification based on the variables age and volume captures the net present value as well as factors of importance for thinning and final felling. Large samples may allow for the use of additional stratification variables. However, the major gains from stratification are normally achieved with a few strata and a few stratification variables (Cochran, 1977; Jacobsson, 1986).

The stratification is performed using the following technique:

1. The number of strata is approximately determined as half the number of sampled compartments. At least two sampled compartments per stratum are necessary to allow for the estimation of standard error.

2. The areas of all compartments are tabled in a matrix table based on age and volume (Table 2, p. 30).

3. An approximate average area per stratum is determined as the total area divided by the number of strata.

4. The cells in the matrix table are manually and subjectively joined to strata with the aim of reaching the average area calculated in step No. 3, and at the same time minimizing the variation in age and in volume per hectare within each stratum.

The result of a stratification is shown as an example in Table 2. This is a teaching exercise case, in which the number of compartments has been chosen with an eye to the educational value of this particular example.

Samples are allotted to strata with the same aim as stratification variables and stratum boundaries are chosen.

To estimate the yield potential with high accuracy, a Neyman-allocation (Cochran, 1977), using the net present value as an allocation variable, is suitable. The variation in net present value within a stratum can be approximated as a linear function of the mean volume per hectare (Jacobsson, 1986), implying more intensive sampling in strata with a higher volume.

The selection of samples within a stratum is done as a PPS-sampling, using compartment area as the size variable. Alternatively, a simple random sample could be employed, using compartment area as a guiding variable for a ratio estimate. However, the PPS-sampling scheme yields estimators that are very simple and, as a rule, also somewhat more efficient.

Allocation, size, and delineation of sample plots

The sample plots are systematically allocated to the sampled compartment in the form of a quadratic grid. Systematic sample-plot allocation is a simple and generally effective method (Nyyssönen, Kilkki & Mikkola, 1967; Lindgren, 1984). All points in the compartment must have the same probability of being sampled if the estimates according to the section "Estimation methods for circular-plot survey of compartments" (p. 33) are to be unbiased. Two problems arise in this context:

1. During fieldwork, it may not be possible to position the sample plot centers where they should be according to the map. Some plots may even fall outside the compartment.

2. Some sample plots will fall exactly on the compartment boundary, i.e. become boundary plots.

These problems are approached in the following way:

1. A sample-plot grid is allocated to the map, oriented along the north-south axis. All sample plots lying inside the compartment boundary are marked on the map.

2. In the field, only sample plots that are marked on the map are visited, with the help of compass and a measuring tape. Sample plots outside the compartment boundary on the map, but which might fall inside the boundary during field work, are thus not surveyed.

3. If the center of a marked sample plot (inside on the map) should fall outside a distinct compartment boundary in the field, then it is reflected into the compartment (Schmid, 1969). This serves as a compensation for the fact that all sample plots just outside the boundary on the map have already been discarded at the planning stage (deleted from the map) (Figure 11).

4. When boundary plots are measured in the terrain, that part of the plot which is outside the boundary is reflected into the compartment.

These procedures minimize the walking time during the survey. No time is spent in the field on checking sample plots outside the compartment boundary on the map. At the same time, points within the boundary zone on the map attain the same probability of being sampled as all other points in the compartment. This is summarized in Figure 11.

Still, in survey practice, sample plots will sometimes fall outside the compartment. This happens when the surveyor, without realizing it, passes through a compartment boundary be-

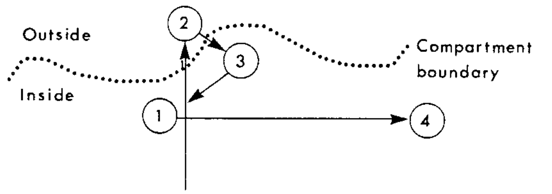

Fig. 11. Plot positions in the case of a misorientation causing a marked plot (inside on the map) to fall outside a clear compartment boundary in the field. (1) The sample plot according to the map. (2) The sample plot entirely outside the compartment boundary in the field according to compass and measuring tape (due to a misorientation). (3) A mirror-plot laid out inside the compartment perpendicular to the compartment boundary; it is measured. (4) An adjustment to the position (1) in order to come straight to the next plot (4).

cause it is indistinct. However, an indistinct compartment boundary is usually indistinct regardless of the direction from which it is approached. Thus, there is some probability that a sample plot from an adjacent compartment in a similar way may compensate for the loss of sample plots in the zone around indistinct boundaries. If the whole sampling procedure is considered, including the selection of sampled compartments and the allocation of sample plots in the field, the only case in which the boundary zones can be misrepresented is when a boundary zone is indistinct from one direction of approach only.

The sample plot is generally circular, with a radius of 10 metres. This radius was chosen because uniformity with the Swedish National Forest Survey was desired with regard to variables that depend on plot size, e.g. basal area per hectare and the diameter of the largest tree.

When stand density makes it difficult to perform measurements on such a large sample plot, a 5-metre radius may be employed. Lindgren (1984) has shown that from the viewpoint of efficiency, a relatively large latitude can be permitted in the choice of plot size.

The boundary of the sample plot has hitherto been located with the help of a special optical instrument (Jonsson & Matérn, 1982) and a measuring tape. However, an automatic distance meter, using ultrasound, is now employed (Figure 12). This instrument measures distance with a resolution of one centimetre, even when the line of sight is blocked by trees (Jonsson, 1991; Jonsson, Holm & Kallur, 1992).

Measurements on sample plots in forests with the major portion of the trees larger than 50 mm at breast height

The observations made on the sample plots may be divided into four types:

1. Description of site characteristics;
2. Description of stand characteristics;
3. Enumeration of all trees except undergrowth of no development potential;
4. Measurements on sample trees.

Geographic region, latitude, and altitude are common variables for the sample plots and are thus recorded at compartment level. Site index is determined using the Swedish College of Forestry method. At each sample plot, the method of estimating site index based on site properties (Hägglund & Lundmark, 1977) is used. On sample plots where the stand is deemed an appropriate indicator, site index is also determined using height development curves (Hägglund, 1979), or by the intercept method (Hägglund, 1976). The latter two methods yield support for a calibration of the site properties method (see section "Calibration of site-index estimates based on site factors", p. 35).

In addition to the site index, the fraction of nonproductive area, soil type, vegetation type, and soil moisture are recorded for each sample plot.

The stand on the sample plot is described with regard to the following factors:

- fertilization history
- thinning history
- estimated basal-area weighted mean age at breast height
- estimated degree of age variation
- estimated basal-area weighted mean quality for timber-quality trees; separate for pine and spruce.

The estimates of mean age and mean quality are used as a basis for imputing an age and a quality to individual trees (see section "Age imputation", p. 34).

The count of trees on a plot is done indirectly in conjunction with measurement of the diameter at breast height of all trees on the plot. Species code and diameter in millimetres are recorded automatically with the aid of an electronic data caliper and a data recorder (Jonsson, 1981, 1991; Jonsson et al., 1992). The latest

Table 2. *Example of partitioning, numbering of strata and number of sampled compartments in each stratum*
Partitioning and numbering of strate

Age classes, year		Area distribution according to the compartment register, hectare Volume classes, m³sk/ha 0-10		11-30	31-60		61-100	101-140		141-180	181-220		221-260		261 +	Total
Clearing	1.	35.0		0.0	0.0		0.0	0.0		0.0	0.0		0.0		0.0	35.0
3–10	2.	201.4		2.2	0.0		0.0	0.0		0.0	0.0		0.0		0.0	203.6
11–20		53.6		82.7	18.7	4.	1.4	0.0		0.0	0.0		0.0		0.0	156.4
21–30		0.0	3.	2.2	27.6		66.9	4.8		2.9	0.0		0.0		0.0	104.4
31–40		0.0		0.0	0.0		19.8	10.2	7.	28.7	1.3		0.0		0.0	60.0
41–50		0.0		0.0	0.0	5.	1.5	0.0		27.4	21.1	10.	19.4	12.	41.9	111.3
51–60		0.0		0.0	0.0		3.7	11.1	8.	25.1	14.3		21.3	13.	76.1	151.6
61–70		0.0		0.0	0.0	6.	0.0	24.3		3.6	19.9	11.	52.9		36.2	136.9
71–80		0.0		0.0	0.0		9.5	3.8	9.	46.3	28.6		5.7	14.	33.8	127.7
81–90		0.0		0.0	0.0		0.0	12.4		11.4	0.0		0.0		12.7	36.5
91–100		0.0		0.0	0.0		0.0	0.0		0.0	0.0		0.0		0.0	0.0
101–110		0.0		0.0	0.0		0.0	2.6		0.0	0.0		0.0		0.0	2.6
Total		290.0		87.1	46.3		102.8	69.2		145.4	85.2		99.3		200.7	1126.0

Number of sampled compartments in each stratum

Stratum No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	Total
Number of compartments	7	4	2	2	2	2	3	3	4	2	3	2	4	4	44

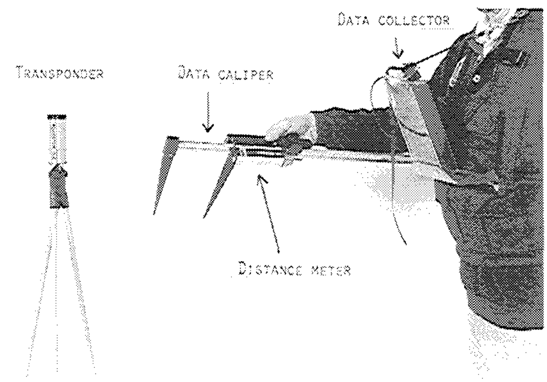

Fig. 12. Electronic data caliper, distance meter with transponder, and data collector.

version of the caliper uses cordless transmission of data between caliper and data recorder. This caliper is also equipped with a distance measuring device (Figure 12).

The data recorder uses a random procedure for choosing sample trees, which are measured for the following characteristics:
- tree species
- diameter at breast height
- tree height
- height to first live branch
- thickness of bark
- age (number of annual rings at breast height)
- quality of butt log.

The sample trees are selected automatically (directly in connection with calipering) with the aid of a random-number generator (Jonsson, 1991); this procedure is called Poisson sampling. Selection is based on probabilities proportional to the basal area of the calipered trees.

A number of instruments have been developed for measuring sample trees (Jonsson, 1991; Jonsson et al., 1992), e.g. electronic hypsometer, annual-ring measuring instrument, bore extractor, bore supporter, and bark measuring instrument.

Measurements on sample plots in young forests

The predicted development of basal area is less reliable when the growth functions are extrapolated below the minimum diameter of 50 millimetres. In young stands, more reliable predictions can be made based on height development. On sample plots with trees predominantly smaller than 50 mm in diameter, the tree count is done by measuring and recording the height of the trees.

The heights of main and secondary trees are recorded separately. Diameter is estimated indirectly, based on height and stem number per hectare. The need for regeneration and cleaning measures is judged for each sample plot, to support estimation of future silvicultural costs.

