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Set-Up, Estimation, and Forecasting Performance

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All remaining errors in this study are, of course, mine.

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Chapter 1

Introduction

Originally, the class of Dynamic Stochastic General Equilibrium (DSGE) models dates back to KYDLAND & PRESCOTT (1982) who first introduced real shocks into the neoclassical growth model under the assumption of completely flexible prices in order to create business cycle fluctuations. Hence, this class of models is also known as Real Business Cycle (RBC) models. Over the past two decades, DSGE models have become state of the art in the (Quantitative) Macroeconomic literature, varying in their degrees of complexity as well as in their specific focus of application.

ROTEMBERG & WOODFORD (1997) were the first to enhance these DSGE models by monopolistic competition in spirit of DIXIT & STIGLITZ (1977) and by nominal rigidities in terms of CALVO (1983) contracts. Differing from the original RBC literature, these two assumptions render money non-neutral (in the short run), which has crucial implications on the effectiveness of monetary policy as laid out by CLARIDA et al. (1999). The state-of-the-art reference on the methodology of these so-called New Keynesian models still is the celebrated research monograph by WOODFORD (2003). Meanwhile, a very accessible textbook for graduate students on New Keynesian modeling is available with GALÍ (2008).

With their famous redux model, OBSTFELD & ROGOFF (1995) pioneered New Open Economy Macroeconomics by enriching the traditional Open Economy Macroeconomic literature of the Mundell-Fleming-Dornbusch type as they first applied the DSGE structure to a two-country setting. Numerous authors have contributed to this topic, which continues to appeal to a wide range of scholars. In consequence, various survey articles and even survey book chapters on open-economy DSGE models have come up over time such as OBSTFELD & ROGOFF (1996, chapter 10), LANE (2001), ENGEL (2002), WALSH (2003, chapter 6), or GALÍ (2008, chapter 7).

Chapter 2 of this dissertation develops a small-scale two-country DSGE model that

is characterized by diverging interest-rate rules. These diverging interest-rate rules represent the differing statutes of the two central banks under scrutiny, the European Central Bank (ECB) and the Federal Reserve System (Fed).

Both RBC and New Keynesian models are micro-founded, i.e. most of the model equations are derived from optimizing behavior on the part of the economy's agents within a dynamic setting. Hence, the models' structural or *deep* parameters directly stem, e.g., from the households' utility function or the firms' profit function. Estimating these structural parameters should then mitigate the LUCAS (1976) critique as opposed to traditional large-scale Macroeconometric models which consisted of a collection of purely ad-hoc equations.

Due to the effectiveness of monetary policy in New Keynesian models, customized models such as the New Area-Wide Model (NAWM) for the Euro area (see CHRISTOF-FEL et al. 2008) are used nowadays by virtually every central bank in the world. These institutions are usually interested in the applications of empirical policy analysis (see, e.g., SMETS & WOUTERS 2003), forecasting (see, e.g., SMETS & WOUTERS 2004), or both (see ADOLFSON et al. 2007 for a prominent example for an open-economy model). In those applications of DSGE models, Bayesian estimation techniques play a major role (see AN & SCHORFHEIDE 2007 for a survey).

Chapters 3 and 4 of this dissertation will cover these two important empirical applications in more detail for two different pairs of countries and with respect to different Bayesian and classical (vector) autoregressive benchmarks.

In GALÍ (2008, chapter 8), the reader can find possible theoretical extensions to the basic closed-economy New Keynesian framework, which may, in principle, be intriguing to open economy researchers, too: the topics mentioned are state-dependent pricing, labor market frictions and unemployment, imperfect information and learning, endogenous capital accumulation, financial market imperfections, and the zero lower bound on nominal interest rates (for the references on these topics see GALÍ 2008, pp. 191-193).

Besides this general introduction, the present dissertation entitled *Monetary DSGE Models of Two Countries: Set-Up, Estimation, and Forecasting Performance* contains three more chapters.

In **Chapter 2**, we investigate within a small-scale two-country DSGE set-up whether there exists a determinate rational expectations equilibrium and what are the implications on key macroeconomic variables of the European Union (EU) and the United States (US) if we assume that the ECB and the Fed stick to their differing legal statutes, which is reflected by diverging interest-rate rules.

For a calibrated version of the model we find that a unique stationary solution exists

and that positive realizations of all types of simulated macroeconomic disturbances have a negative impact on output of both economies. Expansionary monetary policy shocks always have a *prosper thyself* and *beggar thy neighbor* effect with respect to the terms of trade as these shocks influence the terms of trade for the benefit of EU (US) resident households by decreasing them below (raising them above) their zero-inflation steady-state value.

Moreover, we find that if the ECB implemented the interest-rate rule proposed in this chapter, it would encounter lower fluctuations in EU producer-price-index inflation compared to an interest-rate rule as proposed for the Fed. This is consistent with the ECB's paramount objective of price stability. However, this advantage only holds at the expense of relatively high fluctuations in the EU output gap.

In **Chapter 3**, we estimate and forecast with [1] the two-country DSGE model with diverging interest-rate rules developed in Chapter 2 while employing Bayesian techniques and [2] a atheoretical vector autoregressive (VAR) model while employing standard ordinary least squares. In doing so, we use quarterly OECD data ranging from 1994Q1 until 2009Q1 for the Euro area on the one hand and the US on the other.

We find that the estimated DSGE model qualitatively reproduces the findings of the calibrated one from Chapter 2 with respect to most of the parameter values and to impulse responses on the various exogenous error terms. Estimating an unconstrained VAR(1) does not yield the identical causal relationships as implied by the DSGE and impulse responses based on the VAR(1) sometimes differ from the ones obtained for the DSGE. Both models as well as the additive seasonal Holt-Winters method, a simple univariate extrapolation method serving as a benchmark, are not able to predict the severeness or, at least, the evolution of the economic and financial crisis for the forecasting period from 2007Q2 until 2009Q1.

Finally, we obtain the result that the accuracy of one-step-ahead DSGE forecasts can compete well with the accuracy of VAR(1), Holt-Winters, and uniformly combined forecasts under regular economic conditions. In two cases, the DSGE is able to significantly outperform some of the rival forecasting models, but only at the 10% level.

In **Chapter 4**, we shift the focus from the two large open economies to the interior of one of them, namely to two economically integrated EU members that share a common border: Austria and Hungary. We want to compare the forecasting accuracy of new variants of the two-country DSGE model with several Bayesian and classical (vector) autoregressive benchmarks for the subsequent four variables of interest: Austrian and Hungarian output gaps as well as consumer-price-index inflation rates. In particular, we address the forecasting performance [1] of the original open-economy

DSGE characterized by producer-price-index inflation under the interest-rate rules, [2] of an open-economy DSGE with consumer-price-index inflation under the interest-rate rules, [3] and of the closed-economy DSGE with consumer-price-index inflation under the interest-rate rule that is nested in the open-economy structure. In doing so, we use quarterly Eurostat and OECD data ranging from 2000Q1 until 2009Q3 for Austria and Hungary.

We obtain the result that Bayesian and classically estimated (vector) autoregressive benchmarks deliver the most accurate one-step-ahead forecasts for all four endogenous variables with respect to the different variants of the two-country DSGE model, but cannot significantly outperform them. For three out of four variables, open-economy models perform best compared to other single forecasts. If we additionally calculate various combined forecasts, again for three out of four variables open-economy forecast combinations perform best with respect to other combined forecasts.

In summary, even if single DSGE forecasts were not able to deliver the most accurate one-step-ahead forecasts, the additional information provided by these forecasting models seems to be valuable for uniform forecast combination for two variables. Since open-economy models deliver the lowest forecasting error for three out of four variables across single and combined forecasts, taking into account the non-negligible impact of economic interrelations between Austria and Hungary indeed leads to a more accurate prediction of most of the macro variables we take into consideration.

Interestingly, an evolution somewhat comparable to the one of the small-scale two-country DSGE model from above was already perceivable in the literature for a medium-scale closed-economy DSGE model by SMETS & WOUTERS (2003, 2004, 2007): In SMETS & WOUTERS (2003), the model is set up and estimated using Bayesian techniques for Euro area data. In SMETS & WOUTERS (2004), the estimated model is used to perform out-of-sample forecasts of Euro area variables and to address the model's forecasting performance. Finally, in SMETS & WOUTERS (2007), a new variant of the model is estimated and forecasted with for a different country, namely the US.

Chapter 2

A two-country DSGE model with diverging interest-rate rules

2.1 Introduction

If one studies scholarly articles that deal with monetary models of two countries such as CORSETTI & PESENTI (2001), CLARIDA et al. (2002), PAPPA (2004), or BENIGNO & BENIGNO (2006), one usually encounters that the countries' monetary authorities are modeled as perfectly symmetric institutions.

This gives rise to the question to which extent these models are able to capture real-world features and to which extent policy recommendations based on the results of these models are applicable. The reason why this is questionable is that, in general, two different central banks may each obey a differing and legally binding statute. More specifically, let us think of the two monetary authorities under examination as the ECB on the one hand and the Fed on the other.

Article 2 of the Protocol on the Statute of the European System of Central Banks and of the European Central Bank (1992, 2004) states the following:[1]

> "In accordance with Article 105(1) of this Treaty, the primary objective of the ESCB shall be to maintain price stability. Without prejudice to the objective of price stability, it shall support the general economic policies in the Community with a view to contributing to the achievement of the objectives of the Community as laid down in Article 2 of this Treaty. The ESCB shall act in accordance with the principle of an open market economy

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[1]More precisely, the responsible body for the monetary policy of the EU is the European System of Central Banks (ESCB), which comprises the ECB and the national central banks of all 27 EU members (in 2010).

with free competition, favouring an efficient allocation of resources, and in compliance with the principles set out in Article 4 of this Treaty."

However, Section 2a of the Federal Reserve Act (1977, 2000) reads:

"The Board of Governors of the Federal Reserve System and the Federal Open Market Committee shall maintain long run growth of the monetary and credit aggregates commensurate with the economy's long run potential to increase production, so as to promote effectively the goals of maximum employment, stable prices, and moderate long-term interest rates."

As the reader can conclude from these diverging statutes, the paramount objective of the ECB is price stability, whereas for the Fed this goal is just one out of many. In order to model monetary policy of each central bank consistent with their diverging statutes, we will incorporate these differing institutional features into their respective policy functions.

It is worth noting that in the time from 1999 to 2004, the Fed revised its short-run nominal interest rate more than twice as often as the ECB (see SAHUC & SMETS 2008, pp. 505-506). This difference in central bank activism can, at least, partly be explained by differing sensitivities to the respective inflation rate and output gap in estimated interest-rate rules for the ECB and the Fed within two separate closed-economy DSGE models. The differing sensitivities, in turn, may be the empirically validated implication of the diverging statutes of both monetary authorities as laid out above (see SAHUC & SMETS 2008, pp. 512-514). We extend SAHUC & SMETS (2008) by investigating the issue of differing statutes within a two-country DSGE framework.

In line with this observation, our main purpose is to investigate whether there exists a determinate rational expectations equilibrium and what are the implications of this outcome for key macroeconomic variables of the EU and the US such as output gaps, producer-price-index inflation rates, terms of trade, and short-run nominal interest rates if we assume that the ECB and the Fed stick to their diverging statutes. These implications will be expressed in terms of simulated impulse responses of the various endogenous variables to identical aggregate productivity, cost-push, and monetary policy shocks.

Formally, we carry out the analysis by introducing diverging interest-rate rules into a canonical log-linear representation of a variant of the two-country DSGE framework by OBSTFELD & ROGOFF (2001), which will be extended by CALVO (1983) pricing, a more subtle form of nominal rigidities within a New Keynesian model than the one used in the original article.

OBSTFELD & ROGOFF (2001) mostly concentrate on the issue of risk premia on nominal exchange rates while CORSETTI & PESENTI (2001) explore the international transmission mechanism and the welfare properties of different types of money supply and government spending shocks. Given the assumption of diverging interest-rate rules, we will focus on the dynamic interrelations of two large open economies expressed by the international transmission mechanism of various macroeconomic shocks.

A major part of the literature on two-country DSGE models such as CLARIDA et al. (2002), PAPPA (2004), BENIGNO & BENIGNO (2006), or ENGEL (2009) however, cover the issue of optimal monetary policy in its various facets. Given the assumption of producer currency pricing, CLARIDA et al. (2002) find that there are welfare gains from monetary policy cooperation with respect to non-cooperation unless utility of consumption is logarithmic and that the policy problem under non-cooperation is isomorphic to the case of a closed economy. Assuming local currency pricing and co-operation, ENGEL (2009) finds that optimal monetary policy should not only target inflation and the output gap, but also currency misalignment.

In addition to cooperation and non-cooperation, PAPPA (2004) investigates the welfare properties of the *intermediate* case of a monetary union. BENIGNO & BENIGNO (2006) show that it is possible to design *specific targeting rules* for non-cooperating central banks, which have the property to assign the incentive for independent central banks to replicate the cooperative allocation such that possible welfare losses from non-cooperation can be avoided.

There are also numerous articles dealing with optimal monetary policy within a *small-open-economy* DSGE setting like CLARIDA et al. (2001) or GALÍ & MONACELLI (2005). CLARIDA et al. (2001) show that the log-linear representation of a small open economy is isomorphic to a closed economy since all structural equations of a small open economy are identical to their closed-economy counterparts, except that they are related to the terms of trade. GALÍ & MONACELLI (2005) explore the size of welfare losses of suboptimal monetary policies compared to the benchmark case of optimal monetary policy as implied by the model structure.

The main results of a calibrated version of the model under scrutiny, for which a determinate rational expectations equilibriums exists, are summarized in the following.

Simulated aggregate productivity shocks have a negative impact on EU and US output, a result already described for the closed economy by GALÍ (2002). Simulated cost-push as well as contractionary monetary policy shocks also have a negative impact on EU and US output.

In contrast to CORSETTI & PESENTI (2001), expansionary monetary policy shocks

always have a *prosper thyself* and *beggar thy neighbor* effect since they influence the terms of trade for the benefit of EU (US) resident households by decreasing them below (raising them above) their zero-inflation steady-state value. In addition, this effect would induce a rise of both domestic and foreign output above their flexible-price values.

If the ECB implemented the interest-rate rule proposed in the present article, it would encounter lower fluctuations in EU producer-price-index inflation compared to an interest-rate rule as proposed for the Fed. This is consistent with the ECB's paramount objective of price stability. However, this advantage only holds at the expense of relatively high fluctuations in the EU output gap; a trade-off commonly observed in the Monetary Policy literature.

The remainder of this chapter is structured as follows: Section 2.2 outlines the discrete-time two-country DSGE model, Section 2.3 presents the equilibrium conditions on all markets under flexible prices, Section 2.4 introduces the New Keynesian framework, and Section 2.5 derives a locally unique rational expectations equilibrium for a calibrated version of the model. The analysis is completed by an impulse-response analysis in Section 2.6. Finally, Section 2.7 concludes. Detailed derivations of the model equations are given in Appendix 2.8.

2.2 A New Open Economy Macroeconomic model

The subsequent model is based on the OBSTFELD & ROGOFF (2001) two-country DSGE framework, which extends the basic OBSTFELD & ROGOFF (1995) model by introducing uncertainty.

2.2.1 Preferences, consumption and price indices

Suppose world population is constant over time and consists of a continuum of unit mass of infinitely lived atomistic households characterized by identical preferences. Assume further perfect information and rational expectations on the part of all agents. There are two countries, where domestic households live on the segment $[0, n]$ of the unit interval while foreign households live on the remaining segment $(n, 1]$. Even though we will take up the idea of the home country representing the EU and the foreign country representing the US again in Section 2.4, we shall abstract from this thought for the moment since the present framework can possibly be applied to other pairs of countries, too (see Chapter 4).

The discounted stream of expected period utilities of the representative domestic household reads as follows:[2]

$$U_t = E_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \left[\frac{C_s^{1-\rho}}{1-\rho} + \frac{\chi}{1-\varepsilon} \left(\frac{M_s}{P_s} \right)^{1-\varepsilon} - \frac{\gamma}{1-\xi} L_s^{1-\xi} \right] \right\}. \tag{2.1}$$

The above utility function is a constant elasticity of substitution (CES) composite separable in its arguments real consumption C, real money balances M/P (where P denotes the domestic consumer price index (CPI)), and leisure $-L$ such that the partial derivatives of the utility function with respect to one variable are independent of all other variables. β denotes an intertemporal discount factor ($0 < \beta < 1$). Moreover, the following shall hold for the various parameters: $\chi, \gamma > 0$, $0 < \rho, \varepsilon < 1$, and $\xi < 0$. ρ is the coefficient of relative risk aversion in consumption and the modulus of ξ denotes the inverse of the elasticity of labor supply.[3]

Since (2.1) is a function in real money balances, the model is a variant of the SIDRAUSKI (1967) and BROCK (1974) money-in-the-utility-function (MIU) models, in which the inclusion of real money balances in the utility function is justified by assuming that the use of money facilitates transactions. This modeling shortcut guarantees the usage of money even though holding money per se does not yield a positive real return.[4]

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[2]Note that a possible superscript i to distinguish individual variables is suppressed throughput the analysis for the sake of better legibility.

[3]Therefore, we obtain the subsequent first and second partial derivatives of the utility function (2.1) with respect to the single variables:

$$\begin{aligned}
\frac{\partial U}{\partial C} &= C^{-\rho} > 0, & \frac{\partial^2 U}{\partial C^2} &= (-\rho)C^{-\rho-1} < 0, \\
\frac{\partial U}{\partial(\frac{M}{P})} &= \chi \left(\frac{M}{P} \right)^{-\varepsilon} > 0, & \frac{\partial^2 U}{\partial(\frac{M}{P})^2} &= (-\varepsilon)\chi \left(\frac{M}{P} \right)^{-\varepsilon-1} < 0, \\
\frac{\partial U}{\partial(-L)} &= \gamma L^{-\xi} > 0, & \frac{\partial^2 U}{\partial(-L)^2} &= \xi\gamma L^{-\xi-1} < 0.
\end{aligned}$$

1 minus each of the parameters represents the elasticity of the partial utility function in one of the three arguments, denoted by the respective subscript, with respect to this very argument:

$$\begin{aligned}
\epsilon_{U_C,C} &:= \frac{\partial U}{\partial C} \frac{C}{U_C} = C^{-\rho} \frac{C}{\frac{C^{1-\rho}}{1-\rho}} = 1 - \rho, \\
\epsilon_{U_{\frac{M}{P}},\frac{M}{P}} &:= \frac{\partial U}{\partial(\frac{M}{P})} \frac{\frac{M}{P}}{U_{\frac{M}{P}}} = \chi \left(\frac{M}{P} \right)^{-\varepsilon} \frac{\frac{M}{P}}{\frac{\chi}{1-\varepsilon} \left(\frac{M}{P} \right)^{1-\varepsilon}} = 1 - \varepsilon, \\
\epsilon_{U_{(-L)},(-L)} &:= \frac{\partial U}{\partial(-L)} \frac{(-L)}{U_{(-L)}} = \gamma L^{-\xi} \frac{L}{\frac{\gamma}{1-\xi} L^{1-\xi}} = 1 - \xi.
\end{aligned}$$

[4]Note, however, that some New Open Economy Macroeconomic models abstract from explicitly modeling liquidity services provided by the use of money (see, e.g., CLARIDA et al. 2002, p. 882).

The utility function of the representative foreign household is the same as (2.1), except that C^* may differ from C, as well as M^* from M, P^* from P, χ^* from χ, γ^* from γ, and L^* from L.[5]

Moreover, the total domestic consumption index C from above is defined as a population-weighted per-capita Cobb-Douglas composite of domestic and foreign commodity bundles, which implicitly assumes that all consumption goods are tradable and that there are no trading costs:[6]

$$C_t := \frac{C_{t,H}^n C_{t,F}^{1-n}}{n^n (1-n)^{1-n}}. \tag{2.2}$$

The commodity bundles C_H and C_F are CES composites of differentiated final goods produced at home (C_H) or abroad (C_F) as in DIXIT & STIGLITZ (1977):[7]

$$C_{t,H} := \left[\left(\frac{1}{n} \right)^{\frac{1}{\theta}} \int_0^n C_t(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}, \tag{2.3}$$

$$C_{t,F} := \left[\left(\frac{1}{1-n} \right)^{\frac{1}{\theta}} \int_n^1 C_t(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}. \tag{2.4}$$

The preference for differentiated goods expresses the households' love of variety. As one can see from (2.2), the elasticity of substitution between domestic and foreign commodity bundles σ_{C_H, C_F} equals 1 (Cobb-Douglas specification). From (2.3) and (2.4) we see that the elasticity of substitution across two individual goods z, z' produced within a country $\sigma_{C(z), C(z')}$ equals θ (CES specification, $\theta > 1$ for an equilibrium to exist).[8]

Note further that in the present analysis domestic households are assumed to hold and derive utility from holding domestic money only, whereas foreign households are assumed to hold and derive utility from using foreign money only.

[5] As one can see here, real and nominal foreign variables are denoted by a superscript asterisk. In addition, nominal foreign variables are denominated in foreign currency. This holds except for internationally traded bonds, where foreign bond holdings indexed by a superscript asterisk are denominated in *domestic* currency.

[6] As a consequence, there is no source for the Harrod-Balassa-Samuelson effect as described in OBSTFELD & ROGOFF (1996, pp. 210-216). Furthermore, the total domestic consumption index (2.2) is population-weighted for the CPI (2.5) below to have the usual form rather than a form such as, e.g., in CLARIDA et al. (2002, p. 882).

[7] Alternatively, we could treat imported goods as production factors rather than consumption goods as in McCALLUM & NELSON (2001), which we will not consider in our analysis.

[8]

$$\left| \sigma_{C_H, C_F} \right| := \left| \frac{\mathrm{d} \left(\frac{C_H}{C_F} \right)}{\mathrm{d} \left(\frac{\mathrm{d}C_H}{\mathrm{d}C_F} \right)} \frac{\frac{\mathrm{d}C_H}{\mathrm{d}C_F}}{\frac{C_H}{C_F}} \right| = 1,$$

The domestic CPI is again a Cobb-Douglas composite of domestic and foreign producer price indices (PPIs):

$$P_t = P_{t,H}^n P_{t,F}^{1-n}, \tag{2.5}$$

where these subindices are CES composites of domestic and foreign final goods prices:

$$P_{t,H} = \left[\frac{1}{n} \int_0^n P_t(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}}, \tag{2.6}$$

$$P_{t,F} = \left[\frac{1}{1-n} \int_n^1 P_t(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}}. \tag{2.7}$$

For a derivation of the domestic CPI (2.5), the domestic PPI (2.6) as well as the domestic demand functions for individual and composite domestic goods, see Appendix 2.8. The foreign PPI in domestic currency (2.7), domestic demand curves for individual and composite foreign goods as well as all foreign indices can be derived analogously.

Assume that the law of one price holds for consumers across all individual goods at all times:

$$P_t(z) = S_t P_t^*(z) \quad \forall z \in [0,1], \tag{2.8}$$

where S denotes the endogenously determined nominal exchange rate in price quotation (domestic currency units in terms of foreign currency units).

Thus, as domestic and foreign households are characterized by identical preferences, the law of one price implies that absolute purchasing power parity (PPP) always holds for the CPI (2.5), even though relative PPP (stating that *changes* in domestic and foreign price levels should be equal in the long run) would be the more realistic statement (see OBSTFELD & ROGOFF 1996, pp. 200-202):

$$P_t = S_t P_t^*. \tag{2.9}$$

The demand functions of the representative domestic household for individual domestic

$$|\sigma_{C(z),C(z')}| := \left| \frac{d \left[\frac{C(z)}{C(z')} \right] \frac{dC(z)}{dC(z')}}{d \left[\frac{dC(z)}{dC(z')} \right] \frac{C(z)}{C(z')}} \right| = \theta.$$

$C(h)$ and foreign goods $C(f)$ read as follows:

$$C_t(h) = \frac{1}{n} \left[\frac{P_t(h)}{P_{t,H}} \right]^{-\theta} C_{t,H}, \tag{2.10}$$

$$C_t(f) = \frac{1}{1-n} \left[\frac{P_t(f)}{P_{t,F}} \right]^{-\theta} C_{t,F}, \tag{2.11}$$

where $z = h \in [0, n]$ denotes a typical differentiated good z produced at home and $z' = f \in (n, 1]$ another typical differentiated good z' produced abroad.

As we can see from equations (2.10) and (2.11), demand for individual goods is decreasing in its own price relative to the respective domestic or foreign PPI.[9] Note that θ does not only denote the elasticity of substitution between any two individual goods, but also the price elasticity of demand for any individual good faced by each producer.[10]

Equation (2.2) implies that the demand curves for the composite domestic and foreign goods, C_H and C_F, are given by:

$$C_{t,H} = n \left(\frac{P_{t,H}}{P_t} \right)^{-1} C_t, \tag{2.12}$$

$$C_{t,F} = (1-n) \left(\frac{P_{t,F}}{P_t} \right)^{-1} C_t. \tag{2.13}$$

Now we make use of the fact that world consumption C^w equals the population weighted sum of total domestic and total foreign consumption, where C^w then denotes per capita as well as total world consumption since world population is normalized to 1:

$$C_t^w := nC_t + (1-n)C_t^*. \tag{2.14}$$

Combining (2.14) with equations (2.8), (2.10), (2.11), (2.12), and (2.13) we finally obtain the *global* demand functions for individual domestic and foreign goods in terms

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[9]

$$\frac{\partial C(h)}{\partial P(h)} = (-\theta) \frac{1}{n} \left[\frac{P(h)}{P_H} \right]^{-\theta-1} \frac{C_H}{P_H} < 0.$$

[10]

$$\epsilon_{C(h),P(h)} := \frac{\partial C(h)}{\partial P(h)} \frac{P(h)}{C(h)} = (-\theta) \frac{1}{n} \left[\frac{P(h)}{P_H} \right]^{-\theta-1} \frac{C_H}{P_H} P(h) n \left[\frac{P(h)}{P_H} \right]^{\theta} C_H^{-1} = -\theta.$$

of (total) world consumption:[11]

$$C_t^w(h) = \left[\frac{P_t(h)}{P_{t,H}}\right]^{-\theta}\left(\frac{P_{t,H}}{P_t}\right)^{-1}C_t^w, \tag{2.15}$$

$$C_t^w(f) = \left[\frac{P_t(f)}{P_{t,F}}\right]^{-\theta}\left(\frac{P_{t,F}}{P_t}\right)^{-1}C_t^w. \tag{2.16}$$

2.2.2 Households

The representative domestic household maximizes her objective functional (2.1) subject to the following sequence of intertemporal budget constraints (in nominal terms) with respect to her decision variables $C_t, M_t, B_t,$ and L_t:

$$W_t L_t + (1 + i_{t-1})B_{t-1} + M_{t-1} + \Gamma_t(h) \geq P_t C_t + M_t + B_t + P_t \tau_t. \tag{2.17}$$

As an example for a typical flow budget constraint, inequality (2.17) states that the household's period t expenditure (right-hand side) must not exceed period t income (left-hand side).[12] W denotes the endogenously determined nominal wage being the remuneration for supplying labor, which is identical across households ($L = L(h)$), on the perfectly competitive labor market. i_{t-1} denotes the (short-run) nominal interest rate between period $t-1$ and period t on riskless one-period non-government bonds B_{t-1} carried over from period $t-1$. These nominal bonds are denominated in domestic currency and are supposed to be internationally tradable.[13]

Money holdings M_{t-1} can also be transferred from $t-1$ to t, but yield no nominal return. Consumption goods, however, are perishable and cannot be stored. $\Gamma_t(h)$ are instantaneous profits of the representative household acting as a producer of an individual, differentiated domestic good h, which will be explained in more detail below. Finally, let τ denote non-distortionary real lump-sum taxes.

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[11]

$$C_t^w(h) = nC_t(h) + (1-n)C_t^*(h) = \left[\frac{P_t(h)}{P_{t,H}}\right]^{-\theta}\left(\frac{P_{t,H}}{P_t}\right)^{-1}[nC_t + (1-n)C_t^*] = \left[\frac{P_t(h)}{P_{t,H}}\right]^{-\theta}\left(\frac{P_{t,H}}{P_t}\right)^{-1}C_t^w.$$

[12]

$$P_t C_t = P_{t,H}C_{t,H} + P_{t,F}C_{t,F} = \int_0^n P_t(h)C_t(h)dh + \int_n^1 P_t(f)C_t(f)df.$$

[13]Note that one could generalize this formulation by assuming that domestic and foreign households had access to a complete portfolio of state-contingent Arrow-Debreu securities, both domestically and internationally tradable, as in CLARIDA et al. (2002) in order to guarantee for the completeness of (international) financial markets.

Again, for the representative foreign household the intertemporal budget constraint is the same as (2.17). Since internationally traded bonds are supposed to be denominated in domestic currency, foreign bond holdings in domestic currency B^*, however, first have to be divided by the nominal exchange rate before they enter the foreign intertemporal budget constraint: B^*/S. Moreover, W^* may differ from W, i^* from i, $\Gamma^*(f)$ from $\Gamma(h)$, as well as τ^* from τ. Hence, the sequence of foreign intertemporal budget constraints (in nominal terms) reads as follows:

$$W_t^* L_t^* + (1 + i_{t-1}^*)\frac{B_{t-1}^*}{S_{t-1}} + M_{t-1}^* + \Gamma_t^*(f) \geq P_t^* C_t^* + M_t^* + \frac{B_t^*}{S_t} + P_t^* \tau_t^*.$$

Similar to CORSETTI & PESENTI (2001, p. 427), this equation implies that the realized nominal return on internationally traded bonds at the beginning of period t in foreign currency is given by:

$$(1 + i_{t-1}^*) = \frac{S_{t-1}}{S_t}(1 + i_{t-1}). \tag{2.18}$$

It is useful to introduce the domestic real interest rate r, which can be obtained via the subsequent Fisher relation:

$$(1 + r_{t-1}) \equiv \frac{P_{t-1}}{P_t}(1 + i_{t-1}).$$

Note that an analogous equation to the preceding one also holds for the foreign real interest rate r^*. If we substitute for $1 + i_{t-1}^*$ by (2.18) in the foreign analog to the above equation, we will obtain that real interest rates are equal across countries: $r = r^*$.[14]

The maximization of the utility function (2.1) subject to the budget constraint (2.17) then holding with equality is undertaken by maximizing the corresponding Lagrangian and yields the subsequent first order conditions for a utility maximum:

$$\frac{C_t^{-\rho}}{P_t} = \beta(1 + i_t)E_t\left[\frac{C_{t+1}^{-\rho}}{P_{t+1}}\right]. \tag{2.19}$$

This is the intertemporal Euler equation for real consumption stating that the marginal rate of substitution between real consumption in t and in $t+1$ equals their discounted relative prices.

Moreover, we obtain that in a utility maximum the marginal rate of substitution between real money balances and real consumption equals the opportunity costs of

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[14]This is the reason why the home and the foreign country can be treated as symmetric countries, even though there is only one, *domestic*-currency denominated bond in this world.

holding money:

$$\chi \frac{\left(\frac{M_t}{P_t}\right)^{-\varepsilon}}{C_t^{-\rho}} = \frac{i_t}{1 + i_t}. \tag{2.20}$$

Note that equation (2.20) can be rearranged in order to get the following money demand equation:

$$\left(\frac{M_t}{P_t}\right)^{\varepsilon} = \chi \frac{1 + i_t}{i_t} C_t^{\rho},$$

where we can see that the higher total real consumption, the higher is demand for real money balances.

Finally, we also get the subsequent labor supply equation:

$$\gamma \frac{L_t^{-\xi}}{C_t^{-\rho}} = \frac{W_t}{P_t}, \tag{2.21}$$

which states that the marginal rate of substitution between labor and real consumption equals their relative prices, the real consumer wage. For a derivation of conditions (2.19), (2.20), and (2.21) see Appendix 2.8.

Note that analogous equations to (2.19), (2.20), and (2.21) also hold abroad.

2.2.3 Firms

Let us assume further that agents at home and abroad do not only act as utility maximizing households, but also as profit maximizing producers of final goods, which are producible without the input of intermediate goods. In contrast to their role as households whose preferences are assumed to be identical, all commodities are differentiated in order to satisfy the households' love of variety.

Hence, there is the possibility to raise individual goods' prices $P(h), P(f)$ above marginal cost without the risk of dropping out of the market. In other words, non-zero profits are feasible in this model of monopolistic competition.

Let individual domestic output be produced according to the following linear production function:

$$Y_t(h) = A_t L_t(h). \tag{2.22}$$

This is a production function in labor only. For the sake of simplicity, physical capital

is omitted as additional input factor throughout the analysis. This step can be justified by the short- to medium-run perspective of the model. A is a random variable denoting an exogenous aggregate productivity shock, which can be interpreted as a transitory process innovation.

Households need not be self-employed, but it is assumed that domestic firms can employ domestic labor only as well as foreign firms shall be allowed to employ foreign labor only. In other words, there is no migration in this world.

Individual foreign output is produced using the same technology (2.22) as at home. Nonetheless, $Y^*(f)$ may differ from $Y(h)$, A^* from A, as well as $L^*(f)$ from $L(h)$.

Producers' instantaneous profits $\Gamma_t(h)$ are given by:

$$\Gamma_t(h) = P_t(h)Y_t(h) - W_tL_t(h). \tag{2.23}$$

Relative to the producer's own price, equation (2.23) rearranges to:

$$\frac{\Gamma_t(h)}{P_t(h)} = Y_t(h) - \frac{W_t}{P_t(h)}L_t(h) = Y_t(h) - \frac{W_t}{P_t(h)}\frac{Y_t(h)}{A_t} = Y_t(h) - \kappa_tY_t(h), \tag{2.24}$$

where we have made use of the production function (2.22). In (2.24) $\kappa := W/[P(h)A]$ is defined as individual real marginal production cost.

For now, assume all goods prices to be flexible. Then each domestic producer charges the same price denoted by the domestic PPI ($P_H = P(h)$). Thus, instantaneous profits rearrange to:

$$\Gamma_t(h) = P_{t,H}Y_t(h) - W_tL_t(h). \tag{2.25}$$

Maximizing equation (2.25) with respect to $Y(h)$ and using the fact that in case of goods market clearing the output of a single producer equals global demand for the differentiated good ($Y(h) = C^w(h)$), we get the standard first order condition for a profit maximum in a model of monopolistic competition:

$$\begin{aligned}
\frac{\partial \Gamma_t(h)}{\partial Y_t(h)} &= P_{t,H} + Y_t(h)\frac{\partial P_{t,H}}{\partial Y_t(h)} - W_t\frac{\partial L_t(h)}{\partial Y_t(h)} = P_{t,H}\left(1 + \frac{1}{\epsilon_{C(h),P(h)}}\right) - W_t\frac{1}{A_t} \\
&= P_{t,H}\left(1 + \frac{1}{-\theta}\right) - W_t\frac{1}{A_t} = 0 \\
\Rightarrow \frac{W_t}{P_{t,H}A_t} &= \frac{\theta - 1}{\theta} := \kappa_t^{flex}. \tag{2.26}
\end{aligned}$$

Note that an analogous equation to (2.26) also holds abroad and that $\kappa^{flex} = (\kappa^*)^{flex} =$

$(\theta - 1)/\theta$.

Equation (2.26) states that in a profit maximum associated with flexible prices, the corresponding real marginal production cost defined as κ^{flex} equals $(\theta - 1)/\theta$.[15]

2.3 Market clearing under flexible prices

Before introducing nominal rigidities in Section 2.4, we will first consider the benchmark case of market equilibria in a world with completely flexible prices.

2.3.1 World bond and goods markets

For the derivation of the subsequent equations and their relation to one another see Appendix 2.8.

Let us begin with the equilibrium conditions on the world markets for domestic and foreign goods denoted in domestic currency:

$$P_{t,H}Y_t = P_t C_t^w, \tag{2.27}$$
$$P_{t,F}Y_t^* = P_t C_t^w, \tag{2.28}$$

where the left-hand side of equation (2.27) denotes global supply of and the right-hand side global demand for domestic goods.

Note that an analogous interpretation for (2.28) also holds abroad.

Equations (2.27) and (2.28) immediately collapse to the definition of the terms of trade (TOT):

$$T_t := \frac{P_{t,F}}{P_{t,H}} = \frac{S_t P_{t,F}^*}{P_{t,H}} = \frac{Y_t}{Y_t^*}, \tag{2.29}$$

which is the ratio of imported goods' and exported goods' prices from the home country's perspective.

Using the domestic intertemporal budget constraint (2.17) plus further manipulations eventually yield the domestic and foreign balance of payment identities:

$$P_{t,H}Y_t - P_t C_t + i_{t-1}B_{t-1} \equiv B_t - B_{t-1}, \tag{2.30}$$
$$P_{t,F}Y_t^* - P_t C_t^* + i_{t-1}B_{t-1}^* \equiv B_t^* - B_{t-1}^* \tag{2.31}$$

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[15]Note that if we solved equation (2.26) for P_H, we would obtain the domestic PPI as a mark-up on marginal unit labor costs W/A: $P_H = [\theta/(\theta - 1)]W/A$ with $\theta/(\theta - 1) = 1/\kappa^{flex}$ denoting the flexible-price mark-up factor.

with the left-hand side of equation (2.30) representing the home country's current account and the right-hand side its capital account.

Note that an analogous interpretation for (2.31) also holds abroad.

Internationally tradable bonds are supposed to be in zero net world supply:

$$nB_t + (1 - n)B_t^* = 0. \tag{2.32}$$

Assuming that international bond holdings have initially been zero $B_0 = B_0^* = 0$ together with (2.14), (2.30), (2.31), and (2.32) implies that $B_t = B_t^* = 0$ at all times according to CORSETTI & PESENTI (2001, pp. 430-432) and OBSTFELD & ROGOFF (2001, p. 8).[16] Then equations (2.30) and (2.31) simplify to the following:

$$C_t = \frac{P_{t,H}Y_t}{P_t}, \tag{2.33}$$

$$C_t^* = \frac{P_{t,F}Y_t^*}{P_t}. \tag{2.34}$$

Using the definition of the TOT (2.29) the preceding equations can be rewritten as:

$$C_t = T_t^{n-1}Y_t, \tag{2.35}$$

$$C_t^* = T_t^n Y_t^*. \tag{2.36}$$

These are the conditions for domestic and foreign goods market clearing, which imply that households across countries always consume exactly their real incomes (see OBSTFELD & ROGOFF 2001, p. 8).

