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Für meine Eltern

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Abstract

This thesis introduces several portfolio insurance strategies and presents the results of an analysis of the implicit cost and reliability of two widely adopted strategies. The first part is an introduction into portfolio insurance and explains why there is a demand for such strategies and how they emerged. It is followed by a chapter on different risk measures, which will be used in the course of the analysis. Following their classification into static and dynamic strategies the properties of several portfolio insurance strategies are described. The static strategies presented are 1) Buy&Hold, 2) Stop-Loss, 3) Protective Put and 4) its equivalent using call options. The dynamic strategies presented are 1) Synthetic Put, 2) Modified Stop-Loss and 3) Constant Proportion Portfolio Insurance (CPPI).

The main part of this thesis is a detailed analysis of the Synthetic Put and the CPPI strategies using Monte Carlo simulation. Specifically, considering the resulting probability distribution of the insured portfolio returns, the implicit cost and reliability will be assessed. The analysis proves that both strategies have the desired property of creating an asymmetric return distribution that is skewed toward positive returns. Thus, they effectively limit a portfolios downside to a prespecified floor, while retaining the ability to participate in favourable market movements. However, the analysis also reveals the implicit cost of this property, which is a lower mean and median return as compared to an uninsured portfolio. The Synthetic Put proved to be reliable in securing a desired floor when the volatility estimate was accurate. When the Synthetic Put was at the money while approaching expiration a minor “protection error” appeared in the simulation, which was negligible in its magnitude. When volatility is underestimated the Synthetic Put fails to offer the desired protection. The CPPI strategy was fully reliable under the tested parameter settings. Even though higher multipliers resulted in higher mean and median returns of the CPPI strategy, the probability distribution appeared to be very skewed with half of the probability mass lying in a range of modest returns below the risk-free rate.

1. Introduction to Portfolio Insurance

1.1. Characteristics of the Stock Market

The past performance of a broad stock-index such as the S&P 500[1] shows how rewarding an investment in equities can be when compared to the performance of less risky investments such as Treasury Bills[2]. However, by holding equities in the portfolio an investor is exposed to the booms and busts of the stock markets as illustrated in Figure 1, which can result not only in large gains but can also have a severe drag on the portfolio value. The last boom started in 2001/02 after the burst of the dotcom bubble and lasted roughly the next five years. This bull market was brought to end in October 2007 with the breakout of the subprime crisis, which consecutively spread out into a global financial crisis. After reaching an all-time high in October 2007 the S&P 500 dropped by more than 50%, giving up almost all of its gains it accumulated since the burst of the dotcom bubble.[3]

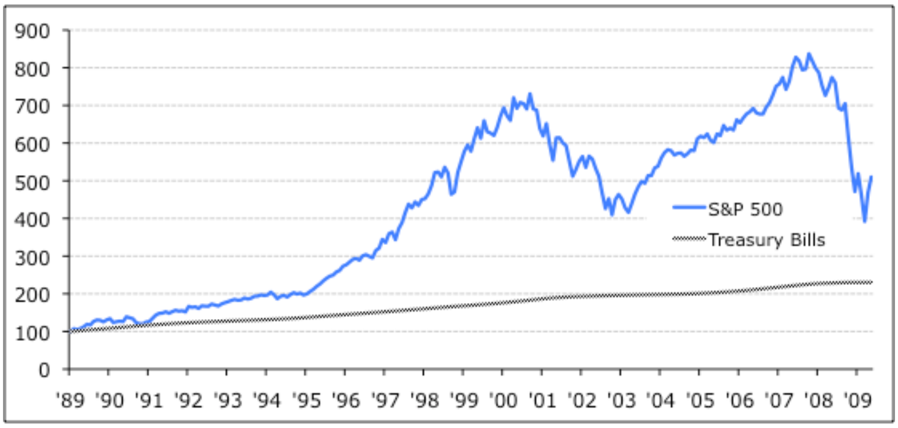

Figure 1: Performance of the S&P 500 and Treasury-Bills from January 1989 to May 2009; Source: Thomson Datastream, own calculation

Considering the possible magnitude of losses, a demand for asset allocation strategies, which allow for participation in boom periods and offer downside protection in adverse

[1] The S&P 500 is a capitalization-weighted index which covers 75% of the U.S. equity market.
[2] Treasury Bills refer to 90-days US Treasury Bills. In most academic analyses US Treasury Bills are used as the risk-free asset, the author will adhere to this practice. See Bodie/Kane/Marcus (2005), p. 200
[3] For an analysis comparing the financial crisis of 2008 to past financial crises see Cogman/Dobbs (2008)

markets, seems natural. This demand led to the development of a specific set of dynamic asset allocation strategies, which offer the desired properties. [4]

