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On the Higgs triplet extension of the Standard Model

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Kurzfassung

In dieser Diplomarbeit untersuchen wir die Higgs Triplett Erweiterung des Standardmodells der Elementarteilchenphysik, in welcher der skalare Sektor mit einem komplexen skalaren Triplet Δ mit Hyperladung $Y = 2$ erweitert wird. Der zustzliche Term in der Lagrangedichte des Yukawa Sektors induziert einen Majorana Massenterm für Neutrinos.

Desweiteren werden Prozesse, in denen die doppelt geladene Komponente H^{--} des Higgs Tripletts involviert ist, untersucht. Nachdem wir die totalen Wirkungsquerschnitte der Leptonflavor verletzenden Prozesse $e^-e^- \to \gamma^-\delta^-$ (für $\gamma, \delta = e, \mu, \tau$) und des Leptonzahl verletzenden Prozesses $e^-e^- \to W^-W^-$ ausrechnen, berechnen wir mehrere partielle Zerfallsbreiten des doppelt geladenen Skalars, nämlich $\Gamma(H^{--} \to \mu^-\mu^-)$, $\Gamma(H^{--} \to \gamma^-\delta^-)$, $\Gamma(H^{--} \to W^-W^-)$, $\Gamma(H^{--} \to W^-H^-)$ und $\Gamma(H^{--} \to W^-H^-Z^0)$.

Abstract

In this thesis we study the Higgs triplet extension of the Standard Model of particle physics where the scalar sector is extended by a complex scalar triplet Δ with hypercharge $Y = 2$. The additional term in the Lagrangian of the Yukawa sector induces a Majorana mass term for neutrinos.

Further on, processes involving the doubly charged component H^{--} of the Higgs triplet are studied. After calculating the total cross sections of the lepton flavor violating process $e^-e^- \to \gamma^-\delta^-$ (for $\gamma, \delta = e, \mu, \tau$) and the lepton number violating process $e^-e^- \to W^-W^-$, we compute several partial decay widths of the doubly charged scalar viz. $\Gamma(H^{--} \to \mu^-\mu^-)$, $\Gamma(H^{--} \to \gamma^-\delta^-)$, $\Gamma(H^{--} \to W^-W^-)$, $\Gamma(H^{--} \to W^-H^-)$ and $\Gamma(H^{--} \to W^-H^-Z^0)$.

Contents

1 Introduction

Although the Standard Model of particle physics (SM) is a well-established theory which describes the electromagnetic, weak and strong interactions with great accuracy, it is far away from being a complete theory which explains all phenomena of nature. There are many problems in particle physics which the SM is not able to describe. On the one hand the SM does not contain a quantum theory of gravity, which is essential for a full theory, and on the other hand it is not able to explain unsolved problems of particle physics like baryon asymmetry[1], dark matter[2], neutrino masses, the strong CP-problem[3] or the hierarchy problem[4]. It is obvious that the theory has to be extended in some way to describe those phenomena, where the SM does not offer an explanation.

There are several possibilities to extend the SM. While some extensions modify the SM in a fundamental way, for example by introducing a larger gauge group than $SU(3)_C \times SU(2)_L \times U(1)_Y$ like in grand unifying theories or by the assumption of extra dimensions like in string theories, there are particular extensions which focus merely on augmentation of a single sector of the SM, like the scalar sector. The scalar sector can be extended by introducing additional Higgs multiplets such as in the two Higgs doublet model [1], the Higgs triplet model [17–28], the Zee model [2, 3] or the Zee-Babu model [4].

So what is the motivation for extending the scalar sector of the SM? While in the SM the Higgs mechanism generates the masses of the fermions and weak gauge bosons via spontaneous symmetry breaking (SSB) of the $SU(2)_L \times U(1)_Y$ gauge symmetry, neutrinos stay massless due to the absence of right-handed neutrino fields. However, recent experiments have discovered the phenomenon of neutrino flavor oscillations, which violates lepton flavor conservation [5–11]. But since the probability amplitude for the transition $\nu_\alpha \to \nu_\beta$ is a function of the squared mass difference Δm_{ij}^2, flavor oscillations are only possible if neutrinos have non-zero and different masses. So it is obvious that the SM has to be extended with a neutrino mass mechanism.

One possibility to introduce neutrino masses is the so-called type-II see-saw mechanism, in which a heavy complex scalar triplet with $Y = 2$, the Higgs triplet, is added to the scalar sector [17–28]. The Lagrangian of the Yukawa sector is enhanced with a gauge invariant coupling $\mathcal{L}_\Delta$ between the Higgs triplet and the lepton doublets, which automatically leads to a Majorana mass term proportional to v_T (the vacuum expectation value (VEV) of the neutral component of the Higgs triplet) for neutrinos at tree-level. One expects v_T to be much smaller than the VEV of the Higgs doublet, viz. $|v_T| \ll v$, since a larger triplet VEV would destroy the tree-level relation $M_W = M_Z \cos(\theta_W)$ between the W and Z^0 boson masses and the Weinberg angle and the precision measurements place a stringent bound on v_T [12]. Due to different scales of v_T and v, the Higgs triplet model gives a clue why the neutrino masses are smaller than the masses of the charged leptons although it does not explain their extraordinary smallness.

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[1]The Baryon asymmetry problem refers to the obvious fact that there is an imbalance between baryonic matter and antibaryonic matter in the universe [11].

[2]Dark matter is a currently unknown type of matter hypothesized to account for a large part of the total mass in the universe. It was proposed to explain the discrepancy between the mass of large astronomical objects determined from their gravitational effects and mass calculated from the luminous matter they contain [11].

[3]The strong CP-problem is the puzzling question why quantum chromodynamics (QCD) does not seem to break the charge-parity-symmetry [11].

[4]The hierarchy problem refers to the fact that quantum contributions to the square of the Higgs-mass inevitably make the mass huge, comparable to the scale at which new physics appears, unless there is an incredible fine-tuning cancellation between the quadratic radiative corrections and the bare mass [11].

This work is organized as follows. In section 2 the Higgs triplet model and its characteristics are introduced. Dirac and Majorana mass terms are reviewed in section 3. In sections 4, 5 and 6 we will calculate the total cross sections $\sigma(e^-e^- \to \mu^-\mu^-)$, $\sigma(e^-e^- \to W^-W^-)$ and $\sigma(\alpha^-\beta^- \to \gamma^-\delta^-)$ (for $\alpha, \beta, \gamma, \delta = e, \mu, \tau$), respectively. The partial decay widths $\Gamma(H^{--} \to \mu^-\mu^-)$, $\Gamma(H^{--} \to \gamma^-\delta^-)$, $\Gamma(H^{--} \to W^-W^-)$, $\Gamma(H^{--} \to W^-H^-)$ and $\Gamma(H^{--} \to W^-H^-Z^0)$, respectively are calculated in sections 7, 8, 9, 10 and 11. The latter is evaluated numerically with a FORTRAN program named "RAMBOC" based on the program "RAMBO" (random momenta booster) [35]. Conclusions are made in section 12. At last, in the appendix in chapter 13 definitions of field operators, auxiliary calculations, important formulas and the source code of "RAMBOC" are given.

2 The Higgs triplet model

The idea of adding a scalar triplet to the SM was first mentioned in a work of W. Konetschny and W. Kummer in the late 70's. In their work the role of scalar bosons in connection with non-conservation of lepton number was studied. It was shown that the addition of scalar singlets S^+ and S^{++} and a triplet $\vec{\Phi} = (\phi^{++}, \phi^+, \phi^0)$ permit Yukawa couplings, which allow lepton flavor violating transitions like $\mu \to e\gamma$ and $\mu \to 3e$ [13].

This idea of adding a scalar triplet to the SM was also used in a work of G.B. Gelmini and M. Roncadelli in 1981. In their minimal SM extension they enlarged the Higgs sector with a Higgs triplet, in order to introduce neutrino masses. Since right-handed neutrinos are still absent in this extension, a Dirac mass term is forbidden. However, a lepton number violating Majorana mass term is possible, viz.

$$\mathcal{L}_{mass} = m_\nu \overline{(\nu_L)^C} \nu_L + \text{H.c.} \tag{1}$$

which can be rewritten as

$$\mathcal{L}_{mass} = -m_\nu \bar{\nu}\nu, \tag{2}$$

with $\nu \equiv \nu_L + (\nu_L)^C = \nu^C$[5]. Therefore the neutrino ν is considered as a Majorana particle in this model. The mass term $\mathcal{L}_{mass}$ has to be introduced by the Higgs mechanism through an invariant Yukawa coupling of $L_L^T \equiv (\nu_L, e_L)$ and L_L^C with a new scalar field, $\vec{\Phi}$, a triplet under $SU(2)_L$ [6] [14].
In the Gelmini-Roncadelli model the Yukawa Lagrangian in the lepton sector is given by [14,15]

$$\mathcal{L}_Y = \sum_{\alpha,\beta} \{-c_{\alpha\beta} \bar{l}_{\alpha R} \phi^\dagger L_{\beta L} + \frac{1}{2} f_{\alpha\beta} L_{\alpha L}^T C^{-1} i\tau_2 \Delta L_{\beta L}\} + \text{H.c.}, \tag{3}$$

where $L_{\alpha L} = (\nu_{\alpha L}, l_{\alpha L})$ denotes the left-handed lepton doublets, l_{aR} the right-handed lepton singlets and ϕ the Higgs doublet, for the flavor indices α and β. $c_{\alpha\beta}$ and $f_{\alpha\beta}$ are Yukawa coupling matrices, the latter is symmetric, i.e. $f_{\alpha\beta} = f_{\beta\alpha}$. Furthermore, C is the charge-conjugation matrix, τ_2 the second pauli matrix and Δ the 2×2 representation of the Higgs triplet which has a neutral, single charged and doubly charged component. The multiplets from (3) transform under $U \in SU(2)$ as

$$L_{\alpha L} \to U L_{\alpha L}, \quad l_{\alpha R} \to l_{\alpha R}, \quad \phi \to U\phi, \quad \Delta \to U \Delta U^\dagger. \tag{4}$$

Their $U(1)$ transformation properties are determined by the hypercharges:

$$\begin{array}{c|cccc} & L_a & \ell_{aR} & \phi & \Delta \\ \hline Y & -1 & -2 & 1 & 2 \end{array} \tag{5}$$

The VEVs of the Higgs multiplets consistent with electric charge conservation are given by [15]

$$\langle\phi\rangle_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \quad \text{and} \quad \langle\Delta\rangle_0 = \begin{pmatrix} 0 & 0 \\ v_T & 0 \end{pmatrix}. \tag{6}$$

[5] The charge conjugate spinor is defined by $(\psi)^c = C\gamma_0^T \psi^*$

[6] Charge conservation rules out the possibility of a neutral singlet.

The relation between the triplet $\vec{\Phi}$, the 2×2 matrix Δ and the charged and neutral scalars contained in the triplet is found as [15]

$$\Delta = \vec{\Phi}\cdot\vec{\tau} = \begin{pmatrix} H^+ & \sqrt{2}H^{++} \\ \sqrt{2}H^0 & -H^+ \end{pmatrix} \tag{7}$$

with

$$\vec{\Phi} = \begin{pmatrix} \frac{1}{\sqrt{2}}(H^0 + H^{++}) \\ \frac{-i}{\sqrt{2}}(H^0 - H^{++}) \\ H^+ \end{pmatrix}. \tag{8}$$

The charge eigenfields are given by

$$H^{++} = \frac{1}{\sqrt{2}}(H_1 - iH_2), \quad H^+ = H_3, \quad H^0 = \frac{1}{\sqrt{2}}(H_1 + iH_2). \tag{9}$$

The matrices τ_j (j=1,2,3) are the pauli matrices. In (6) we have set $\langle H^0\rangle_0 = \frac{v_T}{\sqrt{2}}$. The most general Higgs Potential involving ϕ and Δ is written as [15]

$$\begin{aligned} &V(\phi,\Delta) = \\ &a\phi^\dagger\phi + \frac{b}{2}\mathrm{Tr}(\Delta^\dagger\Delta) + c(\phi^\dagger\phi)^2 + \frac{d}{4}\big(\mathrm{Tr}(\Delta^\dagger\Delta)\big)^2 \\ &+ \frac{e-h}{2}\phi^\dagger\phi\mathrm{Tr}(\Delta^\dagger\Delta) + \frac{f}{4}\mathrm{Tr}(\Delta^\dagger\Delta^\dagger)\mathrm{Tr}(\Delta\Delta) \\ &+ h\phi^\dagger\Delta^\dagger\Delta\phi + \big(t\phi^\dagger\Delta\tilde{\phi} + \mathrm{H.c.}\big), \end{aligned} \tag{10}$$

where $\tilde{\phi} \equiv i\tau_2\phi^*$. Under the assumption that the lepton number is conserved one has to assign lepton number -2 to the Higgs triplet and 0 to the Higgs doublet [14] (see (3)). This lepton number is explicitly broken by the last term in the Higgs potential (10). Otherwise, this Higgs potential agrees with the one given in [14] with the same definition of the coupling constants. All parameters in the Higgs potential are real except t which is complex in general.

The VEV of the Higgs doublet v can always be chosen real and positive, by performing a global $U(1)$ transformation. Because of the t-term in the potential we do not have a second global symmetry, the lepton number [14], to make v_T real. Furthermore, t can also be complex and, therefore, in general we write $t = |t|e^{i\omega}$ and $v_T = we^{i\gamma}$ with $w \equiv |v_T|$. The following orders of magnitude for the parameters in the potential are assumed:

$$a,\ b \sim v^2\,; \quad c,\ d,\ e,\ f,\ h \sim 1\,; \quad |t| \ll v\,. \tag{11}$$

The potential as a function of the VEVs is given by [15]

$$\begin{aligned} V(\langle\phi\rangle_0, \langle\Delta\rangle_0) &= \frac{1}{2}av^2 + \frac{1}{2}bw^2 + \frac{1}{4}cv^4 + \frac{1}{4}dw^4 \\ &+ \frac{1}{4}(e-h)v^2w^2 + v^2w|t|\cos(\omega+\gamma)\,. \end{aligned} \tag{12}$$

In order to obtain relations between the VEVs and the parameters of the Higgs potential, we minimize $V(\langle\phi\rangle_0, \langle\Delta\rangle_0)$ as a function of the three parameters v, w, γ. We minimize with respect to γ, the phase of v_T and get

$$\frac{\partial V}{\partial\gamma} = -v^2w|t|\sin(\omega+\gamma) = 0, \tag{13}$$

which implies that the minimum is at

$$\omega + \gamma = \pi \tag{14}$$

or

$$v_T = -we^{-i\omega} \quad \text{and} \quad v_T t = -w|t|. \tag{15}$$

The minimization with respect to v gives

$$\frac{\partial V}{\partial v} = av + cv^3 + \frac{e-h}{2} vw^2 + 2vw|t|\cos(\omega + \gamma) = 0. \tag{16}$$

When we divide by v and insert (14) we get the second minimum condition

$$a + cv^2 + \frac{e-h}{2} w^2 - 2w|t| = 0. \tag{17}$$

The minimization with respect to w gives

$$\frac{\partial V}{\partial w} = b + dw^3 + \frac{e-h}{2} v^2 w + v^2 |t| \cos(\omega + \gamma) = 0, \tag{18}$$

which after dividing by w and inserting (14) becomes the third minimum condition

$$b + dw^2 + \frac{e-h}{2} v^2 - \frac{v^2|t|}{w} = 0. \tag{19}$$

With the assumption made in (11) we can approximate (17) and (19) and get

$$v^2 \simeq -\frac{a}{c} \quad \text{and} \quad w \simeq |t| \frac{v^2}{b + (e-h)v^2/2} \tag{20}$$

Hence we see that $w \sim |t|$, i.e., the triplet VEV is of the order of the parameter $|t|$ in the Higgs potential. The fine-tuning to get a small triplet VEV is therefore simply given by $|t| \ll v$, which should find an explanation in a more complete theory which has the Gelmini-Roncadelli model as a low energy limit.[7] This is the analoguous situation as with the Standard Model and the see-saw mechanism for light neutrino masses, where the large mass scale of the right-handed neutrino singlets is assumed to come, e.g., from Grand Unification.

Mass terms for the charged leptons and neutrinos are induced by (3) and (6):

$$-\left(\bar{l}_R \mathcal{M}_l l_L + \text{H.c.}\right) \quad \text{with} \quad \mathcal{M}_l = \frac{v}{\sqrt{2}} (c_{\alpha\beta}), \tag{21}$$

$$\frac{1}{2} \nu_L^T C^{-1} \mathcal{M}_\nu \nu_L + \text{H.c.} \quad \text{with} \quad \mathcal{M}_\nu = v_T (f_{\alpha\beta}). \tag{22}$$

[7] Alternatively, one could use $b \gg v^2$ to get a small triplet VEV [17].

3 Neutrino mass terms

3.1 Dirac versus Majorana neutrinos

In this section we will discuss the differences between Dirac and Majorana neutrinos. Aside from the vanishing electric charge of Majorana particles, the bigger part of the established extensions of the SM suggest that neutrinos have Majorana nature. There are several interesting features of Majorana particles, which make them quite different from Dirac particles. Unfortunately, in consequence of the smallness of neutrino masses, it is quite difficult to distinguish between both natures in experiments: Neutrinoless $\beta\beta$-decay seems to be the only prospective road so far. In any case, neutrino oscillations do not distinguish between Dirac and Majorana neutrinos, since both for neutrinos and antineutrinos negative and positive helicity, respectively, remains unchanged during flavor transition.

Though family lepton numbers must be violated for transitions $\nu_\alpha \to \nu_\beta$, the total number remains conserved [8]. With two independent chiral 4-spinor fields $\Psi_{L,R}$ one can construct a Dirac mass term by writing a *Lorentz-invariant* bilinear for Dirac fields [8]:

$$\text{Dirac:} \quad -m\big(\bar{\psi}_R\psi_L + \text{H.c.}\big) = -m\bar{\psi}\psi \quad \text{with} \quad \psi = \psi_L + \psi_R. \tag{23}$$

On the contrary, with only one chiral 4-spinor ψ_L a *Lorentz-invariant* bilinear can still be constructed with the help of the charge-conjugation matrix C. This bilinear is the so-called Majorana mass term [8]:

$$\text{Majorana:} \quad \frac{1}{2}m\psi_L^T C^{-1}\psi_L + \text{H.c.} = -\frac{1}{2}m\bar{\psi}\psi \quad \text{with} \quad \psi = \psi_L + \big(\psi_L\big)^c. \tag{24}$$

Note that from ψ_L one obtains a right-handed field with the charge-conjugation operation $\big(\psi_L\big)^c \equiv C\gamma_0^T\psi_L^*$. In contrast to the Dirac case, this right-handed field is *not* independent of the left-handed field. The equation for free Dirac or Majorana fields in terms of the above defined spinors is the same:

$$\text{Dirac and Majorana:} \quad \big(i\gamma^\mu\partial_\mu - m\big)\psi = 0. \tag{25}$$

In the formalism used here the Majorana nature is hidden in the

$$\text{Majorana condition} \quad \psi = \psi^c. \tag{26}$$

Thus the solution of (25) is found in the same way for both fermion types. However, for Majorana neutrinos the condition (26) is imposed on the solution. This leads to the following observations.

Dirac fermions: The field ψ contains annihilation and creation operators $b, d^\dagger$, respectively, with independent operators b, d, and thus a Dirac fermion field has particles and antiparticles with positive and negative helicities.

Majorana fermions: Because of the condition (26) the annihilation and creation operators are $a, a^\dagger$, respectively, and we have only particles with positive and negative helicities.

The mass terms (23) and (24) are written in the way as they appear in the Lagrangian. The field equation (25) is obtained from the Lagrangian by variation with respect to the independent fields. In this procedure the factor $\frac{1}{2}$ in the Majorana mass term is cancelled by the factor of 2 which occurs in the variation of the mass term because the fields to the left and to the right of C^{-1} are identical (see the first term in (24)). Up to now we have discussed the case of one neutrino with mass $m > 0$. Let us now consider n neutrinos. Then in the Dirac case we have the following mass term:

$$\text{Dirac:} \quad (\bar{\nu}_R \mathcal{M} \nu_L + \text{H.c.}) = \bar{\nu}' \hat{m} \nu'. \tag{27}$$

Here, $\mathcal{M}$ is an *arbitrary* complex $n \times n$ matrix and the fields $\nu_{L,R}$ are vectors containing n 4-spinors. In order to arrive at the diagonal and positive matrix $\hat{m}$ we use the following theorem concerning bidiagonalization in linear algebra [8].

Theorem 1 *If $\mathcal{M}$ is an arbitrary complex $n \times n$ matrix, then there exist unitary matrices $U_{L,R}$ with $U_R^\dagger \mathcal{M} U_L = \hat{m}$ diagonal and positive.*
Applying this theorem we obtain the

$$\text{physical Dirac fields} \quad \nu' = \nu'_L + \nu'_R \quad \text{with} \quad \nu_{L,R} = U_{L,R} \nu'_{L,R}. \tag{28}$$

Note that the $U(1)$ invariance of the mass term under $\nu_{L,R} \to e^{i\alpha} \nu_{L,R}$ corresponds to total lepton number conservation, provided the rest of the Lagrangian respects this symmetry as well.

Switching to the Majorana case we have the following mass term:

$$\text{Majorana:} \quad \frac{1}{2} \nu_L^T C^{-1} \mathcal{M} \nu_L + \text{H.c.} = \frac{1}{2} \nu'^T_L C^{-1} \hat{m} \nu'_L + \text{H.c.} = -\frac{1}{2} \bar{\nu}' \hat{m} \nu'. \tag{29}$$

Now the mass matrix $\mathcal{M}$ is a complex matrix which fulfills

$$\mathcal{M}^T = \mathcal{M}. \tag{30}$$

This follows from the anticommutation property of the fermionic fields and $C^T = -C$. The diagonalization of the mass term proceeds now via a theorem of I. Schur [33].

Theorem 2 *If $\mathcal{M}$ is a complex symmetric $n \times n$ matrix, then there exists a unitary matrix U_L with $U_L^T \mathcal{M} U_L = \hat{m}$ diagonal and positive.*
With this theorem we obtain the

$$\text{physical Majorana fields} \quad \nu' = \nu'_L + \left(\nu'_L\right)^c \quad \text{with} \quad \nu_L = U_L \nu'_L. \tag{31}$$

The mass term (29) not only violates individual lepton family numbers just as the Dirac mass term (27), but it also violates the total lepton number $L = \sum_\alpha L_\alpha$.

3.2 The mixing matrix U_{PMNS}

Neutrino production and detection is described via charged current (CC) interaction with the Hamiltonian [8]

$$\mathcal{H}_{cc} = \frac{g}{\sqrt{2}} W_\rho^- \sum_{\alpha=e,\mu,\tau} \bar{l}_\alpha \gamma^\rho \nu_{\alpha L} + \text{H.c.} \tag{32}$$

Flavor transitions are induced by neutrino mixing:

$$\nu_{\alpha L} = \sum_j U_{\alpha j} \nu_{jL}. \tag{33}$$

Where $\nu_{\alpha L}$ are neutrino flavor eigenfields , ν_{jL} neutrino mass eigenfields and $U_{\alpha j}$ the Pontecorvo-Maki-Nakagawa-Sakata matrix (also called *lepton mixing matrix* or *PMNS matrix*). It is given by

$$U = U_L^{(l)^\dagger} U_L^{(\nu)} \tag{34}$$

where $U_L^{(l)^\dagger}$ is the matrix which bi-diagonalizes the mass matrix of the charged lepton fields, i.e. $U_R^{(l)^\dagger} \mathcal{M}^{(l)} U_L^{(l)} = \hat{m}^{(l)}$ and $U_L^{(\nu)}$ the matrix which diagonalizes the neutrino mass matrix, i.e $U_L^{(\nu)^\dagger} \mathcal{M}^{(\nu)} U_L^{(\nu)} = \hat{m}^{(\nu)}$. The lepton mixing matrix is usually parametrized by [11]

$$\begin{aligned}
U_{PMNS} &= \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = U_{23} U_{13} U_{23} \\
&= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ -s_{23} & c_2 3 & \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{\frac{\alpha_1}{2}} & 0 & 0 \\ 0 & e^{\frac{\alpha_2}{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
&= \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta} & c_{23} c_{13} \end{pmatrix} \\
&\times \begin{pmatrix} e^{\frac{\alpha_1}{2}} & 0 & 0 \\ 0 & e^{\frac{\alpha_2}{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix},
\end{aligned} \tag{35}$$

where $c_{ij} = \cos(\theta_{ij})$ and $s_{ij} = \sin(\theta_{ij})$. The phase factor δ is non-zero only if neutrino oscillation violates charge-parity (CP) symmetry. In case neutrinos are Majorana particles the phase factors α_1 and α_2 are physically meaningful [11].

4 The cross section $\sigma(e^-e^- \to \mu^-\mu^-)$

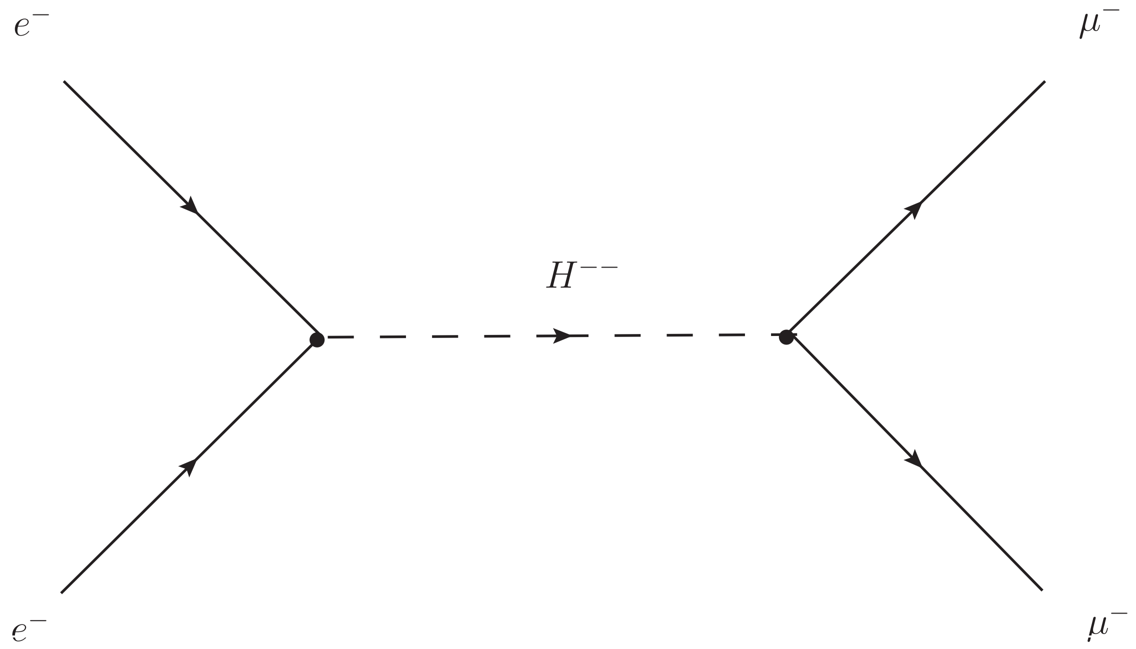

Figure 1: s-channel diagram for $e^-e^- \to \mu^-\mu^-$

In this section we will compute the total cross section of the lepton flavor violating process $e^-e^- \to \mu^-\mu^-$, shown in figure 1. Let us at first compute the S-Matrix element of this tree-level process. The n-th order of the S-matrix series is given by [16]

$$S_n = \frac{(-i)^n}{n!} \int d^4x_1 \dots d^4x_n \mathcal{T}[\mathcal{H_W}(t_1) \dots \mathcal{H_W}(t_n)], \tag{36}$$

where $\mathcal{T}$ denotes the time-ordering operator and $\mathcal{H_W} = -\mathcal{L_W}$ the interaction Hamilton density. In our case we have a second order S-Matrix element

$$S_2 = i^2 \int d^4x d^4y \langle 0 \mid b_{\mu^-}(p_3, s_3) b_{\mu^-}(p_4, s_4) \mathcal{L}(x) \mathcal{L}(y) b^\dagger_{e^-}(p_1, s_1) b^\dagger_{e^-}(p_2, s_2) \mid 0 \rangle. \tag{37}$$

For the computation of the cross section of $e^-e^- \to \mu^-\mu^-$ via virtual H^{--} we need the Higgs triplet interaction term of the Lagrangian from (3)

$$\mathcal{L}_\Delta = \sum_{\alpha,\beta} \frac{1}{2} f_{\alpha\beta} L^T_{\alpha L} C^{-1} i\tau_2 \Delta L_{\beta L} + \text{H.c.} \tag{38}$$

With

$$i\tau_2\Delta = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} H^+ & \sqrt{2}H^{++} \\ \sqrt{2}H^0 & -H^+ \end{pmatrix} = \begin{pmatrix} \sqrt{2}H^0 & -H^+ \\ -H^+ & \sqrt{2}H^{++} \end{pmatrix} \tag{39}$$

the relevant part of the Lagrangian becomes

$$\mathcal{L}'_\Delta = \frac{1}{\sqrt{2}} H^{++} [f_{ee} e^T_L C^{-1} e_L + f_{\mu\mu} \mu^T_L C^{-1} \mu_L] + \frac{1}{\sqrt{2}} H^{--} [f^*_{ee} e^\dagger_L C e^*_L + f^*_{\mu\mu} \mu^\dagger_L C \mu^*_L]. \tag{40}$$

Since we have two e^- in the initial state and two μ^- in the final state we can ignore the second and fourth term and (40) becomes

$$\mathcal{L}''_\Delta = \frac{1}{\sqrt{2}} H^{++} f_{ee} e_L^T C^{-1} e_L + \frac{1}{\sqrt{2}} H^{--} f^*_{\mu\mu} \mu_L^\dagger C \mu_L^*. \tag{41}$$

With the Dirac field operators from (A1) and (A2) for e_L and μ_L and the corresponding Lagrangians of (41) the second order S-Matrix element of (37) becomes

$$\begin{aligned} S_2 = i^2 \int & d^4x d^4y \langle 0 \mid b_{\mu^-}(p_3, s_3) b_{\mu^-}(p_4, s_4) f_{ee} f^*_{\mu\mu} \frac{1}{2} H^{--}(x) H^{++}(y) \\ \times \sum_{r_1, r_2} & \int \frac{d^3k_1}{\sqrt{(2\pi)^3 2E_{k_1}}} \int \frac{d^3k_2}{\sqrt{(2\pi)^3 2E_{k_2}}} e^{ik_1x} u_L^\dagger(k_1, r_1) b^\dagger_{\mu^-}(k_1, r_1) C e^{ik_2x} u_L^*(k_2, r_2) b^\dagger_{\mu^-}(k_2, r_2) \\ \times \sum_{t_1, t_2} & \int \frac{d^3q_1}{\sqrt{(2\pi)^3 2E_{q_1}}} \int \frac{d^3q_2}{\sqrt{(2\pi)^3 2E_{q_2}}} e^{-iq_1y} u_L^T(q_1, t_1) b_{e^-}(q_1, t_1) C^{-1} e^{-iq_2y} u_L(q_2, t_2) b_{e^-}(q_2, t_2) \\ \times\ & b^\dagger_{e^-}(p_1, s_1) b^\dagger_{e^-}(p_2, s_2) \mid 0\rangle. \end{aligned} \tag{42}$$

Let us take a look at the terms $u_L^T(q_1, t_1) C^{-1} u_L(q_2, t_2)$ and $u^\dagger(k_1, r_1) C u^*(k_2, r_2)$ of (42). As in appendix (C) shown, the relations

$$u_L^T(q_1, t_1) C^{-1} u_L(q_2, t_2) = -v(q_1, t_1) P_L u(q_2, t_2) \tag{43}$$

and

$$u^\dagger(k_1, r_1) C u^*(k_2, r_2) = \bar{u}(k_2, r_2) P_R v(k_1, r_1) \tag{44}$$

are valid. We will insert them into (42) later on. At next let us take a closer look at the creation and annihilation operators in the S-matrix element in (42). As in (B3) calculated, the product of the operators can be rewritten as

$$\begin{aligned} & b_{e^-}(q_1, t_1) b_{e^-}(q_2, t_2) b^\dagger_{e^-}(p_1, s_1) b^\dagger_{e^-}(p_2, s_2) \mid 0\rangle \\ & = [\delta^{(3)}(\vec{q_2} - \vec{p_1}) \delta_{t_2 s_1} \delta^{(3)}(\vec{q_1} - \vec{p_2}) \delta_{t_1 s_2} - \delta^{(3)}(\vec{q_1} - \vec{p_1}) \delta_{t_1 s_1} \delta^{(3)}(\vec{q_2} - \vec{p_2}) \delta_{t_2 s_2}] \mid 0\rangle \end{aligned} \tag{45}$$

The product of the muon operators in (42) can be rewritten analogously to (B3).

$$\begin{aligned} & b_{\mu^-}(p_3, s_3) b_{\mu^-}(p_4, s_4) b^\dagger_{\mu^-}(k_1, r_1) b^\dagger_{\mu^-}(k_2, r_2) \mid 0\rangle \\ & = [\delta^{(3)}(\vec{p_4} - \vec{k_1}) \delta_{s_4 r_1} \delta^{(3)}(\vec{p_3} - \vec{k_2}) \delta_{s_3 r_2} - \delta^{(3)}(\vec{p_4} - \vec{k_2}) \delta_{s_4 r_2} \delta^{(3)}(\vec{p_3} - \vec{k_1}) \delta_{s_3 r_1}] \mid 0\rangle. \end{aligned} \tag{46}$$

At next we will compute the integrals of the S-matrix in (42). The space-time integrals over the exponential functions of the lepton fields and the contraction of the doubly charged Higgsfields give the following expression

$$\begin{aligned} & \int d^4x \int d^4y \int \frac{d^4p_H}{(2\pi)^4} e^{ik_1x} e^{ik_2x} e^{-iq_1y} e^{-iq_2y} e^{-ip_H(y-x)} \langle 0 \mid H^{--}(x) H^{++}(y) \mid 0\rangle \\ & = \int \frac{d^4p_H}{(2\pi)^4} (2\pi)^4 \delta^{(4)}(k_1 + k_2 + p_H) (2\pi)^4 \delta^{(4)}(-q_1 - q_2 - p_H) \frac{1}{p_H^2 - m^2 + i\epsilon} \\ & = (2\pi)^4 \frac{\delta^{(4)}(k_1 + k_2 - q_1 - q_2)}{(k_1 + k_2)^2 - m^2 + i\epsilon} = (2\pi)^4 \frac{\delta^{(4)}(k_1 + k_2 - q_1 - q_2)}{s - m^2 + i\epsilon}. \end{aligned} \tag{47}$$

In the next to last step we treat the term $\delta^{(4)}(-q_1 - q_2 - p_H)\frac{1}{p_H^2 - m^2 + i\epsilon}$ as an ordinary function and use the definition of the delta distribution and in the last step we use the definition of the Mandelstam variable $s = (k_1 + k_2)^2$.