Moreover, $B_0 = B_0^* = 0$ together with (2.14), (2.30), (2.31), and (2.32) also implies that $C_t = C_t^* = C_t^w$ at all times such that

$$C_t = C_t^* = C_t^w = nC_t + (1 - n)C_t^* = nT_t^{n-1}Y_t + (1 - n)T_t^n Y_t^* = Y_t^n(Y_t^*)^{1-n},$$

while making use of (2.35) and (2.36).

As a consequence, consumption shares across countries are not only time-constant but even equal (see OBSTFELD & ROGOFF 2001, p. 8). Since current and capital accounts between the two countries are in balance at all times and in all possible states of the world, the mechanism of adjustment to shocks in the world economy will only be

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[16]More precisely, Cobb-Douglas preferences for the domestic and foreign commodity bundles as in (2.2) together with producer-currency pricing and the absence of preference shocks imply under the assumption of completely flexible prices that any shock that reduces the supply of output of a country will increase its price in equal proportion. Thus, the value of its real income remains unchanged and the allocation under complete markets can be achieved without trade in bonds.

represented by movements in the TOT, but not by changes in the countries' net asset positions.

2.3.2 National money markets and world currency market

The government is assumed to set its expenditures equal to its revenues at all times such that its budget is always in balance and no seignorage can occur (see OBSTFELD & ROGOFF 1996, p. 523):[17]

$$M_t - M_{t-1} + P_t \tau_t = 0. \tag{2.37}$$

Note that an analogous equation to (2.37) also holds abroad.

Equation (2.37) describes domestic money supply. Combining (2.37) with (2.20) and using the condition for domestic goods market clearing (2.35), one obtains two equations in M, which can be set equal and eventually solved for P:

$$P_t = \frac{M_{t-1}}{\chi^{\frac{1}{\varepsilon}} \left(\frac{1+i_t}{i_t}\right)^{\frac{1}{\varepsilon}} (T_t^n Y_t^*)^{\frac{\rho}{\varepsilon}} + \tau_t}.$$

Making use of (2.9), an analogous equation in P can be computed abroad such that both equations can be set equal and finally solved for S:

$$S_t = \frac{M_{t-1} \left[(\chi^*)^{\frac{1}{\varepsilon}} \left(\frac{1+i_t^*}{i_t^*}\right)^{\frac{1}{\varepsilon}} (T_t^n Y_t^*)^{\frac{\rho}{\varepsilon}} + \tau_t^* \right]}{M_{t-1}^* \left[\chi^{\frac{1}{\varepsilon}} \left(\frac{1+i_t}{i_t}\right)^{\frac{1}{\varepsilon}} (T_t^{n-1} Y_t)^{\frac{\rho}{\varepsilon}} + \tau_t \right]}.$$

As we can see from the above formula, the current equilibrium nominal exchange rate S_t positively depends on past domestic nominal money balances M_{t-1}, current domestic opportunity costs of holding money $i_t/(1+i_t)$, current foreign output Y_t^*, and current foreign real lump-sum taxes τ_t^*. The dependence on the remaining variables is of opposite sign, except for the current TOT T_t, whose influence is ambiguous. An increase of S illustrates a depreciation of the domestic currency, whereas a decrease characterizes an appreciation.

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[17]One could extend the model by introducing government spending (shocks) (see OBSTFELD & ROGOFF 2001, pp. 37-38), which we will not consider in our analysis.

2.3.3 National labor markets

Notice from equations (2.21) and (2.26) that the real wage differs between consumers and producers because they use different price indices. The ratio between real producer and real consumer wage is known as one type of *wedge* in Labor Economics (see, e.g., LANDMANN & JERGER 1999, pp. 136-138) and equals $P_H/P_H^n P_F^{1-n} = (P_H/P_F)^{1-n} = T^{n-1}$ in the present set-up.

Nonetheless, by combining (2.21), (2.26), and (2.35) with the CPI (2.5) we obtain two equations in $W/P = (W/P_H)T^{n-1}$ which can be solved for L:

$$L_t = T_t^{\frac{(n-1)(\rho-1)}{\xi}} \left(\frac{A_t}{\gamma}\right)^{-\frac{1}{\xi}} \left(\frac{\theta-1}{\theta}\right)^{-\frac{1}{\xi}} Y_t^{\frac{\rho}{\xi}}. \tag{2.38}$$

Equation (2.38) states that in an equilibrium on the perfectly competitive labor market, domestic employment positively depends on the aggregate productivity shock A and flexible-price real marginal production cost $(\theta-1)/\theta$, but negatively on the TOT T and domestic output Y.

Note that an analogous equation to (2.38) also holds abroad.

Combining equation (2.38) with the production function (2.22) and solving for Y, we finally obtain the domestic flexible-price equilibrium output Y^{flex}:

$$Y_t^{flex} = T_t^{\frac{(n-1)(\rho-1)}{\xi-\rho}} A_t^{\frac{\xi-1}{\xi-\rho}} \left(\frac{\theta}{\theta-1}\right)^{\frac{1}{\xi-\rho}} \gamma^{\frac{1}{\xi-\rho}}. \tag{2.39}$$

The domestic flexible-price equilibrium output positively depends on the aggregate productivity shock A, yet negatively on the TOT T and the flexible-price mark-up factor $\theta/(\theta-1)$.

Note that an analogous equation to (2.39) also holds abroad.

2.4 A New Keynesian framework

After having drawn the DSGE set-up and derived optimality conditions for both households and firms (Section 2.2) as well as market clearing conditions under flexible prices (Section 2.3), let us now turn to the New Keynesian framework. In order to establish this type of framework, we have to introduce some form of nominal rigidity in addition to the assumption of monopolistic competition. In the present case, we will concentrate on sticky prices and forego sticky nominal wages as done, e.g., by CORSETTI & PESENTI (2001).

Log-linearizing the alternative market clearing and optimality conditions in the neighborhood of the non-stochastic zero-inflation steady state will lead to a canonical representation of the equilibrium of the model consisting of a dynamic IS curve, a New Keynesian Phillips curve (NKPC), and some form of monetary policy rule for each country, as well as an equation for the TOT.

As there are two countries, altogether we will obtain a system of seven log-linear equations. This form makes the model analytically tractable, especially for empirical applications.[18]

Finally, we assume the monetary policy rules introduced below to be *different* across countries. This is one of the crucial assumptions of the present analysis.

2.4.1 Dynamic IS curves

It is straightforward to derive the dynamic IS curves for both countries by log-linearizing the domestic intertemporal Euler equation for real consumption (2.19) and its foreign analog around the non-stochastic zero-inflation steady state as shown in Appendix 2.8.

Accordingly, we obtain:

$$\hat{y}_t = E_t[\hat{y}_{t+1}] + \frac{1}{\rho}\{E_t[\pi_{t+1}] - \hat{i}_t\} - (1-n)E_t[\Delta t_{t+1}], \tag{2.40}$$

$$\hat{y}_t^* = E_t[\hat{y}_{t+1}^*] + \frac{1}{\rho}\{E_t[\pi_{t+1}^*] - \hat{i}_t^*\} + nE_t[\Delta t_{t+1}]. \tag{2.41}$$

Note that except for all types of interest rates, lower-case Latin letters denote natural logarithms of the corresponding variables. Usually, the hats above these log variables signify percentage deviations from their zero-inflation steady-state values. However, for all types of interest rates, these hats denote deviations measured in percentage points. The zero-inflation steady-state values themselves are denoted by upper bars. Note further that $\bar{i} = \bar{i}^* = \bar{r} = \bar{r}^* = (1-\beta)/\beta$ holds for the zero-inflation steady-state nominal and real interest rates, both at home and abroad.[19]

These two dynamic IS curves represent aggregate demand in both countries, where (2.40) can be interpreted as follows: current domestic demand is higher than its zero-inflation steady-state value if the expected domestic output deviation $E_t[\hat{y}_{t+1}]$ is positive (interpretable as an expected peak in the domestic business cycle). There is also a clear

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[18]In contrast to CORSETTI & PESENTI (2001) who present a closed-form solution of their (deterministic) model, the log-linear approximation used here is considered to be advantageous since the link to empirical applications is immediate.

[19]This can easily be obtained by solving the zero-inflation steady-state version of the domestic intertemporal Euler equation for real consumption (2.19) and its foreign analog for $\bar{i}$ and $\bar{i}^*$, respectively $(C_t^{-\rho} = E_t[C_{t+1}^{-\rho}] = \bar{C}^{-\rho}, P_t = E_t[P_{t+1}] = \bar{P})$.

positive relation of current demand to expected CPI inflation $E_t[\pi_{t+1}] := E_t[p_{t+1}] - p_t$ (households consume more today if prices are expected to augment in the future) and a negative relation to current deviations from the zero-inflation steady-state nominal interest rate $\hat{i}_t$ (investing in nominal bonds is relatively attractive compared to buying consumption goods).

Moreover, there are also spill-over effects from abroad, which affect current domestic demand through expected movements in the TOT $E_t[\Delta t_{t+1}]$: current domestic demand negatively depends on an expected increase in the TOT since TOT expected to augment mean that imported goods become more expensive relative to domestic goods.[20] $(1 - n)$ denotes the degree of openness of the home country to the foreign country (see GALÍ 2008, pp. 155-156). Since the degree of openness coincides with the size of the foreign country due the definition of the domestic CPI (2.2), there is no home bias in consumption, different to what is discussed in PAPPA (2004, pp. 770-771).

Note that an analogous interpretation for (2.41) also holds abroad.

2.4.2 New Keynesian Phillips curves

The NKPCs for both countries can be derived by log-linearizing the price-setting equations of domestic and foreign firms around the non-stochastic zero-inflation steady-state as shown in Appendix 2.8. In order to obtain the short-run *trade-off* between PPI inflation and the output gap represented by a Phillips curve it is necessary to assure price stickiness in addition to monopolistic competition.

This will be done by introducing CALVO (1983) contracts, which means that each producer is only allowed to reset her price with probability $(1 - \delta)$ in any given period, independent of the time since the last adjustment. Therefore, a measure of $(1 - \delta)$ of firms reset theirs prices each period, while a measure of δ of firms keep their prices constant and simply adjust their individual output in order to meet demand. $1/(1 - \delta)$ then captures the average duration of a price (see GALÍ 2008, p. 43):

$$\pi_{t,H} = \beta E_t[\pi_{t+1,H}] + \frac{(1-\delta)(1-\delta\beta)}{\delta}\hat{\kappa}_t, \tag{2.42}$$

$$\pi_{t,F}^* = \beta E_t[\pi_{t+1,F}^*] + \frac{(1-\delta^*)(1-\delta^*\beta)}{\delta^*}\hat{\kappa}_t^*. \tag{2.43}$$

In equation (2.42), $\pi_{t,H} := p_{t,H} - p_{t-1,H}$ is defined as current domestic PPI inflation, which typically differs from domestic CPI inflation in an open economy. The NKPC

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[20]The TOT are expected to increase over time if either the domestic currency is expected to depreciate or if expected foreign PPI inflation will be higher than expected domestic PPI inflation, where these rates of inflation will be discussed below in more detail.

(2.42) states that current domestic PPI inflation $\pi_{t,H}$ is an increasing function of both expected domestic PPI inflation $E_t[\pi_{t+1,H}]$ and the deviation of current domestic real marginal production cost from its zero-inflation steady-state value $\hat{\kappa}_t := \kappa_t - \kappa_t^{flex}$.

Note that an analogous interpretation for (2.43) also holds abroad. However, δ^* is assumed to differ from δ and κ^* also from κ, although $\kappa^{flex} = (\kappa^*)^{flex} = (\theta - 1)/\theta$. Furthermore, let us assume that *setting a new price at home* and *setting a new price abroad* are stochastically independent events. As domestic and foreign firms both set theirs prices in the currency of the countries where they are located, the present model features producer currency pricing, which is one of the possible occurrences of pricing to market.[21]

Nonetheless, we want to express equations (2.40), (2.41), (2.42), and (2.43) in terms of the output gap, which shall be defined as the difference between actual and flexible-price output deviations: $x_t := \hat{y}_t - \hat{y}_t^{flex}$ and $x_t^* := \hat{y}_t^* - (\hat{y}_t^*)^{flex}$. In order to rewrite equations (2.42) and (2.43) in terms of x and x^*, respectively, we have to take a closer look at the ratio of the sticky-price real marginal production cost κ_t and its flexible-price counterpart κ_t^{flex} as given by (2.25):

$$\frac{\kappa_t}{\kappa_t^{flex}} = \frac{\frac{W_t}{P_{t,H}A_t}}{\frac{\theta-1}{\theta}} = \frac{\theta W_t T_t^{1-n}}{(\theta-1)P_t A_t}. \tag{2.44}$$

Combining equation (2.44) with the labor supply curve (2.21), the production function (2.22), and the condition for domestic goods market clearing (2.35), we obtain:

$$\frac{\kappa_t}{\kappa_t^{flex}} = \frac{\theta\gamma\left(\frac{Y_t}{A_t}\right)^{-\xi} T_t^{1-n}}{(\theta-1)(T_t^{n-1}Y_t)^{-\rho}A_t} = \frac{\theta}{\theta-1}\gamma A_t^{\xi-1}T_t^{(n-1)(\rho-1)}Y_t^{\rho-\xi} = \left(\frac{Y_t}{Y_t^{flex}}\right)^{\rho-\xi}, \tag{2.45}$$

where Y_t^{flex} denotes the domestic flexible-price equilibrium output as given by equation (2.39). Log-linearizing this expression around the zero-inflation steady-state yields:

$$\hat{\kappa}_t = (\rho - \xi)(\hat{y}_t - \hat{y}_t^{flex}) = (\rho - \xi)x_t. \tag{2.46}$$

Hence, by using (2.46) equations (2.40), (2.41), (2.42), and (2.43) rearrange to:

$$x_t = E_t[x_{t+1}] + \frac{1}{\rho}\{E_t[\pi_{t+1}] - \hat{i}_t\} - (1-n)E_t[\Delta t_{t+1}] + E_t[\hat{y}_{t+1}^{flex}] - \hat{y}_t^{flex}, \tag{2.47}$$

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[21]This specification has already been adopted in the theoretical literature (see, e.g., CLARIDA et al. 2002, p. 885) and can also be justified by empirical evidence for most of the G7 countries (see LEITH & MALLEY 2007, p. 420).

$$x_t^* = E_t[x_{t+1}^*] + \frac{1}{\rho}\{E_t[\pi_{t+1}^*] - \hat{i}_t^*\} + nE_t[\Delta t_{t+1}] + E_t[(\hat{y}_{t+1}^*)^{flex}] - (\hat{y}_t^*)^{flex}, \quad (2.48)$$

$$\pi_{t,H} = \beta E_t[\pi_{t+1,H}] + \mu x_t + u_t, \quad (2.49)$$

$$\pi_{t,F}^* = \beta E_t[\pi_{t+1,F}^*] + \mu^* x_t^* + u_t^* \quad (2.50)$$

with $\mu := [(1-\delta)(1-\delta\beta)(\rho-\xi)]/\delta$ ($\mu > 0$) and $\mu^* := [(1-\delta^*)(1-\delta^*\beta)(\rho-\xi)]/\delta^*$ ($\mu^* > 0$) representing the slope coefficients of the NKPCs with respect to the domestic (foreign) output gap. In addition, u_t denotes an exogenously given, stationary AR(1) process of the form $u_t = \zeta_u u_{t-1} + \eta_{u,t}$ ($0 < \zeta_u < 1$) with the exogenous error term η_u assumed to be i.i.d. $\sim N(0, \sigma_{\eta_u}^2)$. This AR(1) process can be interpreted as a transitory cost-push shock reflecting determinants of real marginal production cost which do not move proportionally with the output gap (see CLARIDA et al. 2001, pp. 250-251).

The two NKPCs represent aggregate supply in both countries and are isomorphic to their closed-economy counterparts, where (2.49) can be interpreted as follows: the positive short-run *trade-off* between current domestic PPI inflation $\pi_{t,H}$ and the current domestic output gap x_t can be seen.[22] However, this is not really a trade-off to be exploited since $\pi_{t,H}$ is also positively related to (discounted) expected domestic PPI inflation $\beta E_t[\pi_{t+1,H}]$.[23]

Note that an analogous interpretation for (2.50) also holds abroad. However, u^* is uncorrelated with u such that domestic and foreign cost-push shocks are country-specific.

It is useful that the following holds for $E_t[\hat{y}_{t+1}^{flex}] - \hat{y}_t^{flex}$ in case one makes use of the log-linear version of the current domestic flexible-price equilibrium output according to (2.39) and its expected counterpart:

$$
\begin{aligned}
E_t[\hat{y}_{t+1}^{flex}] - \hat{y}_t^{flex} &= E_t[y_{t+1}^{flex}] - y_t^{flex} \\
&= \frac{(n-1)(\rho-1)}{\xi-\rho}E_t[t_{t+1}] + \frac{\xi-1}{\xi-\rho}E_t[a_{t+1}] + \frac{1}{\xi-\rho}\ln\left(\frac{\theta}{\theta-1}\right) + \frac{1}{\xi-\rho}\ln\gamma \\
&\quad - \frac{(n-1)(\rho-1)}{\xi-\rho}t_t - \frac{\xi-1}{\xi-\rho}a_t - \frac{1}{\xi-\rho}\ln\left(\frac{\theta}{\theta-1}\right) - \frac{1}{\xi-\rho}\ln\gamma \\
&= \frac{(n-1)(\rho-1)}{\xi-\rho}E_t[\Delta t_{t+1}] + \frac{\xi-1}{\xi-\rho}E_t[\Delta a_{t+1}], \quad (2.51)
\end{aligned}
$$

where a_t is assumed to follow an exogenously given, stationary AR(1) process of the

---

[22]Note that NKPCs such as (2.49) and (2.50) in terms of the output gap sometimes are referred to as aggregate supply (AS) curves (see CLARIDA et al. 2001, p. 250).

[23]If, for instance, some institution had the power to raise domestic output above its flexible-price value (given the deviations from its zero-inflation steady-state value) by raising $\pi_{t,H}$, not only $\pi_{t,H}$ but also $\beta E_t[\pi_{t+1,H}]$ would have to rise for (2.49) to hold with equality. This means that if output were kept on this artificially high level for an extended period of time, the respective expected inflation rates would continue to rise at accelerating speed, which is described by the so-called acceleration theorem.

form $a_t = \zeta_a a_{t-1} + \eta_{a,t}$ $(0 < \zeta_a < 1)$ with the exogenous error term η_a assumed to be i.i.d. $\sim N(0, \sigma_{\eta_a}^2)$.

Note that an analogous equation to (2.51) also holds abroad. However, a^* is uncorrelated with a such that domestic and foreign aggregate productivity shocks are country-specific.

In consequence, the dynamic IS curves (2.47) and (2.48) rearrange to:

$$x_t = E_t[x_{t+1}] + \frac{1}{\rho}\{E_t[\pi_{t+1}] - \hat{i}_t\} + \frac{(n-1)(\xi-1)}{\xi-\rho}E_t[\Delta t_{t+1}] + \frac{\xi-1}{\xi-\rho}E_t[\Delta a_{t+1}], \quad (2.52)$$

$$x_t^* = E_t[x_{t+1}^*] + \frac{1}{\rho}\{E_t[\pi_{t+1}^*] - \hat{i}_t^*\} + \frac{n(\xi-1)}{\xi-\rho}E_t[\Delta t_{t+1}] + \frac{\xi-1}{\xi-\rho}E_t[\Delta a_{t+1}^*]. \quad (2.53)$$

Finally, we prefer to express these dynamic IS curves in terms of PPI rather than CPI inflation, which can be achieved by using the subsequent log-linear representation of the TOT (2.29): $t_t := s_t + p_{t,F}^* - p_{t,H}$. Subtracting this expression from its expected analog we get: $E_t[\Delta t_{t+1}] = E_t[\Delta s_{t+1}] + E_t[\pi_{t+1,F}^*] - E_t[\pi_{t+1,H}] = E_t[\pi_{t+1,F}^*] - E_t[\pi_{t+1,H}]$. Combining this outcome with the log-linear versions of the domestic CPI (2.5) and its foreign equivalent, we obtain the following relations between expected CPI and expected PPI inflation at home and abroad:

$$E_t[\pi_{t+1}] \equiv E_t[\pi_{t+1,H}] - (n-1)E_t[\Delta t_{t+1}], \quad (2.54)$$

$$E_t[\pi_{t+1}^*] \equiv E_t[\pi_{t+1,F}^*] - nE_t[\Delta t_{t+1}]. \quad (2.55)$$

Substituting for $E_t[\pi_{t+1}]$ by (2.54) and for $E_t[\pi_{t+1}^*]$ by (2.55), the dynamic IS curves (2.52) and (2.53) change to the following:

$$x_t = E_t[x_{t+1}] + \frac{1}{\rho}\{E_t[\pi_{t+1,H}] - \hat{i}_t\} + \vartheta E_t[\Delta t_{t+1}] + \frac{\xi-1}{\xi-\rho}E_t[\Delta a_{t+1}], \quad (2.56)$$

$$x_t^* = E_t[x_{t+1}^*] + \frac{1}{\rho}\{E_t[\pi_{t+1,F}^*] - \hat{i}_t^*\} + \vartheta^* E_t[\Delta t_{t+1}] + \frac{\xi-1}{\xi-\rho}E_t[\Delta a_{t+1}^*], \quad (2.57)$$

where $\vartheta := [(n-1)(\xi\rho - \xi)]/[(\xi - \rho)\rho]$ $(\vartheta > 0)$ and $\vartheta^* := [n(\xi\rho - \xi)]/[(\xi - \rho)\rho]$ $(\vartheta^* < 0)$ holds for the slope coefficients of the dynamic IS curves with respect to the expected movements in the TOT. In consequence, we need an equation that expresses these movements as a function of the remaining endogenous variables. Let us use the log-linear version of equation (2.18), which reads $\hat{i}_{t-1} = \Delta s_t + \hat{i}_{t-1}^*$, in order to substitute for Δs_t in the log-linear representation of current movements in the TOT $\Delta t_t = \Delta s_t + \pi_{t,F}^* - \pi_{t,H}$. Hence, we obtain:

$$\Delta t_t = \hat{i}_{t-1} - \hat{i}_{t-1}^* + \pi_{t,F}^* - \pi_{t,H}. \quad (2.58)$$

2.4.3 Monetary policy rules

With the derivation of equations (2.49), (2.50), (2.56), (2.57), and (2.58) we have derived a system of five log-linear expectational difference equations. However, with $x, x^*, \pi_H, \pi_F^*, \Delta t, \hat{i}, \hat{i}^*$ we have seven endogenous variables, which gives us two more variables than equations. Therefore, we need two more equations representing domestic and foreign monetary policy as TAYLOR (1993) type interest-rate rules in order to obtain a determined system of equations.

Following WOODFORD (2003, pp. 90-101), these interest-rate rules comprise a feedback from (some of) the endogenous variables. In WOODFORD (2003), those interest-rate rules are first incorporated into a Neo-Wicksellian cashless economy, but WOODFORD (2003, pp. 101-106) also shows that rules of this form produce equivalent results in case of monetary frictions, e.g., in case of a MIU model such as given by equation (2.1). Even though the WOODFORD (2003) results have been derived for the closed economy, they are supposed to hold for the open economy, too, which is due to the isomorphism of the models.

The feedback is introduced to circumvent price level (and inflation) indeterminacy as shown by SARGENT & WALLACE (1975), which is typically associated with purely exogenous interest-rate targets (see WOODFORD 2003, p. 86). In case the latter type of modeling is avoided, the monetary aggregate is not a superior policy instrument in comparison to the short-run nominal interest rate. Moreover, it is assumed that the central banks are committed to their rules rather than they implement new *rules* on a period-by-period basis. This is done to overcome time inconsistency of monetary policy.[24]

Hereinafter, the two monetary authorities will be called ECB at home and Fed abroad. As a consequence, the home country will be denoted EU and the foreign country US.

These two non-optimizing central banks are assumed to conduct their monetary policies autonomously while taking as given the policy actions of the respective other monetary authority. This assumption differs, for instance, from CLARIDA et al. (2002), PAPPA (2004), BENIGNO & BENIGNO (2006), or ENGEL (2009) who discuss cooperative and non-cooperative equilibria of optimizing central banks among other possibilities. A particular reason why we neglect cooperative solutions is the finding that there are only quantitatively negligible welfare gains from cooperation between the ECB and the Fed for empirically plausible parameter constellations (see PAPPA 2004, pp. 770-774).

---

[24]For a debate on discretion versus commitment in monetary policy and possible welfare gains from the latter see, e.g., CLARIDA et al. (1999, pp. 1670-1671) or GALÍ (2008, chapter 5).

Even though the central banks' targeting rules fulfill a similar purpose in the EU and the US, it can be justified to assume that they differ up to a certain degree. This is due to the diverging statutes of the ECB and the Fed.

Therefore, the interest-rate rules differ to the extent that the Fed is supposed to conduct its monetary policy by considering current US PPI inflation $\pi_{t,F}^*$ *and* the current US output gap x_t^*, while, for the sake of simplicity, the ECB imposes its monetary policy by taking into account current EU PPI inflation $\pi_{t,H}$ *only*. This difference is due to the fact that all conceivable policy goals of the ECB besides price stability can be interpreted as secondary.

Hence, the two interest-rate rules read:

$$\hat{i}_t = \alpha\pi_{t,H} + \omega\hat{i}_{t-1} + v_t \tag{2.59}$$
$$\Leftrightarrow i_t = \alpha\pi_{t,H} + \omega i_{t-1} + (1-\omega)\bar{i} + v_t,$$
$$\hat{i}_t^* = \alpha^*\pi_{t,F}^* + \iota^*x_t^* + \omega^*\hat{i}_{t-1}^* + v_t^* \tag{2.60}$$
$$\Leftrightarrow i_t^* = \alpha^*\pi_{t,F}^* + \iota^*x_t^* + \omega^*i_{t-1}^* + (1-\omega^*)\bar{i}^* + v_t^*.$$

The ECB's interest rate rule (2.59) can be interpreted as follows: α $(\alpha > 0)$ denotes the sensitivity of the ECB to current domestic PPI inflation $\pi_{t,H}$. Since past decisions cannot be ignored under commitment (see PAPPA 2004, p. 754), the rule incorporates some degree of inertia of the monetary policy instrument i itself, which is measured by the parameter ω $(0 < \omega < 1)$. The parameter $1 - \omega$, however, measures the degree of adjustment to the zero-inflation steady-state value of the nominal interest rate $\bar{i}$.

Note that the feature of interest-rate inertia is rather an empirical finding than an implication of the statutes of the central banks (see WOODFORD 2003, pp. 95-96). Following GALÍ & MONACELLI (2005, p. 723), both rules (2.59) and (2.60) could also be denoted PPI (or domestic) inflation-based Taylor rules (DITR) as opposed to CPI inflation-based Taylor rules (CITR) or a credible peg for the nominal exchange rate. These are the three suboptimal monetary policies compared to the optimal monetary policy benchmark in the GALÍ & MONACELLI (2005) model. We will not take up these other possibilities of monetary policy design here, but will employ a CITR later on in Chapter 4.

In (2.59), v_t denotes an exogenously given, stationary AR(1) process of the form $v_t = \zeta_v v_{t-1} + \eta_{v,t}$ $(0 < \zeta_v < 1)$ with the exogenous error term η_v assumed to be i.i.d. $\sim N(0, \sigma_{\eta_v}^2)$. This AR(1) process can be interpreted as a transitory monetary policy shock, where a positive realization of η_v denotes a contractionary shock (see GALÍ 2008, p. 51).

Note that an analogous interpretation for (2.60) also holds for the US. However, α^* may differ from α as well as ω^* from ω. Moreover, v^* is uncorrelated with v such that domestic and foreign monetary policy shocks are country-specific. ι^* ($\iota^* > 0$) denotes the sensitivity of the Fed to the current foreign output gap x_t^*, where $\iota = 0$ is assumed to hold for the ECB. Since the signs of the elasticities of the central banks' policy instruments to endogenous variables are all positive so that they react anti-cyclically to their changes, the policies can alternatively be characterized as having a *lean against the wind* property (see CLARIDA et al. 1999, p. 1672).

2.5 Determinacy of the rational expectations equilibrium

To investigate whether there exists a determinate rational expectations equilibrium to the system of expectational difference equations (2.49), (2.50), (2.56), (2.57), (2.58), (2.59), and (2.60) we have to rearrange it in matrix form:

$$\mathbf{A}\mathbf{y} = \mathbf{B}\mathbf{x} + \mathbf{u}, \tag{2.61}$$

where the vectors of unknowns $\mathbf{y}, \mathbf{x}$ and the vector of disturbance terms $\mathbf{u}$ read as follows:

$$
\mathbf{y} := \begin{bmatrix} x_t \\ x_t^* \\ \pi_{t,H} \\ \pi_{t,F}^* \\ \Delta t_t \\ \hat{i}_{t-1} \\ \hat{i}_{t-1}^* \end{bmatrix}, \mathbf{x} := \begin{bmatrix} E_t[x_{t+1}] \\ E_t[x_{t+1}^*] \\ E_t[\pi_{t+1,H}] \\ E_t[\pi_{t+1,F}^*] \\ E_t[\Delta t_{t+1}] \\ \hat{i}_t \\ \hat{i}_t^* \end{bmatrix}, \mathbf{u} := \begin{bmatrix} \frac{\xi-1}{\xi-\rho} E_t[\Delta a_{t+1}] \\ \frac{\xi-1}{\xi-\rho} E_t[\Delta a_{t+1}^*] \\ u_t \\ u_t^* \\ 0 \\ -\omega^{-1} v_t \\ -(\omega^*)^{-1} v_t^* \end{bmatrix}.
$$

The coefficient matrices $\mathbf{A}, \mathbf{B}$, however, read:

$$
\mathbf{A} := \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 \\
-\mu & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & -\mu^* & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & 1 & -1 & 1 \\
0 & 0 & \frac{\alpha}{\omega} & 0 & 0 & 1 & 0 \\
0 & \frac{\iota^*}{\omega^*} & 0 & \frac{\alpha^*}{\omega^*} & 0 & 0 & 1
\end{bmatrix},
$$

$$
\mathbf{B} := \begin{bmatrix}
1 & 0 & \rho^{-1} & 0 & \vartheta & -\rho^{-1} & 0 \\
0 & 1 & 0 & \rho^{-1} & \vartheta^* & 0 & -\rho^{-1} \\
0 & 0 & \beta & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \beta & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \omega^{-1} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & (\omega^*)^{-1}
\end{bmatrix}.
$$

To determine the eigenvalues of the system of equations (2.61), it has to be rearranged in the following form:

$$
\mathbf{y} = \mathbf{M}\mathbf{x} + \mathbf{v}, \tag{2.62}
$$

where $\mathbf{M} := \mathbf{A}^{-1}\mathbf{B}$ and $\mathbf{v} := \mathbf{A}^{-1}\mathbf{u}$. Moreover, $\mathbf{A}^{-1}$ denotes the inverse of $\mathbf{A}$, which exists because $\mathbf{A}$ is quadratic and $\det(\mathbf{A}) = 1$.

The matrices $\mathbf{A}^{-1}$ and $\mathbf{M}$ and the vector $\mathbf{v}$ read as follows:

$$
A^{-1}=\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\[2pt]
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\[2pt]
\mu & 0 & 1 & 0 & 0 & 0 & 0 & 0\\[2pt]
0 & \mu^{*} & 0 & 1 & 0 & 0 & 0 & 0\\[4pt]
-\dfrac{\mu(\omega+\alpha)}{\omega} & \dfrac{\mu\omega^{*}+\mu^{*}\alpha^{*}+\iota^{*}}{\omega^{*}} & -\dfrac{\mu(\omega+\alpha)}{\omega} & \dfrac{\mu\omega^{*}+\mu^{*}\alpha^{*}+\iota^{*}}{\omega^{*}} & \dfrac{\alpha+\alpha^{*}}{\omega^{*}} & 1 & 1 & -1\\[8pt]
-\dfrac{\mu\alpha}{\omega} & 0 & -\dfrac{\mu\alpha}{\omega} & 0 & -\dfrac{\alpha}{\omega} & 0 & 1 & 0\\[8pt]
0 & -\dfrac{\mu^{*}\alpha^{*}+\iota^{*}}{\omega^{*}} & 0 & -\dfrac{\mu^{*}\alpha^{*}+\iota^{*}}{\omega^{*}} & -\dfrac{\alpha^{*}}{\omega^{*}} & 0 & 0 & 1
\end{bmatrix},
$$

$$
M=\begin{bmatrix}
\rho^{-1} & 0 & 0 & \vartheta & -\rho^{-1} & 0\\[4pt]
0 & \rho^{-1} & \rho^{-1} & \vartheta^{*} & 0 & -\rho^{-1}\\[4pt]
\dfrac{\mu}{\rho}+\beta & 0 & 0 & \mu\vartheta & -\dfrac{\mu}{\rho} & 0\\[6pt]
0 & \dfrac{\mu^{*}}{\rho}+\beta & \dfrac{\mu\omega^{*}+\mu^{*}\alpha^{*}+\iota^{*}}{\omega^{*}\rho}+\beta & \mu^{*}\vartheta^{*} & 0 & -\dfrac{\mu^{*}}{\rho}\\[8pt]
-\dfrac{\mu(\omega+\alpha)}{\omega\rho}-\dfrac{(\omega+\alpha)\beta}{\omega} & \dfrac{\mu\omega^{*}+\mu^{*}\alpha^{*}+\iota^{*}}{\omega^{*}\rho}+\dfrac{(\omega^{*}+\alpha^{*})\beta}{\omega^{*}} & 0 & -\dfrac{\mu(\omega+\alpha)\vartheta}{\omega}+\dfrac{(\mu\omega^{*}+\mu^{*}\alpha^{*}+\iota^{*})\vartheta^{*}}{\omega^{*}} & \dfrac{\mu(\omega+\alpha)}{\omega\rho}+\omega^{-1} & \dfrac{\mu\omega^{*}+\mu^{*}\alpha^{*}+\iota^{*}}{\omega^{*}\rho}-(\omega^{*})^{-1}\\[10pt]
-\dfrac{\mu\alpha}{\omega\rho}-\dfrac{\alpha\beta}{\omega} & -\dfrac{\mu^{*}\alpha^{*}+\iota^{*}}{\omega^{*}\rho}-\dfrac{\alpha^{*}\beta}{\omega^{*}} & 0 & -\dfrac{(\mu^{*}\alpha^{*}+\iota^{*})\vartheta^{*}}{\omega^{*}} & \dfrac{\mu\alpha}{\omega\rho} & \dfrac{\mu^{*}\alpha^{*}+\iota^{*}}{\omega^{*}\rho}+(\omega^{*})^{-1}
\end{bmatrix},
$$

$$
v=\begin{bmatrix}
\dfrac{(\xi-1)E_t[\Delta a_{t+1}]}{\xi-\rho}\\[10pt]
\dfrac{(\xi-1)E_t[\Delta a^{*}_{t+1}]}{\xi-\rho}\\[10pt]
\dfrac{\mu(\xi-1)E_t[\Delta a_{t+1}]}{\xi-\rho}+u_t\\[10pt]
\dfrac{\mu^{*}(\xi-1)E_t[\Delta a^{*}_{t+1}]}{\xi-\rho}+u_t\\[10pt]
-\dfrac{\mu(\omega+\alpha)(\xi-1)E_t[\Delta a_{t+1}]}{\omega(\xi-\rho)}+\dfrac{(\mu\omega^{*}+\mu^{*}\alpha^{*}+\iota^{*})(\xi-1)E_t[\Delta a_{t+1}]}{\omega^{*}(\xi-\rho)}-\dfrac{(\omega+\alpha)u_t}{\omega}+\dfrac{(\omega^{*}+\alpha^{*})u_t}{\omega^{*}}-\dfrac{v_t}{\omega}+\dfrac{v^{*}_t}{\omega^{*}}\\[12pt]
-\dfrac{\mu\alpha(\xi-1)E_t[\Delta a_{t+1}]}{\omega(\xi-\rho)}-\dfrac{\alpha\,u_t}{\omega}-\dfrac{v_t}{\omega}\\[12pt]
-\dfrac{(\mu^{*}\alpha^{*}+\iota^{*})(\xi-1)E_t[\Delta a^{*}_{t+1}]}{\omega^{*}(\xi-\rho)}-\dfrac{\alpha^{*}u_t}{\omega^{*}}-\dfrac{v^{*}_t}{\omega^{*}}
\end{bmatrix},
$$

36

where $\det(\mathbf{M}) = 0$.

The impact of the single disturbances contained in $\mathbf{v}$ on the structural equations of the model can be characterized as follows. The aggregate productivity shocks a, a^* affect the respective dynamic IS curves, NKPCs, and interest rate rules. The cost-push shocks u, u^*, however, only influence the respective NKPCs and interest-rate rules. The monetary policy shocks v, v^* have a sole impact on the respective interest-rate rules. All macroeconomic shocks spill over abroad since they explicitly affect the TOT equation (2.58).

2.5.1 Analytical solution

The system of equations (2.62) consists of two predetermined variables $(\hat{i}_{t-1}, \hat{i}^*_{t-1})$ and five non-predetermined ones $(x_t, x^*_t, \pi_{t,H}, \pi^*_{t,F}, \Delta t_t)$. Comparable to the case discussed for the closed economy in GALÍ (2008, p. 56), there is a unique stationary solution of (2.62) *if and only if* the coefficient matrix $\mathbf{M}$ has five eigenvalues k inside and two eigenvalues k' on or outside the complex unit circle (sufficient condition for equilibrium determinacy). If there were more than five stable eigenvalues, there would be multiple stationary solutions (indeterminacy). If there were more than two unstable eigenvalues instead, no stationary solution would exist at all (non-existence).

