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Foreword

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IV Table of abbreviations

Abbreviation	Description
R^I	Set of inbound transportation requests
R^O	Set of outbound transportation requests
R	Set of all transportation requests, $R = R^I \cup R^O$
B	Set of hospital wards
O	Set of medical examination rooms
D	Set of depots
P	Set of patients
N	Set of all nodes at the hospital, $N = B \cup O \cup D$
R^*	Set of all transportation requests and depots, $R^* = R \cup D$
E_i	Critical time point for transportation request *i*
T_{ij}	Travel time between two nodes *i* and *j* (in minutes)
D(s)	Home depot of a porter *s*
S_i^T	Service time of transportation request *i*
α	Penalty for time when porter travels empty
β	Penalty for waiting time (of patient)
A(i)	Patient to whom transportation request i refers to
a_i	Arrival time to the pickup location of request *i*
b_i	Start time of the transport of request *i*
c_i	Arrival time to the delivery location of request *i*
d_i	Time point when porter leaves the delivery location of request *i*
x_{ijs}	Binary flow variable
y_{is}	Binary assignment variable
t_{ijs}	Time travelled between request *i* and *j* by porter *s*
w_i^H	Time that porter spends in her home depot after request *i*
w_i	Binary variable equals 1 if porter is sent back home temporarily after request *i*
T^W	Minimal waiting time porter needs to spend at home depot
γ	Penalty, if porter has to wait idle between transport requests
$\|p\|$	Number of patients

Abbreviation	Description
|po|	Number of porters
N	Instance number
C	Class of Instances
f	Value of the objective function
tt	Total travel time (empty)
w (s)	Total porters’ waiting time
w (p)	Total patients’ waiting time
w^H	Time porter spends in her home depot
w^i	Number of times porter is sent to her home depot
No (p)	Number of porters that are assigned to one patient
GAP	Gap between best solution found so far and the best bound in %
No (OS)	Number of optimal solutions found
time	Elapsed run time until termination
LP	Linear program
MIP	Mixed Integer Problem
VRP	Vehicle Routing Problem
VSP	Vehicle Scheduling Problem
PDP	Pickup and Delivery Problem
GPDP	General Pickup and Delivery Problem

1 Introduction

In the recent past transportation, scheduling and supply chain management oriented problems for health care related applications have gained increased attention in the scientific community. In large hospitals, where different hospital units are typically spread across the site in so called pavilions, routing operations come at high costs. Those costs typically include pure routing (i.e. distance/travel time) related costs, but may also include additional costs. Each time a patient misses or comes late to the appointment, the entire schedule is affected and the remaining appointments scheduled afterwards have to be delayed. Moreover, inefficient usage of hospital resources, such as inefficient assignment of hospital staff to tasks, is the cause for unnecessary costs. Furthermore, as the hospital’s service quality is measured through patient satisfaction, it is also necessary to provide high quality service where patients' inconvenience, which is measured through waiting times and number of medical assistants (porters) in use per patient, is minimized. Hence finding good solutions to the underlying routing operations is highly essential.

Routing problems emerge from the need to escort in-patients who are admitted to the hospital on their in-house transportation. Patients have fixed appointments, such as x-ray, ultrasonic, blood testing or surgery. Due to medical reasons they may not be able to go on their own, but rather they will be escorted there (and back) by porters. Hence two transportation requests from the porters’ point of view need to be scheduled, such that their routes are optimized.

The aim of this thesis is to develop a model that has to find such routes, that will minimize the travel costs between hospital units, but that will also take the patient inconvenience (in the sense of long waiting times) into consideration. Therefore, a model needs to be created that will capture the above mentioned objectives. The model should also be adaptable to change accordingly if some other goals are taken into account. From the patients’ point of view, it would be desirable to have, if possible, only one porter who is responsible for them, i.e. that only one porter escorts the patient to her appointment and picks her up afterwards. On the other hand, the hospital management would be interested finding a way to use the resources more efficiently (i.e. by minimizing the unnecessary porters' waiting times that occur somewhere at hospital compound).

The remainder of the paper is organized as follows. In Section 2 a literature review is given. A detailed problem description as well as a mathematical formulation is given in Section 3. The description starts with the formulation of a general model and is later on extended accordingly in order to capture the above-mentioned features from the patients' and hospital managers' point of view. Section 4 gives a description of the data that have been used to test the model, as well as detailed numerical results. The thesis is concluded with summary of the managerial implications and core findings of the investigated model in Section 5.

2 Literature review

In the recent years, as the demand for health-care related services has increased, the associated costs for medical treatments have also increased, which asks for the efficient management of all resources in a hospital. For that reason, transportation, scheduling and supply chain management oriented problems for health care related applications are gaining increased attention in the scientific community. The papers addressing problems in health-care can be grouped together according to the method used.

2.1 Methods used

Among the papers addressing health-care problems, the majority have used heuristic solution methods or hybrids. Due to the complexity of the problem, exact solution techniques could be used only under certain constraints.

Exact solution techniques were used in the cases when the problem settings were deterministic or for small instances tested. Branch and Price and Branch and Cut were used with great success by [6], [8] and [15]. Branch and Cut can be seen as a generalization of Branch and Bound. If the solution, obtained after solving LP relaxation of a Mixed Integer Problem (MIP), does not satisfy the integrality constraint, the violated cut has to be found. If one or more violated cuts are found, they have to be added to the LP formulation. The LP has to be solved again and this procedure has to be repeated until no more violated cuts can be found. At this point the branching starts. On the other hand, Branch and Price use the pricing problem to check the optimality of the LP solution. The columns have to be checked for profitable reduced cost columns. If such columns are found, they have to be added to the LP relaxation and the LP has to be solved again.

For large instances tested a heuristic or a combination of exact solution technique and a heuristic have been used. In [1] a two phase heuristic has been proposed, where in the first phase an insertion scheme is used to generate a solution and in the second phase this solution is improved by using Tabu search. Tabu search is a heuristic used for exploring solution space, where some search moves, i.e. to the points that have been visited in the recent past, are forbidden for some time periods. In this way, cycling is avoided, and good solutions can be obtained in short time. Also [22] used Tabu search in their research. They have used LINGO modeling language, which uses Branch and Bound algorithm to solve MIP, for small instances tested, and Tabu search for larger instances. On the other hand, in [18] an Ant Colony Optimization Algorithm has been proposed for solving nurse scheduling problem.

2.2 Applications

The underlying problems can formally be modeled in terms of combinatorial optimization problems coming from scheduling (for both personnel ([2],[16]), resources and rooms such as operating theatres ([2],[5]), transportation routing (of ambulances [1], nurses and doctors) and supply chain management (supply, delivery, reverse logistics of medical waste). The classical Vehicle Routing Problem (VRP) finds a set of minimum cost vehicle routes, that each starts at the depot, visits some customer locations and returns to the depot. VRP has many variations depending on the characteristics of the vehicles, the facilities and the customers. As stated in [13] and [31], the vehicles may be either identical or differ with respect to capacity limits; the problem may be concerned with delivery only, pick-up only, or both delivery and pickup; the vehicles might be restricted to serve each customer in a given time window and/or serve according to precedence relations of the customers. In addition, the problem may involve a single facility or multiple facilities. On the other hand, in a scheduling problem one has to find time slots in which activities (or jobs) should be processed under given constraints. The main constraints are resource constraints and precedence constraints between activities. The most common scheduling problem is the Machine Scheduling Problem. The Machine Scheduling Problem is concerned with finding an efficient schedule for a set of machines to process the set of jobs. Another case of Scheduling Problem is the Vehicle Scheduling Problem (VSP) that is concerned with determining a set of vehicle schedules to operate a given timetable at the lowest possible cost, as in [3] and [4]. In some variations, there are constraints such as depot capacity and upper and lower bounds on the number of vehicles at each depot.

The problem belongs to the family of *pickup and delivery* problems. In general pickup and delivery problem (GPDP), the vehicles need to be assigned to complete different transportation requests and the optimal vehicle routes have to be generated. Each vehicle has a fix start and end location and each transportation request has a specified pickup and delivery location. The customers (or load) have to be transported from the pickup to the delivery location in that way, that there are no transshipments at other locations. The pickup and delivery problem, as a special case of GPDP, is specified with a central depot, from which all tours start and end. The first papers to address this issue have been [26] and [28]. Their optimization was focused on minimizing the transportation costs. Client-centered aspects (such as waiting times observed by real persons) have been included by [21] and [30]. In their approaches maximum ride time restrictions have been included in the optimization. A formal description closely related to real-world patient transportation has been proposed in [30]. For a recent survey on different formulations see [25], [10] and [11]. A decision support system for real-world patient transportation has been described in [20], based on the problem and solution techniques proposed in [1]. Several variants for this problem have been studied recently (see [14], [19] and [22]). True Pareto optimization considering the two conflicting objectives (costs vs. user inconvenience) has been proposed in [24].

The problem under consideration is a special case of the classical *dial-a-ride* problem, arising within hospitals. The idea of the dial-a-ride problem is to pick up the customer (patient) at her door and to bring her to the desired location (particular hospital unit). The goal is to minimize the travel costs and maximize the service quality, where the service quality is measured through the satisfaction of the customers (patients). This idea can be applied to the problem considered in this Thesis. The capacity of the vehicle (i.e. the porter) is set to one. Furthermore transportation requests are paired in a sense that every patient triggers two transportation requests and their inconvenience is supposed to be minimized. From the patients point of view it would be beneficial if the very same porter could escort them on both resulting transportation requests. Additional extensions under consideration are the porters themselves. Besides executing transportation requests they are also bound to fulfilling other tasks beyond the scope of this model. Hence their duty time should be compact in a sense, that they can also be deployed for those tasks for time slots of sufficient length.

Alternatively the problem at hand can also be considered a special type of the *stacker crane problem* (see [7], [16] and [27]) whereas the latter is an application of full-truck load movements. The stacker crane problem typically arises in port operations where containers need to be moved between different stacks, such that the costs associated with empty movements of the corresponding transporting device are minimized. In the context of our application containers correspond to patients, who need to be transported between known locations. Porters would correspond to the moving device (i.e. the crane) whose empty movements are supposed to be optimized.

To the best of my knowledge so far only classical patient routing problems (such as dial-a-ride problems) have been considered in the scientific community. See [1], [11], [20], [24], [26], [29] and [28] for additional details.

3 Model

In this section the patient routing and scheduling problem at a pavilion structured hospital is going to be described. Next, the mathematical model for solving the general routing and scheduling problem is proposed. This model is later analyzed in detail through two extensions.

3.1 Problem description

The problem focuses on the in-house transportation of patients, where patients have to be transported between different hospital units. The patients are stationed at the hospital wards. Each patient has one medical examination or surgery scheduled at one of the hospital units. Patients have to be transported from the hospital ward in which they are stationed to the hospital unit where their medical examination is scheduled for that day. After the medical examination or the surgery is finished, the patient has to be brought back to her bed at the hospital ward. The transportation requests of the patients are done by so called porters, who are non-medical staff members responsible for logistical operations within the hospital.

The model presented within this thesis focuses on pavilion structured hospitals that are characterized by locally dispersed hospital units. These hospital units can be grouped into several hospital wards, where only patient beds are stationed, and several facilities where medical examinations (like blood tests, X-rays and other) or surgeries take place. Furthermore there are designated buildings where porters are located and may fulfill additional tasks whenever they are idle. The author will refer to them as *home depots*.

It can be distinguished between two different kinds of transportation requests for each patient. The first transportation request is an inbound request, where the patient has to be brought from her hospital ward to the facility where the medical examination is scheduled. After the medical examination ends, the patient has to be picked up at this facility and be brought back to the hospital ward. This kind of transportation requests will be referred to as an outbound request of the patient. The inbound and outbound requests are pictured in Figure 1.

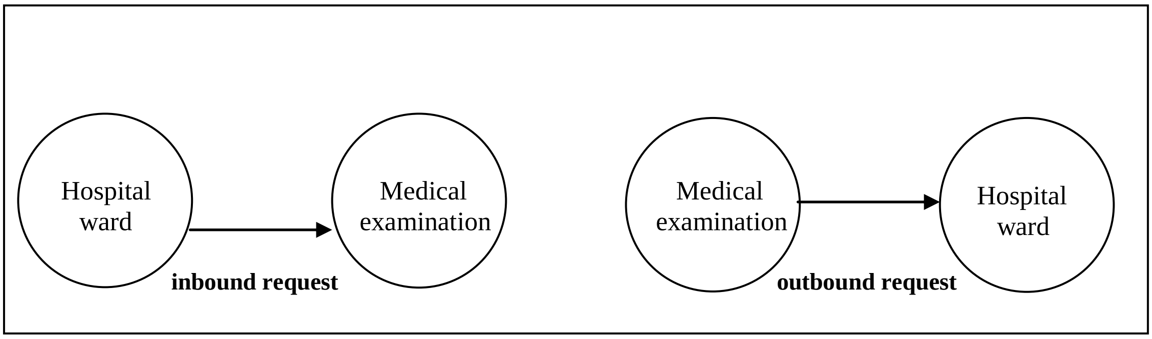

Figure 1: Inbound and outbound transportation request

The porters, besides their other duties, are assigned to escort patients on their in-house transportation. This duty of the porters is very important, especially in case of elderly or disabled persons. The patients are transported either in stretchers or in wheelchairs, depending on their condition. The typical work day of a porter may be scheduled as following: the porter is assigned to complete few transportation requests, hence she has to go from her home depot to complete the first transportation request, after that she proceeds to the next transportation requests and at the end, when all of her requests are completed, she returns to her home depot, as pictured in Figure 2.

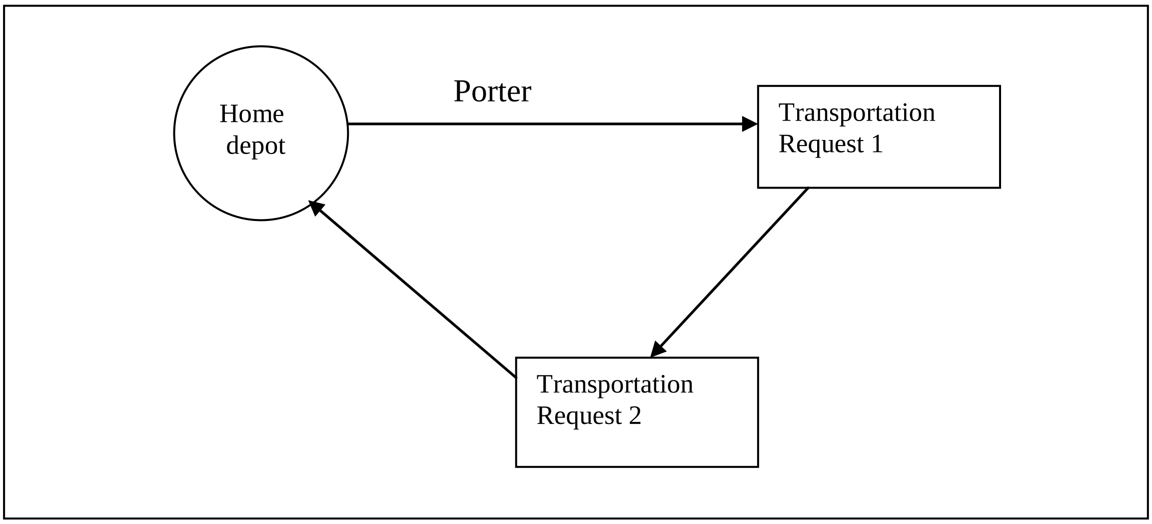

Figure 2: Porter's assignment

If we want to depict the actual route that porter has to traverse, then we obtain the following picture:

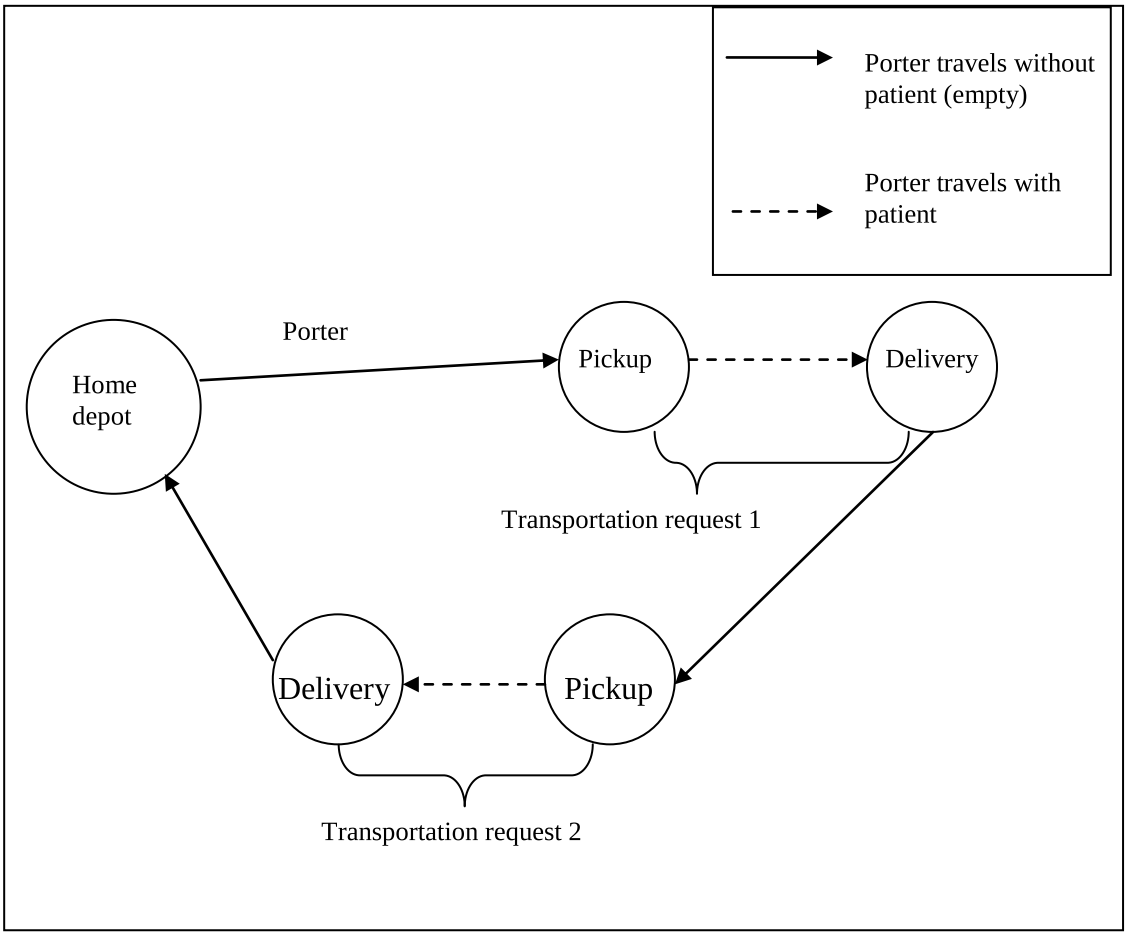

Figure 3: Porter's route

The porter starts from her home depot and goes to the pickup location of her first transportation request. There she picks up the patient and escorts her to her delivery location. From that delivery location the porter proceeds to the pickup location of her next request and escorts the next patient to her delivery location. When the porter has completed all the transportation requests that were scheduled for that day, she has to go back to her home depot.

Depending on the type of the patient's transportation request, the pickup and delivery location can be either hospital ward or the facility where the patient's medical examination is taking place. In the case of inbound request, the pickup location of the patient is going to be her hospital ward and the delivery location will be the hospital unit where the medical examination is scheduled. Similarly, in the case of an outbound request, the pickup location is going to be the hospital unit and the delivery location is going to be the hospital ward of the patient.

Although it may seem that the service provided by porters is very simple and manageable, it actually acquires careful planning. The porters should be scheduled to transportation requests in the way that the patients' waiting time is minimized, however, this scheduling needs to be done in the efficient way, so that porters could also be assigned to complete other duties whenever they are able. Planning of porters' schedule affects costs that arise in the hospital on one hand and the quality of hospital service on the other hand. Furthermore, low quality hospital service imposes additional costs; e.g. if a patient is delivered late to the hospital unit where the medical examination is being scheduled, the hospital resources are going to be underutilized. Moreover, a late delivery of a patient affects the hospital schedule that has been planned ahead. If one patient arrives after the scheduled due date, then the following medical examinations for other patients have to be delayed, this leads to the increase in the waiting time for other patients.

Currently, the porters' scheduling to transportation requests in hospitals is mainly done manually. In the morning, the list with patient names and their appointments that are scheduled for that day is provided at each hospital ward. An example of such list is given in Table 1. The nurses then have to telephone the porters and assign the transportation requests to them, if they are currently available. This way of scheduling is not efficient, as the patients have to wait until one porter is found who is not currently assigned to any other transportation requests. According to unofficial hospital statistics, the necessary time to find the according porter can last up to 10 minutes, due to many reasons (i.e. the porter cannot be reached by telephone, all porters are currently assigned to other requests) and the time spent on this administrative work is approximately one hour a day. This way of planning leads to delays in patients' appointments and to inefficient use of resources (time unnecessary spent on administrative work).

Patient's name	Pavillion number	Due date
Patient A	25	10^{00}
Patient B	27	10^{10}
Patient C	28	10^{10}
Patient D	26	10^{20}

Table 1: List with patient information

The poor management of patient transportation in hospitals leads to patient inconvenience and to increase in the costs. Although its role is very important, it is has not been paid much attention to logistics in healthcare. The aim of this work is to introduce a new model for intra-hospital routing of patients, considering both client- and management related issues. On one hand, the client related issues are concerned with reduction of patient waiting time and patient inconvenience. On the other hand, management related issues are concerned with trying to find the optimal schedule so that the costs are reduced and the hospital resources are used in the best possible way. The underlying objectives are typically conflicting by nature. Therefore, a tradeoff between these conflicting objectives has to be found. In order to do so, the author has used a weighted sum approach, where each term (sum) in the objective function has been multiplied by a different coefficient, depending on how important this particular goal is.

The purpose of this work is to find a model that will, using the provided data, find the optimal assignment of porters to the transportation requests at the hospital on one hand and that will find the optimal routes that porters have to traverse at the hospital on the other hand. However, it can be assumed that this problem can be applied to many hospitals with pavilion structure and help to increase patient convenience and reduce costs and inefficient usage of resources.

The general model developed to solve the assignment and routing problem is described in the next section. However, this general model can be extended in order to discuss the possibilities of further contribution to the patient convenience on one hand or to the even better utilization of the hospital resources on the other hand. Hence, the general model is therefore complemented with two extensions in the following sections.

3.2 General model

The general problem concentrates on finding the optimal assignment under consideration to minimize the travel times between different nodes at a pavilion structured hospital and to reduce patient inconvenience imposed by long waiting times. The total travel times at the hospital consist of the two types of travel times: the travel times when porters travel with the patient from the pickup to the delivery location, and when the porters travel without the patient (so called empty travel times). As the travel times between the pickup and delivery location of the transportation request are fix, they are not considered in the optimization process. Therefore, the travel times that occur in the objective function are only empty travel times. The problem is how to optimally assign porters to complete the transportation requests at the hospital and how to create the optimal routes for porters.

In the general problem, all medical assistants are in their home depots at the beginning of the work day. The transportation requests that are scheduled for that day are known in advance. Therefore it is possible for hospital management to make the optimal assignment and the optimal tours for the porters ahead. The porters leave the depot and go to complete their transportation requests in the order that was computed by the hospital manager. After they have completed their last transportation request in the tour, they go back to their home depots.

In this thesis the problem of offline or static data, where the data are already known, has been studied. All information about transportation requests is randomly generated. Pickup and delivery locations for each transportation request and the associated time point when pickup or delivery should take place are already given.

As already mentioned, transportation requests can be divided into two types, inbound and outbound requests, with respect to whether the patient is being escorted to her medical examination or being picked up afterwards. Division of the transportation requests into these two types was done because it is important to distinguish between two different time windows. In the first case, a patient cannot be brought to the delivery location after the given time point, resp. the patient cannot come late to her medical examination. This restriction is very important because it is the aim of the model to hinder the delay in patients' medical examination, respectively to reduce the waiting time for patients that would be imposed by this delay. The patient can however be brought to their medical examination earlier. In this case the patient will have to wait for her examination. As one objective of the model presented in this work is to reduce the patient inconvenience, each minute the patient has to wait is going to be penalized.

In the second case, when an outbound request is considered, an assistant should pick up the patient at the given time when the patient's medical examination ends. If an assistant comes late, patient would have to wait for her. Patients' waiting time will also result in penalty. In case of an outbound request, however, an assistant is allowed to come earlier to the pickup location and to wait there for the patient. Porters' waiting time is not considered and penalized in the general model.

In this work, all these facts have been taken into consideration, and a model is going to be presented that relies on these limitations and objectives. Prior to presenting the mathematical model, the required notation is introduced. Different sets and numbers used are listed in Table 1.

Abbreviation	Description
R^I	Set of inbound transportation requests
R^O	Set of outbound transportation requests
R	Set of all transportation requests, $R= R^I \cup R^O$
B	Set of hospital wards
O	Set of medical examination rooms
D	Set of depots
P	Set of patients
N	Set of all nodes at the hospital, N = B U O U D
R^*	Set of all transportation requests and depots, $R^* = R \cup D$

Table 2: Notation for sets used

Transportation requests and nodes at the hospital are usually addressed with index *i*. However, if two consecutive transportation requests are addressed in the model, then for addressing the first transportation request index *i* is used, and for addressing the second one index *j* is used. Porters are addressed with index *s*.

Furthermore, as shown in Table 2, the following data associated with time and penalties are used in the model:

Abbreviation	Description
E_i	Critical time point for transportation request *i*
T_{ij}	Travel time between two nodes *i* and *j* (in minutes)
D(s)	Home depot of a porter *s*
S_i^T	Service time of transportation request *i*
α	Penalty for time when porter travels empty
β	Penalty for waiting time (of patient)
A(i)	Patient to whom transportation request i refers to

Table 3: Notation associated with input parameters

With E_i the critical time point for transportation request *i* is represented. In the case of inbound request, E_i is the start time of the medical procedure, and in the case of outbound request, E_i is the time point when the medical procedure ends. Empty travel time between nodes at the hospital (when porter travels without the patients) is represented with T_{ij}. The service time is represented with S_i^T. Travel time between the pickup and the delivery location of patient's transportation request, together with the time that is necessary to prepare the patient for the transportation request, are included in the service time.

In the model, several different decision variables have been used. In the next step the binary variables that stand for routing and scheduling decisions, assignment decisions and penalties will be introduced.

The routing and scheduling decisions are represented by the binary flow variable x_{ijs}, which is equal to 1 if the porter *s* is assigned to transportation request *j* after she completes the transportation request *i*, and equal to 0 otherwise. The binary variable x_{ijs} is defined for $i, j \in \mathbf{R}^*$ and $s \in \mathbf{S}$. In the case when $i \in \mathbf{D(s)}$ ($j \in \mathbf{D(s)}$) the variable x_{ijs} represents the assignment of the porter *s* to her fist transportation request (her last transportation request).

The assignment binary variable y_{is} serves to represent the assignment of transportation requests to porters. The variable is defined for $i \in \mathbf{R}^*$ and $s \in \mathbf{S}$ and is equal to 1 if porter *s* is assigned to transportation request *i*, and is equal to 0 otherwise.

Besides the binary variables, the author has also used variables that are mostly associated with time. For a better understanding, all of those variables are pictured in Figure 4.

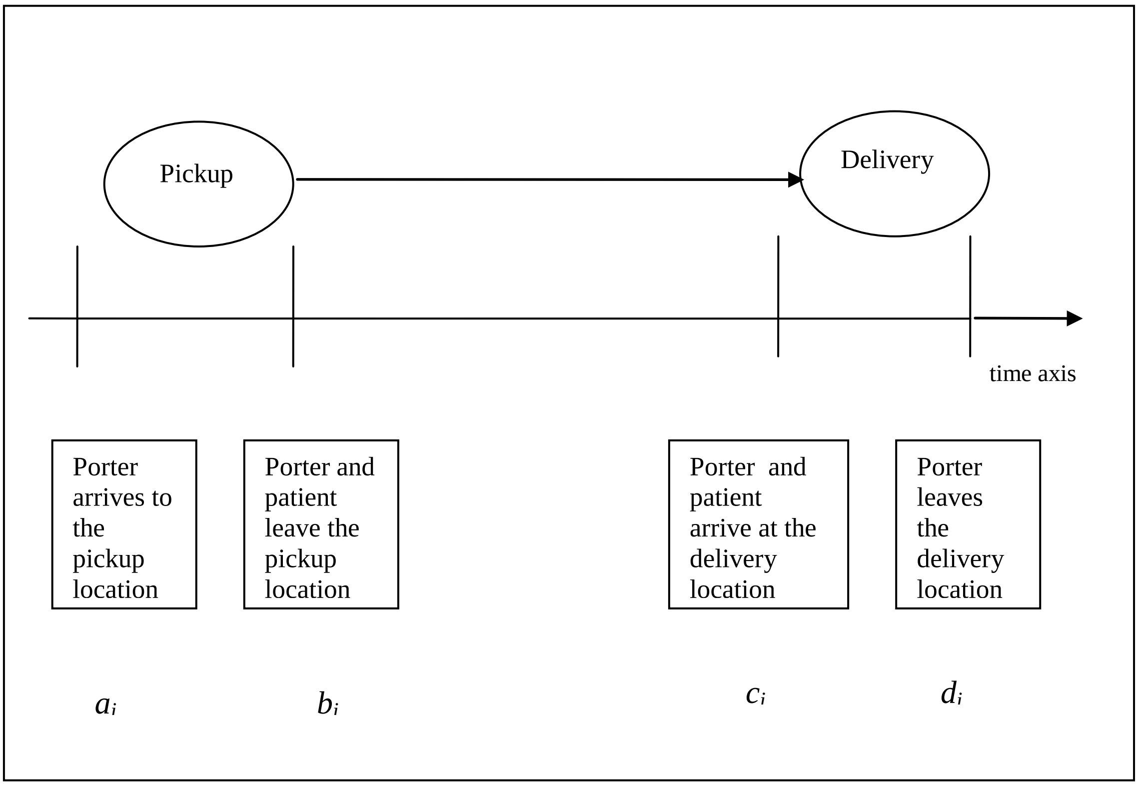

Figure 4: Variables associated with time

With a_i the porter's arrival time to the pickup node of a transportation request *i* is represented. After the porter has arrived to the pickup location she picks up the patient, if the patient is ready (e.g. if the medical examination has already ended in the case of outbound transportation request), the actual transportation request can start. The time when the transportation request *i* starts (when the porter leaves the pickup location together with the patient) is represented with the variable b_i. The actual travel time between pickup and delivery node of the transportation request *i* is included in the service time for each request. The arrival time of porter and patient to the delivery location of the transportation request is represented with c_i. After the porter has escorted the patient to her delivery location, she can leave the delivery node. The time when the porter leaves the delivery node of the transportation request *i* is represented with d_i.

As already mentioned, transportation requests of the patients can be grouped into inbound and outbound requests which is important in order to determine time windows. Now, after the decision variables have been introduced, these time windows can be more deeply explained and graphically illustrated. The time windows and associated variables by an inbound transportation request are depicted in Figure 5.