Estimation of current forest conditions

Estimation methods for stratified PPS-sampling

A detailed fieldwork manual (in Swedish) for circular-plot survey by compartments has been produced and is available from the Department of Biometry and Forest Management at the Swedish University of Agricultural Sciences.

The estimation of totals and variances within strata is arrived at using estimators found in Cochran (1977, pp. 252 and 254).

$$\hat{Y}_h = \frac{1}{n_h} \cdot \sum_{i=1}^{n_h} \frac{y_{hi}}{P_{hi}}$$

$$\hat{V}[\hat{Y}_h] = \frac{1}{n_h \cdot (n_h - 1)} \cdot \sum_{i=1}^{n_h} \left(\frac{y_{hi}}{P_{hi}} - \hat{Y}_h \right)^2,$$

where

$\hat{Y}_h$ denotes estimate of a total value in stratum h. The "total" may refer to forest state variables such as volume or yield variables such as net revenue

n_h denotes number of sampled compartments in stratum h

y_{hi} denotes value of the sampled compartment i in stratum h

P_{hi} denotes sampling probability of the sampled compartment i in stratum h.

The sampling probability P_{hi} for the population of compartments is calculated from area information available from the compartment register.

$$P_{hi} = \frac{a_{hi}}{\sum_{i=1}^{m_h} a_{hi}},$$

where

a_{hi} denotes area according to compartment register for compartment i in stratum h

m_h denotes number of all compartments in stratum h.

The estimators assume PPS-sampling with replacement. Estimates of the corresponding totals and variances for the entire forest holding are arrived at by simple summations over strata:

$$\hat{Y}_{tot} = \sum_{h=1}^{L} \hat{Y}_h,$$

$$\hat{V}[\hat{Y}_{tot}] = \sum_{h=1}^{L} \hat{V}[\hat{Y}_h]$$

where

$\hat{Y}_{tot}$ denotes total for the entire forest holding

L denotes number of strata.

The calculation of y_{hi}, i.e. the value of the sampled compartment i in stratum h, is based on the value per hectare multiplied by the actual area of the compartment according to its representation on the map. Usually, this area is the same as the area reported in the compartment register. However, deviations may occur. Small, unproductive patches may, for example, have been deducted in the compartment register. Regardless of any deviations, the original selection probability, based on the area reported in the compartment register, should be used in the estimators.

To simplify and clarify the computations, the concept of "represented" area is useful. This area is an attribute to each sampled compartment and is calculated as:

$$AR_{hi} = \frac{\sum_{k=1}^{m_h} a_{hk}}{n_h} \cdot \frac{A_{hi}}{a_{hi}}.$$

where

AR_{hi} denotes represented area for sampled compartment i in stratum h

m_h denotes number of all compartments in stratum h

n_h denotes number of sampled compartments in stratum h

a_{hi} denotes register area for the sampled compartment i in stratum h

A_{hi} denotes actual area according to the map for the sampled compartment i in stratum h.

If the actual area according to the map is the same as the register area, all sampled compartments in a stratum will have the same represented area.

Estimates of totals for the entire forest holding are conveniently arrived at by the multiplication of the value per hectare (as arrived at via data from the circular plots) by the represented area of the compartment. Applied to Model (5), this means that the problem is reduced from having dealt with all compartments in the forest holding to only dealing with a limited number of sampled compartments, each of which "represents" a larger part of the forest holding.

The basic structure of the problem, however, remains unchanged.

Analogously, a represented area can be calculated for every plot in the sample by dividing the represented area of the compartment by the number of sample plots. This area may be used when the holding is to be described at the sample-plot level.

Estimation methods for circular-plot survey of compartments

In the foregoing we assumed that y_{hi} is observed. However, it is estimated on the basis of samples of circular plots. The value of a characteristic y of compartment i in stratum h is then estimated as:

$$\hat{y}_{hi} = A_{hi} \cdot \frac{\sum_{j=1}^{n_{hi}} y_{hij}}{n_{hi} \cdot sa_{hi}}.$$

where

y_{hij} denotes the value of sample plot j in compartment i in stratum h,

A_{hi} denotes actual area according to the map of compartment i in stratum h,

n_{hi} denotes number of sample plots in compartment i in stratum h,

sa_{hi} denotes area of a sample plot in compartment i in stratum h.

A_{hi}, i.e. the actual area of the compartment, is estimated by means of a careful areal measurement of the compartment's image on the map.

$\hat{y}_{hi}$ is a substitute for y_{hi} in the formulas above. The value y_{hij} is generally estimated by means of simple summations of the values for the trees on the sample plot. This applies among other things to volumes, revenues, and costs.

Calibration of height and form height

All trees on a sample plot are measured for diameter and thus also for basal area. The height and form height of the trees are estimated by means of static functions (Söderberg, 1986), having diameter as the most influential variable. Volume is estimated in a secondary way by multiplication of basal area and form height. The static functions have a high degree of precision. Söderberg (1986) reports that estimates produced by the single-tree functions have a standard deviation of 11 per cent for Scots pine, 13 per cent for Norway spruce, and 15 per cent for birch. The magnitude of these deviations decreases significantly at more aggregated levels, such as those of compartment and forest holding (Lindgren, 1984).

The functions for height and form height are subject to calibration in the Forest Management Planning Package, to eliminate even these local systematic deviations. A species-wise calibration ratio for form height FHC, is estimated for each stratum as follows:

$$\widehat{FHC}_{hs} = \frac{\sum_{i=1}^{m_{hs}} \frac{Vol_{hsi}}{P_{hsi}}}{\sum_{i=1}^{m_{hs}} \frac{FH_{hsi} \cdot BA_{hsi}}{P_{hsi}}},$$

where

FHC_{hs} denotes calibration ratio for the form height of species s in stratum h

m_{hs} denotes number of sample trees of species s in stratum h

Vol_{hsi} denotes volume according to volume functions based on data from sample-tree measurements on sample tree i of species s in stratum h

P_{hsi} denotes selection probability for sample tree i of species s in stratum h

FH_{hsi} denotes form height according to a static form-height function for sample tree i of species s in stratum h

BA_{hsi} denotes basal area for sample tree i of species s in stratum h.

Tree height is calibrated analogously.

Experience has shown that the value of the calibration ratios is very close to 1 when the total holding is considered. The calibration process thus has the character of a safety measure. It is fully reasonable, from the viewpoint of cost and efficiency, to abstain from the sample-tree measurements and rely exclusively on the static functions.

The calibration ratios are used in the estimation of the initial state of the forest as well as in growth predictions. This serves to increase precision in the latter (Söderberg, 1986).

Functions for volume over bark

All volume estimates for standing trees are given in forest cubic metres (m^3sk); thus, they include the whole trunk with bark. The value for a particular tree is estimated through the value of model trees.

The model trees are evaluated by means of simulated merchandising of the timber under bark. The volume over bark of the prototype trees is subsequently estimated as a function of the volume under bark

$$V_{ob} = \beta_0 \cdot Vol_{ub} + \beta_1 \cdot Vol_{ub}^2,$$

where

Vol_{ob} denotes volume over bark

Vol_{ub} denotes volume under bark

β_0, β_1 denote parameters.

The parameters in the functions are estimated on the basis of sample-tree values.

Age imputation

Tree age is an important independent variable in the functions for growth, height, and form height of individual trees. For reasons of cost, age cannot be measured for all trees. Instead, age data for the sample trees are used to impute an age to all trees whose diameter has been measured (cf. Jonsson, 1974*b*; Holm, Hägglund & Mårtensson, 1979). This imputation is done according to the following procedure (Figure 13):

1. For every sample plot, the basal-area weighted mean age at breast height, *MA*, is estimated based on age values from a minimum of two measured trees. In cases where an insufficient number of sample trees has been selected on the plot, a few supplementary ones are selected in a subjective manner.

2. The age at breast height, A_i, is determined on every randomly selected sample tree and, in

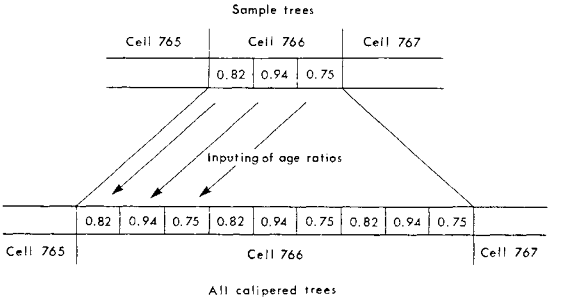

Fig. 13. Imputation of age ratios from sample trees to all trees measured for diameter but not for age. Cell 766 may denote, for instance, spruces growing in pine forests with a normal spread in age, classified as age class 60-80 years with a diameter ratio of 0.6-0.9.

cases where site index is determined by the height development method, on the indicator trees as well; *i* denotes such sample and indicator trees.

3. An age ratio, A_i/MA, is calculated for the above-mentioned trees measured for age. These ratios are sorted into a table divided with regard to:
- the degree of age homogeneity in the sample plot stand
- diameter ratio
- age class
- the species of individual trees
- the species mix on the sample plot.

4. Values for trees on which diameter, but not age, has been measured, are sorted into the same table. These trees are then successively imputed age ratios from the sample trees belonging to the same cell in the table. Sample tree values are obtained from neighboring cells, if a cell should happen not to include any such value.

5. Finally, the age of particular trees is estimated by multiplying the estimated basal-area weighted mean age on the sample plot to which the tree belongs, by the age ratio estimated in step 4, above.

This procedure means that age ratios are imputed in a way that compensates for any possible systematic errors that might occur in the estimates of basal-area weighted mean age produced in the field (see e.g. Ståhl, 1992). Should there be a systematic positive error, the age ratios used will be of a lesser magnitude than the theoretically true age ratios. The procedure further ensures that age variation in the stand is reflected appropriately – necessary to allow the distribution of growth among the trees to be estimated accurately.

Calibration of site-index estimates based on site factors

The method developed by the Swedish College of Forestry for site-factor based estimation of site quality, is used on all sample plots. When this method was constructed, the need for local calibration, based on practical experience, was foreseen (Hägglund, 1979).

For that reason, site index is estimated with the aid of height development curves or using the intercept method on all sample plots where the stand is suitable for use as a site-index indicator.

The resulting pairs of observations provide a basis for calibration of the site-factor method for site index estimation. The calibration is done by means of ratio-, difference-, or regression estimation. An example of results of paired observations, and a calibration based on regression estimation, are shown in Figure 14.

The calibrated estimates of site index for the sample plots are then used as independent variables in growth-forecasts, etc.