In the following we have to combine (43), (44), (45) and (46). Lets take a look at integrals and sums over the spinors with the Dirac- and Kroneckerdeltas of (42), which with the results of (43), (44), (45) and (46) now becomes

$$\begin{aligned}
&\sum_{r_1,r_2}\int\frac{d^3k_1}{\sqrt{(2\pi)^3 2E_{k_1}}}\int\frac{d^3k_2}{\sqrt{(2\pi)^3 2E_{k_2}}}\bar{u}(k_2,r_2)P_R v(k_1,r_1)\\
&\times\sum_{t_1,t_2}\int\frac{d^3q_1}{\sqrt{(2\pi)^3 2E_{q_1}}}\int\frac{d^3q_2}{\sqrt{(2\pi)^3 2E_{q_2}}}(-v(q_1,t_1)P_L u(q_2,t_2))\\
&\times[\delta^{(3)}(\vec{p_4}-\vec{k_1})\delta_{s_4 r_1}\delta^{(3)}(\vec{p_3}-\vec{k_2})\delta_{s_3 r_2} - \delta^{(3)}(\vec{p_4}-\vec{k_2})\delta_{s_4 r_2}\delta^{(3)}(\vec{p_3}-\vec{k_1})\delta_{s_3 r_1}]\\
&\times[\delta^{(3)}(\vec{q_2}-\vec{p_1})\delta_{t_2 s_1}\delta^{(3)}(\vec{q_1}-\vec{p_2})\delta_{t_1 s_2} - \delta^{(3)}(\vec{q_1}-\vec{p_1})\delta_{t_1 s_1}\delta^{(3)}(\vec{q_2}-\vec{p_2})\delta_{t_2 s_2}]
\end{aligned}$$

$$\begin{aligned}
&= \frac{1}{\sqrt{(2\pi)^{12}16E_{k_1}E_{k_2}E_{q_1}E_{q_2}}}\\
&\times[-\bar{u}(p_3,s_3)P_R v(p_4,s_4)\bar{v}(p_2,s_2)P_L u(p_1,s_1)\\
&+\bar{u}(p_3,s_3)P_R v(p_4,s_4)\bar{v}(p_1,s_1)P_L u(p_2,s_2)\\
&+\bar{u}(p_4,s_4)P_R v(p_3,s_3)\bar{v}(p_2,s_2)P_L u(p_1,s_1)\\
&-\bar{u}(p_4,s_4)P_R v(p_3,s_3)\bar{v}(p_1,s_1)P_L u(p_2,s_2)]\\
&= \frac{1}{\sqrt{(2\pi)^{12}16E_{k_1}E_{k_2}E_{q_1}E_{q_2}}}\\
&\times[\bar{v}(p_2,s_2)P_L u(p_1,s_1)-\bar{v}(p_1,s_1)P_L u(p_2,s_2)]\\
&\times[\bar{u}(p_4,s_4)P_R v(p_3,s_3)-\bar{u}(p_3,s_3)P_R u(p_4,s_4)].
\end{aligned} \tag{48}$$

In the first step we use the properties of the Dirac- and Kronecker-deltas to change the momentum- and spin-variables, and in the second step we rewrite the spinor terms as a product since it has the structure of $-ab + ac + db - dc = b(d-a) - c(d-a) = (d-a)(b-c)$.

With the results of (47) and (48) the second order S-matrix element of (42) can now be rewritten as

$$\begin{aligned}
S_2 = i^2\frac{f_{ee}f^*_{\mu\mu}}{2}&\frac{1}{\sqrt{(2\pi)^{12}16E_{k_1}E_{k_2}E_{q_1}E_{q_2}}}(2\pi)^4\frac{\delta^{(4)}(k_1+k_2-q_1-q_2)}{(k_1+k_2)^2-m^2+i\epsilon}\\
\times&[\bar{v}(p_2,s_2)P_L u(p_1,s_1)-\bar{v}(p_1,s_1)P_L u(p_2,s_2)]\\
\times&[\bar{u}(p_4,s_4)P_R v(p_3,s_3)-\bar{u}(p_3,s_3)P_R u(p_4,s_4)].
\end{aligned} \tag{49}$$

In order to perform the calculation of the cross section we have to extract the invariant matrix element $\mathcal{M}$ of the S-matrix by the method described in appendix (E). In the following we will again abbreviate terms like $u(p_i, s_i)$ or $v(p_i, s_i)$ as u_i or v_i. In this case the invariant matrix element $\mathcal{M}$ is:

$$\begin{aligned}
\mathcal{M} &= \frac{1}{2}\frac{f_{ee}f_{\mu\mu}}{s-m^2+i\epsilon}[\bar{v}(p_2,s_2)P_L u(p_1,s_1)-\bar{v}(p_1,s_1)P_L u(p_2,s_2)]\\
&\times[\bar{u}(p_4,s_4)P_R v(p_3,s_3)-\bar{u}(p_3,s_3)P_R u(p_4,s_4)]\\
&= \frac{1}{2}\frac{f_{ee}f_{\mu\mu}}{s-m^2+i\epsilon}[\bar{v}_2 P_L u_1 - \bar{v}_1 P_L u_2][\bar{u}_4 P_R v_3 - \bar{u}_3 P_R u_3] = \frac{1}{2}\frac{f_{ee}f_{\mu\mu}}{s-m^2+i\epsilon}A_1 A_2
\end{aligned} \tag{50}$$

Now we want to average over all spins in the initial state and sum over all spins in the end state. Then we take the absolute square value of $\mathcal{M}$ respectively $A_1 A_2$ and get

$$\begin{aligned}
&\frac{1}{2}\frac{1}{2}\sum_{spins}|A_1A_2|^2 = \frac{1}{4}\sum_{spins_1} A_1A_1^* \sum_{spins_2} A_2A_2^* \\
&= \frac{1}{4}\sum_{s_1,s_2,r_1,r_2} [\bar{v}_2P_Lu_1 - \bar{v}_1P_Lu_2][\bar{v}_2P_Lu_1 - \bar{v}_1P_Lu_2]^\dagger \\
&\times \sum_{s_1,s_2,r_1,r_2} [\bar{u}_4P_Rv_3 - \bar{u}_3P_Rv_4][\bar{u}_4P_Rv_3 - \bar{u}_3P_Rv_4]^\dagger \\
&= \frac{1}{4}\sum_{s_1,s_2,r_1,r_2} [\bar{v}_2P_Lu_1 - \bar{v}_1P_Lu_2][\bar{u}_1P_Rv_2 - \bar{u}_2P_Rv_1] \\
&\times \sum_{s_1,s_2,r_1,r_2} [\bar{u}_4P_Rv_3 - \bar{u}_3P_Rv_4][\bar{v}_3P_Lu_4 - \bar{v}_4P_Lu_3]
\end{aligned}$$

$$\begin{aligned}
&= \frac{1}{4}\sum_{s_1,s_2,r_1,r_2} [\bar{v}_2P_Lu_1\bar{u}_1P_Rv_2 - \bar{v}_1P_Lu_2\bar{u}_1P_Rv_2 - \bar{v}_2P_Lu_1\bar{u}_2P_Rv_1 + \bar{v}_1P_Lu_2\bar{u}_2P_Rv_1] \\
&\times \sum_{s_1,s_2,r_1,r_2} [\bar{u}_4P_Rv_3\bar{v}_3P_Lu_4 - \bar{u}_3P_Rv_4\bar{v}_3P_Lu_4 - \bar{u}_4P_Rv_3\bar{v}_4P_Lu_3 + \bar{u}_3P_Rv_4\bar{v}_4P_Lu_3].
\end{aligned} \tag{51}$$

Before we use the method of trace technique to convert the spinors we have to rewrite the terms $-\bar{v}_1P_Lu_2\bar{u}_1P_Rv_2$ and $-\bar{v}_2P_Lu_1\bar{u}_2P_Rv_1$ respectively $-\bar{u}_3P_Rv_4\bar{v}_3P_Lu_4$ and $-\bar{u}_4P_Rv_3\bar{v}_4P_Lu_3$ in the last two lines of (51).
We can rewrite

$$\begin{aligned}
&\bar{v}_1P_Lu_2 = \bar{v}_1^{cc}P_Lu_2^{cc} = \bar{u}_1^cP_Lv_2^c = (C\gamma_0^Tu_1^*)^\dagger\gamma_0P_L(C\gamma_0^Tv_2^*) = u_1^T\gamma_0^TC^\dagger\gamma_0P_LC\gamma_0^Tv_2^* \\
&= -u_1^TP_L^T\gamma_0^Tv_2^* = -v_2^\dagger\gamma_0P_Lu_1 = -\bar{v}_2P_Lu_1.
\end{aligned} \tag{52}$$

In the fifth step we used some properties of γ_5 and the charge-conjugation matrix C, viz. $C^\dagger\gamma_5C = \gamma_5^T$ and $C^\dagger\gamma_0 = -\gamma_0^\dagger C$ respectively $C^\dagger\gamma_0P_LC = -\gamma_0^TP_L^T$. Analogously to (52) the relation

$$\bar{v}_2P_Lu_1 = -\bar{v}_1P_Lu_2 \tag{53}$$

is true. With (52) and (53) we see that in relation (51) the second and third term in the both brackets are identical to the first and fouth term.
Equation (51) now becomes

$$\begin{aligned}
&\frac{1}{2}\frac{1}{2}\sum_{spins}|A_1A_2|^2 = \frac{1}{4}\sum_{s_1,s_2,r_1,r_2} [2\bar{v}_2P_Lu_1\bar{u}_1P_Rv_2 + 2\bar{v}_1P_Lu_2\bar{u}_2P_Rv_1] \\
&\times \sum_{s_1,s_2,r_1,r_2} [2\bar{u}_4P_Rv_3\bar{v}_3P_Lu_4 + 2\bar{u}_3P_Rv_4\bar{v}_4P_Lu_3].
\end{aligned} \tag{54}$$

Now we have to use the so called "Feynman trace technique" [16]

$$\sum_{spins} \bar{u}(p')\Gamma u(p)\bar{u}(p)\gamma_0\Gamma^\dagger\gamma_0u(p') = \mathrm{Tr}[\Gamma(\not{p}+m)\gamma_0\Gamma^\dagger\gamma_0(\not{p}'+m)]. \tag{55}$$

Where Γ is an arbitrary product of gamma matrices, and $\not{p} = p_\mu \gamma^\mu = p^\mu \gamma_\mu$.
In our case we will take the ultrarelativsitic limit so we can neglect the rest mass. So (55) becomes

$$\sum_{spins} \bar{u}(p')\Gamma u(p)\bar{u}(p)\gamma_0 \Gamma^\dagger \gamma_0 u(p') = \mathrm{Tr}[\Gamma \not{p} \gamma_0 \Gamma^\dagger \gamma_0 \not{p}']. \tag{56}$$

It should be mentioned that in this case it doesn't matter whether we have a u-spinor or a v-spinor in our expression.
When we apply the trace technique on (54) we get

$$\begin{aligned} &\mathrm{Tr}(P_L \not{p}_1 P_R \not{p}_2) = \mathrm{Tr}(\not{p}_1 \not{p}_2 P_L P_L) = \mathrm{Tr}(\not{p}_1 \not{p}_2 P_L) = \mathrm{Tr}(\not{p}_1 \not{p}_2 \frac{\mathbb{1} - \gamma_5}{2}) \\ &= \frac{1}{2}\mathrm{Tr}(\not{p}_1 \not{p}_2) - \frac{1}{2}\mathrm{Tr}(\not{p}_1 \not{p}_2 \gamma_5) = \frac{1}{2}\mathrm{Tr}(\not{p}_1 \not{p}_2) = 2p_1 \cdot p_2. \end{aligned} \tag{57}$$

Now (54) becomes

$$\begin{aligned} &\frac{1}{2}\frac{1}{2}\sum_{spins} |A_1 A_2|^2 = \frac{1}{4}[4p_1 \cdot p_2 + 4p_2 \cdot p_1][4p_3 \cdot p_4 + 4p_4 \cdot p_3] \\ &= 16(p_1 \cdot p_2)(p_3 \cdot p_4). \end{aligned} \tag{58}$$

The squared amplitude $|\mathcal{M}|^2$ is now

$$|\mathcal{M}|^2 = \frac{1}{4}\frac{|f_{ee}|^2 |f_{\mu\mu}|^2}{(s - M_{H^{--}}^2)^2 + M_{H^{--}}^2 \Gamma_H^2} 16(p_1 \cdot p_2)(p_3 \cdot p_4). \tag{59}$$

We replaced m with the mass of the doubly charged Higgs $M_{H^{--}}$ and used the Breit-Wigner formula [16] to rewrite $|\mathcal{M}|^2$ in terms of the total width Γ_H.

Now we can finally calculate the cross section $\sigma(e^- e^- \to \mu^- \mu^-)$. We insert the invariant matrix element from (59) into the cross section formula of (E2). For the integration over the two-body phase space we use the result of (D14) and let $m_1, m_2 \to 0$ since we take the ultrarelativistic limit. The cross section is

$$\begin{aligned} &\sigma(e^- e^- \to \mu^- \mu^-) = \frac{1}{2!}\frac{1}{4p_1 \cdot p_2}\frac{\pi}{2} 16(p_1 \cdot p_2)(p_3 \cdot p_4)\frac{1}{(2\pi)^6}(2\pi)^4 \frac{1}{4}\frac{|f_{ee}|^2 |f_{\mu\mu}|^2}{(s - M_{H^{--}}^2)^2 + M_{H^{--}}^2 \Gamma_H^2} \\ &= \frac{1}{2!} 4 \frac{s}{2}\frac{1}{8\pi}\frac{1}{4}\frac{|f_{ee}|^2 |f_{\mu\mu}|^2}{(s - M_{H^{--}}^2)^2 + M_{H^{--}}^2 \Gamma_H^2} = \frac{|f_{ee}|^2 |f_{\mu\mu}|^2}{32\pi}\frac{s}{(s - M_{H^{--}}^2)^2 + M_{H^{--}}^2 \Gamma_H^2}. \end{aligned} \tag{60}$$

This result was also obtained in [29, 31].

5 The cross section $\sigma(e^-e^- \to W^-W^-)$

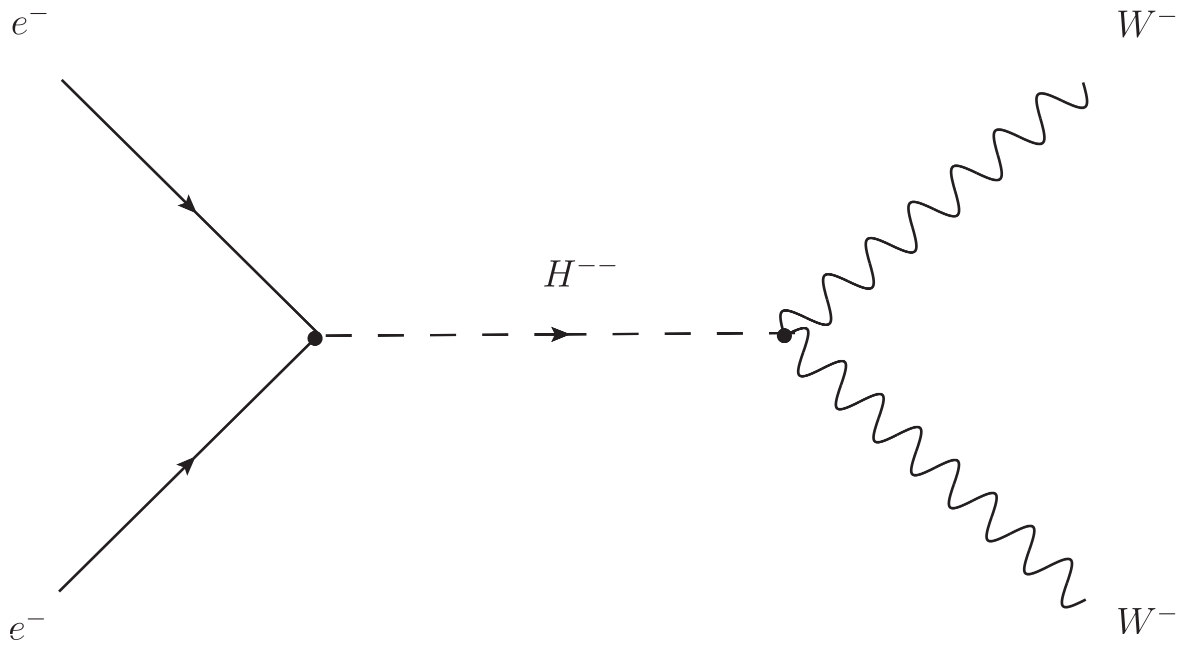

Figure 2: s-channel diagram for $e^-e^- \to W^-W^-$

Another process which is mediated by a doubly charged Higgs is the so-called inverse neutrinoless double beta decay $e^-e^- \to W^-W^-$ [29,30], shown in figure 2. In order to compute the cross section of this process we have to start again with a Lagrangian.

While in the first vertex the electron part of the Lagrangian from equation (38) applies, we need to compute another Lagrangian for the second vertex in which the Higgs triplet couples to gauge bosons. Based on the gauge invariance the Lagrangian has the form

$$\mathcal{L}_{\Delta gauge} = \frac{1}{2}\mathrm{Tr}\{(D_\mu\Delta)^\dagger(D^\mu\Delta)\}. \tag{61}$$

Where Δ again is the 2×2 representation of the Higgs triplet as defined in (7) and D_μ is the covariant derivative, which is defined by

$$D_\mu\Delta = \partial_\mu\Delta + ig[\frac{1}{2}\vec{\tau}\cdot\vec{W}_\mu, \Delta] + ig'B_\mu\Delta, \tag{62}$$

where g and g' are coupling constants, $\vec{\tau}$ the vector of the pauli matrices, $\vec{W}_\mu = (W_\mu^1, W_\mu^2, W_\mu^3)^T$ the W-field and B_μ B-field . In the following we will calculate the commutator and use the definitions of the physical fields $W_\mu^\pm$, which are defined as

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp iW_\mu^2). \tag{63}$$

Where the field W_μ^+ creates a W^- particle or annihilates a W^+ particle, and the field W_μ^-

creates a W^+ particle or annihilates a W^- particle. The commutator can now be calculated as

$$\begin{aligned}
&[\vec{\tau}\cdot\vec{W}_\mu, \Delta] \\
&= \left[\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} W^1_\mu + \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix} W^2_\mu + \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} W^3_\mu, \begin{pmatrix}H^+ & \sqrt{2}H^{++}\\ \sqrt{2}H^0 & -H^+\end{pmatrix}\right] \\
&= \left[\begin{pmatrix}W^3_\mu & W^1_\mu - iW^2_\mu\\ W^1_\mu + iW^2_\mu & -W^3_\mu\end{pmatrix}, \begin{pmatrix}H^+ & \sqrt{2}H^{++}\\ \sqrt{2}H^0 & -H^+\end{pmatrix}\right] \\
&= \begin{pmatrix}W^3_\mu & \sqrt{2}W^+_\mu\\ \sqrt{2}W^-_\mu & -W^3_\mu\end{pmatrix} \cdot \begin{pmatrix}H^+ & \sqrt{2}H^{++}\\ \sqrt{2}H^0 & -H^+\end{pmatrix} \\
&- \begin{pmatrix}H^+ & \sqrt{2}H^{++}\\ \sqrt{2}H^0 & -H^+\end{pmatrix} \cdot \begin{pmatrix}W^3_\mu & \sqrt{2}W^+_\mu\\ \sqrt{2}W^-_\mu & -W^3_\mu\end{pmatrix} \\
&= \begin{pmatrix}\sqrt{2}H^0\sqrt{2}W^+_\mu - \sqrt{2}H^{++}\sqrt{2}W^-_\mu & -2H^+\sqrt{2}W^+_\mu + 2\sqrt{2}H^{++}W^3_\mu\\ 2H^+\sqrt{2}W^-_\mu - 2\sqrt{2}H^0 W^3_\mu & -\sqrt{2}H^0\sqrt{2}W^+_\mu + \sqrt{2}H^{++}\sqrt{2}W^-_\mu\end{pmatrix}.
\end{aligned} \tag{64}$$

Since in the second vertex of the process only a H^{--}boson and two W^- bosons participate we just have to compute the terms of the corresponding fields. We set all fields zero which are not involved and the covariant derivative becomes

$$\begin{aligned}
(D_\mu\Delta)' &= \begin{pmatrix}igH^0W^+_\mu - igH^{++}W^-_\mu & 0\\ 0 & -igH^0W^+_\mu + igH^{++}W^-_\mu\end{pmatrix} \\
&= \begin{pmatrix}a_\mu & 0\\ 0 & -a_\mu\end{pmatrix}.
\end{aligned} \tag{65}$$

In the last step we define

$$a_\mu = igH^0W^+_\mu - igH^{++}W^-_\mu. \tag{66}$$

Now with (65) and (66) the gauge Lagrangian of (61) becomes

$$\begin{aligned}
\mathcal{L}_{\Delta gauge} &= \frac{1}{2}\mathrm{Tr}\{(D_\mu\Delta)^\dagger(D^\mu\Delta)\} \\
&= \frac{1}{2}\mathrm{Tr}\left(\begin{pmatrix}a^*_\mu & 0\\ 0 & -a^*_\mu\end{pmatrix} \cdot \begin{pmatrix}a^\mu & 0\\ 0 & -a^\mu\end{pmatrix}\right) \\
&= a^*_\mu a^\mu = g^2(\frac{|\upsilon_T|^2}{2}W^{\mu+}W^-_\mu - \frac{\upsilon_T}{\sqrt{2}}H^{--}W^{\mu+}W^+_\mu - \frac{\upsilon^*_T}{\sqrt{2}}H^{++}W^{\mu-}W^-_\mu \\
&+ H^{++}H^{--}W^{\mu-}W^+_\mu).
\end{aligned} \tag{67}$$

We replaced the field H^0 its VEV $\frac{\upsilon_T}{\sqrt{2}}$ and again we can simplify the Lagrangian of (67) since in the second vertex of the process two W^- particles are created and one H^{--} is annihilated. The simplified Lagrangian for the second vertex is

$$\mathcal{L}'_{\Delta gauge} = g^2\frac{\upsilon_T}{\sqrt{2}}H^{--}W^{\mu+}W^+_\mu. \tag{68}$$

For the first vertex we take the relevant part of the Lagrangian in (41)

$$\mathcal{L}'_{\Delta e^-e^-} = \frac{f_{ee}}{\sqrt{2}}H^{++}e^T_L C^{-1}e_L. \tag{69}$$

Using the definition of the S-matrix element of (36) we get

$$S_2 = \int d^4x d^4y \langle 0 \mid a_-(k_3,\lambda)a_-(k_4,\sigma)\mathcal{L}'_{\Delta e^- e^-}(x)\mathcal{L}'_{\Delta gauge}(y) \times b^\dagger_{e^-}(k_1,s_1)b^\dagger_{e^-}(k_2,s_2) \mid 0\rangle. \tag{70}$$

Where k_1 and k_2 denote the 4-momenta of the electron in the initial states and k_3 and k_4 the 4-momenta of the W-bosons in the final states. With the corresponding fields for W^+_μ and e_L of (A6) respectively (A1) the second order S-matrix element becomes

$$\begin{aligned} S_2 = i^2 \int d^4x d^4y \langle 0 \mid a_{(-)}(k_3,\lambda)a_{(-)}(k_4,\sigma)\frac{g^2 v_T f_{ee}}{2}H^{--}(x)H^{++}(y) \\ \times \sum_{\rho,\tau}\int \frac{d^3p_1}{\sqrt{(2\pi)^3 2E_{p_1}}}\int \frac{d^3p_2}{\sqrt{(2\pi)^3 2E_{p_2}}} a^\dagger_{(-)}(\vec{p_1},\rho)\epsilon^{*\mu}(\vec{p_1},\rho)e^{ip_1x}a^\dagger_{(-)}(\vec{p_2},\tau)\epsilon^{*\mu}(\vec{p_2},\tau)e^{ip_2x} \\ \times \sum_{t_1,t_2}\int \frac{d^3q_1}{\sqrt{(2\pi)^3 2E_{q_1}}}\int \frac{d^3q_2}{\sqrt{(2\pi)^3 2E_{q_2}}} b_{e^-}(q_1,t_1)u^T_L(q_1,t_1)e^{-iq_1y}C^{-1}b_{e^-}(q_2,t_2)u_L(q_2,t_2)e^{-iq_2y} \\ \times b^\dagger_{e^-}(k_1,s_1)b^\dagger_{e^-}(k_2,s_2) \mid 0\rangle. \end{aligned} \tag{71}$$

At next we will take a look at the creation and annihilation operators in the S-matrix element in (71). According to (B6) the term with the W-boson creation and annihilation operators in (71) can be can be rewritten as

$$\begin{aligned} &a_{(-)}(\vec{k_3},\lambda)a_{(-)}(\vec{k_4},\sigma)a^\dagger_{(-)}(\vec{p_1},\rho)a^\dagger_{(-)}(\vec{p_2},\tau) \mid 0\rangle \\ &= [\delta^{(3)}(\vec{k_4}-\vec{p_1})\delta_{\sigma\rho}\delta^{(3)}(\vec{k_3}-\vec{p_2})\delta_{\lambda\tau} + \delta^{(3)}(\vec{k_3}-\vec{p_1})\delta_{\lambda\rho}\delta^{(3)}(\vec{k_4}-\vec{p_2})\delta_{\sigma\tau}] \mid 0\rangle. \end{aligned} \tag{72}$$

The term with the electron creation and annihilation operators can be rewritten as well as in (B3), viz.

$$\begin{aligned} &b_{e^-}(q_1,t_1)b_{e^-}(q_2,t_2)b^\dagger_{e^-}(k_1,s_1)b^\dagger_{e^-}(k_2,s_2) \mid 0\rangle \\ &= [\delta^{(3)}(\vec{q_2}-\vec{k_1})\delta_{t_2s_1}\delta^{(3)}(\vec{q_1}-\vec{k_2})\delta_{t_1s_2} - \delta^{(3)}(\vec{q_1}-\vec{k_1})\delta_{t_1s_1}\delta^{(3)}(\vec{q_2}-\vec{(p_2)})\delta_{t_2s_2}] \mid 0\rangle. \end{aligned} \tag{73}$$

It should be mentioned that the integrals of (71) are the same as in (47).

Now we can compute the contraction of the of the polarization vectors ϵ_μ with the Dirac- and Kronecker deltas. We sum over ρ and τ and get

$$\begin{aligned} &\sum_{\rho,\tau}\int \frac{d^3p_1}{\sqrt{(2\pi)^3 2E_{p_1}}}\int \frac{d^3p_2}{\sqrt{(2\pi)^3 2E_{p_2}}}\epsilon^{\mu*}(\vec{p_1},\rho)\epsilon^*_2(\vec{p_2},\tau) \\ &\times [\delta^{(3)}(\vec{k_4}-\vec{p_1})\delta_{\sigma\rho}\delta^{(3)}(\vec{k_3}-\vec{p_2})\delta_{\lambda\tau} + \delta^{(3)}(\vec{k_3}-\vec{p_1})\delta_{\lambda\rho}\delta^{(3)}(\vec{k_4}-\vec{p_2})\delta_{\sigma\tau}] \\ &= \frac{1}{\sqrt{(2\pi)^6 4E_{p_1}E_{p_2}}}[\epsilon^{*\mu}(\vec{k_4},\sigma)\epsilon^*_\mu(\vec{k_3},\lambda) + \epsilon^*_\mu(\vec{k_3},\lambda)\epsilon^{*\mu}(\vec{k_4},\sigma)] \\ &= \frac{1}{\sqrt{(2\pi)^6 4E_{p_1}E_{p_2}}}[2\epsilon^{*\mu}(\vec{k_4},\sigma)\epsilon^*_\mu(\vec{k_3},\lambda)]. \end{aligned} \tag{74}$$

The other integral term gives

$$
\begin{aligned}
&\sum_{t_1,t_2}\int\frac{d^3q_1}{\sqrt{(2\pi)^3 2E_{q_1}}}\int\frac{d^3q_2}{\sqrt{(2\pi)^3 2E_{q_2}}}(-v(q_1,t_1)P_L u(q_2,t_2))\\
&\times[\delta^{(3)}(\vec{q_2}-\vec{k_1})\delta_{t_2 s_1}\delta^{(3)}(\vec{q_1}-\vec{k_2})\delta_{t_1 s_2}-\delta^{(3)}(\vec{q_1}-\vec{k_1})\delta_{t_1 s_1}\delta^{(3)}(\vec{q_2}-\vec{(p_2)})\delta_{t_2 s_2}]\\
&=\frac{1}{\sqrt{(2\pi)^6 4E_{q_1}E_{q_2}}}[\bar{v}(k_1,s_1)P_L u(k_2,s_2)-\bar{v}(k_2,s_2)P_L u(k_1,s_1)].
\end{aligned}
\tag{75}
$$

With (74),(75) and (47) we can now rewrite the S-matrix element in (71) as

$$
\begin{aligned}
S_2 &= \frac{g^2 v_T f_{ee}}{2}\frac{1}{\sqrt{(2\pi)^{12}16E_{p_1}E_{p_2}E_{q_1}E_{q_2}}}(2\pi)^4\frac{\delta^{(4)}(k_1+k_2-q_1-q_2)}{s-m^2+i\epsilon}\\
&\times[2\epsilon^{*\mu}(\vec{k_4},\sigma)\epsilon^*_\mu(\vec{k_3},\lambda)][\bar{v}(k_1,s_1)P_L u(k_2,s_2)-\bar{v}(k_2,s_2)P_L u(k_1,s_1)].
\end{aligned}
\tag{76}
$$

When we define

$$
G = 2g^2 v_T f_{ee} \tag{77}
$$

we can write the invariant matrix element $\mathcal{M}$ as

$$
\begin{aligned}
\mathcal{M} &= \frac{G}{s-m^2+i\epsilon}[\epsilon^{*\mu}(\vec{k_4},\sigma)\epsilon^*_\mu(\vec{k_3},\lambda)]\\
&\times[\bar{v}(k_1,s_1)P_L u(k_2,s_2)-\bar{v}(k_2,s_2)P_L u(k_1,s_1)] = \frac{G}{s-m^2+i\epsilon}A_1 A_2.
\end{aligned}
\tag{78}
$$

Now we average over the spins and sum over the polarizations and take the absolute square and get

$$
\begin{aligned}
\frac{1}{4}\sum_{spins,polarization}|A_1A_2|^2 &= \frac{1}{4}\sum_{polarization}A_1A_1^*\sum_{spins}A_2A_2^*\\
&= \frac{1}{4}\sum_{\lambda\sigma}\epsilon^{*\mu}(\vec{k_4},\sigma)\epsilon^*_\mu(\vec{k_3},\lambda)\epsilon^{*\nu}(\vec{k_4},\sigma)\epsilon^*_\nu(\vec{k_3},\lambda)\sum_{spins}A_2A_2^*.
\end{aligned}
\tag{79}
$$

With the orthogonality relation

$$
\sum_\lambda \epsilon_\mu(\vec{k},\lambda)\epsilon^*_\nu(\vec{k},\lambda) = -g_{\mu\nu}+\frac{k_\mu k_\nu}{m_W^2} \tag{80}
$$

we can rewrite (79) as

$$
\begin{aligned}
\frac{1}{4}\sum_{spins,polarization}|A_1A_2|^2 &= \frac{1}{4}(-g^{\mu\nu}+\frac{k_4^\mu k_4^\nu}{m_W^2})(-g_{\mu\nu}+\frac{k_{3\mu}k_{3\nu}}{m_W^2})\sum_{spins}A_2A_2^*\\
&= \frac{1}{4}[4-\frac{k_3^2}{m_W^2}-\frac{k_4^2}{m_W^2}+\frac{(k_3\cdot k_4)^2}{m_W^4}]\sum_{spins}A_2A_2^*\\
&= \frac{1}{4}[2+\frac{(k_3\cdot k_4)^2}{m_W^4}]\sum_{spins}A_2A_2^*.
\end{aligned}
\tag{81}
$$

The term $k_3 \cdot k_4$ can be rewritten as

$$k_3 \cdot k_4 = \frac{1}{2}(k_3 + k_4)^2 - m_W^2 = \frac{s}{2} - m_W^2. \tag{82}$$

Since

$$2 + \frac{1}{m_W^4}(\frac{1}{2}s - m_W^2)^2 = \frac{1}{4}\Big[\frac{1}{m_W^4}(s - 2m_W^2)^2 + 8\Big] \tag{83}$$

relation (81) can now be rewritten as

$$\frac{1}{4}\sum_{spins,polarization}|A_1A_2|^2 = \frac{1}{16}\Big[\frac{(s-2m_W^2)^2 + 8m_W^4}{m_W^4}\Big]\sum_{spins}A_2A_2^*. \tag{84}$$

With (78),(84),(57) and(58) we get

$$|\mathcal{M}|^2 = \frac{G^2}{(s - M_{H--}^2)^2 + M_{H--}^2\Gamma_H^2}\frac{1}{16}\Big[\frac{(s-2m_W^2)^2 + 8m_W^4}{m_W^4}\Big]8(p_1 \cdot p_2). \tag{85}$$

We replaced m with the mass of the doubly charged Higgs M_{H--} and used the Breit-Wigner formula [16] to rewrite $|\mathcal{M}|^2$ in terms of the total width Γ_H.