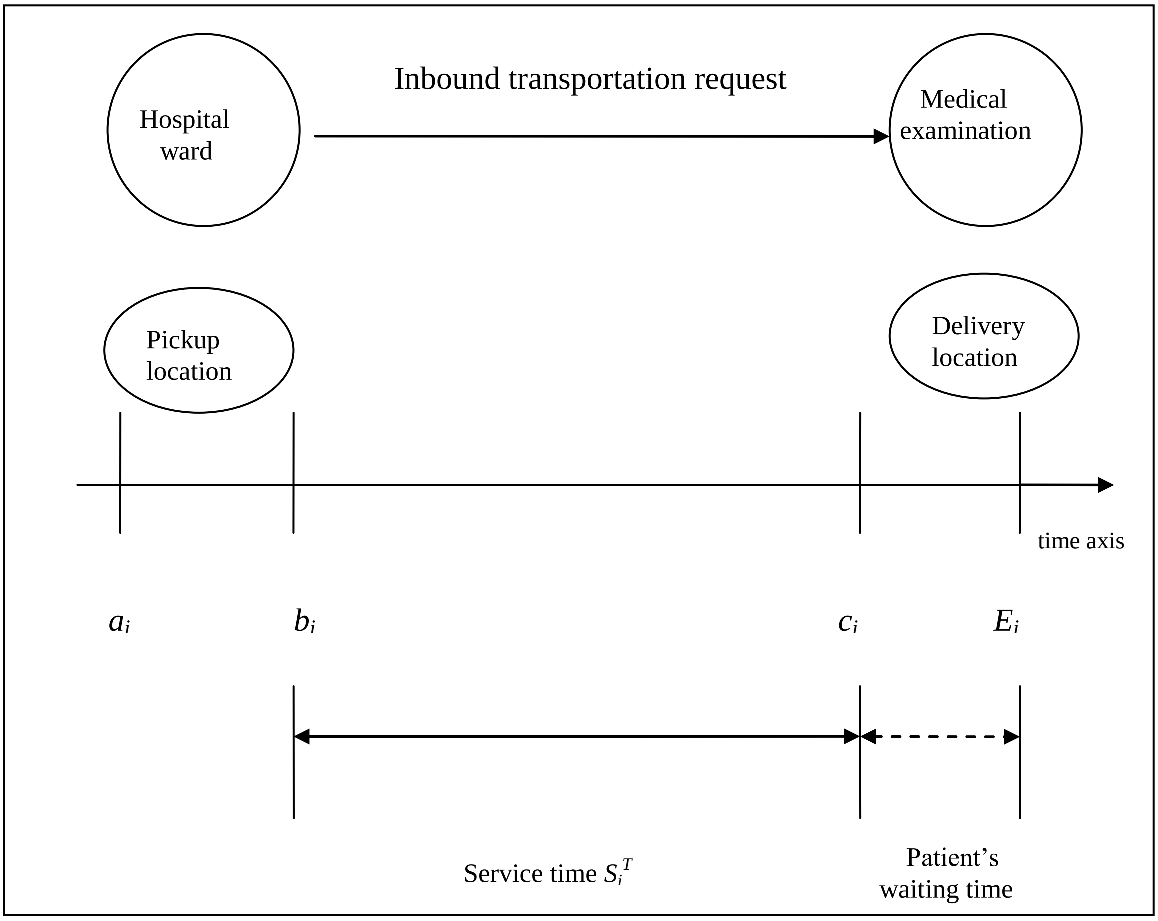

Figure 5: Time window by inbound transportation request

In the case of inbound transportation request, it is important that the patient is brought on time for her medical examination. Therefore the time point when porter arrives with the patient at the delivery location (in this case, the hospital unit where the medical examination is scheduled), c_i, must come before the actual due date of the medical examination, E_i. However, the patient should not be brought to her medical examination too early. In this case, waiting time for the patient is imposed. The aim of this work is to try to minimize this waiting time, i.e. to minimize the difference between the due date of the medical examination and the arrival point to the delivery location. In the figure, this difference is illustrated with dashed line.

On the other hand, in the case of an outbound transportation request, the waiting time occurs at the pickup location, where the critical time point is the end point of the medical examination. Waiting time and decision variables associated with time in the case of an outbound transportation request are pictured in Figure 6.

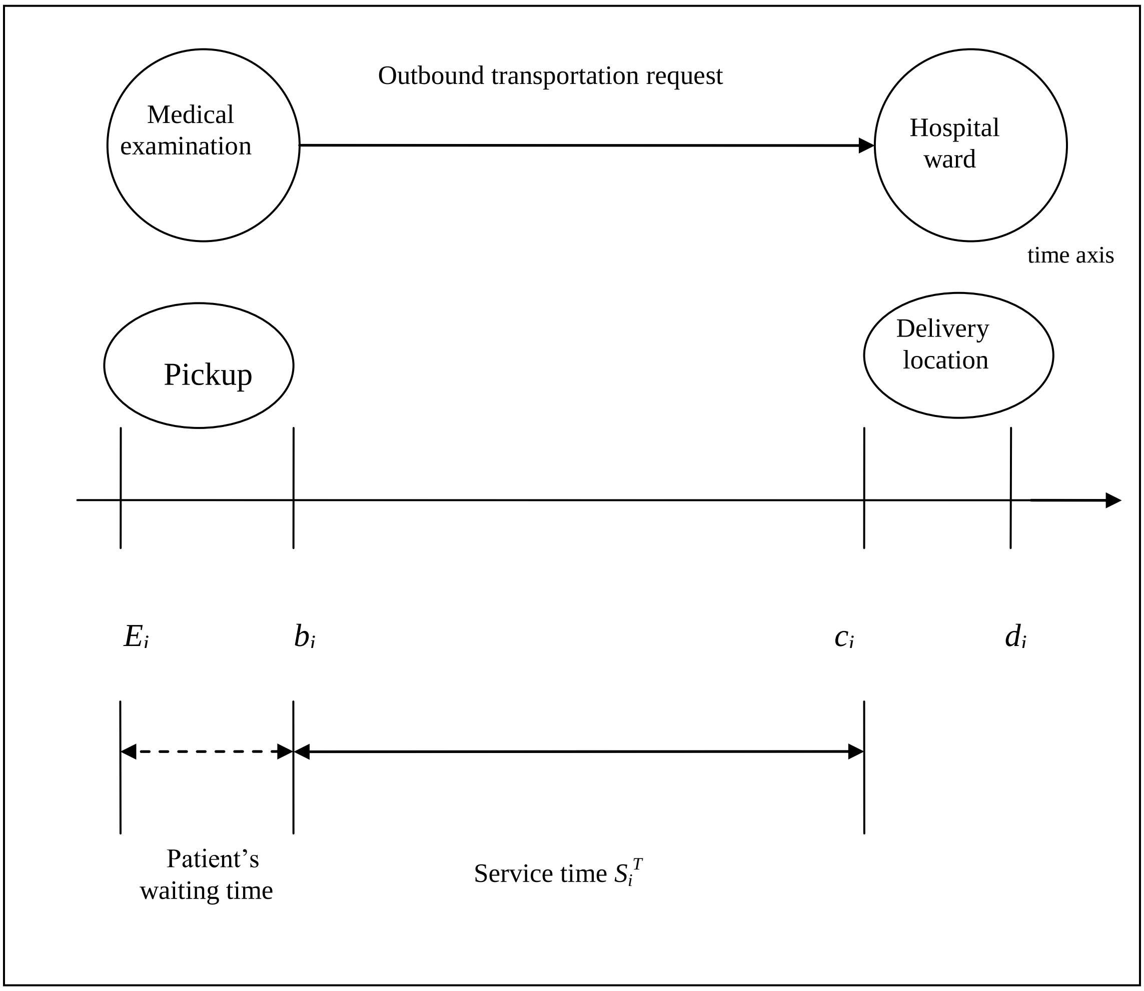

Figure 6: Time window by outbound transportation request

For outbound transportation requests the critical time point is when the medical examination ends. The porter can arrive at the pickup location either before the medical examination ends or after. In case that the porter arrives earlier, she will have to wait until the medical procedure ends, so that the transportation request can start, but this waiting time is of no importance. However, if she arrives late, i.e. if the beginning time point of the transportation request, b_i, lays far beyond the end of medical examination, then the patient inconvenience is going to be increased. Therefore it is necessary to minimize the imposed patient's waiting time in the model, i.e. to minimize the difference between the time point when porter and patient leave the pickup location and the time point when the medical examination ends.

All decision variables that were used in the model are listed in Table 4.

Abbreviation	Description
a_i	Arrival time to the pickup location of request *i*
b_i	Start time of the transport of request *i*
c_i	Arrival time to the delivery location of request *i*
d_i	Time point when porter leaves the delivery location of request *i*
x_{ijs}	Binary flow variable
y_{is}	Binary assignment variable

Table 4: Decision variables

After all the data and indices used have been introduced, and the decision variables explained, the mathematical model can be presented:

Minimize

$$\alpha * \sum_{i \in \boldsymbol{R}^*} \sum_{j \in \boldsymbol{R}^*} \sum_{s \in \boldsymbol{S}} x_{ijs} T_{ij} + \beta * \left(\sum_{i \in \boldsymbol{R}^I} (E_i - c_i) + \sum_{i \in \boldsymbol{R}^O} (b_i - E_i) \right) \qquad (1)$$

subject to

$$\sum_{s \in \boldsymbol{S}} y_{is} = 1 \qquad \forall i \in \boldsymbol{R} \qquad (2)$$

$$y_{\boldsymbol{D}(s)s} = 1 \qquad \forall s \in \boldsymbol{S} \qquad (3)$$

$$y_{is} = \sum_{j \in \boldsymbol{R}^*} x_{ijs} \qquad \forall i \in \boldsymbol{R}^*, \forall s \in \boldsymbol{S} \qquad (4)$$

$$y_{is} = \sum_{j \in \boldsymbol{R}^*} x_{jis} \qquad \forall i \in \boldsymbol{R}^*, \forall s \in \boldsymbol{S} \qquad (5)$$

$$c_i = b_i + S_i^T \qquad \forall i \in \boldsymbol{R} \qquad (6)$$

$$a_i \le b_i \qquad \forall i \in \boldsymbol{R} \qquad (7)$$

$$c_i \le d_i \qquad \forall i \in \boldsymbol{R} \qquad (8)$$

$$c_i \le E_i \qquad \forall i \in \boldsymbol{R}^I \qquad (9)$$

$$E_i \leqslant b_i \qquad \forall i \in \boldsymbol{R}^O \qquad (10)$$

$$d_i + T_{ij} \leqslant a_j + M(1 - x_{ijs}) \qquad \forall i,j \in \boldsymbol{R}^*, \; s \in \boldsymbol{S} \qquad (11)$$

$$d_i + T_{ij} \geqslant a_j - M(1 - x_{ijs}) \qquad \forall i,j \in \boldsymbol{R}^*, s \in \boldsymbol{S} \qquad (12)$$

$$x_{ijs} \in \{1,0\} \qquad \forall i,j \in \boldsymbol{R}^*, \forall s \in \boldsymbol{S} \qquad (13)$$

$$y_{is} \in \{1,0\} \qquad \forall i \in \boldsymbol{R}^*, \forall s \in \boldsymbol{S} \qquad (14)$$

$$a_i, b_i, c_i, d_i \geqslant 0 \qquad \forall i \in \boldsymbol{R}^* \qquad (15)$$

To determine the objective function (1), the weighted sum approach has been used. Each of three different terms that are considered in the objective function have to be multiplied with a penalty coefficient, depending on how important a particular goal is. However, these terms can be divided into two categories, so there are only two different coefficients. The penalty coefficients were subjectively estimated by the author.

The empty travel time of porters is represented with the first term of the objective function. The optimal route should be created so that the travel times of porters are as short as possible. As already mentioned, the pickup and delivery node of one transportation request are paired, that means that the time travelled between a pickup location and its associated delivery location is fixed and cannot be changed or improved anymore. With the first term in the objective function, the total travel time of porters, i.e. the travel time between the delivery node of transportation request *i* to the pickup node of transportation request *j*, is measured. However, porter *s* must be assigned to transportation request *j* after request *i*. As x_{ijs} is defined for R*, the first term also contains the travel times from the porter's home depot to the first transportation request (or rather to the pickup location of her first transportation request) and from the delivery location of her last request to her home depot.

With the other group of terms it is made sure that the comfortableness of the patients is also considered in the optimal plan. The second term is used for inbound requests. With this sum the total waiting time for patients when they arrive at this facility until the time point when their medical procedure is scheduled is minimized. Similarly, the third term serves for the outbound requests, and ensures that patients have to wait as short as possible to be picked up after their medical procedure and brought back to their bed. Both terms are multiplied with the same penalty coefficient in the objective function, as it is equally important for us that patients don't have to wait long in both cases.

In order to obtain the optimal solutions, some restrictions have to be considered. Constraints (2) make sure that each transportation request is served once. With (3) it is ensured that each porter has to visit her home depot. Constraints (4) and (5) are in and out degree constraints.

Restrictions that make sure that every transportation request is completed punctually also need to be considered. With (6) the arrival time at the delivery location is linked to the arrival time at the delivery location. As depicted in Figure 7, the service time includes the time needed to prepare the patient for the transport and the travel time between pickup and delivery location of patient's transportation request. Inequalities (7) and (8) ensure that the start of the transportation request cannot begin if the porter has not arrived yet and that the porter cannot be available for the next transportation request before she has delivered the patient to the target location. A porter can be responsible for only one transportation request at a time. In case of inbound request, with inequality (9) is guaranteed that the patient must not arrive at the delivery location after the time point for which the medical procedure is scheduled, i.e. the patient should not come late for her medical procedure. On the other hand, in case of outbound request, constraint (10) makes sure that the beginning of the transport is after the end point of the surgery, i.e. the porter and the patient can leave the hospital unit only after the medical procedure has ended. If the porter comes earlier to pick up the patient after the medical procedure, she has to wait until the procedure has been done, and the transport must not start until then. Inequalities (11) and (12) enable the connection between two consequent transportation requests. Constraints (13), (14) and (15) are binary constraints and constraint (16) is a non-negativity constraint.

The described mathematical model was implemented in XPRESS-Solver. In Section 4 the author is going to introduce and explain the data used and the solutions obtained. There will be more words on how different data sets were generated, which types of problems were considered and different solutions will be discussed.

3.3 Patient-centered extension

In this section the first extension of the model is going to be introduced. For this extension the general model has been used and the necessary changes were applied.

The extension is based on the wish to provide the best service for patients. The quality of hospital service is measured through patients' satisfaction. Therefore their convenience and well-being should be of high priority for hospital management. In the general model the main goal was to reduce the patients' waiting time. Now the question imposes what else could be done in order to increase the patients' convenience.

It was already mentioned that every patient triggers two transportation requests, one from her hospital ward to the medical examination room and the other one from this examination room back to the hospital ward. As the medical examinations are stressful for the patients by itself, and the necessary transportation further increases this stress, it would be reasonable to try to reduce this inconvenience by assigning the *same* porter to take care of the patient. If the same person picks up the patient and escorts her to her medical examination and afterwards picks her up and delivers back to bed, the patient could develop a feeling of trust toward this person. Knowing that there is one porter who is responsible for them could increase the comfort for the patient.

Therefore, the author wanted to extend the model that would find the assignment in such way, that each patient is assigned to same porter. In order to obtain the resulting assignments and routes, the necessary changes need to be implemented in the model. As this extension has been based on the general model, only the changes will be stated.

The data and indices used as well as the decision variables remain the same as in the general model. There are also no changes in the objective function. However, additional constrain is needed that will make sure that the same porter is assigned to one patient. This can be done with the following constraint:

$$y_{is} = y_{js} \qquad \forall s \in \boldsymbol{S},\ i,j \in \boldsymbol{R} \cap \boldsymbol{A(i)} = \boldsymbol{A(j)} \qquad (16)$$

Constraints (16) make sure that if *i* and *j* are transportation requests of the same patient, then porter *s* has to take care of both requests. In this case, *A(i)* stands for a patient to whom transportation request *i* refers to.

This model has also been implemented in XPRESS and solved as the general model. At the end, all models are compared. The analysis of the solution obtained is stated in Section 4.

3.4 Hospital-centered extension

In this section the second extension of the model, which considers management related issues, is going to be introduced. The second extension is also based on the general model which was then extended and adjusted.

So far it was assumed that medical assistants are assigned to complete the transportation requests in the following way: they start their workday at their home depot, they perform the transportation requests they were assigned for and after they have completed all of them, they go back to their home depots. This model however doesn’t take into consideration the time that porters have to wait empty between two consecutive transportation requests. It is assumed that a porter completes one transportation request, picks up a patient and delivers her at the desired location and then goes to the pickup location of her (porter's) next transportation request. If the porter arrives there earlier than planned, she has to wait. From the managements' point of view this however is suboptimal. Porters are also responsible for executing additional tasks, beyond the transportation requests. So far (empty) waiting times occurred somewhere in the hospital compound and the management is not able to use their resources efficiently. Hence it would be advisable for hospital management to consider this issue explicitly. By sending porters temporarily back home to their home depots, porters could be assigned other tasks there. This may lead to a deterioration of the solution with respect to the distance travelled empty by porters as they may encounter a detour via their home depot. On the other side this allows to efficiently use their resources for other tasks (i.e. collection of blood samples, delivery and supply of medical instruments, etc.) Porters however should only be sent back to their home depots if the resulting time they can spend there exceeds a certain minimum time span.

