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HUMAN CAPITAL POLICIES IN A STATIC, TWO-SECTOR ECONOMY WITH IMPERFECT MARKETS

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Titolo: HUMAN CAPITAL POLICIES IN A STATIC, TWO-SECTOR ECONOMY WITH IMPERFECT MARKETS

Human capital policies in a static, two-sector economy with imperfect markets

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The paper studies a two-sector economy with investments in human and physical capital and imperfect labor markets. Workers and firms endogenously select (paying a fixed cost) the sector they are active in, and choose the amount of their investments. The economy is characterized by pecuniary externalities. Given the partition of the agents among the two sectors, at equilibrium there is underinvestment in both human and physical capital, as in Acemoglu (1996). A second externality is induced by the self-selection of the agents in the two sectors. When the difference between total factor productivities (TFP) is sufficiently large, subsidies to investments in education in the low TFP sector and fixed taxes increasing the cost to access the high productivity sector increase expected total surplus, while subsidies to investments in the high TFP sector can actually reduce it. To the contrary, subsidies to the amount of investments in human capital in the high TFP sector may have a positive effect on social welfare when the TFPs are sufficiently close.

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1. INTRODUCTION

Existence and relevance of the externalities associated with investments in human capital have been an important topic of research in the last twenty years, mostly (but not exclusively) in connection with growth theory. Among the attempts to provide a microeconomic foundation for the existence of increasing returns to the investments in human capital is Acemoglu (1996). His paper considers a parametric (Cobb-Douglas production function and quasi-linear utility function) model where firms and workers invest in physical and human capital in an environment characterized by the lack of perfect competition in the labor markets (firms and workers are paid an exogenously given share of the total output) and impossibility to sign, before the investments take place, binding contracts. Equilibria of the model exhibit a pecuniary externality, because an increase in firms' and workers' investments is unambiguously Pareto improving. An important policy implication is that subsidies to the investments of any subset of workers are Pareto improving.

The purpose of our paper is to extend the analysis to a similar (parametric) set-up where, however, firms and workers choose both the *amount* and the *type* of their investments. We consider a two-sector model (*à la* Roy (1951)). Firms are ex-ante identical, while workers are heterogeneous, because they differ in a parameter determining the marginal disutility of their investments in human capital. By choosing a particular type of investment, and paying a fixed cost, agents self-select into the associated industrial sector. Production requires sector-specific labor and capital and is obtained after a random match worker-firm. We do not analyze in detail the investments in education. We simply assume that there are two different kinds of (instantaneous) education (for instance, high school and college). One (costlier) allows to enter the more productive sector of the economy (the one with the higher total factor productivity, TFP). After agents have selected the type of education, they still have to decide how much effort to devote to it. Effort in education translates one-to-one into additional units of human capital. An alternative, and equivalent, interpretation of the model is that workers simply have an elastic labor supply and that their investments in education determine the *type* of labor (economist vs. bricklayer) they can supply. We study and compare the properties of the equilibria in the Walrasian economy and in an economy with labor market frictions. The essential feature of the economy is that the match worker-firm takes place *after* their investments. However, to facilitate comparison with the results of Acemoglu (1996), we also assume that labor markets are non-competitive. We are mostly interested in the inefficiency properties of equilibria. Equilibria in the economy with frictions are evidently Pareto inefficient (but not Pareto inferior to the ones of the Walrasian economy). More relevant, they are also constrained Pareto inefficient when we define the constraints in the planner's problem (maximization of the conditionally expected total surplus) so to mimic the imperfections of the markets. There are two distinct externalities at work in our economy. The first is basically the one pointed out in Acemoglu (1996): Given the threshold δ^* defining the partition of the agents into the two sectors, an increase in the investments in human and physical capital is Pareto improving. Also, given δ^*, if some workers increase their investments, this translates into a benefit for all the workers and the firms. However, and this is the second source of externalities, changes in the value of δ^* also have an impact on social welfare, because, in an environment with heterogeneous workers, they modify the (conditionally) expected behavior of the two sets of agents. This is a general property of two-sector economies with market imperfections. In the analysis of investments in human capital, the "composition effect" as a source of externalities has been first studied by Charlot and Decreuse (2005, see also Charlot, Decreuse, and Granier (2005)) in a random matching, two-sector economy. A fairly general extension of their analysis is in Mendolicchio, Paolini, and Pietra (2008). In these papers, however, the supply of human (and physical capital) is perfectly inelastic.

In the environment considered here, it can be Pareto improving to *increase* the value of δ^* (i.e., to shrink the set of workers active in the high productivity sector). The intuition behind this result is fairly simple. Firms' investments in sector s are chosen (before matches take place) considering the conditional expectation of the amount of human capital of the workers active in s. Under our assumptions, workers with high marginal disutility of the investment are active in the low TFP sector. To increase δ^* means to shift some workers from the high to the low TFP sector. These individuals are the ones with the lowest amount of human capital in the high TFP sector, but they become the ones with the highest amount in the low TFP sector. Hence, an increase in δ^* induces, simultaneously, an increase in the conditional expectation of the amount of human capital in both sectors. This pushes up the investments in physical capital, which, in turn, stimulate investments in human capital in both sectors. At the equilibrium, given that expected producers' surpluses in the two sectors are strictly positive, this mechanism does not necessarily imply (modulo an income redistribution) a Pareto improvement. We show that it does when the TFPs are sufficiently different. Subsidies to the amount of educational investments have two effects: a (always beneficial) direct one and an indirect one, because of their effect on the threshold value δ^*. When the difference in TFPs is sufficiently large, it turns out that both effects are (modulo a redistribution) Pareto improving in the case of subsidies to the amount of educational investment in the low TFP sector, while they push in opposite directions in the case of subsidies in the high TFP sector.

If we interpret the structure of the model in terms of college *vs.* high school education, the main policy implication is that, when the difference in TFP is large, subsidies to investments in education should be concentrated on high school students, where they are unambiguously Pareto improving.

Alternatively, subsidies to the investment *effort* in higher education should go together with sufficiently high fixed tuitions, so to avoid the Pareto worsening effect due to the adverse "composition effect".

There is a large literature on subsidies to education (for an overview, see Carneiro and Heckman (2003)). In our set-up, subsidies are motivated exclusively by efficiency considerations in an environment where initial distribution of wealth is irrelevant, while labor market imperfections are the key issue. On the contrary, in the literature, most commonly, subsidies to education are considered in economies where imperfections in the financial markets are the key source of inefficiency, so that the initial distribution of wealth matters. See, for instance, Caucutt and Kumar (1999), Lewis and Winston (1997), Dynarski (2003), and Charlot and Decreuse (2007)). Closer to our perspective are several papers where the incentive effects of tuitions and subsidies to higher education are analyzed, see, for instance, Blankenau (2004), Blankenau and Camera (2006, 2008), Sahin (2004), and Su (2004). In particular, Sahin (2004) provides some empirical evidence suggesting a "positive relationship between the total time spent on academic activities and the tuition paid by a student controlling for ability and family income. Additionally, students in states with higher public tuition do study harder". This appears to be consistent with our results related to the composition effect (see, also, Garibaldi *et alii* (2007)),.

The aim of our paper is primarily theoretical. However, it is worthwhile to complement our analysis with a few, sketchy information about human capital policies adopted in several countries. The first thing that we can observe is that, at least in OECD countries, total expenditures per student are typically somewhat increasing in the level of education, and, in most cases, the share of the total costs of education paid by private sources is also increasing (see Table 1 below). However, as pointed out in OECD (2008), about 3/4 of the public subsidies to households and other private entities for educational purposes (in average 0.4% of the GDP in 2004) are aimed to tertiary education, and they are about 18% of the total expenditure at this level. The net result is that the actual private expenditure (net of subsidies) in several OECD countries (and in the average) covers a percentage of total costs of tertiary education which does not appear to be very different from the percentage related to pre-tertiary education. The OECD (2008) report indicates that the countries considered can be classified into four groups. In particular, one group includes "Australia, New Zealand, the UK, the US and the Netherlands, where there are potentially quite high financial barriers to entry tertiary-type A education, but also large public subsidies are provided" (p. 238). On the contrary, in most of the countries in continental Europe, there are low financial barriers to entry and low subsidies to tertiary education. Our theoretical results suggest that it would be particularly hard to rationalize the "continental Europe" model in terms of the incentives provided.

Expenditure on education and financial aid, selected countries, 2004

	A1	A2	B1	B2	C1	C2	C3
Austria	9446	95.3	9595	93.7	5.4	0.9	18.1
Belgium	7751	94.9	7920	90.4	4.7	1.9	15.7
Denmark	8849	97.8	11387	96.7	18.6	13.1	30.3
Finland	7441	99.2	7697	96.3	7.3	3.1	16.7
France	8737	92.7	7372	83.9	3.9	3.4	7.9
Germany	7576	81.9	7724	86.4	7.7	4.9	17.9
Greece	5213	93.8	4521	97.9	2.0	0.3	5.2
Ireland	7110	96.4	7445	82.6	10.5	9.2	14.8
Italy	7843	96.1	4812	69.4	4.4	1.6	16.7
Netherland	7541	94.1	8637	77.6	11.8	7.0	27.0
Norway	11109	99.0[a]	10449	93.7[a]	18.5	7.7	40.8
Spain	6701	92.5	6853	75.9	2.9	1.5	7.8
Sweden	8039	99.9	8355	88.4	11.9	5.9	28.2
UK	7090	86.6	8792	69.6	6.1	2.0	23.9
U.S.	9938	91.3	19842	35.4	5.0	--	20.7
Japan	7615	91.3	--	41.2	3.4	0.2	18.2

A1: Expenditure on education per capita, ISCED 2-4
A2: Relative proportion of public expenditures, % , ISCED 1-4
B1: Expenditure per capita, tertiary education (excluding R&D)
B2: Relative proportion of public expenditures, %
Source: OCDE, Education at a Glance 2008, USD converted using PPPs for GDP.
[a]: refers to 1995
Source: Eurostat, Financial Aid to students
C1: Financial aid as % total expenditure ISCED 1-6
C2: Financial aid as % total expenditure ISCED 1-4
C3: Financial aid as % total expenditure ISCED 5-6

Table 1

The structure of the paper is the following. Section 2 discusses the general features of the model. Section 3 and 4 discuss equilibria in the benchmark, Walrasian, economy, and in the economy with imperfect labor markets. Section 5 studies the properties in terms of welfare of the equilibria of the economy with frictions. Most of the proofs are in Appendix 1. Appendix 2 establishes that qualitatively similar results hold for the economy where agents are paid their marginal products, as long as matches take place after the investment decisions.