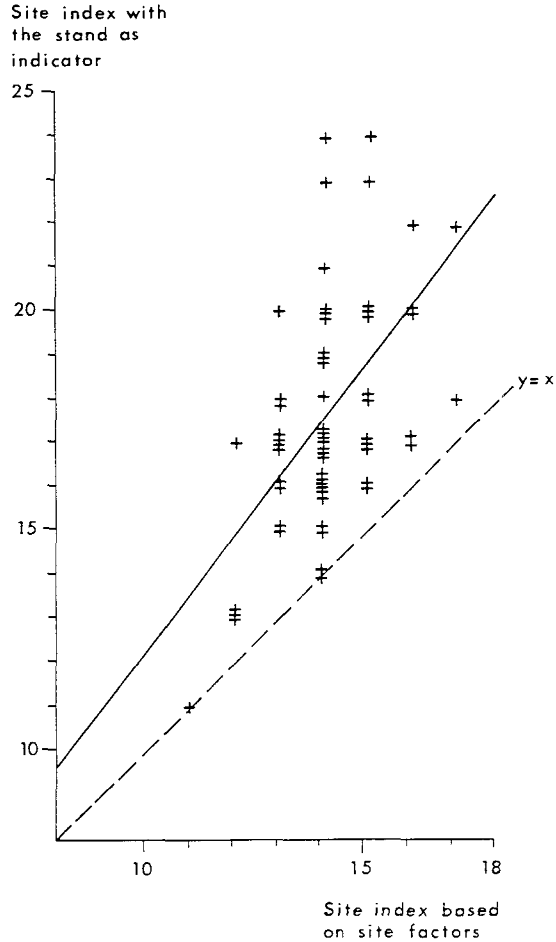

Fig. 14. An example of calibration of site index values estimated using the site-factor method. Regression function: $y = -0.442 + 1.283\ x$ (full line). Extreme discrepancies can be seen between the two methods (from a forest holding in Swedish Lapland).

Calibration of phase 1

The aim of calibration

A phase-1 survey of the entire forest holding is generally associated with errors of systematic and non-systematic character (see e.g. Braun, 1974; Jonsson & Lindgren, 1978).

One is rarely justified in entertaining a definite advance opinion as to the nature of the mechanism underlying the errors in the phase-1 survey. The errors may correlate with the magnitude of a certain variable in linear as well as non-linear fashion. Furthermore, the error of one variable may interact with the value of another variable.

We denote a correct, i.e. free from error, description of a forest holding by V, where V is a $N \times M$ matrix with the elements v_{ij}. In this notation, N is the number of compartments, and M the number of variables per compartment. Thus, v_{ij} denotes the true value taken by the variable j describing the compartment i.

We choose to denote in a similar way an actual compartment register X with the elements x_{ij}.

The error in the register for a certain compartment and variable can thus be represented as the difference $x_{ij} - v_{ij}$.

An often-used summary measure for the magnitude of error for individual variables pertaining to individual compartments, is the mean square error (MSE), which for variable j is defined as:

$$\frac{1}{N}\sum_{i=1}^{N}(x_{ij} - v_{ij})^2.$$

It is usually argued that the losses incurred when using a value from a register increase with the square of the magnitude of error. Blythe

(1948) and Jacobsson (1979, 1986) have indicated several decision situations in forest management in which the quadratic loss function is a reasonable approximation. MSE therefore is a pertinent measure of error.

The mean error in an entire compartment register is

$$\frac{1}{N}\sum_{i=1}^{N}(x_{ij}-v_{ij})$$

i.e. the mean of the error for variable j occurring in the description of individual compartments.

The mean error in subsets of the register is analogously defined.

The aim of calibration is to increase the usefulness of a compartment register for planning purposes. The way to do this is to decrease the magnitude of the errors in the register. Calibration generally results in a decrease in the magnitude of some individual errors, while the magnitudes of others increase. This latter circumstance has led to the questioning of the usefulness of calibration in general (Bergstrand, 1983). In our opinion, a register can be rendered more useful for planning if

1. the average error decreases in the register as a whole, and in such subsets of the register that are of interest for planning purposes; and/or

2. the dispersion in the register decreases for individual compartments.

It is mainly the average error in the whole register and in subsets of the register, the magnitude of which can be reduced by calibration. Dispersion, however, can also be affected, especially if calibration is done with regard to different subsets in the register, e.g. subsets consisting of compartments inventoried by different persons.

In the situation of calibration, it is only possible to take into account those subsets which have been defined by means of available register information. It would, of course, also be desirable to eliminate the average error in subsets defined with the help of the true values, e.g. true age classes. This can indirectly be accomodated if the register values have been *a priori* generated according to a known stochastic model. Li (1988) has studied methods for elimination of the average error, conditioned on the true values, when the error mechanism is a linear function of the true values plus a random variable with a normal distribution.

Calibration methods

The basis for calibration of a compartment register are the differences between the variable estimates in phase 1 and in phase 2. The average error for a variable j in a subset of the compartment register can be estimated as

$$\frac{1}{\sum_{i=1}^{N} a_i \cdot b_i}\sum_{i=1}^{N}(x_{ij}-(v_{ij}+\varepsilon_{ij}))\cdot a_i\cdot b_i, \qquad (6)$$

where

ε_{ij} denotes the error in phase 2 of the variable j in compartment i

a_i denotes area of compartment i

$$b_i = \begin{cases} 1 & \text{when the compartment } i \text{ is included in the sample of compartments} \\ 0 & \text{otherwise.} \end{cases}$$

If phase 2 yields unbiased estimates of v_{ij}, i.e. if the expected value of ε_{ij} is 0, then the estimator (6) gives an unbiased estimate of the average error in the subset in question. This estimate can then be used to calibrate the register.

The division of the register into subsets should be based on knowledge about the mechanism governing the error in the register, and on comparative studies between data from phase 1 and from phase 2. A division with regard to surveyor, survey method, or species for the stands, is often justified. The possibilities for division are limited by the sample size and by the need to retain sufficiently many degrees of freedom in the estimates.

Figures 15 and 16 show examples of comparisons between phase-1 data and phase-2 data.

To economize the degrees of freedom and at the same time differentiate the error estimates with regard to different variables, regression methods can be used.

A general model for multivariate linear calibration has been described by Li (1988).

In a concrete calibration situation, such a general method can be tested. This test can also include non-linear transformations of the variables in the register. Li has shown that considerable gains may result from a multivariate approach.

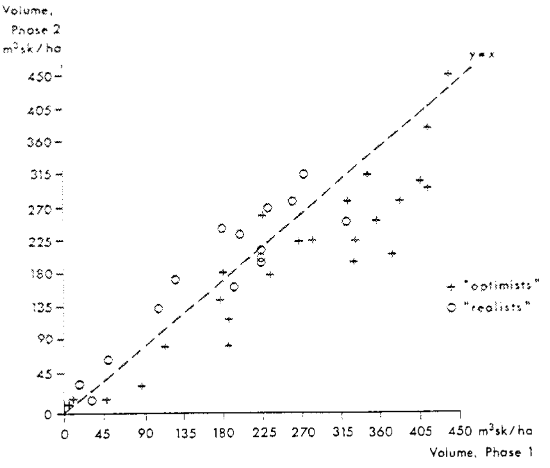

Fig. 15. Comparison of estimates of volume/hectare from phase 1 and phase 2. The register has been divided with regard to two surveyor groups: "optimists" and "realists".

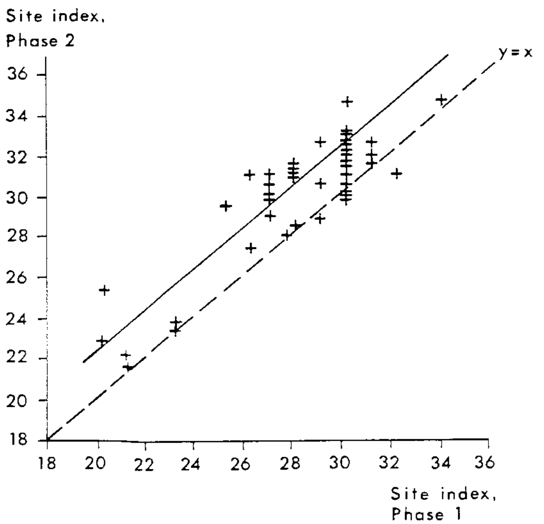

Fig. 16. Comparison of estimates of site index from phase 1 and phase 2. Calibration function based on difference estimation.

Area distributions

Besides means and totals, the area distribution with regard to different variables is important when forest management problems are to be solved. A very elementary description of the management possibilities is, for example, often made using an age-class distribution.

Estimates of the area distribution with regard to different variables depend, among other things, upon:

1. The degree of aggregation.

We assume that the sample plot is the smallest unit of area considered. Sample plots can be aggregated to compartments, which in turn can be aggregated to larger treatment or calculation units.

2. Errors in the data.

In phase 2, the sampling procedure leads to random errors. Phase-1 data contain systematic as well as random errors. The former can be more or less eliminated with the aid of calibration.

The effect of aggregation

All aggregation produces a shift in the value towards the population average. Figure 17 shows age-class distributions based on sample plots and compartments for a large forest holding in northern Sweden. In both cases, the age classification is based on the basal-area weighted mean age. Exactly the same single-tree ages have been used to estimate the plot and the compartment mean ages. The differences in the age class distribution are all due to the effects of aggregation. As shown in Figure 17, the image of the share of older forest is strongly dependent on the degree of aggregation. It can hardly be recommended to base any far-reaching conclusions regarding the management possibilities

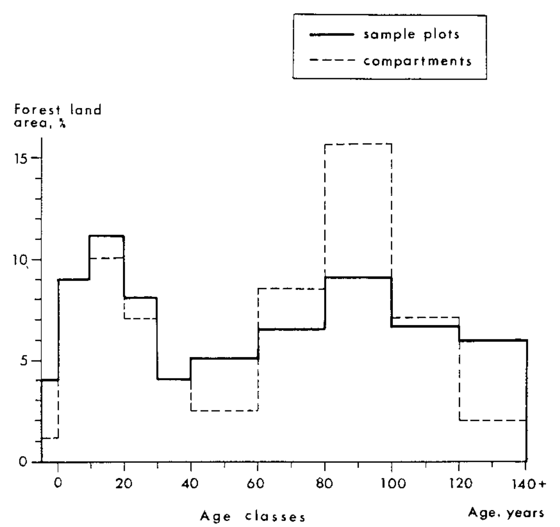

Fig. 17. Distributions of the productive forest land area over 10- or 20-year age classes for the MoDo Örnsköldsvik forest district, according to the methods employed in the Forest Management Planning Package. The distributions shown are based on the age of individual compartments and on the age of individual sample plots within these compartments.

on only one of these distributions. Both of them are correct, but would probably yield very different conclusions.

Hägglund (1982) has demonstrated similar results for area distributions over density classes. The area of compartments with low density is significantly overestimated if the distribution is estimated only from non-corrected sample-plot data. A large fraction of the sample plots with low density is situated in open spaces in otherwise closed stands.

A deeper understanding of the problems and possibilities of forest management requires that several different levels of aggregation can be handled at the same time. A high degree of resolution and a low degree of aggregation are suitable for biological models, particularly models for tree growth. A lower degree of resolution and a higher degree of aggregation are relevant to support decisions concerning the choice between treatment options and the realization of these.