The waiting times that occur from the view point of porters are illustrated in Figure 7. To simplify the problem, it can be assumed that the waiting times for porters occur in two different situations. On one hand, if the porter arrives too early at the pickup location, she will have to wait until the patient is ready for the transport to begin. In this case, the waiting time for porter is the difference between the time when porter arrives at the pickup location, a_i, and the time point when patient and porter leave the pickup location, b_i. From this follows that the waiting time for porter is $(b_i - a_i)$. On the other hand, it may also happen that the porter waits unnecessarily at the delivery location. In order to minimize the waiting times at the delivery location, one must minimize the difference between the time point when the porter leaves the delivery location, d_i, and the time point when she had arrived at the delivery location, c_i. In this case, the waiting time for porter is equal to $(d_i - c_i)$.

At this point, it may be important to mention that in the case of an outbound transportation request, waiting time for porter usually occurs at the pickup node. If the porter arrives at the pickup location before the medical examination has ended, she has to wait until the patient is ready for the transport. These waiting times were mentioned, but not considered in the general models, but the testing that was performed has shown that porters wait between 5 minutes to more than one hour. The unnecessary long waiting times could be put to better use if the porter was sent to her home depot instead.

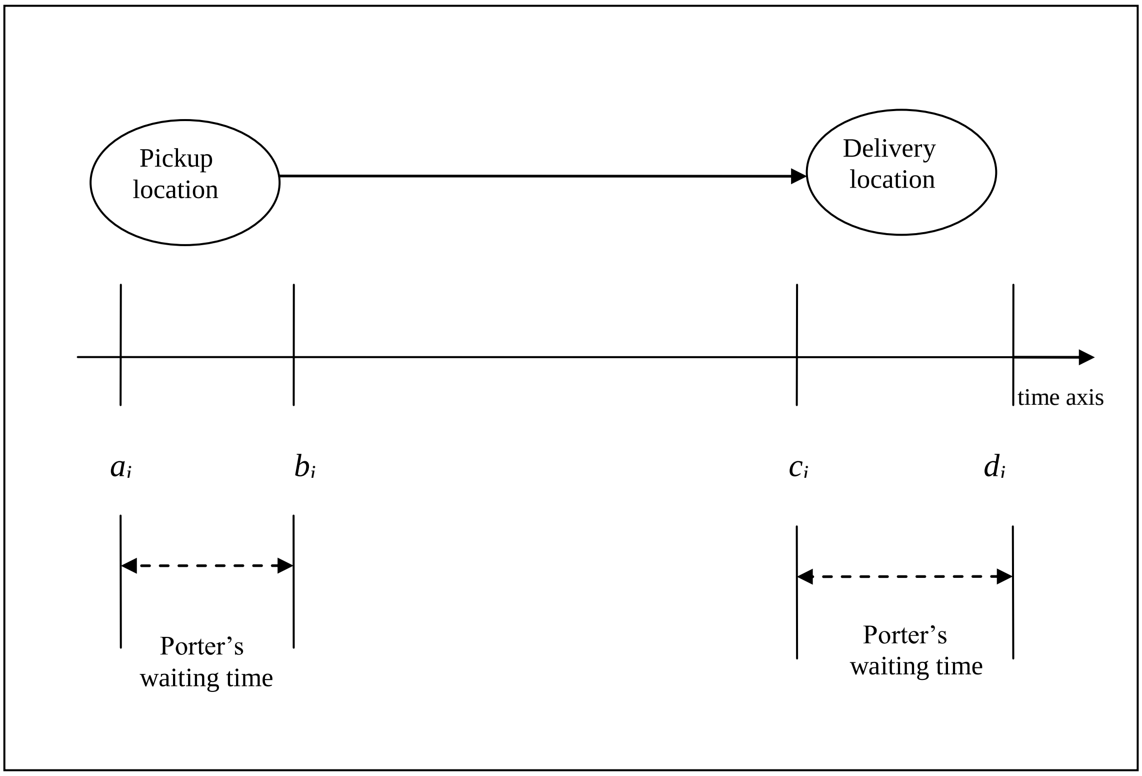

Figure 7: Porter's waiting time

Taking all these information into consideration, the model has been changed and extended accordingly. As the general model was the basis, now only the changes that were necessary to implement in order for the model to work properly are going to be introduced. One of the main changes was to introduce new decision variables that will force the model to send the porter back home if there is enough time and another variable to capture the actual time travelled empty. The new model is graphically illustrated in Figure 8.

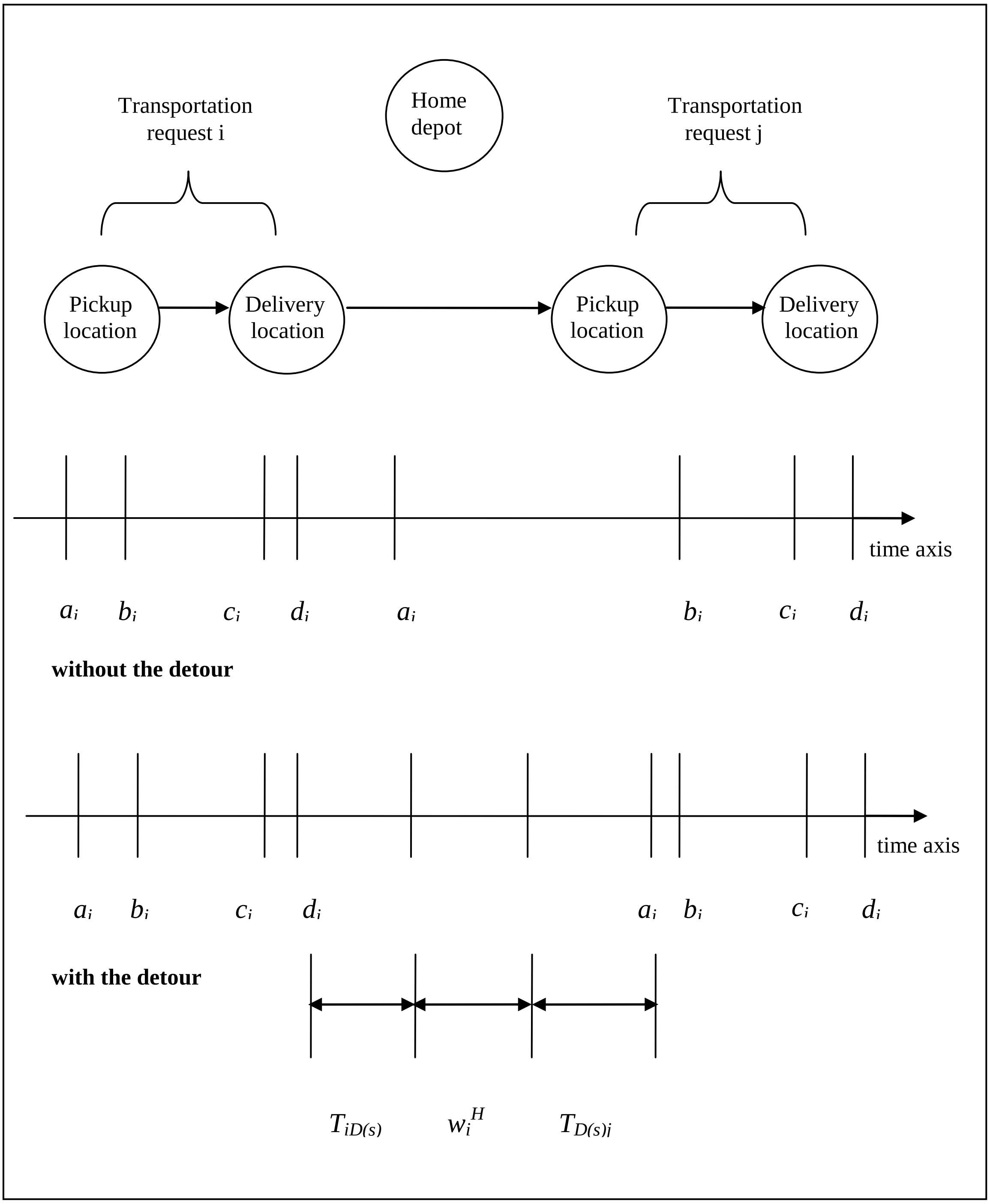

Figure 8: Detour option

If the porter leaves the delivery node of the transportation request *i* at the time point d_i and immediately goes to the pickup location of her next transportation request *j*, then she will have to wait until the transport can start (the waiting time is given as a difference $b_j - a_j$). However, if there is enough time, she could be sent to her home depot in the meanwhile. In this case, the waiting time of the porter will be minimized and the porter could complete some other duties while in her home depot.

The decision whether porter goes back to her home depot between her two consequent transportation requests is represented by a binary variable w_i. This variable is defined for $\forall\, i \in \mathbf{R}^*$ and is equal to 1 if the porter temporarily goes back to her home depot after she has completed her transportation request i. However, w_i is always set to 0 for $\forall\, i \in D$ There is a difference between w_i and $x_{iD(s)s}$, where the latter one refers to the last transportation request i of the porter s, after which completion the porter will finally go back to her home depot and won't be sent to complete any other transportation requests that day. In the first case, porter will only go to her home depot, spend some time there, but then leave the depot in order to complete some other transportation requests.

The actual travelling time spent empty between two transportation requests i and j by porter s is no longer constant. It will be represented by a variable t_{ijs}, capturing an eventual detour via her depot. The new data used are listed in Table 5.

Abbreviation	Description
t_{ijs}	Time travelled between request i and j by porter s
w_i^H	Time that porter spends in her home depot after request i
w_i	Binary variable equals 1 if porter is sent back home temporarily after request i
T^W	Minimal waiting time porter needs to spend at home depot
γ	Penalty, if porter has to wait idle between transport requests

Table 5: Decision variables and parameters for 2nd extension

As far as the mathematical model is concerned, we are now only going to state the changes of the general model that took places and list the additional constraints.

Minimize

$$\alpha * \sum_{i\in \boldsymbol{R}^*} \sum_{j\in \boldsymbol{R}^*} \sum_{s\in \boldsymbol{S}} t_{ijs} + \beta * \left(\sum_{i\in \boldsymbol{R}^I} (E_i - c_i) + \sum_{i\in \boldsymbol{R}^O} (b_i - E_i) \right) + \gamma * \left(\sum_{i\in \boldsymbol{R}} (b_i - a_i) + \sum_{i\in \boldsymbol{R}} (d_i - c_i) \right) \quad (1)$$

subject to

$$w_i + x_{i\boldsymbol{D}(s)s} \leqslant 1 \qquad \forall i \in \boldsymbol{R}, s \in \boldsymbol{S} \quad (17)$$

$$T_{i\boldsymbol{D}(s)} + T_{\boldsymbol{D}(s)j} + w_i^H \leqslant a_j - d_i + M(2 - x_{ijs} - w_i) \qquad \forall i, j \in \boldsymbol{R}^*, s \in \boldsymbol{S} \quad (18)$$

$$T_{i\boldsymbol{D}(s)} + T_{\boldsymbol{D}(s)j} + w_i^H \geqslant a_j - d_i - M(2 - x_{ijs} - w_i) \qquad \forall i \in \boldsymbol{R}, j \in \boldsymbol{R}^*, s \in \boldsymbol{S} \quad (19)$$

$$w_i^H \geqslant T^W * w_i \qquad \forall i \in \boldsymbol{R} \quad (20)$$

$$T_{ij} + d_i \leqslant a_j + M(1 - x_{ijs} + w_i) \qquad \forall i, j \in \boldsymbol{R}^*, s \in \boldsymbol{S} \quad (21)$$

$$T_{ij} + d_i \geqslant a_j - M(1 - x_{ijs} + w_i) \qquad \forall i, j \in \boldsymbol{R}^*, s \in \boldsymbol{S} \quad (22)$$

$$t_{ijs} \geqslant T_{i\boldsymbol{D}(s)} + T_{\boldsymbol{D}(s)j} - M(2 - x_{ijs} - w_i) \qquad \forall i \in \boldsymbol{R}, j \in \boldsymbol{R}^*, s \in \boldsymbol{S} \quad (23)$$

$$t_{ijs} \geqslant T_{ij}(x_{ijs} - w_i) \qquad \forall i, j \in \boldsymbol{R}^*, s \in \boldsymbol{S} \quad (24)$$

$$w_i \in \{1,0\} \qquad \forall i \in \boldsymbol{R} \quad (25)$$

$$w_i^H \geqslant 0 \qquad \forall i \in \boldsymbol{R} \quad (26)$$

$$t_{ijs} \geqslant 0 \qquad \forall i, j \in \boldsymbol{R}^*, \forall s \in \boldsymbol{S} \quad (27)$$

There have been few changes in the objective function. Instead of the first sum in the general model, where the total time travelled by the porter *s* was only depending on the time travelled between her two consequent transportation requests *i* and *j*, the new term calculates the sum of time travelled between two transportation requests, and also adds up the travel time porter needed to go to her home depot and back, if she goes there temporarily between these requests. There was also one more group of goals added. The fourth and the fifth term in the objective function stand for the waiting time of the porter, either before or after having executed a transportation request. These waiting times are penalized with the same coefficient in the objective function, as these waiting times are equally important. However, it is important to say that these waiting times refer only to waiting times between two transportation requests in case when porter doesn't go back to her home depot. The waiting time in the home depot is not penalized, as it is important that porter spends as much time there as possible, so that she could be assigned to some other duties. The fourth term penalizes the waiting times for porters when they arrive at the pickup location and have to wait until the transportation request starts. Similarly, the fifth term penalizes the waiting time at the delivery location, when porter has delivered a patient and waits there to be assigned for the next transportation request.

There have also been few changes among the constraints. Instead of Constraints (11) and (12) in the general model that were used to describe the connection between two consequent transportation requests of a porter, the new constraints (17) to (24) have been added. With (17) it is made sure that porter can either temporarily go home between her two transportation requests, spend some time there and then leave the depot again to complete some other transportation requests, or the porter can finally go to her home depot at the end of her workday. Constraints (18) and (19) make sure that if a porter goes to her home depot temporarily, there is enough time between two successive transportation requests. There should be enough time for porter to go to her home depot after one transportation request, to spend some time there and then go to her next request. Time between two transportation requests is measured as the difference between the time when porter is free from her first transportation request and the time when she should arrive at her next transportation request, if the porter is assigned to complete transportation request j after request i. Inequalities (20) ensure that, if the porter goes back to her home depot after transportation request i, she should spend at least some given time span T^W there. Constraints (21) and (22) make sure that there is a connection between two transportation requests. With (23) and (24) the total travel time between two successive transportation requests is modeled. The travel time will either include the travel time from the delivery location of porter's last transportation request to her home depot and from her home depot to the pickup location of her next request, in case that porter goes back to her home depot after she has completed the transportation request or the travel time will be equal to the travel time from the delivery location of her last transportation request to the pickup location of her next request, in case that the porter goes straight to her next request without visiting the home depot. Constraints (25) are binary constraints, and constraints (26) and (27) are non-negativity constraints.

These changes have also been implemented and the obtained solution as well as the comparison to the general model is going to be discussed in Section 4.

4 Numerical results

In this section, the data used in the model will be introduced and explained. All instances have been tested with XPRESS until the optimal solution was found, or until the termination criterion has been reached. For the purposes of this work, the termination criterion has been set to one hour of computation time. If the optimal solution has not been found by then, the best solution found so far has been used as an objective value and all analysis and comparisons have been made using this value.

4.1 Instances

For the purpose of this work, five different classes of instances have been generated. They differ in the number of transportation requests and the number of porters that are available. For the first three classes, ten different instances have been created and tested. For the fourth class, the number of porters was fixed, and the number of transportation requests has been varied. Finally, for the fifth class, the number of transportation requests has been fixed, and the number of porters has been varied. However, the travel matrix and penalty coefficients remain the same for all instances used. The number of porters and transportation requests, as well as the penalty coefficients, has been set by the author.