2. THE MODEL

The economy is composed by two separate production sectors, denoted by $s \in \{ne, e\}$. Workers (denoted by a subscript i when we refer to individuals, I when we refer to the set) and firms (denoted by j and J, respectively) can choose to enter one of the two sectors, paying a fixed cost. Workers' costs, (c_I^{ne}, c_I^e), are exogenous, and can be interpreted as private, fixed costs of education (tuitions and the like). We denote firms' costs (d_J^{ne}, d_J^e). They are endogenously determined, and will be discussed later on. There are two intervals of equal length of workers, $\Omega_I = (0,1)$, and firms, Ω_J, both endowed with the Lebesgue measure. Let $\nu(\Omega_I^s)$ $(\nu(\Omega_J^s))$ denote the measure of the set Ω_I^s (Ω_J^s, respectively). At equilibrium, each interval is partitioned into two sets, $\{\Omega_I^{ne}, \Omega_I^e\}$ and $\{\Omega_J^{ne}, \Omega_J^e\}$, determined endogenously. In sector s, production requires a firm j (with physical capital k_j^s) and a worker i (with stock of human capital h_i^s). Once the partitions $\Omega_I^p \equiv \{\Omega_I^{ne}, \Omega_I^e\}$ and $\Omega_J^p \equiv \{\Omega_J^{ne}, \Omega_J^e\}$ are given, each sector of the economy reduces to the set-up studied in Acemoglu (1996). The only difference is that his analysis is mostly devoted to the case of homogeneous workers and firms, while we (necessarily) always consider the case of heterogeneous workers. Firms are identical, and choose their investments in physical capital to maximize their expected profits. Workers choose their investments in human capital to maximize their expected utilities.

The economy lasts one period, divided in several subperiods. We consider two versions of the basic model. In the frictionless (or Walrasian) version, in subperiod zero, firms and workers enter (paying a fixed cost) one of the two sectors. In subperiod 1, each firm active in sector s is matched with a worker active in the same sector (we will be more precise on the matching issue later on) and firms and workers can sign binding contracts on the amount of human and physical capital that they will supply. In subperiod 2, investments are carried out. In the final subperiod, 3, exchanges and production take place, and agents are paid on the basis of their marginal product.

In the second version of the model, the total output of each match is split according to the Nash bargaining solution with (exogenous) weights β and $(1-\beta)$ (for a rationalization of this allocation rule in this context, see the Appendix in Acemoglu (1996)). Moreover, and most important, agents cannot commit themselves to a given level of investment, because investments are carried out before the matches. When workers are heterogeneous, the first type of friction in the labor market has a very limited role in determining the efficiency properties of equilibria. The crucial feature is that matches take place *after the investments,* so that firms and workers cannot commit ex-ante to a given amount of them, and must base their choices on the conditional expectation of the investments of the potential future partners. Indeed, as long as random matches take place after the investments, the same qualitative results hold even if the spot labor markets are perfectly competitive[2]. This case is briefly considered in Appendix 2.

To avoid additional complications (not really germane to the main issue

[2] In Acemoglu (1996), the benchmark is an economy with identical workers and firms. Evidently, in this case, if spot labor markets were competitive, we would end up with the Pareto efficient, complete markets allocation.

analyzed here), we want to avoid to introduce the possibility of unemployment. This requires that, at equilibrium, each worker, and each firm, active in a sector is actually matched with a partner. Technologies are described by a pair of Cobb-Douglas production functions with constant returns to scale. Therefore, in the Walrasian set-up, equilibrium profits are zero, entry costs d_j must be zero, so that the equilibrium partition Ω_J^p is essentially arbitrary. Therefore, we can set $\Omega_J^s = \{j \in \Omega_J | j = i, \ i \in \Omega_I^s\}$, each s. On the contrary, in the economy with frictions, the expected producer's surplus is positive in both sectors and, as we will show later on, larger in sector e. We can obtain that the equilibrium value of Ω_J is indeterminate (so that we can put $\Omega_J^s = \{j \in \Omega_J | j = i, \ i \in \Omega_I^s\}$, each s) in at least two alternative ways. First, we can assume that firms cannot move across sectors. A non-null measure of firms is exogenously assigned to each sector. The matching function guarantees that each firm is matched with a worker (and+ conversely) for each non-trivial partition of the workers. As long as there is a continuum of workers and firms in each sector, this can be done. Of course, this approach would break down if we had a finite number of agents and, anyhow, is based on a very *ad hoc* trick. A second approach is to assume that the matching functions guarantee with probability one a match to each agent, provided that $\nu(\Omega_I^s) = \nu(\Omega_J^s)$. Moreover, we need to assume that the technology exploited in sector ne is free, while the one adopted in sector e is protected by a patent, owned by some outside agent (clearly, nothing would change if both technologies were subject to distinct patents)[3]. Rights to use the patent are auctioned off to firms before the match firm-worker obtains. Given that, at an equilibrium, expected profits in both sectors must be identical, the equilibrium royalties (that we identify with d_J) must be equal to the (positive) difference between the expected producer's surpluses in the two sectors. It follows that, at each equilibrium, Ω_J^p is essentially arbitrary. The property we are looking for.

Without any loss of generality, we can take the prices of both kinds of output to be equal to 1 and, therefore, omit them.

Finally, notice that there are always two additional equilibria: the ones where all the workers and the firms are in one of the two sectors. This is because, evidently, if no firm is active in sector s, every worker moves to the other sector (and conversely). As usual, we will ignore these trivial equilibria.

3. THE FRICTIONLESS ECONOMY

When active in sector s, and matched with worker i with human capital h_i^s, firm j has production function

$$y_{ij}^s = f^s\left(h_i^s, k_j^s\right) \equiv A^s h_i^{s\alpha} k_j^{s(1-\alpha)},$$

[3] Of course, any input used only in sector e and with perfectly inelastic supply would do. We consider the case of a patent to simplify as much as possible the model.

with $A^e > A^{ne}$.

Let μ be the unit price of physical capital, that we assume to be equal in the two sectors. This implies some loss of generality, but allows for more straightforward computations. Under suitable restrictions on the values of $\frac{A^s}{\mu^s}$, similar results could be obtained for $\mu^e \neq \mu^{ne}$.

Given a match with worker i, firm j solves optimization problem

$$\text{choose } k_j^s \in \arg\max A^s h_i^{s\alpha} k_j^{s(1-\alpha)} - \mu k_j^s - w_{ij} h_i^s, \tag{Π^{Ws}}$$

where we omit the fixed entry costs d_J^s, because, at equilibrium, they must be zero.

For each individual, the utility function is

$$U_i\left(C_i, h_i\right) = C_i - \frac{1}{\delta_i} \frac{h_i^{(1+\Gamma)}}{1+\Gamma},$$

where C_i denotes consumption, while h_i^s is the amount of human capital. Let c_I^s be the (fixed) cost of the investment in human capital. Then, in the absence of taxes and subsidies, if the worker is active in sector s and matched with firm j, $C_i^s = \left(w_{ij}^s h_i^s - c_I^s\right)$. Workers are heterogeneous because of the parameter δ_i. Without any essential loss of generality, we assume $\delta_i = i$, so that this parameter is uniformly distributed on $(0,1)$. To introduce more general assumptions on the distribution of δ_i and its support would introduce additional computational complexities without changing the essential results.

Evidently, the equilibrium amount of agent i's investment in human capital in sector s is given by

$$H^{Ws}\left(\delta_i\right) \equiv \left[\delta_i \alpha A^{s\frac{1}{\alpha}} \left(\frac{1-\alpha}{\mu}\right)^{\frac{1-\alpha}{\alpha}}\right]^{\frac{1}{\Gamma}}, \tag{1}$$

where the superscript W denotes the frictionless, Walrasian economy. Given that, at the equilibrium, profits are always zero, there is no loss of generality in assuming that firm j is always matched with worker $i = j$. With this convention, the (equilibrium) demand for physical capital of firm $j = i$ is

$$K^{Ws}(\delta_i) \equiv \left[\delta_i A^{s\frac{1+\Gamma}{\alpha}} \alpha \left(\frac{1-\alpha}{\mu}\right)^{\frac{1+\Gamma-\alpha}{\alpha}}\right]^{\frac{1}{\Gamma}}. \tag{2}$$

Let's now consider the equilibrium partition Ω_I^p. For convenience, set $c_I^{ne} = 0$ and $c_I^e > 0$. Worker i chooses to enter sector e if and only if

$$U_i^e\left(H^{We}\left(\delta_i\right), K^{We}\left(\delta_i\right)\right) - U_i^{ne}\left(H^{Wne}\left(\delta_i\right), K^{Wne}\left(\delta_i\right)\right) \geq c_I^e,$$

i.e., by direct computation, if and only if

$$\delta_i \geq \delta^{W*}\left(c_I^e\right) \equiv \left[\frac{(1+\Gamma)\mu^{\frac{(1+\Gamma)(1-\alpha)}{\alpha\Gamma}}}{\Gamma\left[\alpha\left(1-\alpha\right)^{\frac{(1-\alpha)}{\alpha}}\right]^{\frac{1+\Gamma}{\Gamma}}\left[A^{e\frac{1+\Gamma}{\alpha\Gamma}} - A^{ne\frac{1+\Gamma}{\alpha\Gamma}}\right]} c_I^e\right]^{\Gamma}. \quad (3)$$

Hence, for c_I^e positive and sufficiently small, there is a unique threshold value $\delta^{W*}\left(c_I^e\right)$, strictly increasing in c_I^e.

Clearly, for each pair (i,j) the physical-human capital ratio is δ_i-invariant, with $\frac{K^{Ws}(\delta_i)}{H^{Ws}(\delta_i)} = \left(\frac{(1-\alpha)A^s}{\mu}\right)^{\frac{1}{\alpha}}$ and $\frac{K^{We}(\delta_i)}{H^{We}(\delta_i)} > \frac{K^{Wne}(\delta_i)}{H^{Wne}(\delta_i)}$, each i.

4. THE ECONOMY WITH FRICTIONS

Given that matches take place after the investments are carries out, agents' choices must be based on the conditional expectations of the investment amounts of the (potential) partners.

Given any random variables x^s and y^s, with $x^s : \Omega_I^s \to \mathbb{R}$, and $y^s : \Omega_J^s \to \mathbb{R}$, let

$$E_{\Omega_I^s}(x_i^s) \equiv \frac{\int_{\Omega_I^s} x_i^s di}{\nu\left(\Omega_I^s\right)}$$

$\left(E_{\Omega_J^s}(y_j^s)\right)$ be the conditional expectation of x_i^s over the set Ω_I^s (of y_j^s over Ω_J^s). Later on, we will show that, at the equilibrium, it is always $\Omega_I^e = [\delta^*, 1)$. Therefore, in the sequel, the partitions Ω_I^p and Ω_J^p will be defined by the threshold level δ^*. To emphasize it, we will often use the notation $\Omega_J^s\left(\delta^*\right)$ and $\Omega_I^s\left(\delta^*\right)$. We will use δ^{F*} to denote the equilibrium threshold value, where the superscript F denotes the economy with frictions.

For future reference, let's determine the optimal amount of investments assuming that they are subsidized. Let $t^s\left(h\right)$ be the subsidy to the amount invested in human capital, $z^s\left(k\right)$ the one to the amount of the investment in physical capital.

Pick an arbitrary threshold value δ^*. Firm j selects the value of k_j^s solving optimization problem

$$\begin{aligned} & \max_{k_j^s} E_{\Omega_I^s(\delta^*)}\left(\left(1-\beta\right)A^s h_i^{s\alpha} k_j^{s(1-\alpha)} - \mu k_j^s\right) + z^s\left(k_j^s\right) - d_J^s \\ = \quad & \left(1-\beta\right)A^s E_{\Omega_I^s(\delta^*)}\left(h_i^{s\alpha}\right) k_j^{s(1-\alpha)} - \mu k_j^s + z^s\left(k_j^s\right) - d_J^s. \qquad (\Pi^{Fs}) \end{aligned}$$

As mentioned before, we can interpret d_J^s as royalties paid to access the technologies used in the two sectors. At equilibrium, we set $d_J^{ne} = 0$ and d_J^s equal to the (positive, as we will see) difference between the conditional expected producer's surpluses in the two sectors. Therefore, each firm is indifferent between the two sectors (and has non-negative conditional expected profits).