With the two-body phase space integral of (D14) and (D15) and $|\mathcal{M}|^2$ of (85) the total cross section σ becomes now

$$\begin{aligned}
\sigma(e^-e^- \to W^-W^-) &= \frac{1}{2!}\frac{1}{4(p_1\cdot p_2)}(2\pi)^4\frac{1}{(2\pi)^6}\frac{\pi}{2M^2}\\
&\times \frac{G^2}{(s - M_{H--}^2)^2 + M_{H--}^2\Gamma_H^2}\frac{1}{16}[\frac{(s-2m_W^2)^2 + 8m_W^4}{m_W^4}]8(p_1\cdot p_2)\sqrt{\lambda(x,y,z)}\\
&= \frac{1}{128\pi M^2}\\
&\times \frac{G^2}{(s - M_{H--}^2)^2 + M_{H--}^2\Gamma_H^2}\frac{(s-2m_W^2)^2 + 8m_W^4}{m_W^4}\sqrt{s^2 + 2m_W^4 - 2(sm_W^2 + sm_W^2 + m_W^4)}\\
&= \frac{1}{128\pi M^2}\frac{G^2}{(s - M_{H--}^2)^2 + M_{H--}^2\Gamma_H^2}\frac{(s-2m_W^2)^2 + 8m_W^4}{m_W^4}\sqrt{s^2 - 4m_W^2}\\
&= \frac{1}{128\pi M^2}\frac{G^2}{(s - M_{H--}^2)^2 + M_{H--}^2\Gamma_H^2}\frac{(s-2m_W^2)^2 + 8m_W^4}{m_W^4}s\sqrt{1 - \frac{4m_W^2}{s^2}}\\
&= \frac{1}{128\pi}\frac{G^2}{(s - M_{H--}^2)^2 + M_{H--}^2\Gamma_H^2}\frac{(s-2m_W^2)^2 + 8m_W^4}{m_W^4}\sqrt{1 - \frac{4m_W^2}{s^2}}.
\end{aligned} \tag{86}$$

When we define

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_w^2} \tag{87}$$

we can rewrite the cross section of (86) as

$$\begin{aligned}
&\sigma(e^-e^- \to W^-W^-)\\
&\times = \frac{32m_w^4|f_{ee}|^2|v_T|^2}{128\pi}\frac{G_F^2}{(s - M_{H--}^2)^2 + M_{H--}^2\Gamma_H^2}\frac{(s-2m_W^2)^2 + 8m_W^4}{m_W^4}\sqrt{1 - \frac{4m_W^2}{s^2}}\\
&= \frac{1}{4\pi}G_F^2|f_{ee}|^2|v_T|^2\frac{(s-2m_W^2)^2 + 8m_W^4}{(s - M_{H--}^2)^2 + M_{H--}^2\Gamma_H^2}\sqrt{1 - \frac{4m_W^2}{s^2}}.
\end{aligned} \tag{88}$$

This result matches with [29].

6 The cross section $\sigma(\alpha^-\beta^- \to \gamma^-\delta^-)$

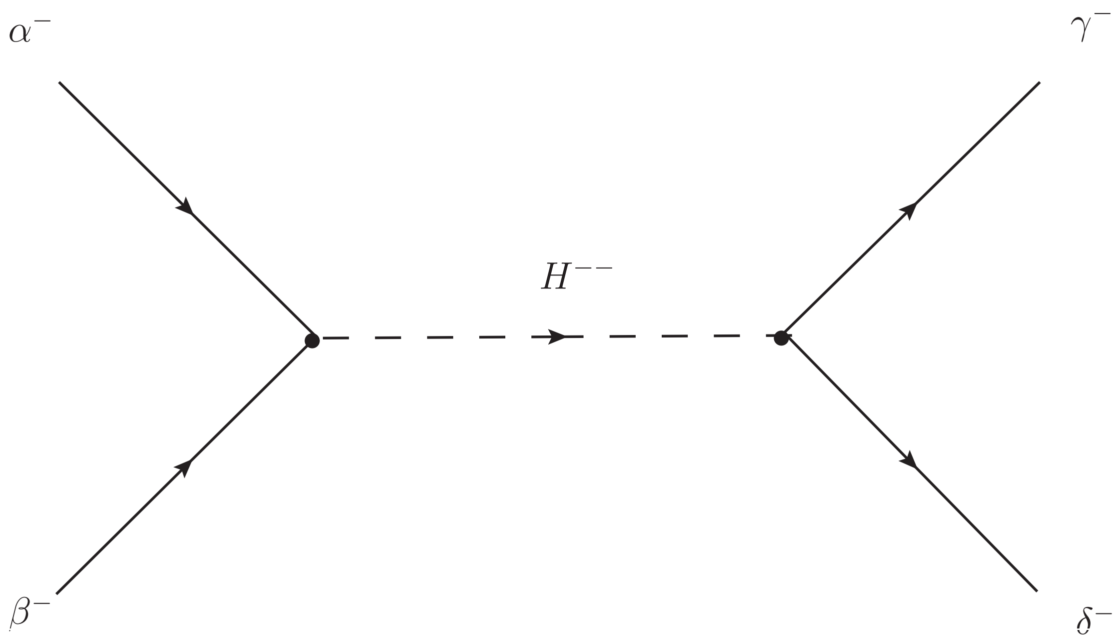

Figure 3: s-channel diagram for $\alpha^-\beta^- \to \gamma^-\delta^-$

A more general expression for the cross section of (60) can be achieved if we consider the general process $\alpha^-\beta^- \to \gamma^-\delta^-$ for $\alpha, \beta, \gamma, \delta = e, \mu, \tau$, shown in figure 3. The generalization of (60) is given by [29]

$$\sigma(\alpha^-\beta^- \to \gamma^-\delta^-) = \frac{|f_{\alpha\beta}|^2|f_{\gamma\delta}|^2}{16\pi(1+\delta_{\gamma\delta})} \frac{s}{(s-M_{H^{--}}^2)^2 + M_{H^{--}}^2\Gamma_H^2}, \tag{89}$$

where $\delta_{\gamma\delta}$ is the Kronecker delta. It should be also mentioned that this formula does not apply if there are two electrons in the final state, since the SM contributions mediated by γ and Z^0 play the dominant role [29].

In the following we will estimate the relative ratio of the cross sections. Therefore let us recall the cross section of (88) and generalize it to

$$\sigma(\alpha^-\beta^- \to W^-W^-) = \frac{1}{4\pi} G_F^2 |f_{\alpha\beta}|^2 |v_T|^2 \frac{(s-2m_W^2)^2 + 8m_W^4}{(s-M_{H^{--}}^2)^2 + M_{H^{--}}^2\Gamma_H^2} \sqrt{1 - \frac{4m_W^2}{s^2}}. \tag{90}$$

When we neglect m_w the cross section can be simplified to

$$\sigma(\alpha^-\beta^- \to W^-W^-) \approx \frac{1}{4\pi} G_F^2 |f_{\alpha\beta}|^2 |v_T|^2 \frac{s^2}{(s-M_{H^{--}}^2)^2 + M_{H^{--}}^2\Gamma_H^2}. \tag{91}$$

The relative ratio of the cross sections is approximately [29]

$$\frac{\sigma(\alpha^-\beta^- \to \gamma^-\delta^-)}{\sigma(\alpha^-\beta^- \to W^-W^-)} \approx \frac{4G_F|v_T|^2(1+\delta_{\gamma\delta})s}{|f_{\gamma\delta}|^2}. \tag{92}$$

7 The partial width $\Gamma(H^{--} \rightarrow \mu^- \mu^-)$

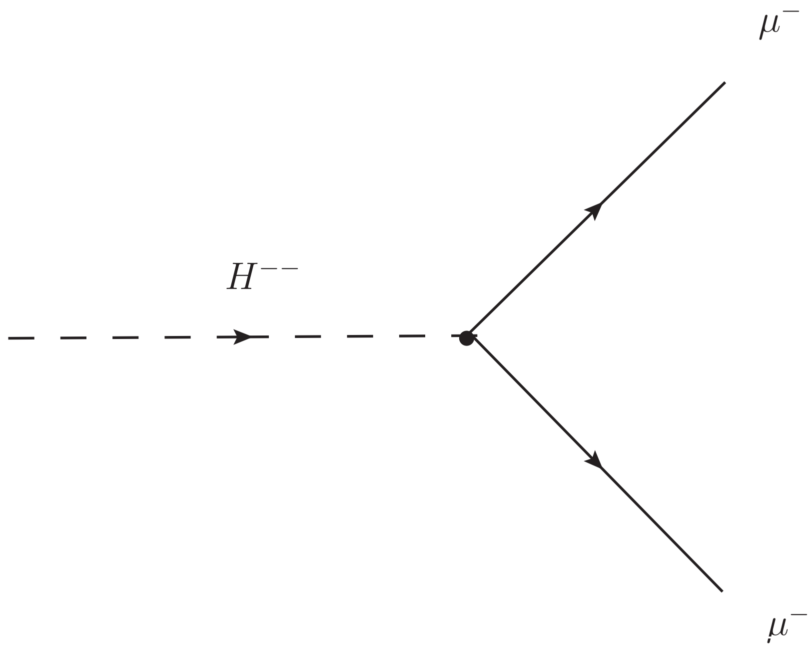

Figure 4: First order Feynman diagram for $H^{--} \rightarrow \mu^- \mu^-$

In this section we calculate the partial width $\Gamma(H^{--} \rightarrow \mu^- \mu^-)$ of the process shown in figure 4. Using the definition of (36) we can write the first order S-matrix element as

$$S_1 = -i \int d^4x \langle 0 \mid b_{\mu^-}(p_3, s_3) b_{\mu^-}(p_4, s_4) \mathcal{L}_{\Delta\mu^-\mu^-}(x) a_H^\dagger(q) \mid 0 \rangle, \tag{93}$$

where p_3 and p_4 denote the 4-momenta of the muons in the final states and q the 4-momentum of the doubly charged Higgs in the inital state. The Lagrangian $\mathcal{L}_{\Delta\mu^-\mu^-}$ is given by the second term of (41)

$$\mathcal{L}_{\Delta\mu^-\mu^-} = \frac{1}{\sqrt{2}} H^{--} f^*_{\mu\mu} \mu_L^\dagger C \mu_L^*. \tag{94}$$

We insert the Lagrangian of (94) together with the scalar field for the doubly charged Higgs and the Dirac fields for the muons defined in (A3) respectively (A2) into our S-matrix element of (93) and get

$$S_1 = -\frac{i f^*_{\mu\mu}}{\sqrt{2}} \int d^4x \langle 0 \mid b_{\mu^-}(p_3, s_3) b_{\mu^-}(p_4, s_4) \frac{1}{\sqrt{(2\pi)^3 2E_{p_H}}} \int d^3p_H a_H(p_H) e^{-ip_H x}$$

$$\times \sum_{r_1 r_2} \int \frac{d^3k_1}{\sqrt{(2\pi)^3 2E_{k_1}}} \int \frac{d^3k_2}{\sqrt{(2\pi)^3 2E_{k_2}}} e^{ik_1 x} u_L^\dagger(k_1, r_1) b^\dagger_{\mu^-}(k_1, r_1) \tag{95}$$
$$\times C e^{ik_2 x} u_L^*(k_2, r_2) b^\dagger_{\mu^-}(k_2, r_2) a_H^\dagger(q) \mid 0 \rangle.$$

The space-time integral gives

$$\int d^4 x e^{-ip_H x} e^{ik_1 x} e^{ik_2 x} = \int d^4 x e^{i(-p_H + k_1 + k_2 x)} = (2\pi)^4 \delta^{(4)}(k_1 + k_2 - p_H). \tag{96}$$

While the factor with the muon creation- and annihilation operators is identical to (46), the operators of the scalar field give

$$a_H(p_H) a_H^\dagger(q) \mid 0\rangle = [a_H(p_H), a_H^\dagger(q)] \mid 0\rangle = \delta^{(3)}(\vec{p_H} - \vec{q}) \mid 0\rangle. \tag{97}$$

The integral over the 3-momentum is

$$\int d^3 p_H \delta^{(3)}(\vec{p_H} - \vec{q}) = 1. \tag{98}$$

With (C13) and (46) the factor $u_L^\dagger(k_1, r_1) C u_L^*(k_2, r_2)$ yields

$$\begin{aligned} &\int d^3 k_1 d^3 k_2 \bar{u}(k_2, r_2) P_R v(k_1, r_1) \\ &\times [\delta^{(3)}(\vec{p_4} - \vec{k_1}) \delta_{s_4 r_1} \delta^{(3)}(\vec{p_3} - \vec{k_2}) \delta_{s_3 r_2} - \delta^{(3)}(\vec{p_4} - \vec{k_2}) \delta_{s_4 r_2} \delta^{(3)}(\vec{p_3} - \vec{k_1}) \delta_{s_3 r_1}] \\ &= \bar{u}(p_3, s_3 P_R v(p_4, s_4) - \bar{u}(p_4, s_4) P_R v(p_3, s_3). \end{aligned} \tag{99}$$

So the S-matrix element of (95) becomes

$$\begin{aligned} S_1 = &-\frac{i f_{\mu\mu}^*}{\sqrt{2}} \frac{1}{\sqrt{(2\pi)^9 8 E_{p_H} E_{k_1} E_{k_2}}} (2\pi)^4 \delta^{(4)}(k_1 + k_2 - p_H) \\ &\times [\bar{u}(p_3, s_3 P_R v(p_4, s_4) - \bar{u}(p_4, s_4) P_R v(p_3, s_3)]. \end{aligned} \tag{100}$$

With the method of (E1) the invariant matrix element can be extracted from (100), viz.

$$\mathcal{M} = -\frac{f_{\mu\mu}^*}{\sqrt{2}} [\bar{u}(p_3, s_3 P_R v(p_4, s_4) - \bar{u}(p_4, s_4) P_R v(p_3, s_3)]. \tag{101}$$

Now we sum over the spins in the final state and take the absolute square. Using the abbreviation $u(p_i, s_i) = u_i$ we get

$$\begin{aligned} |\mathcal{M}|^2 &= \frac{|f_{\mu\mu}|^2}{2} \sum_{spins} [\bar{u}_3 P_R v_4 - \bar{u}_4 P_R v_3][\bar{u}_3 P_R v_4 - \bar{u}_4 P_R v_3]^\dagger \\ &= \frac{|f_{\mu\mu}|^2}{2} \sum_{spins} [\bar{u}_3 P_R v_4 - \bar{u}_4 P_R v_3][\bar{v}_4 P_L u_3 - \bar{v}_3 P_L u_4] \\ &= \frac{|f_{\mu\mu}|^2}{2} \sum_{spins} [2\bar{u}_3 P_R v_4 \bar{v}_4 P_L u_3 + 2\bar{u}_4 P_R v_3 \bar{v}_3 P_L u_4]. \end{aligned} \tag{102}$$

With the method of the trace technique of (55) and (56) we can rewrite $|\mathcal{M}|^2$ in complete analogy to the procedure of (58) and get

$$|\mathcal{M}|^2 = \frac{|f_{\mu\mu}|^2}{2} [4 p_4 \cdot p_3 + 4 p_3 \cdot p_4] = 4 |f_{\mu\mu}|^2 p_3 \cdot p_4 = 2 |f_{\mu\mu}|^2 M_{H^{--}}^2. \tag{103}$$

Now we can compute the partial width using the formula of (E4). In our case for the decay $H^{--} \to \mu^- \mu^-$ the partial width is

$$\Gamma(H^{--} \to \mu^- \mu^-) = \frac{1}{2M_{H^{--}}} \int \frac{d^3p_1}{(2\pi)^3 2E_{p_1}} \frac{d^3p_2}{(2\pi)^3 2E_{p_2}} 2|f_{\mu\mu}|^2 M_{H^{--}}^2 \times (2\pi)^4 \delta^{(4)}(p_H - p_3 - p_4). \tag{104}$$

The two-body phase space integral was solved in detail in (D1) - (D15). We will use the solution of (D15) and (104) becomes

$$\Gamma(H^{--} \to \mu^- \mu^-) = \frac{1}{2M_{H^{--}}} \frac{1}{(2\pi)^6} (2\pi)^4 \frac{\pi}{2M_H^2} 2|f_{\mu\mu}|^2 M_{H^{--}}^2 \sqrt{M_{H^{--}}^4} = \frac{|f_{\mu\mu}|^2}{8\pi} M_{H^{--}}. \tag{105}$$

8 The partial width $\Gamma(H^{--} \to \gamma^-\delta^-)$

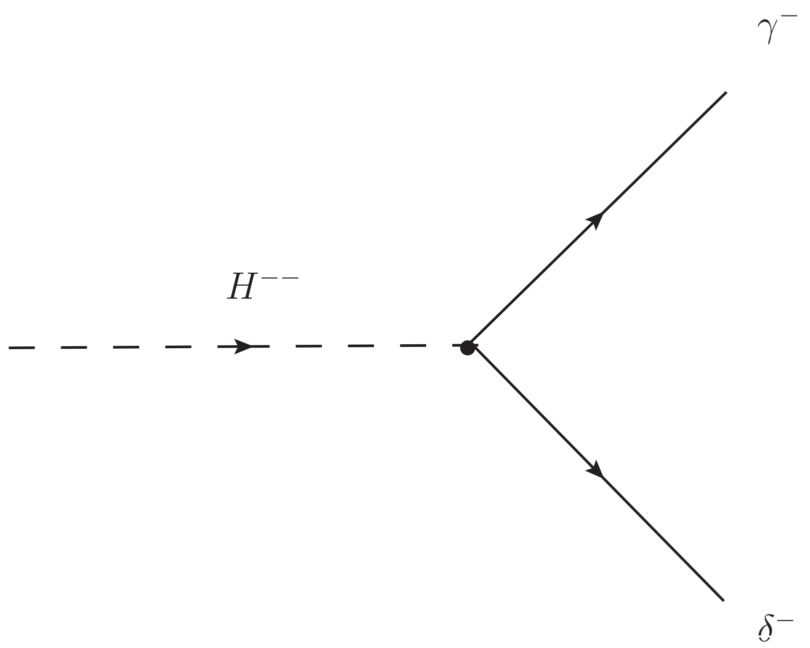

Figure 5: First order Feynman diagram for $H^{--} \to \gamma^-\delta^-$

Since the doubly charged Higgs can decay into any combination of e^-, μ^-, τ^- we can generalize the result of the previous section. The Feynman diagram of the generalized decay is shown in figure 5. For $\gamma, \delta = e, \mu, \tau$ the formula for the decay width in (105) can be generalized to

$$\Gamma(H^{--} \to \gamma^-\delta^-) = \frac{|f_{\gamma\delta}|^2}{4\pi(1+\delta_{\gamma\delta})} M_{H^{--}}, \tag{106}$$

where $\delta_{\gamma\delta}$ is the Kronecker delta.

9 The partial width $\Gamma(H^{--} \to W^- W^-)$

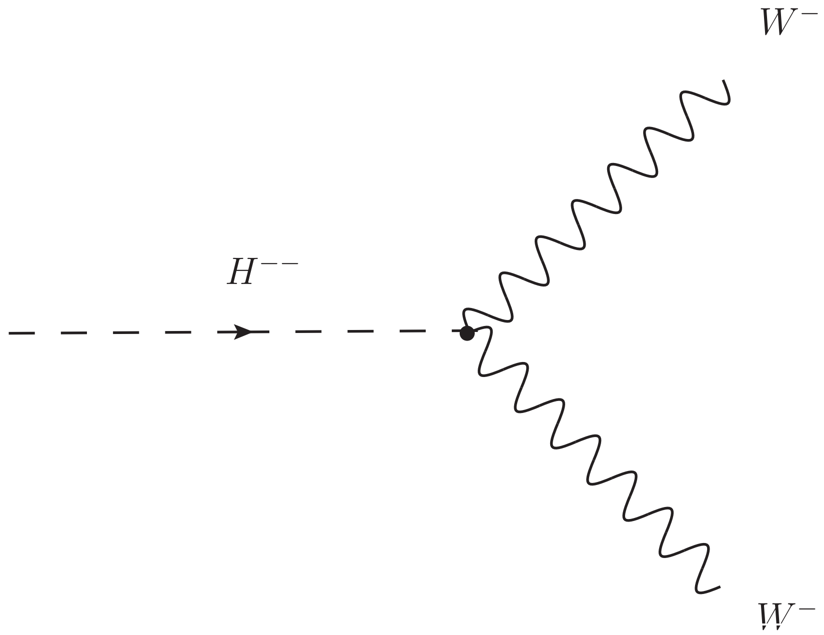

Figure 6: First order Feynman diagram for $H^{--} \to W^- W^-$

In this section we calculate the partial width $\Gamma(H^{--} \to W^- W^-)$ of the process shown in figure 6. As in section (7)we use the definition of (36) to write the first order S-matrix element

$$S_1 = -i \int d^4x \langle 0 \mid a_-(k_3, \lambda) a_-(k_4, \sigma) \mathcal{L}'_{\Delta gauge}(x) a^\dagger_{H^{--}}(q) \mid 0 \rangle. \tag{107}$$

With the Lagrangian $\mathcal{L}'_{\Delta gauge}(x)$ of (69), the W^+_μ field operator and scalar field operator of (A6) and (A3) the S-matrix element of (107) becomes

$$\begin{aligned} S_1 = -i \int d^4x \langle 0 \mid a_-(k_3, \lambda) a_-(k_4, \sigma) \frac{g^2 v_T}{\sqrt{2}} \sum_\rho \int \frac{d^3p_1}{\sqrt{(2\pi)^3 2E_{p_1}}} \\ \times \sum_\tau \int \frac{d^3p_2}{\sqrt{(2\pi)^3 2E_{p_2}}} a^\dagger_-(\vec{p}_1, \rho) \epsilon^{\mu *}(\vec{p}_1, \rho) e^{ip_1x} a^\dagger_-(\vec{p}_2, \tau) \epsilon^*_\mu(\vec{p}_2, \tau) e^{ip_2x} \\ \times \frac{1}{\sqrt{(2\pi)^3 2E_{p_H}}} \int d^3p_H a_H(p_H) e^{-ip_Hx} a^\dagger_H(q) \mid 0 \rangle. \end{aligned} \tag{108}$$

In analogy to (96) the space-time integral gives

$$\int d^4x e^{ip_1x} e^{ip_2x} e^{-ip_Hx} = (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_H). \tag{109}$$

The factor with the gauge boson creation- and annihilation operators is identical to (B6) and the factor with the Higgs boson creation- and annihilation operators is identical to (97). In complete analogy to the last section the S-matrix element of (108) becomes

$$S_1 = -\frac{ig^2 v_T}{\sqrt{2}} \frac{1}{\sqrt{(2\pi)^9 8E_{p_1} E_{p_2} E_{p_H}}} (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_H) \\ \times 2\epsilon^{\mu *}(\vec{k}_4, \sigma)\epsilon^*_\mu(\vec{k}_3, \lambda). \tag{110}$$

Again with the method of (E1) we can extract the invariant matrix element $\mathcal{M}$ and get

$$\mathcal{M} = -\frac{2ig^2 v_T}{\sqrt{2}} 2\epsilon^{\mu *}(\vec{k}_4, \sigma)\epsilon^*_\mu(\vec{k}_3, \lambda). \tag{111}$$

Now we sum over the polarisations in the final state and take the absolute square and get

$$|\mathcal{M}|^2 = 2g^4 |v_T|^2 \sum_{polarizations} \epsilon^{\mu *}(\vec{k}_4, \sigma)\epsilon^{\nu}(\vec{k}_4, \sigma)\epsilon^*_\nu(\vec{k}_3, \lambda)\epsilon_\mu(\vec{k}_3, \lambda). \tag{112}$$

With (80)-(83) we can rewrite (112) and get

$$|\mathcal{M}|^2 = 2g^4 |v_T|^2 \Big[2 + \frac{(k_3 \cdot k_4)^2}{m_w^2} \Big] = 2g^4 |v_T|^2 \frac{1}{4} \frac{(s - 2m_W^2)^2 + 8m_W^4}{m_W^4} \tag{113}$$

With the formula for the partial width of (E4) and the two-body phase space integral (D15) we get

$$\Gamma(H^{--} \to W^- W^-) = \frac{1}{2M_{H^{--}}} 2g^4 |v_T|^2 \frac{1}{(2\pi)^6} (2\pi)^4 \frac{\pi}{2M^2_{H^{--}}} \frac{1}{4} \frac{(M^2_{H^{--}} - 2m_W^2)^2 + 8m_W^4}{m_W^4} \\ \times M^2_{H^{--}} \sqrt{1 - 4\frac{m_W^2}{M^2_{H^{--}}}} = \frac{g^4 |v_T|^2}{32\pi M_{H^{--}}} \frac{(M_H^2 - 2m_W^2)^2 + 8m_W^4}{m_W^4} \sqrt{1 - 4\frac{m_W^2}{M^2_{H^{--}}}} \tag{114}$$

10 The partial width $\Gamma(H^{--} \to W^- H^-)$

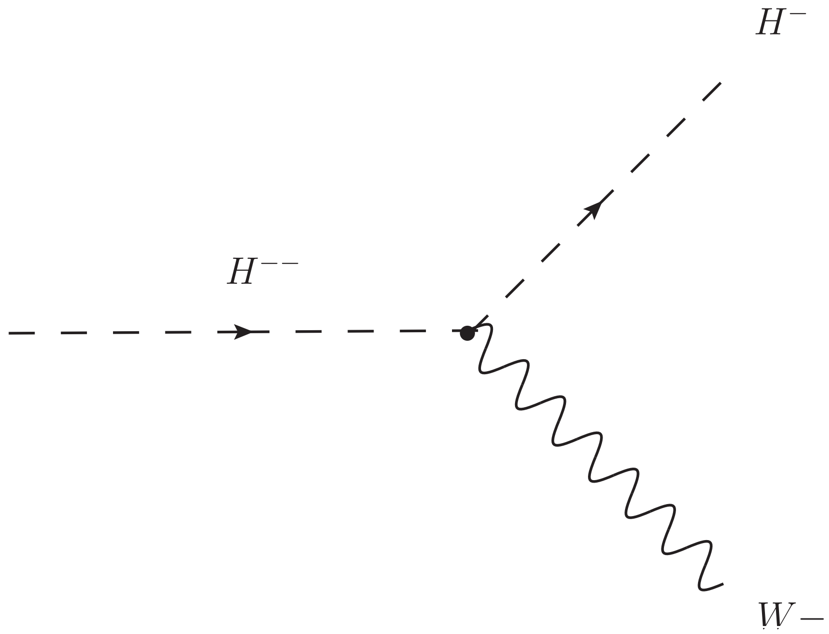

Figure 7: First order Feynman diagram for $H^{--} \to H^- W^-$

In addition to the decays of the previous sections the decay $H^{--} \to W^- H^-$ shown in figure 7 might also occur. In order to compute the width $\Gamma(H^{--} \to W^- H^-)$ we have to start as usual with the corresponding Lagrangian.

Using only the commutator term (64), the Lagrangian $\mathcal{L}_{\Delta gauge}$ does not contain a term proportional to $H^{--}W^+H^+$. So the decay $H^{--} \to W^- H^-$ can only be described correctly if we take the derivative coupling of the covariant derivative of (62) into account. We add the term $\partial_\mu \Delta$ to the commutator term $ig[\frac{1}{2}\vec{\tau} \cdot \vec{W}_\mu, \Delta]$ and the set the W_3 field zero and the covariant derivative becomes

$$D_\mu \Delta = \begin{pmatrix} \partial_\mu H^+ + ig(H^0 W_\mu^+ - H^{++} W_\mu^-) & \partial_\mu \sqrt{2} H^{++} - ig\sqrt{2} H^+ W_\mu^+ \\ \partial_\mu \sqrt{2} H^0 + ig\sqrt{2} H^+ W_\mu^- & -\partial_\mu H^+ - igH^0 W_\mu^+ + igH^{++} W_\mu^- \end{pmatrix}. \tag{115}$$

As usual we set all fields zero which are not participating in the decay $H^{--} \to W^- H^-$ and draw up the index μ and get

$$(D^\mu \Delta) = \begin{pmatrix} \partial^\mu H^+ & -ig\sqrt{2} H^+ W^{\mu +} \\ 0 & -\partial^\mu H^+ \end{pmatrix}. \tag{116}$$

The adjoint term

$$(D_\mu \Delta)^\dagger = \begin{pmatrix} \partial_\mu H^- - ig(H^{0*} W_\mu^- - H^{--} W_\mu^+) & \partial_\mu \sqrt{2} H^{0*} - ig\sqrt{2} H^- W_\mu^+ \\ \partial_\mu \sqrt{2} H^{--} + ig\sqrt{2} H^- W_\mu^- & -\partial_\mu^- + igH^0 W_\mu^- - igH^{--} W_\mu^+ \end{pmatrix}$$

(117)

can also be simplified analogous. We set the non-participating fields zero and get

$$(D_\mu\Delta)^\dagger = \begin{pmatrix} igH^{--}W_\mu^+ & 0 \\ \partial_\mu\sqrt{2}H^{--} & -igH^{--}W_\mu^+ \end{pmatrix}. \tag{118}$$

The Lagrangian becomes now

$$\mathcal{L}(H^{--}, H^+, W^+) = \frac{1}{2}\text{Tr}\{(D_\mu\Delta)^\dagger(D^\mu\Delta)\}$$

$$= \frac{1}{2}\text{Tr}\begin{pmatrix} igH^{--}W_\mu^+(\partial^\mu H^+) & g^2\sqrt{2}H^{--}H^+W_\mu^+W^{\mu+} \\ \sqrt{2}(\partial_\mu H^{--})(\partial^\mu H^+) & -2ig(\partial_\mu H^{--})H^+W^{\mu+} + igH^{--}W_\mu^+(\partial^\mu H^+) \end{pmatrix}$$

$$= ig[H^{--}W_\mu^+(\partial^\mu H^+) - H^+W^{\mu+}(\partial_\mu H^{--})] = \mathcal{L}_1 + \mathcal{L}_2. \tag{119}$$

With the definition of (36) the first part of the S-matrix element becomes now

$$S_1 = g\int d^4x\langle 0 \mid a_w(k_3,\lambda)a_{H^-}(k_4)$$

$$\times \sum_\rho \int \frac{d^3p_1}{\sqrt{(2\pi)^3 2E_{p_1}}} a_w^\dagger(\vec{p_1},\rho)\epsilon_\mu^*(\vec{p_1},\rho) \int \frac{d^3p_2}{\sqrt{(2\pi)^3 2E_{p_2}}} a_{H^-}^\dagger(\vec{p_2})\partial^\mu(e^{ip_2x})$$

$$\times \int \frac{d^3p_{p_{H^{--}}}}{\sqrt{(2\pi)^3 2E_{p_{H^{--}}}}} b_{H^{--}}(\vec{p}_{H^{--}})e^{-ip_{H^{--}}x} b_{H^{--}}(\vec{q}) \mid 0\rangle. \tag{120}$$

The 4-derivative of the exponential function gives

$$\partial^\mu(e^{ip_2x}) = ip_2^\mu(e^{ip_2x}). \tag{121}$$

The integration over the space-time gives

$$\int d^4x e^{ip_1x}e^{ip_2x}e^{ip_{H^{--}}x} = (2\pi)^4\delta^{(4)}(p_1 + p_2 - p_{H^{--}}). \tag{122}$$

All occurring creation and annihilation operators can be rewritten using basic commutator relations. We finally get

$$a_w(k_3,\lambda)a_w^\dagger(p_1,\rho) \mid 0\rangle = \delta^{(3)}(\vec{k}_3 - \vec{p}_1)\delta_{\lambda\rho} \mid 0\rangle, \tag{123}$$

$$a_{H^-}(k_4)a_{H^-}^\dagger(p_2) \mid 0\rangle = \delta^{(3)}(\vec{k}_4 - \vec{p}_2) \mid 0\rangle \tag{124}$$

and

$$b_{H^{--}}(\vec{p}_{H^{--}})b_{H^{--}}^\dagger(\vec{q}) = \delta^{(3)}(\vec{p}_{H^{--}} - \vec{q}) \mid 0\rangle. \tag{125}$$

The integration of the 3-momenta yields

$$\sum_{\rho}\int d^3p_1\delta^{(3)}(\vec{k}_3-\vec{p}_1)\epsilon^*_\mu(\vec{p}_1,\rho)\delta_{\lambda\rho}=\epsilon^*_\mu(\vec{k}_3,\lambda), \tag{126}$$

$$\int d^3p_2p_2^\mu\delta^{(3)}(\vec{k}_4-\vec{p}_2)=k_4^\mu \tag{127}$$

and

$$\int d^3p_{H^{--}}\delta^{(3)}(\vec{p}_{H^{--}}-\vec{q})=1. \tag{128}$$

Finally the first part of the S-matrix element which corresponds to $\mathcal{L}_1$ becomes

$$S_1=\frac{ig}{\sqrt{(2\pi)^9E_{p_1}E_{p_2}E_{p_{H^{--}}}}}(2\pi)^4\delta^{(4)}(p_1+p_2-p_{H^{--}})\epsilon^*_\mu(\vec{k}_3,\lambda)k_4^\mu \tag{129}$$

and in complete analogy to S_1 the second part of the S-matrix element which corresponds to $\mathcal{L}_2$ becomes

$$S_2=\frac{ig}{\sqrt{(2\pi)^9E_{p_1}E_{p_2}E_{p_{H^{--}}}}}(2\pi)^4\delta^{(4)}(p_1+p_2-p_{H^{--}})\epsilon^{\mu*}(\vec{k}_3,\lambda)q_\mu. \tag{130}$$

The total S-matrix is

$$S=S_1+S_2=\frac{ig(2\pi)^4\delta^{(4)}(p_1+p_2-p_{H^{--}})}{\sqrt{(2\pi)^9E_{p_1}E_{p_2}E_{p_{H^{--}}}}}[\epsilon^*_\mu(\vec{k}_3,\lambda)k_4^\mu+\epsilon^{\mu*}(\vec{k}_3,\lambda)q_\mu]. \tag{131}$$

Again with the method of (E1) we can extract the invariant matrix element $\mathcal{M}$ and get

$$\mathcal{M}=ig[\epsilon^*_\mu(k_3,\lambda)k_4^\mu+\epsilon^{\mu*}(k_3,\lambda)q_\mu] \tag{132}$$

and the complex conjugate

$$\mathcal{M}^*=-ig[\epsilon_\mu(k_3,\lambda)k_4^\mu+\epsilon^\mu(k_3,\lambda)q_\mu]. \tag{133}$$

Before we sum over the polarizations in the final states and take the absolute square of $\mathcal{M}$ let us recall the orthogonality relation for the polarization vectors of a massive vector field. It is defined by

$$\sum_\lambda\epsilon^\mu(k,\lambda)\epsilon^{\nu*}(k,\lambda)=-g^{\mu\nu}+\frac{k^\mu k^\nu}{m_w^2}. \tag{134}$$