By computing the characteristic determinant $\det(\mathbf{M} - k\mathbf{I_7})$ we obtain one eigenvalue $k = 0$ and a sixth-degree polynomial in k, which cannot be solved analytically.[25]

In consequence, we have to assign sensible numerical values to the model parameters in order to determine the remaining eigenvalues of $\mathbf{M}$.

2.5.2 Calibration

The numerical exercise is carried out as follows. First, the EU and the US can be treated as approximately equal-sized countries such that $n = 1 - n = 0.5$.[26] $\beta = 0.97$ is assumed to hold for the intertemporal discount factor. This implies $\bar{i} = \bar{i}^* = \bar{r} = \bar{r}^* = (1 - \beta)/\beta \approx 0.03$ for the zero-inflation steady-state nominal and real interest rates across countries. Furthermore, $\xi = -1$ such that $\epsilon_{U_L,L} = \epsilon_{U^*_L,L^*} = 2$ holds for the partial elasticity of the utility function with respect to domestic (foreign) labor. The sensitivity of the Fed to the current foreign output gap shall be fixed ($\iota^* = 0.5$), where this number corresponds to the original value estimated by TAYLOR (1993) for the Fed for the time from 1987 to 1992.

---

[25]This polynomial is not displayed here, but its MATLAB code is available on request.
[26]Note that we would obtain qualitatively similar calibration and simulation results for $n \neq 0.5$.

The Taylor principle, which states that the monetary authority ought to react to an increase in current PPI inflation by augmenting its policy instrument *more* than one for one in order to account for a determinate rational expectations equilibrium (see WOODFORD 2003, p. 40), is assumed to be fulfilled by both central banks ($\alpha = \alpha^* = 1.5$).[27] The degrees of nominal interest-rate inertia across countries shall also be fixed ($\omega = \omega^* = 0.1$) implying that both monetary authorities are supposed to place relatively more weight ($1 - \omega = 1 - \omega^* = 0.9$) on the adjustment of their short-run policy instruments to their common zero-inflation steady-state value.

Moreover, let us set $\rho = 0.8$ such that the intertemporal elasticity of substitution of real consumption $1/\rho = 1.25$. Hence, we get the following for the slope coefficients ϑ, ϑ^* of the dynamic IS curves (2.56) and (2.57) with respect to expected movements in the TOT:

$$
\begin{aligned}
\vartheta &= \frac{(0.5 - 1)[(-1) \cdot 0.8 - (-1)]}{[(-1) - 0.8]0.8} \approx 0.07, \\
\vartheta^* &= \frac{0.5[(-1) \cdot 0.8 - (-1)]}{[(-1) - 0.8]0.8} \approx -0.07.
\end{aligned}
$$

Finally, let us set the degree of price stickiness to $\delta = \delta^* = 0.75$ across countries, which corresponds to an average duration of a price of four periods. This implies the following for the slope coefficients μ, μ^* of the NKPCs (2.49) and (2.50) with respect to the domestic (foreign) output gap:

$$
\mu = \mu^* = \frac{(1 - 0.75)(1 - 0.75 \cdot 0.97)[0.8 - (-1)]}{0.75} \approx 0.16.
$$

Calculating the characteristic determinant $\det(\mathbf{M} - k\mathbf{I_7})$ while using the above parameter configuration yields the subsequent numerical eigenvalues:

$$
\begin{aligned}
k_1 &= 0, \\
k_2 &= 0.5441, \\
k_3 &= 0.8176 + 0.2411i, \\
k_4 &= 0.8176 - 0.2411i, \\
k_5 &= 0.9775,
\end{aligned}
$$

---

[27]Note that the Taylor principle in its purest form is not a necessary condition for equilibrium determinacy for an interest-rate rule of type (2.60). Instead, the condition $\mu^*(\alpha^* - 1) + (1 - \beta)\iota^* > 0$ is a necessary and sufficient condition for equilibrium determinacy in case of a contemporaneous interest-rate rule (see BULLARD & MITRA 2002, pp. 1125-1126). For a graphical representation of determinacy and indeterminacy regions for contemporaneous and forward-looking interest-rate rules see GALÍ (2008, pp. 77-80).

$$k_6 = 13.3372,$$
$$k_7 = 18.2548.$$

Since $\mathbf{M}$ contains five stable (k_1 to k_5) and two unstable eigenvalues (k_6 and k_7), there is a unique stationary solution to the system of equations (2.62) such that the rational expectations equilibrium is determinate.

2.6 Impulse-response analysis

After having assured determinacy of the rational expectations equilibrium, we want to investigate how the endogenous variables of the model react to simulated transitory shocks at home and abroad. This impulse-response analysis can also be viewed as additional robustness test for the goodness of the present model specification.

For this purpose, let us assume the following autocorrelation coefficients of the domestic and foreign productivity, cost-push, and monetary policy shocks: $\zeta_a = \zeta_a^* = \zeta_u = \zeta_u^* = \zeta_v = \zeta_v^* = 0.8.$[28]

Using the above specification and starting from the non-stochastic zero-inflation steady state, we entail impulses in period 1 on the exogenous error terms η_a, η_a^*, η_u, η_u^*, η_v, η_v^* in terms of one standard deviation of $+\sqrt{0.4}$ on their expected value of 0. The impulse-response analysis is carried out by employing the DYNARE preprocessor for MATLAB while performing a Monte Carlo simulation with 10100 draws, where the first 100 draws are used as burn-in draws before computing the usual summary statistics, which are given below.[29]

The six figures below show the responses of the output gaps x, x^*, PPI inflation rates π_H, π_F^*, movements in the TOT Δt, nominal interest rates $\hat{i}, \hat{i}^*$, and the relevant shock variables themselves to (orthogonalized) impulses on the various exogenous error

---

[28]We propose this relatively high serial correlation of the transitory shock variables mainly for illustrative reasons. Qualitatively, we would obtain the same results if we used smaller autocorrelation coefficients.

[29]The software is downloadable in its current version from http://www.cepremap.cnrs.fr/dynare/. For all computations associated with the impulse-response analysis it uses the pure perturbation algorithm developed by SCHMITT-GROHÉ & URIBE (2004, pp. 764-765) as default option. The DYNARE program code for MATLAB is not reported here, but is available on request.

terms for a time range of 40 periods.[30]
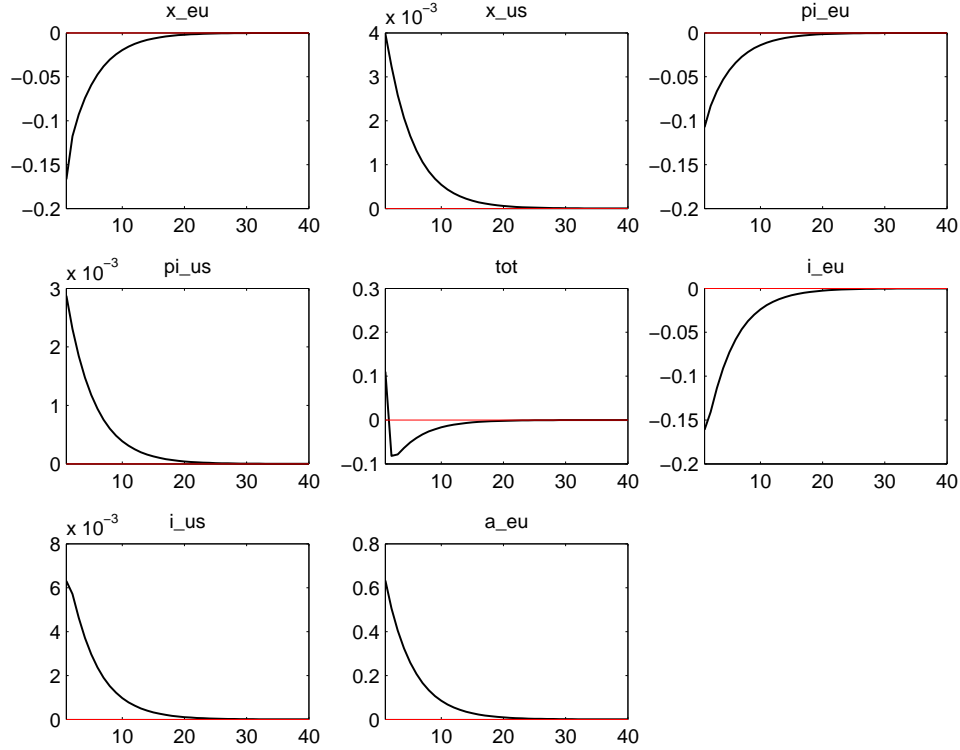
Figure 2.1: EU productivity shock

A detailed interpretation of the results that can be seen from Figures 2.1 to 2.6 can be found in the following.

1. The EU output gap, PPI inflation and nominal interest rates decrease before they return to their zero-inflation steady-state values in response to an impulse on the EU productivity shock (Figure 2.1). The TOT first augment, then drop below their zero-inflation steady-state value until they gradually converge. There is also an impact on all US endogenous variables, which is of opposite sign and quantitatively small.

---

[30]Note that the following is assumed for the variance-covariance matrix of the exogenous error terms:

$$Var(\eta) = \begin{bmatrix} \sigma^2_{\eta_a} & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma^2_{\eta_a^*} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma^2_{\eta_u} & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma^2_{\eta_u^*} & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma^2_{\eta_v} & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma^2_{\eta_v^*} \end{bmatrix}.$$
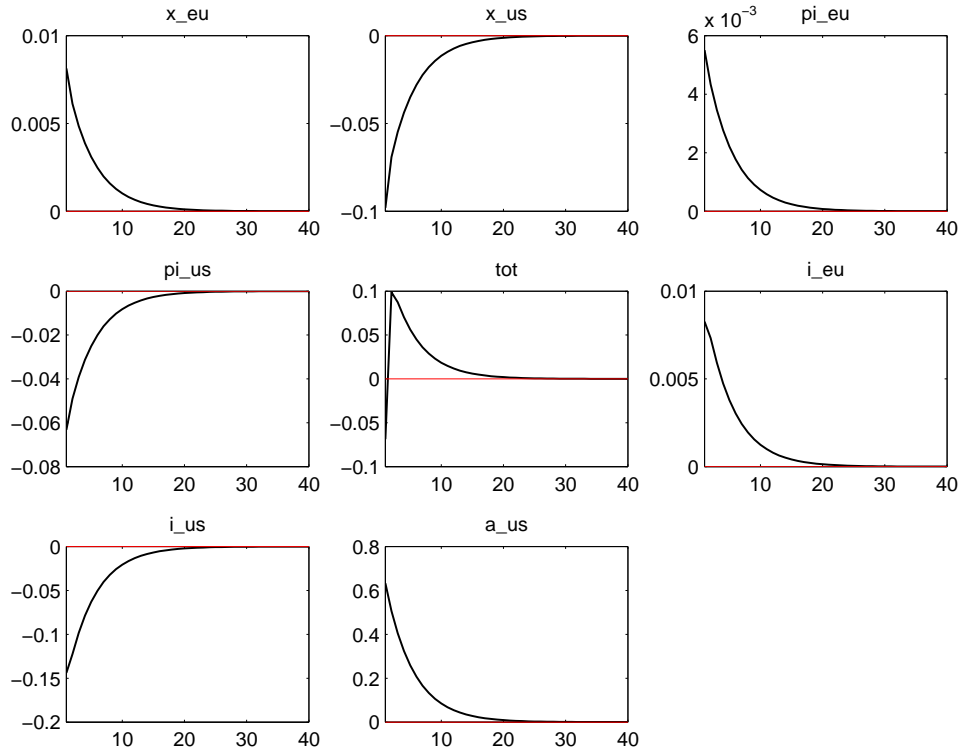
Figure 2.2: US productivity shock

2. The US output gap, PPI inflation and nominal interest rates decrease before they return to their zero-inflation steady-state values in response to an impulse on the US productivity shock (Figure 2.2). The TOT first decrease, then jump above their zero-inflation steady-state value until they gradually converge. There is also an impact on all EU endogenous variables of opposite sign, which is quantitatively larger compared to the impact of the EU productivity shock on foreign variables.

3. The EU output gap decreases, yet the EU PPI inflation and nominal interest rates increase before all endogenous variables return to their zero-inflation steady-state values in response to an impulse on the EU cost-push shock (Figure 2.3). The TOT first plummet, then jump above their zero-inflation steady-state value until they gradually converge. There is also an impact on all US endogenous variables, which is of opposite sign except for the US output gap.

4. The US output gap decreases, yet the US PPI inflation and nominal interest rates
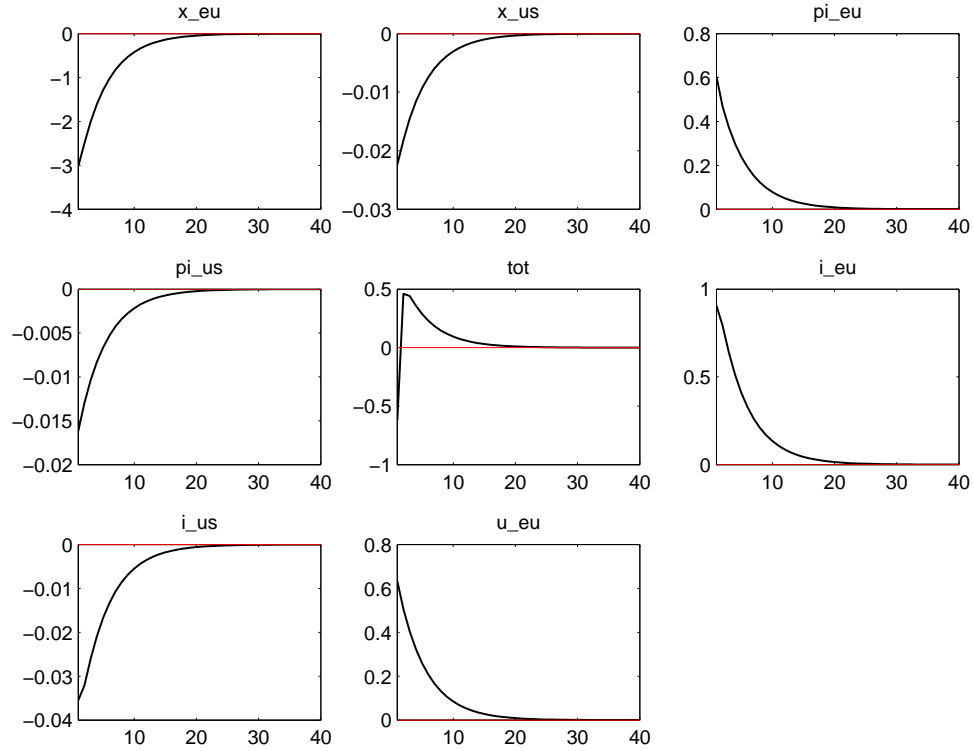
Figure 2.3: EU cost-push shock

increase before all endogenous variables return to their zero-inflation steady-state values in response to an impulse on the US cost-push shock (Figure 2.4). The TOT first augment, then drop below their zero-inflation steady-state value until they gradually converge. There is also an impact on all EU endogenous variables, which is of opposite sign except for the EU output gap.

5. The EU output gap, PPI inflation and nominal interest rates decrease before all endogenous variables return to their zero-inflation steady-state values in response to an impulse on the EU monetary policy shock (Figure 2.5). The TOT augment before they return to their zero-inflation steady-state value without any sudden drops to the negative. There is also an impact on all US endogenous variables, which is of the same sign.

6. The US output gap, PPI inflation and nominal interest rates decrease before all endogenous variables return to their zero-inflation steady-state values in response to an impulse on the US monetary policy shock (Figure 2.6). The TOT plummet
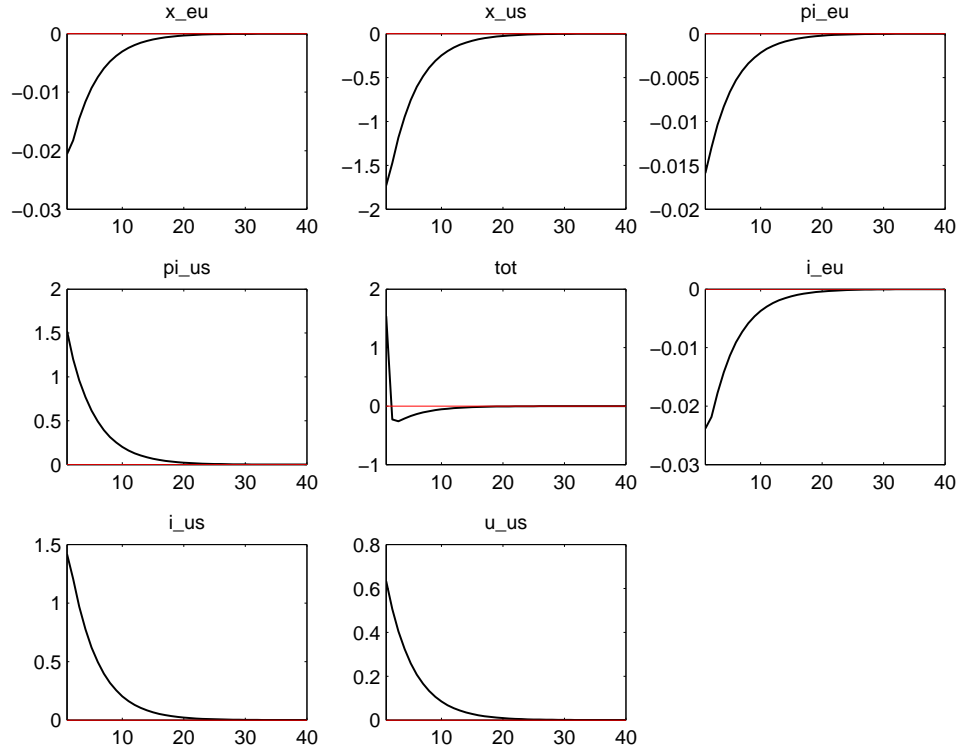
Figure 2.4: US cost-push shock

before they return to their zero-inflation steady-state value without any sudden jumps to the positive. There is also an impact on all EU endogenous variables, which is of the same sign.

The deviation of the EU output gap from the zero-inflation steady state is higher than the one of the US output gap for all three types of disturbances in case they are home-made. However, the picture is ambiguous for PPI inflation and nominal interest rates. None of the shocks discussed above is able to raise output above its flexible-price equilibrium level, neither in the EU nor in the US. Instead, output in both countries drops in response to all entailed impulses before it gradually converges. The negative effect on output is not surprising in context of cost-push shocks, but needs somewhat more explanation concerning the remaining two types of disturbances.

The negative influence of the productivity shocks on the economy contradicts one of the central implications of the standard RBC model, namely a positive correlation of productivity (shocks) and output. Findings for a closed-economy New Keynesian model, which are similar to the present results, are reported, e.g., in GALÍ (2002, pp. 17-18).[31] This finding also seems to be plausible according to empirical studies, which

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[31]The technical reason for this phenomenon is that a positive technology shock hits flexible-price
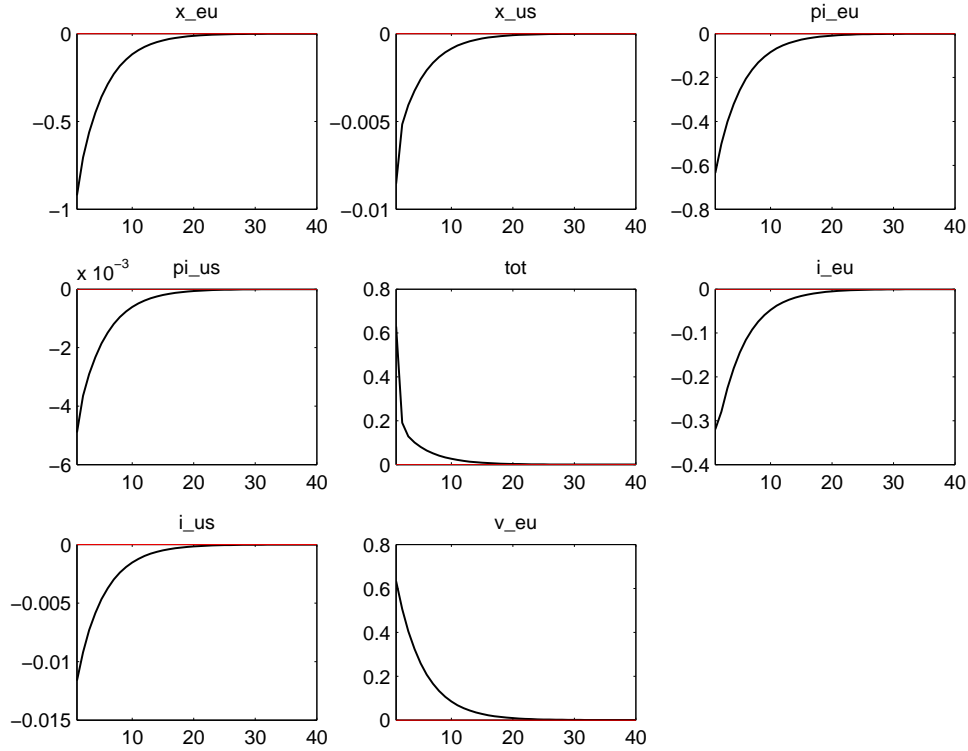
Figure 2.5: EU monetary policy shock

show in addition that technology shocks do not seem to be a significant source for the creation of business cycles at all, which contradicts another central implication of the standard RBC model, namely that technology shocks ought to be the dominant driving force for the creation of business cycles (see GALÍ & RABANAL 2004, pp. 36-39).

Contrary to CORSETTI & PESENTI (2001, pp. 435-439), *negative* realizations of v, v^*, which correspond to expansionary monetary policy shocks, always have a *prosper thyself* and *beggar thy neighbor* effect since they influence the TOT for the benefit of EU (US) resident households by decreasing them below (raising them above) their zero-inflation steady-state value. In addition, this effect induces a rise of both EU and US output above their flexible-price values.[32]

Finally, statistical moments, correlations, and autocorrelations of the simulated endogenous variables are given in Tables 2.1 to 2.3 below.

As one can see from Table 2.1, the explanation why for almost any impulse the

equilibrium output (2.39) harder than actual output (2.22), which needs time to adjust.

[32]Note, however, that monetary policy shocks in CORSETTI & PESENTI (2001) are modeled in terms of permanent and unexpected changes in money supply. Moreover, we interpret the *prosper/beggar* effect somewhat differently since we do not formulate an explicit welfare criterion but stick to the common interpretation of the TOT as laid out in Section 2.3.
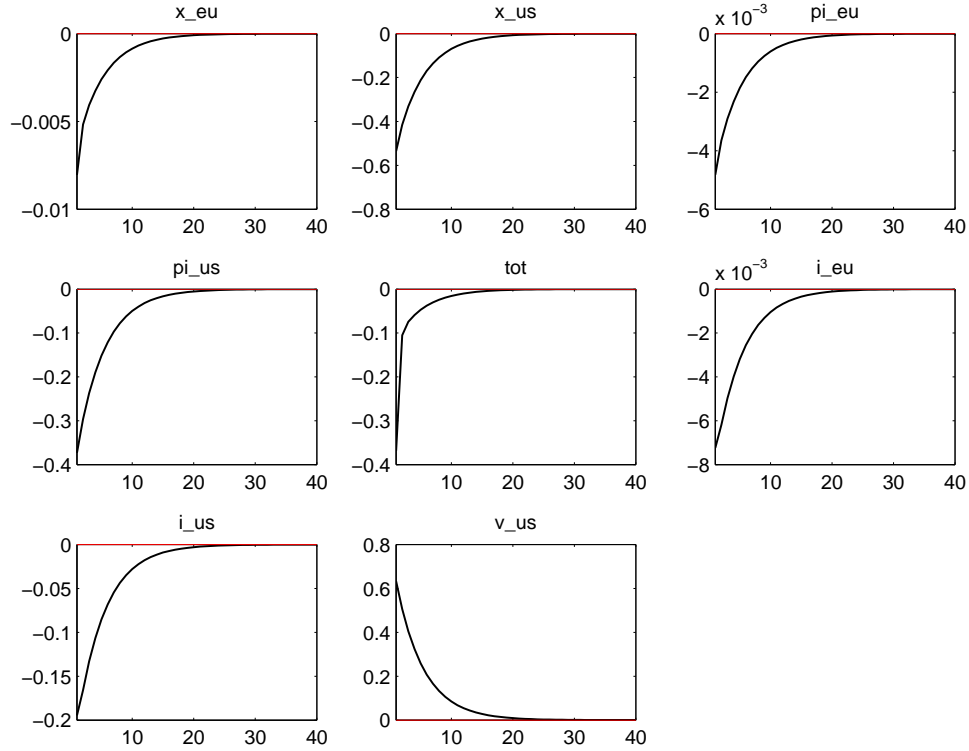
Figure 2.6: US monetary policy shock

Table 2.1: Moments of simulated variables

Variable	Mean	Std. Dev.	Variance	Skewness	Exc. Kur.
x	0.118737	5.457627	29.785693	0.012991	-0.091208
x^*	-0.022633	3.210084	10.304640	0.074824	-0.199552
π_H	0.002329	1.474853	2.175191	-0.019620	0.097960
π_F^*	0.018958	2.635538	6.946059	-0.046841	-0.042459
Δt	-0.025424	2.123355	4.508637	-0.013460	0.046453
$\hat{i}$	-0.016873	1.748576	3.057518	0.000733	0.009221
$\hat{i}^*$	0.024047	2.549748	6.501216	-0.064055	-0.093567

deviation of x is notably higher than of x^* may be found in the differing interest-rate rules for the ECB (2.59) and the Fed (2.60). The positive and fixed sensitivity of the Fed to the current US output gap ceteris paribus absorbs part of the impulses transmitted through the system of equations (2.62). This additional channel works like an attenuator to exogenous disturbances, which does not exist for the EU by assumption. As a consequence, the simulated variance of the domestic output gap $\tilde{\sigma}_x^2$ is almost three

Table 2.2: Correlation of simulated variables

Variable	x	x^*	π_H	π_F^*	Δt	$\hat{i}$	$\hat{i}^*$
x	1.0000	0.0269	-0.4315	0.0360	-0.2352	-0.7875	0.0453
x^*	0.0269	1.0000	0.0001	-0.8655	-0.1809	-0.0011	-0.9041
π_H	-0.4315	0.0001	1.0000	-0.0368	-0.1235	0.8854	-0.0406
π_F^*	0.0360	-0.8655	-0.0368	1.0000	0.2703	-0.0538	0.9924
Δt	-0.2352	-0.1809	-0.1235	0.2703	1.0000	0.0630	0.2258
$\hat{i}$	-0.7875	-0.0011	0.8854	-0.0538	0.0630	1.0000	-0.0620
$\hat{i}^*$	0.0453	-0.9041	-0.0406	0.9924	0.2258	-0.0620	1.0000

Table 2.3: Autocorrelation of simulated variables

Variable	$t-1$	$t-2$	$t-3$	$t-4$	$t-5$
x	0.8131	0.6528	0.5266	0.4261	0.3462
x^*	0.8226	0.6641	0.5340	0.4327	0.3536
π_H	0.7975	0.6489	0.5270	0.4262	0.3480
π_F^*	0.8014	0.6423	0.5125	0.4104	0.3284
Δt	0.1119	0.0553	0.0302	0.0286	0.0236
$\hat{i}$	0.8276	0.6715	0.5432	0.4382	0.3584
$\hat{i}^*$	0.8230	0.6620	0.5296	0.4254	0.3426

times as high as the simulated variance of the foreign output gap $\tilde{\sigma}_{x^*}^2$:

$$\frac{\tilde{\sigma}_x^2}{\tilde{\sigma}_{x^*}^2} = \frac{29.785693}{10.304640} \approx 2.89.$$

On the contrary, the simulated variance of the foreign PPI inflation rate $\tilde{\sigma}_{\pi_F^*}^2$ is more than three times as high as the simulated variance of the domestic PPI inflation rate $\tilde{\sigma}_{\pi_H}^2$:

$$\frac{\tilde{\sigma}_{\pi_F^*}^2}{\tilde{\sigma}_{\pi_H}^2} = \frac{6.946059}{2.175191} \approx 3.19.$$

This means that if the ECB implemented its monetary policy by following the interest-rate rule (2.59), sustaining price stability would be better attainable than if, e.g., it were using an interest-rate rule as proposed for the Fed (2.60) instead. Nonetheless, this advantage can only be reached at the expense of relatively high fluctuations in

the EU output gap, which is a trade-off commonly observed in the Monetary Policy literature.

2.7 Concluding remarks

The main results of the present chapter, which have already been stated in the introductory Section 2.1, shall again be summarized in the following.

Simulated aggregate productivity shocks have a negative impact on EU and US output, a result already described for the closed economy by GALÍ (2002). Simulated cost-push as well as contractionary monetary policy shocks also have a negative impact on EU and US output.

In contrast to CORSETTI & PESENTI (2001), expansionary monetary policy shocks always have a *prosper thyself* and *beggar thy neighbor* effect since they influence the TOT for the benefit of EU (US) resident households by decreasing them below (raising them above) their zero-inflation steady-state value. In addition, this effect would induce a rise of both domestic and foreign output above their flexible-price values.

If the ECB implemented the interest-rate rule proposed in the present article, it would encounter lower fluctuations in EU PPI inflation compared to an interest-rate rule as proposed for the Fed. This is consistent with the ECB's paramount objective of price stability. However, this advantage only holds at the expense of relatively high fluctuations in the EU output gap; a trade-off commonly observed in the Monetary Policy literature.

Besides possible theoretical extensions to the model as summarized, e.g., in GALÍ (2008, chapter 8) and as referred to in Chapter 1, an immediate application of the present New Keynesian framework is an empirical one.

In Chapter 3, we want to do that using quarterly Euro area and US data to estimate the model parameters while employing Bayesian techniques. Moreover, we want to analyze impulse responses based on the estimated model and perform out-of-sample forecasts for the model's endogenous variables in comparison to a classically estimated VAR model and the additive seasonal Holt-Winters method. Finally, we evaluate the forecasting performance of all three forecasting models and of uniformly combined forecasts.

In Chapter 4, we will shift the focus from the two large open economies to the interior of one of them, namely the EU. Here we use quarterly Austrian and Hungarian data to measure the forecast accuracy of new variants of the model in comparison to a number of (Bayesian) (vector) autoregressive benchmarks.

2.8 Appendix to Chapter 2

2.8.1 Price indices and demand curves

The derivation of all price indices and demand curves follows the ideas in OBSTFELD & ROGOFF (1996, pp. 662, 664) for the basic OBSTFELD & ROGOFF (1995) model.

Consumption-based consumer price index and demand curves for composite goods The representative domestic household maximizes

$$C = \frac{C_H^n C_F^{1-n}}{n^n(1-n)^{1-n}}$$

with respect to C_H subject to the budget constraint

$$PC = P_H C_H + P_F C_F.$$

Hence, using the Lagrangian, we get:

$$\Lambda = \frac{C_H^n C_F^{1-n}}{n^n(1-n)^{1-n}} - \lambda(P_H C_H + P_F C_F - PC) \to \max_{C_H}$$

$$\Rightarrow \frac{\partial \Lambda}{\partial C_H} = \frac{n C_H^{n-1} C_F^{1-n}}{n^n(1-n)^{1-n}} - \lambda P_H = 0.$$

Solving this expression for C_H, one obtains the subsequent preliminary demand function for the composite domestic good:

$$C_H = \lambda^{\frac{1}{n-1}} P_H^{\frac{1}{n-1}} \frac{n}{1-n} C_F.$$

Multiplying the preceding equation with P_H, one obtains:

$$P_H C_H = \lambda^{\frac{1}{n-1}} P_H^{\frac{n}{n-1}} \frac{n}{1-n} C_F,$$

with $P_H C_H = nPC$. Now combine the preceding equation with the preliminary demand function from above. Then one gets for C_H equation (2.12):

$$C_H = n\left(\frac{P_H}{P}\right)^{-1} C.$$

Analogously, one gets for C_F equation (2.13):

$$C_F = (1-n)\left(\frac{P_F}{P}\right)^{-1} C.$$

Plugging these two equations into the definition of C, one gets:

$$C = \frac{\left[n\left(\frac{P_H}{P}\right)^{-1} C\right]^n \left[(1-n)\left(\frac{P_F}{P}\right)^{-1} C\right]^{1-n}}{n^n (1-n)^{1-n}} = \left(\frac{P_H}{P}\right)^{-n}\left(\frac{P_F}{P}\right)^{n-1} C.$$

Solving this for P, one finally obtains equation (2.5):

$$P = P_H^n P_F^{1-n}.$$

Consumption-based producer price index and demand curves for individual goods The representative domestic household maximizes

$$C_H = \left[\left(\frac{1}{n}\right)^{\frac{1}{\theta}} \int_0^n C(h)^{\frac{\theta-1}{\theta}} dh\right]^{\frac{\theta}{\theta-1}}$$

with respect to $C(h)$ subject to the budget constraint

$$P_H C_H = \int_0^n P(h)C(h)dh.$$

Hence, using the Lagrangian, we get:

$$\Lambda = \left[\left(\frac{1}{n}\right)^{\frac{1}{\theta}} \int_0^n C(h)^{\frac{\theta-1}{\theta}} dh\right]^{\frac{\theta}{\theta-1}} - \lambda\left[\int_0^n P(h)C(h)dh - P_H C_H\right] \to \max_{C(h)}$$

$$\Rightarrow \frac{\partial \Lambda}{\partial C(h)} = \left[\left(\frac{1}{n}\right)^{\frac{1}{\theta}} \int_0^n C(h)^{\frac{\theta-1}{\theta}} dh\right]^{\frac{1}{\theta-1}} \left(\frac{1}{n}\right)^{\frac{1}{\theta}} C(h)^{-\frac{1}{\theta}} - \lambda P(h) = 0.$$

Solving this expression for $C(h)$, one obtains the subsequent preliminary demand function for individual domestic goods:

$$C(h) = P(h)^{-\theta}\left\{\frac{\lambda}{\left[\left(\frac{1}{n}\right)^{\frac{1}{\theta}} \int_0^n C(h)^{\frac{\theta-1}{\theta}} dh\right]^{\frac{1}{\theta-1}} \left(\frac{1}{n}\right)^{\frac{1}{\theta}}}\right\}^{-\theta}.$$

Multiplying the preceding equation with $P(h)$, one obtains:

$$P(h)C(h) = P(h)^{1-\theta} \left\{ \frac{\lambda}{\left[\left(\frac{1}{n}\right)^{\frac{1}{\theta}} \int_0^n C(h)^{\frac{\theta-1}{\theta}} dh \right]^{\frac{1}{\theta-1}} \left(\frac{1}{n}\right)^{\frac{1}{\theta}}} \right\}^{-\theta}.$$

Taking the integral from 0 to n over both sides of this equation, one gets:

$$\int_0^n P(h)C(h)dh = \int_0^n P(h)^{1-\theta}dh \left\{ \frac{\lambda}{\left[\left(\frac{1}{n}\right)^{\frac{1}{\theta}} \int_0^n C(h)^{\frac{\theta-1}{\theta}} dh \right]^{\frac{1}{\theta-1}} \left(\frac{1}{n}\right)^{\frac{1}{\theta}}} \right\}^{-\theta},$$

with $\int_0^n P(h)C(h)dh = P_H C_H$. Now combine the preceding equation with the preliminary demand function from above. Then one gets for $C(h)$:

$$C(h) = P(h)^{-\theta} \frac{P_H C_H}{\int_0^n P(h)^{1-\theta} dh}.$$

Plugging this into the definition of C_H, one gets:

$$C_H = \left\{ \left(\frac{1}{n}\right)^{\frac{1}{\theta}} \int_0^n \left[P(h)^{-\theta} \frac{P_H C_H}{\int_0^n P(h)^{1-\theta} dh} \right]^{\frac{\theta-1}{\theta}} dh \right\}^{\frac{\theta}{\theta-1}}.$$

Dividing this formula by C_H and raising both sides of the resulting equation to the power of $(\theta-1)/\theta$, we obtain:

$$1^{\frac{\theta-1}{\theta}} = \left(\frac{1}{n}\right)^{\frac{1}{\theta}} \int_0^n \left[P(h)^{-\theta} \frac{P_H}{\int_0^n P(h)^{1-\theta} dh} \right]^{\frac{\theta-1}{\theta}} dh,$$

which can be solved for P_H to finally obtain the domestic PPI given by equation (2.6):

$$P_H = \left[\frac{1}{n} \int_0^n P(h)^{1-\theta} dh \right]^{\frac{1}{1-\theta}}.$$

Plugging this formula into the last given equation in $C(h)$, one eventually gets equation (2.10):

$$C(h) = \frac{1}{n} \left[\frac{P(h)}{P_H} \right]^{-\theta} C_H.$$

2.8.2   First order conditions for a utility maximum

The representative household maximizes

$$U_t = E_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \left[\frac{C_s^{1-\rho}}{1-\rho} + \frac{\chi}{1-\varepsilon} \left(\frac{M_s}{P_s} \right)^{1-\varepsilon} - \frac{\gamma}{1-\xi} L_s^{1-\xi} \right] \right\}$$

with respect to the decision variables C_t, M_t, B_t, L_t subject to the intertemporal budget constraint (in real terms)

$$\frac{W_t}{P_t} L_t + (1 + i_{t-1}) \frac{B_{t-1}}{P_t} + \frac{M_{t-1}}{P_t} + \frac{\Gamma_t(h)}{P_t} = C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} + \tau_t.$$

Hence, using the Lagrangian, we get:

$$\begin{aligned}
\Lambda_t &= E_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \left[\frac{C_s^{1-\rho}}{1-\rho} + \frac{\chi}{1-\varepsilon} \left(\frac{M_s}{P_s} \right)^{1-\varepsilon} - \frac{\gamma}{1-\xi} L_s^{1-\xi} \right] \right. \\
&\quad - \left. \lambda_s \left[\frac{W_s}{P_s} L_s + (1 + i_{s-1}) \frac{B_{s-1}}{P_s} + \frac{M_{s-1}}{P_s} + \frac{\Gamma_s(h)}{P_s} - C_s - \frac{M_s}{P_s} - \frac{B_s}{P_s} - \tau_s \right] \right\} \to \max_{C_t, M_t, B_t, L_t, \lambda_t}
\end{aligned}$$

with $\{\lambda\}_{s=t}^{\infty}$ denoting a sequence of Lagrange multipliers.

$$\begin{aligned}
\Rightarrow \frac{\partial \Lambda_t}{\partial C_t} &= C_t^{-\rho} - \lambda_t(-1) = 0, \\
\frac{\partial \Lambda_t}{\partial M_t} &= \chi \left(\frac{M_t}{P_t} \right)^{-\varepsilon} \frac{1}{P_t} - \lambda_t \left(-\frac{1}{P_t} \right) - \beta E_t \left[\frac{\lambda_{t+1}}{P_{t+1}} \right] = 0, \\
\frac{\partial \Lambda_t}{\partial B_t} &= -\lambda_t \left(-\frac{1}{P_t} \right) - \beta(1 + i_t) E_t \left[\frac{\lambda_{t+1}}{P_{t+1}} \right] = 0, \\
\frac{\partial \Lambda_t}{\partial L_t} &= -\gamma L_t^{-\xi} - \lambda_t \frac{W_t}{P_t} = 0, \\
\frac{\partial \Lambda_t}{\partial \lambda_t} &= -\left[\frac{W_t}{P_t} L_t + (1 + i_{t-1}) \frac{B_{t-1}}{P_t} + \frac{M_{t-1}}{P_t} + \frac{\Gamma_t(h)}{P_t} - C_t - \frac{M_t}{P_t} - \frac{B_t}{P_t} - \tau_t \right] = 0.
\end{aligned}$$

From the first partial derivative one obtains $C_t^{-\rho} = -\lambda_t$ and therefore $C_{t+1}^{-\rho} = -\lambda_{t+1}$. Plugging this into the fourth one, one gets equation (2.21):

$$\gamma \frac{L_t^{-\xi}}{C_t^{-\rho}} = \frac{W_t}{P_t}.$$

Now, by using $C_t^{-\rho} = -\lambda_t$ and $C_{t+1}^{-\rho} = -\lambda_{t+1}$, one obtains from the third partial derivative equation (2.19):

$$\frac{C_t^{-\rho}}{P_t} = \beta(1 + i_t)E_t\left[\frac{C_{t+1}^{-\rho}}{P_{t+1}}\right].$$

Finally, plugging the above expression into the second partial derivative yields equation (2.20):

$$\chi\frac{\left(\frac{M_t}{P_t}\right)^{-\varepsilon}}{C_t^{-\rho}} = \frac{i_t}{1 + i_t}.$$

2.8.3 Equilibrium conditions on world bond and goods markets

The subsequent derivation is based on OBSTFELD & ROGOFF (2001, pp. 7-9), which itself is based on reasoning by CORSETTI & PESENTI (2001, pp. 430-433).