The effect of data error

The following estimator, based on phase-2 data, is used to estimate the area in a certain class, e.g. age or volume class:

$$\sum_{h=1}^{L} \sum_{i=1}^{m_h} AR_{hi} \cdot C_{hi},$$

where

AR_{hi} denotes the represented area of the sampled compartment i in stratum h

$$C_{hi} \begin{cases} = 1 & \text{if the sampled compartment } i \text{ in stratum } h \text{ belongs to the class} \\ = 0 & \text{otherwise.} \end{cases}$$

If this estimate is to be unbiased, it is required that the variable C_{hi} can be established in an unbiased way. Since this variable can take on exclusively the values zero or one, this means that it must be determined without error, which evidently is impossible. However, by making a careful and more intensive survey of individual compartments in phase 2 the estimates of C_{hi} will be improved and thus also the estimates of the area distribution over a set of classes.

Considering only the goal of estimating total values for the forest holding, given a certain highest allowable cost, it would be possible to achieve a higher degree of accuracy by selecting a larger number of sampled compartments, while decreasing the number of sample plots per compartment (Ståhl, 1988, 1992).

Area distributions over a set of classes may also be estimated on the basis of phase-1 data. In this case, either non-calibrated or calibrated data can be used. In both cases the estimates are likely to be biased.

Comparison with traditional systematic circular-plot survey of an entire forest holding

The method described as "Phase 2" has been compared with the traditional method of systematic circular-plot surveys for estimating a number of forest characteristics. These comparisons have been performed as a part of the curriculum in forest survey at the Faculty of Forestry, and also as an experiment at a large forest company in northern Sweden.

Comparisons produced as part of the curriculum

From a theoretical standpoint, the survey method described in section "Phase 2" should produce unbiased estimates of the volume per hectare for a forest holding as a whole. As an alternative, such estimates can also be made using a traditional, systematic circular-plot survey of the entire holding. Independent surveys according to these two methods, have been performed on six forest holdings in southern Sweden. The aim of these surveys was to achieve total comparability of results, as part of a teaching exercise. Figure 18 shows, schematically how the sample-plots were allotted in the two cases.

In the case of the Forest Management Planning Package, total sums and standard errors were estimated according to section "Estimation methods for stratified PPS-sampling" (p. 32). In the case of the traditional circular-plot method, totals were estimated by multiplying the sample-plot value by area factors derived from sample-plot spacing and size (Table 3). Standard errors were estimated according to Matérn (1961).

Comparisons on the holdings of the Svanö Forest Company

During 1986, two totally independent forest inventories were carried out on the holdings of the forest company Svanö, Inc. The holdings

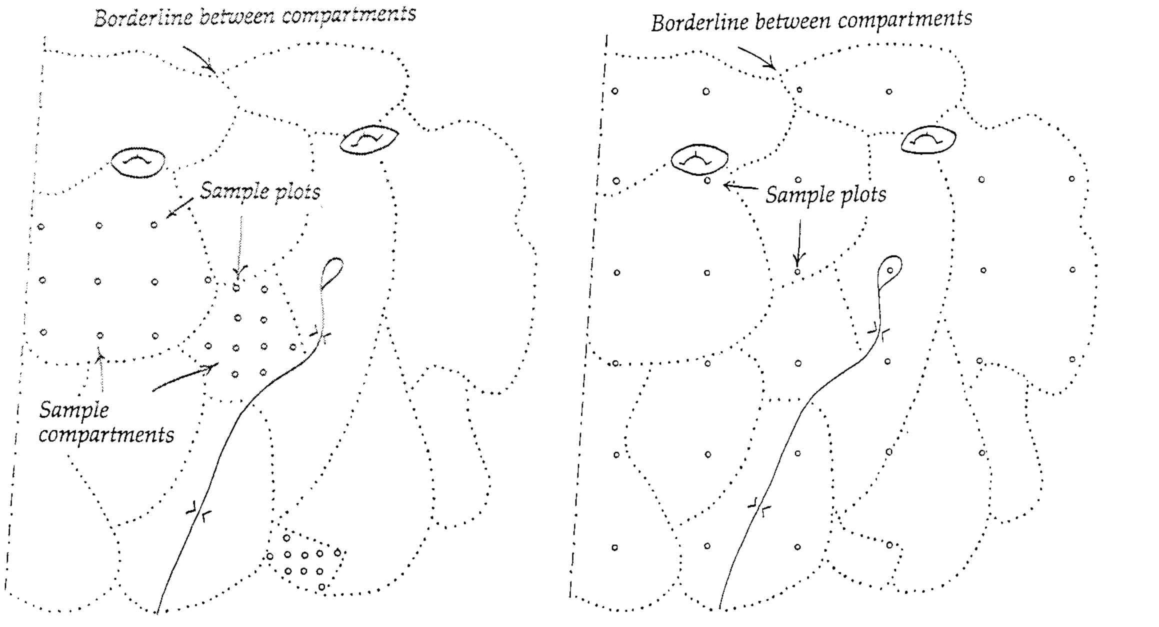

Fig. 18. Sample-plot allocation according to the Forest Management Planning Package on the left and according to the traditional method of circular-plot survey on the right.

Table 3. *Results from a comparison between the survey method employed by the Forest Management Planning Package (FMPP) and a traditional circular-plot survey (Trad).*
Note: in the case of "plots FMPP" the radius is 10 m; in the case of "plots Trad", see the footnotes

Forest holding	Area ha	Number of plots FMPP	Number of plots Trad	Volume per hectare FMPP m^3sk/ha	Volume per hectare Trad	Standard error FMPP %	Standard error Trad
Gräfsnäs + Hjälmared	1 419	680	153[a]	144	146	2.5	4.8
Tagel	1 040	420	120[a]	154	164	6.0	11.0
Barksäter	497	480	137[b]	165	169	5.0	5.1
Remningstorp 1	1 126	440	109[b]	147	150	3.9	6.3
Remningstorp 2	1 209	430	148[b]	162	156	4.3	5.6

[a]10 m radius. [b]7 m radius.

are situated in Northern Sweden and comprise 119,000 hectares of forest land. The two survey methods were:

1. A survey according to the Forest Management Planning Package, consisting of 1480 sample plots within 148 sampled compartments. The stratification was based on an up-dated register from a phase-1 survey made 15 years ago. The sample plot radius was in this case 10 m.

2. A survey according to the method used internally by the Swedish Cellulose Company, Inc. (SCA). This method uses a systematic circular-plot survey based on a tract system. The number of sample plots on timberland was 798. No information from the phase-1 survey was used in this case, either in the selection of tracts or in the estimation procedures. The sample plot radius was in this case 8 m.

The estimated mean errors of the volume estimates were 3.5 per cent for the Forest Management Planning Package, and about 4.1 per cent for the SCA method.

Table 4 displays estimated means and totals for the two methods.

Both of these surveys had the same aim: to support estimates of potential timber cut. In the

Table 4. *Estimates of means and totals for the forests of Svanö, Inc. according to the survey methods used in the Forest Management Planning Package, and in the internal survey routine of the SCA forest company.*
Note: in the FMPP case the sample plot radius is 10 m and in the SCA case 8 m

	Forest Management Planning Package	The SCA Method
Area in forest land, ha	121 000[a]	117 000[a]
Volume/hectare, m^3sk	89[b]	93[b]
Volume fraction in pine, %	36	30
Volume fraction in spruce, %	54	57
Volume fraction in hardwoods, %	10	13
Site index, m	20.3[c]	18.7[c]

[a] 3000 ha had been sold between the two inventories (after the FMPP-inventory).
[b] In the FMPP case: developable trees; in the SCA case: all trees from 0 cm diam.
[c] In the FMPP case: the site index estimated by the site-factor method is calibrated by adding 1.4 m according to the method described above (see e.g. Figure 16). SCA site index is uncalibrated.

SCA case, the HUGIN timber assessment system was used (Bengtsson et al., 1989). The yield-forecasting methods employed in the HUGIN system and in the Forest Management Planning Package are very similar in their essential components (Hägglund, 1981).

Given that the same management program is employed, the outcome of the yield forecasts ought therefore to be similar. A comparison was made, based on exactly the same removal in thinning and final felling, and on the same area subject to fertilization. Remaining aspects of the management program, e.g. choice of compartments, regenerative measures, etc., were designed so as to be as similar as the two timber assessment systems would allow. Total conformity is not possible to achieve. The Forest Management Planning Package is an optimizing system, in which treatments are selected at the compartment level. HUGIN is a simulation system, in which treatments are selected at the sample-plot level. Figure 19 shows how the two compared methods depict the development of the forest, as indicated by the volume per hectare.

The somewhat lower initial inventory level depicted by the Forest Management Planning Package in Figure 19 as compared to the value given in Table 4 is due to the fact that only trees larger than 8 cm in diameter at breast height were included in the final account of the forest development.

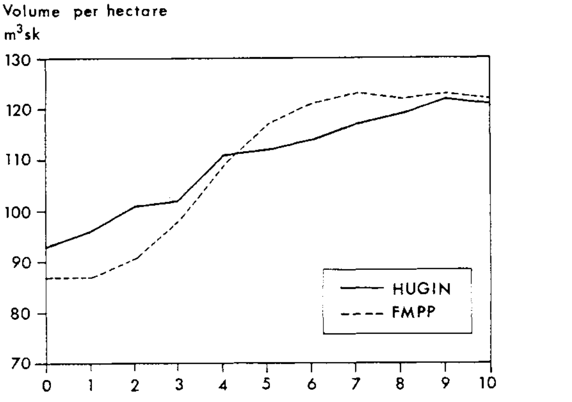

Fig. 19. Change over time of the standing volume per hectare on Svanö Inc. forest holdings. The forecasts shown are produced by the Forest Management Planning Package and the SCA survey method/HUGIN timber assessment system. The same management program is employed in both cases.

Optimization and implementation

The problem of forest management planning has been defined in Model (4) as pertaining to all compartments in a forest holding. However, in our analysis we instead use the sample-based Model (5). The solutions are then primarily used to support strategic decisions pertaining to the entire forest holding. However, it is possible to create operative priority functions by linking the solutions for individual sampled compartments in a second step with their descriptions in the phase-1 survey. These priority functions are directly aimed at taking advantage of the phase-1 data, which are available for all compartments.

Figure 20 shows how the different data sets are used for optimization and creating decision rules (priority functions). The final output is support for both strategic and operative decisions.