Dynamic asset allocation comprises strategies that systematically adjust a portfolio's asset allocation, triggered either by changes inside the portfolio or in overall market conditions.[5] Those specific strategies offering the desired properties of reducing the downside risk, while preserving the upside potential are known as portfolio insurance.[6] The explicit goal of portfolio insurance strategies is to allow the investor to recover at maturity a predetermined amount of his initial investment under adverse market conditions while retaining the ability to participate in favourable market movements.[7]

It is important not to confuse portfolio insurance with a traditional insurance contract. Whereas the concept of an insurance company is to guarantee a certain value in return for an upfront premium and builds upon the pooling of independent risks, the risk of a financial portfolio is common, i.e. the market. Hence, risk pooling is not applicable.[8]

1.2. The Origins of Portfolio Insurance

The origins of portfolio insurance date back to 1976 when Leland became aware of the potential demand for such a financial product and realized that insurance for a portfolio is similar to having a put option on a portfolio.[9] However, as of that time no exchange-traded put-options were available. The first options on individual stocks were traded in June 1977, put options on stock indices were traded not until March 1983.[10]

A viable alternative was revealed with an influential paper on option pricing by Black and Scholes, which provided the necessary tools for synthetically creating insurance for a portfolio.[11] The paper was built around a simple arbitrage argument that described how a call option could be hedged by a short stock position, which resulted in a riskless

[4] See Meyer-Bullerdiek/Schulz (2004), p.13 and Hagen (2002), p. 3
[5] See Trippi/Harriff (1991), p. 19
[6] See Lederman/Klein (1994), p.3
[7] See Prigent/Tahar (2006), p.172
[8] See Leland (1980), p.279 and Leland/Rubinstein (1988), p. 1
[9] See Leland/Rubinstein (1988), p.1
[10] See Bouye (2009), p. 3
[11] See Black/Scholes (1973)

payoff at maturity. Reversing this argument Leland concluded that an option-like payoff could be achieved by a dynamic strategy involving the underlying stock and a riskless asset.[12] Together with his colleague Rubinstein they started to work on the implementation of this strategy, which today is referred to as the synthetic put. The strategy intends to replicate a portfolio consisting of a risky asset and a corresponding put option, by continuously adjusting a portfolio of the same risky asset and a risk-free bond. The adjustment is determined by the hedge ratio, which is derived by the model developed by Black and Scholes.[13]

Having refined their strategy so far as to offer a portfolio insurance product, Leland and Rubinstein, together with John O’Brien founded LOR Associates in 1981. O’Brien joined the company to help market the product to institutional investors, such as pension funds, as he had the required know-how. As their product gained acceptance among large investors, other well-known companies started to offer similar products. The volume of portfolio insurance products increased to around $ 100 billions by 1987, with LOR managing half of it.[14]

However, on 19th October 1987 portfolio insurance was brought to a test, when stock markets plummeted globally. After declining by almost 1000 points over the course of the preceding eight weeks, the Dow Jones dropped by 508 points on 19th October 1987, which corresponds to a decline of 22,6% on a single day. Due to the trading pattern of portfolio insurance programs, which systematically sell stocks as they fall and purchase stocks as they rise, and the widespread implementation of such strategies until that crash, portfolio insurance soon received the blame for having exaggerated an otherwise ordinary market-decline and thus having triggered the market-meltdown. [15]

Leland rejected the suggestion that portfolio insurance might have triggered the crash, as it is a reactive strategy acting only in response to market movements. [16] It had rather been the negative economic news and a rising perception by investors of an

[12] See Leland/Rubinstein (1988), p.2

[13] See Benninga (1990), p. 21 and Bodie/Kane/Marcus (2005), p. 769; The synthetic put strategy is the topic of a separate subchapter in this thesis, where a more detailed description of its mechanics is presented.

[14] See Leland/Rubinstein (1988) and See Bouye (2009), p. 4

[15] See Leland (1988), p.311

[16] See Leland(1988)b, p. 83

overvaluation of the stock market, which caused the slide on that day. Whether portfolio insurance had amplified the decline on the markets cannot be answered unambiguously. Selling by portfolio insurers accounted for 15% of the total volume on that day, which represented merely 0,2% of total stock value. In order to assess the impact of portfolio insurance one would need to determine the amount of stocks portfolio insurers would have sold on that day if they had not used dynamic strategies, which can only be hypothesized.[17]

A point of interest is whether portfolio insurance strategies delivered the promised effect of securing a floor during this turbulent day. To answer this question one has to consider three observations which accompanied the crash: [18] First, on October 19th the volatility of the stock market increased sharply above anyone's expectations, which affected portfolio insurers in particular. The volatility is an important parameter in calculating the hedge-ratio and an underestimation of it resulted in underhedging, which exposed portfolio insurers to more market risk than if volatility had been estimated accurately. Secondly, the sudden increases in volatility resulted in a discontinuity of prices, which made it impossible to execute the necessary transactions on time. The third observation was large mispricing between stock indices and the corresponding stock index futures. As portfolio insurers used futures to implement their strategies, they incurred unanticipated trading costs.