Start with the market clearing condition for a single good z:

$$Y_t(z) = nC_t(z) + (1 - n)C_t^*(z).$$

Assuming, for instance, that good z is a typical domestic good such that $z = h \in [0, n]$ and multiplying the preceding equation with $P_t(h)$ one obtains:

$$P_t(h)Y_t(h) = nP_t(h)C_t(h) + (1 - n)P_t(h)C_t^*(h).$$

Taking the integral from 0 to n and using equations (2.6) and (2.10) yields:

$$\int_0^n P_t(h)Y_t(h)dh = nP_{t,H}C_{t,H} + (1 - n)P_{t,H}C_{t,H}^*.$$

Because of equations (2.12) and (2.14) this expression implies:

$$\int_0^n P_t(h)Y_t(h)dh = n^2 P_t C_t + (1 - n)nP_t C_t^* = nP_t C_t^w,$$

where the right-hand side of the above equation denotes global demand for domestic goods in domestic currency. Since Y denotes domestic per-capita output, the left-hand side of the equation can alternatively be written as $nP_{t,H}Y_t$, which yields the subsequent

equilibrium condition on the world market for domestic goods (2.27):

$$P_{t,H}Y_t = P_t C_t^w.$$

Note that the equilibrium condition on the world market for foreign goods (2.28) can be derived analogously.

Both equations immediately collapse to the definition of the TOT given by equation (2.29):

$$T_t := \frac{P_{t,F}}{P_{t,H}} = \frac{S_t P_{t,F}^*}{P_{t,H}} = \frac{Y_t}{Y_t^*}.$$

Furthermore, substituting equation (2.23) for the domestic household's instantaneous profits into the domestic intertemporal budget constraint (2.17) we get:

$$(1 + i_{t-1})B_{t-1} + M_{t-1} + P_t(h)Y_t(h) = P_t C_t + M_t + B_t + P_t \tau_t.$$

Integrating from 0 to n and using $\int_0^n P_t(h)Y_t(h)dh = nP_{t,H}Y_t$ one obtains:

$$(1 + i_{t-1})B_{t-1} + M_{t-1} + P_{t,H}Y_t = P_t C_t + M_t + B_t + P_t \tau_t.$$

Due to the government's budget constraint (2.37) the preceding equation rearranges to the domestic balance of payments identity (2.30):

$$P_{t,H}Y_t - P_t C_t + i_{t-1}B_{t-1} \equiv B_t - B_{t-1}.$$

Note that the foreign balance of payments identity (2.31) can be derived analogously.

2.8.4 Dynamic IS curves

First rewrite the domestic Euler equation for real consumption (2.19) as follows:

$$C_t^{-\rho} = \beta(1 + i_t)P_t E_t \left[\frac{C_{t+1}^{-\rho}}{P_{t+1}} \right].$$

After having done so, we plug the condition for domestic goods market clearing (2.35) into the preceding equation:

$$(T_t^{n-1}Y_t)^{-\rho} = \beta(1 + i_t)P_t E_t \left[\frac{(T_{t+1}^{n-1}Y_{t+1})^{-\rho}}{P_{t+1}} \right].$$

The non-stochastic zero-inflation steady-state version of this equations reads as follows:

$$(\bar{T}^{n-1}\bar{Y})^{-\rho} = \beta(1+\bar{i})\bar{P}\frac{(\bar{T}^{n-1}\bar{Y})^{-\rho}}{\bar{P}}.$$

The ratio of the last two equations then reads:

$$\left(\frac{T_t^{n-1}Y_t}{\bar{T}^{n-1}\bar{Y}}\right)^{-\rho} = \frac{1+i_t}{1+\bar{i}}P_t\frac{E_t\left[\frac{(T_{t+1}^{n-1}Y_{t+1})^{-\rho}}{P_{t+1}}\right]}{(\bar{T}^{n-1}\bar{Y})^{-\rho}}.$$

By taking the natural logarithm of this ratio, one obtains:

$$\begin{aligned}
-\rho[(n-1)t_t + y_t - (n-1)\bar{t} - \bar{y}] &= \ln(1+i_t) - \ln(1+\bar{i}) + p_t - E_t[p_{t+1}] \\
&\quad - \rho\{(n-1)E_t[t_{t+1}] + E_t[y_{t+1}]\} + \rho[(n-1)\bar{t} + \bar{y}].
\end{aligned}$$

Note that in approximation $\ln(1+i_t) \approx i_t$ and $\ln(1+\bar{i}) \approx \bar{i}$. Moreover, the approximation $\ln E_t[\Psi_{t+1}] \approx E_t[\Psi_{t+1}] - 1 = E_t[\Psi_{t+1} - 1] \approx E_t[\ln\Psi_{t+1}] = E_t\psi_{t+1}$ assures for the exchangeability of the ln and expectations operators for a generic random variable Ψ.

Subsequently, $E_t[\pi_{t+1}] := E_t[p_{t+1}] - p_t$ shall be defined as the expected CPI inflation rate in period $t+1$. In addition, let hatted variables denote the percentage deviations from their zero-inflation steady-state values ($\hat{y}_t := y_t - \bar{y}$, $E_t[\hat{y}_{t+1}] := E_t[y_{t+1}] - \bar{y}$, $\hat{i}_t := i_t - \bar{i}$).

Taking this into account and canceling the term $\rho(n-1)\bar{t}$ on both sides, the last equation rearranges to:

$$-\rho(n-1)t_t - \rho y_t + \rho\bar{y} = \hat{i}_t - E_t[\pi_{t+1}] - \rho(n-1)E_t[t_{t+1}] - \rho E_t[y_{t+1}] + \rho\bar{y}.$$

Solving this for $\hat{y}_t$, one finally obtains the domestic dynamic IS curve (2.40):

$$\hat{y}_t = E_t[\hat{y}_{t+1}] + \frac{1}{\rho}\{E_t[\pi_{t+1}] - \hat{i}_t\} - (1-n)E_t[\Delta t_{t+1}].$$

Note that the foreign dynamic IS curve (2.41) can be derived analogously.

2.8.5 New Keynesian Phillips curves

In period t, a domestic producer willing to reset her price maximizes her expected discounted future profits with respect to $P_t(h)$:

$$E_t\left\{\sum_{s=t}^{\infty}\delta^{s-t}\beta^{s-t}\left(\frac{C_s^w}{C_t^w}\right)^{-\rho}\left[\frac{P_t(h)}{P_{s,H}}Y_s(h) - \kappa_s Y_s(h)\right]\right\} \to \max_{P_t(h)}$$

$\beta^{s-t}(C_s^w/C_t^w)^{-\rho}$ is a stochastic discount factor, which denotes the marginal rate of substitution of real (world) consumption between periods s and t. Note that here one has made use of equation (2.24). In case of goods market clearing output of an individual producer equals global demand for the differentiated good $(Y(h) = C^w(h))$. Note further that the condition $P_t(h) = P_s(h)$ during the length of the contract implies for the global demand function (2.15) for a representative domestic good:

$$C_s^w(h) = \left[\frac{P_t(h)}{P_{s,H}}\right]^{-\theta} \left(\frac{P_{s,H}}{P_s}\right)^{-1} C_s^w.$$

Substituting this into the above equation yields

$$E_t\left\{\sum_{s=t}^{\infty}(\delta\beta)^{s-t}\left(\frac{C_s^w}{C_t^w}\right)^{-\rho}\left[\left(\frac{P_t(h)}{P_{s,H}}\right)^{1-\theta}\left(\frac{P_{s,H}}{P_s}\right)^{-1}C_s^w - \kappa_s\left(\frac{P_t(h)}{P_{s,H}}\right)^{-\theta}\left(\frac{P_{s,H}}{P_s}\right)^{-1}C_s^w\right]\right\} \to \max_{P_t(h)}$$

and gives as the subsequent first order condition:

$$E_t\left\{\sum_{s=t}^{\infty}(\delta\beta)^{s-t}\left(\frac{C_s^w}{C_t^w}\right)^{-\rho}\frac{1}{P_{s,H}}\left[(1-\theta)\left(\frac{P_t(h)}{P_{s,H}}\right)^{-\theta}\left(\frac{P_{s,H}}{P_s}\right)^{-1} + \theta\kappa_s\left(\frac{P_t(h)}{P_{s,H}}\right)^{-\theta-1}\left(\frac{P_{s,H}}{P_s}\right)^{-1}\right]C_s^w\right\} = 0.$$

Solving this for $P_t(h)/P_{t,H}$, one gets the following price-setting equation:

$$\frac{P_t(h)}{P_{t,H}} = \frac{\theta}{\theta-1}\frac{E_t\left\{\sum_{s=t}^{\infty}(\delta\beta)^{s-t}\left[\kappa_s\left(\frac{P_{s,H}}{P_{t,H}}\right)^{\theta}\left(\frac{P_{s,H}}{P_s}\right)^{-1}(C_s^w)^{1-\rho}\right]\right\}}{E_t\left\{\sum_{s=t}^{\infty}(\delta\beta)^{s-t}\left[\left(\frac{P_{s,H}}{P_{t,H}}\right)^{\theta-1}\left(\frac{P_{s,H}}{P_s}\right)^{-1}(C_s^w)^{1-\rho}\right]\right\}}.$$

Now consider the case where everybody resets their prices $(\delta = 0)$. As each producer charges the same price $(P_H = P(h))$, the above equation collapses to the following :

$$\frac{P_t(h)}{P_{t,H}} = \frac{\theta}{\theta-1}\kappa_t = 1.$$

Again we get the real marginal production cost associated with a flexible-price equilibrium κ^{flex}:

$$\kappa_t^{flex} = \frac{\theta-1}{\theta}.$$

Now let us return to the case of sticky prices $(\delta > 0)$. From the domestic PPI (2.6) one gets the subsequent law of motion:

$$P_{t,H}^{1-\theta} = (1-\delta)P_t(h)^{1-\theta} + \delta P_{t-1,H}^{1-\theta}.$$

Log-linearizing the preceding formula around the zero-inflation steady-state price level $\bar{P}_H$ yields the following percentage deviations:

$$\hat{p}_{t,H} = (1-\delta)\hat{p}_t(h) + \delta\hat{p}_{t-1,H}.$$

Now reformulate the price-setting equation as follows:

$$E_t\left\{\sum_{s=t}^{\infty}(\delta\beta)^{s-t}\left[\left(\frac{P_{s,H}}{P_{t,H}}\right)^{\theta-1}\left(\frac{P_{s,H}}{P_s}\right)^{-1}(C_s^w)^{1-\rho}\right]\right\}Q_t$$

$$= \frac{\theta}{\theta-1}E_t\left\{\sum_{s=t}^{\infty}(\delta\beta)^{s-t}\left[\kappa_s\left(\frac{P_{s,H}}{P_{t,H}}\right)^{\theta}\left(\frac{P_{s,H}}{P_s}\right)^{-1}(C_s^w)^{1-\rho}\right]\right\},$$

where $Q_t := P_t(h)/P_{t,H}$.

If one log-linearizes this equation around the zero-inflation steady-state, one finally obtains the subsequent percentage deviations ($\bar{Q}=1, [\theta/(\theta-1)]\kappa_t^{flex}=1$):

$$\ln\left[\frac{(\bar{C}^w)^{1-\rho}}{1-\delta\beta}\right]$$

$$+ \frac{1}{\frac{(\bar{C}^w)^{1-\rho}}{1-\delta\beta}}\left\{\frac{(\bar{C}^w)^{1-\rho}}{1-\delta\beta}\hat{q}_t + \sum_{s=t}^{\infty}(\delta\beta)^{s-t}(\bar{C}^w)^{1-\rho}[(1-\rho)\hat{c}_s^w + (\theta-1)(E_t[\hat{p}_{s,H}]-\hat{p}_{t,H}) + (-1)(E_t[\hat{p}_{s,H}]-E_t[\hat{p}_s])]\right\}$$

$$= \ln\left[\frac{(\bar{C}^w)^{1-\rho}\frac{\theta}{\theta-1}\kappa_t^{flex}}{1-\delta\beta}\right]$$

$$+ \frac{1}{\frac{(\bar{C}^w)^{1-\rho}}{1-\delta\beta}}\left\{\sum_{s=t}^{\infty}(\delta\beta)^{s-t}(\bar{C}^w)^{1-\rho}[(1-\rho)\hat{c}_s^w + E_t[\hat{\kappa}_s] + \theta(E_t[\hat{p}_{s,H}]-\hat{p}_{t,H}) + (-1)(E_t[\hat{p}_{s,H}]-E_t[\hat{p}_s])]\right\},$$

where most of the terms cancel out.

Solving the remainder for $\hat{q}_t + \hat{p}_{t,H}$, one gets:

$$\hat{q}_t + \hat{p}_{t,H} = (1-\delta\beta)\sum_{s=t}^{\infty}(\delta\beta)^{s-t}\{E_t[\hat{p}_{s,H}] + E_t[\hat{\kappa}_s]\}$$

$$= (1-\delta\beta)(\hat{p}_{t,H} + \hat{\kappa}_t) + \delta\beta\{E_t[\hat{q}_{t+1}] + E_t[\hat{p}_{t+1,H}]\}$$

$$\Leftrightarrow \hat{q}_t = (1-\delta\beta)\hat{\kappa}_t + \delta\beta\{E_t[\hat{q}_{t+1}] + E_t[\pi_{t+1,H}]\},$$

where $E_t[\pi_{t+1,H}] := E_t[\hat{p}_{t+1,H}] - \hat{p}_{t,H}$. Due to $\hat{q}_t := \hat{p}_t(h) - \hat{p}_{t,H}$ and $\hat{p}_t(h) = [1/(1-\delta)]\hat{p}_{t,H} - [\delta/(1-\delta)]\hat{p}_{t-1,H}$, it follows that $\hat{q}_t = [\delta/(1-\delta)]\pi_{t,H}$. Plugging this result into the above equation one finally obtains the domestic NKPC (2.42):

$$\pi_{t,H} = \beta E_t[\pi_{t+1,H}] + \frac{(1-\delta)(1-\delta\beta)}{\delta}\hat{\kappa}_t.$$

Note that the foreign NKPC (2.43) can be derived analogously.

Chapter 3

Estimation, structural analysis, and forecasting performance

3.1 Introduction

The present chapter deals with two important applications of DSGE models: estimation and forecasting. These empirical applications are of importance since, e.g., large-scale DSGE models such as the NAWM are regularly used by the ECB and other central banks for macroeconomic projection and policy analysis within the Euro area (see CHRISTOFFEL et al. 2008, p. 4). Despite misspecification, especially those large-scale DSGE models seem to be accurate enough to be used for reliable policy analysis and projection (see DELNEGRO et al. 2005, p. 4).

The small-scale sticky-price two-country DSGE model of the Euro area and the US being our first model under scrutiny is the one developed in Chapter 2 and is characterized by diverging interest-rate rules assigned to the monetary authorities of the two countries, the ECB on the one hand and the Fed on the other. The interest-rate rules differ since they reflect the diverging statutes of these two central banks. The impact of the ECB's and the Fed's differing legal statutes on their monetary policy functions also seems to be empirically plausible (see SAHUC & SMETS 2008, pp. 512-514).

We want to estimate this DSGE model by employing Bayesian inference, which will be discussed in more detail later. *Role-model* closed-economy papers of the Euro area using Bayesian techniques that are frequently cited in the literature are SMETS & WOUTERS (2003) concerning estimation and SMETS & WOUTERS (2004) concerning forecasting. For a general overview of Bayesian inference in DSGE models, the reader is referred to AN & SCHORFHEIDE (2007).

Since VARs of infinite order represent unconstrained versions of DSGE models (see Rubaszek & Skrzypczyński 2008, p. 499), such models are suitable as plausibility tests for the causality structure of DSGE models that is imposed by economic theory. Hence, the second model under scrutiny is of the atheoretical VAR type. Even for VARs of finite order on which we have to rely in practice, Kascha & Mertens (2009) show that those can be good approximations to the dynamic behavior of DSGE models.

Differing from a major part of the literature on the topic (see, e.g., Adolfson et al. 2007, pp. 306-309 or Rubaszek & Skrzypczyński 2008, pp. 505-511), we do not estimate and forecast with a Bayesian VAR (BVAR) for now (see Lütkepohl 2005, pp. 222-229 for a characterization) since we think that assuming the availability of prior information for VAR estimation does not deliver the most unconstrained VAR model possible. Later on in Chapter 4, we will also present an application of several BVAR models.

Besides Smets & Wouters (2007) and Rubaszek & Skrzypczyński (2008) who both investigate the forecasting performance of closed-economy DSGE models of the US, there are many more examples for estimated and/or forecasted DSGE models applying Bayesian techniques such as Adolfson et al. (2007) who investigate the forecasting performance of an open-economy DSGE model of the Euro area. While Smets & Wouters (2004, 2007) and Adolfson et al. (2007) find that their DSGE models are able to forecast well in comparison to (B)VAR benchmarks, Rubaszek & Skrzypczyński (2008) obtain mixed results.

Further analyses are, e.g., Galí & Rabanal (2004) who find that a traditional RBC model does not fit well postwar US data. Moreover, Sahuc & Smets (2008) explore reasons for observed differences in the amplitude of the interest-rate cycles of the ECB and the Fed since 1999 while estimating medium-scale DSGE models of the Euro area and the US separately.

The main results of the analysis based on quarterly OECD data ranging from 1994Q1 until 2009Q1 are summarized in the following.

The estimated DSGE model qualitatively reproduces the findings of the calibrated one from Chapter 2 with respect to most of the parameter values and to impulses responses on the various exogenous error terms. Nonetheless, the degrees of price stickiness in both countries are lower and the degrees of interest-rate inertia across countries are higher than expected prior to estimation.

Estimating an unconstrained VAR(1) does not yield the identical causal relationships as implied by the DSGE. However, Granger causality tests suggest a rather complex causality structure including causalities between real and nominal variables across

countries. In addition, impulse responses based on the VAR(1) sometimes differ from the ones obtained for the DSGE.

Both models and the additive seasonal Holt-Winters method, a simple univariate extrapolation method serving as a benchmark, were not able to predict the severeness or, at least, the evolution of the economic and financial crisis for the forecasting period from 2007Q2 until 2009Q1 since the current crisis is so at odds with regular economic activity.

Finally, we obtain that the accuracy of DSGE forecasts, measured by the root mean squared forecasting error and the mean absolute forecasting error, can compete well with the accuracy of VAR(1), Holt-Winters, and uniformly combined forecasts in times of usual economic performance. In two cases, the DSGE is able to significantly outperform some of the rival forecasting models, but only at the 10% level.

The rest of this chapter is structured as follows. Section 3.2 introduces the two models under scrutiny. Section 3.3 outlines the estimation methodology, followed by the presentation of the OECD data used for launching the estimation in Section 3.4. Section 3.5 states the estimation results for both models. The chapter proceeds with an impulse-response analysis in Section 3.6 and out-of-sample forecasts of the two models and the additive seasonal Holt-Winters method in Section 3.7. In Section 3.8, we continue with measuring the forecast accuracy of all three forecasting models as well as a uniformly combined forecast. Finally, Section 3.9 concludes. Additional tables are given in Appendix 3.10.

3.2 Two models

Subsequently, we introduce the two models that will be under examination throughout the remainder of the present chapter of this dissertation.

3.2.1 A two-country DSGE model with diverging interest-rate rules

The first model under scrutiny is a canonical log-linear representation of the fully micro-founded two-country DSGE framework of the Euro area (EU) and the US developed in Chapter 2. For the reader's convenience, we again state the model's seven structural equations and briefly summarize its properties. Variables indexed by superscript asterisks denote foreign, i.e. US, variables. As we will see in Chapter 4, this model can be applied to other pairs of countries, too. Therefore, we choose to stick to this generic

notation.

$$x_t = E_t[x_{t+1}] + \frac{1}{\rho}\{E_t[\pi_{t+1,H}] - \hat{i}_t\} + \frac{(n-1)(\xi\rho - \xi)}{(\xi + \rho)\rho}E_t[\Delta t_{t+1}] + \frac{\xi + 1}{\xi + \rho}E_t[\Delta a_{t+1}], \quad (3.1)$$

$$x_t^* = E_t[x_{t+1}^*] + \frac{1}{\rho}\{E_t[\pi_{t+1,F}^*] - \hat{i}_t^*\} + \frac{n(\xi\rho - \xi)}{(\xi + \rho)\rho}E_t[\Delta t_{t+1}] + \frac{\xi + 1}{\xi + \rho}E_t[\Delta a_{t+1}^*], \quad (3.2)$$

$$\pi_{t,H} = \beta E_t[\pi_{t+1,H}] + \frac{(1-\delta)(1-\delta\beta)(\rho + \xi)}{\delta}x_t + u_t, \quad (3.3)$$

$$\pi_{t,F}^* = \beta E_t[\pi_{t+1,F}^*] + \frac{(1-\delta^*)(1-\delta^*\beta)(\rho + \xi)}{\delta^*}x_t^* + u_t^*, \quad (3.4)$$

$$\Delta t_t = \hat{i}_{t-1} - \hat{i}_{t-1}^* + \pi_{t,F}^* - \pi_{t,H} + d_t, \quad (3.5)$$

$$\hat{i}_t = \alpha\pi_{t,H} + \omega\hat{i}_{t-1} + v_t, \quad (3.6)$$

$$\hat{i}_t^* = \alpha^*\pi_{t,F}^* + \iota^* x_t^* + \omega^*\hat{i}_{t-1}^* + v_t^*. \quad (3.7)$$

The determined system of expectational difference equations (3.1) to (3.7) consists of Euro area and US dynamic IS curves representing Euro area and US aggregate demand as given by equations (3.1) and (3.2), which can be derived by log-linearizing Euro area and US intertemporal Euler equations for real consumption around the non-stochastic zero-inflation steady state while making use of the goods market clearing conditions for Euro area and US final goods.

In addition, we have Euro area and US NKPCs representing Euro area and US aggregate supply as given by equations (3.3) and (3.4), which can be derived by log-linearizing the price-setting equations of monopolistically competitive Euro area and US final goods firms around the non-stochastic zero-inflation steady-state under the assumption of producer currency pricing. Moreover, we have an equation that relates current movements in the TOT Δt to the other nominal endogenous variables as given by equation (3.5).

The remaining endogenous variables in this two-country model are the Euro area and US output gaps x, x^*, Euro area and US PPI inflation rates π_H, π_F^*, and the Euro area and US short-run nominal interest rates $\hat{i}, \hat{i}^*$, where the latter are determined by the interest-rate rules of the ECB (3.6) and the Fed (3.7), respectively.

In the present case, we assume that the divergence of the central banks' statutes, which has been mentioned in Section 3.1, materializes as follows: besides current US PPI inflation, measured by the sensitivity α^* ($\alpha^* > 0$), the Fed takes into account the current US output gap, measured by the sensitivity ι^* ($\iota^* > 0$), whereas the ECB is supposed to implement its monetary policy by considering current Euro area PPI inflation only, measured by the sensitivity α ($\alpha > 0$). Moreover, ω, ω^* ($0 < \omega, \omega^* < 1$) denote the degrees of interest-rate inertia of the ECB and the Fed, respectively.

From the Euro area and US CES period utility functions we obtain the coefficient

of relative risk aversion in consumption ρ ($0 < \rho < 1$) and the inverse of the elasticity of labor supply ξ ($\xi > 0$), whereby the inverse of ρ denotes the intertemporal elasticity of substitution of real consumption $1/\rho$ in the Euro area and US dynamic IS curves (3.1) and (3.2).[1]

n denotes the country size of the Euro area and $1 - n$ the country size of the US such that world population is normalized to 1. $[(n - 1)(\xi\rho - \xi)]/[(\xi + \rho)\rho] > 0$ denotes the slope coefficient of the Euro area dynamic IS curve (3.1) with respect to expected movements in the TOT. $[n(\xi\rho - \xi)]/[(\xi + \rho)\rho] < 0$, however, denotes the slope coefficient of the US dynamic IS curve (3.2).

In the Euro area and US NKPCs (3.3) and (3.4), we further encounter the intertemporal discount factor β ($0 < \beta < 1$). $[(1 - \delta)(1 - \delta\beta)(\rho + \xi)]/\delta > 0$ represents the slope coefficient of the Euro area NKPC (3.3) with respect to the Euro area output gap. $[(1 - \delta^*)(1 - \delta^*\beta)(\rho + \xi)]/\delta^* > 0$, however, represents the slope coefficient of the US NKPC (3.4). Finally, δ, δ^* ($0 < \delta, \delta^* < 1$) denote the degrees of price stickiness in the Euro area and the US, respectively, and originate from introducing nominal rigidities in terms of CALVO (1983) contracts.

The above framework comprises various transitory macroeconomic disturbances, which read as follows:

$$a_t = \zeta_a a_{t-1} + \eta_{a,t}, \tag{3.8}$$

$$a_t^* = \zeta_a^* a_{t-1}^* + \eta_{a,t}^*, \tag{3.9}$$

$$u_t = \zeta_u u_{t-1} + \eta_{u,t}, \tag{3.10}$$

$$u_t^* = \zeta_u^* u_{t-1}^* + \eta_{u,t}^*, \tag{3.11}$$

$$d_t = \zeta_d d_{t-1} + \eta_{d,t}, \tag{3.12}$$

$$v_t = \zeta_v v_{t-1} + \eta_{v,t}, \tag{3.13}$$

$$v_t^* = \zeta_v^* v_{t-1}^* + \eta_{v,t}^*. \tag{3.14}$$

We assume that $0 < \zeta < 1$ holds for all autocorrelation coefficients ζ such that the stochastic processes (3.8) to (3.14) are all stationary. All exogenous error terms η are assumed to be i.i.d. $\sim N(0, \sigma_\eta^2)$ and uncorrelated, such that $E_t[\eta_t] = 0$, $E_t[\eta_t \eta_t'] = \Sigma_\eta$, and $E_t[\eta_t \eta_s'] = 0$ holds ($s \neq t$), where Σ_η denotes the corresponding variance-covariance matrix.

The single macroeconomic shocks can be interpreted as follows. a, a^* denote country-

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[1]Note that we have to account for ξ having the opposite sign compared to Chapter 2 in equations (3.1) to (3.4) since the prior distribution we wish to assign to this parameter in Section 3.3 is only defined for non-negative real numbers.

specific aggregate productivity shocks. u, u^* denote country-specific cost-push or inflationary shocks. d denotes a global measurement error component comparable to ADOLFSON et al. (2007, p. 297). This assumption differs from Chapter 2 and is mainly introduced at this point since the movements in the TOT will be the variable we will have to construct ourselves in Section 3.4, which is why we introduce a measurement error here. v, v^* denote country-specific monetary policy or interest-rate shocks.

3.2.2 An unconstrained vector autoregressive model

The second model under scrutiny is rather simple. It is an atheoretical VAR model of lag order p, which a priori neither imposes constraints concerning causality between variables nor declares some of the variables exogenous. It reads as follows (see LÜTKEPOHL 2005, p. 13):

$$\mathbf{y}_t = \nu + \mathbf{M}_1\mathbf{y}_{t-1} + ... + \mathbf{M}_i\mathbf{y}_{t-i} + ... + \mathbf{M}_p\mathbf{y}_{t-p} + \varepsilon_t, \quad t = 0, \pm 1, \pm 2, ..., \quad i = 1, ..., p. \quad (3.15)$$

Using the same variables as introduced for the DSGE model above, we have $\mathbf{y}_t := (x_t, x_t^*, \pi_{t,H}, \pi_{t,F}^*, \Delta t_t, i_t, i_t^*)'$, which is a 7×1-dimensional vector of unknowns. Moreover, $\mathbf{M}_i$ are 7×7-dimensional fixed coefficient matrices. $\nu := (\nu_x, \nu_{x^*}, \nu_{\pi_H}, \nu_{\pi_F^*}, \nu_{\Delta t}, \nu_i, \nu_{i^*})'$ is a fixed 7×1-dimensional vector of intercept terms, which allows for a possible non-zero mean $E_t[\mathbf{y}_t]$. Finally, ε_t is a 7×1-dimensional vector of exogenous error terms, for which $E_t[\varepsilon_t] = 0, E_t[\varepsilon_t\varepsilon_t'] = \Sigma_\varepsilon$, and $E_t[\varepsilon_t\varepsilon_s'] = 0$ holds $(s \neq t)$, where Σ_ε denotes the corresponding variance-covariance matrix.

3.3 Estimation methodology

Let us now turn to the basic elements of Bayesian inference, which is commonly used to estimate the structural parameters of DSGE models. Basically, there are three main differences associated with Bayesian estimation in comparison, e.g., to GMM as summarized by AN & SCHORFHEIDE (2007, p. 123). These differences are sometimes seen to be advantageous by adherents of Bayesian techniques.

1. Bayesian estimation is system-based, which means that it fits the solved DSGE model to a vector of aggregate time series. Unlike GMM, it does not depend on specific equilibrium relations such as Euler equations for real consumption.

2. The estimation is based on the likelihood function generated by the DSGE model

rather than on comparing divergences between DSGE and VAR impulse responses.

3. Prior probability density functions (PDF) of the model parameters, which will be introduced below, add information to the estimation procedure.

The question of how to efficiently estimate the unconstrained VAR(p) model (3.15), however, is a straightforward issue. Since there are only lagged values of endogenous variables on the right-hand side of (3.15), we can safely apply standard ordinary least squares (OLS) estimation (see LÜTKEPOHL 2005, p. 71).

3.3.1 Bayesian inference

As laid out in LÜTKEPOHL (2005, pp. 222-223), it is assumed in the Bayesian approach that non-sample information on a generic parameter vector ψ available prior to estimation is summarized in its prior PDF $g(\psi)$. The sample information on ψ, however, is summarized in its sample PDF given by $f(\mathbf{y}|\psi)$, which is algebraically identical to the likelihood function $l(\psi|\mathbf{y})$. Applying Bayes' theorem, we can establish the subsequent relation between these two pieces of information, where $f(\mathbf{y})$ denotes the unconditional sample density:

$$g(\psi|\mathbf{y}) = \frac{f(\mathbf{y}|\psi)g(\psi)}{f(\mathbf{y})}.$$

The preceding equation states that the distribution of ψ conditional on the sample information contained in $\mathbf{y}$ can be summarized by $g(\psi|\mathbf{y})$, which is known as posterior PDF. In other words, the posterior distribution, which contains all information available for the parameter vector ψ, is proportional to the likelihood function times the prior PDF:[2]

$$g(\psi|\mathbf{y}) \propto f(\mathbf{y}|\psi)g(\psi) = l(\psi|\mathbf{y})g(\psi).$$

Since the posterior distribution cannot be determined analytically, we have to adopt a type of Monte-Carlo Markov-Chain (MCMC) sampling algorithm to simulate the distribution of the parameter vector ψ (see CHRISTOFFEL et al. 2008, pp. 34-35). MCMCs are commonly applied to Bayesian inference techniques because they are relatively easy

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[2]By reweighting the likelihood function by a prior, the so-called *dilemma of absurd parameter estimates* (AN & SCHORFHEIDE 2007, pp. 124-125) can be circumvented, which would otherwise result in probably unrealistic posterior means. That is why pure maximum likelihood estimation is not as prominent in DSGE estimation as Bayesian inference.

to implement and less intensive in research time compared to other sampling methods such as importance sampling (see BAUWENS et al. 1999, pp. 83-84). In particular, we adopt the Metropolis-Hastings (MH) algorithm whose steps shall be briefly outlined in the following (see KOOP 2003, pp. 92-94).

1. Choose a starting value ψ^0.

2. Take a candidate draw $\tilde{\psi}$ from the candidate generating density $q = (\psi^{s-1}; \psi)$.

3. Calculate its acceptance probability $Prob(\psi^{s-1}; \tilde{\psi})$.

4. Set $\psi^s = \tilde{\psi}$ with probability $Prob(\psi^{s-1}; \tilde{\psi})$ (candidate draw is accepted) and $\psi^s = \psi^{s-1}$ with probability $1 - Prob(\psi^{s-1}; \tilde{\psi})$ (candidate draw is discarded).

5. Repeat steps 2 to 4 S times.

6. Take the average over the S draws $h(\psi^s)$, $\hat{h}_S = 1/S \sum_{s=1}^{S} h(\psi^s)$, in order to obtain an estimate of $E[h(\psi)|\mathbf{y}]$.

In practice, we have to discard some S_0 initial draws to reduce the impact of our choice for the starting value ψ^0. Moreover, the precise formula for the acceptance probability $Prob(\psi^{s-1}; \tilde{\psi})$ is given by:

$$Prob(\psi^{s-1}; \tilde{\psi}) = \min \left[\frac{g(\psi = \tilde{\psi}|\mathbf{y})q(\tilde{\psi}; \psi = \psi^{s-1})}{g(\psi = \psi^{s-1}|\mathbf{y})q(\psi^{s-1}; \psi = \tilde{\psi})}; 1 \right],$$

where $g(\psi = \tilde{\psi}|\mathbf{y})$ denotes the posterior distribution conditional on the sample information contained in $\mathbf{y}$ evaluated at the point $\psi = \tilde{\psi}$, whereas $q(\tilde{\psi}; \psi = \psi^{s-1})$ denotes the density of the random variable ψ evaluated at the point $\psi = \psi^{s-1}$.

3.3.2 Calibration and prior distributions

In the present framework, the parameters introduced in Section 3.2 constitute the parameter vector $\psi := (n, \beta, \rho, \xi, \delta, \delta^*, \alpha, \alpha^*, \iota^*, \omega, \omega^*, \zeta_a, \zeta_a^*, \zeta_u, \zeta_u^*, \zeta_d, \zeta_v, \zeta_v^*, \sigma_{\eta_a}, \sigma_{\eta_a^*}, \sigma_{\eta_u}, \sigma_{\eta_u^*}, \sigma_{\eta_d}, \sigma_{\eta_v}, \sigma_{\eta_v^*})'$, which is subject to estimation based on quarterly data. The means of

the proposed prior distributions are taken from the calibration section of Chapter 2, whereby the parameters n and β are assumed to be fixed throughout the analysis. We set $n = 1 - n = 0.5$ since the Euro area and the US can be treated as approximately equal-sized countries. Differing from Chapter 2, however, we set $\beta = 0.99$, which implies quarterly zero-inflation steady-state nominal and real interest rates across countries of $\bar{i} = \bar{i}^* = \bar{r} = \bar{r}^* = (1 - \beta)/\beta \approx 0.01$. This corresponds to annualized zero-inflation steady-state interest rates of approximately 4%.