For the first class, the number of porters was set to 2 and the number of patients to 3, respectively. Hence a total number of 6 transportation requests have to be considered. Ten instances were created and tested for general version of the model and for both extensions of the model. The solution for the general model, and also the comparison of the general model solution to the solutions of the two extensions is given in the following sections.

Similarly, for the second (third) class of instances the number of porters was set to 3 (4) and the number of patients was set to 5 (7). For instances belonging to class II additional test runs have been made, to investigate how the solution changes if the minimal acquired time porter has to spend in home depot (T^W) varies. For the fourth class, the number of porters was fixed to 4 and the number of patients has been varied from 8 to 20. The solution and the sensitivity analysis are given in the next section. Finally, for the fifth class, the number of patients has been set to 15, and the number of porters has been varied from 1 to 10.

An overview on the size of problem instances under consideration can be found in Table 6.

Class	Number of porters	Number of patients
I	2	3
II	3	5
III	4	7
IV	4	8-20
V	1-10	15

Table 6: Classes of instances

For all instances the data settings were the same, with only difference in the *critical time points*. Each patient has one appointment scheduled for that day, what is the reason for two patient’s transportation requests. For the inbound transportation request, where the patient has to be brought from her hospital ward to the appointment, the critical point is the beginning of her medical procedure. On the other hand, for outbound transportation request, where the patient has to be picked up after her medical procedure and brought back to her bed, the critical time point is the end of her medical procedure. The medical procedure last approximately 20 to 30 minutes.

The travel time matrix has been drawn from real-world data. A simplified layout of the hospital is given in Figure 9. The travel times were calculated using Manhattan distance.

The penalty coefficient for the time travelled α is set to 1, penalty coefficient χ (waiting time of porters) is set to 2, and the penalty coefficient for the waiting time of patients (β) is set to 3. This can be interpreted as following: one minute that patient (porter) has to wait is three (two) times more important than an additional minute that porter has to travel. These coefficients can easily be adapted by decision makers in order to reflect their true preferences.

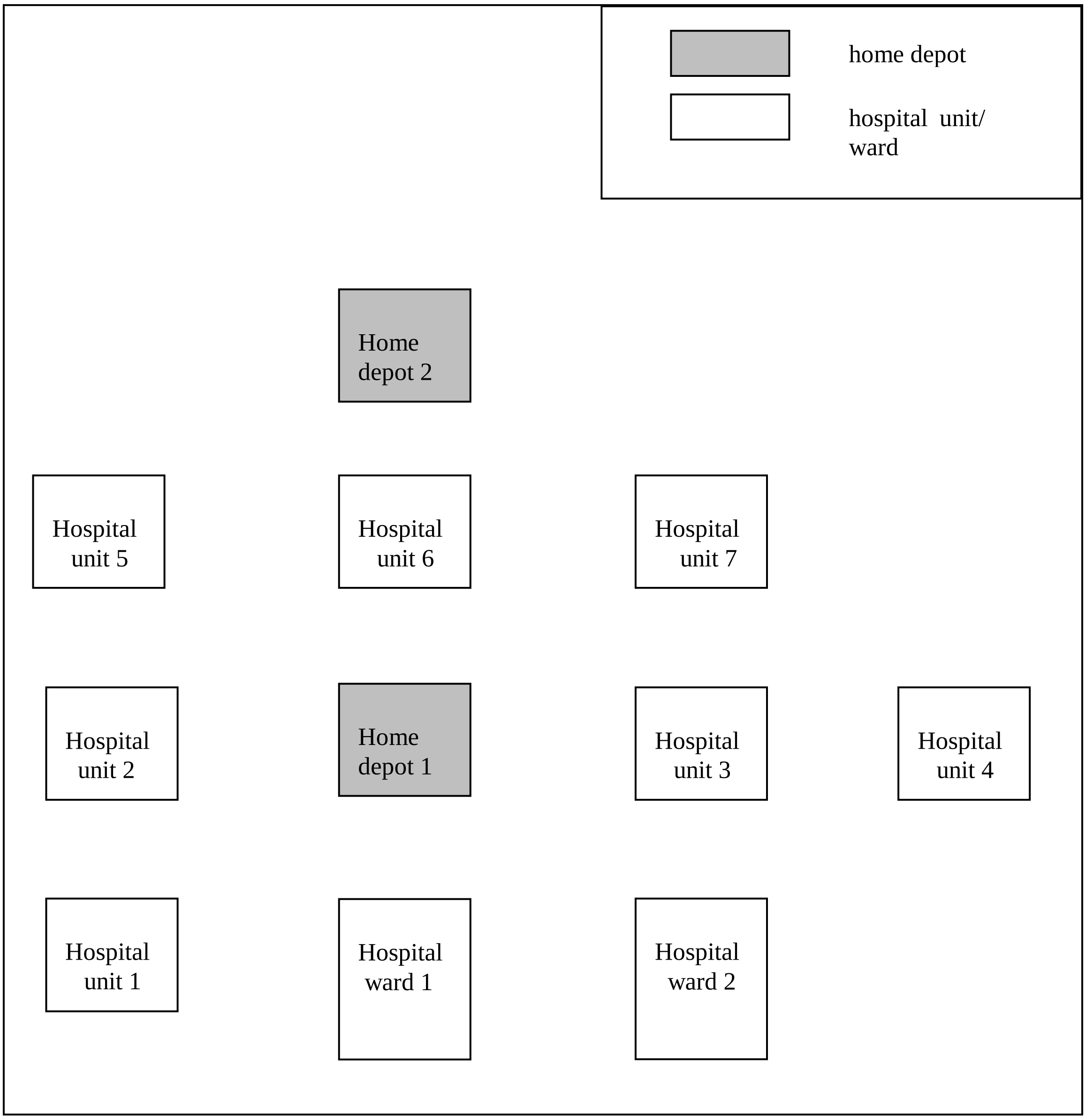

Figure 9: Layout of the hospital

The pickup and delivery nodes have been selected randomly among hospital nodes (hospital nodes and hospital units where medical procedures are being done) depending on the type of the transportation request.

4.2 General model

In order to test the model for the proposed instances for the first three classes the model has been solved using XPRESS. The run time has been set to one hour or until the optimal solution has been found. If the optimal solution has not been found, the best solution found so far has been used as value of the objective function and gap between this value and the current best bound has been calculated.

The obtained solution with XPRESS for the first class is given in Table 7. The table contains all relevant data related to the patients' transportation requests, e.g. average waiting time from both porters' and patients' point of view, total time travelled by porters and average time needed to compute the results. The gap used in the model is calculated as the difference between the best solution found so far and the current best bound, given in percent of the best bound.

N	f	tt	w (s)	w (p)	No (p)	GAP	time
1	28	28	18	0	1.33	0.00%	0.08
2	28	28	39	0	1.00	0.00%	0.06
3	21	21	80	0	1.33	0.00%	0.03
4	28	28	27	0	1.00	0.00%	0.05
5	47	47	11	0	1.33	0.00%	0.11
6	28	28	39	0	1.33	0.00%	0.03
7	36	36	54	0	1.33	0.00%	0.06
8	28	28	64	0	1.33	0.00%	0.03
9	28	28	27	0	1.00	0.00%	0.11
10	21	21	80	0	1.33	0.00%	0.03
avg	29.30	29.30	50.32	0.00	1.23	0.00%	0.06

Table 7: General model (1ST Class)

For the first class of instances tested, average value of the objective value is 29.30, weighted over 10 instances. The total time travelled by porters (headed by tt) is equal to the objective function value. As already mentioned, the objective function (f) only considers the porters' empty travel times (travel times that porters travel without patients) and patients' waiting times (headed by w (s)). In the optimal solution, the patients don't have to wait at all; therefore the value of the objective function is equal to the time travelled by porters. On the other hand, porters do have to wait, either in case that they have to pick up the patient after the medical procedure, where the porter waits at the pickup node, or in case that they have just dropped off a patient, when they wait at the delivery node. However, the porters' waiting time has not been considered explicitly during the stage of optimization, but the solution has been evaluated later. This feature has been considered in Section 4.4. Waiting time for porters is headed by w (p). The average waiting time for the porters is 50.32 seconds. The average number of porters that are assigned to one patient (No (p)) is 1.23, what can be interpreted in the following way: approximately 33% of patients were escorted by different porters, whereas the two thirds were executed by the same porter on both transportation requests to/ from their ward. Please note that also the minimization of the number of porters in use per patient has not been considered during optimization process in the general model. However, in Section 4.3 the number of porters was forced to one. The model has been solved to optimality for all ten instances; hence the gap is equal to zero. The model could be solved in only couple of centiseconds.

The notation used in Table 7 as well as in the following tables is listed in Table 8.

Abbreviation	Description
|p|	Number of patients
|po|	Number of porters
N	Instance number
C	Class of Instances
f	Value of the objective function
tt	Total travel time (empty)
w (s)	Total porters' waiting time
w (p)	Total patients' waiting time
w^H	Time porter spends in her home depot
w^i	Number of times porter is sent to her home depot
No (p)	Number of porters that are assigned to one patient
GAP	Gap between best solution found so far and the best bound in %
No (OS)	Number of optimal solutions found
time	Elapsed run time until termination

Table 8: Notation

After the first class instances have been tested, the instances for the second and the third class have also been created and tested. The obtained results can be found in Appendix A. A comparison between these three classes is been given in Table 9. In the table, C stands for the class number, and with No(OS) the number of optimal solutions found (summed over ten different instances for each class) is represented.

C	f	tt	w (s)	w (p)	No (p)	GAP	No (OS)	time
1	29.30	29.30	50.32	0.00	1.23	0.00%	10	0.06
2	37.30	37.30	129.22	0.00	1.32	0.00%	10	0.30
3	49.40	49.40	111.94	0.00	1.24	0.00%	10	1.97
avg	38.67	38.67	97.16	0.00	1.27	0.00%	10	0.78

Table 9: General model - comparison

It can be concluded that the size of the instances tested influences the computation time. The time needed to find the optimal solution increases with the increase in the size of instances. However, all instances could still been solved to optimality and the optimal solution could be found within few minutes.

4.3 Patient centered extension

After the general model has been tested, the two extensions were also implemented in the XPRESS and solved. The first extension enforces that only one porter can be assigned to the same patient. In this case, the objective function value is slightly increased in comparison to the objective value of the general model. This can be interpreted as, that the costs to increase the patients' convenience and to make sure that they are taken care of by the same porter, are not much higher than in general case. The entire solution obtained for each instance tested is given in Appendix A.

A comparison between these three classes is given in Table 10. The requirement of only using one porter per patient has a minor impact on the quality of the solution obtained, due to mediocre additional empty travel times by porters. On average the resulting empty travel times (headed by tt) increase by 11.7% from 38.67 (obtained for the general model) to 43.2 minutes. From the patients' point of view, however, the situation improves. Waiting times still (w (p)) do not occur. With this extension the average number of porters in use per patient is forced to one (in contrast to 1.27 in the previous general case, where this feature has not been addressed explicitly). This additional constraint has only minor impact on the run times required. On average all instances can be solved to optimality within 0.8 seconds.

C	f	tt	w (s)	w (p)	No (p)	GAP	No (OS)	time
1	30.90	30.90	24.17	0.00	1.00	0.00%	10	0.03
2	42.80	42.80	26.61	0.00	1.00	0.00%	10	0.45
3	56.00	56.00	29.23	0.00	1.00	0.00%	10	2.05
avg	43.23	43.23	26.67	0.00	1.00	0.00%	10	0.84

Table 10: Patient–centered extension - comparison

4.4 Hospital centered extension

Up to now optimization from the porters' view was focused on their time travelled empty. Waiting times, i.e. time slots when they are off-duty, have not been considered explicitly. This, however, seems to be a waste of resources, as porters have other tasks to fulfill. Waiting times occurred between serving consecutive transportation requests somewhere within the hospital complex. Within this extension the author tried to include the minimization of waiting times (i.e. idle times) spent *somewhere* in the compound from the porters' point of view, by sending them back to their *home depots* if they are currently not assigned to a transportation request, where they are supposed to fulfill other tasks. Hence the goal now is to generate schedules for porters where idle times are connected, long enough and occur at the corresponding home depots.

The results for the hospital centered extension are given in Table 11. The minimal time that porters should spend at their home depot, in the case that they goes there between two transportation requests, was set to 15 minutes, for all classes tested. This time should be enough for porters to complete some other tasks that occur at their home depots. However, these tasks may acquire longer time. Therefore, in the next section, additional tests have been made where this minimal time has been extended up to 140 minutes.

C	f	tt	w (s)	w (p)	w^H	w^i	No (p)	GAP	No (OS)	time
1	61.20	57.30	3.90	0.00	107.21	3.10	1.27	0.0%	10	1.03
2	98.00	89.30	8.20	0.50	202.41	4.70	1.52	0.0%	10	373.8
3	138.10	116.80	9.40	11.90	262.72	6.30	1.64	95.9%	0	3792.3
avg	99.10	87.80	7.16	4.13	190.78	4.70	1.48	32.0%	6.67	1389.1

Table 11: Hospital-centered extension - comparison

In this extension, where the porters were sent to their home depots in case that there was enough time, a significant increase in the objective function could be noticed. This increase however is due to the increase in the time travelled empty by porters. What can be noticed is that the porters' average waiting time (headed by s (s)) is decreased significantly in comparison to the general model (7.16 vs. 97.16 minutes). This is due to the fact that porters now can go temporarily back to their home depots between two consecutive transportation requests. The time they spend in the home depot is represented with w^H and the number of times they go to the home depot is represented with w^i. On average porters are sent back to their home depots 4.7 times and spend 190.78 minutes there.

Considering this feature comes at high costs. The solution quality deteriorates in a three-fold way. The average number of porters (No (p)) in use per patient increases from 1.27 (in general model) to 1.5. Patient inconvenience is further increased by additional waiting times that occur before or after the transportation requests. These waiting times (headed by w (p)) increase from 0.0 (in the general model) to 4.13 minutes. Furthermore, empty travel times by porters (tt) increase from 38.67 to 87.8 minutes. This effect is not surprising as sending porters to their home depots leads to increased travel times.

With this extension, however, the underlying combinatorial complexity increases dramatically. In contrast to the previous case this extension heavily influences the solver's capabilities of quickly finding good (optimal) solutions. Especially for the third class it can be noticed, that none of the instances tested could be solved to optimality in one hour of running time. The gap between the best solution found and the best bound after 3600 seconds is still at 95.90%.

4.4.1 Sensitivity analysis

For the second class, a sensitivity analysis has been performed in order to depict how the required waiting time in the home depot affects the value of the objective function. The waiting time has been varied between 5 minutes to 140 minutes. The sensitivity analysis has been performed on one randomly chosen instance among those that were used for the second class. The values obtained are listed in Table 12.

T^W	f	w^H	w^i	w (s)	w (p)	Solution
5	110	193	7	0	0	A
10	110	193	7	0	0	A
15	110	193	7	0	0	A
20	132	185	7	0	7	B
25	149	193	5	17	10	C
30	169	218	5	12	18	D
35	177	225	4	3	24	E
40	177	225	4	3	24	E
45	177	225	4	3	24	E
50	179	223	3	10	24	E
55	179	223	3	10	24	F
60	179	223	3	10	24	F
80	193	168	2	77	11	G
100	201	120	1	101	25	H
120	201	120	1	101	25	H
140	208	0	0	122	9	I

Table 12: Sensitivity analysis

The minimal time that is required for a porter to spend at her home depot is denoted by T^W. The table shows the following: with an increase in the minimum time that needs to be spent at the porter's home depot (if porters are sent back there temporarily between two successive transportation requests), the number of times porters go back home (w^i) decreases. Simultaneously the time spent at home depot (w^H) first increases and starts to decrease starting from T^W = 50, as the total time spent there is offset by the reduced number of visits at the home depot. Similarly, the inconvenience of patients (measured in terms of their waiting times w(p)) first increases and starts to decrease again starting from $T^W = 80$, as it becomes less efficient to send porters back home.

The solutions obtained can be very helpful for the hospital management in order to determine the minimal time that porters should spend in their home depot. However, the hospital managers need to find a trade-off between two conflicting objectives, whether to reduce the patient inconvenience or to increase the time that porters spend at their home depot (i.e. increase the efficiency of porters' usage). A commonly used concept for these kinds of problems, when decision makers are asked to choose between two (or more) conflicting objectives, is Pareto optimality. A solution is Pareto optimal when one objective cannot be further improved unless the other objective deteriorates. The set of Pareto optimal solutions is called Pareto front.