Now we sum over the polarizations in the final state, take the absolute square of $\mathcal{M}$ and use the orthogonality relation of (134) and get

$$|\mathcal{M}|^2=g^2\sum_{pol}\Big[\epsilon^*_\mu(k_3,\lambda)k_4^\mu+\epsilon^{\mu*}(k_3,\lambda)q_\mu\Big]\Big[\epsilon_\nu(k_3,\lambda)k_4^\nu+\epsilon^\nu(k_3,\lambda)q_\nu\Big]$$

$$= g^2 \sum_{pol} \Big[\epsilon^*_\mu(k_3,\lambda) k_4^\mu \epsilon_\nu(k_3,\lambda) k_4^\nu + \epsilon^*_\mu(k_3,\lambda) k_4^\mu \epsilon^\nu(k_3,\lambda) q_\nu$$

$$+ \epsilon^{\mu *}(k_3,\lambda) q_\mu \epsilon_\nu(k_3,\lambda) k_4^\nu + \epsilon^{\mu *}(k_3,\lambda) q_\mu \epsilon^\nu(k_3,\lambda) q_\nu \Big]$$

$$= g^2 \Big[\Big(-g_{\mu\nu} + \frac{k_{3\mu} k_{3\nu}}{m_W^2} \Big) k_4^\mu k_4^\nu + \Big(-g_{\mu\nu} + \frac{k_{3\mu} k_{3\nu}}{m_W^2} \Big) k_4^\mu q^\nu$$

$$+ \Big(-g_{\mu\nu} + \frac{k_{3\mu} k_{3\nu}}{m_W^2} \Big) q^\mu k_4^\nu + \Big(-g_{\mu\nu} + \frac{k_{3\mu} k_{3\nu}}{m_W^2} \Big) q^\mu q^\nu \Big]$$

$$= g^2 \Big[-k_4^2 + \frac{(k_3 \cdot k_4)^2}{m_W^2} - k_4 \cdot q + \frac{(k_3 \cdot k_4)(k_3 \cdot q)}{m_W^2}$$

$$- k_4 \cdot q + \frac{(k_3 \cdot q)(k_3 \cdot k_4)}{m_W^2} - q^2 + \frac{(k_3 \cdot q)^2}{m_W^2} \Big]. \tag{135}$$

We can rewrite the scalar products of the previous equation in terms of the mandelstam variable s and the masses of the involved particles. Since

$$s = (k_3 + k_4)^2 = m_W^2 + 2k_3 \cdot k_4 + m_{H^-}^2 \tag{136}$$

we can write

$$k_3 \cdot k_4 = \frac{s - m_W^2 - m_{H^-}^2}{2}. \tag{137}$$

Due to the conservation of the 4-momentum in the vertex the momentum of the H^{--} is the sum of the momenta of the W^- and H^-. Hence we can rewrite the scalar products

$$k_3 \cdot q = k_3(k_3 + k_4) = m_W^2 + \frac{s - m_W^2 - m_{H^-}^2}{2} = \frac{s + m_W^2 - m_{H^-}^2}{2} \tag{138}$$

and respectively

$$k_4 \cdot q = k_4(k_3 + k_4) = \frac{s - m_W^2 - m_{H^-}^2}{2} + m_{H^-} = \frac{s - m_W^2 + m_{H^-}^2}{2}. \tag{139}$$

We insert (137)-(139) into (135) and get

$$|\mathcal{M}|^2 = g^2 \Big[-m_{H^-}^2 + \frac{(s - m_W^2 - m_{H^-}^2)^2}{4m_W^2} - \frac{s - m_W^2 + m_{H^-}^2}{2}$$

$$+ \frac{s - m_W^2 - m_{H^-}^2}{2} \cdot \frac{s + m_W^2 - m_{H^-}^2}{2} \cdot \frac{1}{m_W^2} - \frac{s - m_W^2 + m_{H^-}^2}{2}$$

$$+\frac{s+m_W^2-m_{H^-}^2}{2}\cdot\frac{s-m_W^2-m_{H^-}^2}{2}\cdot\frac{1}{m_W^2}-s+\frac{(s+m_W^2-m_{H^-}^2)^2}{4}\cdot\frac{1}{m_W^2}\Big].$$

Since in this case

$$s=m_{H^{--}}^2, \tag{140}$$

the invariant matrix element can be rewritten just in terms of the masses of the involved particles. By the use of elementary algebraic conversions we get

$$|\mathcal{M}|^2=g^2\Big[-m_{H^-}^2+\frac{(m_{H^{--}}^2-m_W^2-m_{H^-}^2)^2+(m_{H^{--}}+m_W^2-m_{H^-}^2)^2}{4m_W^2}$$

$$-(m_{H^{--}}^2-m_W^2+m_{H^-}^2)+\frac{(m_{H^{--}}^2-m_{H^-}^2)^2-m_W^4}{2m_W^2}-m_{H^{--}}^2\Big]$$

$$=g^2\Big[-2m_{H^-}^2-2m_{H^{--}}^2+m_W^2+\frac{(m_{H^{--}}^2-m_{H^-}^2)^2+m_W^4}{2m_W^2}+\frac{(m_{H^{--}}^2-m_{H^-}^2)^2-m_W^4}{2m_W^2}\Big]$$

$$=g^2\Big[-2m_{H^-}^2-2m_{H^{--}}^2+m_W^2+\frac{(m_{H^{--}}^2-m_{H^-}^2)^2}{m_W^2}\Big]$$

$$=g^2\Big[-2m_{H^-}^2-2m_{H^{--}}^2+m_W^2+\frac{m_{H^{--}}^4-2m_{H^{--}}^2m_{H^-}^2+m_{H^-}^4}{m_W^2}\Big]$$

$$=g^2\Big[\frac{m_{H^{--}}^4+m_{H^-}^4+m_W^4-2(m_{H^-}^2m_W^2+m_{H^{--}}^2m_W^2+m_{H^{--}}^2m_{H^-}^2)}{m_W^2}\Big]. \tag{141}$$

We see that $|\mathcal{M}|^2$ depends only on the masses of the involved particles. Therefore the integration over the two-body phase space is trivial. With the formula for the partial width of (E4) and the two-body phase space integral (D15) we get

$$\Gamma(H^{--}\to H^-W^-)=\frac{1}{2m_{H^{--}}}\frac{1}{(2\pi)^6}|\mathcal{M}|^2(2\pi)^4\frac{\pi}{2}\frac{1}{m_{H^{--}}^2}$$

$$\times\sqrt{m_{H^{--}}^4+m_{H^-}^4+m_W^4-2(m_{H^{--}}^2m_{H^-}^2+m_{H^{--}}^2m_W^2+m_{H^-}^2m_W^2)}$$

$$=\frac{g^2}{16\pi m_{H^{--}}^3m_W^2}\sqrt{(m_{H^{--}}^4+m_{H^-}^4+m_W^4-2(m_{H^{--}}^2m_{H^-}^2+m_{H^{--}}^2m_W^2+m_{H^-}^2m_W^2))^3}. \tag{142}$$

Using the well-known function $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + xz + yz)$ we can rewrite the result in a more compact form, viz.

$$\Gamma(H^{--} \to H^- W^-) = \frac{g^2}{16\pi m_{H^{--}}^3 m_W^2} [\lambda(m_{H^{--}}^2, m_{H^-}^2, m_W^2)]^{3/2}. \tag{143}$$

This result matches with [32]. We can estimate the order of magnitude by inserting rough numerical values like
$m_{H^{--}} = 800$ GeV,
$m_{H^-} = 700$ GeV,
$m_W = 80$ GeV,
$g = \sqrt{4\pi\alpha} = \sqrt{4\pi \times \frac{1}{137}}$ and get

$$\begin{aligned} &\Gamma(H^{--} \to H^- W^-) \\ &= \frac{4\pi \times \frac{1}{137}}{16\pi \times 800^3 \times 80^2} \\ &\times \left[800^4 + 700^4 + 80^4 - 2(800^2 \times 700^2 + 800^2 \times 80^2 + 700^2 \times 80^2)\right]^{3/2} = 0.404 \ldots \end{aligned} \tag{144}$$

We see that in dependence on the values of $m_{H^{--}}$ and m_{H^-} the partial decay width lies in a range of a few GeV.

11 The partial width $\Gamma(H^{--} \to W^- H^- Z^0)$

Another possible decay of the doubly charged Higgs boson is $H^{--} \to W^- H^- Z^0$. In this case four Feynman diagrams are contributing to the decay viz. the so called contact graph and three graphs each with an intermediate H^- respectively H^{--} and W^-. The width $\Gamma(H^{--} \to W^- H^- Z^0)$ depends on all invariant matrix elements of the four contributions.

We will denote the 4-momentum of the initial state of the H^{--} as $k_{H^{--}}$ and the 4-momenta of the final states of H^-, W^- and Z^0 as k_{H^-}, k_W and k_Z.

11.1 Contact graph

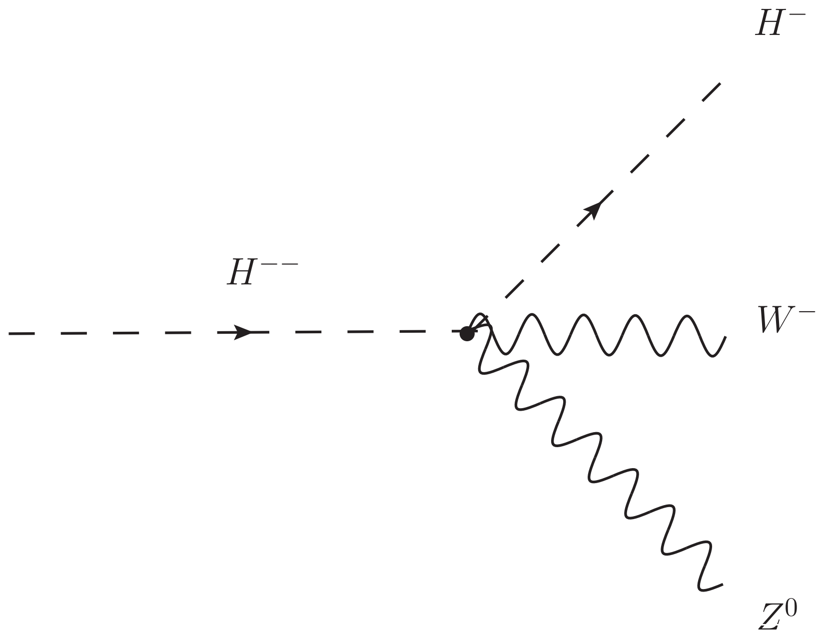

Figure 8: Contact graph contribution for $H^{--} \to H^- H^- Z^0$

In the following we will compute the matrix element $\mathcal{M}_C$ of the so-called contact graph shown in figure 8. As usual we have to start with the Lagrangian. We can expect $\mathcal{L} \propto H^{--} H^+ Z_\mu W^{\mu +}$. Therefore we will leave out all the terms which are not contributing. Let us recall the covariant derivative of (62)

$$D_\mu \Delta = \partial_\mu \Delta + \frac{ig}{2}[\vec{\tau} \cdot \vec{W}_\mu, \Delta] + ig' B_\mu \Delta. \tag{145}$$

The gauge field B_μ and the W_μ^3 component of $\vec{W}_\mu$ can be written as linear combination of Z_μ and A_μ viz.

$$B_\mu = -\sin(\theta_w) Z_\mu + \cos(\theta_w) A_\mu = -s_w Z_\mu + c_w A_\mu \tag{146}$$

and

$$W_\mu^3 = \cos(\theta_w) Z_\mu + \sin(\theta_w) A_\mu = c_w Z_\mu + s_w A_\mu. \tag{147}$$

While the relation between the coupling constant g and g' is given by

$$g\sin(\theta_w) = g'\cos(\theta_w). \tag{148}$$

We insert the equations (146), (147), (148) into the covariant derivative of (145), draw up the index μ, leave out non-participating fields like $W^{\mu-}$ and H^{++} and get

$$(D^\mu\Delta) = \begin{pmatrix} igH^0W^{\mu+} - ig\frac{s_w^2}{c_w}H^+Z^\mu + igs_wH^+A^\mu & -ig\sqrt{2}H^+W^{\mu+} \\ -\frac{g\sqrt{2}}{c_w}H^0Z^\mu & -igH^0W^{\mu+} - ig\frac{s_w}{c_w}H^+ + ig\frac{s_w^2}{c_w}H^+Z^\mu - igs_wH^+A^\mu \end{pmatrix}. \tag{149}$$

Analogous we get the adjoint term by leaving out the H^- field

$$(D_\mu\Delta)^\dagger = \begin{pmatrix} igH^{--}W_\mu^+ & i\frac{g\sqrt{2}}{c_w}H^{0*}Z_\mu \\ ig\sqrt{2}(\frac{s_w^2}{c_w} - c_w)H^{--}Z_\mu - ig2\sqrt{2}s_wH^{--}A_\mu & -igH^{--}W_\mu^+ \end{pmatrix}. \tag{150}$$

The Lagrangian becomes now

$$\mathcal{L} = \frac{1}{2}\mathrm{Tr}\{(D_\mu\Delta)^\dagger(D^\mu\Delta)\} = \frac{1}{2}(A_{11} + A_{22}) \tag{151}$$

with

$$A_{11} = -g^2H^0H^{--}W_\mu^+W^{\mu+} + \frac{g^2s_w^2}{c_w}H^+H^{--}W_\mu^+Z^\mu - g^2s_wH^+H^{--}W_\mu^+A^\mu + \frac{2g^2}{c_w^2}|H^0|Z_\mu Z^\mu \tag{152}$$

and

$$A_{22} = \frac{2g^2}{c_w}(1-2c_w^2)H^{--}H^+Z_\mu W^{\mu+} - 4g^2s_wH^{--}H^+A_\mu W^\mu - g^2H^0H^{--}W_\mu^+W^{\mu+}$$

$$+ g^2\frac{s_w^2}{c_w}H^{--}H^+W_\mu^+Z^\mu - g^2s_wH^{--}H^+W_\mu^+A^\mu. \tag{153}$$

After combining similar terms and leaving out $\frac{2g^2}{c_w^2}|H^0|Z_\mu Z^\mu$ since it does not contribute to the H^{--} decay we get

$$\mathcal{L} = -g^2H^0H^{--}W_\mu^+W^{\mu+} + g^2\frac{s_w^2}{c_w}H^{--}H^+W_\mu^+Z^\mu$$

$$- 3s_wH^{--}H^+W_\mu^+A^\mu + g^2\frac{(1-2c_w^2)}{c_w}H^{--}H^+Z_\mu W^{\mu+}. \tag{154}$$

Since we analyse the decay $H^{--} \to H^-W^-Z^0$ we need only terms proportional to $H^{--}H^+Z_\mu W^{\mu+}$ which are the second and fourth term of (154). Because of

$$\frac{s_w^2}{c_w} + \frac{1}{c_w}(1-2c_w^2) = \frac{s_w^2}{c_w} + \frac{1}{c_w}(1-2(1-s_w^2)) = \frac{s_w^2}{c_w} + \frac{1}{c_w}(-1+2s_w^2) = \frac{1}{c_w}(-1+3s_w^2) \tag{155}$$

our Lagrangian is now

$$\mathcal{L}(H^{--}, H^{+}, W^{+}, Z^{0}) = g^2 \frac{(-1+3s_w^2)}{c_w} H^{--} H^{+} Z_\mu W^{\mu+}. \tag{156}$$

As usual we get the S-matrix element by inserting our Lagrangian into the definition (36). Keep in mind that our S-matrix element is of the 1st order. Since the relation $\mathcal{H} = -\mathcal{L}$ is valid, we get an additional minus sign in contrast to a 2nd order S-matrix element where both minus signs cancel each other. We get

$$\begin{aligned} S_1 = ig^2 \frac{(-1+3s_w^2)}{c_w} \int d^4x \langle 0 \mid a_W(\vec{k}_W, \lambda) a_Z(\vec{k}_Z, \sigma) a_{H^-}(\vec{k}_{H^-}) \\ \times \sum_\rho \int \frac{d^3p_W}{\sqrt{(2\pi)^3 2E_{p_W}}} a_W^\dagger(\vec{p}_W, \rho) \epsilon_\mu^*(\vec{p}_W, \rho) e^{ip_W x} \sum_\tau \int \frac{d^3p_Z}{\sqrt{(2\pi)^3 2E_{p_Z}}} a_Z^\dagger(\vec{p}_Z, \tau) \epsilon_\mu^*(\vec{p}_Z, \tau) e^{ip_Z x} \\ \times \int \frac{d^3p_{H^-}}{\sqrt{(2\pi)^3 2E_{p_{H^-}}}} a_{H^-}^\dagger(\vec{p}_{H^-}) e^{ip_{H^-}x} \int \frac{d^3p_{H^{--}}}{\sqrt{(2\pi)^3 2E_{p_{H^{--}}}}} b_{H^{--}}(\vec{p}_{H^{--}}) e^{-ip_{H^{--}}x} b_{H^{--}}^\dagger(\vec{k}_{H^{--}}) \mid 0\rangle. \end{aligned} \tag{157}$$

The integration over space-time gives

$$\int d^4x e^{ip_W x} e^{ip_Z x} e^{ip_{H^-}x} e^{-ip_{H^{--}}x} = (2\pi)^4 \delta^{(4)}(p_W + p_{H^-} + p_Z - p_{H^{--}}). \tag{158}$$

All occurring creation and annihilation operators can be rewritten using basic commutator relations. We finally get

$$a_W(\vec{k}_W, \lambda) a_w^\dagger(\vec{p}_W, \rho) \mid 0\rangle = \delta^{(3)}(\vec{k}_W - \vec{p}_W) \delta_{\lambda\rho} \mid 0\rangle, \tag{159}$$

$$a_Z(\vec{k}_Z, \sigma) a_Z^\dagger(\vec{p}_Z, \tau) \delta_{\sigma\tau} \mid 0\rangle = \delta^{(3)}(\vec{k}_Z - \vec{p}_Z) \delta_{\sigma\tau} \mid 0\rangle, \tag{160}$$

$$a_{H^-}(\vec{k}_{H^-}) a_{H^-}^\dagger(\vec{p}_{H^-}) \mid 0\rangle = \delta^{(3)}(\vec{k}_{H^-} - \vec{p}_{H^-}) \mid 0\rangle, \tag{161}$$

and

$$b_{H^{--}}(\vec{p}_{H^{--}}) b_{H^{--}}^\dagger(\vec{k}_{H^{--}}) \mid 0\rangle = \delta^{(3)}(\vec{p}_{H^{--}} - \vec{k}_{H^{--}}) \mid 0\rangle. \tag{162}$$

The integration over the 3-momenta gives

$$\sum_\rho \int d^3p_W \delta^{(3)}(\vec{k}_W - \vec{p}_W) \epsilon_\mu^*(\vec{p}_W, \rho) \delta_{\lambda\rho} = \epsilon_\mu^*(\vec{k}_W, \lambda), \tag{163}$$

$$\sum_\tau \int d^3p_Z \delta^{(3)}(\vec{k}_Z - \vec{p}_Z) \epsilon^{\mu*}(\vec{p}_Z, \tau) \delta_{\sigma\tau} = \epsilon^{\mu*}(\vec{k}_Z, \sigma), \tag{164}$$

$$\int d^3p_{H^-} \delta^{(3)}(\vec{k}_{H^-} - \vec{p}_{H^-}) = 1, \tag{165}$$

and

$$\int d^3 p_{H^{--}} \delta^{(3)}(\vec{p}_{H^{--}} - \vec{k}_{H^{--}}) = 1. \tag{166}$$

Finally the S-matrix element becomes

$$\begin{aligned} S_1 = ig^2 \frac{(-1+3s_w^2)}{c_w} & \frac{1}{\sqrt{(2\pi)^{12} 16 E_{p_W} E_{p_Z} E_{p_{H^-}} E_{p_{H^{--}}}}} \\ & \times (2\pi)^4 \delta^{(4)}(p_W + p_Z + p_{H^-} - p_{H^{--}}) \epsilon^*_\mu(\vec{k}_W, \lambda) \epsilon^{\mu *}(\vec{k}_Z, \sigma) \end{aligned} \tag{167}$$

Again with the method of (E1) we can extract the invariant matrix element $\mathcal{M}$ of (167) and get

$$\mathcal{M}_C = \frac{g^2(-1+3s_w^2)}{c_w} \epsilon^*_\mu(\vec{k}_W, \lambda) \epsilon^{\mu *}(\vec{k}_Z, \sigma) \tag{168}$$

11.2 Virtual H^- graph

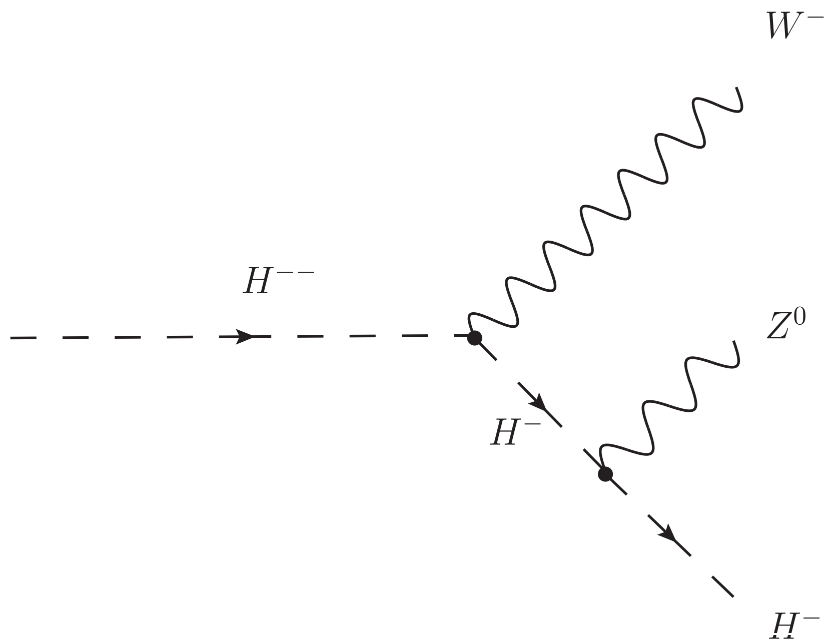

Figure 9: Virtual H^- contribution for $H^{--} \to W^- H^- Z^0$

At next we will compute the matrix element $\mathcal{M}_{H^-}$ of the contribution to $H^{--} \to H^- W^- Z$ via virtual H^- shown in figure 9. While the Lagrangian for the $H^{--}W^+H^+$ vertex is the same as in (119), we have to compute the Lagrangian for the H^-H^+Z vertex. Again we use the covariant derivative of (62) and leave out all non-participating fields and get

$$\begin{aligned} (D_\mu \Delta) &= \partial_\mu \Delta + \frac{ig}{2}[\vec{\tau} \cdot \vec{W}_\mu, \Delta] + ig' B_\mu \Delta \\ &= \begin{pmatrix} \partial_\mu H^+ & 0 \\ 0 & -\partial_\mu H^+ \end{pmatrix} + \frac{ig}{2}\left[\begin{pmatrix} W^3_\mu & 0 \\ 0 & -W^3_\mu \end{pmatrix}, \begin{pmatrix} H^+ & 0 \\ 0 & -H^+ \end{pmatrix} \right] + ig' \begin{pmatrix} B_\mu H^+ & 0 \\ 0 & -B_\mu H^+ \end{pmatrix}. \end{aligned} \tag{169}$$

The commutator term of course vanishes and since no photons are involved in this process we can set $B_\mu = -s_w Z_\mu$ and get

$$(D_\mu \Delta) = \begin{pmatrix} \partial_\mu H^+ - ig' s_w Z_\mu H^+ & 0 \\ 0 & -\partial_\mu H^+ + ig' s_w Z_\mu H^+ \end{pmatrix}. \tag{170}$$

The adjoint term is

$$(D_\mu \Delta)^\dagger = \begin{pmatrix} \partial_\mu H^- + ig' s_w Z_\mu H^- & 0 \\ 0 & -\partial_\mu H^- - ig' s_w Z_\mu H^- \end{pmatrix}. \tag{171}$$

Therewith we get the Lagrangian

$$\begin{aligned}
&\mathcal{L}(H^-, H^+, Z) = \frac{1}{2}\mathrm{Tr}\{(D_\mu\Delta)^\dagger (D^\mu \Delta)\} = \frac{1}{2}(A_{11} + A_{22}) \\
&= \frac{1}{2}\big[(\partial_\mu H^- + ig' s_w Z_\mu H^-)(\partial^\mu H^+ - ig' s_w Z^\mu H^+) \\
&+ (-\partial_\mu H^- - ig' s_w Z_\mu H^-)(-\partial^\mu H^+ + ig' s_w Z^\mu H^+)\big] \\
&= \big[(\partial_\mu H^-)(\partial^\mu H^+) - ig' s_w (\partial_\mu H^-) Z^\mu H^+ + ig' s_w (\partial^\mu H^+) Z_\mu H^- + g'^2 {s_w}^2 H^- H^+ Z_\mu Z^\mu\big].
\end{aligned} \tag{172}$$

Since we know that the Lagrangian for the $H^- H^+ Z$-vertex can only contain terms $\propto H^- H^+ Z$ we leave out all other terms and finally get

$$\begin{aligned}
\mathcal{L}(H^-, H^+, Z) &= ig' s_w \big[H^- (\partial^\mu H^+) Z_\mu - H^+ (\partial_\mu H^-) Z^\mu\big] \\
&= \frac{ig {s_w}^2}{c_w}\big[H^- (\partial^\mu H^+) Z_\mu - H^+ (\partial^\mu H^-) Z_\mu\big].
\end{aligned} \tag{173}$$

Together with the Lagrangian $\mathcal{L}(H^{--}, H^+, W^+)$ from (119) and $\mathcal{L}(H^-, H^+, Z)$ from (173) the S-matrix element becomes now

$$\begin{aligned}
&S = i^2 \int d^4x d^4y \langle 0 \mid a_W(\vec{k}_W, \lambda) a_{H^-}(\vec{k}_{H^-}) a_Z(\vec{k}_Z, \sigma) \\
&\times \mathcal{L}(H^{--}, H^+, W^+)(x) \mathcal{L}(H^-, H^+, Z)(y) b^\dagger_{H^{--}}(\vec{k}_{H^{--}}) \mid 0\rangle \\
&= i^4 \frac{g^2 {s_w}^2}{c_w} \int d^4x d^4y \langle 0 \mid a_W(\vec{k}_W, \lambda) a_{H^-}(\vec{k}_{H^-}) a_Z(\vec{k}_Z, \sigma) \\
&\times \sum_\rho \int \frac{d^3 p_W}{\sqrt{(2\pi)^3 2E_{p_W}}} a^\dagger_W(\vec{p}_W, \rho) \epsilon^*_\mu(\vec{p}_W, \rho) e^{ip_W x} \\
&\times \int \frac{d^3 p_{H^{--}}}{\sqrt{(2\pi)^3 2E_{p_{H^{--}}}}} b_{H^{--}}(\vec{p}_{H^{--}}) e^{-ip_{H^{--}} x} H^+(x) [ip^\mu_{H^-} + ip^\mu_{H^{--}}] \\
&\times \sum_\tau \int \frac{d^3 p_Z}{\sqrt{(2\pi)^3 2E_{p_Z}}} a^\dagger_Z(\vec{p}_Z, \tau) \epsilon^*_\nu(\vec{p}_Z, \tau) e^{ip_Z y} \int \frac{d^3 q_{H^-}}{\sqrt{(2\pi)^3 2E_{q_{H^-}}}} a^\dagger_{H^-}(\vec{q}_{H^-}) e^{iq_{H^-} y} \\
&\times H^-(y) [iq^\nu_{H^-} + ip^\nu_{H^-}] b^\dagger_{H^{--}}(\vec{k}_{H^{--}}) \mid 0\rangle.
\end{aligned} \tag{174}$$

The integration over space-time gives

$$\int d^4x e^{ip_w x} e^{-ip_{H^{--}} x} e^{ip_{H^-} x} = (2\pi)^4 \delta^{(4)}(p_W + p_{H^-} - p_{H^{--}}) \tag{175}$$

and respectively

$$\int d^4 y e^{ip_Z y} e^{iq_{H^-} y} e^{-ip_{H^-} y} = (2\pi)^4 \delta^{(4)}(p_Z + q_{H^-} - p_{H^-}). \tag{176}$$

All occurring creation and annihilation operators can be rewritten using basic commutator relations. We finally get

$$a_W(\vec{k}_W, \lambda) a_W^\dagger(\vec{p}_W, \rho) \mid 0\rangle = \delta^{(3)}(\vec{k}_W - \vec{p}_W)\delta_{\lambda\rho} \mid 0\rangle \tag{177}$$

$$a_{H^-}(\vec{k}_{H^-}) a_{H^-}^\dagger(\vec{q}_{H^-}) \mid 0\rangle = \delta^{(3)}(\vec{k}_{H^-} - \vec{q}_{H^-}) \mid 0\rangle \tag{178}$$

$$a_Z(\vec{k}_Z, \sigma) a_Z^\dagger(\vec{p}_Z, \tau) \mid 0\rangle = \delta^{(3)}(\vec{k}_Z - \vec{p}_Z)\delta_{\sigma\tau} \mid 0\rangle \tag{179}$$

$$b_{H^{--}}(\vec{p}_{H^{--}}) b_{H^{--}}^\dagger(\vec{k}_{H^{--}}) \mid 0\rangle = \delta^{(3)}(\vec{p}_{H^{--}} - \vec{k}_{H^{--}}) \mid 0\rangle. \tag{180}$$

Now we can integrate over the 3-momenta and get

$$\sum_\rho \int d^3 p_W \delta^{(3)}(\vec{k}_W - \vec{p}_W) \epsilon_\mu^*(p_W, \rho)\delta_{\lambda\rho} = \epsilon_\mu^*(k_W, \lambda), \tag{181}$$

$$\int d^3 p_{H^{--}} \delta^{(3)}(\vec{p}_{H^{--}} - \vec{k}_{H^{--}})[ip_{H^-}^\mu + ip_{H^{--}}^\mu] = [ip_{H^-}^\mu + ik_{H^{--}}^\mu], \tag{182}$$

$$\sum_\tau \int d^3 p_Z \delta^{(3)}(\vec{k}_Z - \vec{p}_Z) \epsilon_\nu^*(p_Z, \tau)\delta_{\sigma\tau} = \epsilon_\nu^*(k_Z, \sigma) \tag{183}$$

and

$$\int d^3 q_{H^-} \delta^{(3)}(\vec{k}_{H^-} - \vec{q}_{H^-})[iq_{H^-}^\nu + ip_{H^-}^\nu] = [ik_{H^-}^\nu + ip_{H^-}^\nu]. \tag{184}$$

Furthermore, we contract $H^+(x)$ and $H^-(y)$ and get the H^- propagator

$$\langle 0 \mid H^+(x) H^-(y) \mid 0\rangle = \int \frac{d^4 p_{H^-}}{(2\pi)^4} \frac{i}{p_{H^-}{}^2 - m_{H^-}^2 + i\epsilon}. \tag{185}$$

Now we integrate over $d^4 p_{H^-}$ and get

$$\begin{aligned} &\int \frac{d^4 p_{H^-}}{(2\pi)^4} (2\pi)^4 \delta^{(4)}(p_W + p_{H^-} - p_{H^{--}})(2\pi)^4 \delta^{(4)}(p_Z + q_{H^-} - p_{H^-}) \frac{i}{p_{H^-}{}^2 - m_{H^-}^2 + i\epsilon} \\ &= i(2\pi)^4 \frac{\delta^{(4)}(p_W + p_Z + q_{H^-} - p_{H^{--}})}{(k_{H^-} + k_Z)^2 - m_{H^-}^2 + i\epsilon}. \end{aligned} \tag{186}$$

Finally the S-matrix element becomes

$$\begin{aligned} S = i^7 \frac{g^2 s_w^2}{c_w} &\frac{1}{\sqrt{(2\pi)^{12} 16 E_{p_W} E_{p_{H^{--}} E_{p_Z} E_{q_H^-}}}} (2\pi)^4 \frac{\delta^{(4)}(p_W + p_Z + q_{H^-} - p_{H^{--}})}{(k_{H^-} + k_Z)^2 - m_{H^-}^2 + i\epsilon} \\ &\times \epsilon_\mu^*(k_W, \lambda)(p_{H^-}^\mu + k_{H^{--}}^\mu) \epsilon_\nu^*(k_Z, \sigma)(k_{H^-}^\nu + p_{H^-}^\nu). \end{aligned} \tag{187}$$

Now we can extract the invariant matrix element with the method of (E1) and get

$$i\mathcal{M}_{H^-} = -i \frac{g^2 s_w^2}{c_w} \frac{\epsilon_\mu^*(k_W, \lambda) \epsilon_\nu^*(k_Z, \sigma)(p_{H^-}^\mu + k_{H^{--}}^\mu)(k_{H^-}^\nu + p_{H^-}^\nu)}{(k_{H^-} + k_Z)^2 - m_{H^-}^2 + i\epsilon}. \tag{188}$$

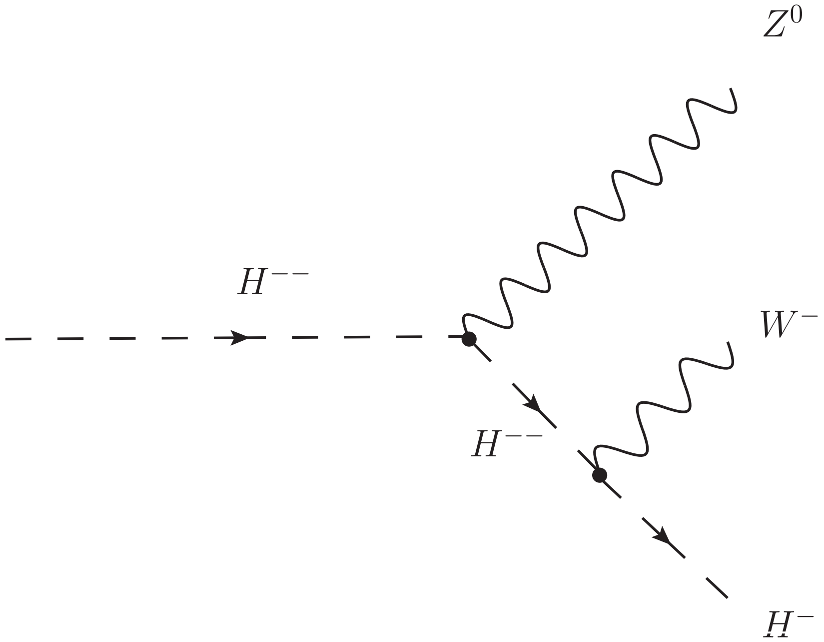

Figure 10: Virtual H^{--} contribution for $H^{--} \to W^- H^- Z^0$

11.3 Virtual H^{--} graph

At next we will compute the matrix element $\mathcal{M}_{H^{--}}$ of the contribution to $H^{--} \to H^- W^- Z$ via virtual H^{--} shown in figure 10. While the Lagrangian for the $H^{--}H^+W^+$ vertex is the same as in (119), we have to compute the Lagrangian for the $H^{--}H^{++}Z$ vertex. As usual we use the covariant derivative of (62) and leave out all non-participating fields an get

$$\begin{aligned} D_\mu \Delta &= \partial_\mu \Delta + ig[\frac{1}{2}\vec{\tau} \cdot \vec{W}_\mu, \Delta] + ig' B_\mu \Delta \\ &= \begin{pmatrix} 0 & \sqrt{2}\partial_\mu H^{++} \\ 0 & 0 \end{pmatrix} + \frac{ig}{2} \begin{pmatrix} 0 & 2\sqrt{2} W_\mu^3 H^{++} \\ 0 & 0 \end{pmatrix} + ig'(-s_w Z_\mu) \begin{pmatrix} 0 & \sqrt{2} H^{++} \\ 0 & 0 \end{pmatrix} \\ &= \begin{pmatrix} 0 & (\partial_\mu + igc_w Z_\mu - ig\frac{s_w^2}{c_w} Z_\mu)\sqrt{2} H^{++} \\ 0 & 0 \end{pmatrix}. \end{aligned} \tag{189}$$