What makes the present calibration still differ from well-known articles of the empirical DSGE literature such as SMETS & WOUTERS (2003, pp. 1142-1143) or RUBASZEK & SKRZYPCZYŃSKI (2008, p. 505) is that with $\rho = 0.8, \xi = 1, \alpha = \alpha^* = 1.5$, and $\omega = \omega^* = 0.1$ we assume that the prior means of some of the parameters to be estimated are a bit lower than usually done. Concerning the chosen value for the coefficient of relative risk aversion in consumption ρ, the domain of this parameter $0 < \rho < 1$ as implied by the theoretical model from Chapter 2 prevents us from setting it equal to $\rho^{SW} = 1$. In addition, $\rho^{SW} = 1$ would result in canceling out the TOT effects in the dynamic IS curves (3.1) and (3.2).

Not setting the inverse of the elasticity of labor supply equal to $\xi^{SW} = 2$ implies a somewhat high elasticity of labor supply of 1 prior to estimation. However, a value of $\xi = 1$ is seen as a good compromise between the micro Labor and macro Business Cycle literature (see GALÍ et al. 2007, p. 47). If the posterior means of these and the other parameters were to differ substantially from their prior means, this will be indicated by the estimation results anyway.

Table 3.1 given below summarizes the prior information on the various parameters contained in the vector ψ, where the choice of prior PDFs follows [1] common suggestions in the literature as laid out in AN & SCHORFHEIDE (2007, pp. 127-130), and [2] the domain of the various parameters as implied by theory. The different prior PDFs are documented, e.g., in BAUWENS et al. (1999, pp. 290-294). The inverse gamma prior PDF is extensively used in the literature for residual variances according to BAUWENS et al. (1999, p. 292) and reflects the fact that there is only little prior information available for these parameters (see CHRISTOFFEL et al. 2008, p. 40).

Employing the DYNARE preprocessor for MATLAB, we will use the inverse gamma-1 rather than the inverse gamma prior and choose to estimate standard deviations rather than variances, where an infinite standard deviation around the mean of the standard deviation to be estimated assures a closed-form solution. This is necessary in order to be able to derive the scale and shape parameters of the inverse gamma-1 prior PDF (see ADJEMIAN 2007, p. 7). Note that similar to Chapter 2, there is a unique stationary

solution to the system of equations (3.1) to (3.7) for the selected parameter values since there are five eigenvalues inside and two eigenvalues on or outside the complex unit circle of the corresponding coefficient matrix, which is the sufficient condition for equilibrium determinacy in case of two predetermined variables $(\hat{i}_{t-1}, \hat{i}^*_{t-1})$ and five non-predetermined ones $(x_t, x^*_t, \pi_{t,H}, \pi^*_{t,F}, \Delta t_t)$.

Table 3.1: Prior information

Parameter	Domain	Prior PDF	Mean	Std. Dev.
ρ	$[0,1)$	Beta	0.8	0.025
ξ	$[0,+\infty)$	Gamma	1	0.25
δ	$[0,1)$	Beta	0.75	0.1
δ^*	$[0,1)$	Beta	0.75	0.1
α	$(-\infty,+\infty)$	Normal	1.5	0.1
α^*	$(-\infty,+\infty)$	Normal	1.5	0.1
ι^*	$(-\infty,+\infty)$	Normal	0.5	0.25
ω	$[0,1)$	Beta	0.1	0.05
ω^*	$[0,1)$	Beta	0.1	0.05
ζ_a	$[0,1)$	Beta	0.8	0.05
ζ^*_a	$[0,1)$	Beta	0.8	0.05
ζ_u	$[0,1)$	Beta	0.8	0.05
ζ^*_u	$[0,1)$	Beta	0.8	0.05
ζ_d	$[0,1)$	Beta	0.8	0.05
ζ_v	$[0,1)$	Beta	0.8	0.05
ζ^*_v	$[0,1)$	Beta	0.8	0.05
σ_{η_a}	$[0,+\infty)$	Inv. Gamma-1	0.02	$+\infty$
$\sigma_{\eta_a^*}$	$[0,+\infty)$	Inv. Gamma-1	0.02	$+\infty$
σ_{η_u}	$[0,+\infty)$	Inv. Gamma-1	0.02	$+\infty$
$\sigma_{\eta_u^*}$	$[0,+\infty)$	Inv. Gamma-1	0.02	$+\infty$
σ_{η_d}	$[0,+\infty)$	Inv. Gamma-1	0.02	$+\infty$
σ_{η_v}	$[0,+\infty)$	Inv. Gamma-1	0.02	$+\infty$
$\sigma_{\eta_v^*}$	$[0,+\infty)$	Inv. Gamma-1	0.02	$+\infty$

3.4 Quarterly data for the Euro area and the US

We want to estimate quarterly models of the Euro area and US economies using OECD data for the time from 1994Q1 until 2009Q1, which means that we have 61 observations altogether. Note that we have to limit our analysis to the time after 1994Q1 because there are no data available on Euro area short-run nominal interest rate prior to this

date.[3] The OECD data are either taken from OECD (2008) (output gaps) or from OECD (2010) (all other variables).

- The Euro area and US output gaps x, x^* are calculated from the natural logarithm of the actual volume at constant prices of the respective seasonally and working-day adjusted gross domestic product (GDP) minus the natural logarithm of the volume at constant prices of the respective potential output of the total economy.

- The Euro area and US PPI inflation rates π_H, π_F^* are proxied by the percentage change to the previous period of the domestic producer price indices in manufacturing. We restrict ourselves to these prices since we assume that firms employ producer currency pricing and that only final goods are produced and traded.

- We have to construct the TOT Δt ourselves, which is done by employing the following formula (see Chapter 2): $\Delta t_t = \Delta s_t + \pi_{t,F}^* - \pi_{t,H}$, where Δs_t denotes the first difference of the natural logarithm of the monthly average of the nominal exchange rate of Euro (from 1994Q1 until 1998Q4 of European Currency Units, ECU) per US Dollar.

- The nominal interest rates in per cent per annum of the ECB and the Fed i, i^* are proxied by the subsequent time series: the three-month Euro Interbank Offered Rate (EURIBOR) in case of i and the three-month nationally traded certificates of deposit issued by commercial banks in case of i^*.[4] Both interest rates have to be subtracted by their common annualized zero-inflation steady-state value ≈ 0.04 before entering the DSGE estimation in order to obtain the desired percentage-point deviations $\hat{i}, \hat{i}^*$. In the case of the VAR(p), this constraint on the nominal interest rates is not imposed by the model, but will also be accounted for to assure

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[3]Note that we choose to employ interbank rates as a reasonable compromise between central bank and retail interest rates since, e.g., households usually decide for their asset holdings according to the interest rates offered by commercial banks.

[4]We are aware of the fact that the ECB has been operating no sooner than January 1999. This means that the time series used for the Euro area short-run nominal interest rate during the period from 1994Q1 until 1998Q4 is a synthetic rate calculated by using national rates, London Interbank Offered Rate (LIBOR) where available, weighted by GDP (see OECD 2010). A rejection of this approximation would shorten the sample. It would be possible to even extend the sample prior to 1994Q1 by employing the former German short-run nominal interest rate FIBOR (Frankfurt Interbank Offered Rate) as *predecessor* of the EURIBOR since also the Deutsche Bundesbank could be seen as *predecessor* of the ECB. Since there are many contrary opinions on this issue, we will stick to our original suggestion.

comparability of the two models. Moreover, the actual means of the nominal interest rates in both countries roughly correspond to 0.04 during the estimation period, anyway ($\bar{i} = 0.0395$ for the Euro area and $\bar{i}^* = 0.0430$ for the US).

The subsequent graphs plot the various variables in levels for the period under scrutiny, whereby EU is used to indicate Euro area variables in all graphs and tables below.
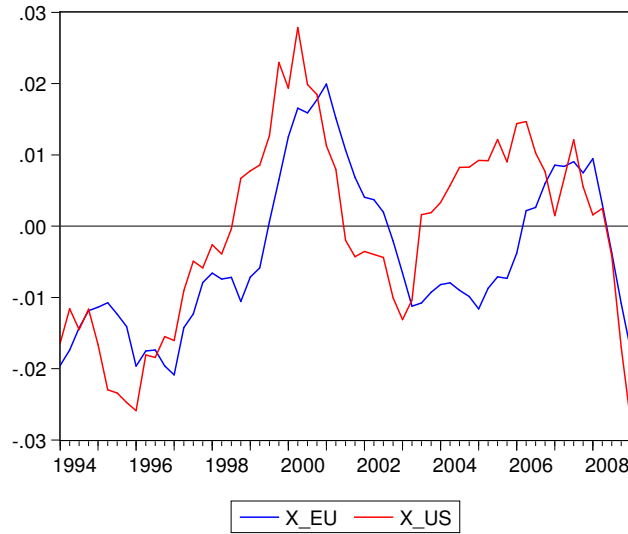
Figure 3.1: Euro area and US output gaps

As we can conclude from Figures 3.1 and 3.2, the output gaps and PPI inflation rates are negative in both the Euro area and the US from 2008Q3 (output gaps) or 2008Q4 (PPI inflation rates), respectively, onwards. This reflects the global economic downturn associated with the present economic and financial crisis. The comovement of the graphs of the output gaps of both economies indicates a positive correlation of Euro area and US business cycles. The TOT as given in Figure 3.3 have been beneficial for US resident households from 2008Q3 onwards. The values of the Euro area and US nominal interest rates as given in Figure 3.4 have already been below their common zero-inflation steady-state value since 2008Q1 (US) or 2009Q1 (Euro area), which may reflect countercyclical monetary policy by both central banks designed to mitigate the impact of the crisis. This countercyclical policy seems to have already positively affected the interbank markets across countries, too.

In our opinion, the current crisis is so much at odds with regular economic behavior that including the observations during the period from 2007Q2 until 2009Q1 would harm the goodness of the estimation and of the structural analysis thereafter. Hence,

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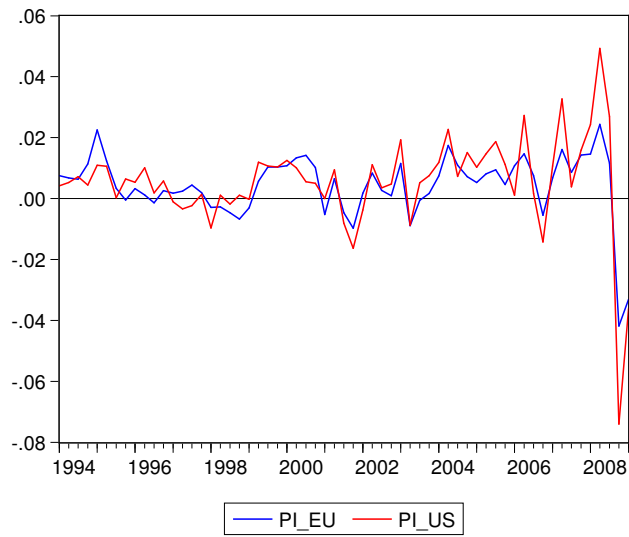
Figure 3.2: Euro area and US PPI inflation rates
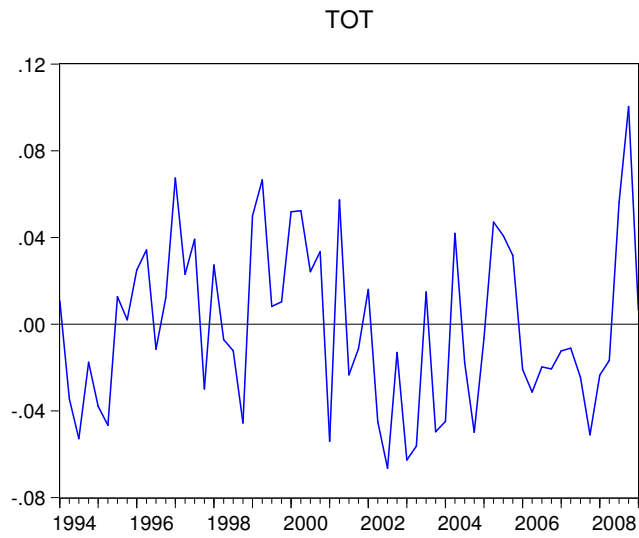
Figure 3.3: Terms of trade

we restrict our estimation sample to the period from 1994Q1 until 2007Q1, which leaves us 53 observations.
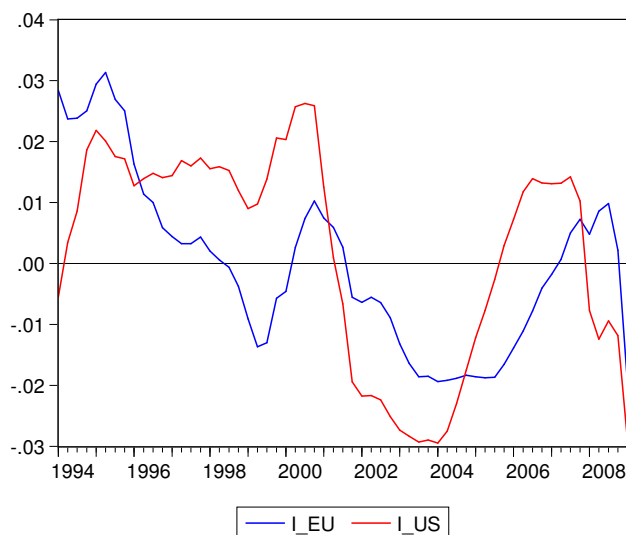
Figure 3.4: Euro area and US nominal interest rates

The year 2007 is typically associated with the initial point of the crisis (see, e.g., the timeline by the Federal Reserve Bank of St. Louis).[5] Chow break-point tests in the work of ČIHÁK et al. (2009, p. 13) further suggest August 2007 as split point between pre-crisis and crisis for the ECB's short-run and several market interest rates of the Euro area. If we observe the data underlying Figures 3.1 to 3.4, we can conclude that at least in 2007Q1 no unusual economic activity is recognizable regarding our variables of interest. Therefore, we choose to keep the observations of 2007Q1 for our estimation sample. Nonetheless, having the actual observations for the period of crisis will allow us to compare them with out-of-sample forecasts of the two rival models and a benchmark later on in Section 3.7.

3.5 Estimation results

In the following, we want to summarize the estimation results based on the above OECD data both for the DSGE (3.1) to (3.14) and the VAR(p) (3.15).

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[5]See http://timeline.stlouisfed.org/index.cfm?p=timeline.

3.5.1 DSGE

Employing the DYNARE preprocessor for MATLAB we obtain the subsequent results for estimating the DSGE model (3.1) to (3.14) while using Bayesian techniques for 53 observations ranging from 1994Q1 to 2007Q1. The options specified in the `estimation` command associated with the MH algorithm whose selected values differ from the default values, shall be briefly discussed in the following (see MANCINI-GRIFFOLI 2007, pp. 52-53).[6]

- `mh_replic=50000` denotes the number of replications within each MH simulation S, whereby $S_0 = 25000$ initial replications are discarded as burn-in draws.

- `mh_nblocks=5` states the number of runs of MH simulations or the number of parallel MCMCs, respectively.

- `mh_jscale=0.25` denotes the scale to be used for the candidate generating density $q = (\psi^{s-1}; \psi)$ in the MH algorithm.

- `mh_init_scale=0` gives us the scale to be used for drawing the initial value ψ^0 of all the MCMCs.

The estimation results in terms of the posterior means and a 90% confidence interval around these means are summarized in Table 3.2 below.

We will briefly discuss the most important results (see Table 3.2) in the following. The posterior means of the parameters stemming from the utility function (ρ, ξ) and of the sensitivities of both central banks towards current PPI inflation (α, α^*) are a bit higher than their prior means. The posterior mean of ι^*, however, is a bit lower. There seem to be higher degrees of interest-rate inertia in both the Euro area and the US (ω, ω^*) than believed prior to the estimation. I.e. both central banks put more weight on past realizations of the nominal interest rates (i_{t-1}, i^*_{t-1}) than on their common zero-inflation steady-state value $(\bar{i} = \bar{i}^*)$.

Interestingly, the posterior means of δ, δ^* suggest that the degrees of price stickiness in both countries are much lower than expected, maybe because producer prices are able to react more flexible to business cycle fluctuations than consumer prices. The posterior means of the Euro area and US degrees of price stickiness both imply an average

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[6]The DYNARE program code for MATLAB is not reported here, but is available on request.

Table 3.2: DSGE estimation results

Parameter	Prior Mean	Post. Mean	90% Conf. Int.		Prior PDF	Prior Std. Dev.
ρ	0.8000	0.8436	0.8123	0.8776	Beta	0.0250
ξ	1.0000	1.0313	0.5931	1.4597	Gamma	0.2500
δ	0.7500	0.3706	0.2956	0.4439	Beta	0.1000
δ^*	0.7500	0.4007	0.3161	0.4863	Beta	0.1000
α	1.5000	1.5890	1.4340	1.7422	Normal	0.1000
α^*	1.5000	1.5710	1.4128	1.7224	Normal	0.1000
ι^*	0.5000	0.4161	0.0270	0.7993	Normal	0.2500
ω	0.1000	0.4381	0.3155	0.5650	Beta	0.0500
ω^*	0.1000	0.4304	0.3028	0.5626	Beta	0.0500
ζ_a	0.8000	0.8392	0.7864	0.8945	Beta	0.0500
ζ_a^*	0.8000	0.8179	0.7613	0.8793	Beta	0.0500
ζ_u	0.8000	0.8929	0.8533	0.9355	Beta	0.0500
ζ_u^*	0.8000	0.8584	0.8044	0.9141	Beta	0.0500
ζ_d	0.8000	0.7015	0.6097	0.7916	Beta	0.0500
ζ_v	0.8000	0.6434	0.5544	0.7314	Beta	0.0500
ζ_v^*	0.8000	0.6312	0.5289	0.7349	Beta	0.0500
σ_{η_a}	0.0200	0.0376	0.0262	0.0490	Inv. Gamma-1	$+\infty$
$\sigma_{\eta_a^*}$	0.0200	0.0449	0.0324	0.0574	Inv. Gamma-1	$+\infty$
σ_{η_u}	0.0200	0.0076	0.0049	0.0102	Inv. Gamma-1	$+\infty$
$\sigma_{\eta_u^*}$	0.0200	0.0088	0.0052	0.0126	Inv. Gamma-1	$+\infty$
σ_{η_d}	0.0200	0.0412	0.0343	0.0474	Inv. Gamma-1	$+\infty$
σ_{η_v}	0.0200	0.0104	0.0085	0.0123	Inv. Gamma-1	$+\infty$
$\sigma_{\eta_v^*}$	0.0200	0.0152	0.0124	0.0180	Inv. Gamma-1	$+\infty$

duration of a price of approximately two quarters: $1/(1 - \delta) \approx 1.5888$, $1/(1 - \delta^*) \approx 1.6686$.

As pointed out by PICHLER (2008, pp. 10-11), we have to interpret the parameter estimates with a grain of salt. This is due to the fact that the likelihood function of some of the parameters may be almost flat or it may feature several local maxima, two problems typically associated with the estimation of DSGE models using Bayesian inference. One solution to these issues could be to calibrate rather than estimate more of the parameters, especially those that are said to be weakly identified. In Chapter 4, we will pick up this idea and keep the parameters stemming from the utility function, ρ and ξ, fixed during estimation.

Figures 3.5 to 3.7 below display the prior distributions (solid gray line), the posterior distributions (solid black line), as well as the posterior modes that are calculated from the numerical optimization of the posterior kernel (dashed green line) for the various

parameters. According to MANCINI-GRIFFOLI (2007, p. 59), the fact that the mode expressed by the dashed green line and the mode of the posterior distribution are almost equal gives us confidence in the goodness of the estimation. The fact that the posterior distributions of some of the parameters differ from their prior distributions signifies that in these cases the observed data are particularly able to provide additional information (see CHRISTOFFEL et al. 2008, p. 42).
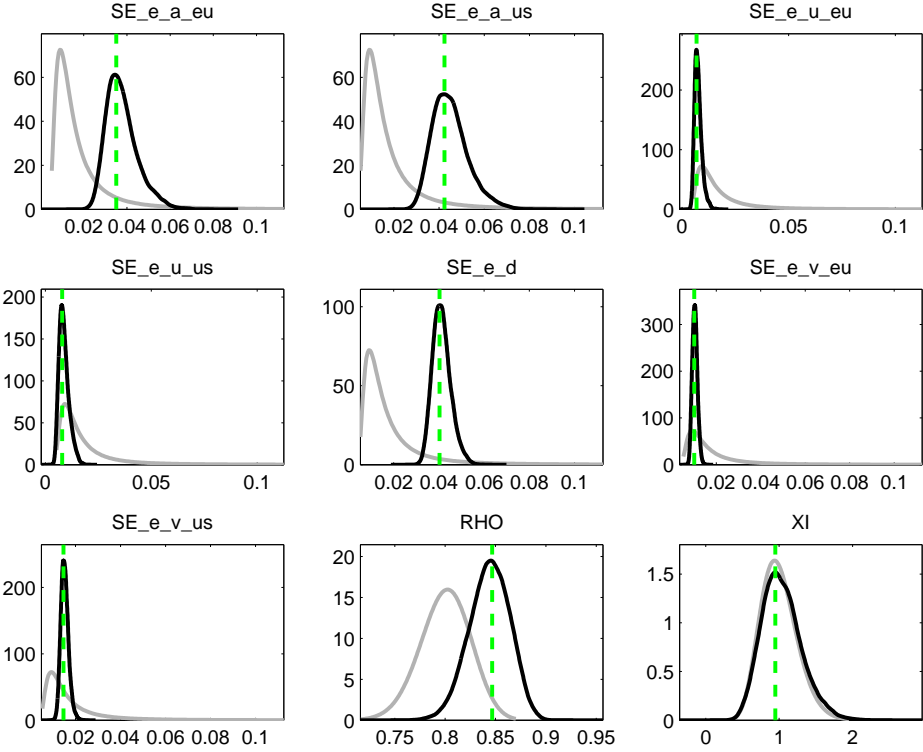
Figure 3.5: Priors and posteriors I

Finally, Figure 3.8 below gives us additional confidence that the aggregated estimation results based on the eigenvalues of the variance-covariance matrix of each parameter over the five parallel MCMCs are sensible, since both the red and the blue line, which are based on differing initial values, remain (almost) constant over all draws for the three displayed *specific measures* and converge in the end. `interval` reports an 80% confidence interval around the parameter mean, `m2` a *specific measure* of variance, and `m3` a *specific measure* of skewness (see MANCINI-GRIFFOLI 2007, p. 58). For each of the runs of MH simulations we obtain an acceptance probability $Prob(\psi^{s-1}; \tilde{\psi})$ of about 50%.
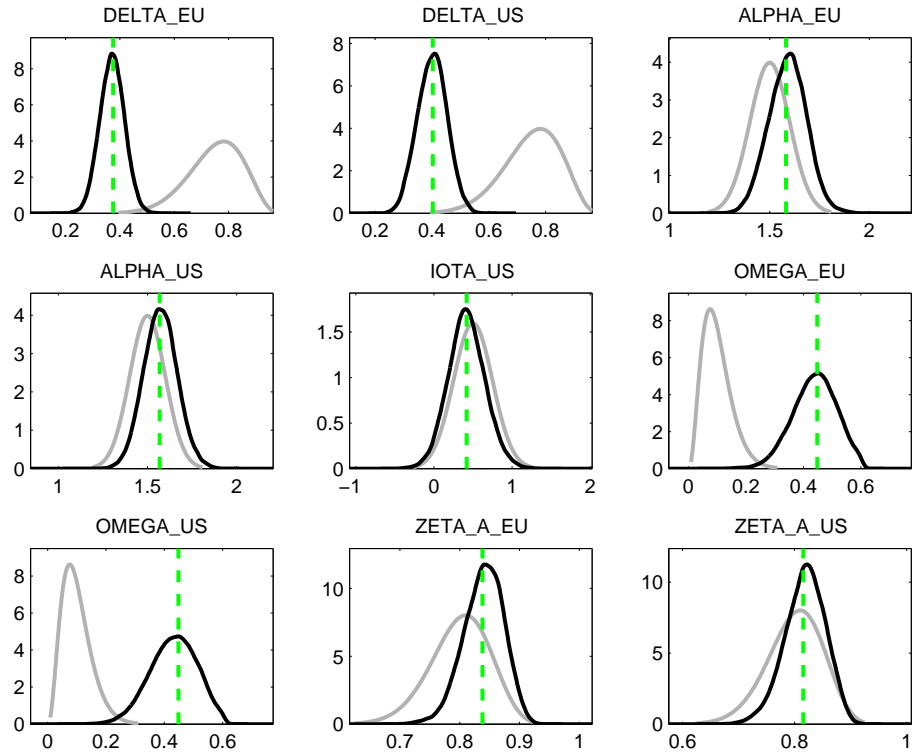
Figure 3.6: Priors and posteriors II
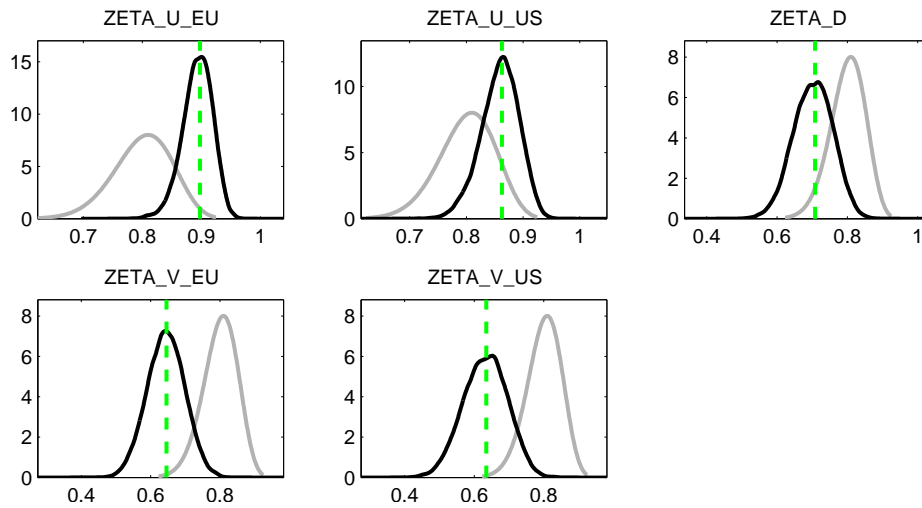
Figure 3.7: Priors and posteriors III

3.5.2 VAR

Before we estimate the VAR(p) model (3.15), we first have to determine the appropriate lag order. The maximum lag order supported by EVIEWS for VAR estimation for
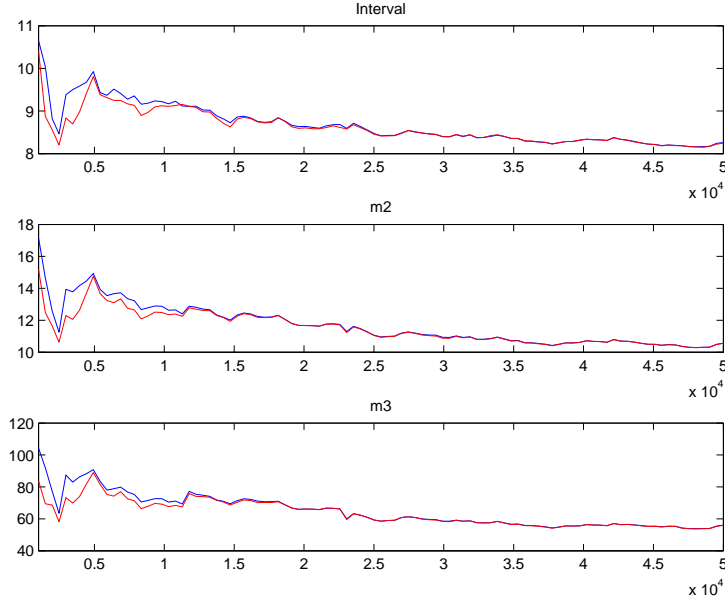
Figure 3.8: MCMC multivariate diagnostics

the given number of variables is $p = 6$. The Schwarz (SC) and Hannan-Quinn (HQ) information criteria for this preliminary estimation, however, suggest that the optimal lag order is $p = 1$ as can be seen from Table 3.7 in Appendix 3.10. Unlike, e.g., Akaike's information criterion (AIC), SC and HQ are strongly consistent information criteria (see LÜTKEPOHL 2005, pp. 150-151). Since we lose as little information as possible by using only one lag, we follow their recommendation and restrict our attention to the analysis of the subsequent simple VAR(1) model, which enables us to use 52 observations ranging from 1994Q2 until 2007Q1, one observation less than in case of the DSGE (3.1) to (3.14), for launching the estimation:

$$\mathbf{y}_t = \nu + \mathbf{M}_1 \mathbf{y}_{t-1} + \varepsilon_t. \tag{3.16}$$

The OLS estimation results of the VAR(1) model (3.16) including standard errors, t-statistics, and the usual summary statistics are given in Table 3.3 below. Significant t-statistics are indicated by boldface numbers.

As we can conclude from the t-statistics in Table 3.3, VAR(1) estimation suggests that there is a statistically significant, positive influence of past values of Euro area and US output gaps on their respective current values, which is not too surprising. There is also a statistically significant, negative impact of the past Euro area output gap on the current US nominal interest rate. We further observe a statistically significant, positive

Table 3.3: VAR(1) estimation results

	X_EU	X_US	PI_EU	PI_US	TOT	I_EU	I_US
X_EU(-1)	0.766383	-0.136643	-0.298464	-0.446885	-0.440509	-0.039116	-0.423885
	(0.04827)	(0.08861)	(0.13424)	(0.17722)	(0.74646)	(0.05341)	(0.07161)
	[15.8774]	[-1.54216]	[-2.22342]	[-2.52160]	[-0.59013]	[-0.73241]	[-5.91930]
X_US(-1)	0.173376	0.871283	0.384487	0.473280	0.481585	0.064219	0.238397
	(0.05944)	(0.10910)	(0.16529)	(0.21822)	(0.91914)	(0.06576)	(0.08818)
	[2.91706]	[7.98591]	[2.32615]	[2.16882]	[0.52395]	[0.97653]	[2.70363]
PI_EU(-1)	0.208025	-0.071884	0.493498	0.274085	-2.911130	0.369345	0.267275
	(0.07693)	(0.14122)	(0.21395)	(0.28246)	(1.18971)	(0.08512)	(0.11413)
	[2.70404]	[-0.50902]	[2.30664]	[0.97036]	[-2.44693]	[4.33906]	[2.34178]
PI_US(-1)	-0.191877	-0.000978	-0.164483	-0.127614	1.457977	-0.165792	-0.081844
	(0.05995)	(0.11006)	(0.16673)	(0.22013)	(0.92717)	(0.06634)	(0.08895)
	[-3.20039]	[-0.00888]	[-0.98650]	[-0.57973]	[1.57251]	[-2.49925]	[-0.92015]
TOT(-1)	0.006889	0.006172	0.004321	4.55E-05	0.084019	-0.004914	0.011651
	(0.00908)	(0.01666)	(0.02524)	(0.03332)	(0.14036)	(0.01004)	(0.01346)
	[0.75899]	[0.37043]	[0.17120]	[0.00136]	[0.59862]	[-0.48935]	[0.86531]
I_EU(-1)	-0.031524	-0.200199	0.183641	0.154181	-0.366408	0.896498	0.038799
	(0.04936)	(0.09061)	(0.13727)	(0.18122)	(0.76331)	(0.05461)	(0.07323)
	[-0.63868]	[-2.20957]	[1.33784]	[0.85078]	[-0.48003]	[16.4154]	[0.52984]
I_US(-1)	0.052954	0.071359	-0.123698	-0.186613	1.027763	0.057739	0.938502
	(0.02991)	(0.05490)	(0.08317)	(0.10980)	(0.46249)	(0.03309)	(0.04437)
	[1.77065]	[1.29983]	[-1.48728]	[-1.69950]	[2.22222]	[1.74488]	[21.1522]
C	-0.000602	-0.000356	0.002438	0.003498	-0.001024	-0.001794	-0.002181
	(0.00042)	(0.00077)	(0.00117)	(0.00154)	(0.00649)	(0.00046)	(0.00062)
	[-1.43442]	[-0.46258]	[2.08933]	[2.27062]	[-0.15774]	[-3.86417]	[-3.50274]
R-squared	0.964201	0.922108	0.336724	0.249272	0.309584	0.976353	0.972490
Adj. R-squared	0.958506	0.909716	0.231202	0.129838	0.199745	0.972591	0.968113
Sum sq. resids	0.000208	0.000701	0.001610	0.002806	0.049978	0.000255	0.000458
S.E. equation	0.002175	0.003993	0.006049	0.007986	0.033635	0.002407	0.003227
F-statistic	169.2995	74.41225	3.191050	2.087108	2.818529	259.5268	222.2027
Log likelihood	249.3569	217.7720	196.1706	181.7246	106.9523	244.0965	228.8451
Akaike AIC	-9.282959	-8.068153	-7.237332	-6.681717	-3.805859	-9.080635	-8.494044
Schwarz SC	-8.982768	-7.767961	-6.937141	-6.381526	-3.505667	-8.780444	-8.193852
Mean dependent	-0.004399	-0.000339	0.004652	0.005402	-0.002205	-0.001036	0.003124
S.D. dependent	0.010677	0.013287	0.006898	0.008561	0.037599	0.014536	0.018070
Determinant resid cov. (df adj.)	1.09E-33						
Determinant resid cov.	3.40E-34						
Log likelihood	1487.210						
Akaike information criterion	-55.04656						
Schwarz criterion	-52.94522						

impact of past Euro area PPI inflation on the current Euro area nominal interest rate, which may indicate the relatively high inflation awareness of the ECB. Past observations of US PPI inflation influence the current Euro area output gap significantly negatively. Moreover, there is a statistically significant, positive impact of past values of the area and US nominal interest rates on on their respective current values, which corroborates our idea of the existence of interest-rate smoothing from Chapter 2. In addition, only the means of the Euro area and the US nominal interest rates seem to significantly differ from zero.

By performing autocorrelation LM tests with 10 lags as summarized in Table 3.8 in Appendix 3.10, we cannot reject the null hypothesis of no serial correlation of residuals

for most of the lags. White heteroskedasticity tests without cross terms as given in Table 3.9 in Appendix 3.10 cannot reject the null hypothesis of homoskedasticity of the residual variances. However, as all equations in (3.16) have the same regressors, the presence of autocorrelation and/or heteroskedasticity would not render OLS estimation inefficient since it would be equivalent to generalized least squares (GLS) estimation (see LÜTKEPOHL 2005, p. 71). We further conclude from Table 3.10 in Appendix 3.10 that Jarque-Bera tests cannot reject the null hypothesis that residuals are normally distributed for the single variables with two degrees of freedom each.

Moreover, we want to investigate whether there are causal relationships between (some of) the endogenous variables according to the data. We check for pairwise Granger causality between the variables, whereby causality within this concept means that if some generic random variable z is said to cause another generic random variable z', preceding information on the former variable should help to improve forecasting the latter (see LÜTKEPOHL 2005, p. 41).

As shown in Table 3.11 in Appendix 3.10, Granger causality tests with ten lags reject the null hypothesis of no Granger causality for the subsequent relationships (at least at the 10% significance level): $x \to \hat{i}, x^* \to \pi_H, \hat{i} \to x^*, \pi_F^* \to \pi_H, \hat{i} \to \pi_H, \hat{i}^* \to \pi_F^*, \hat{i} \to \hat{i}^*$. Besides these seven direct causal relationships, there may be even more indirect ones, which cannot be immediately retrieved. Thus, the causality structure as a whole may be quite complex. From the seven identified causal relationships, three are between nominal and real variables.

3.6   Impulse-response analysis

Impulse-response analysis together with Granger causality testing in a VAR(p) setting is sometimes summarized under the term structural analysis (see LÜTKEPOHL 2005, pp. 41-66). Although testing for Granger causality in a DSGE framework is unnecessary, comparing impulse responses based on estimated DSGEs to those based on estimated VARs is quite common in the literature (see, e.g., CHRISTIANO et al. 2005 or DELNE-GRO et al. 2005).

The impulse-response analysis for both the DSGE (3.1) to (3.14) and the VAR(1) (3.16) is carried out in terms of one standard deviation on the respective residuals ($\hat{\eta}$ or $\hat{\varepsilon}$). Due to possible cross-correlations of residuals, Cholesky decompositions are applied to the residual variance-covariance matrices while using the subsequent ordering of variables: $x, x^*, \pi_H, \pi_F^*, \Delta t, \hat{i}, \hat{i}^*$.

3.6.1 DSGE

In case of the DSGE (3.1) to (3.14), Figures 3.9 to 3.15 below show the posterior distributions of Bayesian impulse-response functions of the various endogenous variables, which are computed from the estimated parameter values and residual variances as obtained from their respective posterior distributions (see MANCINI-GRIFFOLI 2007, p. 54).
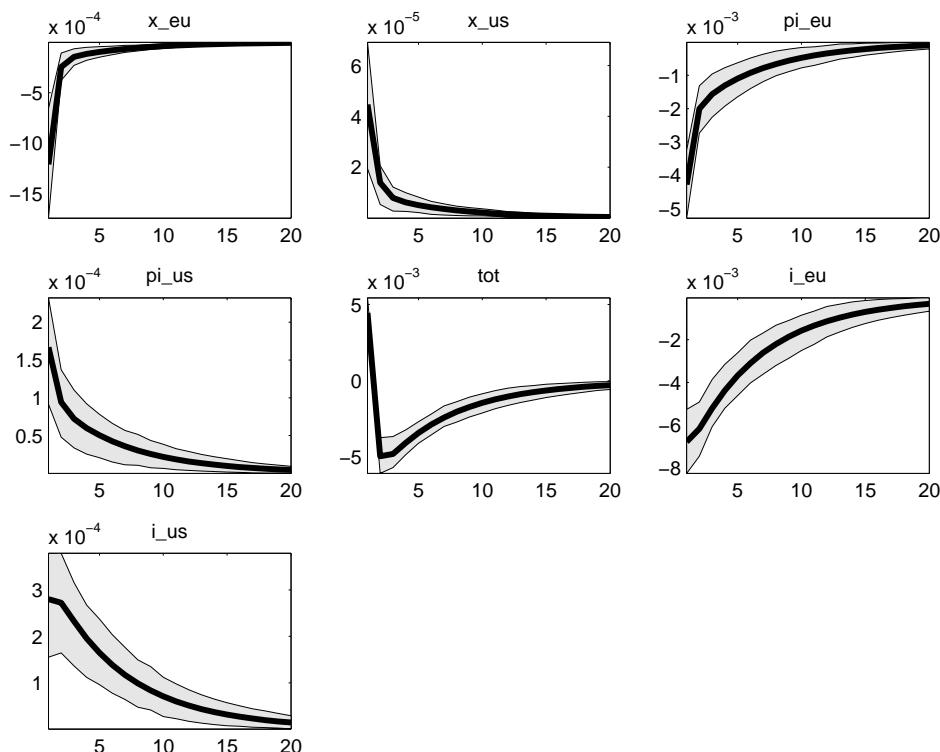
Figure 3.9: Euro area productivity shock

Qualitatively, the results obtained from Figures 3.9 to 3.12, which shall be summarized in the following for the reader's convenience, are very similar to those obtained in Chapter 2 for the calibrated model including the negative impact of a home-made productivity shock on own output and the undesirable response of the TOT to a home-made contractionary monetary policy shock. The major difference to Chapter 2, however, is the presence of responses to an impulse on the global measurement error component as shown by Figure 3.13.