These conditions were a serious challenge to portfolio insurers, with some of them aborting the strategy prematurely. The ones who committed themselves to stay with their programs experienced a certain amount of underperformance of their strategies. However, considering the special conditions the underperformance had not been severe and had been definitely lower than the loss incurred if a portfolio insurance strategy had not been implemented.[19]

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[17] See Leland(1988)b, p. 84
[18] See Leland (1988), p.312
[19] See Leland (1988), p.313

1.3. Structure of the Thesis

The following chapter gives an overview on how risk is assessed in asset-management and how a portfolio's exposure to market-risk can be reduced. Portfolio insurance is one such method that has unique properties, which will be presented in more detail. The consecutive chapters of this thesis are structured according to the classification of portfolio insurance strategies in figure 6: [20]

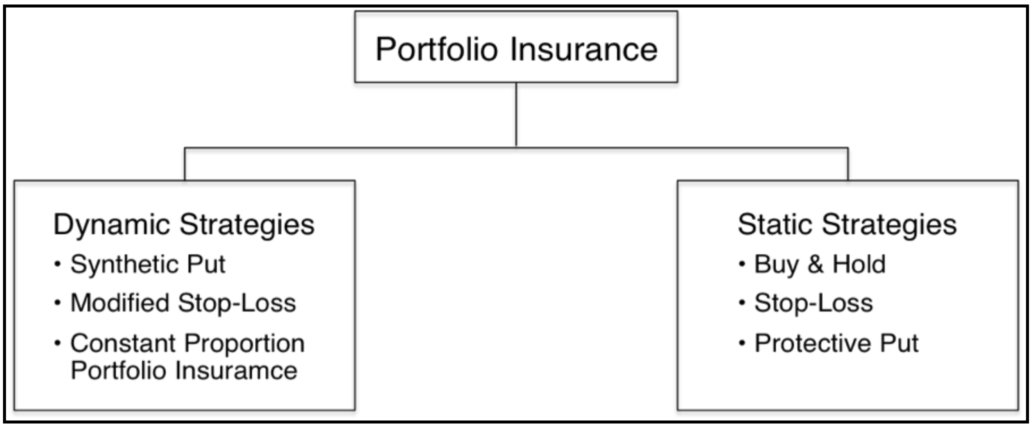


Figure 2: Classification of Portfolio Insurance Strategies.

Static portfolio insurance strategies are characterised by an asset-allocation decision at the beginning of the insurance program. Until the expiration of the insurance program no active adaptation to the initial asset-allocation is done. In contrast, dynamic portfolio insurance strategies rely on continuous reallocation of funds between risky assets and a risk-free asset in order to achieve the convex performance profile. [21]

The next chapter will start out with a detailed description of static portfolio insurance strategies. It will be followed by a thorough analysis of dynamic portfolio insurance strategies. Apart from the theoretical concepts the chapters will be supplemented by results from a simulation to further illustrate the properties of each strategy.

The final chapter will present a thorough evaluation of dynamic portfolio insurance strategies with respect to the change in the probability distribution of the insured

[20] See Meyer-Bullerdiek/Schulz (2004), p. 32 and Bossert/Burzin (2002), p. 135
[21] See Bossert/Burzin (2002), p.135, p.137

portfolio returns. The aim is to illustrate the costs of portfolio insurance by considering the whole distribution, similar to the work done by Clarke and Arnott.[22]

The analysis in this thesis is conducted by means of stochastic modelling. Analyses based on historical results depend very much on the chosen time-period and show only one possible path of events, which does not offer much insight into the return distribution of the analyzed strategies.[23] More explicitly, Monte Carlo simulation[24] is used to generate a return distribution for the presented strategies, which allows for a detailed analysis of the benefits and costs accompanying portfolio insurance.

All calculations and presentations in this thesis use S&P 500 data for the risky asset and 90-days US Treasury Bills data for the riskless asset, from the period January 1961 to May 2009. Calculations involving the S&P 500 are conducted on a total return basis, i.e. dividend payments from companies underlying the index are reinvested. As indicated at any one time, daily and weekly data is used.

2. Risk Measurement

2.1. Volatility as a Measure of Risk

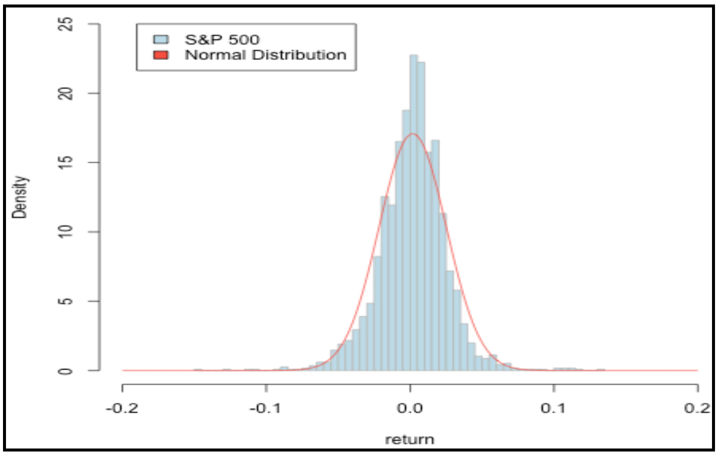
The most prominent measure for assessing the risk of an investment is the volatility. It corresponds to the standard deviation, which is the square root of the variance of the rate of returns. The volatility is a symmetric measure and measures the squared deviation of realized returns from its mean return[25]:

[22] See Clarke/Arnott (1988)
[23] See Clarke/Arnott (1987), p.35
[24] For an introduction to Monte Carlo simulation see Hull (2005), pp. 410-412.
[25] See Bruns/Meyer-Bullerdiek (2008), pp. 9-11

$$\sigma = \sqrt{\frac{1}{n-1} * \sum_{t=1}^{n} (r_t - \mu)^2} \quad \text{with} \quad \mu = \frac{1}{n} \sum_{t=1}^{n} r_t$$

σ	...volatility
μ	...mean return
t	...time of observation
n	...number of observations

It does not distinguish between positive and negative deviations and thus is only a useful measure if the distribution of returns is symmetric around its mean. In fact, were the returns normally distributed, the mean return and the volatility would entirely describe the distribution of returns.[26] An empirical analysis of past S&P 500 continuously compounded returns[27] reveals a distribution that is very close to a normal distribution. Figure 2 shows the frequency distribution and its statistics for the S&P 500 weekly returns, which is contrasted with a normal distribution having the same mean and standard deviation.



	S&P 500	Normal Distribution
Mean	0,18%	0,18%
Std.	2,33%	2,33%
Median	0,25%	0,18%
Max	13,01%	-
Min	-14,85%	-
Skewness	-0,11	0,00
Kurtosis	6,62	3,00

Figure 3: Frequency distribution of S&P 500 weekly returns vs. normal distribution.

However, a tighter concentration of returns around the mean is evident, which is documented by a kurtosis of 6,62 for the S&P 500 returns compared with a kurtosis of 3 for a normal distribution.[28] The higher kurtosis of the S&P 500 returns implies a higher

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[26] See Bodie/Kane/Marcus (2005), p. 144

[27] $r_t = \ln\left(\frac{Price_t}{Price_{t-1}}\right)$; See Bodie/Kane/Marcus (2005), p. 163

[28] Analyses of the S&P 500 of periods covering 1987 are sensitive to that years 43rd week's return (October), when the index dropped by -31,8%. The results reported in this thesis are based on calculations excluding this particular week. If the return of that week is included the kurtosis of the return-distribution changes from 6,62 to 18,65. See Corrado/Su (1997)

probability for returns at the margins of the distribution than in the case of a normal distribution. Additionally, the frequency distribution appears to be slightly negatively skewed having a coefficient of skewness of -0,11.

2.2. Components of Risk – the Effect of Diversification

The preceding illustration of volatility as a measure of risk that is incurred by an investor was based on the S&P 500 index. This corresponds to an investor holding a portfolio of individual stocks comprised by the index, with each stock being weighted according to its proportion in the index. A central point of modern portfolio theory is that the volatility of such a portfolio of stocks is less than the sum of the volatilities of each individual stock.[29] It was Markowitz who emphasized the effect of diversification in the construction of efficient portfolios.[30] Based on his work Sharpe developed a single-factor model for describing the return of a security.[31] According to this model the risk of a security can be split up into two components:

$$\sigma_i^2 = \underbrace{\beta_i^2 * \sigma_M^2}_{\text{market risk}} + \underbrace{\sigma_\varepsilon^2}_{\text{firm specific risk}}$$

σ_i ...volatility of security i
β_i ...sensitivity to market movements of security i
σ_M ...volatility of the market
σ_ε ...firm specific risk

The effect of diversification materializes by increasing the number of stocks to be included in the portfolio, which causes the firm-specific part of securities risk to vanish. [32] The market risk remains however, regardless of the number of securities included. Thus, it is the market risk a holder of a diversified portfolio has to bear. In fact, the Capital Asset Pricing Model reveals that an investor is rewarded only for the market

[29] See Bodie/Kane/Marcus (2005), p. 244
[30] See Markowitz (1952) and (1959)
[31] See Sharpe (1963); for a summary see Frantzmann (2002), pp. 52-53
[32] See Bodie/Kane/Marcus (2005), p. 324

risk he bears.[33] More explicitly, the return in excess of the riskless rate an investor can expect is proportional to the excess return on the market. The proportionality factor is beta, which is the sensitivity of a security's return to movements in the market. This relationship reveals the risk-return trade off an investor faces: A higher return is attainable only in exchange for accepting a higher exposure to market risk.