In the last step we used $W_\mu^3 = c_w Z_\mu + s_w A_\mu$ and left out the A_μ since no photons are involved in the process. The coupling constant g' was converted with the relation $g' = \frac{g s_w}{c_w}$. The adjoint term is

$$(D_\mu \Delta)^\dagger = \begin{pmatrix} 0 & 0 \\ (\partial_\mu - igc_w Z_\mu + ig\frac{s_w^2}{c_w} Z_\mu)\sqrt{2} H^{--} & 0 \end{pmatrix} \tag{190}$$

Therewith we get the Lagrangian for the $H^{--}H^{++}Z$ vertex

$$\begin{aligned}
&\mathcal{L}(H^{--},H^{++},Z)=\frac{1}{2}Tr\{(D_\mu\Delta)^\dagger(D^\mu\Delta)\}\\
&=\frac{1}{2}\mathrm{Tr}\begin{pmatrix}0 & 0\\ 0 & (\partial_\mu-igc_wZ_\mu+ig\frac{s_w^2}{c_w}Z_\mu)H^{--}(\partial_\mu+igc_wZ_\mu-ig\frac{s_w^2}{c_w}Z_\mu)2H^{++}\end{pmatrix}\\
&=[(\partial_\mu H^{--})+(-igc_w+\frac{igs_w^2}{c_w})Z_\mu H^{--}][(\partial^\mu H^{++})+(igc_w-\frac{igs_w^2}{c_w})Z^\mu H^{++}]\\
&=(\partial_\mu H^{--})(\partial^\mu H^{++})+(\partial_\mu H^{--})(c_w-\frac{s_w^2}{c_w})igZ^\mu H^{++}+(-c_w+\frac{s_w^2}{c_w})igZ_\mu H^{--}(\partial^\mu H^{++})\\
&+(-c_w+\frac{s_w^2}{c_w})igZ_\mu H^{--}(c_w-\frac{s_w^2}{c_w})igZ^\mu H^{--}.
\end{aligned}\tag{191}$$

But only terms $\propto H^{--}H^{++}Z$ contribute to $\mathcal{L}(H^{--}H^{++}Z)$ therefore we get

$$\begin{aligned}
&\mathcal{L}(H^{--},H^{++},Z)=ig(\frac{2c_w^2-1}{c_w})(\partial_\mu H^{--})Z^\mu H^{++}+ig(\frac{1-2c_w^2}{c_w})(\partial^\mu H^{++})Z_\mu H^{--}\\
&=ig(\frac{2c_w^2-1}{c_w})[(\partial_\mu H^{--})Z^\mu H^{++}-(\partial_\mu H^{++})Z^\mu H^{--}].
\end{aligned}\tag{192}$$

The prefactor was rewritten using the trigonometric identity $s_w^2+c_w^2=1$.
Let us now perform the calculation of the S-matrix element. In addition to $\mathcal{L}(H^{--},H^{++},Z)$ which we just have calculated we use $\mathcal{L}(H^{--},H^{+},W^{+})$ from (119) and the S-matrix element becomes now

$$\begin{aligned}
&S=i^2\int d^4xd^4y\langle 0\mid a_W(\vec{k}_W,\lambda)a_{H^-}(\vec{k}_{H^-})a_Z(\vec{k}_Z,\sigma)\\
&\times\mathcal{L}(H^{--},H^{++},Z)(x)\mathcal{L}(H^{--},H^{+},W^{+})(y)b^\dagger_{H^{--}}(\vec{k}_{H^{--}})\mid 0\rangle\\
&=i^4g^2\frac{2c_w^2-1}{c_w}\int d^4xd^4y\langle 0\mid a_W(\vec{k}_W,\lambda)a_{H^-}(\vec{k}_{H^-})a_Z(\vec{k}_Z,\sigma)\\
&\times\sum_\rho\int\frac{d^3p_Z}{\sqrt{(2\pi)^32E_{p_Z}}}a_Z^\dagger(\vec{p}_Z,\rho)\epsilon^*_\mu(\vec{p}_Z,\rho)e^{ip_Zx}\int\frac{d^3q_{H^{--}}}{\sqrt{(2\pi)^32E_{q_{H^{--}}}}}b_{H^{--}}(\vec{q}_{H^{--}})e^{-iq_{H^{--}}x}\\
&\times H^{++}(p_{H^{--}})(x)[-iq^\mu_{H^{--}}-ip^\mu_{H^{--}}]\sum_\tau\int\frac{d^3p_W}{\sqrt{(2\pi)^32E_{p_W}}}a_W^\dagger(\vec{p}_W,\tau)\epsilon^*_\nu(\vec{p}_W,\tau)e^{ip_Wy}\\
&\times\int\frac{d^3p_{H^-}}{\sqrt{(2\pi)^32E_{p_{H^-}}}}a^\dagger_{H^-}(\vec{p}_{H^-})e^{ip_{H^-}y}H^{--}(p_{H^{--}})(y)[ip^\nu_{H^-}+ip^\nu_{H^{--}}]b_{H^{--}}(\vec{k}_{H^{--}})\mid 0\rangle.
\end{aligned}\tag{193}$$

The integration over space-time gives

$$\int d^4xe^{ip_Zx}e^{-iq_{H^{--}}x}e^{ip_{H^{--}}x}=(2\pi)^4\delta^{(4)}(p_Z+p_{H^{--}}-q_{H^{--}})\tag{194}$$

and respectively

$$\int d^4ye^{ip_Wy}e^{ip_{H^-}y}e^{-ip_{H^{--}}y}=(2\pi)^4\delta^{(4)}(p_W+p_{H^-}-p_{H^{--}}).\tag{195}$$

All occurring creation and annihilation operators can be rewritten using basic commutator relations. We finally get

$$a_W(\vec{k}_W,\lambda)a^\dagger_W(\vec{p}_W,\tau)\mid 0\rangle = \delta^{(3)}(\vec{k}_W-\vec{p}_W)\delta_{\lambda\tau}\mid 0\rangle \tag{196}$$

$$a_{H^-}(\vec{k}_{H^-})a^\dagger_{H^-}(\vec{p}_{H^-})\mid 0\rangle = \delta^{(3)}(\vec{k}_{H^-}-\vec{p}_{H^-})\mid 0\rangle \tag{197}$$

$$a_Z(\vec{k}_Z,\sigma)a^\dagger_Z(\vec{p}_Z,\rho)\mid 0\rangle = \delta^{(3)}(\vec{k}_Z-\vec{p}_Z)\delta_{\sigma\rho}\mid 0\rangle \tag{198}$$

$$b_{H^{--}}(\vec{q}_{H^{--}})b^\dagger_{H^{--}}(\vec{k}_{H^{--}})\mid 0\rangle = \delta^{(3)}(\vec{q}_{H^{--}}-\vec{k}_{H^{--}})\mid 0\rangle. \tag{199}$$

Now we can integrate over the 3-momenta and get

$$\sum_\rho \int d^3p_Z\delta^{(3)}(\vec{k}_Z-\vec{p}_Z)\epsilon^*_\mu(p_Z,\rho)\delta_{\sigma\rho} = \epsilon^*_\mu(k_Z,\sigma), \tag{200}$$

$$\int d^3q_{H^{--}}\delta^{(3)}(\vec{q}_{H^{--}}-\vec{k}_{H^{--}})[iq^\mu_{H^{--}}+ip^\mu_{H^{--}}] = [ik^\mu_{H^{--}}+ip^\mu_{H^{--}}], \tag{201}$$

$$\sum_\tau \int d^3p_W\delta^{(3)}(\vec{k}_W-\vec{p}_W)\epsilon^*_\nu(p_W,\tau)\delta_{\lambda\tau} = \epsilon^*_\nu(k_W,\lambda) \tag{202}$$

and

$$\int d^3p_{H^-}\delta^{(3)}(\vec{k}_{H^-}-\vec{p}_{H^-})[-iq^\nu_{H^-}-ip^\nu_{H^{--}}] = [-ik^\nu_{H^-}-ip^\nu_{H^{--}}]. \tag{203}$$

Furthermore, we contract $H^{++}(x)$ and $H^{--}(y)$ and get the H^{--} propagator

$$\langle 0\mid H^{++}(x)H^{--}(y)\mid 0\rangle = \int \frac{d^4p_{H^{--}}}{(2\pi)^4}\frac{i}{{p_{H^{--}}}^2-m^2_{H^{--}}+i\epsilon}. \tag{204}$$

Now we integrate over $d^4p_{H^{--}}$ and get

$$\begin{aligned}&\int \frac{d^4p_{H^{--}}}{(2\pi)^4}(2\pi)^4\delta^{(4)}(p_Z+p_{H^{--}}-q_{H^{--}})(2\pi)^4\delta^{(4)}(p_W+p_{H^-}-p_{H^{--}})\frac{i}{{p_{H^{--}}}^2-m^2_{H^{--}}+i\epsilon}\\ &= i(2\pi)^4\frac{\delta^{(4)}(p_Z+p_W+p_{H^-}-q_{H^{--}})}{(k_W+k_{H^-})^2-m^2_{H^{--}}+i\epsilon}.\end{aligned} \tag{205}$$

Finally the S-matrix element becomes

$$\begin{aligned}S &= i^4\frac{g^2(2c_w^2-1)}{c_w}\frac{i}{\sqrt{(2\pi)^{12}16E_{p_Z}E_{q_{H^{--}}}E_{p_W}E_{p_{H^-}}}}(2\pi)^4\frac{\delta^{(4)}(p_Z+p_W+p_{H^-}-q_{H^{--}})}{(k_W+k_{H^-})^2-m^2_{H^{--}}+i\epsilon}\\ &\times \epsilon^*_\mu(k_Z,\sigma)i(k^\mu_{H^{--}}+p^\mu_{H^{--}})\epsilon^*_\nu(k_W,\lambda)i(-k^\nu_{H^-}-p^\nu_{H^{--}}).\end{aligned} \tag{206}$$

Now we can extract the invariant matrix element with the method of (E1) and get

$$i\mathcal{M}_{H^{--}} = -i^7\frac{g^2(2c_w^2-1)}{c_w}\frac{\epsilon^*_\mu(k_Z,\sigma)\epsilon^*_\nu(k_W,\lambda)(k^\mu_{H^{--}}+p^\mu_{H^{--}})(k^\nu_{H^-}+p^\nu_{H^{--}})}{(k_W+k_{H^-})^2-m^2_{H^{--}}+i\epsilon}. \tag{207}$$

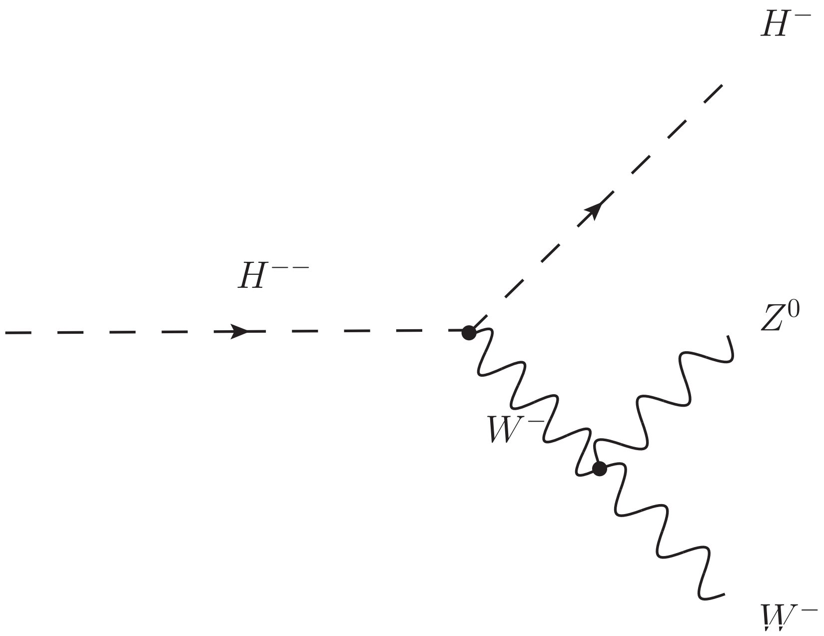

Figure 11: Virtual W^- contribution for $H^{--} \to W^- H^- Z^0$

11.4 Virtual W^- graph

At last we will compute the matrix element $\mathcal{M}_{W^-}$ of the contribution to $H^{--} \to H^- W^- Z$ via virtual W^- shown in figure 11.

While the Lagrangian for the $H^{--}H^+W^+$ is the same as in (119), we use the Lagrangian from [34] for the W^-W^+Z vertex. The Lagrangian is

$$\begin{aligned}
&\mathcal{L}(W^-, W^+, Z) = igc_w[Z_\mu W^-_\nu \overleftrightarrow{\partial^\mu} W^{+\nu} + W^-_\mu W^+_\nu \overleftrightarrow{\partial^\mu} Z^\nu + W^+_\mu \overleftrightarrow{\partial^\mu} W^{-\nu}] \\
&= igc_w[Z_\mu(W^-_\nu(\partial^\mu W^{+\nu}) - (\partial^\mu W^-_\nu)W^{+\nu}) + W^-_\mu(W^+_\nu(\partial^\mu Z^\nu) - (\partial^\mu W^+_\nu)Z^\nu) \\
&+ W^+_\mu(Z_\nu(\partial^\mu W^{-\nu}) - (\partial^\mu Z_\nu)W^{-\nu})] \\
&= igc_w[Z_\mu W^-_\nu W^{+\nu}(ip^\mu_W + iq^\mu_W) + W^-_\mu W^+_\nu Z^\nu(ip^\mu_Z - ip^\mu_W) + W^+_\mu Z_\nu W^{-\nu}(-iq^\mu_W - ip^\mu_Z)] \\
&= \mathcal{L}_A(W^-, W^+, Z) + \mathcal{L}_B(W^-, W^+, Z) + \mathcal{L}_C(W^-, W^+, Z).
\end{aligned} \tag{208}$$

We used the definition for the $\overleftrightarrow{\partial^\mu}$ symbol, which is $A \overleftrightarrow{\partial^\mu} B = A(\partial^\mu B) - (\partial^\mu A)B$. At next we will compute the matrix elements for the three terms of $\mathcal{L}(W^-W^+Z)$ for the virtual W^- graph. We start with the matrix element corresponding to $\mathcal{L}_A(W^-, W^+, Z)$ which we will denote as $\mathcal{M}_{AW^-}$.

Calculation of $\mathcal{M}_{AW^-}$
For the calculation of $\mathcal{M}_{AW^-}$ we need the Lagrangians

$$\mathcal{L}(H^{--}, H^+, W^+) = igH^{--}H^+W^+_\mu[ip^\mu_{H^-} + ip^\mu_{H^{--}}] \tag{209}$$

from (119) and

$$\mathcal{L}_A(W^-, W^+, Z) = igc_w Z_\alpha W^-_\beta W^{+\beta}[ip^\alpha_W + iq^\alpha_W] \tag{210}$$

from (208). The S-matrix becomes now

$$\begin{aligned}
S &= i^2 \int d^4x d^4y \langle 0 \mid a_W(\vec{k}_W, \lambda) a_{H^-}(\vec{k}_{H^-}) a_Z(\vec{k}_Z, \sigma) \mathcal{L}(H^{--}, H^+, W^+) \\
&\times \mathcal{L}_A(W^-, W^+, Z) b^\dagger_{H^{--}}(\vec{k}_{H^{--}}) \mid 0 \rangle \\
&= i^4 g^2 c_w \frac{1}{\sqrt{(2\pi)^{12} 16 E_{p_{H^{--}}} E_{p_{H^-}} E_{q_W} E_{p_Z}}} \int d^4x d^4y \langle 0 \mid W^+_\mu(x)(ik^\mu_{H^-} + ik^\mu_{H^{--}}) \\
&\times e^{-ik_{H^{--}}x} e^{ik_{H^-}x} W^-_\beta(y) e^{ik_W y} \epsilon^{*\beta}_W(k_W, \lambda) e^{ik_Z y} \epsilon^*_{Z\alpha}(k_Z, \sigma)(ik^\alpha_W + iq^\alpha_W) \mid 0 \rangle.
\end{aligned} \tag{211}$$

Furthermore, we contract $W^+_\mu(x)$ and $W^-_\beta(y)$ and get the W^- propagator

$$\langle 0 \mid W^+_\mu(x) W^-_\beta(y) \mid 0 \rangle = \int \frac{d^4q_W}{(2\pi)^4} \frac{-i}{q^2_W - m^2_W + i\epsilon}(g_{\mu\beta} - \frac{q_{W\mu} q_{W\beta}}{m^2_W}) e^{-iq_W(y-x)}. \tag{212}$$

The integration over the space-time gives

$$\int d^4x e^{-ik_{H^{--}}x} e^{ik_{H^-}x} e^{iq_W x} = (2\pi)\delta^{(4)}(k_{H^-} + q_W - k_{H^{--}}) \tag{213}$$

and

$$\int d^4y e^{ik_W y} e^{ik_Z y} e^{-iq_W y} = (2\pi)\delta^{(4)}(k_W + k_Z - q_W). \tag{214}$$

Now we integrate over d^4q_W and get

$$\begin{aligned}
&\int \frac{d^4q_W}{(2\pi)^4} (2\pi)^4 \delta^{(4)}(k_{H^-} + q_W - k_{H^{--}})(2\pi)^4 \delta^{(4)}(k_W + k_Z - q_W) \\
&\times \frac{-i}{{q_W}^2 - m^2_{W^-} + i\epsilon}(g_{\mu\beta} - \frac{q_{W\mu} q_{W\beta}}{m^2_W})(ik^\alpha_W + iq^\alpha_W) \\
&= -i(2\pi)^4 \frac{\delta^{(4)}(k_W + k_Z + k_{H^-} - k_{H^{--}})}{(k_W + k_Z)^2 - m^2_W + i\epsilon}(g_{\mu\beta} - \frac{(k_{W\mu} + k_{Z\mu})(k_{W\beta} + k_{Z\beta})}{m^2_W})(2ik^\alpha_W + ik^\alpha_Z)
\end{aligned} \tag{215}$$

The S-matrix becomes now

$$\begin{aligned}
S &= i^4 g^2 c_w \frac{1}{\sqrt{(2\pi)^{12} 16 E_{p_{H^{--}}} E_{p_{H^-}} E_{p_W} E_{p_Z}}} (2\pi)^4 \frac{\delta^{(4)}(k_W + k_Z + k_{H^-} - k_{H^{--}})}{(k_Z + k_W)^2 - m^2_W + i\epsilon} \\
&\times (-i)[g_{\mu\beta} - \frac{(k_{W\mu} + k_{Z\mu})(k_{W\beta} + k_{Z\beta})}{m^2_W}] \epsilon^*_{Z\alpha}(k_Z, \sigma) \epsilon^{*\beta}_W(k_W, \lambda)(ik^\mu_{H^-} + ik^\mu_{H^{--}})(2ik^\alpha_W + ik^\alpha_Z)
\end{aligned} \tag{216}$$

Now we can extract the invariant matrix element with the method of (E1) and get

$$\begin{aligned}
i\mathcal{M}_{AW^-} &= -i^7 g^2 c_w \frac{[g_{\mu\beta} - \frac{(k_{W\mu}+k_{Z\mu})(k_{W\beta}+k_{Z\beta})}{m^2_W}]}{(k_W + k_Z)^2 - m^2_W + i\epsilon} \\
&\times \epsilon^*_{Z\alpha}(k_Z, \sigma) \epsilon^{*\beta}_W(k_W, \lambda)(k^\mu_{H^-} + k^\mu_{H^{--}})(2k^\alpha_W + k^\alpha_Z) \\
&= ig^2 c_w \frac{[g_{\mu\beta} - \frac{(k_{W\mu}+k_{Z\mu})(k_{W\beta}+k_{Z\beta})}{m^2_W}]}{(k_W + k_Z)^2 - m^2_W + i\epsilon} \\
&\times \epsilon^*_{Z\alpha}(k_Z, \sigma) \epsilon^{*\beta}_W(k_W, \lambda)(k^\mu_{H^-} + k^\mu_{H^{--}})(2k^\alpha_W + k^\alpha_Z).
\end{aligned} \tag{217}$$

Calculation of $\mathcal{M}_{BW^-}$
For the computation of $\mathcal{M}_{BW^-}$ we need the Lagrangians

$$\mathcal{L}(H^{--}, H^+, W^+) = igH^{--}H^+W^+_\mu[ip^\mu_{H^-} + ip^\mu_{H^{--}}] \tag{218}$$

from (119) and

$$\mathcal{L}_B(W^-, W^+, Z) = igc_w W^-_\alpha W^+_\beta Z^\beta[ip^\alpha_Z - ip^\alpha_W] \tag{219}$$

from (208). The S-matrix becomes now

$$\begin{aligned} S &= i^2 \int d^4x d^4y \langle 0 \mid a_W(\vec{k}_W, \lambda) a_{H^-}(\vec{k}_{H^-}) a_Z(\vec{k}_Z, \sigma) \mathcal{L}(H^{--}, H^+, W^+) \\ &\times \mathcal{L}_B(W^-, W^+, Z) b^\dagger_{H^{--}}(\vec{k}_{H^{--}}) \mid 0\rangle \\ &= i^4 g^2 c_w \frac{1}{\sqrt{(2\pi)^{12} 16 E_{p_{H^{--}}} E_{p_{H^-}} E_{p_W} E_{p_Z}}} \int d^4x d^4y \langle 0 \mid W^+_\mu(x)(ik^\mu_{H^-} + ik^\mu_{H^{--}}) \\ &\times e^{-ik_{H^{--}}x} e^{ik_{H^-}x} W^-_\alpha(y) e^{ik_W y} \epsilon^*_{W\beta}(k_W, \lambda) e^{ik_Z y} \epsilon^{*\beta}_Z(k_Z, \sigma)(ik^\alpha_Z - ik^\alpha_W) \mid 0\rangle. \end{aligned} \tag{220}$$

Furthermore, we contract $W^+_\mu(x)$ and $W^-_\alpha(y)$ and get the W^- propagator

$$\langle 0 \mid W^+_\mu(x) W^-_\alpha(y) \mid 0\rangle = \int \frac{d^4q_W}{(2\pi)^4} \frac{-i}{q^2_W - m^2_W + i\epsilon}(g_{\mu\alpha} - \frac{q_{W\mu}q_{W\alpha}}{m^2_W}) e^{-iq_W(y-x)}. \tag{221}$$

The integration over the space-time gives

$$\int d^4x e^{-ik_{H^{--}}x} e^{ik_{H^-}x} e^{iq_W x} = (2\pi)\delta^{(4)}(k_{H^-} + q_W - k_{H^{--}}) \tag{222}$$

and

$$\int d^4y e^{ik_W y} e^{ik_Z y} e^{-iq_W y} = (2\pi)\delta^{(4)}(k_W + k_Z - q_W). \tag{223}$$

Now we integrate over d^4q_W and get

$$\begin{aligned} &\int \frac{d^4q_W}{(2\pi)^4}(2\pi)^4\delta^{(4)}(k_{H^-} + q_W - k_{H^{--}})(2\pi)^4\delta^{(4)}(k_W + k_Z - q_W) \\ &\times \frac{-i}{{q_W}^2 - m^2_{W^-} + i\epsilon}(g_{\mu\alpha} - \frac{q_{W\mu}q_{W\alpha}}{m^2_W})(ip^\alpha_Z - iq^\alpha_W) \\ &= -i(2\pi)^4 \frac{\delta^{(4)}(k_W + k_Z + k_{H^-} - k_{H^{--}})}{(k_W + k_Z)^2 - m^2_W + i\epsilon}(g_{\mu\alpha} - \frac{(k_{W\mu} + k_{Z\mu})(k_{W\alpha} + k_{Z\alpha})}{m^2_W})(ik^\alpha_Z - ik^\alpha_W). \end{aligned} \tag{224}$$

The S-matrix becomes now

$$\begin{aligned} S &= i^4 g^2 c_w \frac{1}{\sqrt{(2\pi)^{12} 16 E_{p_{H^{--}}} E_{p_{H^-}} E_{p_W} E_{p_Z}}}(2\pi)^4 \frac{\delta^{(4)}(k_W + k_Z + k_{H^-} - k_{H^{--}})}{(k_Z + k_W)^2 - m^2_W + i\epsilon} \\ &\times (-i)[g_{\mu\alpha} - \frac{(k_{W\mu} + k_{Z\mu})(k_{W\alpha} + k_{Z\alpha})}{m^2_W}]\epsilon^*_{Z\alpha}(k_Z, \sigma)\epsilon^{*\beta}_W(k_W, \lambda)(ik^\mu_{H^-} + ik^\mu_{H^{--}})(ik^\alpha_Z - ik^\alpha_W). \end{aligned} \tag{225}$$

Now we can extract the invariant matrix element with the method of (E1) and get

$$
\begin{aligned}
i\mathcal{M}_{BW^-} &= -i^7 g^2 c_w \frac{[g_{\mu\alpha} - \frac{(k_{Z\mu}+k_{W\mu})(k_{Z\alpha}+k_{W\alpha})}{m_W^2}]}{(k_W+k_Z)^2 - m_W^2 + i\epsilon} \\
&\times \epsilon^*_{W\beta}(k_W,\lambda)\epsilon_Z^{*\beta}(k_Z,\sigma)(ik^\mu_{H^-} + ik^\mu_{H^{--}})(k_Z^\alpha - k_W^\alpha) \\
&= ig^2 c_w \frac{[g_{\mu\alpha} - \frac{(k_{Z\mu}+k_{W\mu})(k_{Z\alpha}+k_{W\alpha})}{m_W^2}]}{(k_W+k_Z)^2 - m_W^2 + i\epsilon} \\
&\times \epsilon^*_{W\beta}(k_W,\lambda)\epsilon_Z^{*\beta}(k_Z,\sigma)(ik^\mu_{H^-} + ik^\mu_{H^{--}})(k_Z^\alpha - k_W^\alpha).
\end{aligned} \tag{226}
$$

Calculation of $\mathcal{M}_{CW^-}$

For the calculation of $\mathcal{M}_{CW^-}$ we need the Lagrangians

$$\mathcal{L}(H^{--}, H^+, W^+) = igH^{--}H^+W^+_\mu[ip^\mu_{H^-} + ip^\mu_{H^{--}}] \tag{227}$$

from (119) and

$$\mathcal{L}_C(W^-, W^+, Z) = igc_w W^+_\alpha Z_\beta W^{-\beta}[-p^\alpha_{W^-} - ip^\alpha_Z] \tag{228}$$

from (208).

The S-matrix element becomes now

$$
\begin{aligned}
S &= i^2 \int d^4x d^4y \langle 0 \mid a_W(\vec{k}_W,\lambda) a_{H^-}(\vec{k}_{H^-}) a_Z(\vec{k}_Z,\sigma)\mathcal{L}(H^{--},H^+,W^+) \\
&\times \mathcal{L}_C(W^-,W^+,Z) b^\dagger_{H^{--}}(\vec{k}_{H^{--}}) \mid 0\rangle \\
&= i^4 g^2 c_w \frac{1}{\sqrt{(2\pi)^{12} 16 E_{p_{H^{--}}} E_{p_{H^-}} E_{p_W} E_{p_Z}}} \int d^4x d^4y \langle 0 \mid W^+_\mu(x)[ik^\mu_{H^-} + ik^\mu_{H^{--}}] \\
&\times e^{-ik_{H^{--}}x} e^{ik_{H^-}x} W^-_\beta(y)[-iq^\alpha_W - ik^\alpha_Z] e^{ik_W y}\epsilon^*_{W\alpha}(k_W,\lambda) e^{ik_Z y}\epsilon^\beta_Z(k_Z,\sigma) \mid 0\rangle.
\end{aligned} \tag{229}
$$

Furthermore, we contract $W^+_\mu(x)$ and $W^-_\beta(y)$ and get the W^- propagator

$$\langle 0 \mid W^+_\mu(x) W^-_\beta(y) \mid 0\rangle = \int \frac{d^4q_W}{(2\pi)^4} \frac{-i}{q_W^2 - m_W^2 + i\epsilon}(g_{\mu\beta} - \frac{q_{W\mu}q_{W\beta}}{m_W^2}) e^{-iq_W(y-x)}. \tag{230}$$

The integration over the space-time gives

$$\int d^4x e^{ik_{H^-}x} e^{iq_W x} e^{-ik_{H^{--}}x} = (2\pi)\delta^{(4)}(k_{H^-} + q_W - k_{H^{--}}) \tag{231}$$

and

$$\int d^4y e^{ik_W y} e^{ik_Z y} e^{-iq_W y} = (2\pi)\delta^{(4)}(k_W + k_Z - q_W) \tag{232}$$

Now we integrate over d^4q_W and get

$$
\begin{aligned}
&\int \frac{d^4q_W}{(2\pi)^4}(2\pi)^4\delta^{(4)}(k_{H^-} + q_W - k_{H^{--}})(2\pi)^4\delta^{(4)}(k_W + k_Z - q_W) \\
&\times \frac{-i}{{q_W}^2 - m^2_{W^-} + i\epsilon}(g_{\mu\beta} - \frac{q_{W\mu}q_{W\beta}}{m_W^2})(-iq^\alpha_W - ik^\alpha_Z) \\
&= -i(2\pi)^4 \frac{\delta^{(4)}(k_W + k_Z + k_{H^-} - k_{H^{--}})}{(k_W+k_Z)^2 - m_W^2 + i\epsilon}(g_{\mu\beta} - \frac{(k_{W\mu}+k_{Z\mu})(k_{W\beta}+k_{Z\beta})}{m_W^2})(-ik^\alpha_W - 2ik^\alpha_Z)
\end{aligned} \tag{233}
$$

The S-matrix element becomes now

$$
\begin{aligned}
S &= i^4 g^2 c_w \frac{1}{\sqrt{(2\pi)^{12} 16 E_{p_{H^{--}}} E_{p_{H^-}} E_{p_W} E_{p_Z}}} (2\pi)^4 \frac{\delta^{(4)}(k_W + k_Z + k_{H^-} - k_{H^{--}})}{(k_W + k_Z)^2 - m_W^2 + i\epsilon} \\
&\times (-i)[g_{\mu\beta} - \frac{(k_{W\mu} + k_{Z\mu})(k_{W\beta} + k_{Z\beta})}{m_W^2}] \epsilon^*_{W\alpha}(k_W, \lambda) \epsilon^*_{Z\beta}(k_Z, \sigma)(-ik_W^\alpha - 2ik_Z^\alpha)[ik^\mu_{H^-} + ik^\mu_{H^{--}}]
\end{aligned}
\tag{234}
$$

Now we can extract the invariant matrix element with the method of (E1) and get

$$
\begin{aligned}
i\mathcal{M}_{CW^-} &= i^4 g^2 c_w \frac{-i}{(k_W + k_Z)^2 - m_W^2 + i\epsilon} [g_{\mu\beta} - \frac{(k_{W\mu} + k_{Z\mu})(k_{W\beta} + k_{Z\beta})}{m_W^2}] \\
&\times \epsilon^*_{W\alpha}(k_w, \lambda) \epsilon^{\beta *}_Z(k_Z, \sigma)(-ik_W^\alpha - 2ik_Z^\alpha)(ik^\mu_{H^-} + ik^\mu_{H^{--}}) \\
&= i^7 g^2 c_W \frac{[g_{\mu\beta} - \frac{(k_{W\mu} + k_{Z\mu})(k_{W\beta} + k_{Z\beta})}{m_W^2}]}{(k_W + k_Z)^2 - m_W^2 + i\epsilon} (k^\mu_{H^-} + k^\mu_{H^{--}})(k_W^\alpha + 2k_Z^\alpha) \epsilon^*_{W\alpha}(k_w, \lambda) \epsilon^{\beta *}_Z(k_Z, \sigma) \\
&= -ig^2 c_W \frac{[g_{\mu\beta} - \frac{(k_{W\mu} + k_{Z\mu})(k_{W\beta} + k_{Z\beta})}{m_W^2}]}{(k_W + k_Z)^2 - m_W^2 + i\epsilon} (k^\mu_{H^-} + k^\mu_{H^{--}})(k_W^\alpha + 2k_Z^\alpha) \epsilon^*_{W\alpha}(k_w, \lambda) \epsilon^{\beta *}_Z(k_Z, \sigma).
\end{aligned}
\tag{235}
$$

11.5 Calculation of $\Gamma(H^{--} \to H^- W^- Z^0)$ and $|\sum_i \mathcal{M}_i|^2$

In order to calculate the partial width $\Gamma(H^{--} \to H^- W^- Z)$ we have to square the sum of all contributing invariant matrix elements. The squared sum can be written as

$$
|\sum_i \mathcal{M}_i|^2 = \sum_i |\mathcal{M}_i|^2 + 2Re \sum_{i<j} \mathcal{M}_i \mathcal{M}_j^*, \tag{236}
$$

consisting of $6 + \binom{6}{2} = 6 + 15 = 21$ summands. Since most of the matrix elements consists of several summands, it can be estimated that expression in (236) contains over a hundred terms. Therefore the evaluation of $\Gamma(H^{--} \to H^- W^- Z)$ is very tedious. It will be evaluated numerically in section 11.6.