A Pareto front for the solutions obtained in Table 12 is depicted in Figure 10. The decision makers (in this case the hospital management) can compare different solutions, can track how her decision influences the two variables and can decide which objective to favor. The goal of the hospital management is to find the optimal trade-off between patient waiting times and the time porters spend in their home depots. The solutions are "better" if the waiting time of patients is as short as possible and if the time porters spend in their home depot is as long as possible. The figure shows that solution H is worse than solution F (solution H is dominated by solution F) because patient waiting times are greater for solution H and the time that porters spend in their home depots are shorter. Hence, the hospital management would use the option F over option H. However, the solution F is weakly dominated by solution E. The waiting times for patients are the same for both solutions, but the time that porters spend in their home depot is greater for solution E. Similarly, solution A is better than solutions B, C and G. The hospital management would therefore have to choose between solutions A, D and E, as neither of these solutions dominates the others. The solution A is better from the patients' point of view, as the patients' waiting times are equal to zero. However, the solutions D and E are better from the porters' point of view, as the time they spend in their home depots are greater. The decision makers have to decide which goal is more important in order to choose one solution.

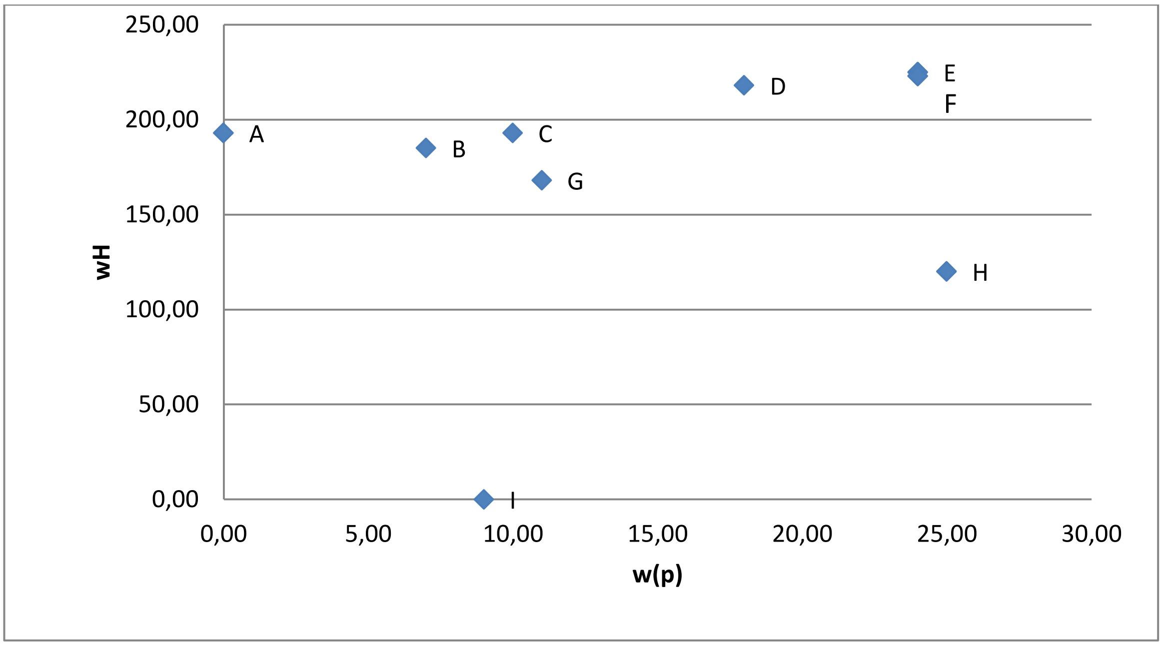

Figure 10: Pareto front

4.4.2 Evaluation of the solution quality

To show how the value of the objective function and the best bound change over time, a random instance is chosen among those that were used for the third class of instances. The relation is shown in Figure 11.

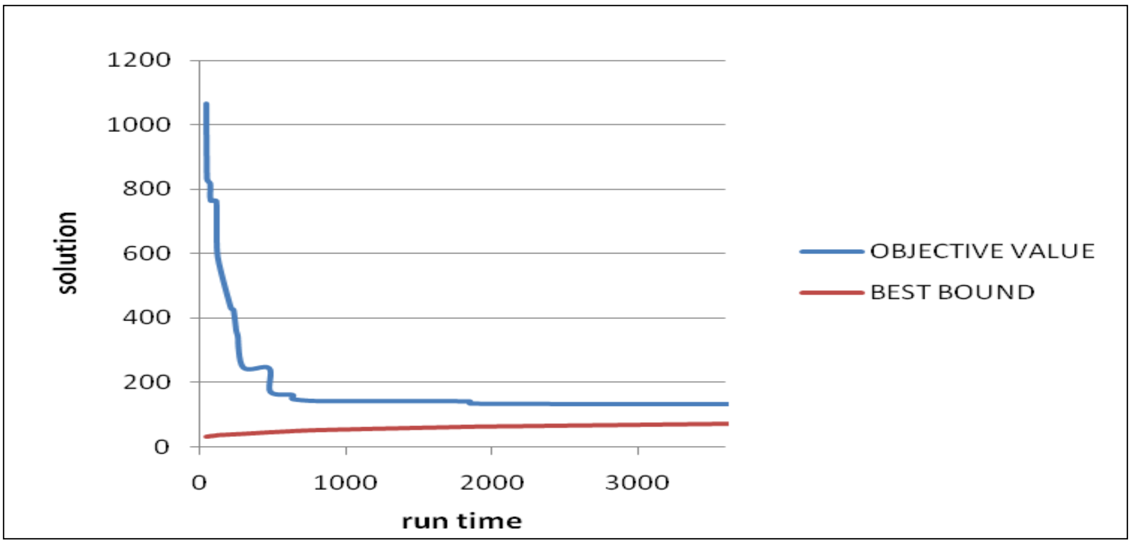

Figure 11: Objective value and Best Bound vs. time

Instances within the third class where among the largest instances under consideration, where the number of porters was set to four and 14 transportation requests had to be considered. For this particular instance the first feasible solution was found after 45.66 seconds, with an initial gap of 3363%. Within the first half of the total run time the solution quality could be improved by 86.6% to 142 and the resulting gap decreased to 125%. Within the last 1800 seconds the solution (gap) could only be improved marginally. The problem under consideration is static and operational by nature. Hence the optimization could be executed over night in order to generate a solution for the next day. It could be observed, however, that solutions of reasonable quality already could be obtained within half an hour.

4.5 General model vs. extension – comparison

In order to graphically illustrate the solutions obtained for in-house transportation of porters and to visualize the routes that porters are assigned to, one instance has been chosen and is now going to be examined explicitly. Instance 10 from the first class has been chosen for this purpose. In the first step, the data used for this instance are going to be stated. The problem captured in this instance is how to find the optimal assignment for a case when 2 porters have to be assigned to complete 6 different transportation requests, i.e. to accompany 3 patients on their transportation. In Table 13, critical time points and pickup and delivery locations for these six requests are given.

	inbound request			**outbound request**		
patient	1	2	3	1	2	3
transportation request	1	2	3	4	5	6
due date	9^{04}	11^{09}	10^{02}	9^{31}	11^{30}	10^{23}
pickup location	hospital ward 1	hospital ward 2	hospital ward 1	hospital unit 2	hospital unit 4	hospital unit 6
delivery location	hospital unit 2	hospital unit 4	hospital unit 6	hospital ward 1	hospital ward 2	hospital ward 1

Table 13: Transportation requests - example

The transportation requests are also graphically shown in Figure 12. The first patient has to be brought from hospital ward 1 to hospital unit 2. Similarly, the patient 2 (3) has to be brought from hospital ward 2 (1) to the hospital unit 4 (6). As all requests are paired, i.e. if the patient is escorted to the examination, she must also be picked up there afterwards and brought back to her bed, from the first three transportation requests (which are all inbound transportation requests) follow the remaining three transportation requests (the outbound transportation requests).

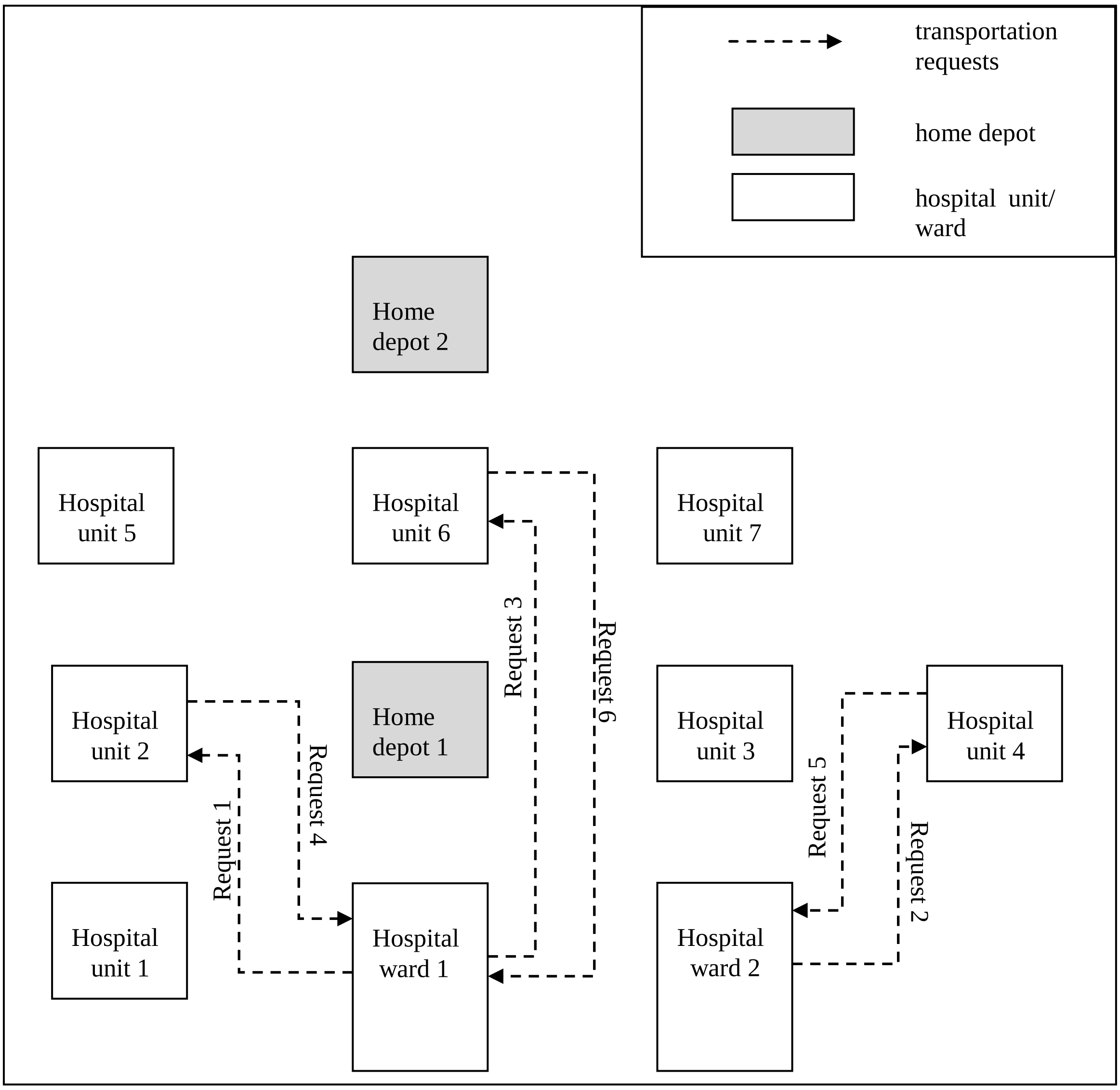

Figure 12: Transportation requests

The instances have been previously solved to optimality with XPRESS. The solution obtained for the general model and for both extensions is shown in Table 14.

Model	f	tt	w (s)	w (p)	wH	wi	No (p)	GAP	time
general	21	21	80	0	0	0	1.33	0.00%	0.03
1st extension	28	28	77	0	0	0	1.00	0.00%	0.03
2nd extension	54	46	8	0	51	3	1.00	0.00%	1.09

Table 14: Instance 10 - solution

The optimal assignment and the resulting routes for porters for the general model for the instance tested are shown in the table below.

Porter	order of transportation requests
1	Home depot – 1 – 4 – 3 – 6 – 2 – Home depot
2	Home depot – 5 – Home depot

Table 15: Optimal results – general model

The first porter starts at the depot, executes transportation request 1 (escorts patient 1), waits there until the medical procedure has ended, then escorts the patient back to her bed (executes transportation request 4). The porter then proceeds to execute transportation requests 3, 6 and 2 and then returns to her home depot. On the other side, porter 2 leaves the depot only to execute transportation request 5, after which she goes back to her home depot. The optimal routes for both porters are colored accordingly. *Dashed line* has been used for the paths (travel distances) that do not occur in the objective function. *Continuous line* has been used to accentuate the paths (travel distances) that are included in the objective function and that have been selected optimally.

The time points when porters leave one location and when they arrive to the other locations (a, b, c and d) are also depicted in Figure 13. Therefore, the resulting travel time for porters (time when porters travel empty) can be calculated and is equal to 21. Patients are always brought to the medical examination and picked up afterwards on time, so the patient waiting time is equal to zero. Porter 2 is only assigned to one transportation request and doesn’t have to wait at all. However, porter 1 has to wait 27 (21) minutes at the hospital unit 2 (6), 8 (21) minutes between requests 4 and 3 (6 and 2) and 3 minutes at hospital ward 1. Therefore, the total waiting time for porters is 80.

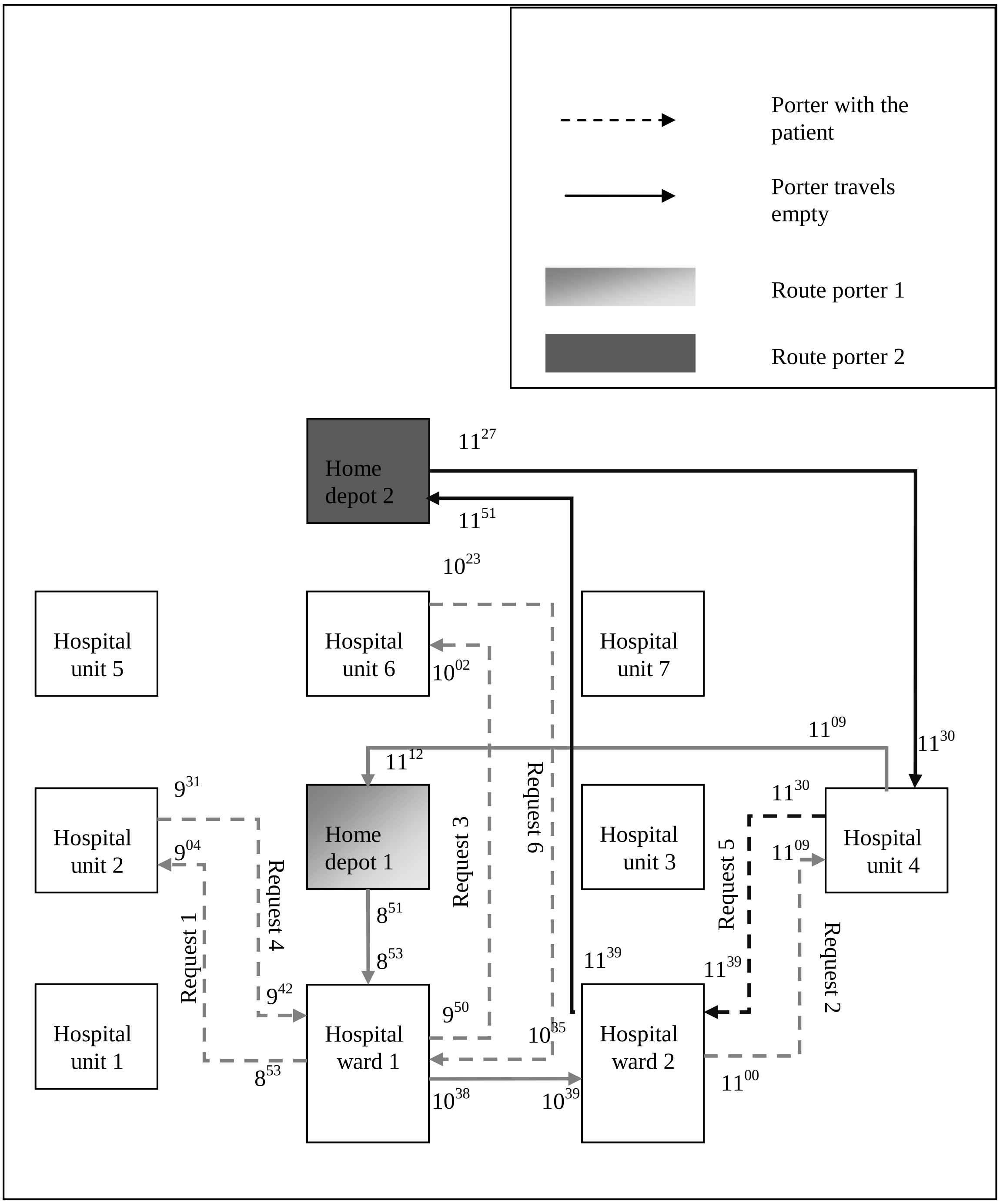

Figure 13: Porters' routes (general model)

When instance 10 was tested for the second extension of the model, i.e. patient-centered extension, the following assignment resulted:

Porter	order of transportation requests
1	Home depot – 1 – 4 – 3 – 6 – Home depot
2	Home depot – 2 – 5 – Home depot

Table 16: Optimal routes – patient-centered extension

The optimal assignment and the resulting porters' routes are depicted in Figure 14.