2.3. Alternative Risk Measures

Even though the volatility is the most prominent measure for risk, it is not undisputed. As a symmetric measure, which does not distinguish between positive and negative deviations, critics contest whether it appropriately resembles the common notion of risk as something undesired.[34] Alternative measures are suggested which focus more on the downside-risk of a return-distribution and thus better capture an investor's risk perception. A special class of risk measures usually discussed in this context are lower partial moments (LPM), which consider only negative deviations from a required target return:[35]

$$LPM_m = \sum_{i=0}^{n^*} p_i * \left(r_{\text{target}} - r_i^-\right)^m$$

LPM_m ...lower partial moment of order m
n^* ...number of realized returns $< r_{\text{target}}$
p_i ...probability of deviation
r_{target} ...target return
r_i^- ...realized return $< r_{\text{target}}$

The order m reflects the weight an investor puts on the negative deviation. The most common LPMs used are of order 0 to 2. The LPM of order 2 is referred to as the semi-variance, which is also considered by Markowitz as an alternative risk measure for the

[33] Based on Markowitz's portfolio selection model Sharpe (1964), Lintner (1965) and Mossin (1966) developed the capital asset pricing model which describes expected returns on risky assets. For a summary see Bodie/Kane/Marcus (2005), p. 282
[34] See Unser (2000)
[35] See Bruns/Meyer-Bullerdiek (2008), p. 22

construction of efficient portfolios. [36] The LPM_0 and the LPM_1 correspond to the probability of loss and the expected loss, respectively.

3. Managing the Exposure to Market Risk

3.1. Diversification across Asset-Classes

The relationship between risk and return introduced in the previous chapter constraints a risk-sensible investor in his choice of portfolio-construction. In a desire to achieve a maximum return with the least amount of risk, he faces the earlier mentioned trade-off between risk and return. This implies a higher return is attainable only in exchange for incurring a higher risk. Wishing to reduce the risk of his equity portfolio the investor can diversify his portfolio across asset-classes with different risk-profiles. For instance by shifting some of the funds of a pure equity-portfolio into a riskless asset, the investor reduces the overall riskiness of his portfolio at the cost of a lower return.[37] The resulting effect is illustrated in figure 3, which compares the return distribution of an equity portfolio fully invested into the S&P 500 to a portfolio with 30% of funds invested into the S&P500 and 70% into riskless treasury bills (30-70 portfolio).

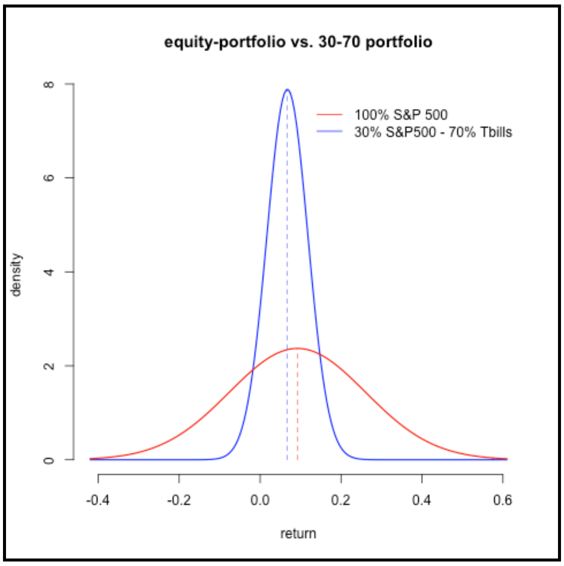

Figure 4: Distribution of a pure equity portfolio vs. a 30-70 portfolio.

[36] See Markowitz (1959), p.188

[37] See Bossert/Burzin (2002), pp. 132-133

The return distribution of the 30-70 portfolio is much less dispersed than the one of the pure equity portfolio, indicating the reduction of risk. However, the mean of the 30-70 portfolio depicted as the dashed line has shifted to the left, which illustrates the reduction in the portfolios expected return. Although this approach offers a reduction of risk, lowering the probability for both large negative and positive returns, the distribution still remains symmetrical. This means, even though less probable, negative returns can still occur.

3.2. Portfolio Insurance

Portfolio insurance strategies aim to alter the return distribution in such a way as to limit the downside while preserving the upside. The effect of portfolio insurance is to create an asymmetric distribution of returns as depicted in figure 4.[38]