Nevertheless, the invariant matrix elements, which we have calculated before can be written the general form

$$
\mathcal{M}_i = A_i^{\alpha\beta} \epsilon^*_{Z\alpha}(k_Z, \sigma) \epsilon^*_{W\beta}(k_W, \lambda) \tag{237}
$$

where the coefficients $A_i^{\alpha\beta}$ have the general form

$$
A_i^{\alpha\beta} = a k_W^\alpha k_Z^\beta + b k_{H^-}^\alpha k_Z^\beta + c k_W^\alpha k_{H^-}^\beta + d k_{H^-}^\alpha k_{H^-}^\beta + e g^{\alpha\beta}. \tag{238}
$$

All other possible scalar products vanish since $\epsilon^*_{Z\beta}(k_Z, \lambda) k_Z^\beta = 0$ and $\epsilon^*_{W\beta}(k_W, \lambda) k_W^\beta = 0$. At next we will rewrite the matrix elements $\mathcal{M}_C$, $\mathcal{M}_{H^-}$, $\mathcal{M}_{H^{--}}$, $\mathcal{M}_{AW^-}$, $\mathcal{M}_{BW^-}$ and $\mathcal{M}_{CW^-}$ in the form of (237).
The matrix element of the contact term in (168) can be rewritten as

$$
\begin{aligned}
\mathcal{M}_C &= g^2 \frac{(-1 + 3s_w^2)}{c_w} \epsilon^*_{W\beta}(\vec{k}_W, \lambda) \epsilon^{*\beta}_Z(\vec{k}_Z, \sigma) = g^2 \frac{(-1 + 3s_w^2)}{c_w} g^{\alpha\beta} \epsilon^*_{Z\alpha}(\vec{k}_Z, \sigma) \epsilon^*_{W\beta}(\vec{k}_W, \lambda) \\
&= A_C^{\alpha\beta} \epsilon^*_{Z\alpha}(\vec{k}_Z, \sigma) \epsilon^*_{W\beta}(\vec{k}_W, \lambda).
\end{aligned}
\tag{239}
$$

At next we will rewrite the matrix element of the virtual H^- contribution in (188). Using $p_{H^-} = k_{H^-} + k_Z$ and $k_{H^{--}} = k_{H^-} + k_Z + k_W$, $\mathcal{M}_{H^-}$ can be rewritten as

$$\begin{aligned}\mathcal{M}_{H^-} &= -\frac{g^2 s_w^2}{c_w}\frac{\epsilon^*_{Z\alpha}(k_Z,\sigma)\epsilon^*_{W\beta}(k_W,\lambda)(2k^\beta_{H^-}+2k^\beta_Z+k^\beta_W)(2k^\alpha_{H^-}+k^\alpha_Z)}{(k_{H^-}+k_Z)^2-m^2_{H^-}+i\epsilon} \\ &= -\frac{g^2 s_w^2}{c_w}\frac{\epsilon^*_{Z\alpha}(k_Z,\sigma)\epsilon^*_{W\beta}(k_W,\lambda)}{(k_{H^-}+k_Z)^2-m^2_{H^-}+i\epsilon}[4k^\alpha_{H^-}k^\beta_{H^-}+4k^\alpha_{H^-}k^\beta_Z+k^\alpha_{H^-}k^\beta_W+2k^\alpha_Z k^\beta_{H^-}+2k^\alpha_Z k^\beta_Z+k^\alpha_Z k^\beta_W]\end{aligned} \tag{240}$$

As mentioned before some terms vanish since $\epsilon^*_{Z\beta}(k_Z,\lambda)k^\beta_Z = 0$ and $\epsilon^*_{W\beta}(k_W,\lambda)k^\beta_W = 0$. Therefore we get

$$\begin{aligned}\mathcal{M}_{H^-} &= -\frac{g^2 s_w^2}{c_w}\frac{\epsilon^*_{Z\alpha}(k_Z,\sigma)\epsilon^*_{W\beta}(k_W,\lambda)}{(k_{H^-}+k_Z)^2-m^2_{H^-}+i\epsilon}[4k^\alpha_{H^-}k^\beta_{H^-}+4k^\alpha_{H^-}k^\beta_Z] \\ &= -\frac{4g^2 s_w^2}{c_w}\frac{[k^\alpha_{H^-}k^\beta_{H^-}+k^\alpha_{H^-}k^\beta_Z]}{(k_{H^-}+k_Z)^2-m^2_{H^-}+i\epsilon}\epsilon^*_{Z\alpha}(k_Z,\sigma)\epsilon^*_{W\beta}(k_W,\lambda) = A^{\alpha\beta}_{H^-}\epsilon^*_{Z\alpha}(k_Z,\sigma)\epsilon^*_{W\beta}(k_W,\lambda)\end{aligned} \tag{241}$$

At next we will rewrite the matrix element of the virtual H^{--} contribution in (207). Using $p_{H^{--}} = k_{H^-} + k_W$ and $k_{H^{--}} = k_{H^-} + k_Z + k_W$ and leaving out all vanishing terms $\mathcal{M}_{H^{--}}$ can be rewritten as

$$\begin{aligned}\mathcal{M}_{H^{--}} &= \frac{g^2(2c_w^2-1)}{c_w}\frac{\epsilon^*_{Z\alpha}(k_Z,\sigma)\epsilon^*_{W\beta}(k_W,\lambda)}{(k_W+k_{H^-})^2-m^2_{H^{--}}+i\epsilon}(2k^\alpha_{H^-}+2k^\alpha_W+k^\alpha_Z)(2k^\beta_{H^-}+k^\beta_W) \\ &= \frac{g^2(2c_w^2-1)}{c_w}\frac{[4k^\alpha_{H^-}k^\beta_{H^-}+4k^\alpha_W k^\beta_{H^-}]}{(k_W+k_{H^-})^2-m^2_{H^{--}}+i\epsilon}\epsilon^*_{Z\alpha}(k_Z,\sigma)\epsilon^*_{W\beta}(k_W,\lambda) \\ &= \frac{4g^2(2c_w^2-1)}{c_w}\frac{[k^\alpha_{H^-}k^\beta_{H^-}+k^\alpha_W k^\beta_{H^-}]}{(k_W+k_{H^-})^2-m^2_{H^{--}}+i\epsilon}\epsilon^*_{Z\alpha}(k_Z,\sigma)\epsilon^*_{W\beta}(k_W,\lambda) \\ &= A^{\alpha\beta}_{H^{--}}\epsilon^*_{Z\alpha}(k_Z,\sigma)\epsilon^*_{W\beta}(k_W,\lambda)\end{aligned} \tag{242}$$

Now we will rewrite the remaining matrix elements $\mathcal{M}_{AW^-}$, $\mathcal{M}_{BW^-}$ and $\mathcal{M}_{CW^-}$. Starting with $\mathcal{M}_{AW^-}$ from (217) and using $k_{H^{--}} = k_{H^-} + k_W + k_Z$ we get

$$\begin{aligned}\mathcal{M}_{AW^-} &= g^2 c_w\frac{[g_{\mu\beta}-\frac{(k_{W\mu}+k_{Z\mu})(k_{W\beta}+k_{Z\beta})}{m_W^2}]}{(k_W+k_Z)^2-m_W^2+i\epsilon} \\ &\times \epsilon^*_{Z\alpha}(k_Z,\sigma)\epsilon^{*\beta}_W(k_W,\lambda)(2k^\mu_{H^-}+k^\mu_W+k^\mu_Z)(2k^\alpha_W+k^\alpha_Z) \\ &= g^2 c_w\frac{1}{(k_W+k_Z)^2-m_W^2+i\epsilon} \\ &\times [2k_{H^-\beta}+k_{W\beta}+k_{Z\beta}-\frac{2k_W k_{H^-}+k_W k_Z+m_W^2+2k_Z k_{H^-}+m_Z^2+k_Z k_W}{m_W^2}(k_{Z\beta}+k_{W\beta})] \\ &\times 2k^\alpha_W\epsilon^*_{Z\alpha}(k_Z,\sigma)\epsilon^{*\beta}_W(k_W,\lambda) \\ &= g^2 c_w\frac{1}{(k_W+k_Z)^2-m_W^2+i\epsilon}[2k_{H^-\beta}+(1-A)k_{Z\beta}]2k^\alpha_W\epsilon^*_{Z\alpha}(k_Z,\sigma)\epsilon^{*\beta}_W(k_W,\lambda) \\ &= 4g^2 c_w\frac{1}{(k_W+k_Z)^2-m_W^2+i\epsilon}[k^\alpha_W k^\beta_{H^-}+\frac{(1-A)}{4}k^\alpha_W k^\beta_Z]\epsilon^*_{Z\alpha}(k_Z,\sigma)\epsilon^*_{W\beta}(k_W,\lambda) \\ &= A^{\alpha\beta}_{AW^-}\epsilon^*_{Z\alpha}(k_Z,\sigma)\epsilon^*_{W\beta}(k_W,\lambda).\end{aligned} \tag{243}$$

Of course terms like $k_i^\alpha \epsilon_\alpha(k_i, \lambda)$ vanish. Furthermore, we introduced the useful variable A which is defined by

$$A = \frac{2k_Z k_{H^-} + 2k_W k_{H^-} + 2k_W k_Z + m_Z^2 + m_W^2}{m_W^2} \tag{244}$$

Let us now continue with $\mathcal{M}_{BW^-}$ from (226). Again we use $k_{H^{--}} = k_{H^-} + k_W + k_Z$ and we get

$$\begin{aligned}
\mathcal{M}_{BW^-} &= g^2 c_w \frac{1}{(k_W + k_Z)^2 - m_W^2 + i\epsilon}[g_{\mu\alpha} - \frac{(k_{Z\mu} + k_{W\mu})(k_{Z\alpha} + k_{W\alpha})}{m_W^2}] \\
&\times \epsilon^*_{W\beta}(k_W, \lambda)\epsilon_Z^{*\beta}(k_Z, \sigma)(2k^\mu_{H^-} + k^\mu_W + k^\mu_Z)(k^\alpha_Z - k^\alpha_W) \\
&= g^2 c_w \frac{1}{(k_W + k_Z)^2 - m_W^2 + i\epsilon} \\
&\times [2k_{H^-\alpha} + k_{W\alpha} + k_{Z\alpha} - A(k_{Z\alpha} + k_{W\alpha})](k^\alpha_Z - k^\alpha_W)\epsilon^*_{W\beta}(k_W, \lambda)\epsilon_Z^{*\beta}(k_Z, \sigma) \\
&= g^2 c_w \frac{1}{(k_W + k_Z)^2 - m_W^2 + i\epsilon} \\
&\times [2k_{H^-}k_Z + k_W k_Z + m_Z^2 - 2k_{H^-}k_W - m_W^2 - k_W k_Z - A(m_Z^2 - m_W^2)]\epsilon^*_{W\beta}(k_W, \lambda)\epsilon_Z^{*\beta}(k_Z, \sigma) \\
&= 2g^2 c_w \frac{1}{(k_W + k_Z)^2 - m_W^2 + i\epsilon} \\
&\times [k_{H^-}k_Z - k_{H^-}k_W + \frac{(1-A)}{2}(m_Z^2 - m_W^2)]g^{\alpha\beta}\epsilon^*_{Z\alpha}(k_Z, \sigma)\epsilon^*_{W\beta}(k_W, \lambda) \\
&= A^{\alpha\beta}_{BW^-}\epsilon^*_{Z\alpha}(k_Z, \sigma)\epsilon^*_{W\beta}(k_W, \lambda).
\end{aligned} \tag{245}$$

At last we will rewrite $\mathcal{M}_{CW^-}$ from (235). Using $k_{H^{--}} = k_{H^-} + k_W + k_Z$ we get

$$\begin{aligned}
\mathcal{M}_{CW^-} &= -g^2 c_w \frac{1}{(k_W + k_Z)^2 - m_W^2 + i\epsilon}[g_{\mu\beta} - \frac{(k_{W\mu} + k_{Z\mu})(k_{W\beta} + k_{Z\beta})}{m_W^2}] \\
&\times \epsilon^*_{W\alpha}(k_w, \lambda)\epsilon_Z^{*\beta}(k_Z, \sigma)(k^\alpha_W + 2k^\alpha_Z)(2k^\mu_{H^-} + k^\mu_Z + k^\mu_W) \\
&= -g^2 c_w \frac{1}{(k_W + k_Z)^2 - m_W^2 + i\epsilon} \\
&\times [2k_{H^-\beta} + k_{Z\beta} + k_{W\beta} - A(k_{W\beta} + k_{Z\beta})] \\
&\times 2k^\alpha_Z \epsilon^*_{W\alpha}(k_w, \lambda)\epsilon_Z^{*\beta}(k_Z, \sigma) \\
&= -g^2 c_w \frac{1}{(k_W + k_Z)^2 - m_W^2 + i\epsilon}[2k_{H^-\beta} + (1-A)k_{W\beta}]2k_{Z\alpha}\epsilon^*_{W\alpha}(k_w, \lambda)\epsilon_Z^{*\beta}(k_Z, \sigma) \\
&= -2g^2 c_w \frac{1}{(k_W + k_Z)^2 - m_W^2 + i\epsilon}[2k^\alpha_Z k^\beta_{H^-} + (1-A)k^\alpha_Z k^\beta_W]\epsilon^*_{W\alpha}(k_w, \lambda)\epsilon^*_{Z\beta}(k_Z, \sigma).
\end{aligned} \tag{246}$$

Finally we redefine the indices $\alpha \leftrightarrow \beta$ to be consistent with (237) and get

$$\begin{aligned}
\mathcal{M}_{CW^-} &= -4g^2 c_w \frac{1}{(k_W + k_Z)^2 - m_W^2 + i\epsilon}[k^\alpha_{H^-}k^\beta_Z + \frac{(1-A)}{2}k^\alpha_W k^\beta_Z]\epsilon^*_{Z\alpha}(k_Z, \sigma)\epsilon^*_{W\beta}(k_w, \lambda) \\
&= A^{\alpha\beta}_{CW^-}\epsilon^*_{Z\alpha}(k_Z, \sigma)\epsilon^*_{W\beta}(k_w, \lambda).
\end{aligned} \tag{247}$$

The calculation of $\Gamma(H^{--} \to H^- W^- Z^0)$ by using (236) would lead to hundreds of terms, which have to be integrated over the 3-body phase space. Since most of the terms contains H^--, H^{--}-

or W^--propagators, we get rational functions, whose antiderivative can not be written in terms of elementary functions. Furthermore, an approximation of the integral is not obvious because $m_{H^{--}}$ and the masses of the weak gauge bosons m_W and m_Z are on a different mass scale. Since the contraction of the polarization vectors $\epsilon_{Z\alpha}, \epsilon_{W\beta}$ yields terms proportional to $\frac{1}{m_W^2}$ or $\frac{1}{m_Z^2}$, it is not evident how to perform the limit $m_W, m_Z \to 0$. Therefore we have to evaluate occurring 3-body phase space integral numerically.

11.6 Nummeriacal evaluation of $\Gamma(H^{--} \to H^- W^- Z^0)$

For the numerical evaluation of $\Gamma(H^{--} \to H^- W^- Z^0)$ we have written the program "RAMBOC" (see appendix F) based on "RAMBO" (random momenta beautifully organized) written by R. Kleiss, W.J. Stirling and S.D. Ellis [35]. "RAMBO" is based on the Monte Carlo algorithm (MC), which relies on repeated random sampling to evaluate numerical problems. The basic idea of the program is that integration over the phase space can be replaced by a number of random choices of the integration variable. Random momenta are generated according to the phase-space distribution in the phase-space volume in (E4). Every event is given a weight factor, which is multiplied with the squared matrix element $|\mathcal{M}|^2 = |\mathcal{M}_C + \mathcal{M}_{H^-} + \mathcal{M}_{H^{--}} + \mathcal{M}_{AW} + \mathcal{M}_{BW} + \mathcal{M}_{CW}|^2$ of the decay. At last the mean of all values of the phase-space volumes is taken. The phase-space volume for this decay results in

$$U = U + \frac{SQ * W}{FLOAT(IC)}, \tag{248}$$

where U is the phase-space volume after a certain number of calls of "RAMBO", SQ the squared matrix element of the decay, W the weight factor of the event and $FLOAT(IC)$ the number of calls of the subroutine "RAMBO". In order to get $\Gamma(H^{--} \to H^- W^- Z^0)$ the result of (248) is divided by $2m_{H^{--}}$.

The program "RAMBOC" consists of several subroutines and functions. The subroutine "RAMBOS" reads in the mass $m_{H^{--}}$ of the decaying particle as well as the masses of the decay products m_{H^-}, m_W and m_Z. Random numbers are generated by the subroutine "RANMAR", which are used by "RAMBOS" together with m_{H^-}, m_W and m_Z to output random 4-momenta. These random momenta are given out in form of a 4×3 matrix $P(k_H, k_W, k_z)$. The corresponding entries of $P(k_H, k_W, k_z)$ are assigned to the 4-momenta of H^-, W^- and Z^0, viz. k_H, k_W and k_z. Furthermore, the subroutines "GETMC", "GETMH", "GETMH2", "GETMAW", "GETMBW" and "GETMCW" use the 4-momenta k_H, k_W, k_z to give out $\mathcal{M}_C$, $\mathcal{M}_{H^-}$, $\mathcal{M}_{H^{--}}$, $\mathcal{M}_{AW}$, $\mathcal{M}_{BW}$ and $\mathcal{M}_{CW}$, which are summed up and squared in the function "QUADS".

In the following we see $\Gamma(H^{--} \to H^- W^- Z^0)$ plotted against m_{H^-} for several values of $m_{H^{--}}$.

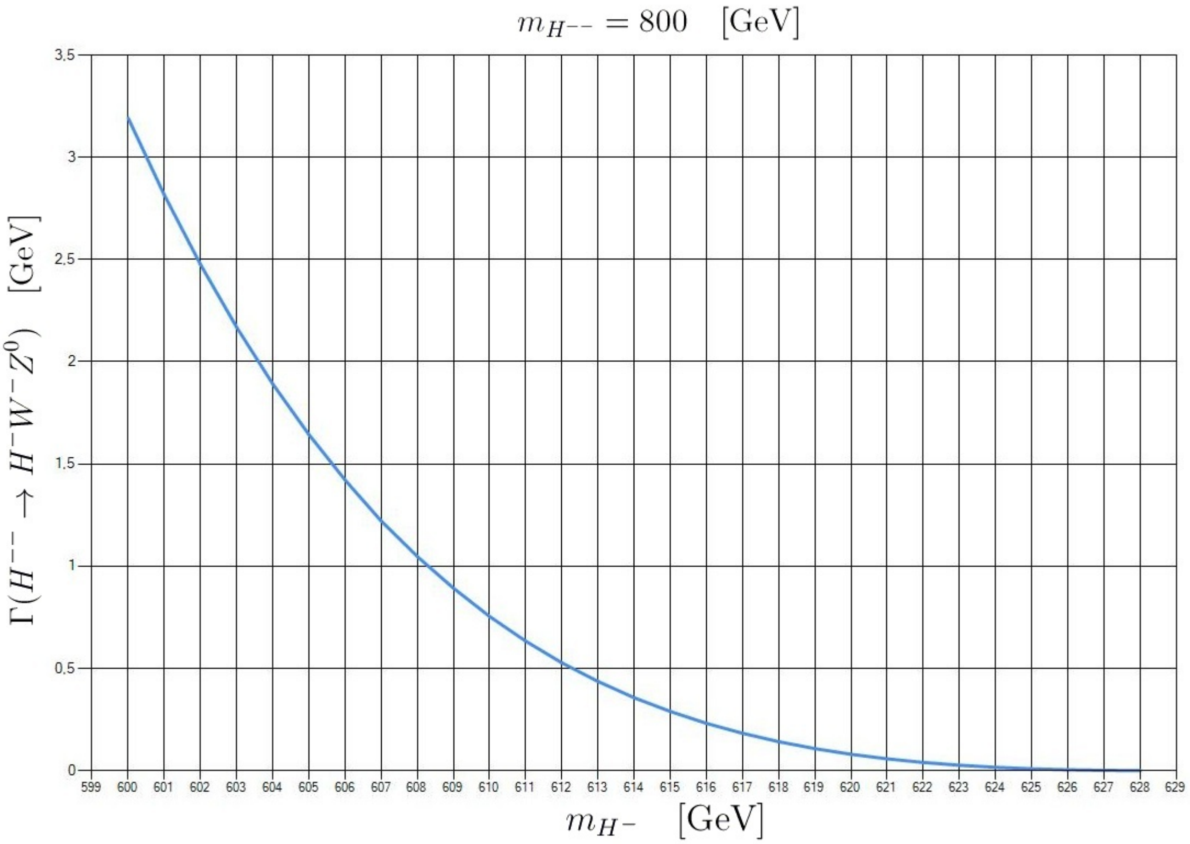

Figure 12: The partial decay width $\Gamma(H^{--} \to H^- W^- Z^0)$ of the doubly charged Higgs boson is plotted as a function of m_{H^-} for $m_{H^{--}} = 800$ GeV. As we see $\Gamma(H^{--} \to H^- W^- Z^0)$ strongly depends on the mass difference $m_{H^{--}} - m_{H^-}$. Therefore we restricted the range of m_{H^-} to the interval $[0.75 m_{H^{--}}, m_{H^{--}} - m_W - m_Z]$.

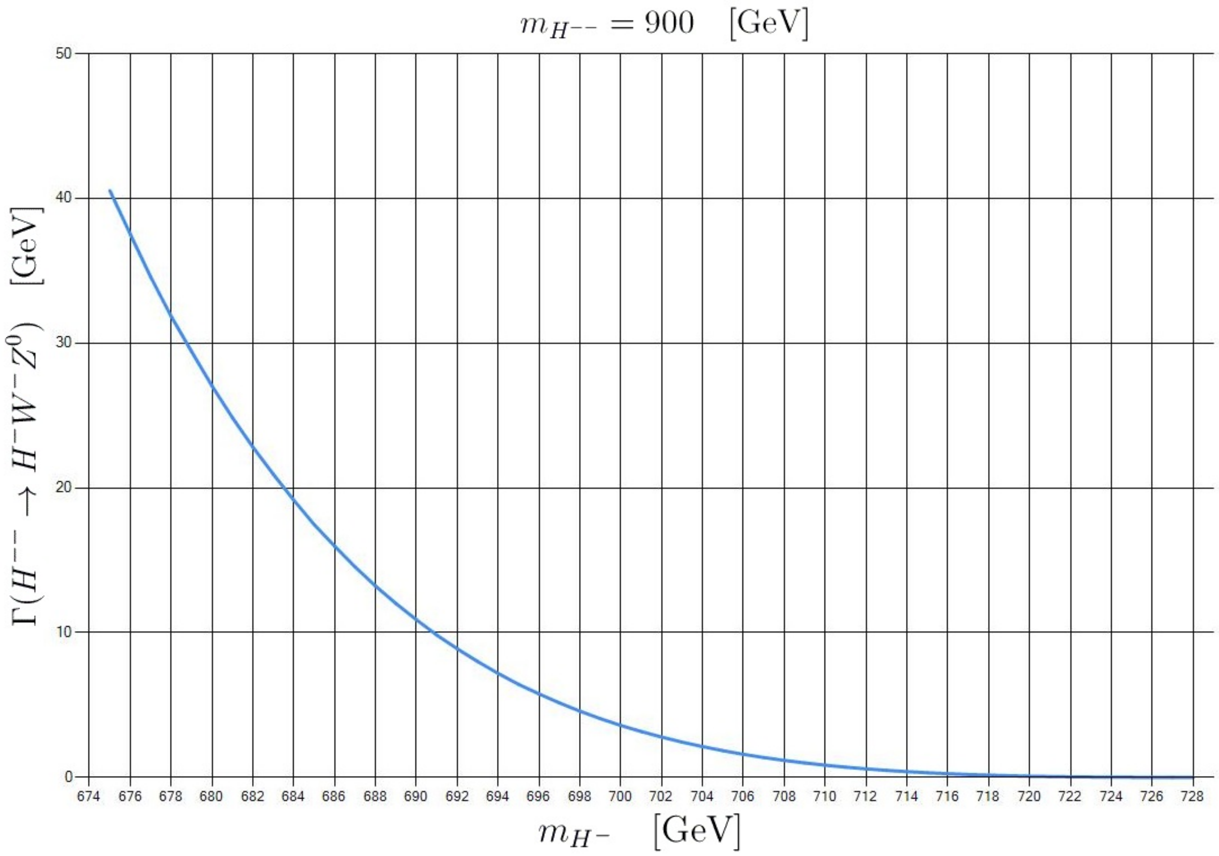

Figure 13: The partial decay width $\Gamma(H^{--} \to H^- W^- Z^0)$ of the doubly charged Higgs boson for is plotted as a function of m_{H^-} for $m_{H^{--}} = 900$ GeV. Like in the previous figure, $\Gamma(H^{--} \to H^- W^- Z^0)$ increases strongly with the mass difference $m_{H^{--}} - m_{H^-}$. The range of m_{H^-} is again restricted to the interval $[0.75 m_{H^{--}}, m_{H^{--}} - m_W - m_Z]$.

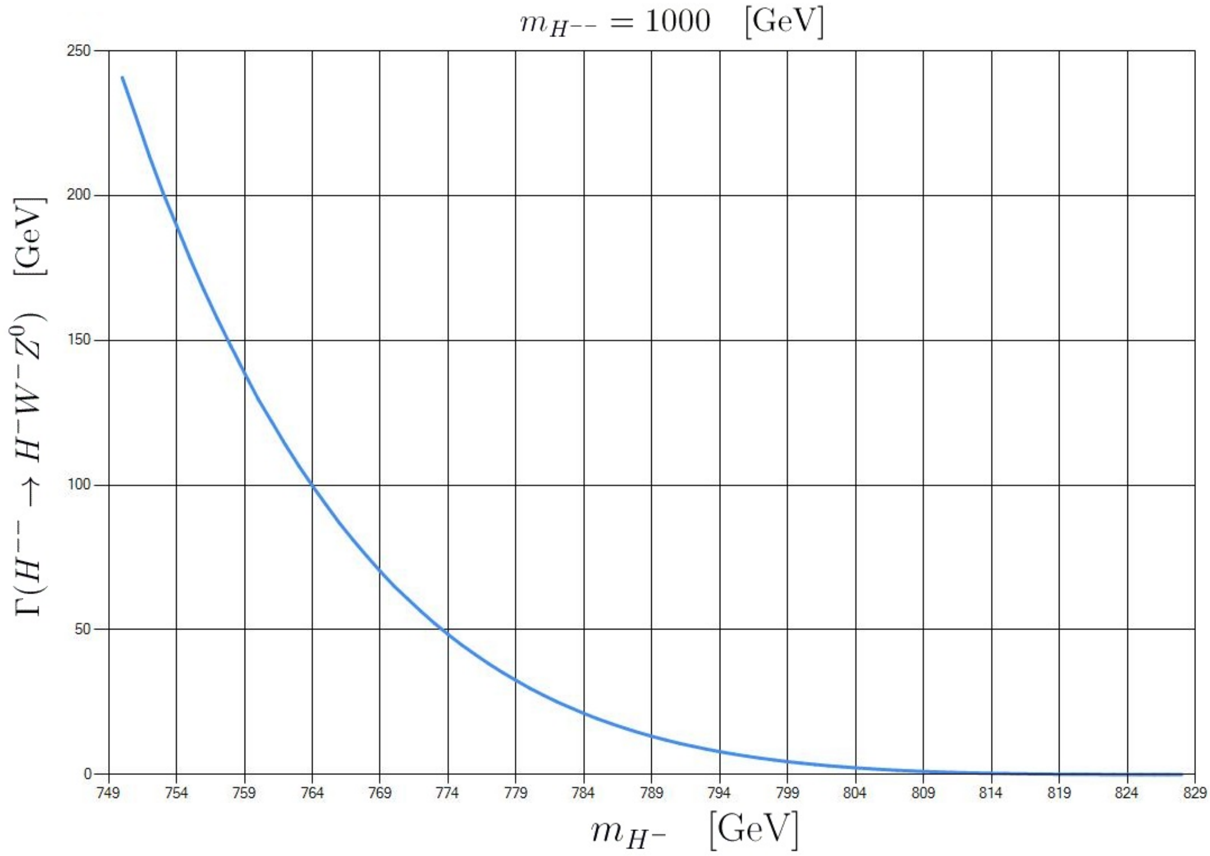

Figure 14: The partial decay width $\Gamma(H^{--} \to H^- W^- Z^0)$ of the doubly charged Higgs boson for is plotted as a function of m_{H^-} for $m_{H^{--}} = 1000$ GeV. Like in previous figures, $\Gamma(H^{--} \to H^- W^- Z^0)$ increases strongly with the mass difference $m_{H^{--}} - m_{H^-}$. The range of m_{H^-} is again restricted to the interval $[0.75 m_{H^{--}}, m_{H^{--}} - m_W - m_Z]$.

It should be mentioned that a value of a few GeV for $\Gamma(H^{--} \to H^- W^- Z^0)$ is expected. In order to keep the partial width realistically small the mass difference $m_{H^{--}} - m_{H^-}$ has to be very small, e.g. a few percent of $m_{H^{--}}$ or even smaller. Therefore we restrict $\Gamma(H^{--} \to H^- W^- Z^0) < 10$ GeV in the next plot for a few values of $m_{H^{--}}$.

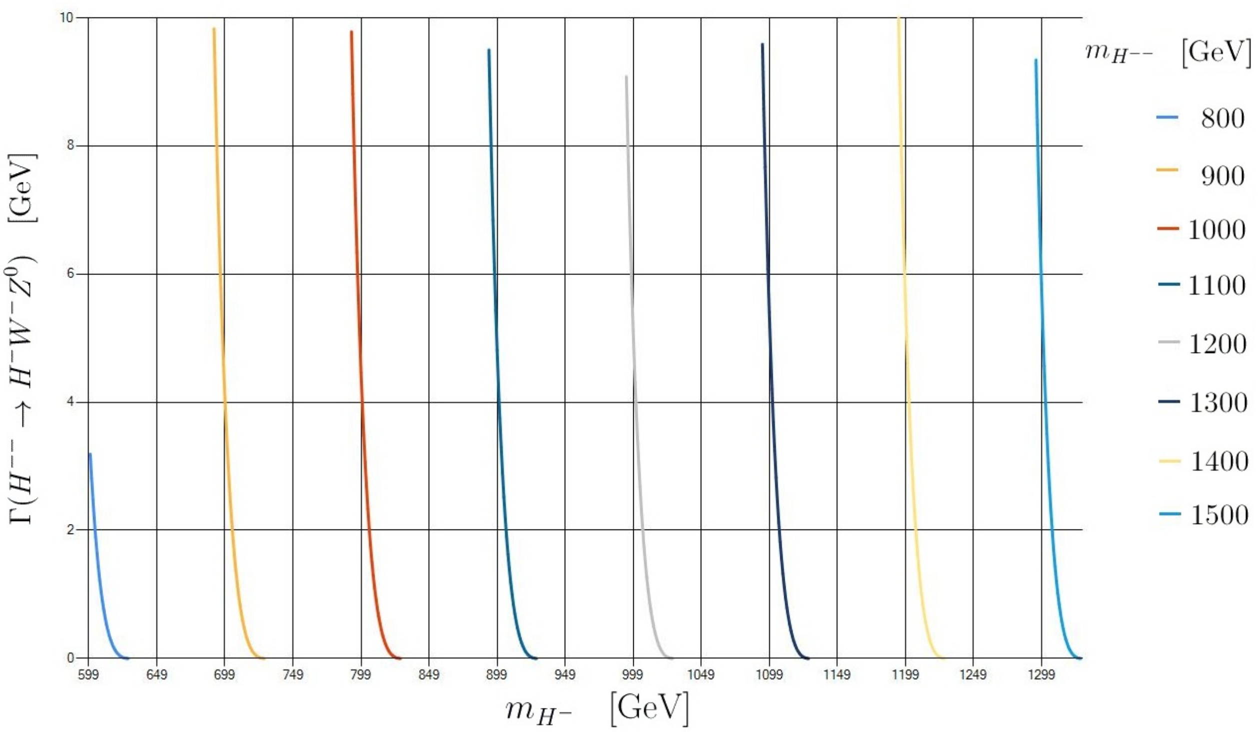

Figure 15: The partial decay width $\Gamma(H^{--} \to H^- W^- Z^0)$ of the doubly charged Higgs boson for is plotted as a function of m_{H^-} for $m_{H^{--}} = 800, 900, 1000, 1100, 1200, 1300, 1400, 1500$ GeV. $\Gamma(H^{--} \to H^- W^- Z^0) < 10$GeV was assumed. The range of m_{H^-} is again restricted to the interval $[0.75 m_{H^{--}}, m_{H^{--}} - m_W - m_Z]$ for each value of m_{H^-}.

12 Conclusions

In this work we studied the Higgs triplet extension of the SM, where the scalar sector was extended by a complex scalar triplet Δ with hypercharge $Y = 2$, in order to introduce neutrino masses. The additional term in the Yukawa Lagrangian $\mathcal{L}_Y$ in (3), which describes the coupling between the Higgs triplet and the lepton doublet induced a lepton number violating Majorana mass term for neutrinos. The Majorana mass term was found as $\frac{1}{2}\nu_L^T C^{-1}\mathcal{M}_\nu \nu_L$, where the neutrino mass matrix was given by $\mathcal{M}_\nu = v_T(f_{\alpha\beta})$, with v_T the VEV of the neutral component of the Higgs triplet and $f_{\alpha\beta}$ the complex and symmetric Yukawa coupling matrix.

The most general Higgs potential $V(\langle\phi\rangle_0, \langle\Delta\rangle_0)$ as a function of the VEVs of the Higgs doublet ϕ and the Higgs triplet Δ (see (12)) was minimized in order to gain relations between the VEVs and the parameters of the potential. Since the VEV of the Higgs triplet was expected to be much smaller then the VEV of the Higgs doublet, viz. $|v_T| \ll v$, in order to maintain the tree-level relation $M_W = M_Z \cos(\theta_W)$, we needed to perform fine-tuning by assuming $|t| \ll v$, where t was the coefficient of the t-term ($t\phi^\dagger\Delta\phi\ +$ H.c) in the Higgs potential in (10).

Furthermore, the question whether neutrinos have Dirac or Majorana nature arose. Besides the Higgs triplet model, most other SM extensions assume the neutrino to be a Majorana fermion. We reviewed Dirac and Majorana mass terms for neutrinos and the normal form theorem for both cases. Moreover, the topic of neutrino mixing and the Pontecorvo-Maki-Nakagawa-Sakata matrix U_{PMNS} was briefly covered.