1. The EU output gap, PPI inflation and nominal interest rates decrease before they return to their zero-inflation steady-state values in response to an impulse on the EU productivity shock (Figure 3.9). The TOT first augment, then drop below
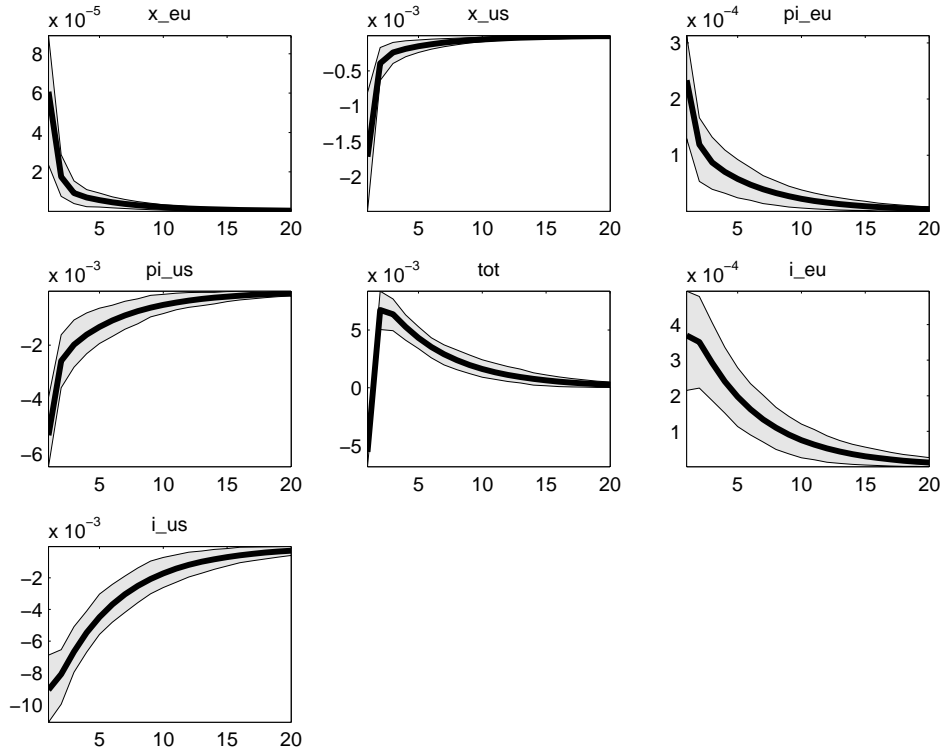
Figure 3.10: US productivity shock

their zero-inflation steady-state value until they gradually converge. There is also an impact on all US endogenous variables, which is of opposite sign and quantitatively small.

2. The US output gap, PPI inflation and nominal interest rates decrease before they return to their zero-inflation steady-state values in response to an impulse on the US productivity shock (Figure 3.10). The TOT first decrease, then jump above their zero-inflation steady-state value until they gradually converge. There is also an impact on all EU endogenous variables of opposite sign, which is quantitatively larger compared to the impact of the EU productivity shock on foreign variables.

3. The EU output gap decreases, yet the EU PPI inflation and nominal interest rates increase before all endogenous variables return to their zero-inflation steady-state values in response to an impulse on the EU cost-push shock (Figure 3.11). The TOT first plummet, then jump above their zero-inflation steady-state value until they gradually converge. There is also an impact on all US endogenous variables,
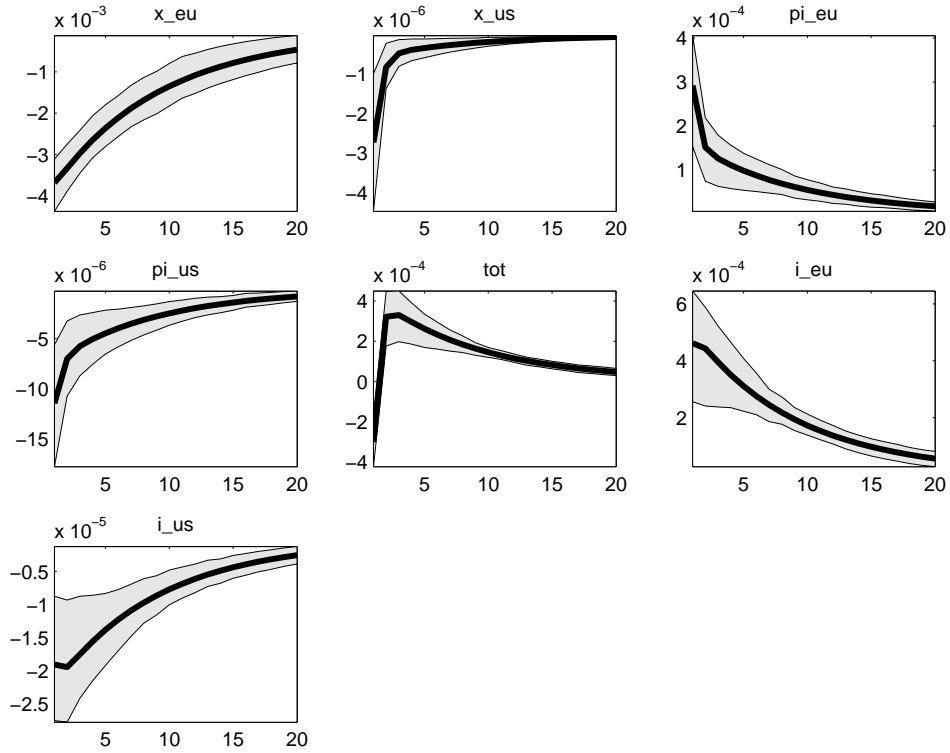
Figure 3.11: Euro area cost-push shock

which is of opposite sign except for the US output gap.

4. The US output gap decreases, yet the US PPI inflation and nominal interest rates increase before all endogenous variables return to their zero-inflation steady-state values in response to an impulse on the US cost-push shock (Figure 3.12). The TOT first augment, then drop below their zero-inflation steady-state value until they gradually converge. There is also an impact on all EU endogenous variables, which is of opposite sign except for the EU output gap.

5. The Euro area output gap, PPI inflation and nominal interest rates increase before all endogenous variables return to their zero-inflation steady-state values in response to an impulse on the global measurement error component (Figure 3.13). The TOT augment before they return to their zero-inflation steady-state value without any sudden drops to the negative. The impact on all US endogenous variables is of opposite sign.

6. The EU output gap, PPI inflation and nominal interest rates decrease before all
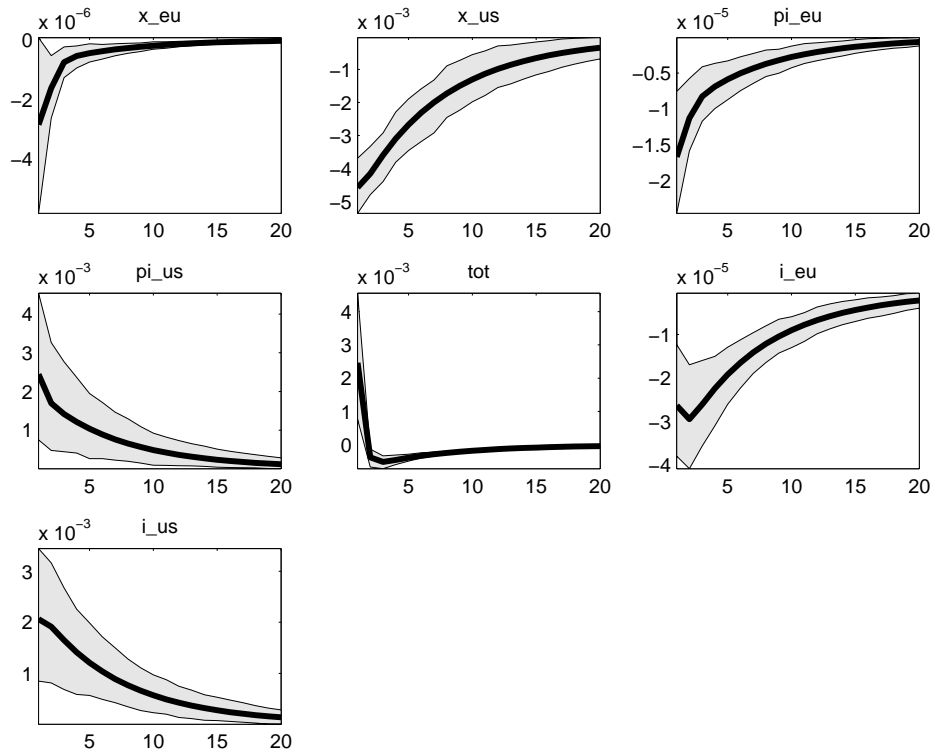
Figure 3.12: US cost-push shock

endogenous variables return to their zero-inflation steady-state values in response to an impulse on the EU monetary policy shock (Figure 3.14). The TOT augment before they return to their zero-inflation steady-state value without any sudden drops to the negative. There is also an impact on all US endogenous variables, which is of the same sign.

7. The US output gap, PPI inflation and nominal interest rates decrease before all endogenous variables return to their zero-inflation steady-state values in response to an impulse on the US monetary policy shock (Figure 3.15). The TOT plummet before they return to their zero-inflation steady-state value without any sudden jumps to the positive. There is also an impact on all EU endogenous variables, which is of the same sign.

3.6.2 VAR

Figure 3.16 below shows the responses of the various endogenous variables on the seven different impulses for the VAR(1) (3.16) in combined graphs.
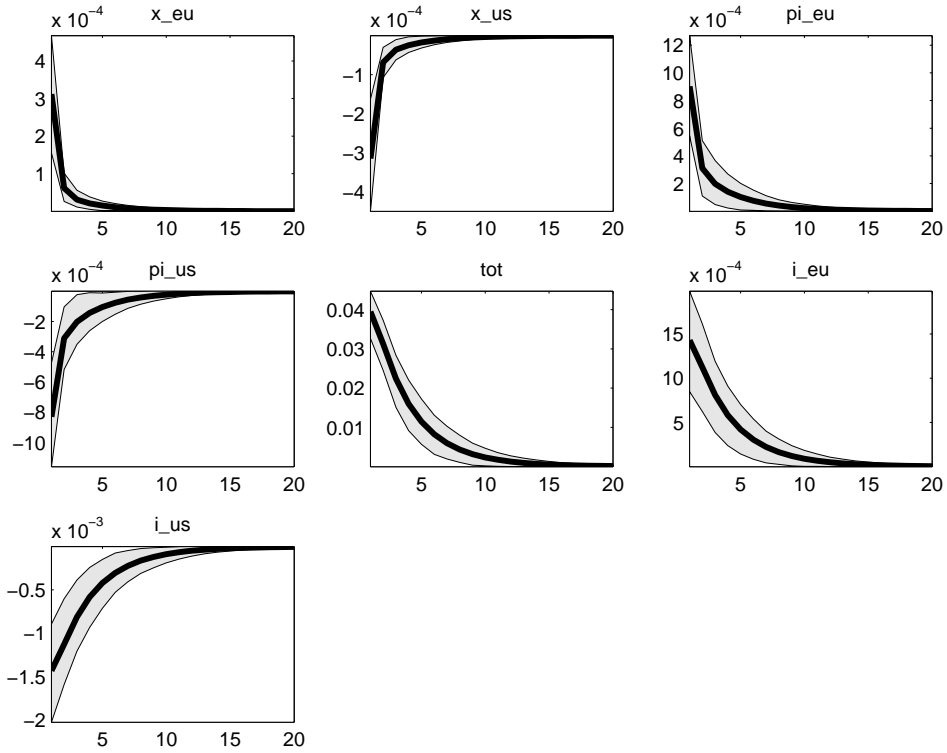
Figure 3.13: Global measurement error component

We will forego to interpret every single impulse response of the VAR(1) model, but as far as the results obtained from Figure 3.16 substantially differ from those obtained from Figures 3.9 to 3.15, they shall be briefly discussed in the following. What most VAR(1) impulse responses have in common and what makes them differ from the DSGE impulse responses, however, is that they show oscillating behavior and that it takes them almost twice as long to converge.

From an economic theory perspective, the difference in impulse responses on Euro area and US productivity shocks between the DSGE (3.1) to (3.14) and the VAR(1) (3.16) is of special interest. The negative impact of productivity shocks on actual output across countries in case of the DSGE reproduces the findings of the calibrated DSGE from Chapter 2, which in turn is in line with GALÍ (2002, pp. 17-18) and GALÍ & RABANAL (2004, pp. 36-37). The positive impact of productivity shocks on actual output across countries in case of the VAR(1), however, is closer to the ideas of the RBC literature (blue and red lines in Figure 3.16).

This *make-believe contradiction* can be explained as follows: since potential (or flexible-price equilibrium) output is assumed to be hit more severe than actual output by a positive technology shock because of the structure of the DSGE model (see Chapter
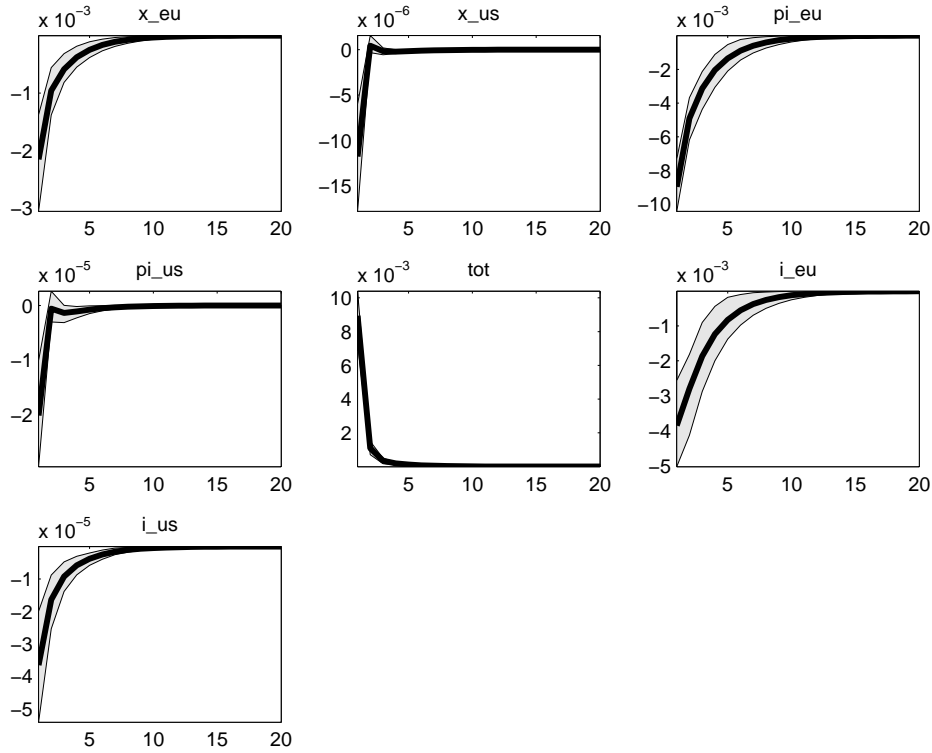
Figure 3.14: Euro area monetary policy shock

2), actual output simply needs time to adjust. Because of the different structure of the VAR(1) model, this phenomenon is not present in the latter case.

Again contrary to the DSGE (3.1) to (3.14) there does not seem to be any noteworthy effect at all of Euro and US nominal interest rates on the TOT so that *prosper thyself* or *beggar thy neighbor* effects in spirit of CORSETTI & PESENTI (2001) are not observable for VAR(1) (3.16) impulse responses.

3.7 Out-of-sample forecasting

Starting from the forecast origin 2007Q1, we perform out-of-sample mean forecasts of the DSGE (3.1) to (3.14) on the one hand and the VAR(1) (3.16) on the other for a forecast horizon of two years or eight quarters, i.e. until 2009Q1, to check if one of the models were able to predict the severeness or, at least, the evolution of the economic and financial crisis.

The reason why we employ out-of-sample forecasting is that using all information available up to the forecast origin, the maximum information set possible, assures optimal prediction in the sense that the associated mean squared forecast error (MSFE),
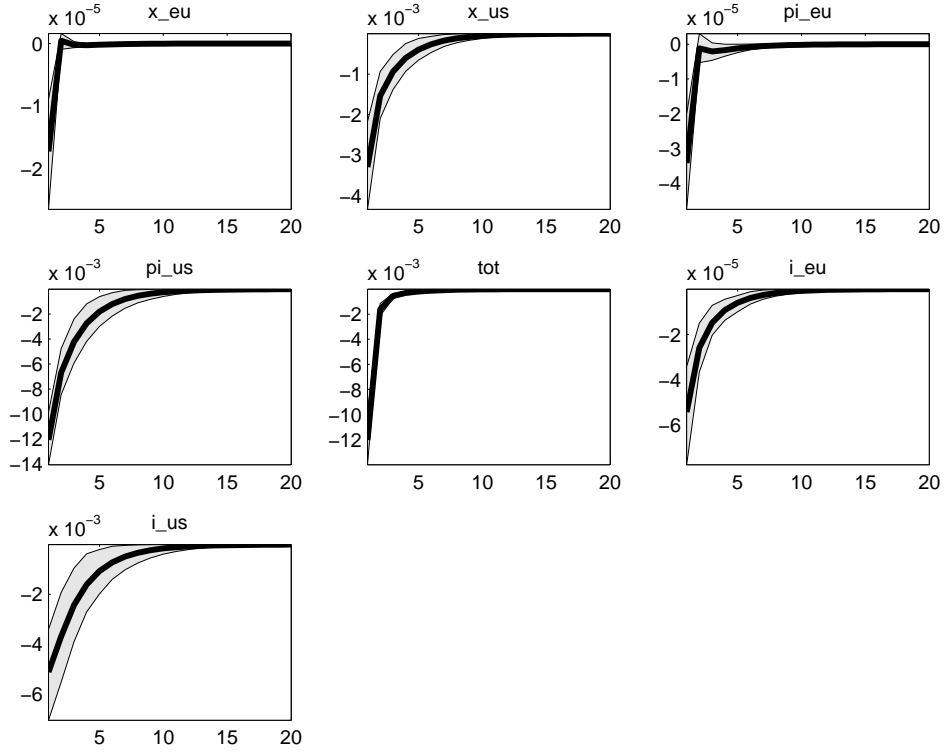
Figure 3.15: US monetary policy shock

which is interpretable as the squared loss of forecasting, is minimized (see LÜTKEPOHL 2005, pp. 32-35).

In addition, we want to compare the forecast accuracy of the two models under scrutiny with the forecast accuracy of one of the many model-free extrapolation methods to serve as a benchmark (see CHATFIELD 2001, pp. 97-98): the additive seasonal Holt-Winters method (HW), which is a generalization of the various univariate exponential smoothing procedures accounting for possible trend and seasonal components:[7]

$$L_{t,z} = \tau_{1,z}(z_{t,z} - S_{t-s,z}) + (1 - \tau_{1,z})(L_{t-1,z} + T_{t-1,z}), \tag{3.17}$$

$$T_{t,z} = \tau_{2,z}(L_{t,z} - L_{t-1,z}) + (1 - \tau_{2,z})T_{t-1,z}, \tag{3.18}$$

$$S_{t,z} = \tau_{3,z}(z_{t,z} - L_{t,z}) + (1 - \tau_{3,z})S_{t-s,z}. \tag{3.19}$$

In (3.17) to (3.19), z_t denote actual values of the various variables, L_z the corresponding local level, T_z the corresponding local trend, and S_z the corresponding additive seasonal factor. As we investigate quarterly data, we have the following length of the seasonal

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[7]Note that other univariate benchmarks, e.g. from the AR(I)MA class, would also be feasible and will be taken up in Chapter 4.
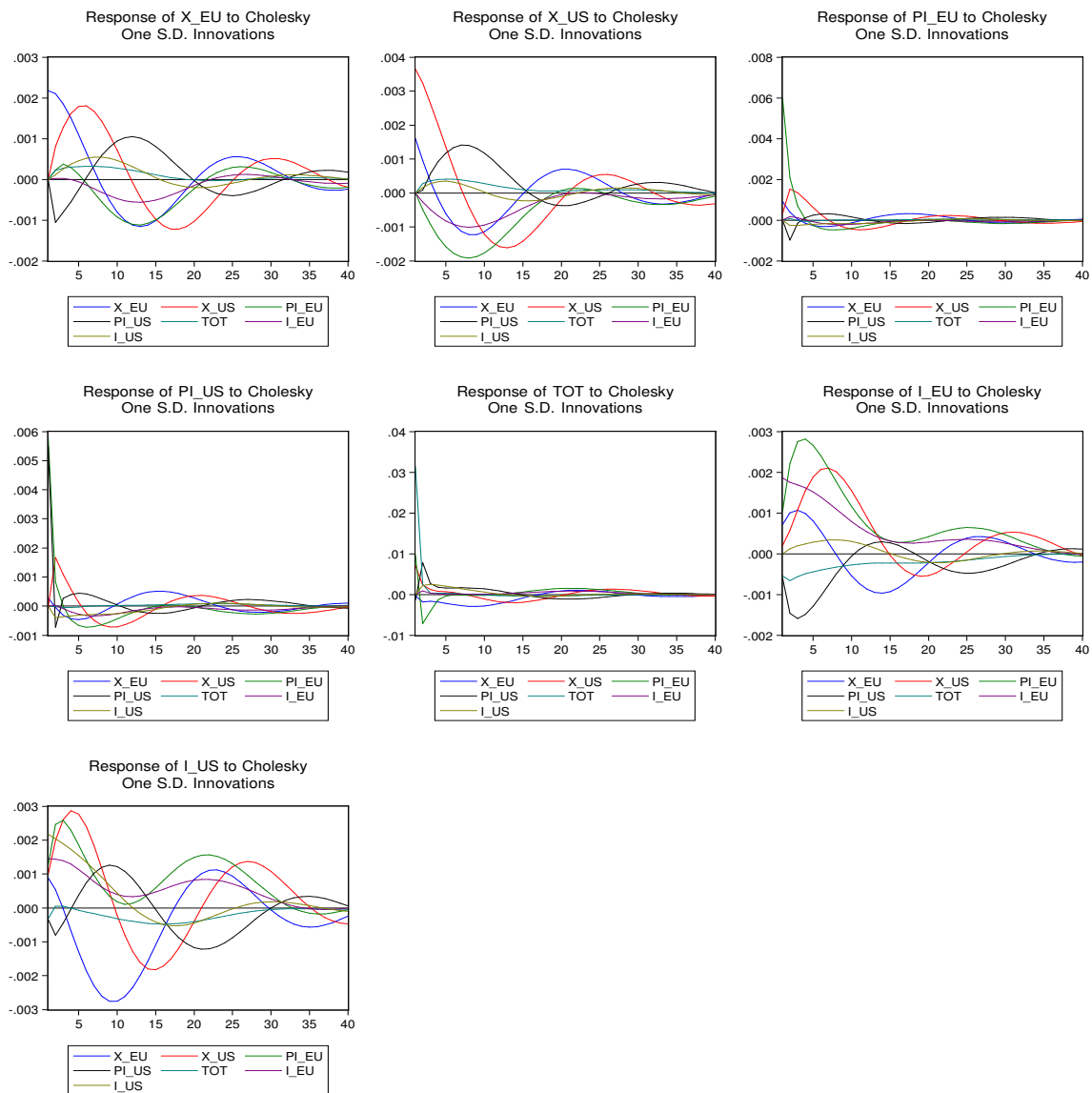
Figure 3.16: VAR(1) impulse responses

cycle: $s = 4$. The various smoothing parameters $\tau_{1,z}, \tau_{2,z}, \tau_{3,z}$ $(0 < \tau_{1,z}\tau_{2,z}, \tau_{3,z} < 1)$ are estimated in EVIEWS by minimizing the sum of squared errors.

Figures 3.17 to 3.20 below plot once more the actual values (blue lines) of the various endogenous variables for the time period from 2007Q2 until 2009Q1 and also the eight-step-ahead predictors of the three forecasting models: DSGE (3.1) to (3.14) (red lines), VAR(1) (3.16) (orange lines), and HW (3.17) to (3.19) (green lines).

As we can see from Figure 3.17, only in case of the Euro area output gap the two
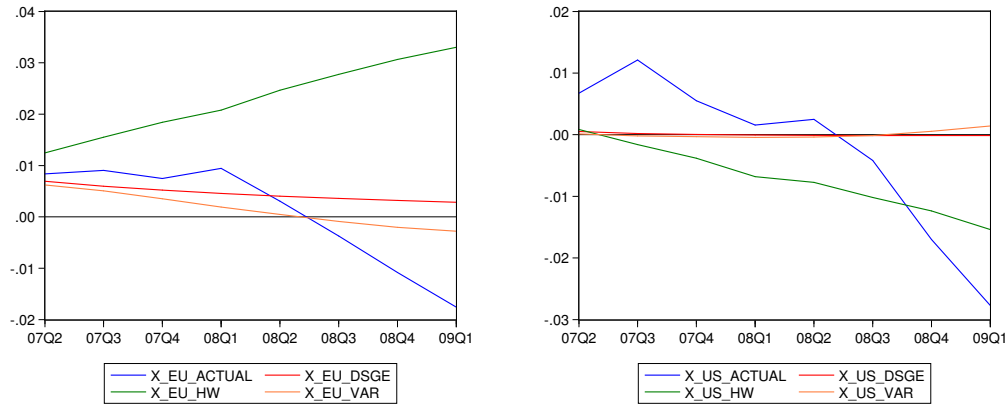
Figure 3.17: Euro area and US output gaps – actual and forecasted values
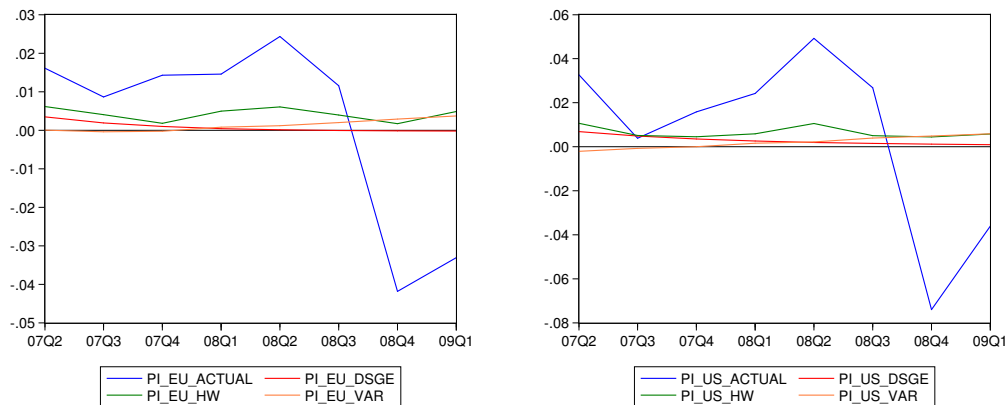
Figure 3.18: Euro area and US PPI inflation rates – actual and forecasted values

models could identify the downward swing of the business cycle, whereas for the US output gap the extrapolation method performed better. In the case of PPI inflation and the TOT *none* of the three methods was able to replicate the actual behavior of these variables across countries (see Figures 3.18 and 3.19). Taking a look at the nominal interest rates across countries, again the two models could at least indicate the downward trend in the given period (see Figure 3.20).

Nonetheless, we have to conclude that none of the forecasting models was able to predict at least the evolution of the crisis as a whole: the output gaps across countries were way more negative than the three forecasting models were able to predict and so were PPI inflation rates. Also the nominal interest rates were closer to their zero-lower bound towards the end of the forecast horizon than the forecasting models could
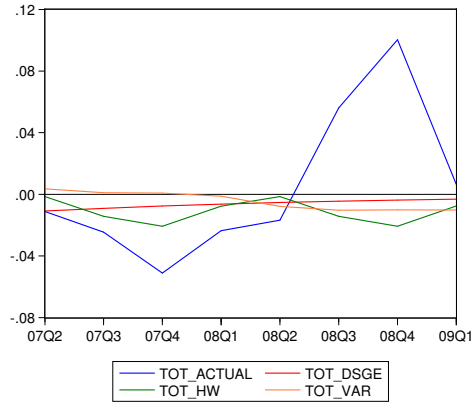
Figure 3.19: Terms of trade – actual and forecasted values
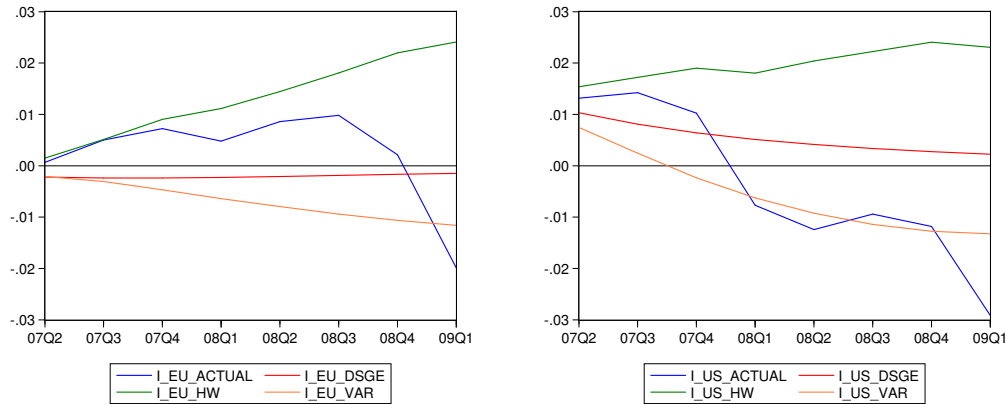
Figure 3.20: Euro area and US nominal interest rates – actual and forecasted values

predict. As a consequence, shortening the sample by leaving out this period of irregular economic behavior could indeed have added to improve the goodness of the estimation and of the structural analysis in Sections 3.5 and 3.6.

3.8 Measuring forecast accuracy

If we now assess the forecasting performance of the DSGE (3.1) to (3.14), the VAR(1) (3.16), and the HW (3.17) to (3.19) by quantitative measures of forecast accuracy we have to rely on out-of-sample forecasts for a test set that lies within a period of regular economic activity. Therefore, we want to compare the accuracy of the one-step-ahead predictor for the time period from 2006Q2 until 2007Q1. In general, a better forecast

accuracy of the DSGE (3.1) to (3.14) relative to the VAR(1) (3.16) would justify the constraints in the DSGE as imposed by economic theory relative to the unconstrained VAR specification (see RUBASZEK & SKRZYPCZYŃSKI 2008, p. 499).

We evaluate the forecasting performance of the one-step-ahead predictor for 2006Q2, 2006Q3, 2006Q4, and 2007Q1.[8] Hence, each of the four training sets comprises 49 observations so that we obtain four predictions based on estimations of equal quality: 1994Q1-2006Q1, 1994Q2-2006Q2, 1994Q3-2006Q3, and 1994Q4-2006Q4.

One commonly used quantitative forecast measure is the already mentioned MSFE, which reads as follows for each of the seven variables (see CHATFIELD 2001, p. 150):

$$
MSFE := \frac{1}{m} \sum_{t=N-m+1}^{N} \left[z_t - \hat{z}_{t-1}(1) \right]^2,
$$

where z_t again denote actual and $\hat{z}_{t-1}(1)$ forecasted values of the various variables and $m = 4$ the number of one-step-ahead forecasts.

Nonetheless, we want to indicate the loss of forecasting in terms of standard deviations rather than variances, which could ease the interpretation. Therefore, we introduce the root mean squared forecast error (RMSFE), which simply is the square root of the MSFE:

$$
RMSFE := \sqrt{\frac{1}{m} \sum_{t=N-m+1}^{N} \left[z_t - \hat{z}_{t-1}(1) \right]^2}.
$$

Another commonly used linear measure of forecast accuracy is the mean absolute forecast error (MAFE), which is given in the following (see CHATFIELD 2001, p. 150):

$$
MAFE := \frac{1}{m} \sum_{t=N-m+1}^{N} \left| z_t - \hat{z}_{t-1}(1) \right|.
$$

This measure is more sensitive to small deviations from zero compared to the other ones presented, but less sensitive to large deviations since it is not computed based on squared losses. Besides the RMSFE and the MAFE used here, there are several other measures of forecast accuracy (see, e.g., CHATFIELD 2001, chapter 6 for an overview).

Finally, empirical studies often suggest that combined forecasts yield more accurate forecasts than the underlying single forecasts that are built on possibly misspecified forecasting models (see CHATFIELD 2001, pp. 101-102). Hence, we choose to calculate

---

[8]Due to the computational burden associated with DSGE estimation, we restrict ourselves to re-estimate all three forecasting models only four times on a quarterly basis.

the various measures of forecast accuracy for an average of the forecasted values based on the DSGE (3.1) to (3.14), the VAR(1) (3.16), and HW (3.17) to (3.19). As opposed to multiple encompassing tests in spirit of HARVEY & NEWBOLD (2000), we simply assign uniform weights since recent simulation experiments have shown little evidence for the superiority of test-based combination compared to uniform weighting for the one-step-ahead predictor in typical forecasting situations (see COSTANTINI et al. 2010, p. 11):

$$f_{COMB} = \frac{1}{3} f_{DSGE} + \frac{1}{3} f_{VAR(1)} + \frac{1}{3} f_{HW}. \tag{3.20}$$

Subsequently, Table 3.4 shows the RMSFE, whereas Table 3.5 shows the MAFE as obtained for the DSGE (3.1) to (3.14), the VAR(1) (3.16), HW (3.17) to (3.19), and the combined forecast (3.20) for the various endogenous variables, where the smallest RMSFE and MAFE values for the single variables are given in boldface numbers.

Table 3.4: RMSFE of rival models

Model	x	x^*	π_H	π_F^*	Δt	$\hat{i}$	$\hat{i}^*$
$DSGE$	0.0044	**0.0034**	0.0085	0.0198	**0.0077**	0.0026	0.0042
$VAR(1)$	**0.0006**	0.0062	**0.0069**	**0.0153**	0.0338	0.0021	0.0021
HW	0.0025	0.0042	0.0075	0.0161	0.0280	**0.0003**	0.0037
$COMB$	0.0021	0.0042	0.0074	0.0166	0.0197	0.0013	**0.0014**

Table 3.5: MAFE of rival models

Model	x	x^*	π_H	π_F^*	Δt	$\hat{i}$	$\hat{i}^*$
$DSGE$	0.0040	**0.0025**	0.0075	0.0191	**0.0064**	0.0024	0.0034
$VAR(1)$	**0.0006**	0.0058	**0.0060**	**0.0143**	0.0318	0.0021	0.0016
HW	0.0021	0.0039	0.0066	0.0150	0.0192	**0.0002**	0.0033
$COMB$	0.0018	0.0035	0.0064	0.0161	0.0170	0.0011	**0.0014**

Observing Tables 3.4 and 3.5, we can derive the following. The RMSFE and the MAFE qualitatively deliver the same results. The DSGE (3.1) to (3.14) performs best for the US output gap and for the TOT. The VAR(1) (3.16) outperforms all other models in case of the Euro area output gap and both PPI inflation rates. The HW

benchmark (3.17) to (3.19) is able to predict best the Euro area nominal interest rate. Only in the case of the US nominal interest rate the combined forecast (3.20) delivers the most accurate result.

Finally, we want to investigate if the forecasting model characterized by the lowest RMSFE/MAFE per variable can *significantly* outperform the remaining three forecasting models. In doing so, we employ DIEBOLD & MARIANO (1995) (DM) tests. The null hypothesis of this test is that the loss differentials (square of true minus forecasted values) of two different forecasting models $LD(Model1), LD(Model2)$ are identical. The DM test statistic is calculated as follows and is asymptotically $\sim N(0, 1)$ distributed:

$$DM := \frac{LD(Model1) - LD(Model2)}{\sqrt{Var(Numerator)}}.$$

The forecasting model under examination, i.e. $Model1$, would indeed significantly outperform another model if the corresponding DM test statistic were significantly different from zero and negative. In order to interpret the results from Table 3.6 below correctly, we have to check whether the modulus of the DM test statistic is significantly greater than zero as indicated by the one-sided p-value. Significance at the 10% level is indicated by (*), at the 5% level by (**), and at the 1% level by (***).

Taking a look at Table 3.6, we can conclude that we indeed see statistically significant outperformance of the forecasting model characterized by the lowest RMSFE/MAFE per variable in some cases. The VAR(1) improves at the 10% level on the DSGE for x as well as on the combined forecast for π_F^*. The DSGE, however, improves at the 10% level on the VAR(1) for x^* and Δt. The HW benchmark significantly outperforms the DSGE at the 10% level and the VAR(1) even at the 1% level for $\hat{i}$. In case of π_H and $\hat{i}^*$ the best-performing models per variable (VAR(1) for π_H, combined forecast for $\hat{i}^*$) seem to forecast equally well as the remaining three forecasting models.