For computational convenience, assume that $z^s\left(k_j^s\right) = z^s k_j^{s(1-\alpha)}$. The stated functional form is obviously selected for computational convenience. It could be, arbitrarily closely, approximated using a step-linear subsidy schedule. Also for computational convenience, express the amount of the subsidy as

$$z^s \equiv \zeta^s \left(1-\beta\right) A^s E_{\Omega_I^s\left(\delta^{F*}\right)}\left(h_i^{s*\alpha}\right),$$

where $E_{\Omega_I^s\left(\delta^{F*}\right)}\left(h_i^{s\alpha}\right)$ is the equilibrium value of $E_{\Omega_I^s\left(\delta^{*}\right)}\left(h_i^{s\alpha}\right)$. To avoid misunderstandings: In the sequel, we will consider equilibria associated with $\zeta = 0$, and study the effects of the introduction of subsidies. Hence, it is perfectly legitimate to consider ζ as an exogenous variable even if, for the sake of computational easiness, we express it as a function of an (endogenous) equilibrium value $E_{\Omega_I^s\left(\delta^{F*}\right)}\left(h^{s\alpha}\right)$ (or of some other target level).

At the equilibrium[4], the first order conditions (FOCs in the sequel) of $\left(\Pi^{Fs}\right)$ imply that

$$k_{ji}^s = \left[\frac{\left(1-\beta\right)\left(1-\alpha\right)\left(1+\zeta^s\right) A^s E_{\Omega_I^s\left(\delta^{F*}\right)}\left(h_i^{s\alpha}\right)}{\mu}\right]^{\frac{1}{\alpha}}. \qquad (4)$$

Given that firms in sector s are, ex-ante, identical, $k_j^s = k^s$, and $E_{\Omega_j^s\left(\delta^{F*}\right)}\left(k_j^{s(1-\alpha)}\right) = k^{s(1-\alpha)}$. Using this fact, at the equilibrium, the optimization problem of worker i (if s) is

$$\max_{h_i^s} E_{\Omega_J^s\left(\delta^{F*}\right)}\left(U_i\left(.\right)\right) = \beta A^s h_i^{s\alpha} k^{s*(1-\alpha)} - \frac{1}{\delta_i}\frac{h_i^{s(1+\Gamma)}}{1+\Gamma} + t^s h_i^{s\alpha} - c_I^s, \; \left(U^{Fs}\right)$$

where $t^s\left(h_i^s\right) = t^s h_i^{s\alpha}$ is the subsidy to amount of the investments in human capital. As above, for computational convenience, we express the amount of the subsidy as $t^s \equiv \left(\tau^s \beta A^s k^{s*(1-\alpha)}\right)$, where k^{s*} is the equilibrium value of k^s.

At equilibrium, the FOCs of optimization problem $\left(U^{Fs}\right)$ imply that

$$h_i^s\left(k^{s*}\right) = \left[\left(1+\tau^s\right)\delta_i \alpha\beta A^s k^{s*(1-\alpha)}\right]^{\frac{1}{1+\Gamma-\alpha}}. \qquad (5)$$

Solving (4) and (5), we obtain

$$\begin{aligned} K^{Fs}(\delta^{F*}) &= \left[\frac{\left(1-\alpha\right)\left(1+\zeta^s\right)\left(1-\beta\right)}{\mu} E_{\Omega_I^s\left(\delta^{F*}\right)}\left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}}\right)\right]^{\frac{1+\Gamma-\alpha}{\alpha\Gamma}} \\ &\quad \times \left[\left(1+\tau^s\right)\alpha\beta\right]^{\frac{1}{\Gamma}} A^{s\,\frac{1+\Gamma}{\alpha\Gamma}}, \qquad (6) \end{aligned}$$

[4] Of course, the FOCs are well-defined out of equilibrium. We focus on their equilibrium values to avoid unnecessary notational complexities.

and

$$H^{Fs}\left(\delta_i,\delta^{F*}\right) = \left[\frac{(1-\alpha)(1+\zeta^s)(1-\beta)}{\mu}E_{\Omega_I^s\left(\delta^{F*}\right)}\left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}}\right)\right]^{\frac{1-\alpha}{\alpha\Gamma}} \times\delta_i^{\frac{1}{1+\Gamma-\alpha}}\left[(1+\tau^s)\alpha\beta\right]^{\frac{1}{\Gamma}}A^{s\frac{1}{\alpha\Gamma}}. \quad (7)$$

>From (6) and (7), we obtain that the equilibrium (conditional) expected utility is

$$E_{\Omega_J^s\left(\delta^{F*}\right)}\left(U_i(\delta_i,\delta^{F*})\right) = \left[\frac{(1-\alpha)(1+\zeta^s)(1-\beta)}{\mu}E_{\Omega_I^s\left(\delta^{F*}\right)}\left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}}\right)\right]^{\frac{(1+\Gamma)(1-\alpha)}{\alpha\Gamma}}\times \delta_i^{\frac{\alpha}{1+\Gamma-\alpha}}\beta^{\frac{1+\Gamma}{\alpha\Gamma}}A^{s\frac{1+\Gamma}{\alpha\Gamma}}\left[(1+\tau^s)\alpha\right]^{\frac{1}{\Gamma}}\frac{1+\Gamma-(1+\tau^s)\alpha}{1+\Gamma}, \quad (8)$$

while the (conditional) expected producer' surplus is

$$E_{\Omega_I^s\left(\delta^{F*}\right)}\left(\Pi^F(\delta_i)\right) = \left[\frac{(1-\alpha)(1+\zeta^s)(1-\beta)}{\mu}E_{\Omega_I^s\left(\delta^{F*}\right)}\left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}}\right)\right]^{\frac{1+\Gamma-\alpha}{\alpha\Gamma}} \times\frac{\mu a\left[(1+\tau^s)\alpha\beta\right]^{\frac{1}{\Gamma}}A^{s\frac{1+\Gamma}{\alpha\Gamma}}}{1-\alpha}. \quad (9)$$

Notice that, at $\tau^s=\zeta^s=0$, each s, given that $E_{\Omega_I^e\left(\delta^{F*}\right)}\left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}}\right) > E_{\Omega_I^{ne}\left(\delta^{F*}\right)}\left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}}\right)$, and $A^e > A^{ne}$, we always have $E_{\Omega_I^e\left(\delta^{F*}\right)}\left(\Pi^{Fe}(\delta_i)\right) > E_{\Omega_I^{ne}\left(\delta^{F*}\right)}\left(\Pi^{Fne}(\delta_i)\right)$, as claimed above.

Worker i enters sector e if and only if

$$E_{\Omega_J^e\left(\delta^{F*}\right)}(U_i(.)) - E_{\Omega_J^{ne}\left(\delta^{F*}\right)}(U_i(.)) \geq c_I^e,$$

i.e., by direct computation, if and only if $F\left(\delta_i,\delta^{F*},\tau,\zeta,c_I^e\right)\geq 0$, where

$$F\left(\delta_i,\delta^{F*},\tau,\zeta,c_I^e\right) \equiv \sum_s(-1)^{\chi(s)}\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}}\left(A^s\left((1+\zeta^s)E_{\Omega_I^s\left(\delta^{F*}\right)}\left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}}\right)\right)^{(1-\alpha)}\right)^{\frac{1+\Gamma}{\alpha\Gamma}} \times(1+\tau^s)^{\frac{1}{\Gamma}}\left(1+\Gamma-(1+\tau^s)\alpha\right)-c_I^{e'}, \quad (10)$$

where $\chi(s)=1$ if $s=ne$, $\chi(s)=2$ if $s=e$, and

$$c_I^{e'} \equiv \frac{1+\Gamma}{\alpha^{\frac{1}{\Gamma}}\beta^{\frac{1+\Gamma}{\Gamma}}}\left[\frac{\mu}{(1-\alpha)(1-\beta)}\right]^{\frac{(1+\Gamma)(1-\alpha)}{\alpha\Gamma}}c_I^e.$$

The equilibrium threshold value δ^{F*} is then obtained solving $F\left(\delta^{F*},\delta^{F*},\tau,\zeta,c_I^e\right) = 0$.

The following Proposition summarizes the fundamental properties of the equilibria. The proof is in Appendix 1.

Proposition 1. *Given* $(A, \Gamma, \alpha, \beta, \tau, \zeta)$, *with* $\tau = \zeta = 0$, *there is* $\widetilde{C} > 0$ *such that, for each* $c_I^e \in \left(0, \widetilde{C}\right)$, *there is an equilibrium with threshold value* $\delta^{F*}(., c_I^e) \in (0,1)$. *Moreover, given* $(A^{ne}, \Gamma, \alpha, \beta)$, *there is* A^{e*} *such that, for each* $A^e > A^{e*}$, *at* $(\tau, \zeta) = 0$, *the equilibrium is unique,* $\frac{\partial F(.)}{\partial \delta^{F*}} > 0$ *and the equilibrium map* $\delta^{F*}(., c_I^e)$ *satisfies* $\frac{\partial \delta^{F*}}{\partial \tau^e}|_{\tau^e=0} < 0$, $\frac{\partial \delta^{F*}}{\partial \tau^{ne}}|_{\tau^{ne}=0} > 0$ *and* $\frac{\partial \delta^{F*}}{\partial c_I^e}|_{c_I^e=0} > 0$.

In the sequel, we will mostly consider the leading case where $\frac{\partial F(.)}{\partial \delta^{F*}} > 0$ at each equilibrium threshold. The crucial role of this condition is that it guarantees that $\frac{\partial \delta^{F*}}{\partial \tau^e} < 0$ and $\frac{\partial \delta^{F*}}{\partial \tau^{ne}} > 0$, when evaluated at the equilibrium without subsidies, so that the direct and indirect effects of an increase in τ^{ne} work in the same direction, an empirically plausible restriction. Also, notice that, if there is a threshold δ^{F*1} where $\frac{\partial F(.)}{\partial \delta^{F*}}|_{\delta^{*1}=\delta^{F*}} < 0$, there must also be (at least) one second (lower) equilibrium threshold, δ^{F*2}, with $\frac{\partial F(.)}{\partial \delta^{F*}}|_{\delta^{*2}=\delta^{F*}} > 0$, because $F(.)$ is continuous and $F(0,0,c_I^e) < 0$.

Some considerations on the properties of the equilibrium allocations in the two classes of economies (with and without frictions, and without taxes and subsidies) may be of some interest. First, the physical/human capital ratio is given by

$$\frac{K^{Fs}(\delta^{F*})}{H^{Fs}\left(\delta_i, \delta^{F*}\right)} = \frac{K^{Ws}(\delta_i)}{H^{Ws}(\delta_i)} \frac{(1-\beta)^{\frac{1}{\alpha}} E_{\Omega_I^s(\delta^{F*})}\left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}}\right)^{\frac{1}{\alpha}}}{\delta_i^{\frac{1}{1+\Gamma-\alpha}}}.$$

Evidently, the second term is always greater than one, for sufficiently small δ_i. This immediately implies that the frictionless equilibrium allocation is not Pareto superior to the one of the economy with frictions: Agents with a sufficiently low δ_i are always better off in the last one.