In the main part of this work cross sections and decay widths of possible processes involving the doubly charged component H^{--} of the Higgs triplet were calculated. Whereas the extended Yukawa Lagrangian $\mathcal{L}_Y$ in (3) allowed lepton flavor violating processes like $e^-e^- \to \mu^-\mu^-$ and the generalization $\alpha^-\beta^- \to \gamma^-\delta^-$ (for $\alpha, \beta, \gamma, \delta = e, \mu, \tau$), respectively, the gauge coupling $\mathcal{L}_{\Delta gauge}$ of the Higss triplet in (61) permitted the so-called inverse neutrinoless double beta decay $e^-e^- \to W^-W^-$. The amplitude of the latter process was suppressed by v_T, i.e. $\sigma(e^-e^- \to W^-W^-) \propto |v_T|^2$.

Further on, decays of the doubly charged Higgs H^{--} were studied. The width of the decay into a like-sign lepton pair was found to be proportional to the squares of the elements of the Yukawa coupling matrices and the mass of the doubly charged Higgs, viz. $\Gamma(H^{--} \to \mu^-\mu^-) \propto |f_{\mu\mu}|^2 M_{H^{--}}$ and $\Gamma(H^{--} \to \gamma^-\delta^-) \propto |f_{\gamma\delta}|^2 M_{H^{--}}$, respectively. On the other hand, the width of the decay into a pair of W-bosons was suppressed by v_T, i.e. $\Gamma(H^{--} \to W^-W^-) \propto |v_T|^2$. Moreover, it turned out that the decay width $\Gamma(H^{--} \to W^-H^-)$ was increasing with $M_{H^{--}}^3$. Our rough estimation gave a range of a few GeV.

At last, the decay $H^{--} \to W^-H^-Z^0$ was studied. The four contributions of this process would have lead to a 3-body phase space integral over more than a hundred terms. Therefore we evaluated the width $\Gamma(H^{--} \to W^-H^-Z^0)$ numerically with the FORTRAN program "RAMBOC". The width was plotted against the allowed values of M_{H^-} for certain values of $M_{H^{--}}$ and showed a strong dependency on the mass difference $M_{H^{--}} - M_{H^-}$. In order to keep the width realistically small, for example $\Gamma(H^{--} \to W^-H^-Z^0) < 10$GeV, the mass difference $M_{H^{--}} - M_{H^-}$ must lie the range of approximately 170 to 210 GeV. (see Figure 15).

13 Appendix

A Field operators

Dirac fields

Charged spin 1/2 particles are described by Dirac field operator, which general expression is given by

$$\Psi(x) = \sum_s \int \frac{d^3k}{\sqrt{(2\pi)^3 2E}} [e^{-ikx} u(k,s) b(k,s) + e^{+ikx} v(k,s) d^\dagger(k,s)]. \tag{A1}$$

The adjoint operator is given by

$$\Psi^\dagger(x) = \sum_s \int \frac{d^3k}{\sqrt{(2\pi)^3 2E}} [e^{+ikx} u^\dagger(k,s) b^\dagger(k,s) + e^{-ikx} v^\dagger(k,s) d(k,s)]. \tag{A2}$$

Scalar fields

Charged Spin 0 particles are described by non-hermitian scalar field operators which general expression is given by

$$\Phi^-(x) = \frac{1}{\sqrt{(2\pi)^3 2E_p}} \int d^3p [a(p) e^{-ipx} + b^\dagger(p) e^{+ipx}]. \tag{A3}$$

The adjoint operator is given by

$$\Phi^+(x) = \frac{1}{\sqrt{(2\pi)^3 2E_p}} \int d^3p [a(p)^\dagger e^{+ipx} + b(p) e^{-ipx}]. \tag{A4}$$

Vector fields

Spin 1 particles are described by vector field operators which general expression is given by

$$W^-_\mu = \int \frac{d^3k}{\sqrt{(2\pi)^3 2E_k}} \sum_\lambda [a_{(-)}(\vec{k},\lambda) \epsilon_\mu(\vec{k},\lambda) e^{-ikx} + b^\dagger_{(-)}(\vec{k},\lambda) \epsilon^*_\mu(\vec{k},\lambda) e^{ikx}]. \tag{A5}$$

The adjoint operator is given by

$$W^+_\mu = \int \frac{d^3k}{\sqrt{(2\pi)^3 2E_k}} \sum_\lambda [a^\dagger_{(-)}(\vec{k},\lambda) \epsilon^*_\mu(\vec{k},\lambda) e^{ikx} + b_{(-)}(\vec{k},\lambda) \epsilon_\mu(\vec{k},\lambda) e^{-ikx}]. \tag{A6}$$

B Anticommutators and commutators

Anticommuators

The term $b_{e^-}(q_1,t_1)b_{e^-}(q_2,t_2)b^\dagger_{e^-}(p_1,s_1)b^\dagger_{e^-}(p_2,s_2)\mid 0\rangle$ in (42) can be rewritten by using the basic anticommutator relation for creation and annihilation operators, which is given by

$$\{b(q_1,t_1),b^\dagger(q_2,t_2)\}=\delta^{(3)}(\vec{q_1}-\vec{q_2})\delta_{t_1t_2} \tag{B1}$$

and can rewritten as

$$b(q_1,t_1)b^\dagger(q_2,t_2)=\delta^{(3)}(\vec{q_1}-\vec{q_2})\delta_{t_1t_2}-b^\dagger(q_2,t_2)b(q_1,t_1). \tag{B2}$$

We insert (B2) several times into the product of electron creation- and annihilation-operators of (42) and get

$$\begin{aligned}
&b_{e^-}(q_1,t_1)b_{e^-}(q_2,t_2)b^\dagger_{e^-}(p_1,s_1)b^\dagger_{e^-}(p_2,s_2)\mid 0\rangle\\
&=b_{e^-}(q_1,t_1)[\delta^{(3)}(\vec{q_2}-\vec{p_1})\delta_{t_2s_1}-b^\dagger_{e^-}(p_1,s_1)b_{e^-}(q_2,t_2)]b^\dagger_{e^-}(p_2,s_2)\mid 0\rangle\\
&=\delta^{(3)}(\vec{q_2}-\vec{p_1})\delta_{t_2s_1}b_{e^-}(q_1,t_1)b^\dagger_{e^-}(p_2,s_2)\mid 0\rangle\\
&-b_{e^-}(q_1,t_1)b^\dagger_{e^-}(p_1,s_1)b_{e^-}(q_2,t_2)b^\dagger_{e^-}(p_2,s_2)\mid 0\rangle\\
&=\delta^{(3)}(\vec{q_2}-\vec{p_1})\delta_{t_2s_1}[\delta^{(3)}(\vec{q_1}-\vec{p_2})\delta_{t_1s_2}-b^\dagger_{e^-}(p_2,s_2)b_{e^-}(q_1,t_1)]\mid 0\rangle\\
&-b_{e^-}(q_1,t_1)b^\dagger_{e^-}(p_1,s_1)[\delta^{(3)}(\vec{q_2}-\vec{p_2})\delta_{t_2s_2}-b^\dagger_{e^-}(p_2,s_2)b_{e^-}(q_2,t_2)]\mid 0\rangle\\
&=\delta^{(3)}(\vec{q_2}-\vec{p_1})\delta_{t_2s_1}\delta^{(3)}(\vec{q_1}-\vec{p_2})\delta_{t_1s_2}\mid 0\rangle\\
&-\delta^{(3)}(\vec{q_2}-\vec{p_1})\delta_{t_2s_1}b^\dagger_{e^-}(p_2,s_2)b_{e^-}(q_1,t_1)\mid 0\rangle\\
&-b_{e^-}(q_1,t_1)b^\dagger_{e^-}(p_1,s_1)\delta^{(3)}(\vec{q_2}-\vec{p_2})\delta_{t_2s_2}\mid 0\rangle\\
&+b_{e^-}(q_1,t_1)b^\dagger_{e^-}(p_1,s_1)b^\dagger_{e^-}(p_2,s_2)b_{e^-}(q_2,t_2)\mid 0\rangle\\
&=\delta^{(3)}(\vec{q_2}-\vec{p_1})\delta_{t_2s_1}\delta^{(3)}(\vec{q_1}-\vec{p_2})\delta_{t_1s_2}\mid 0\rangle\\
&-b_{e^-}(q_1,t_1)b^\dagger_{e^-}(p_1,s_1)\delta^{(3)}(\vec{q_2}-\vec{p_2})\delta_{t_2s_2}\mid 0\rangle\\
&=\delta^{(3)}(\vec{q_2}-\vec{p_1})\delta_{t_2s_1}\delta^{(3)}(\vec{q_1}-\vec{p_2})\delta_{t_1s_2}\mid 0\rangle\\
&-[\delta^{(3)}(\vec{q_1}-\vec{p_1})\delta_{t_1s_1}-b^\dagger_{e^-}(p_1,s_1)b_{e^-}(q_1,t_1)]\delta^{(3)}(\vec{q_2}-\vec{p_2})\delta_{t_2s_2}\mid 0\rangle\\
&=\delta^{(3)}(\vec{q_2}-\vec{p_1})\delta_{t_2s_1}\delta^{(3)}(\vec{q_1}-\vec{p_2})\delta_{t_1s_2}\mid 0\rangle-\delta^{(3)}(\vec{q_1}-\vec{p_1})\delta_{t_1s_1}\delta^{(3)}(\vec{q_2}-\vec{p_2})\delta_{t_2s_2}\mid 0\rangle\\
&=[\delta^{(3)}(\vec{q_2}-\vec{p_1})\delta_{t_2s_1}\delta^{(3)}(\vec{q_1}-\vec{p_2})\delta_{t_1s_2}-\delta^{(3)}(\vec{q_1}-\vec{p_1})\delta_{t_1s_1}\delta^{(3)}(\vec{q_2}-\vec{p_2})\delta_{t_2s_2}]\mid 0\rangle.
\end{aligned} \tag{B3}$$

Commuators

The term $a_{(-)}(\vec{k}_3,\lambda)a_{(-)}(\vec{k}_4,\sigma)a^\dagger_{(-)}(\vec{p}_1,\rho)a^\dagger_{(-)}(\vec{p}_2,\tau)\mid 0\rangle$ in (71) can be rewritten using the basic commutator relation for creation and annihilation operators, which is given by

$$[a(\vec{k},\lambda),a^\dagger(\vec{p},\rho)]=\delta^{(3)}(\vec{k}-\vec{p})\delta_{\lambda\rho} \tag{B4}$$

and can be rewritten as

$$a(\vec{k},\lambda)a^\dagger(\vec{p},\rho)=\delta^{(3)}(\vec{k}-\vec{p})\delta_{\lambda\rho}+a^\dagger(\vec{p},\rho)a(\vec{k},\lambda). \tag{B5}$$

We insert (B5) several times into the product of W-boson creation- and annihilation-operators of (71) and get in complete analogy to (B3)

$$\begin{aligned} & a_{(-)}(\vec{k_3},\lambda)a_{(-)}(\vec{k_4},\sigma)a^\dagger_{(-)}(\vec{p_1},\rho)a^\dagger_{(-)}(\vec{p_2},\tau) \mid 0\rangle \\ & = [\delta^{(3)}(\vec{k_4}-\vec{p_1})\delta_{\sigma\rho}\delta^{(3)}(\vec{k_3}-\vec{p_2})\delta_{\lambda\tau} + \delta^{(3)}(\vec{k_3}-\vec{p_1})\delta_{\lambda\rho}\delta^{(3)}(\vec{k_4}-\vec{p_2})\delta_{\sigma\tau}] \mid 0\rangle. \end{aligned} \tag{B6}$$

C Rewriting terms with u- and v-spinors

In this appendix we will rewrite the terms $u_L^T(q_1,t_1)C^{-1}u_L(q_2,t_2)$ and $u^\dagger(k_1,r_1)Cu^*(k_2,r_2)$ of (42). In order to get a better readability we will use the abbreviations $u_L^T(q_1,t_1)C^{-1}u_L(q_2,t_2) = u_{1L}^T C^{-1}u_{2L}$ and $u^\dagger(k_1,r_1)Cu^*(k_2,r_2) = u^\dagger_{1L}Cu^*_{2L}$. With the definitions of left-handed spinors, chiral projectors and the properties of γ_5 viz.

$$u_L = P_L u, \tag{C1}$$

$$P_L = \frac{\mathbb{1}-\gamma_5}{2}, \tag{C2}$$

$$C^{-1}\gamma_5 C = \gamma_5^T, \tag{C3}$$

$$CP_L^T C^{-1} = C\frac{\mathbb{1}-\gamma_5^T}{2}C^{-1} = \frac{\mathbb{1}-\gamma_5}{2} \tag{C4}$$

we can rewrite

$$\begin{aligned} & u_{1L}^T C^{-1}u_{2L} = u_1^T P_L^T C^{-1} P_L u_2 = u_1^T C^{-1} C P_L^T C^{-1} P_L u_2 = u_1^T C^{-1} P_L P_L u_2 = \\ & = u_1^T C^{-1} P_L u_2. \end{aligned} \tag{C5}$$

Since

$$C^{-1}\gamma_0 = C^{-1}\gamma_0 C C^{-1} = -\gamma_0^T C^{-1} \tag{C6}$$

and

$$u^T\gamma_0^* C^{-1}\gamma_0 = \bar{v} \tag{C7}$$

we can combine those relations and get

$$-u^T\gamma_0^*\gamma_0^T C^{-1} = \bar{v}. \tag{C8}$$

Since $\gamma_0^*\gamma_0^T = (\gamma_0^\dagger)^T\gamma_0^T = \gamma_0^T\gamma_0^T = \mathbb{1}$ we get the relation

$$-u^T C^{-1} = \bar{v}. \tag{C9}$$

When we insert (C9) into the last step of (C5) we get

$$u_{1L}^T C^{-1}u_{2L} = -v_1 P_L u_2. \tag{C10}$$

Now let us do the analogous procedure for the term $u^\dagger(k_1,r_1)Cu^*(k_2,r_2)$ in (42) abbreviated as $u^\dagger_{1L}Cu^*_{2L}$.

$$\begin{aligned} u^\dagger_{1L}Cu^*_{2L} &= ((u^\dagger_{1L}Cu^*_{2L})^*)^* = (u^T_{1L}C^*u_{2L})^* = (u^T_1 P_L C^* P_L u_2)^* \\ &= (u^T_1 C^{-1}CP^T_L(-C^{-1})P_L u_2)^* = (-u^T_1 C^{-1}P_L P_L u_2)^* = (-u^T_1 C^{-1}P_L u_2)^* \end{aligned} \tag{C11}$$

In the fourth step we used $C^* = -C^{-1}$ and in the fifth step we used $P_L = \frac{\mathbb{1}-\gamma_5}{2} = C\frac{\mathbb{1}-\gamma_5^T}{2}C^{-1}$. Using (C9) in the last step of (C11) we get the relation

$$u^\dagger_{1L}Cu^*_{2L} = (-u^T_1 C^{-1}P_L u_2)^* = (\bar{v}_1 P_L u_2)^*. \tag{C12}$$

Finally we get

$$\begin{aligned} &u^\dagger_{1L}Cu^*_{2L} = ((u^\dagger_{1L}Cu^*_{2L})^*)^* = (\bar{v}_1 P_L u_2)^* = (\bar{v}_1 P_L u_2)^\dagger = u^\dagger_2 P_L \gamma_0 v_1 \\ =&\bar{u}_2 P_R v_1. \end{aligned} \tag{C13}$$

In the third step we used the fact that $(\bar{v}_1 P_L u_2)^*$ is a $(1\times 4)(4\times 4)(4\times 1) = 1\times 1$ matrix respectively a number. Since $(number)^* = (number)^\dagger$ we can write a $\dagger$ instead of $*$.

D The calculation of the two-body phase space integral

For the calculation of total cross sections of two body scattering processes, the so called two-body phase space integral is needed, which is given by the following expression.

$$\begin{aligned} I_2 &= \int \frac{d^3k_1}{2k_1^0}\frac{d^3k_2}{2k_1^0}\delta^{(4)}(P-k_1-k_2)f(k_1\cdot k_2) \\ &= \int \frac{d^3k_1}{2k_1^0}\frac{d^3k_2}{2k_1^0}\delta(E-k_1^0-k_2^0)\delta^{(3)}(-\vec{P}+\vec{k_1}+\vec{k_2})f(k_1\cdot k_2). \end{aligned} \tag{D1}$$

Where P denotes a timelike 4-vector. In the rest frame this extension becomes

$$\begin{aligned} I_2 &= \int \frac{d^3k_1}{2k_1^0}\frac{d^3k_2}{2k_1^0}\delta(M-k_1^0-k_2^0)\delta^{(3)}(\vec{k_1}+\vec{k_2})f(k_1\cdot k_2) \\ &= \int \frac{d^3k_1}{2k_1^0 2k_2^0}\delta(M-k_1^0-k_2^0)f(Q). \end{aligned} \tag{D2}$$

Since $\vec{k}_2 = -\vec{k}_1$ the relation $(k_2^0)^2 = \vec{k}_1^2 + m_2^2$ is valid. With the volume element in spherical coordinates $d^3k_1 = k_1^2 dk_1 d\Omega$ our integral of (D2) can be rewritten as

$$I_2 = \int_0^{4\pi} d\Omega f(Q)\int_0^\infty \frac{dk_1}{4k_1^0k_2^0}k_1^2\delta\Big(M-\sqrt{\vec{k}_1^2+m_1^2}-\sqrt{\vec{k}_2^2+m_2^2}\Big). \tag{D3}$$

As we have a delta distribution of functions in the integrand we have use the well known formula

$$\delta(f(x)) = \frac{1}{|f'(x)|}\delta(x-x_0). \tag{D4}$$

Now we have to calculate the the zeros of the delta distribution's argument. We can again use $\vec{k}_2 = -\vec{k}_1$ and the delta distribution's argument of (D3) becomes

$$M - \sqrt{\vec{k}_1^2 + m_1^2} - \sqrt{\vec{k}_1^2 + m_2^2} = 0. \tag{D5}$$

Adding $\sqrt{\vec{k}_1^2 + m_2^2}$ to both sides of the equation and taking the square gives

$$M^2 - 2M\sqrt{\vec{k}_1^2 + m_1^2} + \vec{k}_1^2 + m_1^2 = \vec{k}_1^2 + m_2^2, \tag{D6}$$

which can be simplified to

$$M^2 - 2M\sqrt{\vec{k}_1^2 + m_1^2} + m_1^2 = m_2^2. \tag{D7}$$

After adding $2M\sqrt{\vec{k}_1^2 + m_1^2}$ and subtracting m_2^2 from both sides of the equation, we take the square and get

$$(M^2 + m_1^2 - m_2^2)^2 = 4M^2(\vec{k}_1^2 + m_1^2). \tag{D8}$$

Expanding the left side of the equation and dividing both sides by $4M^2$ gives

$$\frac{M^4 + 2M^2(m_1^2 - m_2^2) + (m_1^2 - m_2^2)^2}{4M^2} = (\vec{k}_1^2 + m_1^2). \tag{D9}$$

We subtract m_1^2 on both sides and reduce the terms of the left side to a common denominator and get

$$\frac{M^4 + 2M^2(m_1^2 - m_2^2) + (m_1^2 - m_2^2)^2 - 4M^2 m_1^2}{4M^2} = \vec{k}_1^2, \tag{D10}$$

which can be simplified to

$$\frac{M^4 - 2M^2(m_1^2 + m_2^2) + (m_1^2 - m_2^2)^2}{4M^2} = \vec{k}_1^2. \tag{D11}$$

Taking the square root of both sides gives us the zero of the delta distribution's argument in (D3)

$$k_1 = \sqrt{\frac{M^4 - 2M^2(m_1^2 + m_2^2) + (m_1^2 - m_2^2)^2}{4M^2}} = p_0. \tag{D12}$$

The derivative gives

$$\frac{1}{|f'(x)|} = \frac{k_1^0(p_0)k_2^0(p_0)}{p_0(k_1^0(p_0) + k_2^0(p_0))} = \frac{k_1^0(p_0)k_2^0(p_0)}{p_0 M}. \tag{D13}$$

With (D12) and (D13) the integral of (D3) becomes

$$I_2 = \int \frac{dk_1}{4k_1^0 k_2^0} \frac{k_1^0(p_0)k_2^0(p_0)}{p_0 M} k_1^2 \delta(k_1 - p_0) 4\pi f(Q)$$

$$= \frac{\pi}{2M^2}\sqrt{M^4 - 2M^2(m_1^2 + m_2^2) + (m_1^2 - m_2^2)^2} f(Q). \tag{D14}$$

This result can be rewritten with the familiar function $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + xz + yz)$ with $x = M^2$, $y = m_1^2$, $z = m_2^2$:

$$I_2 = \frac{\pi}{2M^2} f(Q)\sqrt{\lambda(x, y, z)}. \tag{D15}$$

E Total cross section and width

Total cross section

Once the S-matrix element is calculated the invariant matrix element $\mathcal{M}$ is extracted by the following method [16]:
The S-Matrix: $S = \mathbb{1} + iT$
The matrixelement $\mathcal{M}$:

$$\begin{aligned} &\langle \vec{k_1}, \ldots, \vec{k_n} \mid iT \mid \vec{p_1}, \ldots, \vec{p_n} \rangle = i(2\pi)^4 \delta^{(4)}(p_1 + p_2 - \sum_{j=1}^{n} k_j) \\ &\times \frac{1}{(2\pi)^{\frac{3}{2}} \sqrt{2p_1^0}} \times \frac{1}{(2\pi)^{\frac{3}{2}} \sqrt{2p_2^0}} \times \prod_{j=1}^{n} \frac{1}{(2\pi)^{\frac{3}{2}} \sqrt{2k_j^0}} \times \mathcal{M}(\vec{k_1}, \ldots, \vec{k_n} \leftarrow \vec{p_1}, \vec{p_2}) \end{aligned} \tag{E1}$$

The cross section for $\vec{p_1} \parallel \vec{p_2}$ is now calculated as

$$\begin{aligned} &\sigma(B \leftarrow \vec{p_1}, \vec{p_2}) = \frac{\mathcal{S}}{4\sqrt{(p_1.p_2)^2 - m_1^2 m_2^2}} \int_B \frac{d^3k_1}{(2\pi)^3 2k_1^0} \cdots \frac{d^3k_n}{(2\pi)^3 2k_n^0} \\ &(2\pi)^4 \delta^{(4)}(p_1 + p_2 - \sum_{j=1}^{n} k_j) |\mathcal{M}|^2 \end{aligned} \tag{E2}$$

With the statistical factor $\mathcal{S}$:
$\sum_k m_k = n$ for m_k identical particles in the final state

$$\mathcal{S} = \prod_k \frac{1}{m_k!} \tag{E3}$$

Width

The width is given by [16]

$$\Gamma = \frac{1}{2m_A} \int (\prod_f \frac{d^3p_f}{(2\pi)^3 2E_f}) |\mathcal{M}(m_A \to \{p_f\})|^2 (2\pi)^4 \delta^{(4)}(P_A - \sum_f p_f). \tag{E4}$$