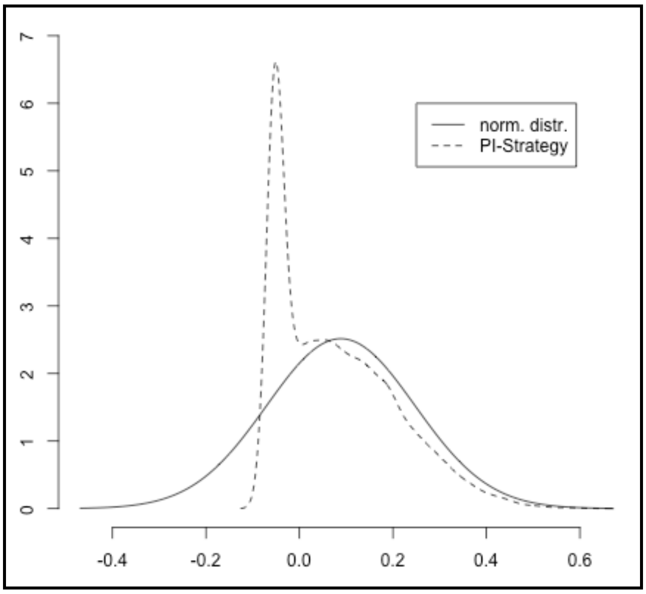

Figure 5: symmetric vs. asymmetric distribution

The insured portfolio has clearly a limited downside as returns below the floor are eliminated. However, the insurance comes at a cost, which is reflected in the lower mean of the distribution of insured returns.

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[38] See Clarke/Arnott (1987), p.38

The participation in upward markets and the maintenance of a floor in downward markets results in a convex performance profile of insured portfolios.[39] Thus, the payoff is very similar to a combination of a portfolio and a protective put option, with a strike price set correspondingly as to guarantee the desired floor. [40] Figure 5 shows the performance of an insured portfolio, with a strike price set at a level guaranteeing a floor of the initial investment, i.e. a portfolio return not less than zero.

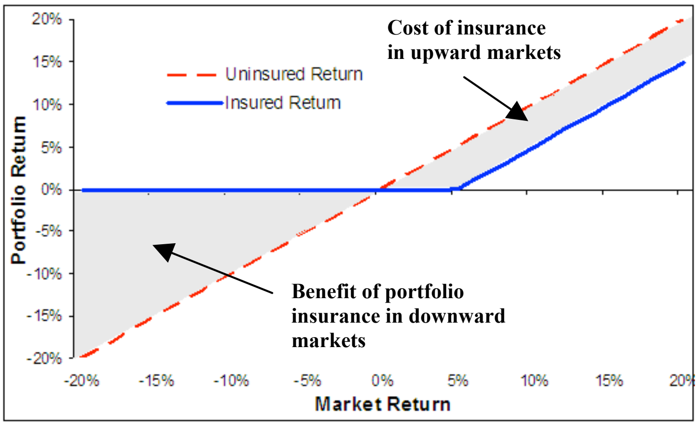

Figure 6: Performance of an insured portfolio vs. an uninsured portfolio. (See Clarke/Arnott (1987), p.37)

The figure makes it clear, in a downmarket where an uninsured portfolio realizes negative returns, the insured portfolio stays at the predetermined floor level. On the other hand, during rising markets the portfolio value of the insured portfolio increases in value. However, it does not gain in the same amount as an uninsured portfolio. This observation, which is referred to as "loss of upside capture" is the main cost incurred by an insurance strategy and is approximately equal to the initial price of a put option. The amount of this cost is sensitive, among other factors, to the chosen floor level, the volatility of the market, the risk free rate and the time horizon of the insurance strategy. Some of those factors, such as the floor level and the time horizon lie in the scope of the investor. Other factors, such as the volatility or the risk free rate are external factors, which can have a significant impact on the cost of an insurance strategy. [41]

[39] See Meyer-Bullerdiek/Schulz (2004), p.30

[40] The required strike price K correspondent with a desired floor return r_f: $K=(1+r_f)*(S+P)-D$; with S being the current stock price, P the price of the put option and D the dividend of the stock paid before maturity. See Clarke/Arnott (1987), p.47

[41] See Clarke/Arnott (1987), p.36

Referring to costs accompanying the implementation of a portfolio insurance strategy one can distinguish between implicit and explicit costs. The already mentioned loss of upside capture is an implicit cost. The explicit cost is the transaction cost arising from an increase in portfolio of a dynamic insurance strategy. [42] Even though explicit costs are not to be neglected, the focus of this thesis lies on the implicit costs of a portfolio insurance strategy, which result from the change in the shape of the return distribution. Hence, the effect on transaction costs is not incorporated in the subsequent analysis.

A portfolio insurance strategy, where the payoff of any profitable position at expiration is a predictable percentage of the underlying uninsured portfolio, is said to be path independent.[43] More specifically, a strategy is path independent if the payoff at expiration does not depend on the path taken but is determined only by the value of the underlying portfolio at the time of the payoff. It is an additional attribute of portfolio insurance strategies that is desired by an investor, as he doesn't want the payoff at expiration to be influenced by intermediate levels of the portfolio. The payoff should be determined by the level of the underlying portfolio at the end of the investment horizon only.[44] In the following chapters different portfolio insurance strategies will be introduced. Not all of which have the desired property of being path independent.