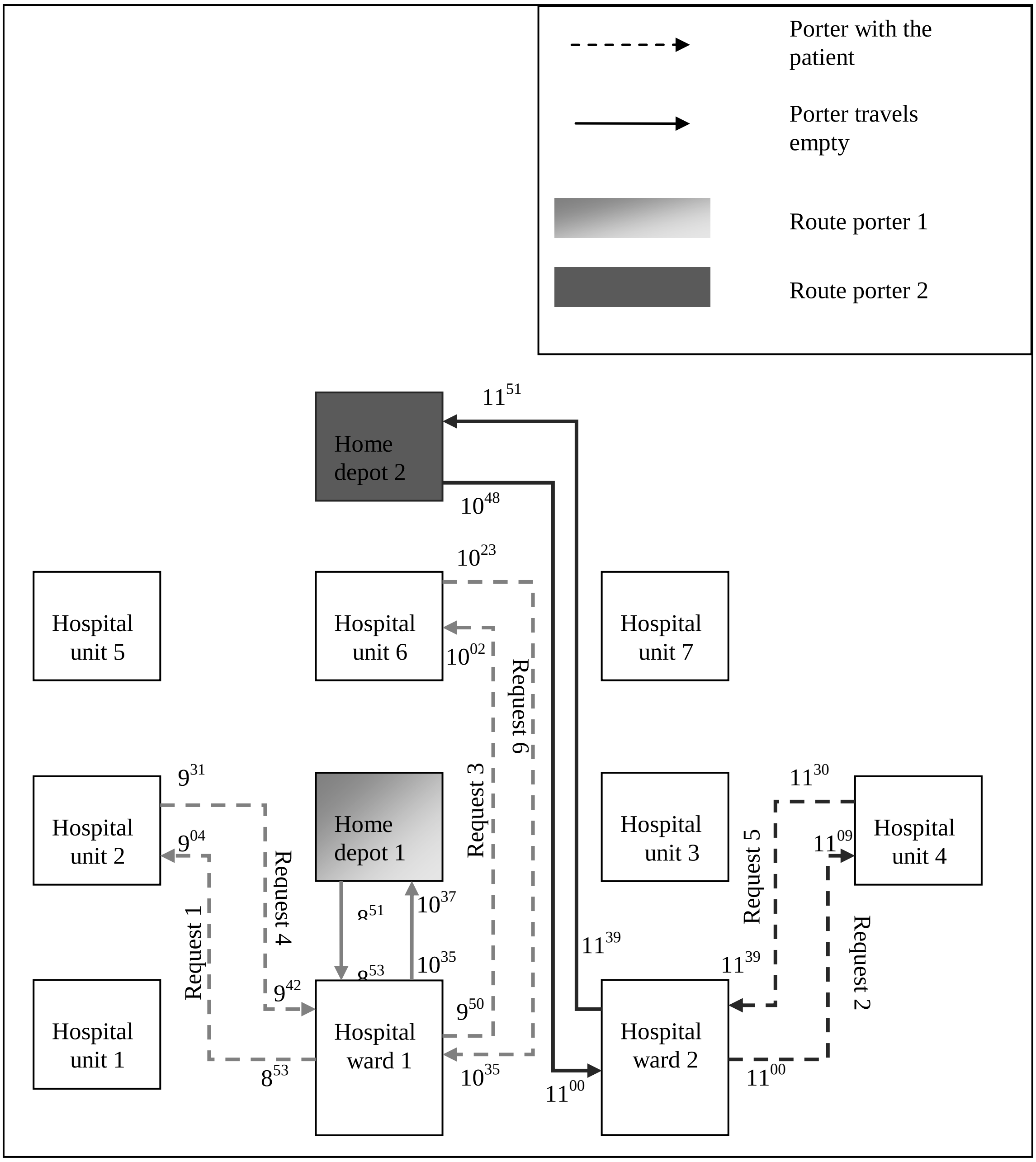

Figure 14: Porters' routes (patient - centered extension)

In this case the porters' routes look a little bit different. The first porter still starts with transportation request 1 and afterwards completes 4, 3 and 6, but after the last one she goes back to her home depot. Due to the additional constraint that the same porter has to escort the patient to her medical examination and pick her up afterwards, porter 2 is now assigned to patient 2 and therefore has to complete both assignments (transportation requests 2 and 5). The travel times are thus increased from 21 to 28 minutes. The waiting time for patients is still equal to zero. The waiting times for porters are however reduced to 77.

In the previous two extensions, the routing from the porters' point of view is not that efficient. They have to wait 80 (77) minutes in the general model (first extension), however, this waiting times is split over 6 transportation requests. It would be more efficient if the waiting time of porters could be accumulated so that each time that porter has to wait for her next transportation request, there should be enough time for porter to complete some other duties. This idea has been incorporated in the third extension. For the third extension, i.e. hospital-centered extension, the resulting assignment and the porters' routes are shown in Table 17. For this extension, the porters are sent to their home depot between transportation requests if the time slot between two successive transportation requests is long enough for porter to go there, spend at least 15 minutes there and then go and complete the next transportation request. With these additional constraints the waiting time for porters that occurs somewhere at the hospital will be reduced. The porters could then be assigned to complete some other tasks at their home depot.

Porter	order of transportation requests
1	Home depot – 2 – Home depot – 5 – Home depot
2	Home depot – 1 – Home depot – 4 – 3 – Home depot – 6 – Home depot

Table 17: Optimal routes – hospital-centered extension

The changes in the optimal solution are now very different when compared to the general model. The solution has been depicted in Figure 15. *Dot-dash line* represent the porter's temporarily travel to her home depot, after which she returns to complete some other transportation requests.

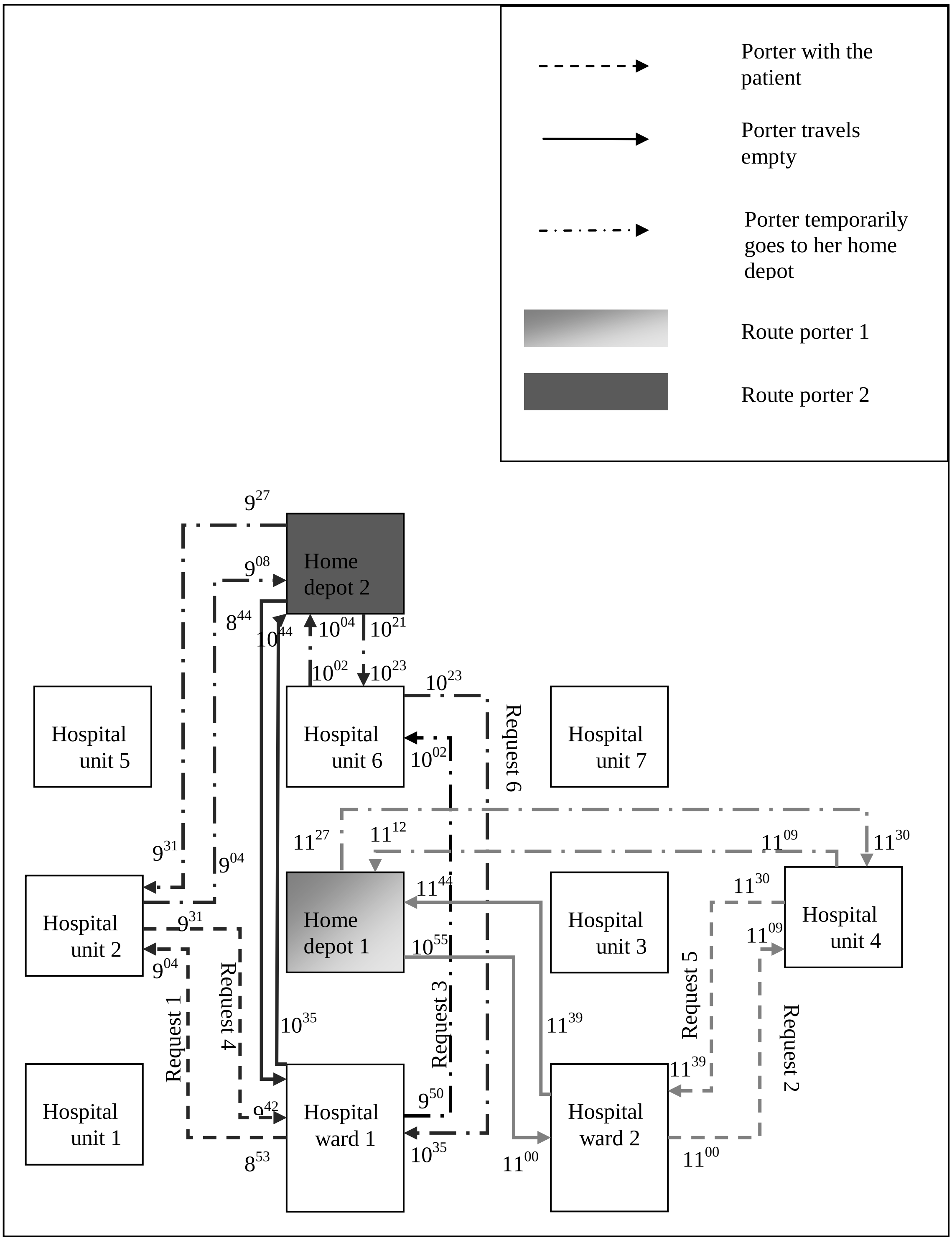

Figure 15: Porters' routes (hospital - centered extension)

Porter 1 is now assigned to complete assignments 2 and 5, but between these requests she is sent to her home depot, where she spends 15 minutes. After completion of transportation request 5 she finally goes back to her home depot. Porter 2 completes the remaining transportation requests and is sent temporarily to her home depot after transportation request 1 (3) to spend 19 (17) minutes there. Porters spend 51 minutes in the home depot in total, and thereby their unnecessary waiting time somewhere else at the hospital compound is reduced to only 8 minutes (and occurs only between requests 4 and 3). However, the efficient usage of the porters' time is costly in the sense of travel times. The total travel times are increased from 21 to 46 due to additional travelling to and from the home depot. Nevertheless, as the additional changes in the model do not cause increase in the patient waiting time (which is still equal to zero), it is beneficial to utilize the chance to efficiently use porters' time. Therefore, porters should be sent to their home depot if there are no pending transportation requests.

4.6 Larger instances

After the initial testing for the first three classes, larger classes have also been tested. In the case of class IV, the number of porters has been fixed and the number of porters has been varied. On the other side, for class V the number of patients has been fixed and only the number of porters has been varied.

4.6.1 Variation of number of patients

In order to test the performance of the solver in use the following experiment has been set up: the number of patients (and the number of resulting transportation requests) has been gradually increased up to 20 (40). For the resulting fourth class of instances, the number of porters still has been set to 4. The best solutions found within one hour of run time can be found in Table 18 and 19. The number of patients is represented with |p|. XPRESS could solve the problem up to 40 transportation requests in one hour of running time. As the computing times were very large, only the general model and the first extension were tested for the fourth class. The solutions obtained for the general model are depicted in Table 18.

|p|	f	tt	w (s)	w (p)	No (p)	GAP	No (OS)	time
8	56	56	108	0	1.4	0.00%	1	2.0
9	40	40	138	0	1.3	0.00%	1	1.5
10	47	47	131	0	1.4	0.00%	1	3.6
11	56	56	53	0	1.2	0.00%	1	18.9
12	56	56	53	0	1.3	0.00%	1	18.0
13	48	48	82	0	1.3	0.00%	1	25.6
14	56	56	51	0	1.1	0.00%	1	27.4
15	56	56	37	0	1.1	0.00%	1	194.8
16	83	83	47	0	1.4	0.00%	1	228.7
17	83	80	34	3	1.4	8.68%	0	3645.3
18	64	64	76	0	1.2	0.00%	1	344.3
19	64	64	30	0	1.1	0.00%	1	2322.8
20	68	68	25	0	1.1	0.00%	1	1786.2

Table 18: 4th class – general model

With the increase in the number of patients, the value of the objective function, as well as the computational time, tend to increase. The problems up to |p| = 15 (up to 30 transportation requests) could be solved within less than 30 seconds. Afterwards the computational time has increased rapidly, but the problem could still be solved to optimality for almost all instances tested (exception is a problem with 17 patients).

The fourth class was also tested with the extended model, namely with the patient centered extension. The obtained results are given in Table 19. Compared to the general model, this extension could be solved to optimality for all instances tested. The optimal solution could be obtained within less than 10 minutes. The values of the objective function (f) deteriorated up to 20% in comparison to the general model, which means that with slight increase in the travel time, the patient convenience (i. e. the same porter to take care of them) could be guaranteed. From the porters' point of view, the solution deteriorates slightly in the sense that the travel time, tt, increases, but the porters' waiting times even decrease for few instances tested.

|p|	f	tt	w (s)	w (p)	No (p)	GAP	No (OS)	time
8	64	64	39	0	1.0	0.00%	1	6.0
9	48	48	39	0	1.0	0.00%	1	0.8
10	56	56	39	0	1.0	0.00%	1	3.8
11	56	56	49	0	1.0	0.00%	1	9.6
12	56	56	47	0	1.0	0.00%	1	8.4
13	56	56	24	0	1.0	0.00%	1	48.5
14	56	56	38	0	1.0	0.00%	1	19.3
15	56	56	47	0	1.0	0.00%	1	26.8
16	83	83	21	0	1.0	0.00%	1	107.3
17	83	80	37	3	1.0	0.00%	1	383.2
18	64	64	31	0	1.0	0.00%	1	167.1
19	64	64	44	0	1.0	0.00%	1	246.2
20	72	72	38	0	1.0	0.00%	1	598.2

Table 19: 4th class – patient-centered extension

Larger instances were also tested with XPRESS, where the computation time was increased to five hours. Already for the problem with 4 porters and 25 patients (i.e. 50 transportation requests) no feasible solution could be found within the given time.

4.6.2 Variation of number of porters

In order to investigate what would be the optimal number of porters for a given number of transportation requests, additional testing has been made. In this case, the number of patients (transportation requests) has been set to 15 (30). The data used were the same as those used for the according instance from class IV. The number of porters has been varied from 1 to 10. The best solutions found in one hour of running time are shown in Table 20 and 21. The number of porters is represented with |po|. Analog to class IV, due to the long computational time, the fifth class was also tested only for general model and the patient-centered extension.