F Source code of RAMBOC

In this appendix the source code of the FORTRAN program "RAMBOC" which was used for the numerical evaluation of $\Gamma(H^{--} \to H^- W^- Z)$ is shown. "RAMBOC" is based on the program "RAMBO" (random momenta booster) [35].

```
      SUBROUTINE INITRN(IJK)
        integer IJK
        ASSEMBLY_INTERFACE(NAME="INITRN")
        CALL RMARIN(IJK,0,0)
      END INITRN

!
   ************************************************************************

      SUBROUTINE RAMBOS(N,ET,XM,P,WT)
!-----------------------------------------------------------------------
!
!                          RAMBO
!
!     RA(NDOM)  M(OMENTA)  B(EAUTIFULLY)  O(RGANIZED)
!
!     A DEMOCRATIC MULTI-PARTICLE PHASE SPACE GENERATOR
!     AUTHORS:  S.D. ELLIS,  R. KLEISS,  W.J. STIRLING
!     THIS IS VERSION 1.0 -  WRITTEN BY R. KLEISS
!
!     N  = NUMBER OF PARTICLES (>1, IN THIS VERSION <101)
!     ET = TOTAL CENTRE-OF-MASS ENERGY
!     XM = PARTICLE MASSES ( DIM=100 )
!     P  = PARTICLE MOMENTA ( DIM=(4,100) )
!     WT = WEIGHT OF THE EVENT
!
!-----------------------------------------------------------------------
      DIMENSION VEC(10)
       real (kind=1) XM(0:)
       real (kind=2) P(0:,0:)
       real*8, INTENT(OUT) :: WT
       !DATA ACC/1.E-14/,ITMAX/6/,IBEGIN/0/,IWARN/5*0/
       real*8 ACC
       integer ITMAX,IBEGIN,IWARN,DEB
      DIMENSION Q(4,100),Z(100),R(4),B(3),P2(100),XM2(100),E(100),V
         (100),IWARN(5)
      ASSEMBLY_INTERFACE(NAME="RAMBOS")
      ACC = 1.E-14
      ITMAX = 6
      IBEGIN = 0
      DEB = 0
```

```
! INITIALIZATION STEP: FACTORIALS FOR THE PHASE SPACE WEIGHT
      IF (DEB == 1) THEN
       write(*,*) "ET: ",ET
       write(*,*) "XM1: ",XM(1)
       write(*,*) "XM2: ",XM(2)
       write(*,*) "XM3: ",XM(3)
      END IF
      IF(IBEGIN.NE.0) GOTO 103
      IBEGIN=1
      TWOPI=8.*ATAN(1.)
      PO2LOG=ALOG(TWOPI/4.)
      Z(2)=PO2LOG
      DO 101 K=3,100
  101 Z(K)=Z(K-1)+PO2LOG-2.*ALOG(FLOAT(K-2))
      DO 102 K=3,100
  102 Z(K)=(Z(K)-ALOG(FLOAT(K-1)))
!
! CHECK ON THE NUMBER OF PARTICLES
  103 IF(N.GT.1.AND.N.LT.101) GOTO 104
      WRITE(2,1001)N
      STOP
!
! CHECK WHETHER TOTAL ENERGY IS SUFFICIENT; COUNT NONZERO MASSES
  104 XMT=0.
      NM=0
      DO 105 I=1,N
      IF(XM(I).NE.0.) NM=NM+1
  105 XMT=XMT+ABS(XM(I))
      IF(XMT.LE.ET) GOTO 201
      WRITE(2,1002)XMT,ET
      STOP
!
! THE PARAMETER VALUES ARE NOW ACCEPTED
!
! GENERATE N MASSLESS MOMENTA IN INFINITE PHASE SPACE
  201 DO 202 I=1,N
      CALL RANMAR(VEC,8)
      C=2.*VEC(1)-1.
      S=SQRT(1.-C*C)
      F=TWOPI*VEC(2)
      Q(4,I)=-ALOG(VEC(3)*VEC(4))
      Q(3,I)=Q(4,I)*C
      Q(2,I)=Q(4,I)*S*COS(F)
  202 Q(1,I)=Q(4,I)*S*SIN(F)
!
! CALCULATE THE PARAMETERS OF THE CONFORMAL TRANSFORMATION
```

```
      DO 203 I=1,4
  203 R(I)=0.
      DO 204 I=1,N
      DO 204 K=1,4
  204 R(K)=R(K)+Q(K,I)
      RMAS=SQRT(R(4)**2-R(3)**2-R(2)**2-R(1)**2)
      DO 205 K=1,3
  205 B(K)=-R(K)/RMAS
      G=R(4)/RMAS
      A=1./(1.+G)
      X=ET/RMAS
!
! TRANSFORM THE Q'S CONFORMALLY INTO THE P'S
      DO 207 I=1,N
      BQ=B(1)*Q(1,I)+B(2)*Q(2,I)+B(3)*Q(3,I)
      DO 206 K=1,3
  206 P(K,I)=X*(Q(K,I)+B(K)*(Q(4,I)+A*BQ))
  207 P(4,I)=X*(G*Q(4,I)+BQ)
!
! CALCULATE WEIGHT AND POSSIBLE WARNINGS
      WT=PO2LOG
      IF(N.NE.2) WT=(2.*N-4.)*ALOG(ET)+Z(N)
      IF(WT.GE.-180.) GOTO 208
      IF(IWARN(1).LE.5) WRITE(2,1004)WT
      IWARN(1)=IWARN(1)+1
  208 IF(WT.LE. 174.) GOTO 209
      IF(IWARN(2).LE.5) WRITE(2,1005)WT
      IWARN(2)=IWARN(2)+1
!
! RETURN FOR WEIGHTED MASSLESS MOMENTA
  209 IF(NM.NE.0) GOTO 210
      WT=EXP(WT)
      RETURN
!
! MASSIVE PARTICLES: RESCALE THE MOMENTA BY A FACTOR X
  210 XMAX=SQRT(1.-(XMT/ET)**2)
      DO 301 I=1,N
      XM2(I)=XM(I)**2
  301 P2(I)=P(4,I)**2
      ITER=0
      X=XMAX
      ACCU=ET*ACC
  302 F0=-ET
      G0=0.
      X2=X*X
      DO 303 I=1,N
```

```
      E(I)=SQRT(XM2(I)+X2*P2(I))
      F0=F0+E(I)
  303 G0=G0+P2(I)/E(I)
      IF(ABS(F0).LE.ACCU) GOTO 305
      ITER=ITER+1
      IF(ITER.LE.ITMAX) GOTO 304
      IF(DEB.NE.0)WRITE(2,1006)ITMAX,I,ACCU
      GOTO 305
  304 X=X-F0/(X*G0)
      GOTO 302
  305 DO 307 I=1,N
      V(I)=X*P(4,I)
      DO 306 K=1,3
  306 P(K,I)=X*P(K,I)
  307 P(4,I)=E(I)
      IF (DEB == 1) THEN
      write(*,*) "-------- P ---------- "
      write(*,*) P(1,1)," - ",P(1,2)," - ", P(1,3)
      write(*,*) P(2,1)," - ",P(2,2)," - ", P(2,3)
      write(*,*) P(3,1)," - ",P(3,2)," - ", P(3,3)
      write(*,*) P(4,1)," - ",P(4,2)," - ", P(4,3)
      END IF
!
! CALCULATE THE MASS-EFFECT WEIGHT FACTOR
      WT2=1.
      WT3=0.
      DO 308 I=1,N
      WT2=WT2*V(I)/E(I)
  308 WT3=WT3+V(I)**2/E(I)
      WTM=(2.*N-3.)*ALOG(X)+ALOG(WT2/WT3*ET)
!
! RETURN FOR  WEIGHTED MASSIVE MOMENTA
      WT=WT+WTM
      IF (DEB == 1) THEN
      write(*,*) "wt: ", WT
      end if
      IF(WT.GE.-180.) GOTO 309
      IF(IWARN(3).LE.5)WRITE(2,1004)WT
      IWARN(3)=IWARN(3)+1
  309 IF(WT.LE. 174.) GOTO 310
      IF(IWARN(4).LE.5) WRITE(2,1005)WT
      IWARN(4)=IWARN(4)+1
  310 WT=EXP(WT)
      RETURN
!
 1001 FORMAT(' RAMBO FAILS: # OF PARTICLES =',I5,' IS NOT ALLOWED')
```

```
1002 FORMAT(' RAMBO FAILS: TOTAL MASS = ',E15.6,' IS NOT',' SMALLER 
   THAN TOTAL ENERGY = ',E15.6)
1004 FORMAT(' RAMBO WARNS: WEIGHT = EXP(',F20.9,') MAY UNDERFLOW')
1005 FORMAT(' RAMBO WARNS: WEIGHT = EXP(',F20.9,') MAY  OVERFLOW')
1006 FORMAT(' RAMBO WARNS: ',I3,' ITERATIONS (',I3,') DIDNT GIVE THE
  ',' DESIRED ACCURACY = ',E15.6)
     END
     SUBROUTINE RANMAR(RVEC,LENV)
! UNIVERSAL RANDOM NUMBER GENERATOR PROPOSED BY MARSAGLIA AND ZAMAN
! IN REPORT FSU-SCRI-87-50
!          MODIFIED BY F. JAMES, 1988 AND 1989, TO GENERATE A VECTOR
!          OF PSEUDORANDOM NUMBERS RVEC OF LENGTH LENV, AND TO PUT IN
!          THE COMMON BLOCK EVERYTHING NEEDED TO SPECIFY CURRRENT
   STATE,
!          AND TO ADD INPUT AND OUTPUT ENTRY POINTS RMARIN, RMARUT.
!!!
   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++

!!!  CALLING SEQUENCES FOR RANMAR:
   ++
!!!       CALL RANMAR (RVEC, LEN)   RETURNS A VECTOR RVEC OF LEN
   ++
!!!                     32-BIT RANDOM FLOATING POINT NUMBERS BETWEEN
   ++
!!!                     ZERO AND ONE.
   ++
!!!       CALL RMARIN(I1,N1,N2)   INITIALIZES THE GENERATOR FROM ONE
   ++
!!!                     32-BIT INTEGER I1, AND NUMBER COUNTS N1,N2
   ++
!!!                    (FOR INITIALIZING, SET N1=N2=0, BUT TO RESTART
   ++
!!!                      A PREVIOUSLY GENERATED SEQUENCE, USE VALUES
   ++
!!!                      OUTPUT BY RMARUT)
   ++
!!!       CALL RMARUT(I1,N1,N2)   OUTPUTS THE VALUE OF THE ORIGINAL
   ++
!!!                    SEED AND THE TWO NUMBER COUNTS, TO BE USED
   ++
!!!                    FOR RESTARTING BY INITIALIZING TO I1 AND
   ++
!!!                    SKIPPING N2*100000000+N1 NUMBERS.
   ++
!!!
   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
```

```
      DIMENSION RVEC(*)
      COMMON/RASET1/U(97),C,I97,J97
      PARAMETER (MODCNS=1000000000)
      SAVE CD, CM, TWOM24, NTOT, NTOT2, IJKL
      DATA NTOT,NTOT2,IJKL/-1,0,0/
!
      IF (NTOT .GE. 0) GO TO 50
!
!         DEFAULT INITIALIZATION. USER HAS CALLED RANMAR WITHOUT
   RMARIN.
      IJKL = 54217137
      NTOT = 0
      NTOT2 = 0
      KALLED = 0
      GO TO 1
!
      ENTRY      RMARIN(IJKLIN, NTOTIN,NTOT2N)
!          INITIALIZING ROUTINE FOR RANMAR, MAY BE CALLED BEFORE
!          GENERATING PSEUDORANDOM NUMBERS WITH RANMAR. THE INPUT
!          VALUES SHOULD BE IN THE RANGES:  0<=IJKLIN<=900 OOO OOO
!                                           0<=NTOTIN<=999 999 999
!                                           0<=NTOT2N<<999 999 999!
! TO GET THE STANDARD VALUES IN MARSAGLIA'S PAPER, IJKLIN=54217137
!                                            NTOTIN,NTOT2N=0
      IJKL = IJKLIN
      NTOT = MAX(NTOTIN,0)
      NTOT2= MAX(NTOT2N,0)
      KALLED = 1
!          ALWAYS COME HERE TO INITIALIZE
    1 CONTINUE
      IJ = IJKL/30082
      KL = IJKL - 30082*IJ
      I = MOD(IJ/177, 177) + 2
      J = MOD(IJ, 177)     + 2
      K = MOD(KL/169, 178) + 1
      L = MOD(KL, 169)
      WRITE(2,'(A,I10,2X,2I10)') ' RANMAR INITIALIZED:',IJKL,NTOT,
         NTOT2
!        PRINT '(A,4I10)', '    I,J,K,L= ',I,J,K,L
      DO 2 II= 1, 97
      S = 0.
      T = .5
      DO 3 JJ= 1, 24
         M = MOD(MOD(I*J,179)*K, 179)
         I = J
```

```
          J = K
          K = M
          L = MOD(53*L+1, 169)
          IF (MOD(L*M,64) .GE. 32) S = S+T
    3     T = 0.5*T
    2 U(II) = S
      TWOM24 = 1.0
      DO 4 I24= 1, 24
    4 TWOM24 = 0.5*TWOM24
      C  =    362436.*TWOM24
      CD =   7654321.*TWOM24
      CM =  16777213.*TWOM24
      I97 = 97
      J97 = 33
!         COMPLETE INITIALIZATION BY SKIPPING
!              (NTOT2*MODCNS + NTOT) RANDOM NUMBERS
      DO 45 LOOP2= 1, NTOT2+1
      NOW = MODCNS
      IF (LOOP2 .EQ. NTOT2+1) NOW=NTOT
      IF (NOW .GT. 0) THEN
        WRITE(2,'(A,I15)') ' RMARIN SKIPPING OVER ',NOW
       DO 40 IDUM = 1, NTOT
       UNI = U(I97)-U(J97)
       IF (UNI .LT. 0.) UNI=UNI+1.
       U(I97) = UNI
       I97 = I97-1
       IF (I97 .EQ. 0) I97=97
       J97 = J97-1
       IF (J97 .EQ. 0) J97=97
       C = C - CD
       IF (C .LT. 0.) C=C+CM
   40  CONTINUE
      ENDIF
   45 CONTINUE
      IF (KALLED .EQ. 1) RETURN
!
!         NORMAL ENTRY TO GENERATE LENV RANDOM NUMBERS
   50 CONTINUE
      DO 100 IVEC= 1, LENV
      UNI = U(I97)-U(J97)
      IF (UNI .LT. 0.) UNI=UNI+1.
      U(I97) = UNI
      I97 = I97-1
      IF (I97 .EQ. 0) I97=97
      J97 = J97-1
      IF (J97 .EQ. 0) J97=97
```

```
      C = C - CD
      IF (C .LT. 0.)  C=C+CM
      UNI = UNI-C
      IF (UNI .LT. 0.) UNI=UNI+1.
      RVEC(IVEC) = UNI
!               REPLACE EXACT ZEROS BY UNIFORM DISTR. *2**-24
         IF (UNI .EQ. 0.)  THEN
         ZUNI = TWOM24*U(2)
!               AN EXACT ZERO HERE IS VERY UNLIKELY, BUT LET'S BE SAFE
  .
         IF (ZUNI .EQ. 0.) ZUNI= TWOM24*TWOM24
         RVEC(IVEC) = ZUNI
         ENDIF
  100 CONTINUE
      NTOT = NTOT + LENV
         IF (NTOT .GE. MODCNS)  THEN
         NTOT2 = NTOT2 + 1
         NTOT = NTOT - MODCNS
         ENDIF
      RETURN
!             ENTRY TO OUTPUT CURRENT STATUS
      ENTRY RMARUT(IJKLUT,NTOTUT,NTOT2T)
      IJKLUT = IJKL
      NTOTUT = NTOT
      NTOT2T = NTOT2
      RETURN
      END

! ----------------------------------------------------------------
! Silverfrost FTN95 for Microsoft Visual Studio
! Free Format FTN95 Source File
! ----------------------------------------------------------------

      SUBROUTINE MASSCHECK(P,XM1,XM2,XM3,MASS)
          real (kind=2) P(0:,0:)
          real*8 XM1,XM2,XM3
          real*8, INTENT(OUT) :: MASS
          ASSEMBLY_INTERFACE(NAME="MASSCHECK")
          Real*8, Dimension(4) :: kw,kh,kz
          real*8 PXM1,PXM2,PXM3
          kw(1) = P(1,1)
          kw(2) = P(2,1)
          kw(3) = P(3,1)
          kw(4) = P(4,1)
          PXM1 = SQRT( -kw(1)**2 - kw(2)**2 - kw(3)**2 + kw(4)**2 )
          kh(1) = P(1,2)
          kh(2) = P(2,2)
```

```
      kh(3) = P(3,2)
      kh(4) = P(4,2)
      PXM2 = SQRT( -kh(1)**2 - kh(2)**2 - kh(3)**2 + kh(4)**2 )
      kz(1) = P(1,3)
      kz(2) = P(2,3)
      kz(3) = P(3,3)
      kz(4) = P(4,3)
      PXM3 = SQRT( -kz(1)**2 - kz(2)**2 - kz(3)**2 + kz(4)**2 )
      Write(*,*) "PXM1: ",PXM1," xm1: ", XM1
      Write(*,*) "PXM2: ",PXM2," xm2: ", XM2
      Write(*,*) "PXM3: ",PXM3," xm3: ", XM3
      MASS = XM1 + XM2 + XM3 - (PXM1 + PXM2 + PXM3);
END


 SUBROUTINE SINGLESQ(MATX,SSQ)
      real (kind=2) MATX (0:,0:)
      real*8 SSQ, resp
      integer i,j
      ASSEMBLY_INTERFACE(NAME="SINGLESQ")
      resp = 0.
      do i = 1, 4
        do j = 1, 4
            resp = resp + MATX(i,j)**2
        enddo
      enddo
      SSQ = resp
 END SINGLESQ

 REAL*8 FUNCTION QUADS(MATRIX,P,XM3,XM2,DE,WWout,ZZout)
      real (kind=2) MATRIX (0:,0:)
      real (kind=2) P (0:,0:)
      real (kind=2) WWout (0:,0:)
      real (kind=2) ZZout (0:,0:)
      integer, optional :: DE
      ASSEMBLY_INTERFACE(NAME="QUADS")
      integer alpha,beta,gamma,delta
      real*8 XM2,XM3,epicendsum
      real*8, Dimension(4,4) :: ZZ,WW,GG
      Real*8, Dimension(4) :: kw,kh,kz
      !XM(1) = mh, XM(2) = mw, XM(3) = mz
      integer DEB
      if(present(DE)) then
        DEB = DE
      else
        DEB = 0
```

```
endif
GG(1,1) = -1.
GG(1,2) = 0.
GG(1,3) = 0.
GG(1,4) = 0.
GG(2,1) = 0.
GG(2,2) = -1.
GG(2,3) = 0.
GG(2,4) = 0.
GG(3,1) = 0.
GG(3,2) = 0.
GG(3,3) = -1.
GG(3,4) = 0.
GG(4,1) = 0.
GG(4,2) = 0.
GG(4,3) = 0.
GG(4,4) = 1.
kw(1) = -1. * P(1,1)
kw(2) = -1. * P(2,1)
kw(3) = -1. * P(3,1)
kw(4) = P(4,1)
kh(1) = P(1,2)
kh(2) = P(2,2)
kh(3) = P(3,2)
kh(4) = P(4,2)
kz(1) = -1. * P(1,3)
kz(2) = -1. * P(2,3)
kz(3) = -1. * P(3,3)
kz(4) = P(4,3)
do alpha=1, 4
  do gamma=1,4
      ZZ(alpha,gamma) = -1.*GG(alpha,gamma) + ((kz(alpha)*
         kz(gamma)) / (XM2**2))
      ZZout(alpha,gamma) = -1.*GG(alpha,gamma) + ((kz(
         alpha)*kz(gamma)) / (XM2**2))
  end do
end do
do beta=1, 4
  do delta=1,4
      WW(beta,delta) = -1.*GG(beta,delta) + (kw(beta)*kw(
         delta) / XM3**2)
      WWout(beta,delta) = -1.*GG(beta,delta) + (kw(beta)*
         kw(delta) / XM3**2)
  end do
end do
epicendsum = 0.
```

```
    do alpha=1, 4
      do beta=1, 4
         do delta=1, 4
             do gamma=1, 4
                  epicendsum = epicendsum + (MATRIX(alpha,beta
                     )*MATRIX(delta,gamma)*ZZ(alpha,delta)*WW(
                     beta,gamma))
              end do
          end do
      end do
    end do
    IF (DEB == 1) THEN
    write(*,*) "———————"
    write(*,*) "——————— EpicEndSum ———————"
    write(*,*) "   ees = ",epicendsum
    write(*,*) "———————"
    write(*,*) "———————"
    write(*,*) "——————— W ———————"
    write(*,*) WW(1,1)," - ",  WW(1,2)," - ",  WW(1,3)," - ",
       WW(1,4)
    write(*,*) WW(2,1)," - ",  WW(2,2)," - ",  WW(2,3)," - ",
       WW(2,4)
    write(*,*) WW(3,1)," - ",  WW(3,2)," - ",  WW(3,3)," - ",
       WW(3,4)
    write(*,*) WW(4,1)," - ",  WW(4,2)," - ",  WW(4,3)," - ",
       WW(4,4)
    write(*,*) "——————————————"
    write(*,*) "——————— Z ———————"
    write(*,*) ZZ(1,1)," - ",  ZZ(1,2)," - ",  ZZ(1,3)," - ",
       ZZ(1,4)
    write(*,*) ZZ(2,1)," - ",  ZZ(2,2)," - ",  ZZ(2,3)," - ",
       ZZ(2,4)
    write(*,*) ZZ(3,1)," - ",  ZZ(3,2)," - ",  ZZ(3,3)," - ",
       ZZ(3,4)
    write(*,*) ZZ(4,1)," - ",  ZZ(4,2)," - ",  ZZ(4,3)," - ",
       ZZ(4,4)
    write(*,*) "——————————————"
    END IF
    QUADS = epicendsum
    Return
end function quads

REAL*8 FUNCTION FSINGLESQ(MATX)
    real (kind=2) MATX (0:,0:)
    real (kind=2) resp
    integer I,J
```

```
      ASSEMBLY_INTERFACE(NAME="FSINGLESQ")
      resp = 0.
      write(*,*) "———————"
      write(*,*) "——————— FSINGLESQ ———————"
      do I = 1, 4
        do J = 1, 4
            resp = real(resp + (MATX(i,j)**2))
            !write(*,*) resp, " + ", (MATX(i,j)**2)
        end do
        write(*,*) MATX(i,1)," - ", MATX(i,2)," - ", MATX(i,3)
          ," - ", MATX(i,4)
      end do
      write(*,*) "———————"
      write(*,*) resp
      write(*,*) "———————"
      FSINGLESQ = resp
      Return
END Function FSINGLESQ


SUBROUTINE MATSQ(P,SQ)
 real (kind=2) P(0:,0:)
 Dimension Q(4,3)
ASSEMBLY_INTERFACE(NAME="MATSQ")
Q = transpose(P)
SQ=MATMUL(P,Q)
RETURN
END

SUBROUTINE GETMC(MMC,DE)
      real (kind=2) MMC(0:,0:)
      Real*8 prefakt, g, sw, cw
      integer, optional :: DE
      ASSEMBLY_INTERFACE(NAME="GETMC")
      integer DEB
      if(present(DE)) then
        DEB = DE
      else
        DEB = 0
      endif
      !sw = sin(28.74)
      sw = SQRT(0.23119)
      cw = SQRT(1-sw**2)
      !cw = cos(28.74)
      !g = 1.602176487e-19 / sw
      g = SQRT(16.*ATAN(1.) * 7.2973525376E-03)
```

```
    MMC(1,1) = -1.
    MMC(1,2) = 0.
    MMC(1,3) = 0.
    MMC(1,4) = 0.
    MMC(2,1) = 0.
    MMC(2,2) = -1.
    MMC(2,3) = 0.
    MMC(2,4) = 0.
    MMC(3,1) = 0.
    MMC(3,2) = 0.
    MMC(3,3) = -1.
    MMC(3,4) = 0.
    MMC(4,1) = 0.
    MMC(4,2) = 0.
    MMC(4,3) = 0.
    MMC(4,4) = 1.
    prefakt = g*g*(-1.+3.*sw*sw)/cw
    MMC = MMC * prefakt
    IF (DEB == 1) THEN
    write(*,*) "--------- MMC ---------"
    write(*,*) MMC(1,1)," - ", MMC(1,2)," - ", MMC(1,3)," - 
       ", MMC(1,4)
    write(*,*) MMC(2,1)," - ", MMC(2,2)," - ", MMC(2,3)," - 
       ", MMC(2,4)
    write(*,*) MMC(3,1)," - ", MMC(3,2)," - ", MMC(3,3)," - 
       ", MMC(3,4)
    write(*,*) MMC(4,1)," - ", MMC(4,2)," - ", MMC(4,3)," - 
       ", MMC(4,4)
    write(*,*) "---------------------"
    End if
RETURN
END

SUBROUTINE GETMH(P,XM1,MMH,DE)
    real (kind=2) MMH(0:,0:)
    Real*8 prefakth, g, sw, cw,nen, zae, XM1
    Real*8, Dimension(4) :: kw,kh,kz
    real (kind=2) p(0:,0:)
    integer alpha,beta
    integer, optional :: DE
    ASSEMBLY_INTERFACE(NAME="GETMH")
    integer DEB
    if(present(DE)) then
      DEB = DE
    else
      DEB = 0
```

```
      endif
      !sw = sin(28.74)
      !cw = cos(28.74)
      !g = 1.602176487e-19 / sw
      g = SQRT(16.*ATAN(1.) * 7.2973525376E-03)
      sw = SQRT(0.23119)
      cw = SQRT(1.-sw**2)
      prefakth = -4.*(g*g*sw*sw/cw)
      kw(1) = P(1,1)
      kw(2) = P(2,1)
      kw(3) = P(3,1)
      kw(4) = P(4,1)
      kh(1) = P(1,2)
      kh(2) = P(2,2)
      kh(3) = P(3,2)
      kh(4) = P(4,2)
      kz(1) = P(1,3)
      kz(2) = P(2,3)
      kz(3) = P(3,3)
      kz(4) = P(4,3)
      nen = -(kh(1)+kz(1))**2 -(kh(2)+kz(2))**2 -(kh(3)+kz(3))
         **2 + (kh(4)+kz(4))**2 -XM1**2
      do alpha=1, 4
        do beta=1, 4
            zae = (kh(int(alpha)) * kh(int(beta)) + kh(int(alpha
               ))* kz(int(beta)) )
           MMH(int(alpha),int(beta)) = zae/nen
        Enddo
      Enddo
     MMH = MMH * prefakth
      IF (DEB == 1) THEN
      write(*,*) "------- MH ------- zae: ", zae, " nen: ", nen
      write(*,*) MMH(1,1)," - ", MMH(1,2)," - ", MMH(1,3)," - 
         ", MMH(1,4)
      write(*,*) MMH(2,1)," - ", MMH(2,2)," - ", MMH(2,3)," - 
         ", MMH(2,4)
      write(*,*) MMH(3,1)," - ", MMH(3,2)," - ", MMH(3,3)," - 
         ", MMH(3,4)
      write(*,*) MMH(4,1)," - ", MMH(4,2)," - ", MMH(4,3)," - 
         ", MMH(4,4)
      write(*,*) "----------------"
      end if
RETURN
END

SUBROUTINE GETMH2(P,ET,MMH2,DE)
```

```
real (kind=2) MMH2(0:,0:)
Real*8 prefakt, g, sw, cw, nen, zae, ET
Real*8, Dimension(4) :: kw,kh,kz
real (kind=2) p(0:,0:)
integer alpha,beta
integer, optional :: DE
ASSEMBLY_INTERFACE(NAME="GETMH2")
integer DEB
if(present(DE)) then
  DEB = DE
else
  DEB = 0
endif
!sw = sin(28.74)
!cw = cos(28.74)
!g = 1.602176487e-19 / sw
g = SQRT(16.*ATAN(1.) * 7.2973525376E-03)
sw = SQRT(0.23119)
cw = SQRT(1-sw**2)
prefakt = 4*g*g*((2*cw*cw-1)/cw)
kw(1) = P(1,1)
kw(2) = P(2,1)
kw(3) = P(3,1)
kw(4) = P(4,1)
kh(1) = P(1,2)
kh(2) = P(2,2)
kh(3) = P(3,2)
kh(4) = P(4,2)
kz(1) = P(1,3)
kz(2) = P(2,3)
kz(3) = P(3,3)
kz(4) = P(4,3)
nen = -(kw(1)+kh(1))**2 - (kw(2)+kh(2))**2 - (kw(3)+kh(3))
   **2 + (kw(4)+kh(4))**2 - ET**2
do alpha=1, 4
  do beta=1, 4
      zae = (kh(int(alpha)) * kh(int(beta)) + kw(int(alpha
         ))* kh(int(beta)) )
      MMH2(int(alpha),int(beta)) = zae/nen
  Enddo
Enddo
MMH2 = MMH2 * prefakt
IF (DEB == 1) THEN
write(*,*) "---------- MH2 ----------zae: ", zae, " nen: ", nen
write(*,*) MMH2(1,1)," - ", MMH2(1,2)," - ", MMH2(1,3),"
   - ", MMH2(1,4)
```

```
      write(*,*) MMH2(2,1),"␣-␣",  MMH2(2,2),"␣-␣",  MMH2(2,3),"
         ␣-␣",  MMH2(2,4)
      write(*,*) MMH2(3,1),"␣-␣",  MMH2(3,2),"␣-␣",  MMH2(3,3),"
         ␣-␣",  MMH2(3,4)
      write(*,*) MMH2(4,1),"␣-␣",  MMH2(4,2),"␣-␣",  MMH2(4,3),"
         ␣-␣",  MMH2(4,4)
      write(*,*) "---------------"
      end if
RETURN
END


SUBROUTINE GETMAW(P,XM1,XM2,ET,MMAW,DE)
      real (kind=2) MMAW (0:,0:)
      Real*8 prefakt, g, sw, cw, nen, zae, A, ET, XM1,XM2
      Real*8, Dimension(4):: kw,kh,kz
      real (kind=2) P (0:,0:)
      integer alpha, beta
      integer, optional :: DE
      ASSEMBLY_INTERFACE(NAME="GETMAW")
      integer DEB
      if(present(DE)) then
        DEB = DE
      else
        DEB = 0
      endif
      IF (DEB == 1) THEN
      write(*,*) "-------␣P␣-------"
      write(*,*) P(1,1),"␣-␣",  P(1,2),"␣-␣",  P(1,3)
      write(*,*) P(2,1),"␣-␣",  P(2,2),"␣-␣",  P(2,3)
      write(*,*) P(3,1),"␣-␣",  P(3,2),"␣-␣",  P(3,3)
      write(*,*) P(4,1),"␣-␣",  P(4,2),"␣-␣",  P(4,3)
      write(*,*) "---------------"
      end if
      !sw = sin(28.74)
      !cw = cos(28.74)
      !g = 1.602176487e-19 / sw
      g = SQRT(16.*ATAN(1.) * 7.2973525376E-03)
      sw = SQRT(0.23119)
      cw = SQRT(1-sw**2)
      prefakt = 4*g*g*cw
      A = (ET**2 - XM1**2)/XM2**2
      kw(1) = P(1,1)
      kw(2) = P(2,1)
      kw(3) = P(3,1)
      kw(4) = P(4,1)
```

```
      kh(1) = P(1,2)
      kh(2) = P(2,2)
      kh(3) = P(3,2)
      kh(4) = P(4,2)
      kz(1) = P(1,3)
      kz(2) = P(2,3)
      kz(3) = P(3,3)
      kz(4) = P(4,3)
      nen = -(kw(1)+kz(1))**2 - (kw(2)+kz(2))**2 - (kw(3)+kz(3))
         **2 + (kw(4)+kz(4))**2 - XM2**2
      do alpha=1, 4
        do beta=1, 4
             zae = (kw(alpha) * kh(beta)) + ((1.-A)/4.)* (kw(
                alpha)* kz(beta) )
            MMAW(alpha,beta) = zae/nen
        Enddo
      Enddo
      MMAW = MMAW * prefakt
      IF (DEB == 1) THEN
      write(*,*) "-------- MAW --------zae: ", zae, " nen: ", nen
      write(*,*) MMAW(1,1)," - ", MMAW(1,2)," - ", MMAW(1,3),"
         - ", MMAW(1,4)
      write(*,*) MMAW(2,1)," - ", MMAW(2,2)," - ", MMAW(2,3),"
         - ", MMAW(2,4)
      write(*,*) MMAW(3,1)," - ", MMAW(3,2)," - ", MMAW(3,3),"
         - ", MMAW(3,4)
      write(*,*) MMAW(4,1)," - ", MMAW(4,2)," - ", MMAW(4,3),"
         - ", MMAW(4,4)
      write(*,*) "----------------"
      end if
RETURN
END


SUBROUTINE GETMMBW(P,ET,XM1,XM2,XM3,MMBW,DE)
      real (kind=2) MMBW (0:,0:)
      Real*8 prefakt, pretemp, pretemp2, g, sw, cw, XM1, XM2,
         XM3, ET, RV
      Real*8, Dimension(4) :: kw,kh,kz
      real (kind=2) P (0:,0:)
      integer, optional :: DE
      ASSEMBLY_INTERFACE(NAME="GETMMBW")
      integer DEB
      if(present(DE)) then
        DEB = DE
      else
```

```
  DEB = 0
endif
RV = (ET**2 - XM1**2)/ (XM2**2)
IF (DEB == 1) THEN
write(*,*) "RV: ", RV
end if
!sw = sin(28.74)
!cw = cos(28.74)
!g = 1.602176487e-19 / sw
g = SQRT(16.*ATAN(1.) * 7.2973525376E-03)
sw = SQRT(0.23119)
cw = SQRT(1.-sw**2)
kw(1) = P(1,1)
kw(2) = P(2,1)
kw(3) = P(3,1)
kw(4) = P(4,1)
kh(1) = P(1,2)
kh(2) = P(2,2)
kh(3) = P(3,2)
kh(4) = P(4,2)
kz(1) = P(1,3)
kz(2) = P(2,3)
kz(3) = P(3,3)
kz(4) = P(4,3)
MMBW(1,1) = -1.
MMBW(1,2) = 0.
MMBW(1,3) = 0.
MMBW(1,4) = 0.
MMBW(2,1) = 0.
MMBW(2,2) = -1.
MMBW(2,3) = 0.
MMBW(2,4) = 0.
MMBW(3,1) = 0.
MMBW(3,2) = 0.
MMBW(3,3) = -1.
MMBW(3,4) = 0.
MMBW(4,1) = 0.
MMBW(4,2) = 0.
MMBW(4,3) = 0.
MMBW(4,4) = 1.
pretemp = -(kw(1)+kz(1))**2 - (kw(2)+kz(2))**2 - (kw(3)+kz
   (3))**2 + (kw(4)+kz(4))**2 - XM2**2
prefakt = 2.*g*g*cw*( 1./ pretemp )
IF (DEB == 1) THEN
write(*,*) "pf: ", prefakt
end if
```

```
    pretemp2 = (-(kh(1)*kz(1)) - (kh(2)*kz(2)) - (kh(3)*kz(3))
         + (kh(4)*kz(4)) )
    pretemp2 = pretemp2 - ( - (kh(1)*kw(1)) - (kh(2)*kw(2)) -
        (kh(3)*kw(3)) + (kh(4)*kw(4)) )
    prefakt = prefakt *(pretemp2 + ((1.-RV)/2.)*(XM3**2 - XM2
        **2))
    !PREFAKTOR ERROR
    MMBW = MMBW * prefakt
    IF (DEB == 1) THEN
    write(*,*) "------- MBW ---------pf: ", prefakt, " pt: ",
        pretemp, " pt2: ", pretemp2
    write(*,*) MMBW(1,1)," - ", MMBW(1,2)," - ", MMBW(1,3),"
        - ", MMBW(1,4)
    write(*,*) MMBW(2,1)," - ", MMBW(2,2)," - ", MMBW(2,3),"
        - ", MMBW(2,4)
    write(*,*) MMBW(3,1)," - ", MMBW(3,2)," - ", MMBW(3,3),"
        - ", MMBW(3,4)
    write(*,*) MMBW(4,1)," - ", MMBW(4,2)," - ", MMBW(4,3),"
        - ", MMBW(4,4)
    write(*,*) "----------------"
    end if
RETURN
END



SUBROUTINE GETMMCW(P,ET,XM1,XM2,MMCW,DE)
 real (kind=2) MMCW (0:,0:)
 Real*8 prefakt, g, sw, cw,nen, zae, XM2, XM1, ET, A
 Real*8, Dimension(4) :: kw,kh,kz
 real (kind=2) P (0:,0:)
 integer alpha, beta
 integer, optional :: DE
 ASSEMBLY_INTERFACE(NAME="GETMMCW")
 integer DEB
    if(present(DE)) then
      DEB = DE
    else
      DEB = 0
    endif
 g = SQRT(16.*ATAN(1.) * 7.2973525376E-03)
 sw = SQRT(0.23119)
 cw = SQRT(1.-sw**2)
 prefakt = -4.*g*g*cw
 A = (ET**2 - XM1**2)/XM2**2
 kw(1) = P(1,1)
```

```
      kw(2) = P(2,1)
      kw(3) = P(3,1)
      kw(4) = P(4,1)
      kh(1) = P(1,2)
      kh(2) = P(2,2)
      kh(3) = P(3,2)
      kh(4) = P(4,2)
      kz(1) = P(1,3)
      kz(2) = P(2,3)
      kz(3) = P(3,3)
      kz(4) = P(4,3)
      nen = -(kw(1)+kz(1))**2 - (kw(2)+kz(2))**2 - (kw(3)+kz(3))**2
     +  + (kw(4)+kz(4))**2 - XM2**2
      !nen = 1.
      do alpha=1, 4
        do beta=1, 4
            zae = kh(alpha) * kz(beta) + ((1.-A)/2.)* kw(alpha)* kz(
               beta)
            !zae = 1.
           MMCW(alpha,beta) = zae/nen
        Enddo
      Enddo
      MMCW = MMCW * prefakt
      IF (DEB == 1) THEN
      write(*,*) "-------MCW--------zae: ", zae, " nen: ", nen
          write(*,*) MMCW(1,1)," - ", MMCW(1,2)," - ", MMCW(1,3),"
             - ", MMCW(1,4)
          write(*,*) MMCW(2,1)," - ", MMCW(2,2)," - ", MMCW(2,3),"
             - ", MMCW(2,4)
          write(*,*) MMCW(3,1)," - ", MMCW(3,2)," - ", MMCW(3,3),"
             - ", MMCW(3,4)
          write(*,*) MMCW(4,1)," - ", MMCW(4,2)," - ", MMCW(4,3),"
             - ", MMCW(4,4)
          write(*,*) "-----------------"
      end if
      RETURN
      END
```

```
  using System;
using System.Collections.Generic;
using System.Linq;
using System.Text;

namespace fortranWrapper
{
    class Matrix
    {
```

```
public Double[,] array, origarray, f1origarray;
public Double[,] MMC, MMH, MH2, MMAW, MMBW, MMCW, SUM6, WW,
   ZZ;
public Double[] kw, kh, kz;
public double ET, XM1, XM2, XM3, XM4, WT, SSQ, epicendsum;
public event EventHandler<MyMatrixEventArgs> newMessage;
int n, m, DEB;

public class MyMatrixEventArgs : EventArgs
{
    private string m_message = "";
    public string message
    {
        get { return m_message; }
    }
    public MyMatrixEventArgs(string i_message)
    {
        m_message = i_message;
    }
}

public Matrix(int n, int m, double ET = 0, double XM1 = 0,
   double XM2 = 0, double XM3 = 0, double WT = 0)
{
    this.n = n;
    this.m = m;
    array = new Double[this.n, this.m];
    kw = new Double[n];
    kh = new Double[n];
    kz = new Double[n];
    this.ET = ET;
    this.XM1 = XM1;
    this.XM2 = XM2;
    this.XM3 = XM3;
    this.WT = WT;
}

private void DoMessageSend(string message)
{
    if (newMessage != null)
    {
        newMessage(this, new MyMatrixEventArgs(message));
    }
}

/*public static Matrix operator +(Matrix a,Matrix b)
```

```
{
    return a.array + b.array;
}*/
public void set(Double[,] input)
{
    array = input;
}
public void setp(Double[,] input, int de = 0)
{
    this.DEB = de;
    DoMessageSend("GOT P, WT");
    origarray = input;
    Double[,] p = new Double[4, 3];
    Double[,] p2 = new Double[4, 3];
    p[0, 0] = input[1, 1];
    p[0, 1] = input[1, 2];
    p[0, 2] = input[1, 3];

    p[1, 0] = input[2, 0];
    p[1, 1] = input[2, 1];
    p[1, 2] = input[2, 2];

    p[2, 0] = input[2, 3];
    p[2, 1] = input[3, 0];
    p[2, 2] = input[3, 1];

    p[3, 0] = input[3, 2];
    p[3, 1] = input[3, 3];
    p[3, 2] = input[4, 0];

    //f1origarray = p;

    p2[0, 0] = p[0, 0];
    p2[1, 0] = p[0, 1];
    p2[2, 0] = p[0, 2];
    p2[3, 0] = p[1, 0];  //

    p2[0, 1] = p[1, 1];
    p2[1, 1] = p[1, 2];
    p2[2, 1] = p[2, 0];
    p2[3, 1] = p[2, 1];

    p2[0, 2] = p[2, 2];
    p2[1, 2] = p[3, 0];
    p2[2, 2] = p[3, 1];
    p2[3, 2] = p[3, 2];
```

```
    array = p2;

    kw[0] = p2[0, 0];
    kw[1] = p2[1, 0];
    kw[2] = p2[2, 0];
    kw[3] = p2[3, 0];
    kh[0] = p2[0, 1];
    kh[1] = p2[1, 1];
    kh[2] = p2[2, 1];
    kh[3] = p2[3, 1];
    kz[0] = p2[0, 2];
    kz[1] = p2[1, 2];
    kz[2] = p2[2, 2];
    kz[3] = p2[3, 2];
    DoMessageSend("Rambotfunctionsdll (FORTRAN95) 
       initialized");
    setMc();
    DoMessageSend("GOT MC");
    setMh();
    DoMessageSend("GOT MH");
    setMh2();
    DoMessageSend("GOT MH2");
    setMaw();
    DoMessageSend("GOT MAW");
    setMbw();
    DoMessageSend("GOT MBW");
    setMcw();
    DoMessageSend("GOT MCW");

    //setssq();
    setSum();
    DoMessageSend("GOT SUM6");
    setepicendsum();
    DoMessageSend("GOT EPICENDSUM");
}


private void setMc()
{
    double[,] re = new double[5, 5];
    Rambotfunctionsdll.GETMC(re,DEB);
    this.MMC = re;
}

private void setMh()
```

```
{
    double[,] re = new double[5, 5];
    Rambotfunctionsdll.GETMH(this.origarray, this.XM1, re,
       DEB);
    this.MMH = re;
}

private void setMh2()
{
    double[,] re = new double[5, 5];
    Rambotfunctionsdll.GETMH2(this.origarray, this.ET, re,
       DEB);
    this.MH2 = re;
}

private void setMaw()
{
    double[,] re = new double[5, 5];
    Rambotfunctionsdll.GETMAW(this.origarray, this.XM1, this
       .XM2, this.ET, re, DEB);
    this.MMAW = re;
}

private void setMbw()
{
    double[,] re = new double[5, 5];
    Rambotfunctionsdll.GETMMBW(this.origarray, this.ET, this
       .XM1, this.XM2, this.XM3, re, DEB);
    this.MMBW = re;
}

private void setMcw()
{
    double[,] re = new double[5, 5];
    Rambotfunctionsdll.GETMMCW(this.origarray, this.ET, this
       .XM1, this.XM2, re, DEB);
    this.MMCW = re;
}

private void setssq()
{
    double assq = 0;
    assq += Rambotfunctionsdll.FSINGLESQ(this.MMC);
    assq += Rambotfunctionsdll.FSINGLESQ(this.MMH);
    assq += Rambotfunctionsdll.FSINGLESQ(this.MH2);
    assq += Rambotfunctionsdll.FSINGLESQ(this.MMAW);
```

```
    assq += Rambotfunctionsdll.FSINGLESQ(this.MMBW);
    assq += Rambotfunctionsdll.FSINGLESQ(this.MMCW);
    this.SSQ = assq;
}

private void setSum()
{
    this.SUM6 = matadd(MMC, MMH);
    this.SUM6 = matadd(this.SUM6, MH2);
    this.SUM6 = matadd(this.SUM6, MMAW);
    this.SUM6 = matadd(this.SUM6, MMBW);
    this.SUM6 = matadd(this.SUM6, MMCW);
}

private void setepicendsum()
{
    this.WW = new double[5, 5];
    this.ZZ = new double[5, 5];
    double RE = Rambotfunctionsdll.QUADS(this.SUM6, this.
       origarray, this.XM3, this.XM2,0,this.WW,this.ZZ);
    this.epicendsum = RE;
}

private double[,] matadd(double[,] a, double[,] b)
{
    double[,] resp = new double[5,5];
    for (int i = 0; i < 5; i++)
    {
        for (int j = 0; j < 5; j++)
        {
            resp[i, j] = a[i, j] + b[i, j];
        }
    }

    return resp;
}
public double[,] getMc()
{
    double[,] re = new double[5,5];
    Rambotfunctionsdll.GETMC(re,0);
    return re;
}

public double[,] getMh(double[,] P, float xm1)
{
    double[,] re = new double[5, 5];
```

```
            Rambotfunctionsdll.GETMH(P,xm1,re,0);
            return re;
        }

        public double[,] getMh2(double[,] P, float xm1, float et)
        {
            double[,] re = new double[5, 5];
            Rambotfunctionsdll.GETMH2(P, et, re,0);
            return re;
        }
    }
}
```

References

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