4. Static Strategies

4.1. Buy&Hold Strategy

The most common strategy for guaranteeing a minimum portfolio value over a fixed investment period is the buy & hold strategy. At the beginning of the investment period an investor allocates all of his funds between risky assets and a risk-free asset. This initial asset-allocation is established in the beginning and held till the end of the investment period without any further transactions, hence the term buy & hold. The portion invested in the risk-free asset determines the floor of this strategy. The portion allocated to risky assets specifies the maximum loss the investor is prepared to bear. In

[42] See Zhu/Kavee (1988), p.52.
[43] See Bookstaber/Langsam (1988), p.16
[44] See Rubinstein (1985), p.12

case of a total loss of the risky investment, the investor retains the risk-free portion of the portfolio.[45]

Figure 7 illustrates three kinds of buy&hold strategies: a maximum return strategy with all funds allocated to risky assets (here the S&P 500 index), a minimum risk strategy with all funds invested in the risk-free asset (here the 3 month treasury bills) and a 60/40 strategy with 60% of the funds allocated to the risk-free asset and 40% to risky assets.

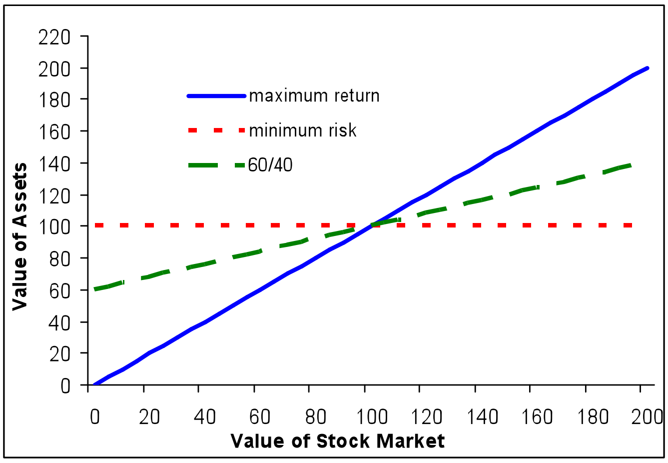

Figure 7: Buy&Hold Strategies: maximum return, minimum risk, 60/40 (See Perold/Sharpe)

The figure shows the linear relationship between the portfolio value and the stock index. The slope corresponds to the initial portion of funds allocated to the risky assets. This means the portfolio value participates in the development of the stock index according to the initial portion invested in stocks. In case of the 60/40 strategy the portfolio participation amounts to 40%, which means a 10% increase in the stock index results in a 4% increase in the portfolio value. Thus the upside potential of the buy & hold strategy is unlimited. Similarly a 10% decrease in the stock index results in a 4% decrease in the portfolio value. However, the portfolio value can never fall below the initial investment in the risk-free asset, which guarantees the floor of this strategy.[46]

The greater the portion initially allocated to risky assets, the better the portfolio performance when the stock index moves upward. Also an increase in the stock index

[45] See Hagen (2002), p.88 and Meyer-Bullerdiek/Schulz (2004), p.35
[46] See Perold/Sharpe (1995), p.150

results in a change of the initial asset-allocation as the portion of the portfolio invested in risky assets increases correspondingly. On the other hand, the greater the portion invested in risky assets, the worse the portfolio performance will turn out, when the market moves downward. Similarly a decrease in the stock index results in a decline of the portion of the portfolio invested in risky assets.[47]

As shown by the performance profile in figure 7 the two border strategies, i.e. maximum return strategy and minimum risk strategy, do not comply fully with the properties of portfolio insurance. The maximum return strategy offers the highest upside potential among all other buy&hold strategies. However, no downside protection is offered as the portfolio value can fall to zero. On the other hand, the minimum risk strategy offers the highest downside protection by fully guaranteeing the initial investment but fails to offer any upside potential.[48]

4.1.1. *Assessment of the Buy&Hold Strategy*

The buy&hold strategy offers a floor to the portfolio value in downward markets, which is determined by the initial allocation of funds to the risk-free asset. The remainder, which initially is the maximum loss the investor is willing to bear is allocated to risky funds and thus enables upside participation. As no further transactions are required apart from the initial asset-allocation, the buy&hold strategy, results in low transaction costs. The fact that no transactions take place over the investment-period implies that no intermediate gains are secured by transferring some of the gains of risky assets to the riskless asset. This might be considered as a drawback of this strategy.[49]

4.2. Stop-Loss Strategy

The stop-loss strategy starts out by initially allocating all funds to risky assets. The floor for the stop-loss strategy is determined by the present value[50] of the minimum portfolio

[47] See Meyer-Bullerdiek/Schulz (2004), p.33
[48] See Faber (2007), p. 28
[49] See Meyer-Bullerdiek/Schulz (2004), p.36
[50] The present value of the minimum portfolio value at the end of the investment horizon is calculated as $PV_t^{Floor}=F*(1+r_f)^{-(T-t)}$; with F being the minimum portfolio value required, r_f being the riskless rate for the period corresponding to the investment-horizon, t being the current point in time of the calculation of

value required at the end of the investment horizon. As soon as the portfolio value drops below the current floor, the stop-loss strategy implies the sale of all risky assets and the investment of all the proceeds in the riskless asset. After the portfolio has been stopped out and all the proceeds have been allocated to the risk-free asset no further transactions take place until the end of the investment period. By the end of the investment horizon those funds will have grown at the risk-free rate to yield the required minimum portfolio value.[51]