In the economy with frictions, the threshold value δ^{F*} can be either lower or higher than its value in the Walrasian economy, as we establish with the following example.

Example 1. Consider the economy with $A^e = 2, A^{ne} = 1$, $\alpha = \beta = 1/2$, and $\Gamma = 1$. By direct computation, we obtain the two following equilibrium maps

$$F^W\left(\delta^{W*}, c_I^e\right) = \frac{15}{32}\delta^{W*} - c_I^e = 0$$

in the Walrasian economy, and

$$F^F\left(\delta^{F*}, c_I^e\right) = \frac{27\sqrt[3]{\delta^{F*}}}{8192}\left(\frac{16\left(\delta^{F*\frac{4}{3}} - 1\right)^2}{\delta^{F*2} - 2\delta^{F*} + 1} - \delta^{F*\frac{2}{3}}\right) - c_I^e = 0$$

in the economy with frictions. One can verify that, for $c_I^e < 0.019$, $\delta^{F*} < \delta^{W*}$, while, for $c_I^e > 0.019$, the opposite occurs. This is shown in Figure 1 describing the two maps $\left(F^W\left(\delta^{F*}, c_I^e\right) + c_I^e\right)$ and $\left(F^F\left(\delta^{F*}, c_I^e\right) + c_I^e\right)$.

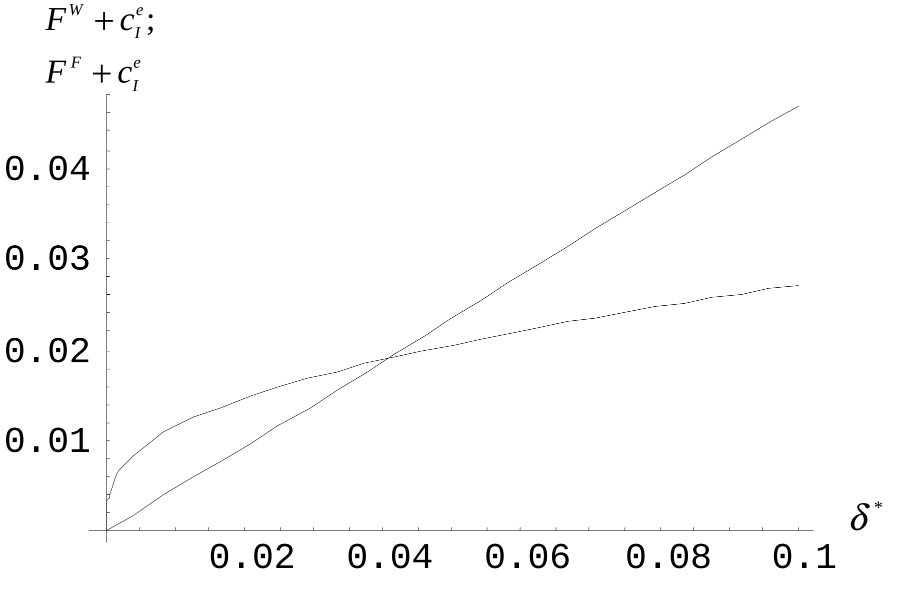

Figure 1

5. INEFFICIENCY PROPERTIES OF THE ECONOMY WITH FRICTIONS

It is trivial to show that the equilibria of the economy with frictions are Pareto inefficient. More interesting is to analyze their inefficiency in terms of its effects on the amount, and the type, of investments. In the sequel, we will mainly refer to the investments in human capital. Similar considerations hold for the ones in physical capital.

In our set-up, inefficiencies can be of two different types. First, an individual can choose an *amount* of investment different from the Pareto optimal one, *given* the partition Ω_I^p. We will refer to this possible source of inefficiency as *underinvestment* (or *overinvestment*) *in educational effort.* Secondly, an agent can choose to invest in a *type* of education different from the one assigned to her at the (constrained) Pareto optimal allocation. We will say that there is *underinvestment in educational type* when agent i invests in education ne, while, at the CPO allocation, she should invest in education level e.

In the one-sector model, equilibria are unambiguously characterized by *underinvestment in educational effort* and in physical capital. In our set up, the same effect is at work: In each sector, given the equilibrium value of δ^{F*}, an increase in the investment of firms and workers leads to a Pareto improvement. The argument is identical to the one exploited by Acemoglu (1996): Consider a small change in h_i^s and k_j^s, each i and j. The changes in utilities and producers' surplus evaluated at the equilibrium pair (h_i^{s*}, k_j^{s*}) (and taking into account that $k_j^{s*} = k^{s*}$, each j and s), are given by

$$0 < \left(\alpha\beta A^s \left[\frac{k^{s*}}{h_i^{s*}} \right]^{1-\alpha} - \frac{1}{\delta_i} h_i^{s*\Gamma} \right) dh + \left((1-\alpha)\beta A^s \left(\frac{h_i^{s*}}{k^{s*}} \right)^{\alpha} \right) dk, \quad (11)$$

and

$$\begin{aligned} 0 \quad < \quad & \left((1-\alpha)(1-\beta) A^s \frac{E_{\Omega_I^s(\delta^{F*})}\left(h_i^{s\alpha}\right)}{k^{s*\alpha}} - \mu \right) dk \\ & + \left(\alpha (1-\beta) A^s \frac{k^{s*(1-\alpha)}}{E_{\Omega_I^s(\delta^{F*})}\left(h_i^{s(1-\alpha)}\right)} \right) dh, \end{aligned} \quad (12)$$

respectively. The inequalities hold because the first terms in parenthesis in both (11) and (12) are zero (at an optimal solution of $\left(\Pi^{Fs}\right)$ and (U^{Fs})), while the second terms are clearly positive. Hence, in each sector, there is underinvestment in both educational effort and physical capital.

In the two-sector case, there is a second potential source of inefficiency, because changes in the value of δ^* may also entail Pareto improvements. An increase in the threshold value δ^* increases the conditional expected amount of human capital in both sectors at the same time and, consequently, induces an increase in the amount of physical investments of firms in both sectors. Indeed,

$$\frac{\partial E_{\Omega_I^s(\delta^{F*})}\left(\delta_i^{\overline{1+\Gamma-\alpha}}\right)}{\partial \delta^{F*}} > 0, \text{ for each } s, \quad (13)$$

and, consequently, using (6) and (7),

$$\frac{\partial H^{Fs}\left(\delta_i, \delta^{F*}\right)}{\partial \delta^{F*}} > 0 \text{ and } \frac{\partial K^{Fs}\left(\delta^{F*}\right)}{\partial \delta^{F*}} > 0, \text{ each } s. \quad (14)$$

More relevant, using (8) and (9),

$$\frac{\partial E_{\Omega_J^s(\delta^{F*})}\left(U_i(.)\right)}{\partial \delta^{F*}} > 0 \text{ and } \frac{\partial E_{\Omega_I^s(\delta^{F*})}\left(\Pi\left(\delta^{F*}\right)\right)}{\partial \delta^{F*}} > 0, \text{ for each } i. \quad (15)$$

These properties do not suffice to establish our claim, because a change in the threshold induces a jump in the producer's surplus for the firms shifting from one sector to the other. However, as we will formally establish below (in Proposition 3), under suitable restrictions on the equilibria, ex-post producer's surplus of the firms matched with agents with their δ_i in some neighborhood of δ^* are strictly positive in sector ne and negative in sector e. Given the fixed costs of education, the utility of the agent with $\delta_i = \delta^{F*}$ is identical in the two sectors, so that the impact of the differences in utility levels is negligible. Therefore, for this subset of economies, sufficiently small increases of the threshold value are Pareto improving, because of the "composition effect".

To complete the analysis of the welfare properties of equilibria, it is convenient to introduce an explicit notion of (constrained) efficiency. As usual in economies with frictions, we consider the metaphor of a benevolent planner choosing an allocation while facing constraints aiming to capture the ones (informational or of other nature) the agents face in the decentralized economy. We provide two results. First, we show that there are constrained Pareto optimal allocations (Proposition 2) and that they can be attained with an appropriate system of taxes and subsidies. Secondly, in Proposition 3, we study the effects of taxes and subsidies taking as given the demand/supply functions of the agents. We show that, at least for a subset of economies, there are systems of taxes and subsidies which increase the aggregate expected surplus.

Bear in mind that, in the sequel, we always consider changes in the conditional expectation of the total surplus. We are not concerned with actual Pareto improvements. However, given that utility functions are quasi-linear, an increase of the expected total surplus immediately translates (modulo an appropriate system of lump-sum taxes and transfers) into a Pareto improvement. Also, given the structure of the economy, it is easy to check that our systems of taxes and transfers can actually be designed so to guarantee a balanced budget.

5.1. Constrained Pareto efficient allocations

The objective function of the planner is given by the sum of the conditional expected utilities and producers' surpluses, i.e.,

$$\begin{aligned} P'\left(h_i^s, k_j^s, \Omega_I^s, \Omega_J^s\right) \equiv & \sum_s \int_{\Omega_I^s} \left[\beta E_{\Omega_J^s}\left(A^s h_i^{s\alpha} k_j^{s(1-\alpha)}\right) - \frac{1}{\delta_i}\frac{h_i^{s(1+\Gamma)}}{1+\Gamma} - c_I^s\right] di \\ & + \sum_s \int_{\Omega_J^s} \left[(1-\beta) E_{\Omega_I^s}\left(A^s h_i^{s\alpha} k_j^{s(1-\alpha)}\right) - \mu k_j^s\right] dj. \end{aligned}$$

The policy instruments are the partitions Ω_I^p and Ω_J^p and a pair of maps $\left(H^{CPOs}\left(\delta_i, \delta^*\right), K^{CPOs}\left(\delta^*\right)\right)$, where we restrict the partitions to have the structure $\Omega_I^e\left(\delta^*\right) = \left\{i \in \Omega_I | \delta_i \geq \delta^*\right\}$, and $\Omega_J^e\left(\delta^*\right) = \left\{j \in \Omega_J | j = i,\ i \in \Omega_I^e\left(\delta^*\right)\right\}$.