3.9   Concluding remarks

Putting things together, we want to summarize once more the key findings obtained from the above analysis, which constitutes an empirical application of DSGE models.

The estimated DSGE model qualitatively reproduces the findings of the calibrated one from Chapter 2 with respect to most of the parameter values and to impulses responses on the various exogenous error terms. Nonetheless, the degrees of price stickiness in both countries are lower and the degrees of interest-rate inertia across countries are higher than expected prior to estimation.

Table 3.6: DM test statistics

Variable	Models under comparison	DM test statistic	One-sided p-value
x	$VAR(1)$ vs. $DSGE$	-1.3009*	0.0966
	$VAR(1)$ vs. HW	-0.9843	0.1625
	$VAR(1)$ vs. $COMB$	-0.9364	0.1745
x^*	$DSGE$ vs. $VAR(1)$	-1.4136*	0.0787
	$DSGE$ vs. HW	-0.4125	0.3400
	$DSGE$ vs. $COMB$	-0.6187	0.2681
π_H	$VAR(1)$ vs. $DSGE$	-0.5962	0.2755
	$VAR(1)$ vs. HW	-0.3029	0.3810
	$VAR(1)$ vs. $COMB$	-0.9935	0.1602
π_F^*	$VAR(1)$ vs. $DSGE$	-0.7880	0.2153
	$VAR(1)$ vs. HW	-0.1872	0.4258
	$VAR(1)$ vs. $COMB$	-1.5907*	0.0558
Δt	$DSGE$ vs. $VAR(1)$	-1.4498*	0.0736
	$DSGE$ vs. HW	-0.6096	0.2711
	$DSGE$ vs. $COMB$	-0.8572	0.1957
$\hat{i}$	HW vs. $DSGE$	-1.5630*	0.0590
	HW vs. $VAR(1)$	-2.8197***	0.0024
	HW vs. $COMB$	-1.1266	0.1300
$\hat{i}^*$	$COMB$ vs. $DSGE$	-0.9064	0.1824
	$COMB$ vs. $VAR(1)$	-0.4139	0.3395
	$COMB$ vs. HW	-1.0396	0.1493

Estimating an unconstrained VAR(1) does not yield the identical causal relationships as implied by the DSGE. However, Granger causality tests suggest a rather complex causality structure including causalities between real and nominal variables across countries. In addition, impulse responses based on the VAR(1) sometimes differ from the ones obtained for the DSGE.

Both models and the additive seasonal Holt-Winters method, a simple univariate extrapolation method serving as a benchmark, were not able to predict the severeness or, at least, the evolution of the economic and financial crisis for the forecasting period from 2007Q2 until 2009Q1 since the current crisis is so at odds with regular economic activity.

Finally, we obtain that the accuracy of DSGE forecasts, measured by the RMSFE and the MAFE, can compete well with the accuracy of VAR(1), Holt-Winters, and uniformly combined forecasts. In two cases, the DSGE is able to significantly outperform some of the rival forecasting models, but only at the 10% level.

3.10 Appendix to Chapter 3

Table 3.7: Lag order selection criteria

VAR Lag Order Selection Criteria
Endogenous variables:
X_EU X_US PI_EU PI_US TOT I_EU I_US
Exogenous variables: C
Date: 05/20/10 Time: 15:43
Sample: 1994Q1 2007Q1
Included observations: 48

Lag	LogL	LR	FPE	AIC	SC	HQ
0	1083.224	NA	7.90e-29	-44.84269	-44.56980	-44.73956
1	1398.260	525.0588	1.24e-33*	-55.92749	-53.74442*	-55.10250*
2	1427.942	40.81247	3.16e-33	-55.12256	-51.02931	-53.57572
3	1466.872	42.17456	6.91e-33	-54.70300	-48.69956	-52.43429
4	1511.555	35.37429	1.88e-32	-54.52313	-46.60951	-51.53256
5	1662.022	75.23355*	1.63e-33	-58.75093*	-48.92712	-55.03850

* indicates lag order selected by the criterion

Table 3.8: Autocorrelation LM tests

VAR Residual Serial Correlation LM Tests
Null Hypothesis: no serial correlation at lag order h
Date: 05/21/10 Time: 10:02
Sample: 1994Q1 2007Q1
Included observations: 52

Lags	LM-Stat	Prob
1	50.58592	0.4107
2	34.66013	0.9396
3	34.67918	0.9393
4	34.72418	0.9386
5	80.48746	0.0031
6	49.84059	0.4397
7	52.48338	0.3406
8	34.20719	0.9462
9	48.48202	0.4940
10	51.35996	0.3814

Probs from chi-square with 49 df.

Table 3.9: White heteroskedasticity tests

VAR Residual Heteroskedasticity Tests: No Cross Terms
Date: 05/21/10 Time: 10:13
Sample: 1994Q1 2007Q1
Included observations: 52

Joint test:

Chi-sq	df	Prob.
410.3690	392	0.2515

Individual components:

Dependent	R-squared	F(14,37)	Prob.	Chi-sq(14)	Prob.
res1*res1	0.221743	0.753009	0.7093	11.53063	0.6439
res2*res2	0.069429	0.197181	0.9989	3.610315	0.9974
res3*res3	0.316009	1.221017	0.3020	16.43244	0.2877
res4*res4	0.347451	1.407195	0.1985	18.06746	0.2037
res5*res5	0.316468	1.223616	0.3003	16.45635	0.2863
res6*res6	0.208172	0.694807	0.7647	10.82492	0.6997
res7*res7	0.224543	0.765270	0.6974	11.67622	0.6323
res2*res1	0.205830	0.684964	0.7738	10.70314	0.7092
res3*res1	0.230656	0.792352	0.6709	11.99412	0.6068
res3*res2	0.300976	1.137926	0.3602	15.65076	0.3352
res4*res1	0.192851	0.631456	0.8213	10.02827	0.7601
res4*res2	0.262143	0.938944	0.5290	13.63143	0.4775
res4*res3	0.313751	1.208305	0.3104	16.31504	0.2945
res5*res1	0.134668	0.411296	0.9617	7.002724	0.9346
res5*res2	0.236050	0.816605	0.6471	12.27459	0.5843
res5*res3	0.285573	1.056413	0.4247	14.84981	0.3885
res5*res4	0.245187	0.858485	0.6061	12.74975	0.5463
res6*res1	0.085973	0.248585	0.9963	4.470574	0.9919
res6*res2	0.199187	0.657359	0.7988	10.35770	0.7356
res6*res3	0.329965	1.301500	0.2528	17.15818	0.2478
res6*res4	0.309660	1.185485	0.3259	16.10233	0.3072
res6*res5	0.321969	1.254983	0.2804	16.74238	0.2702
res7*res1	0.162782	0.513857	0.9095	8.464673	0.8637
res7*res2	0.178314	0.573527	0.8681	9.272339	0.8132
res7*res3	0.289133	1.074937	0.4094	15.03492	0.3758
res7*res4	0.283599	1.046218	0.4332	14.74715	0.3957
res7*res5	0.336272	1.338979	0.2323	17.48613	0.2312
res7*res6	0.212891	0.714818	0.7459	11.07032	0.6805

Table 3.10: Jarque-Bera normality tests

VAR Residual Normality Tests
Orthogonalization: Cholesky (Lutkepohl)
Null Hypothesis: residuals are multivariate normal
Date: 05/21/10 Time: 10:06
Sample: 1994Q1 2007Q1
Included observations: 52

Component	Skewness	Chi-sq	df	Prob.
1	-0.167268	0.242482	1	0.6224
2	0.566265	2.779018	1	0.0955
3	0.047331	0.019415	1	0.8892
4	-0.088543	0.067945	1	0.7944
5	-0.177099	0.271823	1	0.6021
6	-0.083489	0.060410	1	0.8058
7	-0.323550	0.907269	1	0.3408
Joint	4.348363	7	0.7389	

Component	Kurtosis	Chi-sq	df	Prob.
1	1.745038	3.412348	1	0.0647
2	2.420606	0.727344	1	0.3937
3	2.420122	0.728561	1	0.3934
4	2.058128	1.922098	1	0.1656
5	1.947782	2.398853	1	0.1214
6	1.700503	3.658835	1	0.0558
7	2.158985	1.532496	1	0.2157
Joint	14.38053	7	0.0448	

Component	Jarque-Bera	df	Prob.	
1	3.654830	2	0.1608	
2	3.506362	2	0.1732	
3	0.747976	2	0.6880	
4	1.990044	2	0.3697	
5	2.670676	2	0.2631	
6	3.719245	2	0.1557	
7	2.439765	2	0.2953	
Joint	18.72890	14	0.1756	

Table 3.11: Pairwise Granger causality tests

Pairwise Granger Causality Tests
Date: 05/21/10   Time: 10:59
Sample: 1994Q1 2007Q1
Lags: 10

Null Hypothesis:	Obs	F-Statistic	Prob.
X_US does not Granger Cause X_EU	43	0.67939	0.7319
X_EU does not Granger Cause X_US		0.95513	0.5061
PI_EU does not Granger Cause X_EU	43	0.35857	0.9520
X_EU does not Granger Cause PI_EU		1.16930	0.3610
PI_US does not Granger Cause X_EU	43	1.09585	0.4068
X_EU does not Granger Cause PI_US		0.92800	0.5269
TOT does not Granger Cause X_EU	43	1.17440	0.3579
X_EU does not Granger Cause TOT		1.51097	0.2010
I_EU does not Granger Cause X_EU	43	1.35611	0.2633
X_EU does not Granger Cause I_EU		2.64184	0.0276
I_US does not Granger Cause X_EU	43	1.26621	0.3070
X_EU does not Granger Cause I_US		1.31270	0.2837
PI_EU does not Granger Cause X_US	43	0.78169	0.6457
X_US does not Granger Cause PI_EU		2.37630	0.0435
PI_US does not Granger Cause X_US	43	1.25974	0.3104
X_US does not Granger Cause PI_US		0.86239	0.5790
TOT does not Granger Cause X_US	43	0.93548	0.5212
X_US does not Granger Cause TOT		0.86492	0.5770
I_EU does not Granger Cause X_US	43	1.96580	0.0896
X_US does not Granger Cause I_EU		0.98045	0.4872
I_US does not Granger Cause X_US	43	1.04164	0.4433
X_US does not Granger Cause I_US		0.98586	0.4832
PI_US does not Granger Cause PI_EU	43	2.24297	0.0549
PI_EU does not Granger Cause PI_US		1.24243	0.3196
TOT does not Granger Cause PI_EU	43	0.89177	0.5554
PI_EU does not Granger Cause TOT		1.25337	0.3137
I_EU does not Granger Cause PI_EU	43	1.95584	0.0912
PI_EU does not Granger Cause I_EU		0.35167	0.9549
I_US does not Granger Cause PI_EU	43	1.08300	0.4152
PI_EU does not Granger Cause I_US		1.28794	0.2959
TOT does not Granger Cause PI_US	43	1.52785	0.1951
PI_US does not Granger Cause TOT		0.99948	0.4733
I_EU does not Granger Cause PI_US	43	1.59536	0.1732
PI_US does not Granger Cause I_EU		0.54570	0.8390
I_US does not Granger Cause PI_US	43	2.99300	0.0153
PI_US does not Granger Cause I_US		1.13350	0.3828
I_EU does not Granger Cause TOT	43	0.83999	0.5973
TOT does not Granger Cause I_EU		1.37077	0.2567
I_US does not Granger Cause TOT	43	0.89385	0.5537
TOT does not Granger Cause I_US		1.76188	0.1289
I_US does not Granger Cause I_EU	43	1.71166	0.1409
I_EU does not Granger Cause I_US		3.80672	0.0043

Chapter 4

New variants of the DSGE model and a different pair of countries

4.1 Introduction

Two-country DSGE models as the one developed in Chapter 2 are usually applied for modeling economic interrelations between two large open economies such as the EU (or the Euro area) and the US. As laid out in Chapter 3, the parameters of the DSGE structure can be comfortably estimated by applying Bayesian techniques and the estimated model can further be used for forecasting endogenous variables and assessing the model's forecasting performance compared to rival models. In principle, the latter is also possible for small-open-economy DSGE models for which GALÍ & MONACELLI (2005) can be taken as prototypical example.

Meanwhile, there exist estimated open-economy New Keynesian models, too, which are explicitly tuned to the Austrian (see BREUSS & RABITSCH 2009) and the Hungarian economy (see JAKAB & VILÁGI 2008). Similar models respecting country-specific particularities also exist for many other small open economies (see, e.g., the estimated model by CHRISTIANO et al. 2009 for the case of Sweden or the simulated model by CUCHE-CURTI et al. 2009 for the case of Switzerland) and are usually developed and used by the countries' monetary authorities.

After concentrating on a pair of large countries such as the EU and the US, we now shift the focus to the interior of one of these economies, namely the EU. We apply the two-country model from Chapters 2 and 3 to Austria and Hungary, two European countries of approximately equal size that share a common border and whose economies should be treated as small, of course. In the case of small open economies, European or world variables certainly have an impact on Austrian and Hungarian variables, but

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not vice versa. Austria and Hungary are being integrated within the European context since the fall of the Iron Curtain in 1989. The integration is of political as well as of economic nature since both countries are members of the EU (Austria since 1995, Hungary since 2004) and, hence, also of the Common Market.

Alternatively, the subsequent exercise could be performed for two different *regions* within one large economy, e.g. two other EU members or two US states. Basically, there are three reasons why we want to concentrate on Austria and Hungary in particular.

1. As can be seen from recent numbers in Table 4.1 below (see OECD 2010), trade in goods between the two countries (all commodities) is non-negligible compared to trade with the countries' largest trading partners (the EU of 27 for both countries) and with the rest of the world. This may suggest that the impact of Austrian and Hungarian macro variables on one another may be non-negligible as well (see Section 4.3).

2. Austria and Hungary use different currencies (the Euro in Austria, the Forint in Hungary) such that the nominal exchange rate of these two currencies typically is not equal to 1. Besides PPI inflation rates, the nominal exchange rate then constitutes an additional source for movements in the TOT.

3. If we neglect other countries in the analysis, we can isolate the impact Austrian macro variables may have on Hungarian ones and vice versa.[1]

In this chapter, we want to compare the forecasting accuracy of the two-country DSGE model from Chapter 2 and of several new variants thereof in terms of the RMSFE with Bayesian and classical (vector) autoregressive benchmarks. In particular, we will address the forecasting performance [1] of the original open-economy DSGE with PPI inflation under the interest-rate rules, [2] of an open-economy DSGE with CPI inflation under the interest-rate rules, [3] and of the closed-economy DSGE with CPI inflation under the interest-rate rule that is nested in the open-economy structure. Consistently, we employ open-economy as well as closed-economy benchmarks comprising different vectors of endogenous variables.

Our hypothesis is the following: if the impact of economic interrelations between Austria and Hungary were non-negligible, using open-economy models should add in-

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[1]There are other studies on two neighboring countries within the EU that, e.g., explore the effects and transmission channels of macroeconomic shocks between these countries while neglecting the remaining EU members (see PRETTNER & KUNST 2009 for the case of Austria and Germany).

Table 4.1: Commodity trade of Austria and Hungary in 2008

Trade in goods	Austria	Hungary
Imports from the other country (in per cent of imports from the rest of the world)	3.01%	7.10%
Imports from the other country (in per cent of imports from the EU of 27)	3.69%	9.04%
Exports to the other country (in per cent of exports to the rest of the world)	3.86%	5.61%
Exports to the other country (in per cent of exports to the EU of 27)	5.06%	6.26%

formation to the forecasting procedure of the output gaps and CPI inflation rates, the macro variables we are mostly interested in, so that those could be predicted more accurately. These economic interrelations will be directly captured by movements in the TOT and indirectly within the remaining variables. As the reader will see in Section 4.3, pairwise Granger causality tests already suggest that indeed preceding information on some variables of one country should help to improve forecasting the variables of central interest of the other.

As we will further see in Section 4.3, both Austrian and Hungarian variables are not only positively correlated with one another, but are *both* strongly correlated with the corresponding Euro area variables. Thus, distinctly Austrian time series cannot simply be taken as proxies for Euro area time series.

For the short-run nominal interest rate and the currency of Austria we have to rely on Euro area variables, anyway. By doing that, we can account for two things. First, we can mitigate an omitted variable bias possibly caused by completely neglecting the influence of Euro area variables, especially on the Austrian economy. Second, by including Euro area variables we are able to capture some sort of *rest of the world*, which the reader may expect to encounter in a typical small-open-economy analysis.

In addition to the different variants of the two-country DSGE model to be estimated and forecasted with, there are some more differences to Chapter 3. We not only treat a different pair of countries and, hence, use a different dataset, but also employ a different calibration, which is based on BREUSS & RABITSCH (2009) and is more likely

to resemble plausible values for the structural parameters of the Austrian economy. This time, we will concentrate on measuring forecasting accuracy with respect to rival models. Since we only possess a small sample, we will keep the observations during the economic and financial crisis for estimation and forecasting. Despite possible caveats arising from this procedure, it may be interesting to see how well the variants of the DSGE model are able to forecast in times of economic turmoil.

The main results of the analysis based on quarterly Eurostat and OECD data ranging from 2000Q1 until 2009Q3 are summarized subsequently.

Bayesian and classically estimated (vector) autoregressive benchmarks deliver the most accurate one-step-ahead forecasts in terms of the RMSFE for Austrian and Hungarian output gaps and CPI inflation rates with respect to the different variants of the two-country DSGE model. However, the benchmarks cannot significantly outperform the DSGE models. For three out of four variables (Austrian and Hungarian output gaps, Hungarian CPI inflation) open-economy models perform best with respect to other single forecasts.

If we additionally calculate various uniformly combined forecasts, again for three out of four variables (Austrian and Hungarian output gaps, Austrian CPI inflation) open-economy forecast combinations perform best with respect to other combined forecasts. In case of Austrian and Hungarian CPI inflation rates, the combined forecasts perform better than their best single forecasts, where these two combined forecasts also incorporate single DSGE forecasts.

Hence, we conclude that even if single DSGE forecasts were not able to deliver the most accurate one-step-ahead forecasts, the additional information provided by these forecasting models seems to be valuable for uniform forecast combination in case of CPI inflation rates across countries. Since open-economy models deliver the lowest RMSFE for three out of four variables across single and combined forecasts, taking into account the non-negligible impact of economic interrelations between Austria and Hungary indeed leads to a more accurate prediction of most of their macro variables. As a consequence, applying the same forecasting models used here to calculate combined forecasts of other pairs of *regions* within the EU or the US may be appealing for future research.

The rest of this chapter is structured as follows. Section 4.2 briefly outlines the two-country DSGE model and introduces two new variants. Section 4.3 describes the quarterly Eurostat and OECD data ranging from 2000Q1 until 2009Q3 to be used for estimation and forecasting. Section 4.4 proceeds with calibration and prior distributions of the model parameters. Section 4.5 compares the forecast accuracy in terms

of the RMSFE of out-of-sample one-step-ahead forecasts based on the variants of the model with respect to several (B)(V)AR benchmarks and various uniformly combined forecasts. Finally, Section 4.6 concludes. Additional tables are given in Appendix 4.7.

4.2 New variants of the DSGE model

For the reader's convenience, we once again state the two-country DSGE model from Chapters 2 and 3 in the following. Differing from the previous analysis and as mentioned in Section 4.1, we now turn attention to the forecast accuracy of CPI inflation of both countries under scrutiny. Therefore, we have to include equations (4.3) and (4.4), which correspond to equations (2.54) and (2.55) from Chapter 2. e, e^* again denote stationary AR(1) measurement errors in spirit of ADOLFSON et al. (2007) as already introduced for the TOT in Chapter 3.

The domestic country in the present case corresponds to Austria and the foreign country, whose variables are indexed by superscript asterisks, corresponds to Hungary. Besides that, all variables, parameters, and macroeconomic shocks are again defined as in Chapters 2 and 3. One difference to the previous analysis is that from here onwards we do not impose the a-priori restriction that the Austrian National Bank does not react at all to the Austrian output gap, as can be seen in equation (4.7) by $\iota > 0$.

$$x_t = E_t[x_{t+1}] + \frac{1}{\rho}\{E_t[\pi_{t+1,H}] - \hat{i}_t\} + \frac{(n-1)(\xi\rho-\xi)}{(\xi+\rho)\rho}E_t[\Delta t_{t+1}] + \frac{\xi+1}{\xi+\rho}E_t[\Delta a_{t+1}], \quad (4.1)$$

$$x_t^* = E_t[x_{t+1}^*] + \frac{1}{\rho}\{E_t[\pi_{t+1,F}^*] - \hat{i}_t^*\} + \frac{n(\xi\rho-\xi)}{(\xi+\rho)\rho}E_t[\Delta t_{t+1}] + \frac{\xi+1}{\xi+\rho}E_t[\Delta a_{t+1}^*], \quad (4.2)$$

$$\pi_t = \pi_{t,H} - (n-1)\Delta t_t + e_t, \quad (4.3)$$

$$\pi_t^* = \pi_{t,F}^* - n\Delta t_t + e_t^*, \quad (4.4)$$

$$\pi_{t,H} = \beta E_t[\pi_{t+1,H}] + \frac{(1-\delta)(1-\delta\beta)(\rho+\xi)}{\delta}x_t + u_t, \quad (4.5)$$

$$\pi_{t,F}^* = \beta E_t[\pi_{t+1,F}^*] + \frac{(1-\delta^*)(1-\delta^*\beta)(\rho+\xi)}{\delta^*}x_t^* + u_t^*, \quad (4.6)$$

$$\hat{i}_t = \alpha\pi_{t,H} + \iota x_t + \omega\hat{i}_{t-1} + v_t, \quad (4.7)$$

$$\hat{i}_t^* = \alpha^*\pi_{t,F}^* + \iota^* x_t^* + \omega^*\hat{i}_{t-1}^* + v_t^*, \quad (4.8)$$

$$\Delta t_t = \hat{i}_{t-1} - \hat{i}_{t-1}^* + \pi_{t,F}^* - \pi_{t,H} + d_t. \quad (4.9)$$

In addition to the original model above, we introduce two new variants for estimation and measurement of forecast accuracy.

First, we want to replace PPI by CPI inflation in the interest-rate rules (4.7) and (4.8) and hence employ a CITR rather than a DITR (see Chapter 2 or GALÍ & MONA-CELLI 2005, p. 723). Since these equations stem from ad-hoc assumptions anyway, a change in those variables does not alter the consistency of the rest of the model.

Hence, for a CITR, equations (4.7) and (4.8) rearrange as follows:

$$\hat{i}_t = \alpha \pi_t + \iota x_t + \omega \hat{i}_{t-1} + v_t,$$
$$\hat{i}_t^* = \alpha^* \pi_t^* + \iota^* x_t^* + \omega^* \hat{i}_{t-1}^* + v_t^*.$$

Second, we notice that similar to CLARIDA et al. (2001) the closed-economy model of Austria or Hungary, respectively, is incorporated in (4.1) to (4.9): if we set the country size $n = 1$, the TOT effect in equation (4.1) disappears and the shock-free equation (4.3) implies $\pi = \pi_H$.

Hence, for the closed economy the above model collapses to equations (4.1), (4.5), and (4.7) once we have replaced PPI with CPI inflation:

$$x_t = E_t[x_{t+1}] + \frac{1}{\rho}\{E_t[\pi_{t+1}] - \hat{i}_t\} + \frac{\xi + 1}{\xi + \rho} E_t[\Delta a_{t+1}],$$
$$\pi_t = \beta E_t[\pi_{t+1}] + \frac{(1 - \delta)(1 - \delta\beta)(\rho + \xi)}{\delta} x_t + u_t,$$
$$\hat{i}_t = \alpha \pi_t + \iota x_t + \omega \hat{i}_{t-1} + v_t.$$

Altogether, we can now estimate and forecast with three different variants or four different models.

- AUT/HUN_{PPI},

- AUT/HUN_{HCPI},

- AUT_{HCPI}, and HUN_{HCPI}.

4.3 Quarterly data for Austria and Hungary

Our sample of quarterly data for Austria and Hungary for the nine endogenous variables ranges from 2000Q1 to 2009Q3 and only comprises 39 observations. The shortness of the sample is due to the availability of PPI inflation rates for the Austrian economy. The data are either taken from Eurostat (2010) (output gaps) or from OECD (2010) (all other variables). More precisely, we use the subsequent time series.

- The Austrian and Hungarian output gaps x, x^* are calculated from the natural logarithm of the actual volume at constant prices of the respective seasonally and

working-day adjusted GDP minus its Hodrick-Prescott filtered value.

- The Austrian and Hungarian CPI inflation rates π, π^* are given by the percentage change to the previous period of the harmonized consumer price indices.

- The Austrian and Hungarian PPI inflation rates π_H, π_F^* are proxied by the percentage change to the previous period of the domestic producer price indices in manufacturing. We again restrict ourselves to these prices since the model still assumes that firms employ producer currency pricing and that only final goods are produced and traded.

- The Austrian and Hungarian National Banks' short-run nominal interest rates in per cent per annum i, i^* are proxied by the subsequent times series: the three-month EURIBOR serving as official successor of the Vienna Interbank Offered Rate (VIBOR) since January 1999 in case of i and the interest rate on unsecured Hungarian Forint interbank lending transactions of three months' duration in case of i^*. Both interest rates have to be subtracted by their common annualized zero-inflation steady-state value ≈ 0.04 before entering the DSGE estimation in order to obtain the desired percentage-point deviations $\hat{i}, \hat{i}^*$ (see Section 4.4).

- Again, we have to calculate the TOT Δt ourselves, which is done by employing the subsequent formula $\Delta t_t = \Delta s_t + \pi_{t,F}^* - \pi_{t,H}$ (see Chapters 2 and 3), where Δs_t denotes the first difference of the natural logarithm of the monthly average of the nominal exchange rate of Euro per Hungarian Forint.

Note that we are aware that the economic and financial crisis from 2007Q2 onwards has affected Austria and Hungary, too. Differing from Chapter 3, however, we decide to keep the observations after 2007Q1 in our sample because it already is very short. Despite possible caveats arising from this procedure, it may be interesting to see how well the variants of the DSGE model AUT/HUN_{PPI}, AUT/HUN_{HCPI}, AUT_{HCPI}, and HUN_{HCPI} are able to forecast with respect to various benchmarks in times of economic turmoil.

Figures 4.1 and 4.2 below plot the variables of central interest for this chapter: Austrian and Hungarian output gaps as well as CPI inflation rates. If we investigate Figure 4.1, we can conclude that due to the economic and financial crisis, both the Austrian

(since 2009Q1) and the Hungarian (since 2008Q4) output gaps have been negative. Taking a look at Figure 4.2, one can immediately see that Hungarian CPI inflation has been notably higher and more volatile than its Austrian equivalent throughout the sample and that CPI inflation in both countries follows some seasonal pattern.
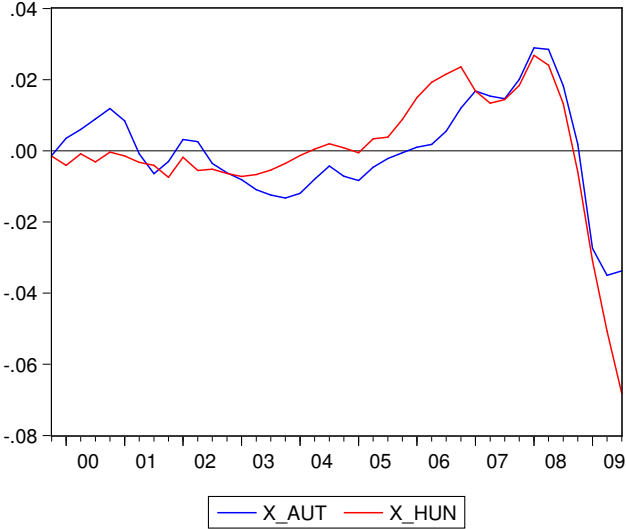

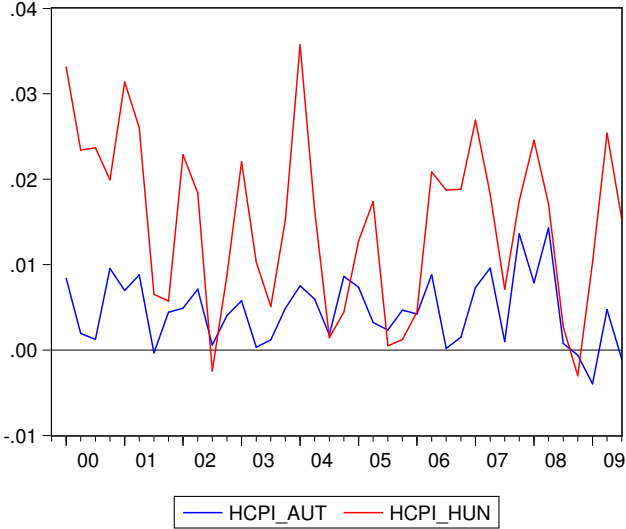Figure 4.1: Austrian and Hungarian output gaps

Figure 4.2: Austrian and Hungarian CPI inflation rates

As can be seen from Table 4.7 in Appendix 4.7, pairwise Granger causality tests with four lags reject the null hypothesis of no Granger causality for the subsequent relationships (at least at the 10% significance level): $x^* \rightarrow x, x \rightarrow \pi, x \rightarrow \hat{i}, \Delta t \rightarrow x, x^* \rightarrow \pi, x^* \rightarrow \pi_H, x^* \rightarrow \pi_F^*, x^* \rightarrow \hat{i}, \pi^* \rightarrow \pi, \pi \rightarrow \hat{i}, \Delta t \leftrightarrow \pi^*, \hat{i} \rightarrow \pi_H, \Delta t \rightarrow \hat{i}^*$ with $\leftrightarrow$ denoting a mutual causality, where cause and effect cannot be clearly distinguished. Hence, the impact of Austrian and Hungarian macro variables on one another indeed seems to be non-negligible in at least eight cases.

Tables 4.8 to 4.11 in Appendix 4.7 further show that not only Austrian and Hungarian variables are positively correlated with one another, but that both are strongly correlated with their Euro area equivalents. The positive correlation of Hungarian with Euro area variables is most prominent for the output gaps and PPI inflation rates, but is also present for CPI inflation and nominal interest rates. In the case of Austrian variables, the correlation with Euro area variables is generally very high. The correlation between Austrian and Euro area nominal interest rates, however, is perfect because we employ the same time series.

Since Hungarian variables themselves are often highly correlated with Euro area variables, Austrian times series cannot be simply interpreted as proxies for Euro area time series. Nonetheless, the use of Euro area time series for some of the Austrian variables (Euro area nominal interest rate and the nominal exchange rate of Euro per Hungarian Forint to calculate the TOT) should mitigate an omitted variable bias, which we would possibly face when completely ignoring Euro area variables.

4.4 Calibration and estimation

Before we assess the forecasting performance of the four models, we first have to calibrate and estimate them using the same Bayesian techniques as laid out in Chapter 3.[2] The calibration itself, however, will be somewhat different and is based on BREUSS & RABITSCH (2009, pp. 139-144) for both countries. Hence, $\alpha = \alpha^*$ etc. Concerning the choice of fixed parameter values as well as prior means and standard deviations for their DSGE model of Austria, these authors are guided by previous studies on New Keynesian models, e.g. SMETS & WOUTERS (2003).

In order to tie in with most of the existing literature we choose to keep those parameters fixed that are said to be weakly identified, i.e. the parameters stemming from the utility function (2.1): $\beta = 0.99, \rho = 0.67, \xi = 1.5$. As before, $\beta = 0.99$

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[2]Note that again all computations associated with Bayesian inference are again carried out using the DYNARE preprocessor for MATLAB. All other computations are performed in EVIEWS.

implies zero-inflation steady-state nominal and real interest rates of approximately 4%
per annum for both countries. Since country size is not an economic parameter, we
fix it to $n = 0.45$, which roughly corresponds to the population of Austria relative to
Hungary in 2008 if the world were just made up of these two countries (Austria: approx.
8.3 million inhabitants, Hungary: approx. 10.0 million inhabitants, see OECD 2010).
The sensitivity of the central banks towards the current output gaps is also fixed to
$\iota = \iota^* = 0.2$ since we only possess a small sample. In consequence, we use the prior
PDFs given in Table 4.2 below for the remaining parameters according to their domain
as implied by economic theory (see Chapter 3).

Table 4.2: Prior information

Parameter	Domain	Prior PDF	Mean	Std. Dev.
δ	$[0, 1)$	Beta	0.6	0.15
α	$(-\infty, +\infty)$	Normal	1.7	0.15
ω	$[0, 1)$	Beta	0.8	0.05
ζ_a	$[0, 1)$	Beta	0.8	0.05
ζ_e	$[0, 1)$	Beta	0.8	0.05
ζ_u	$[0, 1)$	Beta	0.8	0.05
ζ_v	$[0, 1)$	Beta	0.8	0.05
ζ_d	$[0, 1)$	Beta	0.8	0.05
σ_{η_a}	$[0, +\infty)$	Inv. Gamma-1	0.1	$+\infty$
σ_{η_e}	$[0, +\infty)$	Inv. Gamma-1	0.1	$+\infty$
σ_{η_u}	$[0, +\infty)$	Inv. Gamma-1	0.1	$+\infty$
σ_{η_v}	$[0, +\infty)$	Inv. Gamma-1	0.1	$+\infty$
σ_{η_d}	$[0, +\infty)$	Inv. Gamma-1	0.1	$+\infty$

Bayesian estimation is carried out by employing the MH algorithm. The only dif-
ference to Chapter 3 is that we have to set `mh_jscale=0.1` in case of AUT/HUN_{HCPI}
to obtain a reasonable acceptance probability of 25% for each of the runs of MH simu-
lations.[3]

4.5 Measuring forecast accuracy

Differing from Chapter 3, we are only interested in forecasting the output gaps x, x^*
and CPI inflation rates π, π^*. We evaluate the forecasting performance of the one-step-

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[3]Estimation results (posterior means and standard deviations, graphs of posterior PDFs) and con-
vergence results of the five parallel MCMCs for each of the models comparable to Chapter 3 are not
reported here but are available on request.

ahead predictor for 2008Q4, 2009Q1, 2009Q2, and 2009Q3 so that each of the four training sets (2000Q1-2008Q3, 2000Q2-2008Q4, 2000Q3-2009Q1, and 2000Q4-2009Q2) comprises 35 observations.

Besides the four DSGE models AUT/HUN_{PPI}, AUT/HUN_{HCPI}, AUT_{HCPI}, and HUN_{HCPI}, we will estimate and forecast with several multivariate (B)VAR and also univariate AR benchmarks with ad-hoc lag length of $p = 1$ (see, e.g., SMETS & WOUTERS 2004, p. 847, ADOLFSON et al. 2007, p. 309, or PICHLER 2008, p. 19 for other articles using (B)VAR benchmarks with ad-hoc lag order).[4] Since we want to investigate whether taking into account the economic interrelations between the two countries can improve the predictive accuracy of their output gaps and CPI inflation rates, we consistently employ closed- and open-economy (vector) autoregressive benchmarks as done, e.g., by ADOLFSON et al. (2007, p. 306).

We distinguish the forecasts based on classically estimated VARs using OLS from those that are estimated in a Bayesian fashion since the latter may constitute a more *natural* benchmark for DSGEs as they use similar estimation techniques. Moreover, unrestricted VARs may be overparameterized and, hence, may perform poorly in out-of-sample forecasting (see SMETS & WOUTERS 2007, p. 595). For BVAR estimation and forecasting we employ the Minnesota prior, an informative prior developed by DOAN et al. (1984) for an otherwise unconstrained VAR with intercept. For further details on the Minnesota prior see, e.g., BAUWENS et al. (1999, pp. 269-272).

Thus, we employ the subsequent closed- and open-economy benchmarks (all with lag order $p = 1$).

- $AR(x)$, $AR(x^*)$, $AR(\pi)$, $AR(\pi^*)$,

- $(B)VAR(x, \pi)$, $(B)VAR(x^*, \pi^*)$,

- $(B)VAR(x, \pi, \pi_H)$, $(B)VAR(x^*, \pi^*, \pi_F^*)$,

- $(B)VAR(x, \pi, \hat{i})$, $(B)VAR(x^*, \pi^*, \hat{i}^*)$,

- $(B)VAR(x, x^*, \pi, \pi^*)$,

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[4]For the generic VAR(p) model see equation (3.15) in Chapter 3.

- $(B)VAR(x, x^*, \pi, \pi^*, \pi_H, \pi_F^*)$, and

- $(B)VAR(x, x^*, \pi, \pi^*, \hat{i}, \hat{i}^*)$.

Since the MAFE (usually) delivers qualitatively similar results as the RMSFE (see Chapter 3), this time we concentrate on the RMSFE as the only quantitative measure of forecast accuracy. Results of single forecasts are summarized in Table 4.3 below, where the smallest RMSFE values for the single variables are given in boldface numbers.