Solution algorithm

The selection procedure that precedes phase 2 ascertains that the surveyed sample plots will be representative of the entire forest holding as structured in compartments. In Model (5), q_i denotes the area represented by the sampled compartment i. The set of treatment options A

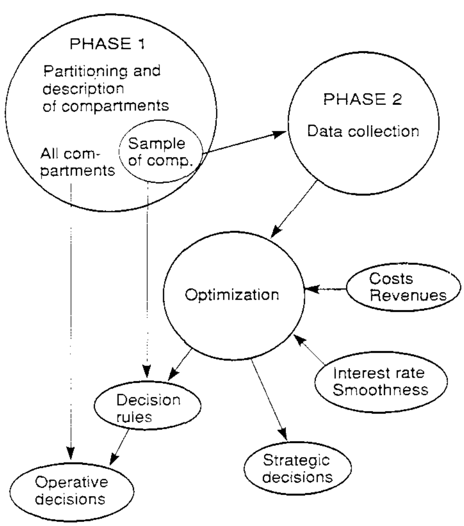

Fig. 20. The principles according to which data are used for optimization in the Forest Management Planning Package.

for the entire holding is replaced by the set of treatment options AS for the surveyed sampled compartments. Thus, we have to find the optimal H among AS by maximizing the model.

This problem includes a non-linear objective function, if $b<1$. A special algorithm has been developed (Jacobsson, 1986) for the solution of this non-linear optimization problem.

First, we define a subset $AS+$ of AS, which includes those treatment options which result in positive net revenues in all time periods. The function $c \cdot N^b$ is defined only for non-negative values of N. A subset $\Omega \in R^{\infty}$ is assigned to the subset $AS+$. The elements in Ω constitute net revenue profiles for the treatment options $H \in AS+$ and are denoted by

$$N_H = (N_{H1}, N_{H2}, \ldots),$$

where $N_{H1}, N_{H2}, \ldots$ denote periodic net revenues from treatment option H during periods 1, 2, ...

We can now write (5) as

$$\max_{N_H \in \Omega} U \quad \text{where } U = \sum_{p=1}^{\infty} e^{-rt_p} \cdot c \cdot (N_{Hp})^b. \tag{7}$$

The solution is obtained in two steps:

1. The search for optimal shadow prices for the periodic net revenues. The shadow prices are expressed as an interest factor that varies over time.

2. Maximization of net present value by compartments with given shadow prices for the periodic net revenues.

The search for optimal shadow prices

Jacobsson (1986) has shown that if

$N_H = (N_{H1}, N_{H2}, \ldots)$ maximizes

$$PV = \sum_{p=1}^{\infty} k_p \cdot N_{Hp} \tag{8}$$

and

$$\frac{N_{H(p+1)}}{N_{Hp}} = \left[\frac{k_{(p+1)} \cdot d_p}{k_p \cdot d_{(p+1)}}\right]^{1/(b-1)} \quad \text{for all } p \geq 1,$$

where

k_p denotes $k_p = \exp[-(2.5 \cdot r + \sum_{i=1}^{p-1} 5 \cdot m_i)]$

which is the shadow price for net revenues in period p based on the varying interest rates m_i; k_p is technically a discounting factor;

d_p denotes $e^{-r \cdot p}$, which is the discounting factor in Model (7).

then $K = (k_1, k_2, ...)$ is the optimal shadow-price vector and N_H is the optimal net revenue profile which maximizes (7).

A heuristic method is applied to the search for shadow prices. This method rests on the possibility of forseeing changes in the shape of the net revenue profile when shadow prices (the interest factor in different periods) change. Hoganson and Rose (1984) use a similar technique for minimizing the production costs for a certain periodically constrained output of harvest volumes.

Dynamic programming at the compartment level

Since the function (8) is additively separable, the maximization can be divided into standard problems of present-value maximization for each sampled compartment. We have chosen backwards recursive dynamic programming as our solution method, with shifts in forest generations as the solution steps.

Step 1

The first step is to maximize the net present value of forest generations that are born after the time period P. The following is valid after this period:

$$\frac{k_{(p+1)}}{k_p} = \frac{d_{(p+1)}}{d_p}; \qquad p > P,$$

which implies that the varying interest rate m_p for $p > P$ is a constant and has a value equal to r. With the support of the Faustmann formula (Faustmann, 1849), the net present-value maximization is performed for a set of J site types by means of a full enumeration. That is

$$\max_{H_j \in M_j} B_j \quad \text{where}$$

$$B_j = \frac{\sum_{a=0}^{u} \exp(r \cdot 5(u-a)) \cdot N_{Ha}(j)}{\exp(r \cdot 5 \cdot u) - 1}, \quad j = 1,...,J,$$

where

B_j	denotes land value for site type j in period $p > P$
H	denotes treatment option
M_j	denotes set of allowed treatment options for site type j
a	denotes stand age, 5-year periods
u	denotes time of rotation expressed as number of 5-year periods; u is given by H_j
$N_{Ha}(j)$	denotes net revenue per area unit at stand age a (periods), when treatment option H is applied to site type j.

Step 2

The maximization according to the Faustmann formula yields a land value as well as an optimal program of treatment for generations born after period P. For generations born in the time period p, where $1 \leq p \leq P$, a somewhat different procedure is used. The optimal treatment option and the land value is determined for every site type and period of birth by the solution of the maximization problem:

$$\max_{H_j \in M_j} B_{jp} \quad \text{where}$$

$$B_{jp} = \sum_{a=0}^{u} \frac{k_{(a+p)}}{k_p} \cdot N_{Ha}(j) + \frac{k_{(u+p)}}{k_p} \cdot B_{j(p+u)},$$

in which B_{jp} denotes land value for site type j in period p.

This maximization is performed backwards for every period of birth, beginning with period P. Since $B_{j(p+u)}$ can be set equal to max B_j from step 1 when $(p+u) > P$, it is possible to solve the problem for all periods $p \leq P$ by total enumeration.

Step 3

In the third and final step, the optimal treatment and the net present value are determined for the stand generation existing in the initial state. The optimization is done by compartments by means of a total enumeration, relying on the expression:

$$\max_{H \in M_i} g_i \quad \text{where } g_i = \sum_{p=1}^{u} k_p \cdot N_{Hp}(i) + k_u \cdot B_{ju},$$

where

g_i denotes net present value for compartment i

$N_{Hp}(i)$ denotes net revenue from compartment i in period p according to treatment option H

u denotes the 5-year period in which final felling occurs

M_i denotes set of treatment options for compartment i in the first stand generation and

B_{ju} is derived from step 2.

Inference from the optimization result to the population of all compartments

The main output from the solution algorithm is the optimal treatment option H^* for the compartments in the sample. Operative planning, however, requires that we make inferences from the optimal treatment of the sampled compartments to all individual compartments in the forest holding. The choice of treatment option in this case has to be supported by the information produced by the phase-1 survey, which covered all stands. The aim is to deviate as little as possible from those treatment options which would have been chosen if all compartments had been subject to a survey in phase 2.

Inoptimality losses

The treatment option H', which implies a deviation from the optimal option H^*, causes the occurrence of an inoptimality loss IL. This loss is defined by the equation

$$U(N_{H^*1} - IL, N_{H^*2},...) = U(N_{H'1}, N_{H'2},...).$$

If H' deviates from H^* by only a small amount, e.g. if the treatment of only one compartment is changed, then IL can be approximated by

$$IL = \sum_{p=1}^{\infty} k_p \cdot (N_{H^*p} - N_{H'p}). \qquad (9)$$

The aim when choosing treatment options for the total population of individual compartments should be to minimize the total inoptimality loss.

The treatment options consist of sequences of treatments that stretch over all periods. What we are doing when deciding on a treatment option is more along the lines of excluding certain options than of selection, once and for all, a final option. We limit the future latitude of choice by deciding what treatments to take during the first period.

Let M_{ih} be the set of treatment options for compartment i, which is such that treatment h will be used in the first period.

In that case, the solution to

$$\max_{H \in M_{ih}} g_i \quad \text{where } g_i = \sum_{p=1}^{u} k_p\, N_{Hp}(i) + k_u\, B_{hu}$$

is the maximum conditional net present value based on treatment h. We denote this value $MCPV_i(h)$.

Analogously with (9), the inoptimality loss for a treatment h in a compartment can then be calculated as the difference

$$MCPV_i(*) - MCPV_i(h),$$

where * denotes the optimal treatment.

The $MCPV_i$ may also be conditional on a set of treatments, such as the set of treatments which excludes final felling in period 1.

Decision trees

The choice of treatments in the first period for all compartments in the forest holding can be illustrated as a decision tree (Figure 21).

To support the decision stated at the decision

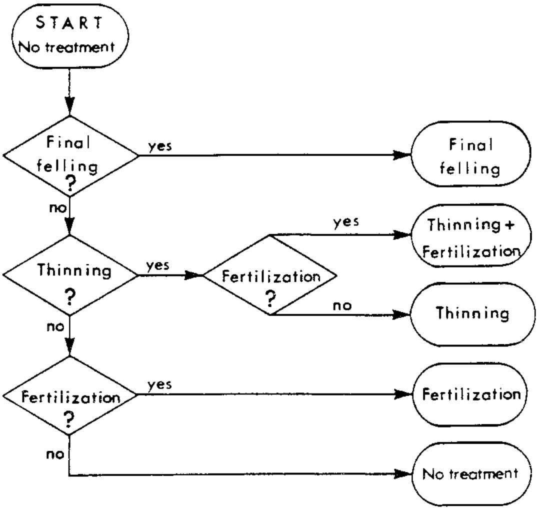

Fig. 21. Operative planning decision tree.

nodes in Figure 21, the following differences are estimated for all compartments:

$$MCPV_i(\text{No}) - MCPV_i(\text{Yes}), \qquad (10)$$

where

No denotes the set of treatments in the first period where the response to the question at the decision node is no;

Yes denotes the set of treatments in the first period where the response to the question at the decision node is yes.

If the value of the differences turns out to be positive, the response to the question at the decision node is no. At each decision node the number of treatments in the first period, which the MCPVi is conditional upon, decreases. Finally, only one treatment in the first period remains. The estimates of the differences (10) for the sampled compartments are made by means of direct calculations in conjunction with the optimization. For the remaining compartments, the estimates are based on priority functions.

Priority functions

One solution to the problem of estimating $MCPV_i(h)$ for all compartments is to base growth forecasts and economic calculations on phase-1 data and the optimal shadow-price vector. This method was used by Kilkki (1985) to transfer results from linear-programming solutions of an aggregated model to results pertaining to individual compartments. The procedure makes it possible to approach the strategic planning problem using aggregated compartment data without losing the link with the operative planning. Eriksson (1987) relies on a reduced-cost index that is analogous to an inoptimality loss. The index is determined by calculation by compartments, using the shadow prices from a linear-planning solution of an aggregated strategic forest management problem. He uses aggregated phase-1 data for solving the strategic problem and phase-1 data by compartments for calculating the reduced-cost index.

The main problem with this procedure is finding calculation models adapted to input data with a low degree of resolution and associated with large errors.

Calculation models are generally deduced from knowledge of biological and economic processes, and assume correct information about the variables used in the models.

However, if the assumption is that compartment data contain errors of an unknown nature, calculation models should then be created by induction from an empirical data set. This data set should include phase-1 data as well as calculated data y_i(phase 2), where y_i(phase 2) denotes differences among $MCPV_i(j)$-values for different treatments h based on phase-2 data.