The solution for the general model is depicted in 20. The value of the objective function improves from 2165 (obtained in the case when only one porter is assigned to complete all the transportation requests) to 56 (obtained already in the case when |po| = 4). From that point on, the value of objective function remains the same for all other instances tested. The computational time also increases, and starting from |po| = 6 the optimal solution cannot be found in one hour of running time. However, the resulting gap of only 0.04% shows that the XPRESS did not manage to prove the optimality in one hour, but the solution found so far cannot be improved anymore. Hence, the conclusion can be made that it is optimal to select 4 porters to complete these 30 transportation requests. Increasing the number of porters would not improve the quality of the solution and it is therefore unnecessary. On the other side, choosing too few porters is also unprofitable; it leads to long waiting times and high travel times. Increasing the number of porters from 1 to 2 leads to an improvement of the objective function value of 89.80%. Further increase in the number of porters (from 2 to 3) decreases the objective function value for 72.8%. Another improvement is also the decrease in the number of porters that are assigned to one patient (from 1.33 to 1.10).

|po|	f	tt	w (s)	w (p)	No (p)	GAP	time
1	2165	134	8	677	1.00	7.13	3710.81
2	221	86	6	45	1.33	0.00	923.797
3	60	54	35	2	1.10	0.00	596.704
4	56	56	37	0	1.10	0.00	369.75
5	56	56	37	0	1.10	0.00	1319.13
6	56	56	37	0	1.10	0.04	3647.06
7	56	56	37	0	1.10	0.04	3657.27
8	56	56	37	0	1.10	0.04	3683.55
9	56	56	37	0	1.10	0.04	3654.64
10	56	56	37	0	1.10	0.04	3636.31

Table 20: 5th class – general model

The solution for the patient centered extension is given in Table 21. The additional constraint that only one porter should be assigned to one patient did not change the solution remarkably. The only difference from the general model is the case when |po| is set to 2 and 3 where the value of objective function and the resulting travel times are slightly higher than in the first case. But starting with |po| = 4 the values for the objective function in the remaining instances are the same as in the general model. The number of porters that are assigned to one patient is forced to be one, so the difference is only in the way the porters are chosen and assigned to transportation requests. However, the porters' waiting times have increased noticeably, from 37 to 47 (for |po| >= 4). On the other side, the computational time is significantly lower than in the general model, and only two instances could not be solved (proved) to optimality within one hour of running time. Therefore, a conclusion can be made that in both cases (in the general model and in the patient centered extension) it would be optimal to choose 4 porters to escort 15 patients on their in-house transportation.

|po|	**f**	**tt**	**w (s)**	**w (p)**	**No (p)**	**GAP**	**time**
1	2165	134	8	677	1	7.13	3705.09
2	251	80	10	45	1	0.00	2041.19
3	65	59	42	2	1	0.00	73.437
4	56	56	47	0	1	0.00	77.984
5	56	56	47	0	1	0.00	180.937
6	56	56	47	0	1	0.00	1811.86
7	56	56	47	0	1	0.00	539.813
8	56	56	47	0	1	0.00	880.156
9	56	56	47	0	1	0.04	3787.95
10	56	56	47	0	1	0.04	4030.78

Table 21: 5^{th} class - patient-centered extension

5 Conclusion and outlook

The scope of this thesis was to present a novel optimization model for intra-hospital routing, a very important aspect prevailing in health-care modeling. Besides considering classical objectives for vehicle routing (i.e. minimizing distance or travel time related costs) the focus was also set on the patients’ inconvenience (i.e. the so called client-centered perspective, consisting of minimizing waiting times before and after their scheduled appointments, as well as the number of porters assigned to patients). One the other hand, the perspective of hospital management has also been taken into consideration. A possible solution for a more efficient use of the hospital resources (in terms of reducing the unnecessary porters’ waiting times somewhere in the hospital compound) has been proposed.

The model proposed could be solved within one hour of computational time. The instances were tested for up to 40 transportation requests. The performed sensitivity analyses have shown that an increase in the number of porters that have to be assigned to transportation requests does not guarantee an improvement in the solution quality. The marginal increase in the number of porters can slightly improve the value of the objective function but strongly influences the computational time.

The solution for reduction in the patients’ inconvenience can easily be implemented. It is necessary to impose a limit on the number of porters in use per patient. The quality of the solution obtained in this case deteriorates slightly when compared to the general model (on average 11.7%). This is due to additional travel times by porters. The effect on run times for obtaining the solution is, however, negligible. Hence the inconvenience of the patient can easily be reduced at low costs and should be considered whenever possible. The number of porters in use per patient can be reduced by 27%.

For the hospital centered extension of the model a following modeling approach has been proposed: the waiting times of porters should be minimized in the way that porters are sent to their home depot whenever possible and assigned to other tasks beyond the scope of this model. From the patients’ point of view the solution deteriorates slightly (patients’ average waiting time increases from 0 to 4.13 minutes and the number of porters pro patient increases by 16%). However one major drawback are the resulting run times. The complexity of the underlying model increases dramatically and the model becomes computationally intractable as the size of the problem instance increases. Within one hour of run time the large instances under consideration could not be solved to optimality. The resulting gap was still at 95.6%.

There are a couple of possible propositions for the further research and extension of this work. First, the model could be solved in reasonable amount of time only for small instances tested. For larger instances, however, the use and development of a (meta-)heuristic becomes unavoidable. Next, the assumption that every patient may only be escorted to/from one single appointment may be relaxed. Instead, a problem where every patient has several sequential appointments could be considered. Moreover, another extension could be that patient appointments have not been scheduled to a particular hospital unit yet (only starting and ending times of appointments are given), so the scheduling of patient appointments and routing of patients (and porters) need to be done simultaneously. Furthermore, it could also be possible to consider the online data, where the information about transformation requests is not known in advance. In this case, besides the in-patients (who are admitted to the hospital and have to stay there overnight), also the out-patients (who are not admitted to the hospital, but only come for the treatment) and their in-house transportation has to be considered.

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Appendix A Detailed results

A – 1 General model (1st, 2nd and 3rd class)

C	N	f	tt	w (s)	w (p)	No (p)	GAP	time
1	1	28	28	18	0	1.33	0.00%	0.08
	2	28	28	39	0	1.00	0.00%	0.06
	3	21	21	80	0	1.33	0.00%	0.03
	4	28	28	27	0	1.00	0.00%	0.05
	5	47	47	11	0	1.33	0.00%	0.11
	6	28	28	39	0	1.33	0.00%	0.03
	7	36	36	54	0	1.33	0.00%	0.06
	8	28	28	64	0	1.33	0.00%	0.03
	9	28	28	27	0	1.00	0.00%	0.11
	10	21	21	80	0	1.33	0.00%	0.03
	avg	29.30	29.30	50.32	0.00	1.23	0.00%	0.06
2	1	30	30	138	0	1.20	0.00%	0.39
	2	46	46	150	0	1.40	0.00%	0.25
	3	46	46	20	0	1.20	0.00%	0.30
	4	38	38	127	0	1.40	0.00%	0.33
	5	30	30	140	0	1.20	0.00%	0.28
	6	30	30	150	0	1.20	0.00%	0.08
	7	30	30	128	0	1.20	0.00%	0.13
	8	45	45	160	0	1.40	0.00%	0.25
	9	40	40	149	0	1.60	0.00%	0.47
	10	38	38	131	0	1.40	0.00%	0.53
	avg	37.30	37.30	129.22	0.00	1.32	0.00%	0.30
3	1	48	48	182	0	1.29	0.00%	1.05
	2	40	40	180	0	1.29	0.00%	0.97
	3	48	48	141	0	1.14	0.00%	1.34
	4	46	46	175	0	1.29	0.00%	1.05
	5	64	64	29	0	1.14	0.00%	1.73
	6	48	48	27	0	1.29	0.00%	1.28
	7	64	64	21	0	1.29	0.00%	4.42
	8	48	48	114	0	1.29	0.00%	3.34
	9	40	40	122	0	1.29	0.00%	3.06
	10	48	48	127	0	1.14	0.00%	1.41
	avg	49.40	49.40	111.94	0.00	1.24	0.00%	1.97

A-2 Patient-centered extension (1st, 2nd and 3rd class)

C	N	f	tt	w (s)	w (p)	No (p)	GAP	time
1	1	28	28	26	0	1	0.00%	0.05
	2	28	28	25	0	1	0.00%	0.05
	3	28	28	27	0	1	0.00%	0.05
	4	28	28	27	0	1	0.00%	0.02
	5	49	49	12	0	1	0.00%	0.03
	6	28	28	24	0	1	0.00%	0.03
	7	36	36	27	0	1	0.00%	0.03
	8	28	28	24	0	1	0.00%	0.05
	9	28	28	27	0	1	0.00%	0.02
	10	28	28	69	0	1	0.00%	0.03
	avg	30.90	30.90	24.17	0.00	1	0.00%	0.03
2	1	38	38	57	0	1	0.00%	0.50
	2	46	46	28	0	1	0.00%	0.58
	3	46	46	24	0	1	0.00%	0.52
	4	46	46	28	0	1	0.00%	0.16
	5	38	38	26	0	1	0.00%	0.55
	6	38	38	27	0	1	0.00%	0.48
	7	38	38	25	0	1	0.00%	0.48
	8	46	46	27	0	1	0.00%	0.45
	9	46	46	26	0	1	0.00%	0.27
	10	46	46	0	0	1	0.00%	0.47
	avg	42.80	42.80	26.61	0.00	1	0.00%	0.45
3	1	56	56	30	0	1.00	0.00%	0.88
	2	48	48	27	0	1.00	0.00%	3.84
	3	56	56	25	0	1.00	0.00%	1.02
	4	56	56	34	0	1.00	0.00%	1.28
	5	64	64	26	0	1.00	0.00%	4.24
	6	56	56	36	0	1.00	0.00%	1.42
	7	64	64	27	0	1.00	0.00%	0.16
	8	56	56	26	0	1.00	0.00%	4.59
	9	48	48	26	0	1.00	0.00%	2.17
	10	56	56	35	0	1.00	0.00%	0.95
	avg	56.00	56.00	29.23	0.00	1.00	0.00%	2.05

A-3 Hospital-centered extension (1st, 2nd and 3rd class)

C	N	f	tt	w (s)	w (p)	wH	wi	No (p)	GAP	time
	1	61	60	1	0	153	3	1.67	0.00%	1.45
	2	73	62	6	0	127	3	1.67	0.00%	1.06
	3	50	46	2	0	62	3	1.00	0.00%	0.84
	4	64	64	0	0	131	4	1.00	0.00%	1.14
1	5	62	53	5	0	26	1	1.00	0.00%	0.92
	6	64	64	0	0	169	4	1.33	0.00%	0.80
	7	64	64	0	0	186	4	1.00	0.00%	0.66
	8	64	64	0	0	107	4	1.33	0.00%	1.33
	9	56	50	3	0	61	2	1.67	0.00%	1.02
	10	54	46	8	0	51	3	1.00	0.00%	1.09
	avg	61.20	57.30	1.95	0.00	107.30	3.10	1.27	0.00%	1.03
	1	92	80	6	0	146	4	1.40	0.00%	368.91
	2	110	110	0	0	193	7	1.60	0.01%	413.77
	3	113	87	13	0	119	5	1.40	0.00%	768.94
	4	100	97	2	0	198	5	1.80	0.00%	343.27
2	5	96	94	1	0	275	6	1.60	0.00%	274.81
	6	108	98	5	0	212	5	1.60	0.00%	946.91
	7	84	72	6	0	82	4	1.40	0.00%	120.24
	8	108	102	3	1	239	5	1.60	0.00%	250.72
	9	95	79	8	0	356	4	1.80	0.00%	226.56
	10	74	74	0	0	264	2	1.00	0.00%	24.02
	avg	98.00	89.30	4.30	0.10	208.40	4.70	1.52	0.00%	373.81
	1	133	130	2	0	273	8	1.57	82.97%	3669.34
	2	141	113	12	5	331	5	1.71	113.47%	3687.20
	3	130	117	7	0	304	4	1.43	68.76%	3713.20
	4	123	103	10	0	225	6	1.43	90.31%	3686.64
3	5	143	112	13	5	166	6	1.57	107.25%	3671.47
	6	132	114	9	1	218	7	1.71	76.83%	3706.03
	7	154	123	15	1	216	6	1.57	108.11%	3703.17
	8	147	124	10	4	305	8	2.00	104.93%	3702.39
	9	148	125	12	0	394	8	1.86	126.74%	3678.94
	10	130	107	11	1	195	5	1.57	79.68%	4704.67
	avg	138.10	116.80	9.80	1.70	262.70	6.30	1.64	95.90%	3792.31

Appendix B Abstract

This thesis deals with the in-house transportation of the patients in pavilion structured hospitals. This topic has gained increased attention over the last years, due to the fact that routing operations come at a high price and that efficient routing plan could not only help reduce the costs, but also to improve the service quality, which is reflected through patients' satisfaction.

The aim of this work is to introduce a new model for intra-hospital routing of patients, considering both client- and management related issues. Patients in a hospital have fixed appointments, such as x-rays or ultrasonic and due to medical reasons they may not be able to walk on their own, so they have to be escorted by porters. In the model logistical costs for the usage of porters and patient inconvenience are minimized. Furthermore, the model is expanded and changed accordingly in order to capture distinguish patient centred issues on one hand and hospital centred issues on the other hand. It has been shown that the different developed model variants are tractable for realistic problem instances in medium-sized hospitals.

The model belongs to the group of pickup and delivery problems. Under some circumstances, the problem can be also seen as a dial-a-ride problem, or a special type of a stacker-crane problem.

Appendix C Zusammenfassung

Diese Magisterarbeit befasst sich mit dem in-house Transport von Patienten in pavillonartig strukturierten Krankenhäusern. Dieses Thema hat in den letzten Jahren an Bedeutung gewonnen, da Routing in Krankenhäusern große Kosten verursacht und ein effizienteres Routing sowohl die Kosten senken, als auch die Servicequalität, gemessen an der Patientenzufriedenheit, erhöhen kann.

Ziel dieser Arbeit ist es, ein neues Modell für das in-house Routing von Patienten in Krankenhäusern zu entwickeln, das die Ansprüche von Patienten und Management gleichermaßen erfüllt. Patienten in Krankenhäusern haben fixe Termine, wie z.B. Röntgen- oder Ultraschalluntersuchungen, und müssen aus medizinischen Gründen oft von Trägern zu diesen Terminen begleitet werden. Im Modell werden die logistischen Kosten der Verwendung von Trägern und die Unannehmlichkeiten von Patienten minimiert. Außerdem gibt es zwei Erweiterungen, in der Ersten stehen primär patientenorientierte Sachverhalte im Vordergrund, in der Zweiten stehen krankenhaus- bzw. managementorientierte Sachverhalte im Vordergrund. Es wird gezeigt, dass die entwickelten Modelle in der Lage sind, reale Problemstellungen in mittelgroßen Krankenhäusern zu lösen.

Das Modell gehört zur Gruppe der Pickup and Delivery-Probleme. Unter bestimmten Umständen kann das Model auch als Dial-a-Ride-Problem oder als spezielle Variante eines Stacker-Crane-Problems gesehen werden.

Appendix D Curriculum vitae

BELMA TURAN

PERSONAL INFORMATION

Name:	Belma Turan
Date of birth:	December 26, 1985
Citizenship:	Bosnian

STUDIES

since 2009	University of Vienna, Vienna, Austria **Magister Degree Programme in Business Administration** Focus on Transportation Logistics und Operations Research Title of Master Thesis: Models for Intra-hospital Patient Routing
2005 - 2009	University of Graz, Graz, Austria **Bachelor Degree Programme in Business Administration** Title of Bachelor Thesis: Qualitätssteigerung durch Trusted Computing Degree: *Bakkalaurea der Sozial- und Wirtschaftswissenschaften* *(Bakk. rer. soc. oec.)*
2003 - 2004	**Hainberg-Gymnasium**, Göttingen, Germany AFS Exchange program
2000 - 2004	**Gymnasium**, Banja Luka, Bosnia and Herzegovina Degree: General qualification Graduation "with honors"

SPECIAL SKILLS

IT-Skills:
MS Office
MATLAB

Optimization:
Xpress-Optimizer

Programming:
C++
Pascal

Languages:
Bosnian/Croatian/Serbian (mother tongue)
German (fluent in written and spoken)
English (fluent in written and spoken)
Spanish (basic knowledge)

INTERESTS

Archery, aerobics, swimming