Given that firms are (ex-ante) identical, and given the informational constraints embedded into the definition of $P'(.)$, and the properties of the (implicit) matching function, to impose this structure on Ω_J^p does not entail any loss of generality. Given that the optimal choice k_j^s is $j-$invariant and that, by assumption, $\nu\left(\Omega_I^s\left(\delta^*\right)\right)=\nu\left(\Omega_J^s\left(\delta^*\right)\right)$, the planner' s objective function can then be rewritten as

$$
\begin{aligned}
P\left(h_i^s, k^s, \delta^*\right) \quad \equiv & \quad \sum_s \int_{\Omega_I^s(\delta^*)}\left[\beta A^s h_i^{s\alpha} k^{s(1-\alpha)}-\frac{1}{\delta_i} \frac{h_i^{s(1+\Gamma)}}{1+\Gamma}\right] di-c_I^s \nu\left(\Omega_I^s\left(\delta^*\right)\right) \\
& +\sum_s(1-\beta) A^s k^{s(1-\alpha)} \int_{\Omega_I^s(\delta^*)} h_i^{s\alpha} di-\mu k^s \nu\left(\Omega_J^s\left(\delta^*\right)\right) \\
= & \quad \sum_s \int_{\Omega_I^s(\delta^*)}\left[A^s h_i^{s\alpha} k^{s(1-\alpha)}-\frac{1}{\delta_i} \frac{h_i^{s(1+\Gamma)}}{1+\Gamma}\right] di \\
& -c_I^s \nu\left(\Omega_I^s\left(\delta^*\right)\right)-\mu k^s \nu\left(\Omega_J^s\left(\delta^*\right)\right).
\end{aligned}
$$

Its optimization problem is, then,

$$
\max_{\left(h_i^s, k^s, \delta^*\right)} P\left(h_i^s, k^s, \delta^*\right). \tag{P}
$$

It is convenient to decompose (P) into three problems. First, given an arbitrary value δ^*, for $s=e, ne$, we determine the maps $\left(H^{CPOs}\left(\delta_i, \delta^*\right), K^{CPOs}\left(\delta^*\right)\right)$ solving, for each s, the optimization problem

$$
\begin{aligned}
\max_{\left(h_i^s, k^s\right)} P_{\delta^*}^s\left(h_i^s, k^s\right) \quad \equiv & \quad \int_{\Omega_I^s(\delta^*)}\left[A^s h_i^{s\alpha} k^{s(1-\alpha)}-\frac{1}{\delta_i} \frac{h_i^{s(1+\Gamma)}}{1+\Gamma}\right] di \\
& -c_I^s \nu\left(\Omega_I^s\left(\delta^*\right)\right)-\mu k^s \nu\left(\Omega_J^s\left(\delta^*\right)\right).
\end{aligned} \tag{P$_{\delta^*}^s$}
$$

Next, given the value functions $P^s(\delta^*)$ of the two problems $(P_{\delta^*}^s)$, $s=e, ne$, we recast problem (P) as

$$
\max_{\delta^*} P^*(\delta^*) \equiv P^e(\delta^*)+P^{ne}(\delta^*), \tag{P'}
$$

finding the optimal value of δ^*, δ^{CPO}. Given that $P^s(\delta^*)$, $s=e, ne$, is a continuous function of δ^*, problem (P') has an optimal solution, either interior or at one of the two boundary points $\{0,1\}$. Hence, a constrained Pareto optimal allocation exists.

Comparing the FOCs associated with the CPO allocation and the ones associated with the equilibrium of the economy with frictions, we can immediately establish that equilibria of the latter economy are constrained Pareto inefficient. The source of inefficiency considered by Acemoglu (1996) reappears in our set-up, because, given any threshold level δ^*, $H^{CPOs}\left(\delta_i, \delta^*\right) > H^{Fs}\left(\delta_i, \delta^*\right)$, and $K^{CPOs}\left(\delta^*\right) > K^{Fs}\left(\delta^*\right)$. On the other hand, the relation between the CPO value of the threshold, δ^{CPO}, and its equilibrium level, δ^{F*}, is not univocal. Example 2 in Appendix 1 shows two economies:

the first has $\delta^{CPO} < \delta^{F*}$, the other $\delta^{CPO} > \delta^{F*}$. This does not contradict our previous claim (that an increase in δ^{F*} can be Pareto improving). There, we were evaluating the reactions of firms and workers according to their actual demand and supply functions. Here, the optimal value δ^{CPO} is determined using the demand/supply functions determined by the planner.

PROPOSITION 2. *Under the maintained assumptions, each economy with frictions has a CPO allocation. Moreover, equilibrium allocations are never CPO and they are characterized by underinvestment in the amount of physical capital and in educational effort. Both under and overinvestments in educational level are possible.*

The details are in Appendix 1.

It is quite obvious that the CPO allocation can be implemented with an appropriate system of subsidies to investments, and of fixed taxes or subsidies.

COROLLARY 1. *There is a (balanced budget) system of taxes and subsidies* $(\Delta c, \Delta c_I, \tau, \zeta)$ *such that the associated equilibrium allocation is CPO.*

5.2. Pareto improving taxes and subsidies

We conclude considering the welfare effects of alternative, active human capital policies. In particular, we consider the case of a general, non-linear, tax/subsidy, $(\Delta c_I, t^s(h^s))$. We show that, depending upon the sign of $\frac{\partial F}{\partial \delta^{F*}}$ (and provided that the threshold level δ^{F*} is not "too high"), distinct subsets of policy instruments have unambiguous welfare improving effects.

If $\frac{\partial F}{\partial \delta^{F*}} > 0$ (a maintained assumption in the sequel), an increase in the cost of education (redistributing the revenues as a lump-sum transfer), or an increase of the subsidies to educational effort in the "low skill" sector ne, always increase expected total surplus. On the contrary, an increase in the subsidy to investments in the high skill sector may decrease it. The (fairly transparent) logic of these results is the following. When δ^* is not "too high", ex-post producer surplus is negative in sector e for δ sufficiently close to δ^{F*}, so that an increase in its value induces an improvement in total social surplus. A subsidy $t^{ne}(h^{ne})$ has a direct, positive, effect on investments in education effort in sector ne, and a positive, indirect, effect on the investment effort in both sectors, because it induces an increase in the equilibrium value of δ^{F*}. For the same reason, a tax on higher education $\Delta c_I > 0$ (whose revenue is redistributed using lump-sum transfers) has an indirect, positive effects on the investments in educational effort in both sectors. Therefore, they always lead (modulo a redistribution) to a Pareto improvement. Given the structure of the preferences, we can always combine the two policies so that the net budget cost is zero.

The third policy, making sector e more attractive to workers, induces some workers with $\delta_i < \delta^{F*}$ to switch sector. This has an unambiguous, negative effect on the welfare of the workers remaining in sector ne. Moreover, the negative effect on the welfare of the workers in sector e, due to the composition effect, may overcome the positive effect of the incentives on the investments in educational effort in this sector too, so that we may end up with a lower expected total surplus. This is established in Proposition 3 and by a final example.

Let's make the previous heuristic argument formal. Consider a planner's optimization problem where we introduce explicit (anonymous) policy instruments: a fixed tax/subsidy on the type of education (denoted Δc_I) and a subsidy to the effort in education. The problem of the planner is to choose the values of $(\Delta c_I, \tau)$, after that, workers and firms choose their individually optimal behavior. Hence, (omitting the superscript F whenever is not useful) we define the following planning problem

$$max_{\tau,\Delta c_I} P(\tau, \Delta c_I) \equiv \sum_s \int_{\Omega_I^s\left(\delta^{F*}\right)} \left(U_i\left(H^s\left(\delta_i, \delta^{F*}\right), K^s\left(\delta^{F*}\right), \tau^s\right) - c_I^s - \Delta c_I^s\right) di$$

$$+\sum_s \int_{\Omega_J^s\left(\delta^{F*}\right)} \Pi_j\left(H^s\left(\delta_i, \delta^{F*}\right), K^s\left(\delta^{F*}\right), \tau^s\right) dj + \Delta c_I \nu\left(\Omega_I^e\left(\delta^{F*}\right)\right), \qquad (\mathrm{P}^t)$$

where the last term reflects the fact that we redistribute equally across all the workers the revenues of the fixed tax Δc_I, charged to the ones active in sector e, so that $\Delta c = \Delta c_I \nu\left(\Omega_I^e\left(\delta^{F*}\right)\right)$.

PROPOSITION 3. *Consider an equilibrium associated with* $(\Delta c, \Delta c_I, \tau) = 0$ *and such that* $\frac{\partial F}{\partial \delta^{F*}} > 0$ *and* $\left(\frac{1+\Gamma}{(1-\alpha)(1+\Gamma-\alpha)} < \left(\frac{1-\delta^{F*\frac{1+\Gamma}{1+\Gamma-\alpha}}}{\delta^{F*\frac{\alpha}{1+\Gamma-\alpha}} - \delta^{F*\frac{1+\Gamma}{1+\Gamma-\alpha}}}\right)\right)$.
Then,
i. $(\Delta c, \Delta c_I)$ *with* $\Delta c_I > 0$, *and sufficiently small, increases the expected total surplus,*
ii. $\tau^{ne} > 0$, *and sufficiently small, increases the expected total surplus,*
iii. $\tau^e > 0$, *and sufficiently small, may decrease the expected total surplus.*

The proof of statements (i, ii) is in Appendix 1, where we also establish that the welfare effect of a subsidy τ^e is, in general, indeterminate. We now provide a strategy to construct economies where an increase in τ^e decreases the total expected surplus. The third statement, therefore, is established by the following example.

EXAMPLE 3. *Welfare-reducing subsidies to investments in human capital*

Using (8) and (9) above, we can express the FOC of optimization problem (P^t) as

$$\frac{\partial P(\tau)}{\partial \tau^e} = \int_{\Omega_I^e\left(\delta^{F*}\right)} \frac{(1+\Gamma-\alpha)\,\Gamma}{1-\alpha} U_i\left(\delta_i, \delta^{F*}, \tau\right) di + \int_{\Omega_j^e\left(\delta^{F*}\right)} \frac{\Pi_j\left(\delta_i, \delta^{F*}, \tau\right)}{\Gamma} dj + \frac{\partial P(\tau)}{\partial \delta^{F*}} \frac{\partial \delta^{F*}}{\partial \tau^e}.$$

We need to construct an economy such that, at the equilibrium, $\frac{\partial P(\tau,\Delta c)}{\partial \tau^e} < 0$. The first two terms of the expression above are clearly bounded above. Hence, the result is established, if we can construct an equilibrium with $\frac{\partial P(\tau)}{\partial \delta^{F*}}$ bounded away from zero, and $\frac{\partial \delta^{F*}}{\partial \tau^e} < 0$ and arbitrarily large in absolute value.

Consider first $\frac{\partial P(\tau)}{\partial \delta^{F*}}$. In view of the proof of Proposition 3 in Appendix 1, we can write it as

$$\frac{\partial P(\tau)}{\partial \delta^{F*}} = \Delta\Pi + \left(\sum_s \int_{\Omega_I^s(\delta^{F*})} \frac{\partial U_i\,(.)}{\partial \delta^{F*}} di + \sum_s \int_{\Omega_J^s(\delta^{F*})} \frac{\partial \Pi_j\,(.)}{\partial \delta^{F*}} dj\right),$$

By (8) and (9), the term in brackets is positive and bounded away from zero. Hence, $\frac{\partial P(\tau)}{\partial \delta^{F*}}$ is bounded away from zero if $\Delta\Pi \geq 0$. In the proof of Proposition 3, we have established that $\Delta\Pi \geq 0$ if

a. $\frac{\partial F(.)}{\partial \delta^{F*}} > 0$;

b. $\frac{1+\Gamma}{(1-\alpha)(1+\Gamma-\alpha)} < \left(\frac{1-\delta^{F*\frac{1+\Gamma}{1+\Gamma-\alpha}}}{\delta^{F*\frac{\alpha}{1+\Gamma-\alpha}} - \delta^{F*\frac{1+\Gamma}{1+\Gamma-\alpha}}}\right)$.