Assume that V is a correct description of a forest holding, and that V denotes a $N \times M$ matrix with the elements v_{ij}. The number of compartments is N and the number of variables is M. Thus, v_{ij} is the true value for variable j in compartment i. By deduction, we create the calculation model

$$y_i = \beta_0 + \sum_{j=1}^{M} \beta_j \cdot v_{ij}, \qquad (9)$$

where

y_i denote the true value of the difference $MCPV_i(\text{No}) - MCPV_i(\text{Yes})$

β_j denote parameters in the model.

At the same time, we have a $N \times M$ matrix X, whose elements ε_{ij} represent the errors in an existing compartment register. As an estimator of y_i, we use

$$\hat{y}_i = b_0 + \sum_{j=1}^{M} b_j \cdot (v_{ij} + \varepsilon_{ij}).$$

Then, the actual error in the estimate $\hat{y}_i - y_i$ is

$$\hat{y}_i - y_i = b_0 - \beta_0 + \sum_{j=1}^{M} (b_j - \beta_j) \cdot v_{ij} + \sum_{j=1}^{M} b_j \cdot \varepsilon_{ij}.$$

The accuracy of this estimate can be measured by the expected value and the variance of the error

$$E[\hat{y} - y] =$$

$$= b_0 - \beta_0 + \sum_{j=1}^{M} (b_j - \beta_j) \cdot v_{ij} + \sum_{j=1}^{M} b_j \cdot E\,[\varepsilon_{ij}]$$

$$V[\hat{y} - y] = \sum_{j=1}^{M} \sum_{k=1}^{M} b_j \cdot b_k \cdot \text{cov}_{jk},$$

where cov_{jk} denotes the covariance between ε_{ij} and ε_{ik}.

Analysis of an empirical data set makes it possible to estimate the parameters b_j in the model with the aim of minimizing the errors $(\hat{y}_i - y_i)$.

If the deduced parameters β_j in Model (9) are used to estimate $\hat{y}_i$(ded) based on data from an existing compartment register $(v_{ij} + \varepsilon_{ij})$, the result is generally a lower level of accuracy than the best one possible to achieve.

To the existing variables j; $j = 1,2,\ldots,M$, we can add j_{M+1}, which is precisely $\hat{y}_i$(ded). With the aid of this extended compartment register and the model

$$y_i = b_0 + \sum_{j=1}^{M+1} b_j \cdot (v_{ij} + \varepsilon_{ij})$$

we can make estimates that are equally as good as or better than $\hat{y}_i$(ded).

The parameters β_j; $j = 0, 1,\ldots, M+1$ can be estimated by weighted multiple linear regression, based on information from the sampled compartments, i.e. by minimizing the expression

$$\sum_{i=1}^{m} (\hat{y}_i - y_i(\text{phase 2}))^2 \cdot w_i,$$

where

m denotes the number of sampled compartments

y_i(phase 2) denotes a calculation of y_i based on phase-2 data

w_i denotes weights, which are assigned low values for sampled compartments in a stage of development in which it is inappropriate to subject them to treatments.

The aim of the weighting procedure is to adapt the estimated model primarily to those compartments for which the sign of the estimated difference $\hat{y}_i$ is not known a priori. A model for supporting the decision of final felling, for example, need not be adapted in such a way that it is valid also for seedling stands.

The essence of this procedure is that a calculation model is created by induction, and that this model utilizes available phase-1 data with the errors and insufficiencies with which they are associated. The reason for this is that it brings the result as close as possible to the result that would have been produced if phase-2 data had been available for all compartments.

Examples of strategic and operational planning

The Forest Management Planning Package has been used for analysis of a large number of forest holdings in Sweden. A total of more than 2.5 million hectares has been analysed between 1983 and 1992, which is equivalent to one-quarter of the area of all forest holdings of sufficient area for an application to be meaningful. The applicable area is about half the size of the total Swedish forest area.

In the following, as an example of a practical application, the result of an analysis with the Forest Management Planning Package of the Remningstorp Research Forest (Jonsson & Kallur, 1989) will be presented.

Strategic planning

Strategic forest-management planning can be supported by a study of what the optimal treatment option will be, given different assumptions concerning the economic context. These assumptions roughly fall into two categories:

1. Assumptions that concern the shape of the net revenue profile, i.e. the rate of interest r and the smoothness parameter b;
2. Assumptions that concern future price changes of inputs and outputs.

Interest rate and sustained net-revenue profile

The shape of the optimal net-revenue profile is influenced by interest rate and by smoothness considerations. If a high rate of interest is desired, the result will be higher net revenues at an early stage and lower net revenues at a later stage. Figure 22 shows the net revenue profiles resulting from an interest rate of 1.5, 2, and 3 per cent. In all cases, the smoothness parameter has the value 0.5 or 1, i.e. we generate profiles with or without a requirement for smoothness.

The basic pattern – high initial net revenues as a response to a high rate of interest – is shown in Figures 22 and 23. However, the effect of the rate of interest is moderated by a requirement for smoothness.

Values of the smoothness parameter other than 1 result in a more or less strong moderation on the profile of net revenues. Figure 24 shows the net-revenue profile given a 1.5 per cent rate

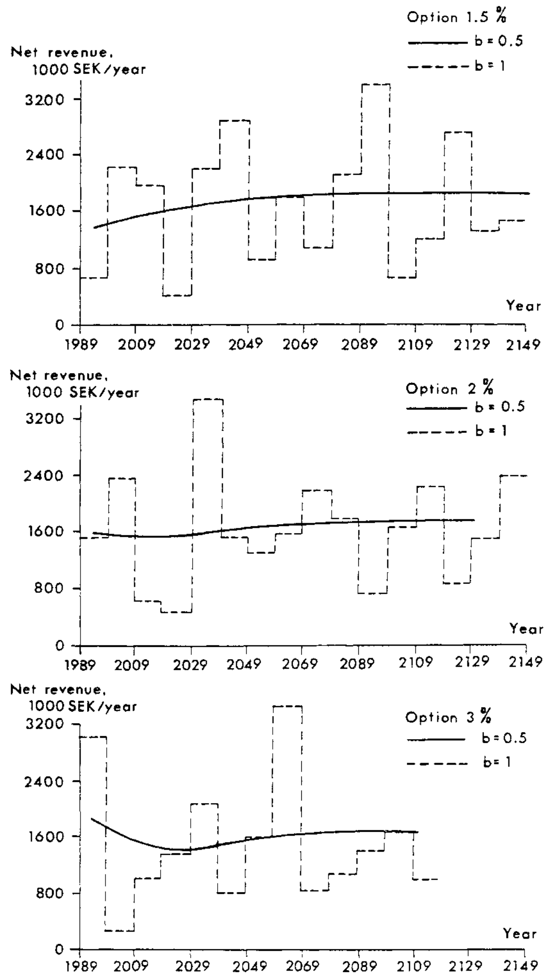

Fig. 22. Net-revenue profiles, given an interest rate of 1.5, 2, and 3 per cent, with or without a requirement for smoothness.

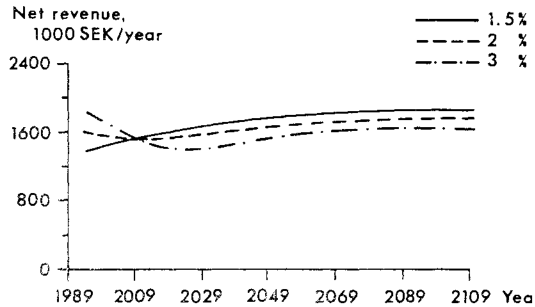

Fig. 23. Comparison of the net-revenue profiles, given an interest rate of 1.5, 2, and 3 per cent and a requirement for smoothness ($b = 0.5$).

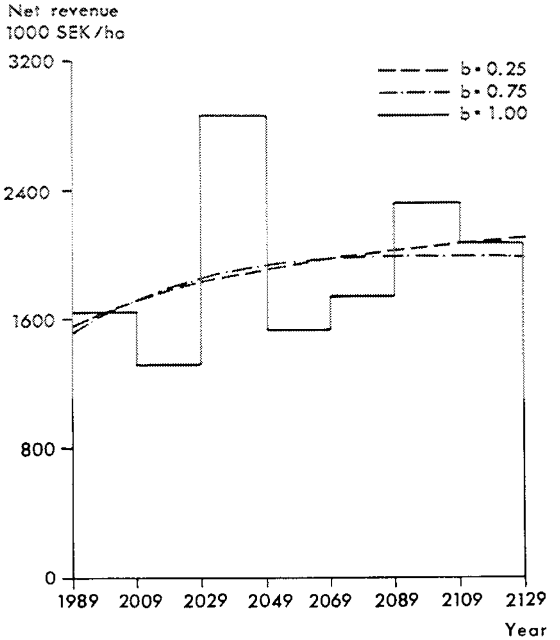

Fig. 24. Net-revenue profiles, given varying requirements for smoothness of revenues at a constant rate of interest. The value of b is 1.0, 0.75, and 0.25, and the value of r is 1.5 per cent.

of interest and three different values of the smoothness parameter: 1.0, 0.75, and 0.25.

The vector of marginal rate of interest is strongly affected by a requirement for smoothness of revenues.

From the optimality condition

$$\frac{N_{H(p+1)}}{N_{Hp}} = \left[\frac{k_{(p+1)} \cdot d_p}{k_p \cdot d_{(p+1)}}\right]^{1/(b-1)},$$

it follows that

$$\left[\frac{N_{H(p+1)}}{N_{Hp}}\right]^{1-b} = \exp\,(5 \cdot (m_p - r)), \qquad (11)$$

where

r denotes the desired rate of interest in Model (10)

m_p denotes varying rate of interest between period p and period $p+1$.

Model (11) shows that the varying rate of interest m_p between the time periods p and $p+1$ has a higher value than the interest rate r called for in the objective function when the net-revenue profile leans upward (the ratio $N_{H(p+1)}/N_{Hp} > 1$).

There is a deviation between the result from a pure present-value maximization with a constant rate of interest r, and the optimal solution with an expressed desire for smoothness of revenues. As could be expected, the magnitude of the deviation increases as the requirement for smoothness increases.

Table 5. *Present value at 1.5 per cent interest rate for different values of the smoothness parameter*

Smoothness parameter b	Present value, million SEK	Present value, relative terms
1	127.9	100
0.75	127.7	99.8
0.50	127.6	99.8
0.25	127.5	99.7

Regardless of the requirement for smoothness, it is possible to calculate the net present value of all solutions at a fixed rate of interest r. Table 5 shows present value at interest rate 1.5 per cent at varying levels of smoothness.

The loss in net present value is reflected as the degree of deviation between the varying interest rate m_p and the fixed interest rate r in Model (5), as shown in Figure 25.