Table 4.3: RMSFE of single forecasts

Model	x	x^*	π	π^*
AUT/HUN_{PPI}	0.0182	0.0240	0.0420	0.0258
AUT/HUN_{HCPI}	0.1055	0.1579	0.0424	0.0283
AUT_{HCPI}, HUN_{HCPI}	0.0178	0.0239	0.0050	0.0166
$AR(x), AR(x^*), AR(\pi), AR(\pi^*)$	0.0174	0.0204	**0.0045**	0.0113
$VAR(x, \pi), VAR(x^*, \pi^*)$	0.0179	0.0204	0.0071	0.0102
$VAR(x, \pi, \pi_H), VAR(x^*, \pi^*, \pi_F^*)$	0.0176	0.0201	0.0074	0.0099
$VAR(x, \pi, \hat{i}), VAR(x^*, \pi^*, \hat{i}^*)$	0.0167	0.0210	0.0069	0.0129
$VAR(x, x^*, \pi, \pi^*)$	0.0161	**0.0180**	0.0071	0.0107
$VAR(x, x^*, \pi, \pi^*, \pi_H, \pi_F^*)$	0.0167	0.0181	0.0076	0.0099
$VAR(x, x^*, \pi, \pi^*, \hat{i}, \hat{i}^*)$	0.0161	0.0185	0.0071	0.0137
$BVAR(x, \pi), BVAR(x^*, \pi^*)$	0.0177	0.0205	0.0059	0.0109
$BVAR(x, \pi, \pi_H), BVAR(x^*, \pi^*, \pi_F^*)$	0.0170	0.0200	0.0065	0.0115
$BVAR(x, \pi, \hat{i}), BVAR(x^*, \pi^*, \hat{i}^*)$	0.0172	0.0212	0.0058	0.0129
$BVAR(x, x^*, \pi, \pi^*)$	0.0160	0.0208	0.0065	0.0107
$BVAR(x, x^*, \pi, \pi^*, \pi_H, \pi_F^*)$	**0.0156**	0.0201	0.0066	0.0104
$BVAR(x, x^*, \pi, \pi^*, \hat{i}, \hat{i}^*)$	0.0161	0.0217	0.0073	**0.0095**

Regarding single forecasts, we can conclude that $BVAR(x, x^*, \pi, \pi^*, \pi_H, \pi_F^*)$ delivers the highest forecast accuracy for x, $VAR(x, x^*, \pi, \pi^*)$ for x^*, $AR(\pi)$ for π, and $BVAR(x, x^*, \pi, \pi^*, \hat{i}, \hat{i}^*)$ for π^*. Thus, Bayesian and classically estimated benchmarks outperform the DSGE models in two cases each. The DSGE AUT/HUN_{HCPI} delivers particularly high RMSFEs for the forecasts of the Austrian and Hungarian output gaps. However, for three out of four variables, open-economy models are preferred over closed-economy models.

In order to check whether the forecasting model characterized by the lowest RMSFE per variable can significantly outperform the four DSGE models, we again employ DM

tests (see Chapter 3). Significance at the 10% level would again be indicated by $(^*)$, at the 5% level by $(^{**})$, and at the 1% level by $(^{***})$. Nonetheless, as we see from Table 4.4, none of the autoregressive benchmarks delivers significantly more accurate forecasts than the four DSGE models, not even with respect to AUT/HUN_{HCPI}.

Table 4.4: DM test statistics of single forecasts

Variable	Models under comparison	DM test statistic	One-sided p-value
x	$BVAR(x,x^*,\pi,\pi^*,\pi_H,\pi_F^*)$ vs. AUT/HUN_{PPI}	-0.6744	0.2500
	$BVAR(x,x^*,\pi,\pi^*,\pi_H,\pi_F^*)$ vs. AUT/HUN_{HCPI}	-0.9167	0.1798
	$BVAR(x,x^*,\pi,\pi^*,\pi_H,\pi_F^*)$ vs. AUT_{HCPI}	-0.6289	0.2647
x^*	$VAR(x,x^*,\pi,\pi^*)$ vs. AUT/HUN_{PPI}	-1.0141	0.1553
	$VAR(x,x^*,\pi,\pi^*)$ vs. AUT/HUN_{HCPI}	-0.7749	0.2192
	$VAR(x,x^*,\pi,\pi^*)$ vs. HUN_{HCPI}	-0.9989	0.1589
π	$AR(\pi)$ vs. AUT/HUN_{PPI}	-1.0153	0.1550
	$AR(\pi)$ vs. AUT/HUN_{HCPI}	-1.1024	0.1351
	$AR(\pi)$ vs. AUT_{HCPI}	-0.1137	0.4547
π^*	$BVAR(x,x^*,\pi,\pi^*,\hat{i},\hat{i}^*)$ vs. AUT/HUN_{PPI}	-0.7923	0.2141
	$BVAR(x,x^*,\pi,\pi^*,\hat{i},\hat{i}^*)$ vs. AUT/HUN_{HCPI}	-0.6771	0.2492
	$BVAR(x,x^*,\pi,\pi^*,\hat{i},\hat{i}^*)$ vs. HUN_{HCPI}	-0.6245	0.2661

Similar to Chapter 3, we calculate the RMSFE of various uniformly combined forecasts. Besides a combined forecast across all forecasting models, we calculate combined forecasts across DSGE models, (V)AR benchmarks, BVAR benchmarks, closed-economy models, and open-economy models.

- $COMB_{ALL}$,

- $COMB_{DSGE}$, $COMB_{VAR}$, $COMB_{BVAR}$,

- $COMB_{CLOSED}$, and $COMB_{OPEN}$.

Results of combined forecasts are given in Table 4.5 below, where the smallest RMSFE values for the single variables are again given in boldface numbers.

Observing Table 4.5 we can conclude that with $COMB_{BVAR}$, $COMB_{VAR}$, and $COMB_{ALL}$ for three out of four variables (x, x^*, π) combinations including open-economy forecasts are preferred. Only for π^* the purely closed-economy combined forecast $COMB_{CLOSED}$ outperforms the other combinations. Moreover, with $COMB_{ALL}$ and

Table 4.5: RMSFE of combined forecasts

Model	x	x^*	π	π^*
$COMB_{ALL}$	0.0184	0.0202	**0.0038**	0.0107
$COMB_{DSGE}$	0.0406	0.0525	0.0269	0.0143
$COMB_{VAR}$	0.0166	**0.0195**	0.0066	0.0108
$COMB_{BVAR}$	**0.0166**	0.0207	0.0062	0.0109
$COMB_{CLOSED}$	0.0173	0.0208	0.0056	**0.0094**
$COMB_{OPEN}$	0.0209	0.0240	0.0078	0.0130

$COMB_{CLOSED}$ two combinations incorporating single DSGE forecasts deliver the lowest RMSFEs among combined forecasts for Austrian and Hungarian CPI inflation rates. Their RMSFEs are even lower than the ones of the single forecasts. Since single DSGE forecasts are not significantly worse than the single most accurate forecasts per variable, the additional information provided by the DSGEs as forecasting models seems to be valuable for uniform forecast combination in case of CPI inflation rates across countries.

Table 4.6 once again summarizes the results, where boldface letters denote the most accurate model type across single and combined forecasts.

Table 4.6: Model type with lowest RMSFE across forecast types

Forecast type	x	x^*	π	π^*
Single forecast	**Open-economy**	**Open-economy**	Closed-economy	Open-economy
Combined forecast	Open-economy	Open-economy	**Open-economy** (including DSGE forecasts)	**Closed-economy** (including DSGE forecasts)

4.6 Concluding remarks

In summary, Bayesian and classically estimated (vector) autoregressive benchmarks deliver the most accurate one-step-ahead forecasts in terms of the RMSFE for Austrian

and Hungarian output gaps and CPI inflation rates with respect to the different variants of the two-country DSGE model. However, the benchmarks cannot significantly outperform the DSGE models. For three out of four variables (Austrian and Hungarian output gaps, Hungarian CPI inflation) open-economy models perform best with respect to other single forecasts.

If we additionally calculate various uniformly combined forecasts, again for three out of four variables (Austrian and Hungarian output gaps, Austrian CPI inflation) open-economy forecast combinations perform best with respect to other combined forecasts. In case of Austrian and Hungarian CPI inflation rates, the combined forecasts perform better than their best single forecasts, where these two combined forecasts also incorporate single DSGE forecasts.

Hence, we conclude that even if single DSGE forecasts were not able to deliver the most accurate one-step-ahead forecasts, the additional information provided by these forecasting models seems to be valuable for uniform forecast combination in case of CPI inflation rates across countries. Since open-economy models deliver the lowest RMSFE for three out of four variables across single and combined forecasts, taking into account the non-negligible impact of economic interrelations between Austria and Hungary indeed leads to a more accurate prediction of most of their macro variables. As a consequence, applying the same forecasting models used here to calculate combined forecasts of other pairs of *regions* within the EU or the US may be appealing for future research.

4.7 Appendix to Chapter 4

Table 4.7: Pairwise Granger causality tests

Null Hypothesis:	Obs	F-Statistic	Prob.
X_HUN does not Granger Cause X_AUT	35	2.96761	0.0382
X_AUT does not Granger Cause X_HUN		1.03371	0.4087
HCPI_AUT does not Granger Cause X_AUT	35	0.23210	0.9178
X_AUT does not Granger Cause HCPI_AUT		2.74571	0.0498
HCPI_HUN does not Granger Cause X_AUT	35	0.06997	0.9905
X_AUT does not Granger Cause HCPI_HUN		0.23346	0.9170
PPI_AUT does not Granger Cause X_AUT	35	0.58883	0.6736
X_AUT does not Granger Cause PPI_AUT		1.61914	0.1994
PPI_HUN does not Granger Cause X_AUT	35	0.87749	0.4909
X_AUT does not Granger Cause PPI_HUN		0.86591	0.4975
I_AUT does not Granger Cause X_AUT	35	0.36359	0.8322
X_AUT does not Granger Cause I_AUT		4.33248	0.0081
I_HUN does not Granger Cause X_AUT	35	1.50376	0.2301
X_AUT does not Granger Cause I_HUN		0.71984	0.5862
TOT does not Granger Cause X_AUT	35	3.00933	0.0364
X_AUT does not Granger Cause TOT		1.32083	0.2885
HCPI_AUT does not Granger Cause X_HUN	35	0.55628	0.6963
X_HUN does not Granger Cause HCPI_AUT		7.77885	0.0003
HCPI_HUN does not Granger Cause X_HUN	35	0.16799	0.9527
X_HUN does not Granger Cause HCPI_HUN		0.34927	0.8421
PPI_AUT does not Granger Cause X_HUN	35	0.61837	0.6534
X_HUN does not Granger Cause PPI_AUT		2.87582	0.0426
PPI_HUN does not Granger Cause X_HUN	35	1.21646	0.3278
X_HUN does not Granger Cause PPI_HUN		3.38574	0.0234
I_AUT does not Granger Cause X_HUN	35	1.55229	0.2166
X_HUN does not Granger Cause I_AUT		3.56339	0.0191
I_HUN does not Granger Cause X_HUN	35	0.67583	0.6148
X_HUN does not Granger Cause I_HUN		0.15053	0.9611
TOT does not Granger Cause X_HUN	35	0.53434	0.7117
X_HUN does not Granger Cause TOT		0.96118	0.4453
HCPI_HUN does not Granger Cause HCPI_AUT	35	2.97913	0.0377
HCPI_AUT does not Granger Cause HCPI_HUN		1.25700	0.3120
PPI_AUT does not Granger Cause HCPI_AUT	35	1.08477	0.3845
HCPI_AUT does not Granger Cause PPI_AUT		2.07903	0.1125
PPI_HUN does not Granger Cause HCPI_AUT	35	1.99800	0.1244
HCPI_AUT does not Granger Cause PPI_HUN		1.90893	0.1390
I_AUT does not Granger Cause HCPI_AUT	35	1.26433	0.3092
HCPI_AUT does not Granger Cause I_AUT		2.24004	0.0922
I_HUN does not Granger Cause HCPI_AUT	35	0.24259	0.9115
HCPI_AUT does not Granger Cause I_HUN		0.80910	0.5307
TOT does not Granger Cause HCPI_AUT	35	0.22060	0.9245
HCPI_AUT does not Granger Cause TOT		0.97802	0.4366
PPI_AUT does not Granger Cause HCPI_HUN	35	0.22803	0.9202
HCPI_HUN does not Granger Cause PPI_AUT		0.76227	0.5593
PPI_HUN does not Granger Cause HCPI_HUN	35	0.92343	0.4654
HCPI_HUN does not Granger Cause PPI_HUN		1.08073	0.3863
I_AUT does not Granger Cause HCPI_HUN	35	0.73397	0.5771
HCPI_HUN does not Granger Cause I_AUT		1.04150	0.4049

I_HUN does not Granger Cause HCPI_HUN	35	0.90426	0.4759
HCPI_HUN does not Granger Cause I_HUN		0.36069	0.8342
TOT does not Granger Cause HCPI_HUN	35	3.54359	0.0195
HCPI_HUN does not Granger Cause TOT		4.44540	0.0072
PPI_HUN does not Granger Cause PPI_AUT	35	1.49548	0.2325
PPI_AUT does not Granger Cause PPI_HUN		0.15134	0.9607
I_AUT does not Granger Cause PPI_AUT	35	3.42068	0.0225
PPI_AUT does not Granger Cause I_AUT		1.91515	0.1379
I_HUN does not Granger Cause PPI_AUT	35	0.74098	0.5727
PPI_AUT does not Granger Cause I_HUN		1.01166	0.4195
TOT does not Granger Cause PPI_AUT	35	1.70936	0.1782
PPI_AUT does not Granger Cause TOT		0.41357	0.7972
I_AUT does not Granger Cause PPI_HUN	35	2.06504	0.1145
PPI_HUN does not Granger Cause I_AUT		2.15964	0.1018
I_HUN does not Granger Cause PPI_HUN	35	0.65340	0.6297
PPI_HUN does not Granger Cause I_HUN		1.00472	0.4230
TOT does not Granger Cause PPI_HUN	35	0.42216	0.7912
PPI_HUN does not Granger Cause TOT		0.80469	0.5334
I_HUN does not Granger Cause I_AUT	35	0.42295	0.7906
I_AUT does not Granger Cause I_HUN		1.97327	0.1283
TOT does not Granger Cause I_AUT	35	1.95685	0.1309
I_AUT does not Granger Cause TOT		0.60899	0.6598
TOT does not Granger Cause I_HUN	35	2.40672	0.0752
I_HUN does not Granger Cause TOT		0.46917	0.7578

Table 4.8: Correlation of output gaps

	x	x^{EURO}	x^*
x	1.000000	0.805056	0.782258
x^{EURO}	0.805056	1.000000	0.689072
x^*	0.782258	0.689072	1.000000

Table 4.9: Correlation of CPI inflation rates

	π	π^{EURO}	π^*
π	1.000000	0.728417	0.488245
π^{EURO}	0.728417	1.000000	0.394072
π^*	0.488245	0.394072	1.000000

Table 4.10: Correlation of PPI inflation rates

	π_H	π_{PPI}^{EURO}	π_F^*
π_H	1.000000	0.857666	0.614582
π_{PPI}^{EURO}	0.857666	1.000000	0.701888
π_F^*	0.614582	0.701888	1.000000

Table 4.11: Correlation of nominal interest rates

	$\hat{i}$	$\hat{i}^{EURO}$	$\hat{i}^*$
$\hat{i}$	1.000000	1.000000	0.554328
$\hat{i}^{EURO}$	1.000000	1.000000	0.554328
$\hat{i}^*$	0.554328	0.554328	1.000000

References

Adjemian, S. (2007): Prior distributions in Dynare. Mimeo.
URL: http://www.cepremap.cnrs.fr/ãdjemian/r/dynare/pdf/Distributions.pdf.

Adolfson, M., Lindé, J., and M. Villani (2007): Forecasting Performance of an Open Economy DSGE Model. Econometric Reviews, Vol. 26, pp. 289–328.

An, S. and F. Schorfheide (2007): Bayesian Analysis of DSGE Models. Econometric Reviews, Vol. 26, pp. 113–172.

Bauwens, L., Lubrano, M., and J. F. Richard (1999): Bayesian Inference in Dynamic Econometric Models. Advanced Texts in Econometrics. Oxford University Press. Oxford and New York.

Benigno, G. and P. Benigno (2006): Designing targeting rules for international monetary policy cooperation. Journal of Monetary Economics, Vol. 53, pp. 473—506.

Board of Governors of the Federal Reserve System (1977, 2000): Federal Reserve Act, Section 2a. 12 USC 225a. As added by act of November 16, 1977 (91 Stat. 1387) and amended by acts of October 27, 1978 (92 Stat. 1897); Aug. 23, 1988 (102 Stat. 1375); and Dec. 27, 2000 (114 Stat. 3028).

Breuss, F. and K. Rabitsch (2009): An estimated two-country DSGE model of Austria and the Euro Area. Empirica, Vol. 36, pp. 123–158.

Brock, W. A. (1974): Money and Growth: The Case of Long Run Perfect Foresight. International Economic Review, Vol. 15, pp. 750—777.

Bullard, J. and K. Mitra (2002): Learning about monetary policy rules. Journal of Monetary Economics, Vol. 49, pp. 1105—1129.

Calvo, G. A. (1983): Staggered prices in a utility-maximizing framework. Journal of Monetary Economics, Vol. 12, pp. 383–398.

Chatfield, C. (2001): Time-Series Forecasting. Chapman & Hall. Boca Raton, London, New York, and Washington, D.C.

Christiano, L. J., Eichenbaum, M., and C. Evans (2005): Nominal rigidities and the dynamic effects of a shock to monetary policy. Journal of Political Economy, Vol. 113, pp. 1–45.

Christiano, L. J., Trabandt, M., and K. Walentin (2009): Introducing Financial Frictions and Unemployment into a Small Open Economy Model. Sveriges Riksbank Working Paper Series No. 214.

Christoffel, K., Coenen, G., and A. Warne (2008): The New Area-Wide Model of the Euro Area: A Micro-Founded Open-Economy Model for Forecasting and Policy Analysis. ECB Working Paper Series No. 944.

Čihák, M., Harjes, T., and E. Stavrev (2009): Euro Area Monetary Policy in Uncharted Waters. IMF Working Paper No. 09185.

Clarida, R., Galí, J., and M. Gertler (1999): The Science of Monetary Policy: A New Keynesian Perspective. Journal of Economic Perspectives, Vol. 37, pp. 1661—1707.

Clarida, R., Galí, J., and M. Gertler (2001): Optimal Monetary Policy in Open versus Closed Economies: An Integrated Approach. American Economic Review, Vol. 91, pp. 248–252.

Clarida, R., Galí, J., and M. Gertler (2002): A simple framework for international monetary policy analysis. Journal of Monetary Economics, Vol. 49, pp. 879–904.

Corsetti, C. and P. Pesenti (2001): Welfare and Macroeconomic Interdependence. Quarterly Journal of Economics, Vol. 116, pp. 421–445.

Costantini, M., Gunter, U., and R. M. Kunst (2010): Forecast Combination Based on Multiple Encompassing Tests in a Macroeconomic DSGE System. Economics Series No. 251, Institute for Advanced Studies, Vienna.

Cuche-Curti, N. A., Dellas, H., and J. M. Natal (2009): DSGE-CH: A dynamic stochastic general equilibrium model for Switzerland. Swiss National Bank Economic Studies No. 5 2009.

DelNegro, M., Schorfheide, F., Smets, F., and R. Wouters (2005): On the Fit and Forecasting Performance of New-Keynesian Models. ECB Working Paper Series No. 491.

Diebold, F. X. and R. S. Mariano (1995): Comparing Predictive Accuracy. Journal of Business and Economic Statistics, Vol. 13, pp. 253–263.

Dixit, A. K. and J. E. Stiglitz (1977): Monopolistic Competition and Optimum Product Diversity. American Economic Review, Vol. 67, pp. 297—308.

Doan, T., Litterman, R. B., and C. A. Sims (1984): Forecasting and conditional projection under realistic prior distributions. Econometric Reviews, Vol. 3, pp. 1–100.

Engel, C. (2002): The Responsiveness of Consumer Prices to Exchange Rates And the Implications for Exchange-Rate Policy: A Survey Of a Few Recent New Open-Economy Macro Models. NBER Working Paper No. 8725.

Engel, C. (2009): Currency Misalignments and Optimal Monetary Policy: A Reexamination. NBER Working Paper No. 14829.

European System of Central Banks (1992, 2004): Protocol on the Statute of the European System of Central Banks and of the European Central Bank, Article 2. Protocol annexed to the Treaty establishing the European Community (OJ C 191, 29.7.1992, p. 68), as amended by the Treaty of Amsterdam (OJ C 340, 10.11.1997, p. 1), the Treaty of Nice (OJ C 80, 10.3.2001, p. 1), Council Decision 2003/223/EC (OJ L 83, 1.4.2003, p. 66) and the Act concerning the conditions of Accession of the Czech Republic, the Republic of Estonia, the Republic of Cyprus, the Republic of Latvia, the Republic of Lithuania, the Republic of Hungary, the Republic of Malta, the Republic of Poland, the Republic of Slovenia and the Slovak Republic and the adjustments to the Treaties on which the European Union is founded (OJ L 236, 23.9.2003, p. 33) – unofficial consolidated version.

Eurostat (2010): National accounts (including GDP).
URL: http://epp.eurostat.ec.europa.eu/portal/page/portal/national_accounts/introduction.

Galí, J. (2002): New Perspectives on Monetary Policy, Inflation, and the Business Cycle. NBER Working Paper No. 8767.

Galí, J. (2008): Monetary Policy, Inflation, and the Business Cycle. Princeton University Press. Princeton and Oxford.

Galí, J., Gertler, M., and J. D. López-Salido (2007): Markups, Gaps, and the Welfare Costs of Business Fluctuations. Review of Economics and Statistics, Vol. 89, pp. 44–59.

Galí, J. and T. Monacelli (2005): Optimal Monetary Policy and Exchange Rate Volatility in a Small Open Economy. Review of Economic Studies, Vol. 72, pp. 707–734.

Galí, J. and P. Rabanal (2004): Technology Shocks and Aggregate Fluctuations: How Well Does the Real Business Cycle Model Fit Postwar U.S. Data? IMF Working Paper No. 04234.

Harvey, D. I. and P. Newbold (2000): Tests for Multiple Forecast Encompassing. Journal of Applied Econometrics, Vol. 15, pp. 471—482.

Jakab, Z. M. and B. Világi (2008): An estimated DSGE model of the Hungarian economy (A magyar gazdaság egy becsült, sztochasztikus, dinamikus általános egyensúlyi modellje). Magyar Nemzeti Bank Working Papers 2008/9.

Kascha, C. and K. Mertens (2009). Business cycle analysis and VARMA models. Journal of Economic Dynamics and Control, Vol. 33, pp. 267–282.

Koop, G. (2003): Bayesian Econometrics. John Wiley & Sons. Chichester.

Kydland, F. E. and E. C. Prescott (1982): Time to Build and Aggregate Fluctuations. Econometrica, Vol. 50, pp. 1345–1370.

Landmann, O. and J. Jerger (1999): Beschäftigungstheorie. Springer. Berlin, Heidelberg, and New York.

Lane, P. R. (2001): The new open economy macroeconomics: a survey. Journal of International Economics, Vol. 54, pp. 235—266.

Leith, C. and J. Malley (2007): Estimated Open Economy New Keynesian Phillips Curves for the G7. Open Economies Review, Vol. 18, pp. 405-–426.

Lucas, R. E. (1976): Econometric Policy Evaluation: A Critique. Carnegie-Rochester Conference Series on Public Policy, Vol. 1, pp. 19—46.

Lütkepohl, H. (2005): New Introduction to Multiple Time Series Analysis. Springer. Berlin, Heidelberg, and New York.

Mancini-Griffoli, T. (2007): DYNARE User Guide. An introduction to the solution & estimation of DSGE models.
URL: http://www.cepremap.cnrs.fr/dynare/download/manual/Dynare_UserGuide_WebBeta.pdf.

McCallum, B. T. and E. Nelson (2001): Monetary Policy for an Open Economy: An Alternative Framework with Optimizing Agents and Sticky Prices. NBER Working Paper No. 8175.

Obstfeld, M. and K. Rogoff (1995): Exchange Rate Dynamics Redux. Journal of Political Economy, Vol. 103, pp. 624-–660.

Obstfeld, M. and K. Rogoff (1996): Foundations of International Macroeconomics. The MIT Press. Cambridge, MA and London.

Obstfeld, M. and K. Rogoff (2001): Risk and Exchange Rates. Conference paper in honor of Assaf Razin, Tel-Aviv University.

Organisation for Economic Co-Operation and Development (2008): OECD Economic Outlook No. 84.

Organisation for Economic Co-Operation and Development (2010): OECD Frequently Requested Statistics.
URL: http://www.oecd.org/document/15/0,3343,en_2649_33715_1873295_1_1_1_1,00.html.

Pappa, E. (2004): Do the ECB and the fed really need to cooperate? Optimal monetary policy in a two-country world. Journal of Monetary Economics, Vol. 51, pp. 753-–779.

Pichler, P. (2008): Forecasting with DSGE Models: The Role of Nonlinearities. The B.E. Journal of Macroeconomics, Vol. 8, Art. 20.

Prettner, K. and R. M. Kunst (2009): The Dynamic Interrelations between Unequal Neighbors: An Austro-German Case Study. Vienna Economics Papers No. 0913.

Rotemberg, J. and M. Woodford (1997): An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy. NBER Macroeconomics Annual, Vol. 12, pp. 297–346.

Rubaszek, M. and P. Skrzypczyński (2008): On the forecasting performance of a small-scale DSGE model. International Journal of Forecasting, Vol. 24, pp. 498—512.

Sahuc, J. G. and F. Smets (2008): Differences in Interest Rate Policy at the ECB and the Fed: An Investigation with a Medium-Scala DSGE Model. Journal of Money, Credit and Banking, Vol. 40, pp. 505–521.

Sargent, T. J. and N. Wallace (1975): 'Rational' Expectations, the Optimal Monetary Instrument, and the Optimal Money Supply Rule. Journal of Political Economy, Vol. 83, pp. 241—254.

Schmitt-Grohé, S. and M. Uribe (2004): Solving dynamic general equilibrium models using a second-order approximation to the policy function. Journal of Economic Dynamics and Control, Vol. 28, pp. 755–775.

Sidrauski, M. (1967): Rational Choice and Patterns of Growth in a Monetary Economy. American Economic Review, Vol. 57, pp. 534—544.

Smets, F. and R. Wouters (2003): An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area. Journal of the European Economic Association, Vol. 1, pp. 1123–1175.

Smets, F. and R. Wouters (2004): Forecasting with a Bayesian DSGE Model: An Application to the Euro Area. Journal of Common Market Studies, Vol. 42, pp. 841–867.

Smets, F. and R. Wouters (2007): Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach. American Economic Review, Vol. 97, pp. 586–606.

Taylor, J. B. (1993): Discretion versus Policy Rules in Practice. Carnegie-Rochester Conferences Series on Public Policy, Vol. 39, pp. 195—214.

Walsh, C. E. (2003): Monetary Theory and Policy, Second Edition. The MIT Press. Cambridge, MA and London.

Woodford, M. (2003): Interest and Prices. Princeton University Press. Princeton and Oxford.

Abstract

Besides an introductory chapter, the present dissertation entitled *Monetary DSGE Models of Two Countries: Set-Up, Estimation, and Forecasting Performance* is divided into three more chapters.

In **Chapter 2**, we develop a two-country DSGE model and investigate what are the implications of diverging interest-rate rules on key macroeconomic variables of the EU and the US in terms of impulse responses. We find that positive realizations of all types of disturbances have a negative impact on output of both economies. Expansionary monetary policy shocks always have a *prosper thyself* and *beggar thy neighbor* effect. Moreover, we find that if the ECB implemented the interest-rate rule proposed in this chapter, it would encounter lower fluctuations in EU PPI inflation compared to an interest-rate rule as proposed for the Fed. This is consistent with the ECB's paramount objective of price stability.

In **Chapter 3**, we estimate and forecast with the two-country DSGE model developed in Chapter 2 and a VAR model using Euro area and US data. We find that the estimated DSGE model qualitatively reproduces most of the findings of the calibrated one from Chapter 2. Estimating the VAR does not yield the identical causal relationships as implied by the DSGE and impulse responses based on the VAR sometimes differ from the ones based on the DSGE. Both models as well as some extrapolation benchmark are not able to predict the severeness or, at least, the evolution of the economic and financial crisis. Finally, we obtain the result that the accuracy of one-step-ahead DSGE forecasts can compete well with the accuracy of VAR, extrapolation, and uniformly combined forecasts in times of regular economic activity.

In **Chapter 4**, we shift the focus to Austria and Hungary. We compare the forecasting accuracy of closed- and open-economy variants of the DSGE model from Chapter 2 for four variables with respect to closed- and open-economy Bayesian and classical (V)AR benchmarks. We obtain the result that these benchmarks deliver the most accurate one-step-ahead forecasts, but cannot significantly outperform the DSGE models. For three out of four variables open-economy models perform best with respect to other single forecasts. If we calculate uniformly combined forecasts, we obtain similar results. Even if single DSGE forecasts were not able to deliver the most accurate one-step-ahead forecasts, this additional information is important for uniform forecast combination for two of the four variables. Taking into account the economic interrelations between Austria and Hungary by using open-economy models leads to a more accurate prediction of most of their macro variables in general.

Zusammenfassung

Die vorliegende Dissertation mit dem Titel *Monetary DSGE Models of Two Countries: Set-Up, Estimation, and Forecasting Performance* beinhaltet neben einem einleitenden noch drei weitere Kapitel.

In **Kapitel 2** entwickeln wir ein Zwei-Länder-DSGE-Modell und untersuchen die Auswirkungen von divergierenden Zinssatzregeln auf wichtige makroökonomische Variablen der EU und der USA in Form von Impuls-Antwort-Funktionen. Positive Realisierungen aller Arten von Schocks haben einen negativen Einfluss auf den Output beider Volkswirtschaften. Expansive geldpolitische Schocks haben immer einen *Prosper-thyself*- bzw. *Beggar-thy-neighbor*-Effekt. Des Weiteren erhalten wir das Ergebnis, dass sich die EZB im Falle der für sie vorgeschlagenen Zinssatzregel einer geringeren Produzentenpreisindex-Inflationsrate gegenüber sieht, als es für eine Zinssatzregel der Fall wäre, wie sie für die Fed vorgeschlagen wird. Dies ist konsistent mit dem vorrangigen Ziel der Preisniveaustabilität der EZB.

In **Kapitel 3** schätzen wir das Zwei-Länder-DSGE-Modell aus Kapitel 2, sowie ein VAR-Modell, indem wir US-amerikanische und Euroraum-Daten verwenden. Das geschätze DSGE-Modell repliziert die Mehrzahl der Ergebnisse des kalibrierten Modells aus Kapitel 2. Das geschätze VAR impliziert nicht immer dieselben Kausalzusammenhänge wie das DSGE und auch die Impuls-Antwort-Funktionen basierend auf dem VAR weichen manchmal von den auf dem DSGE basierenden ab. Beide Modelle, sowie ein extrapolatives Benchmark, sind nicht in der Lage, die Schärfe der Wirtschafts- und Finanzkrise oder zumindest deren Entwicklung vorherzusagen. Unter normalen ökonomischen Rahmenbedingungen weist das DSGE bei einperiodigem Prognosehorizont jedoch eine recht gute Prognosegüte im Vergleich zu VAR-, extrapolativen und einheitlich kombinierten Prognosen auf.

In **Kapitel 4** verlagern wir unser Hauptaugenmerk auf Österreich und Ungarn. Wir untersuchen die Prognosegüte von Varianten des DSGE-Modells geschlossener und offener Volkswirtschaften für vier Variablen in Bezug auf bayesianische und klassische (V)AR-Benchmarks geschlossener und offener Volkswirtschaften. Diese Benchmarks liefern die besten Prognosen bei einperiodigem Prognosehorizont, sind jedoch nicht signifikant besser als die Prognosen des DSGE-Modells. Im Falle einzelner Prognosen sind für drei von vier Variablen Modelle offener Volkswirtschaften genauer. Im Falle einheitlich kombinierter Prognosen erhalten wir ähnliche Ergebnisse. Obwohl einzelne DSGE-Prognosen nicht die besten einperiodigen Prognosen liefern, ist diese zusätzliche Information wichtig für die Berechnung einheitlich kombinierter Prognosen für zwei der vier Variablen. Im Allgemeinen trägt das Einbeziehen der volkswirtschaftlichen Verflechtungen zwischen Österreich und Ungarn durch Verwendung von Modellen offener Volkswirtschaften dazu bei, die Mehrzahl ihrer Makrovariablen genauer zu prognostizieren.

PERSÖNLICHE DATEN

Geburtsdatum und -ort	25. November 1982, Regensburg
Nationalität	Deutschland

AUSBILDUNG

10/2007 - 9/2010   PhD-Studium der Volkswirtschaftslehre (Economics) an der Universität Wien

- Studienabschluss als Doctor of Philosophy (PhD) in Economics (erwartet für 9/2010)
- Dissertation: *Monetary DSGE Models of Two Countries: Set-Up, Estimation, and Forecasting Performance* (Betreuer: Prof. Dr. Gerhard Sorger und Prof. Dr. Robert M. Kunst)

3/2006 - 6/2006   Auslandsstudium an der Leavey School of Business, Santa Clara University, Santa Clara, USA

10/2004 - 7/2007   Studium *Honors Wirtschaftswissenschaften* (www.honors.de) im Elitenetzwerk Bayern (www.elitenetzwerk.bayern.de) des Bayerischen Staatsministeriums für Wissenschaft, Forschung und Kunst and der Universität Regensburg

10/2002 - 7/2007   Diplomstudium der Volkswirtschaftslehre an der Universität Regensburg

- Studienabschluss als Dipl.-Volkswirt Univ. (äquivalent zu MSc with Honors)
- Diplomarbeit: *Geldpolitische Regeln in Woodfords monetärer Makroökonomik* (Betreuer: Prof. Dr. Lutz Arnold)
- Schwerpunkte: Empirische Wirtschaftsforschung, Finanzmarkttheorie, Fortgeschrittene Makroökonomie, Fortgeschrittene Mikroökonomie, Internationale und interregionale Ökonomie

6/2002   Allgemeine Hochschulreife (Abitur) am Werner-von-Siemens-Gymnasium, Regensburg

BERUFSERFAHRUNG

4/2009 - 8/2009   Praktikant bei der Österreichischen Finanzmarktaufsicht, Bereich Integrierte Aufsicht, Abteilung Integrierte Finanzmärkte, Wien, Mitarbeit u. a. am Entwurf eines europäischen Makro-Stresstests für Versicherungsunternehmen, Verfassen einer schriftlichen Ausarbeitung über die Adaption der Ratingmethodologien für Hybridkapital von Banken im Zuge der Finanzkrise

10/2007 - 9/2010   Kollegassistent im Initiativkolleg *Issues in the Global Economy: Dynamics, Governance, and Information* (www.univie.ac.at/vwl/IKHomepage/willkommen.html) am Institut für Volkswirtschaftslehre, Universität Wien

4/2007 - 5/2007   Praktikant bei der Deutschen Bundesbank, Zentralbereich Banken- und Finanzaufsicht, Abteilung Mikro- und Makroprudenzielle Analysen des Bankensektors, Frankfurt (Main), Mitarbeit am Teilprojekt *Untersuchung der Besicherung und der Verlustquote von Interbankkrediten*

| 8/2005 - 10/2005 | Praktikant am Institut für Wirtschaftsforschung Halle, Abteilung Makroökonomik, Halle (Saale), Bearbeitung des Themas *Besteuerung von Unternehmensgewinnen in ausgewählten Ländern der Europäischen Union* |

FORSCHUNGSINTERESSEN (ALPHABETISCH)

(Empirische) Makroökonomie, empirische Wirtschaftsforschung allgemein, Finanzmärkte, Geldtheorie und -politik

WORKING PAPERS

| 2010 | *Forecast combination based on multiple encompassing tests in a macroeconomic DSGE system*, zusammen mit Mauro Costantini und Robert M. Kunst, Economics Series No. 251, Institute for Advanced Studies, Vienna |
| 2009 | *On the Forecasting Performance of a Two-Country DSGE Model*, Mimeo *Macroeconomic Interdependence in a Two-Country DSGE Model under Diverging Interest-Rate Rules*, Vienna Economic Papers No. 0903 |

VORTRÄGE

6/2010	30th International Symposium on Forecasting, San Diego, USA: *Forecast combination based on multiple encompassing tests in a macroeconomic DSGE system*
5/2010	Quantitative Economics Doctorate (QED) Meeting, Alicante, Spanien: *Forecast combination based on multiple encompassing tests in a macroeconomic DSGE system*
9/2009	Public Lecture und Workshop *The Financial Crisis – The End of Globalization?* des Initiativkollegs in Zusammenarbeit mit der Unicredit Bank Austria AG, Wien: *On the Forecasting Performance of a Two-Country DSGE Model*

LEHRE

3/2010 - 4/2010	Lektor für *Introduction to Macroeconomics* (Studierende der BWL im 1. Studienabschnitt, auf Englisch) am Institut für Betriebswirtschaftslehre, Universität Wien
10/2009 - 1/2010	Tutor für *Einführung in die Wirtschaftswissenschaft* (Studierende der Statistik und VWL im 1. Studienabschnitt) am Institut für Volkswirtschaftslehre, Universität Wien
10/2008 - 1/2009	Tutor für *Applied Econometrics - Praktikum aus Empirischer Wirtschaftsforschung* (Studierende der VWL im 2. Studienabschnitt, auf Englisch) am Institut für Volkswirtschaftslehre, Universität Wien
10/2004 - 7/2007	Tutor für *Makroökonomie I und II* (Studierende der BWL und VWL im 1. Studienabschnitt) am Institut für Volkswirtschaftslehre, Universität Regensburg

Persönliche Kompetenzen

Software	Dynare, EViews, LaTeX, Microsoft Office (jeweils sehr gute Anwenderkenntnisse)
	Matlab, SAP R/3 (Modul FI), Stata (jeweils grundlegende Anwenderkenntnisse)
Sprachen	Deutsch (Muttersprache)
	Englisch (verhandlungssicher)
	Französisch (fließend)
	Portugiesisch, Spanisch (jeweils fortgeschritten)
	Japanisch, Türkisch (jeweils Grundkenntnisse)

Sonstiges

9/2009	Teilnahme an der OeNB Summer School 2009 *The Current Financial Crisis: What Can Structural Macro Models tell us?* (Prof. Dr. Tommaso Monacelli und Dr. Mathias Trabandt) am Joint Vienna Institute
Seit 5/2005	Mitglied der Deutsch-Japanischen Gesellschaft Regensburg
Interessen	Kino, Politik, Reisen