Pick a sequence of economies (parameterized by the value of A^e) such that $\frac{\partial F(.)}{\partial \delta^{F*}} > 0$ at the equilibria of the economies with $A^e > \bar{A}^e$, $\frac{\partial F(.)}{\partial \delta^{F*}} < 0$ at (some of the equilibria of) the ones associated with $A^e < \bar{A}^e$. Notice that, if $\frac{\partial F(.)}{\partial \delta^{F*}} > 0$, $\frac{\partial \delta^{F*}}{\partial A^e} > 0$. Hence, if ($b$) is satisfied for some $A^{e^\circ} > \bar{A}^e$, it is also satisfied for each $A^e \in \left(\bar{A}^e, A^{e^\circ}\right)$, because the right hand side of (b) is monotonically increasing in δ^{F*}. It follows that, for each $A^e \in \left(\bar{A}^e, A^{e^\circ}\right)$, $\Delta\Pi \geq 0$ and, therefore, $\frac{\partial P(t,\Delta c)}{\partial \delta^{F*}} \geq \theta$, for some $\theta > 0$. Choose c_I^e so that, for each economy associated with $\mathrm{A}^e > \bar{A}^e$, the threshold value satisfies $\delta^{F*} \geq \xi$, for some $\xi > 0$.

Consider now $\frac{\partial \delta^{F*}}{\partial \tau^e}$. By the implicit function theorem, $\frac{\partial \delta^{F*}}{\partial \tau^e} = -\frac{\frac{\partial F}{\partial \tau^e}}{\frac{\partial F}{\partial \delta^{F*}}}$, where

$$\frac{\partial F}{\partial \tau^e} = \frac{1+\Gamma-\alpha}{\Gamma} \delta^{F*\frac{\alpha}{1+\Gamma-\alpha}} \left(A^e E_{\Omega_I^e(\delta^{F*})} \left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}}\right)^{(1-\alpha)}\right)^{\frac{1+\Gamma}{\alpha\Gamma}}$$

is bounded away from zero for each $\delta^{F*} \geq \xi > 0$. On the other hand, by construction, for any sequence $\{A^{ev}\}_{v=1}^{\infty}$ such that $A^{ev} \to \bar{A}^e$, $A^{ev} > \bar{A}^e$ each v, the associated sequence $\frac{\partial F(A^{ev})}{\partial \delta^{F*}}$ converges to zero. Hence, we can make $|\frac{\partial \delta^{F*}}{\partial \tau^e}|$ arbitrarily large by choosing $A^e > \bar{A}^e$ and sufficiently close to $\bar{A}^e$. Therefore, there are economies such that $\frac{\partial P(\tau)}{\partial \tau^e}|_{\tau^e=0} < 0$.

6. CONCLUSIONS

We have extended the analysis of the externalities related to investments in human capital considering economies with two different types of human capital, and market imperfections. The most natural interpretation of our model is in terms of college *vs.* high school education. Our results partly confirm the conclusions of Acemoglu (1996). Labor market imperfections generate a positive externality of investments in educational effort and physical capital. However, in the two-sector case, there is a second externality, due to the composition effect and induced by the endogenous workers' choice of the type of education to invest in. Under our assumptions, this externality has always a negative effect on the investments: at equilibrium, an increase in the measure of agents getting the higher type of education has a negative effect on the investments in physical capital and educational effort in both sectors. The effects on welfare are somewhat more ambiguous.

The one-sector and the two-sector models have sharply different consequences in terms of policy prescriptions. In the first kind of model, some (possibly small) level of subsidies to investments is always Pareto improving. In the two-sector framework, on the contrary, it is crucial to select the right combination of taxes and subsidies.

Our conclusions rest crucially on the assumption of random matching taking place after the investments are selected. In economies with directed search, they would obviously be different (see, for instance, Acemoglu and Shimer (1999)).

7. APPENDIX 1: PROOFS OF THE PROPOSITIONS

Proof of Proposition 1. Pick the partition $\Omega_I^p(\delta^*)$ induced by an arbitrary δ^*. Assume that there is an agent i such that $F(\delta_{i'}, \delta^*) = 0$, so that $\delta_{i'} = \delta^*$. Evidently, $F(\delta_i, \delta^*) > 0$ for each $\delta_i > \delta^*$ and $F(\delta_i, \delta^*) < 0$ for each $\delta_i < \delta^*$. Hence, each equilibrium partition $\Omega_I^p\left(\delta^{F*}\right)$ such that $\Omega_I^s\left(\delta^{F*}\right) \neq \emptyset$, each s, satisfies $\Omega_I^e\left(\delta^{F*}\right) = \left\{i \in \Omega_I | \delta_i \geq \delta^{F*}\right\}$, where δ^{F*} is the (unique) threshold value.

By direct computation, for each threshold δ^*. Therefore,

$$E_{\Omega_I^e(\delta^*)}\left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}}\right) = \left(\frac{1+\Gamma-\alpha}{1+\Gamma}\frac{1-\delta^{*\frac{1+\Gamma}{1+\Gamma-\alpha}}}{1-\delta^*}\right)$$

and

$$E_{\Omega_I^{ne}(\delta^*)}\left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}}\right) = \left(\frac{1+\Gamma-\alpha}{1+\Gamma}\delta^{*\frac{\alpha}{1+\Gamma-\alpha}}\right).$$

Evidently, both functions are continuous in the (arbitrarily selected) threshold value δ^*. With some abuse of notation, define the function

$$F(\delta^*) \equiv \delta^{*\frac{\alpha}{1+\Gamma-\alpha}} A^{e\frac{1+\Gamma}{\alpha\Gamma}} \left(\frac{1+\Gamma-\alpha}{1+\Gamma} \frac{1-\delta^{*\frac{1+\Gamma}{1+\Gamma-\alpha}}}{1-\delta^*} \right)^{\frac{(1-\alpha)(1+\Gamma)}{\alpha\Gamma}}$$

$$-\delta^{*\frac{\alpha}{1+\Gamma-\alpha}} A^{ne\frac{1+\Gamma}{\alpha\Gamma}} \left(\frac{1+\Gamma-\alpha}{1+\Gamma} \frac{\delta^{*\frac{1+\Gamma}{1+\Gamma-\alpha}}}{\delta^*} \right)^{\frac{(1-\alpha)(1+\Gamma)}{\alpha\Gamma}} - c_I^{e'}.$$

$F(\delta^*)$ is clearly continuous on $(0,1)$. Given that $E_{\Omega_I^s(\delta^*)}\left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}}\right)$, each s, is bounded, $\lim_{\delta^*\to 0} F(\delta^*) < 0$.

Given that $\lim_{\delta^*\to 1} \frac{1-\delta^{*\frac{1+\Gamma}{1+\Gamma-\alpha}}}{1-\delta^*} = \frac{\partial\left(\delta^{*\frac{1+\Gamma}{1+\Gamma-\alpha}}\right)}{\partial\delta^*}|_{\delta^*=1} = \frac{1+\Gamma}{1+\Gamma-\alpha}$,

$$\lim_{\delta^*\to 1} F(\delta^*) = \left(\frac{1+\Gamma-\alpha}{1+\Gamma}\right)^{\frac{(1-\alpha)(1+\Gamma)}{\alpha\Gamma}} \left(A^{e\frac{1+\Gamma}{\alpha\Gamma}} \frac{1+\Gamma}{1+\Gamma-\alpha} - A^{ne\frac{1+\Gamma}{\alpha\Gamma}} \right) \equiv \bar{C} > 0$$

Hence, by the intermediate value theorem, for each c_I^e such that $c_I^{e'} \in \left(0, \bar{C}\right)$, there is an interior equilibrium, whose threshold value δ^{F*} is given by the solution to $F(\delta^*) = 0$.

Evidently, $\frac{\partial F}{\partial c_I^e} < 0$, and, by direct computation, for each s,

$$\frac{\partial F}{\partial \tau^s}|_{\tau^s=0} = \delta^{F*\frac{\alpha}{1+\Gamma-\alpha}} \left(A^s E_{\Omega_I^s(\delta^{F*})} \left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}}\right)^{(1-\alpha)} \right)^{\frac{1+\Gamma}{\alpha\Gamma}} \frac{(1+\Gamma)(1-\alpha)}{\Gamma} > 0.$$

Unfortunately, the sign of $\frac{\partial F}{\partial \delta^{F*}}$ depends upon the specific parameters of the economy. By direct computation, at $\tau = \zeta = 0$, setting $\gamma \equiv \frac{1+\Gamma}{1+\Gamma-\alpha}$,

$$\frac{1}{A^{ne\frac{1+\Gamma}{\alpha\Gamma}}} \frac{\partial F}{\partial \delta^{F*}} = (\gamma - 1) \frac{1}{\delta^{F*}} F\left(\delta^{F*}\right) + \frac{(1-\alpha)(1+\Gamma)}{\alpha\Gamma} \frac{\delta^{F*\gamma-1}}{\delta^{F*}}$$

$$\times \left(\left(\frac{A^e}{A^{ne}}\right)^{\frac{1+\Gamma}{\alpha\Gamma}} E_{\Omega_I^e(\delta^{F*})} \left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}}\right)^{\frac{(1-\alpha)(1+\Gamma)}{\alpha\Gamma}} \eta^e\left(\delta^{F*}\right) - E_{\Omega_I^e(\delta^{F*})} \left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}}\right)^{\frac{(1-\alpha)(1+\Gamma)}{\alpha\Gamma}} \eta^{ne}\left(\delta^{F*}\right) \right),$$

where $\eta^s(\delta^*)$ is the elasticity of $E_{\Omega_I^s(\delta^*)}\left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}}\right)$. By direct computation, $\eta^{ne}(\delta^*) = (\gamma - 1)$, while $\eta^e(\delta^*) = \frac{-\gamma\delta^{*\gamma}(1-\delta^*)+\delta^*(1-\delta^{*\gamma})}{(1-\delta^*)(1-\delta^{*\gamma})}$. With a straightforward manipulation, we obtain that

$$\left(\frac{1+\Gamma-\alpha}{1+\Gamma}\right)^{\frac{(1-\alpha)(1+\Gamma)}{\alpha\Gamma}} \frac{\Gamma}{A^{ne\frac{1+\Gamma}{\alpha\Gamma}}} \frac{\partial F}{\partial \delta^{F*}}$$

$$= \delta^{F*\frac{1-\Gamma}{\Gamma}} \left(\left(\frac{A^e}{A^{ne}}\right)^{\frac{1+\Gamma}{\alpha\Gamma}} \left(\frac{\frac{1-\delta^{F*\gamma}}{1-\delta^{F*}}}{\frac{\delta^{F*\gamma}}{\delta^{F*}}}\right)^{\frac{(1-\alpha)(1+\Gamma)}{\alpha\Gamma}} \left(\frac{\alpha\Gamma}{1+\Gamma-\alpha} + \frac{(1-\alpha)(1+\Gamma)}{\alpha}\eta\left(\delta^{F*}\right)\right) - 1 \right).$$

If $\eta^e(\delta^*) \geq 0$ at each $\delta^* \in (0,1)$, and $\left(\frac{1-\delta^{F*\gamma}}{(1-\delta^{F*})\delta^{F*\gamma-1}}\right)$ is bounded away from zero, the right hand side of the eq. above is always positive, for $\frac{A^e}{A^{ne}}$ sufficiently large. Therefore, $\frac{\partial F}{\partial \delta^{F*}} > 0$ at each equilibrium, for A^e large enough. Evidently, given that $F(0) < 0$, if $\frac{\partial F}{\partial \delta^{F*}} > 0$ at each solution to $F(\delta^*) = 0$, the solution must be unique. Moreover, by the implicit function theorem, $\frac{\partial F}{\partial \delta^{F*}} > 0$ at each equilibrium also implies that $\delta^{F*}(\tau, c_I^e)$ satisfies $\frac{\partial \delta^{F*}}{\partial \tau^e}|_{\tau^e=0} < 0$, $\frac{\partial \delta^{F*}}{\partial \tau^{ne}}|_{\tau^{ne}=0} > 0$ and $\frac{\partial \delta^{F*}}{\partial c_I^e} > 0$. Hence, to conclude the proof we need two additional results:

Fact 1. $\eta^e(\delta^*) \geq 0$, at each $\delta^* \in (0,1)$.