As a rule, the losses in net present value due to a smoothing-out of the net revenues are small. Most forest holdings can be made to generate revenues without large fluctuations in level, by harvesting mature stands either earlier or later than would have been the case without any requirement for smoothness. However, these measures will not prevent the initial state of the forest from causing trends in the development of the net-revenue profile. A requirement for smoothness that does not even allow any trends

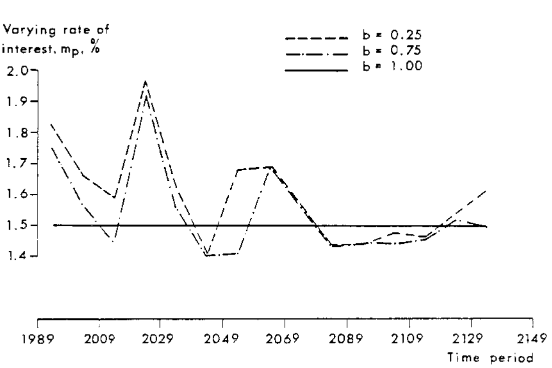

Fig. 25. The influence of the requirement for smoothness of revenues on the varying rate of interest. $r = 1.5$ per cent.

in the yield over time, but instead requires the removals to be the same in all periods, may cause major losses to occur.

Changes in the shape of the net-revenue profile reflect changes in the management strategy for harvesting and other management activities. To change the net-revenue profile, it is necessary to change the management of the entire holding, as well as that of individual compartments. Table 6 shows how the volume of the removals during the first 10-year period varies among six different net-revenue profiles.

The effect of timber-price changes

One of the most difficult problems facing the decision-maker is evaluation of the effect of future timber-price changes on the choice of treatment options. The analysis of the Remningstorp property includes three different price-development scenarios:

Option A
Present prices remain the same in real terms.

Option B
Gradual decline in price level during the first periods. During the fourth 5-year period, the price of sawtimber is 40 per cent below the present level, and the price of pulpwood 20 per cent below. This level remains stable during the subsequent periods. It should be observed in this context that the procedure for tree evaluation rests on the assumption that the trees are divided into logs with regard to the price relations in the price lists.

Option C
Price changes are differentiated by quality. Price levels for different qualities change gradually during the first periods. The following price levels, relative to today's levels, apply during the fourth 5-year period:

Quality	*Pine saw logs, %*	*Spruce saw logs, %*
Unsorted (I–IV)	120	110
Fifth (V)	100	100
Sixth (VI)	80	90

The price of pulpwood remains at the present level. The price level during period four is assumed to persist during the remaining periods.

Figure 26 shows how the levels of removals and standing volume change over time according to the three price options.

Table 7 shows the level of the removals during the first 10-year period, as influenced by the three price-development options.

Table 7 shows essential differences between the price options A and B. The expected future price decrease in option B is met with an increased amount of thinnings in the first 10-year period. The choice of compartments for final felling is also influenced. An expected decrease in timber price results in an immediate and particularly strong decrease in the relative revenue for stands with a low value per unit of volume. Final fellings are thus reassigned to compartments with a higher degree of mixed-in hardwoods, and to compartments with a low stocking level.

The conflict between the calls for a high net present value and a sustained profile of net revenues, is solved in option B by a high initial degree of removals in terms of volume. This is accompanied by an increase in the value per volume-unit harvested in the future, brought about by delayed final felling in high-volume stands.

Table 6. *Removals during the first 10-year period at the Remningstorp holding, given six different net-revenue profiles*

Option		Thinning,		Final felling,		Total removals, m^3sk/year
r	*b*	ha/year	m^3sk/ha/year	ha/year	m^3sk/ha/year	
3	1	24	58	47	303	15 700
2	1	22	54	21	325	8 000
1.5	1	24	54	9	317	4 200
3	0.5	26	59	25	326	9 700
2	0.5	28	59	21	322	8 500
1.5	0.5	26	59	19	315	7 600

Table 7. *Removals during the first 10-year period, assuming three different price developments;* r = *1.5 per cent and* b = *0.5*

Treatment		Option A	B	C
Thinning	area, ha/year	26	42	28
	m^3sk/ha/year	59	58	58
	m^3sk/year, total	1 533	2 469	1 653
Final felling	area, ha/year	19	20	19
	m^3sk/ha/year	315	297	312
	m^3sk/year, total	6 031	5 927	6 035
Total removals	m^3sk/year	7 564	8 396	7 688

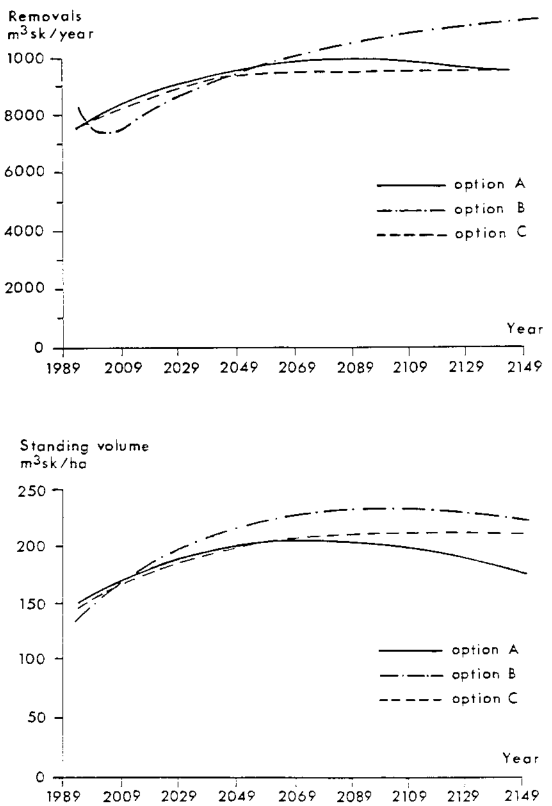

Fig. 26. Change in removals and in standing volume, as influenced by three sets of assumptions regarding price changes; $r = 1.5$ per cent and $b = 0.5$.

The differences between options A and C are marginal. This should be interpreted as evidence that it is difficult to find measures geared to exploiting the increasing price differentiation in option C. The small increase in the share of pine removals, particularly in final fellings, is explained by the fact that the quality of spruce sawtimber in this region generally is higher than that of pine sawtimber. The expected future increase in price differentiation among sawnwood of different qualities is best exploited by delayed final felling in spruce compartments. Obviously, the conclusion will be different if a future price increase for pine alone is expected.

In the long term, all three options result in different forms of optimal normal forests. Table 8 shows the rotation age and level of removals for these.

As expected, the rotation age increases when the price of timber decreases (option B). The price increase for high-quality saw logs also results in a prolonged rotation. The levels of removals and net average production increase with prolonged rotation. The maximum average production is reached at a rotation of approximately 95 years. However, the maximum is flat.

Operative planning

In the operative planning process, the differences among the $MCPV(j)$ values for different activities j are estimated with the aid of regression functions. For the Remningstorp case, two functions have been developed:

1. A function that estimates the difference

$$MCPV_i(j; j \in A) - MCPV_i(j; j \in B),$$

Table 8. *Rotation and level of removals for the future optimal normal forest, assuming three different options of price change;* r = *1.5 per cent*

	Option A	B	C
Rotation, years	84	94	89
Removals, m^3sk/year	9 500	9 600	9 600

where

A denotes the set of treatment options that excludes final felling in period 1;

B denotes set of treatment options that includes final felling in period 1.

This difference is denoted *IL F* (inoptimality loss, final felling). The *IL F* value is used in the process of selecting compartments for final felling.

2. The other function estimates the difference

$MCPV_i(j; j \in C) - MCPV_i(j; j \in D)$,

where

C denotes the set of treatment options excluding fellings in period 1;

D denotes set of treatment options including thinning in period 1.

This difference is denoted by *IL T* (inoptimality loss, thinning). The *IL T* value is used in the process of selecting compartments for thinning, after final-felling compartments have already been selected.

The final-felling function received the following form:

	T-value
$IL\,F = -2351$	0.39
$-34.2 \cdot$ *age*	0.75 (years)
$+0.0082 \cdot$ *stem-number*^{2}	3.33 (stems/ha)
$-0.0646 \cdot$ *volume*^{2}	6.57 (m^3sk/ha)
$+8960 \cdot$ *spruce fraction*	1.39
$+7020 \cdot$ *pine fraction*	1.10

The thinning function received the following form:

	T-value
$IL\,T = +443$	0.66
$-0.000467 \cdot$ *stem-number*^{2}	2.04 (stems/ha)
$+5686 \cdot$ *mean stem volume*	5.29 (m^3sk)
$-8166 \cdot$ *hardwood fraction*	3.08
$+11552 \cdot$ *hardwood fraction*^{2}	3.80

Figure 27 shows the relationship between observed values for *IL F* and *IL T*, and corre-

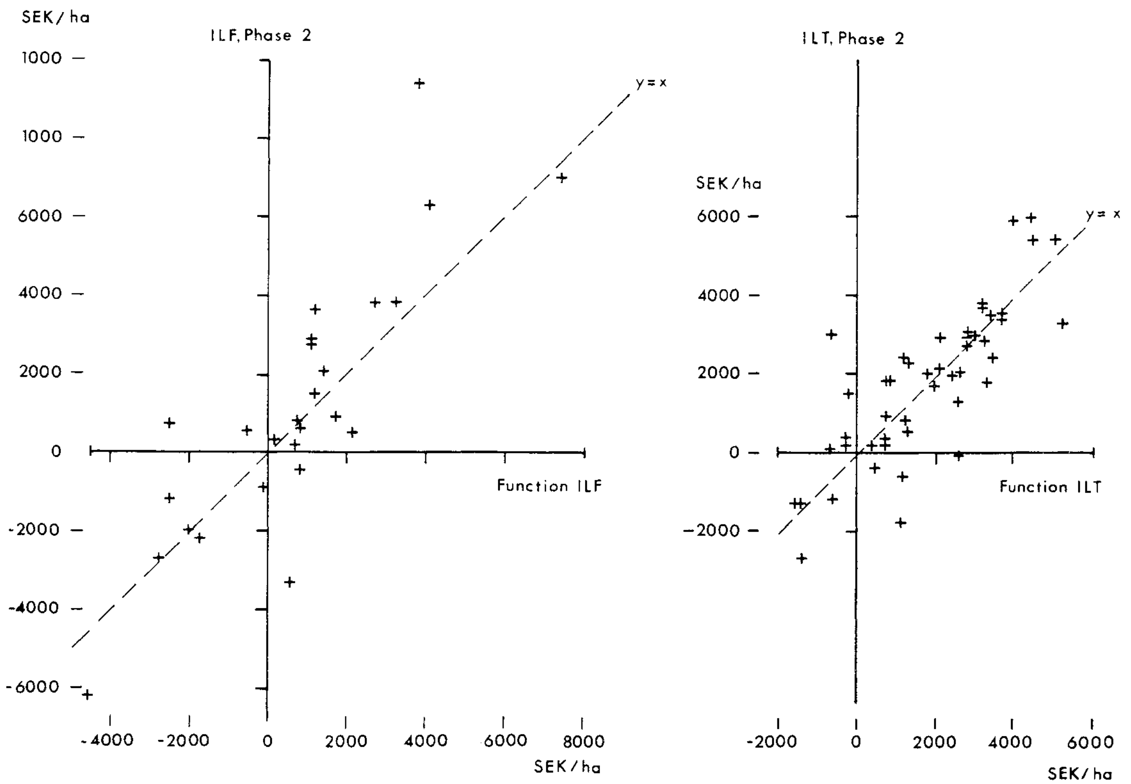

Fig. 27. The relationship between values for *IL F* and *IL T* observed in phase (2), and values estimated with the regresssion functions.

sponding values estimated with the regression functions.