By direct computation, $\eta^e(0) = 0$ and $\eta^e(1) = \frac{\gamma-1}{2} > 0$. Hence, either there is $\delta \in (0,1)$ such that $\eta^e(\delta) = 0$ or $\eta^e(\delta) > 0$ for each $\delta \in (0,1)$, as claimed.

Consider the numerator of $\eta^e(\delta)$, call it $f(\delta)$,

$$f(\delta) = -\gamma\delta^\gamma(1-\delta) + \delta(1-\delta^\gamma).$$

Evidently, $\eta^e(\delta) = 0$ if and only if $f(\delta) = 0$.

Clearly, $f(0) = f(1) = 0$. Given that $\frac{\partial f}{\partial \delta} = 1 - \gamma^2\delta^{\gamma-1} + (\gamma^2-1)\delta^\gamma$, $\frac{\partial f}{\partial \delta}|_{\delta=0} > 0$ and $\frac{\partial f}{\partial \delta}|_{\delta=1} = 0$. Moreover, $\frac{\partial^2 f}{\partial \delta^2}|_{\delta=1} = \gamma(\gamma^2-1)\delta^{\gamma-1} - \gamma^2(\gamma-1)\delta^{\gamma-2} = \gamma(\gamma-1) > 0$, so that $\delta = 1$ is a local minimum of $f(\delta)$. This implies that, if there is a $\delta' \in (0,1)$ such that $f(\delta') = 0$, there must also be a $\delta" \in (0,1)$ such that $f(\delta") = 0$ and $\frac{\partial f}{\partial \delta}|_{\delta=\delta"} > 0$. Given that, by assumption, $\delta" \in (0,1)$, $\delta" \neq 0$, and, therefore, $\frac{f(\delta")}{\delta"} = 0$, and $\left(\frac{\partial f}{\partial \delta}|_{\delta=\delta"} - \frac{f(\delta")}{\delta"}\right) > 0$. However,

$$\begin{aligned}\frac{\partial f}{\partial \delta}|_{\delta=\delta"} - \frac{f(\delta")}{\delta"} &= -\gamma^2\delta"^{\gamma-1} + (\gamma^2-1)\delta"^\gamma + \gamma\delta"^{\gamma-1}(1-\delta") + \delta"^\gamma \\ &= (\gamma-\gamma^2)(1-\delta")\delta"^{\gamma-1} < 0,\end{aligned}$$

because $\gamma > 1$. Hence, $f(\delta^*) > 0$ and therefore $\eta(\delta^*) > 0$, at each $\delta^* \in (0,1)$.

Fact 2. Let $G(\delta^*) \equiv \left(\frac{1-\delta^{*\gamma}}{1-\delta^*}\frac{\delta^*}{\delta^{*\gamma}}\right)$. Then, $G(\delta^*) > 1$, for each $\delta^* \in (0,1)$.

Given that $\gamma > 1$, the result is quite obvious. Using Taylor's expansion, for each $\delta^* \in (0,1)$, we can write $\delta^{*\gamma} = 1 + (\gamma\delta'^{\gamma-1})(\delta^*-1)$, for some $\delta' \in (\delta^*, 1)$. Similarly, $\delta^{*\gamma} = 0 + \left(\gamma\delta"^{\gamma-1}\right)\delta^*$ for some $\delta" \in (0, \delta^*)$. Hence, $G(\delta^*) = \frac{\gamma\delta'^{\gamma-1}}{\gamma\delta"^{\gamma-1}} > 1$, because $\frac{\delta'}{\delta"} > 1$.

EXAMPLE 1. The following example shows that it can actually be the case that $\frac{\partial F}{\partial \delta^{F*}} < 0$. Assume that $\alpha = \frac{1}{2}, \Gamma = 10, A^{ne} = 1, A^{e} = 11/10$. Then, by direct computation,

$$F(\delta^{*}, .) + c_{I}^{e'} = \delta^{*\frac{1}{21}} \left(\left(\frac{11}{10} \left(\frac{105}{110} \frac{1-\delta^{*\frac{110}{105}}}{1-\delta^{*}} \right)^{\frac{1}{2}} \right)^{\frac{11}{5}} - \left(\frac{105}{110} \delta^{*\frac{5}{110}} \right)^{\frac{11}{10}} \right)$$

with $\frac{\partial F(\delta^{*},.)}{\partial \delta^{*}}$ strictly positive for δ^{*} sufficiently small, and negative for all δ^{*} larger than some critical value δ^{+}. For instance, one can check that $\lim_{\delta^{*} \to 1} \frac{\partial F(\delta^{*},.)}{\partial \delta^{*}} < -0.003$. Obviously, by choosing appropriately c_{I}^{e}, every δ^{*} can be made to become the equilibrium δ^{F*}.

Proof of Proposition 2. Given that optimization problem $(P_{\delta^{*}}^{s})$ is concave, each s, its solution is completely characterized by the FOCs:

i. $\frac{\partial P_{\delta^{*}}^{s}(h_{i}^{s},k^{s})}{\partial h_{i}} = \alpha A^{s} k^{s(1-\alpha)} h_{i}^{s(\alpha-1)} - \frac{1}{\delta_{i}} h_{i}^{s\Gamma} = 0,$

ii. $\frac{\partial P_{\delta^{*}}^{s}(h_{i}^{s},k^{s})}{\partial k} = (1-\alpha) A^{s} k^{s(-\alpha)} \int_{\Omega_{I}^{s}(\delta^{*})} h^{s\alpha} - \mu \int_{\Omega_{I}^{s}(\delta^{*})} di = 0,$

which imply

a. $K^{CPOs}(\delta^{*}) = A^{s\frac{1+\Gamma}{\Gamma\alpha}} \alpha^{\frac{1}{\Gamma}} \left(\frac{(1-\alpha)}{\mu} E_{\Omega_{I}^{s}} \left(\delta_{i}^{\frac{\alpha}{1+\Gamma-\alpha}} \right) \right)^{\frac{1+\Gamma-\alpha}{\alpha\Gamma}},$

b. $H^{CPOs}(\delta_{i}, \delta^{*}) = \delta_{i}^{\frac{1}{1+\Gamma-\alpha}} \alpha^{\frac{1}{\Gamma}} A^{s\frac{1}{\alpha\Gamma}} \left(\frac{(1-\alpha)}{\mu} E_{\Omega_{I}^{s}} \left(\delta_{i}^{\frac{\alpha}{1+\Gamma-\alpha}} \right) \right)^{\frac{1-\alpha}{\alpha\Gamma}}.$

It follows that $K^{CPOs}(\delta^{*}) > K^{Fs}(\delta^{*})$ and $H^{CPO}(\delta_{i}, \delta^{*}) > H^{Fs}(\delta_{i}, \delta^{*})$, for each value δ^{*}. Therefore, equilibria are always characterized by under-investment in physical capital and in the effort in education.

Demand and supply functions are clearly well-defined and continuous at each $\delta^{*} \in (0,1)$ and $P^{s}(\delta^{*})$ has the same properties. Hence, problem (P') has a solution, either internal or at one of the boundary points, and, therefore, CPO allocations exist.

Compare a market allocation and any CPO allocation. If they have the same threshold value δ^{*}, $K^{CPOs}(\delta^{*}) \neq K^{Fs}(\delta^{*})$ and the market allocation is not CPO. Otherwise, the threshold values are different, and constrained Pareto inefficiency follows immediately.

EXAMPLE 2. By tedious computation, one can verify that δ^{CPO} is the solution to

$$\begin{aligned} & \frac{1+\Gamma}{1+\Gamma-\alpha} \left(\frac{\mu}{1-\alpha} \right)^{\frac{(1+\Gamma)(1-\alpha)}{\alpha\Gamma}} \frac{1}{\alpha^{\frac{1}{\Gamma}}} c_{W}^{e} \\ = \; & A^{e\frac{1+\Gamma}{\Gamma\alpha}} \delta^{CPO\frac{a}{1+\Gamma-\alpha}} E_{\Omega_{W}^{e}(\delta^{CPO})} \left(\delta_{i}^{\frac{\alpha}{1+\Gamma-\alpha}} \right)^{\frac{(1+\Gamma)(1-\alpha)}{\alpha\Gamma}} \left(1 - (1-\alpha) \frac{1-\delta^{CPO\frac{1+\Gamma}{1+\Gamma-\alpha}}}{\delta^{CPO\frac{a}{1+\Gamma-\alpha}} (1-\delta^{CPO})} \right) \\ & -\alpha A^{ne\frac{1+\Gamma}{\Gamma\alpha}} \alpha^{\frac{1}{\Gamma}} \delta^{CPO\frac{a}{1+\Gamma-\alpha}} E_{\Omega_{W}^{ne}(\delta^{CPO})} \left(\delta_{i}^{\frac{\alpha}{1+\Gamma-\alpha}} \right)^{\frac{(1+\Gamma)(1-\alpha)}{\alpha\Gamma}}, \end{aligned}$$

while δ^{F*} is the solution to

$$\delta^{F*\frac{\alpha}{1+\Gamma-\alpha}} A^{e\frac{1+\Gamma}{\alpha\Gamma}} E_{\Omega_I^e(\delta^{F*})} \left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}}\right)^{\frac{(1-\alpha)(1+\Gamma)}{\alpha\Gamma}} - \delta^{F*\frac{\alpha}{1+\Gamma-\alpha}} A^{ne\frac{1+\Gamma}{\alpha\Gamma}} E_{\Omega_I^{ne}(\delta^{F*})} \left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}}\right)^{\frac{(1-\alpha)(1+\Gamma)}{\alpha\Gamma}}$$

$$= \frac{1+\Gamma}{1+\Gamma-\alpha} \left(\frac{\mu}{1-\alpha}\right)^{\frac{(1+\Gamma)(1-\alpha)}{\alpha\Gamma}} \frac{1}{\alpha^{\frac{1}{\Gamma}}} \left(\frac{1}{\beta(1-\beta)^{\frac{1-\alpha}{\alpha}}}\right)^{\frac{1+\Gamma}{\Gamma}} c_I^e.$$

It is easy to construct economies with $\delta^{CPO} > \delta^{F*}$ and with $\delta^{F*} < \delta^{CPO}$. Assume that $\Gamma = \mu = A^{ne} = 1$, while $\alpha = \beta = \frac{1}{2}$, $c_I^e = \frac{1}{8}$. Then, the two conditions above reduce to

$$\frac{1}{3} = A^{e4} \delta^{CPO\frac{1}{3}} \left(\frac{4}{3}\frac{1-\delta^{CPO\frac{4}{3}}}{1-\delta^{CPO}}\right)^2 \left(1 - \frac{1}{2}\frac{1-\delta^{CPO\frac{4}{3}}}{\delta^{CPO\frac{1}{3}}(1-\delta^{CPO})}\right) - \delta^{CPO\frac{1}{3}} \left(\frac{4}{3}\delta^{CPO\frac{1}{3}}\right)^2$$

and

$$\frac{16}{3} = \delta^{F*\frac{1}{3}} A^{e4} \left(\frac{4}{3}\frac{1-\delta^{F*\frac{4}{3}}}{1-\delta^{F*}}\right)^2 - \delta^{F*\frac{1}{3}} \left(\frac{4}{3}\delta^{F*\frac{1}{3}}\right)^2.$$

For $A^e = 2, \delta^{CPO} = 0.18290$ while $\delta^{F*} = 0.00643$. At $A^e = \frac{13}{10}, \delta^{CPO} = 0.39928$, while $\delta^{F*} = 0.57283$.