The magnitudes of deviation in Figure 27 include the errors in the phase-2 survey. This means that the deviations between the estimated values and the true values of the inoptimality losses are overestimated.

With the aid of the priority functions, the main features of the optimization results based on phase-2 data can be transferred to all compartments.

Experience from practical applications

In this final chapter we will give examples of changed strategies in Swedish forestry caused by the use of the Forest Management Planning Package. Additional examples could be given (see Jacobsson & Jonsson, 1991).

The potential of forestry according to our analysis

Until about five years ago, forestry discussions in Sweden were dominated by an overwhelming problem – "the wood-supply dip". In large parts of Sweden, especially in the north and in company forests in southern Sweden, the age-class distribution is markedly uneven (see for example the age-class distribution for the forests of Svanö, Inc. – Fig. 28).

It is evident that if these stands were to be managed according to "age-norms" for thinning and final felling, the result would indeed be a wood-supply dip. Generally, however, it is possible to eliminate the effect of an uneven age-class distribution without a great deal of difficulty.

The supply of wood today and in the future essentially depends on how much we cut in relation to growth. A wood-supply dip will only be produced if the level of cut exceeds growth for a prolonged period.

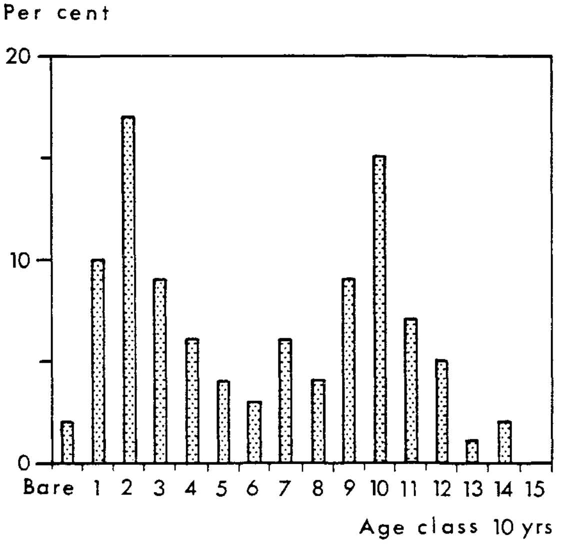

Fig. 28. Age-class distribution in the forests of the forest company Svanö, Inc.

The proper cutting level is primarily an economic problem. The age-class distribution has only an indirect effect, insofar as it can say anything about the economic maturity of the forest. A good supply of old stands generally justifies a high cutting level in relation to growth.

Biological age, and economic cutting maturity, are two different things. As a rule, there are many biologically old stands which are not at all economically mature for cutting. At the same time, most of the middle-aged or young stands today will be economically mature for cutting at a considerably lower age than that to which we are currently accustomed.

Our experience from applying the Forest Management Planning Package indicates that, as a rule, it is possible to cut almost the entire growth and at the same time, neutralise the wood-supply dip, without incurring serious losses in net present value. This is achieved by reasonable prolongations of the rotation ages and premature cutting, in combination with adaptation of the thinnings in older forest.

Figure 29 shows the development for some variables of interest, according to an optimized timber assessment made for the forest of the Svanö, Inc. forest company.

The optimization was made at the required interest rate of 2.0 per cent, combined with a strong requirement for even net revenues over time. The variation of the marginal rate of interest over time about the normal 2 per cent level shown in Figure 29, illustrates the degree of deviation from an optimization without a required even net revenue.

The heavy solid lines in Figure 29 represent a management option which maximizes gross revenue without regard to costs and interest rate in the analysis; the costs are considered afterwards. This option is rather similar to the management program used by the Svanö, Inc. forest company and is consistent with Swedish forestry practice in recent decades.

We note that the future "interest bulge", as the wood-supply dip should rightfully be named, causes premature harvesting of stands

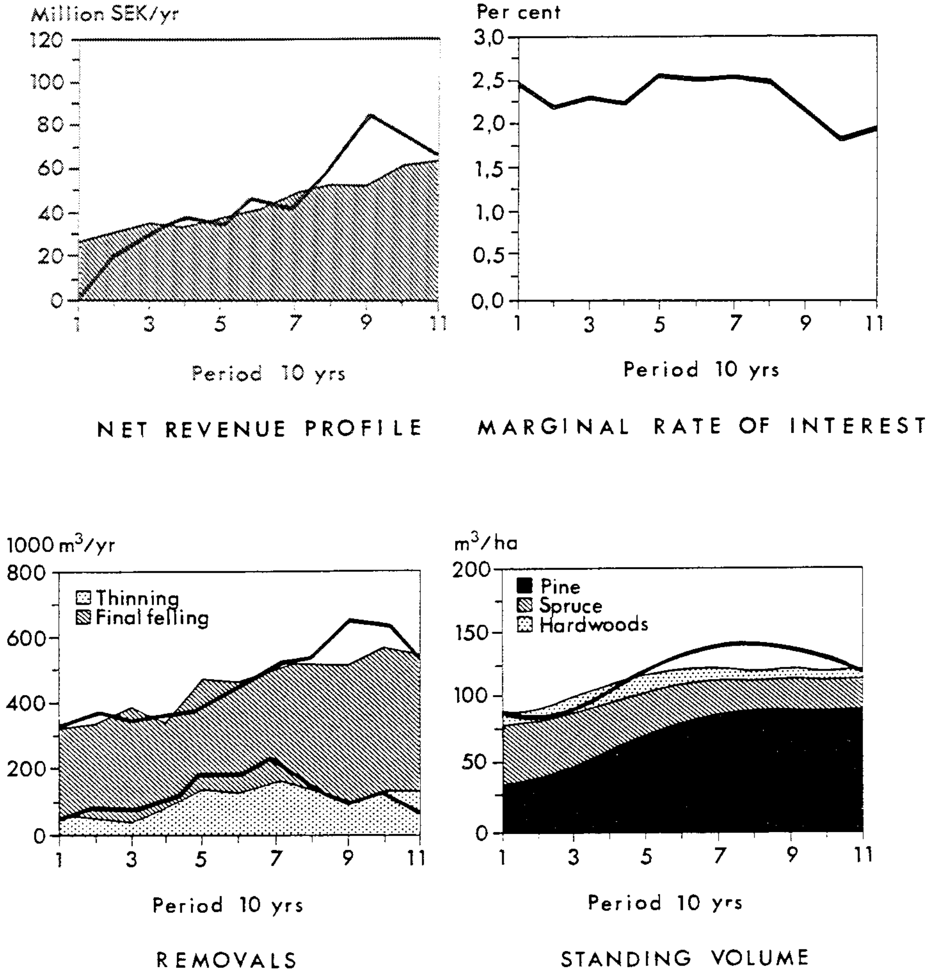

Fig. 29. Development of the forest company Svanö, Inc.'s forests at a required long-term 2 per cent rate of interest. Heavy solid lines represent a management option which maximizes gross revenue without regard to costs and interest rate in the analysis (see text).

with a maximum yield of 2.6 per cent, in order to achieve a smoothing-out over time of net revenues and harvested volumes. The degree of prematurity of the harvest is not more pronounced in the interest bulge than it is in the initial state. The total loss of net revenue due to this smoothing-out amounts to 0.6 per cent of the total net present value.

Analysis must replace rule of thumb

Figure 30 shows the relationship in principle between the economic need for final felling and the volume per hectare at a given stand age. Economic need for final felling is here understood to mean the net present-value gain of an immediate final felling when the required rate of interest on investments amounts to a very few per cent. At a very low volume per hectare, the stand is not capable of yielding a sufficient rate of return in relation to the land value, and at a very high volume it is not capable of yielding a sufficient rate of return on the high value of the standing crop (stumpage value).

In practice, decisions about final felling are often founded on rules of thumb, which are established and adjusted in forestry schools and calibrated on forest excursions. The professional "forester's eye" is trained to focus on objects such as those on the left in Figure 30, i.e. stands which do not fully utilise the land.

If these stands do not exist, or occur only

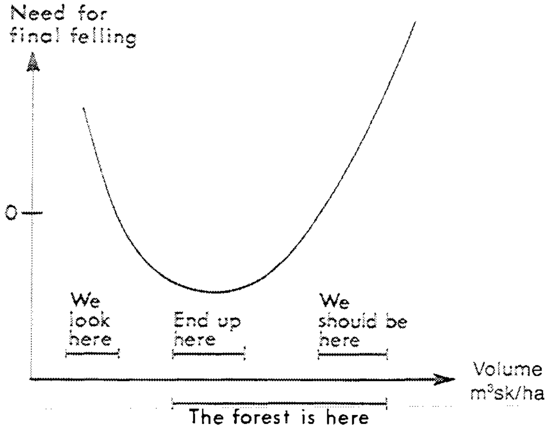

Fig. 30. Relationship in principle between the economic need for final felling and the volume per hectare at a given stand age.

very sparsely, something else must be selected. Surely it is then difficult to realise that what should be considered is a totally different type of stand – namely, the most well-stocked ones, with the biggest trees. Neverthless, what usually happens is that a selection of half-stocked, half-size stands is made, somewhere in the middle of the density scale – exactly those stands which give the greatest rate of return at a given stand age. With only slight exaggeration, one could claim that this is tantamount to final felling of well-thinned stands.

The Forest Management Planning Package as a tool for better understanding

The Forest Management Planning Package is a tool. Like all tools, it can be used and abused. We do believe, however, that a well-sharpened axe is a good thing, even if it can be used for murderous purposes.

For the Forest Management Planning Package to function well, however, it is necessary that the decision-makers take an active interest in forest management. This applies to all systems for strategic forest management planning. No forest management system could ever be created, which allows the decision-maker to specify a management policy in a simple and conclusive way, and then to rely on the system to do the rest. Application of the Forest Management Planning Package is an iterative search process, in which the results prompt re-evaluation of the assumptions, which in turn will lead to new results, and so on (see Figure 3). In the end, this search process leads to a treatment option close to the optimal one. During the work, the actively engaged decision-maker will gain insights and knowledge about the potential of the actual forest and how it can be optimally used.

In the end, it is only such insights and knowledge which can motivate application of the system.

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