Proof of Proposition 3. The (necessary) FOCs of problem (P^t) are given by

$$\frac{\partial P(\tau, \Delta c)}{\partial \tau^s} = \int_{\Omega_I^s(\delta^{F*})} \frac{\partial U_i\left(\delta_i, \delta^{F*}, \tau\right)}{\partial \tau^s} di + \int_{\Omega_j^s(\delta^{F*})} \frac{\Pi_j\left(\delta_i, \delta^{F*}, \tau\right)}{\partial \tau^s} dj + \frac{\partial P(\tau, \Delta c)}{\partial \delta^{F*}} \frac{\partial \delta^{F*}}{\partial \tau^s} = 0$$

$$\frac{\partial P(\tau, \Delta c)}{\partial \Delta c_I^e} = -\nu\left(\Omega_I^e\left(\delta^{F*}\right)\right) + \nu\left(\Omega_I^e\left(\delta^{F*}\right)\right) + \frac{\partial P(\tau, \Delta c)}{\partial \delta^{F*}} \frac{\partial \delta^{F*}}{\partial \Delta c_I^e} = 0,$$

where

$$\begin{aligned}\frac{\partial P(\tau, \Delta c)}{\partial \delta^{F*}} = & \left(-U_i\left(H^e\left(\delta^{F*}, \delta^{F*}\right), K^e\left(\delta^{F*}\right), \tau\right) + c_I^e + U_i\left(H^{ne}\left(\delta^{F*}, \delta^{F*}\right), K^{ne}\left(\delta^{F*}\right), \tau\right)\right) \\ & - \left(\Pi_j\left(H^{ne}\left(\delta^{F*}, \delta^{F*}\right), K^e\left(\delta^{F*}\right)\right)\right) - \left(\Pi_j\left(H^{ne}\left(\delta^{F*}, \delta^{F*}\right), K^{ne}\left(\delta^{F*}\right)\right)\right) \\ & + \sum_s \int_{\Omega_I^s(\delta^{F*})} \frac{\partial U_i(.)}{\partial \delta^{F*}} di + \sum_s \int_{\Omega_J^s(\delta^{F*})} \frac{\partial \Pi_j(.)}{\partial \delta^{F*}} dj,\end{aligned}$$

while the term $\left(-\nu\left(\Omega_I^e\left(\delta^{F*}\right)\right) + \nu\left(\Omega_I^e\left(\delta^{F*}\right)\right)\right)$ reflects the redistribution of the tax revenues. By definition of δ^{F*}, the first term in brackets is zero. We have already established that the last two terms are positive. Hence, $\frac{\partial P(\tau,\Delta c)}{\partial \delta^{F*}}$ is certainly positive if

$$\Delta\Pi \equiv \left(\Pi_j\left(H^e\left(\delta^{F*}, \delta^{F*}\right), K^e\left(\delta^{F*}\right)\right)\right) - \left(\Pi_j\left(H^{ne}\left(\delta^{F*}, \delta^{F*}\right), K^{ne}\left(\delta^{F*}\right)\right)\right) < 0,$$

where

$$\begin{aligned}
&\Pi_j\left(H^s\left(\delta^{F*},\delta^{F*}\right),K^s\left(\delta^{F*}\right)\right)\\
=\ &\left((1-\beta)\,[\alpha\beta]^{\frac{1}{\Gamma}}\left(\frac{(1-\alpha)\,(1-\beta)}{\mu}\right)^{\frac{(1+\Gamma)(1-a)}{\alpha\Gamma}}\right)A^{s\,\frac{1+\Gamma}{\alpha\Gamma}}E_{\Omega_I^s\left(\delta^{F*}\right)}\left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}}\right)^{\frac{(1+\Gamma)(1-a)}{\alpha\Gamma}}\\
&\times\left(\delta^{F*\,\frac{a}{1+\Gamma-\alpha}}-(1-\alpha)\,E_{\Omega_I^s\left(\delta^{F*}\right)}\left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}}\right)\right).
\end{aligned}$$

In sector ne,

$$\left(\delta^{F*\,\frac{a}{1+\Gamma-\alpha}}-(1-\alpha)\,E_{\Omega_I^{ne}\left(\delta^{F*}\right)}\left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}}\right)\right)=\delta^{F*\,\frac{a}{1+\Gamma-\alpha}}\left(\alpha\frac{2+\Gamma-\alpha}{1+\Gamma}\right)>0.$$

In sector e, we have

$$\delta^{F*\,\frac{a}{1+\Gamma-\alpha}}\left(1-(1-\alpha)\,\frac{1+\Gamma-\alpha}{1+\Gamma}\left(\frac{1-\delta^{F*\,\frac{1+\Gamma}{1+\Gamma-\alpha}}}{\delta^{F*\,\frac{a}{1+\Gamma-\alpha}}-\delta^{F*\,\frac{1+\Gamma}{1+\Gamma-\alpha}}}\right)\right),$$

which is negative by assumption (this implies that δ^{F*} is not "too high"). Hence, $\frac{\partial P(\tau,\Delta c)}{\partial\delta^{F*}}>0$.

When $\frac{\partial F}{\partial\delta^{F*}}>0$, $\frac{\partial\delta^{F*}}{\partial\Delta c_I^e}>0$ and $\frac{\partial\delta^{F*}}{\partial\tau^{ne}}>0$, so that $\frac{\partial P(\tau,\Delta c)}{\partial\tau^{ne}}>0$ and $\frac{\partial P(\tau,\Delta c)}{\partial\Delta c_I^e}>0$. It follows that a subsidy to education effort in sector ne, and/or an increase in the fixed cost of education c_I^e, increase the expected total surplus .

On the other hand, $\frac{\partial\delta^{F*}}{\partial\tau^e}<0$ and, therefore, the sign of $\frac{\partial P(\tau,\Delta c)}{\partial\tau^s}$ is undefined.

8. APPENDIX 2: COMPETITIVE SPOT LABOR MARKETS

We use the same notation as above. We start solving for the ex-post competitive equilibrium, contingent on the amount of investments in physical capital. A straightforward computation shows that the equilibrium wage map is defined by $w^s\left(\delta_i,k_j^s\right)=\frac{\left(\delta_i\alpha A^s k_j^{s(1-\alpha)}\right)^{\frac{\Gamma}{1+\Gamma-\alpha}}}{\delta_i}$, and the investment in educational effort by $h^s\left(\delta_i,k_j^s\right)=\left(\delta_i\alpha A^s k_j^{s(1-\alpha)}\right)^{\frac{1}{1+\Gamma-\alpha}}$. Ex-post producer's surplus is given by $R\left(\delta_i,k_j^s\right)=(1-\alpha)\,\alpha^{\frac{\alpha}{1+\Gamma-\alpha}}A^{s\,\frac{1+\Gamma}{1+\Gamma-\alpha}}\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}}k_j^{\frac{(1+\Gamma)(1-\alpha)}{1+\Gamma-\alpha}}$. Consider now the ex-ante optimization problem of the firm,

$$\begin{aligned}
&\max_{k_j^s}E_{\Omega_I^s}\left(R\left(\delta_i,k_j^s\right)\right)-\mu k_j^s\\
=\ &(1-\alpha)\,\alpha^{\frac{\alpha}{1+\Gamma-\alpha}}A^{s\,\frac{1+\Gamma}{1+\Gamma-\alpha}}E_{\Omega_I^s}\left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}}\right)k_j^{\frac{(1+\Gamma)(1-\alpha)}{1+\Gamma-\alpha}}-\mu k_j^s,
\end{aligned}$$

with, j-invariant, optimal solution

$$K^s(\delta^*) = \left(\left(\frac{(1+\Gamma)(1-\alpha)}{1+\Gamma-\alpha}\frac{1-\alpha}{\mu}\right)\alpha^{\frac{\alpha}{1+\Gamma-\alpha}}A^{s\frac{1+\Gamma}{1+\Gamma-\alpha}}E_{\Omega_I^s(\delta^*)}\left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}}\right)\right)^{\frac{1+\Gamma-\alpha}{\alpha\Gamma}}.$$

Let $B \equiv \left(\frac{(1+\Gamma)(1-\alpha)}{1+\Gamma-\alpha}\frac{1-\alpha}{\mu}\right)$. Then, replacing $K^s(\delta^*)$ into $h^s\left(\delta_i, k_j^s\right)$, and simplifying, we obtain

$$H^s(\delta_i, \delta^*) = \delta_i^{\frac{1}{1+\Gamma-\alpha}}\alpha^{\frac{1}{\Gamma}}A^{s\frac{1}{\alpha\Gamma}}\left(BE_{\Omega_I^s(\delta^*)}\left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}}\right)\right)^{\frac{1-\alpha}{\alpha\Gamma}}.$$

Therefore,

$$\begin{aligned} E_{\Omega_J^s(\delta^*)}\left(U(\delta_i, \delta^*)\right) &= \frac{\Gamma}{1+\Gamma}\frac{1}{\delta_i}\left(\delta_i^{\frac{1}{1+\Gamma-\alpha}}\alpha^{\frac{1}{\Gamma}}A^{s\frac{1}{\alpha\Gamma}}\left(BE_{\Omega_I^s(\delta^*)}\left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}}\right)\right)^{\frac{1-\alpha}{\alpha\Gamma}}\right)^{1+\Gamma} \\ &= \frac{\Gamma}{1+\Gamma}\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}}\alpha^{\frac{1+\Gamma}{\Gamma}}A^{s\frac{1+\Gamma}{\alpha\Gamma}}\left(BE_{\Omega_I^s(\delta^*)}\left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}}\right)\right)^{\frac{(1+\Gamma)(1-\alpha)}{\alpha\Gamma}}, \end{aligned}$$

and the map defining the threshold is

$$\begin{aligned} 0 &= \delta^{*\frac{\alpha}{1+\Gamma-\alpha}}\left(A^e E_{\Omega_I^e(\delta^*)}\left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}}\right)^{(1-\alpha)}\right)^{\frac{(1+\Gamma)}{\alpha\Gamma}} \\ &\quad -\delta^{*\frac{\alpha}{1+\Gamma-\alpha}}\left(A^e E_{\Omega_I^e(\delta^*)}\left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}}\right)^{(1-\alpha)}\right)^{\frac{(1+\Gamma)}{\alpha\Gamma}} - \underset{\sim}{c}{}_I^e \end{aligned}$$

Modulo the product by a positive scalar, this is the condition $F(\delta^*) = 0$ in the text. Evidently, the qualitative properties of the equilibria are identical.

9. REFERENCES

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