It is not possible to plot a general performance profile for the stop loss strategy, similar to the one depicted for the buy&hold strategy. The reason for this is that its payoff at the end of the investment-horizon does not depend solely on the level of the stock market index, as it is with the buy&hold strategy. It rather depends on the specific path taken by the stock index over the investment period. Has the floor been breached over the course of the investment period, the terminal value of the insured portfolio will just be the required minimum portfolio value, regardless if the stock market recovered afterwards and has risen above the floor level by the end of the respective period. Hence, the stop-loss strategy is path dependent.[52]

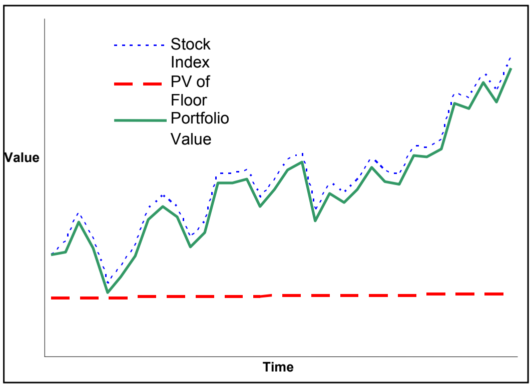

Figure 8: Stop-Loss strategy without breach of the floor

Figure 8 above illustrates a scenario where the stock index stays above the floor over the whole investment period. The performance of the insured portfolio will equal the one of the stock index.

the floor and T being the ending point of the investment period. (See Meyer-Bullerdiek/Schulz (2004), p.36)

[51] See Bossert/Burzin (2002), p.136

[52] See Bookstaber/Langsam (1988), p.16

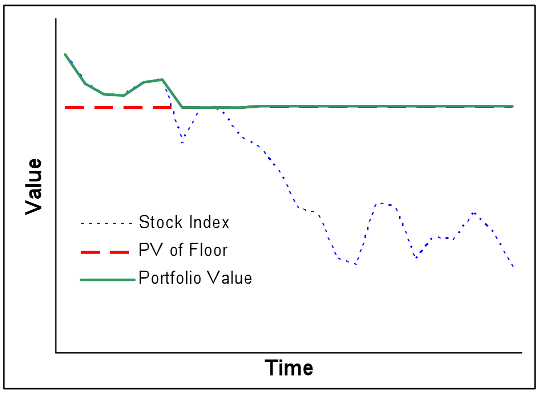

Figure 9: Stop-Loss strategy with breach of the floor

Figure 9 shows an alternative scenario in which the stock index breaches the floor over the course of the investment period. Here the stock index does not recover and closes below the required minimum portfolio value at the end of the respective period. However, as all the funds have been transferred to the risk-free asset at the time when the stock index breached the floor, the portfolio value has grown at the risk-free rate to yield the required minimum portfolio value at the end of the period.

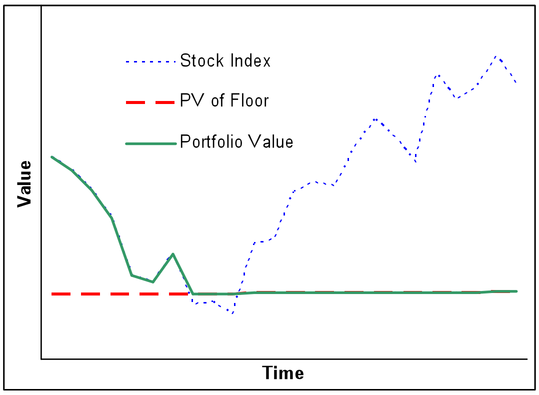

Figure 10: Stop-Loss strategy with breach of the floor and recovery of the index

Figure 10 illustrates the little desired path dependency of the stop-loss strategy. At a given point, halfway across the investment period, the stock index drops bellow the floor value triggering the transfer of all funds to the risk-free asset. Shortly after, the stock index recovers and closes above the required minimum portfolio value at the end of the investment period. However, as all the funds have been transferred to the risk-free asset and no further transactions are prescribed by the stop-loss strategy, the portfolio cannot participate in the later recovery and yields only the required minimum value.[53]

4.2.1. *Assessment of the Stop-Loss Strategy*

The stop-loss strategy entails the initial placement of all funds to risky assets and a transfer of all funds to the risk-free asset in case the stock index falls bellow the floor. By transferring all funds to the risk-free asset in case of a breach of the floor, the minimum portfolio value required at the end of the investment horizon can be secured. However, the path dependency of this strategy may result in unsatisfactory results at the end of the investment period, in the case where the portfolio has been stopped out halfway through the period and the stock index recovers afterwards.

Another issue affecting stop-loss strategies are jumps in security prices. A so-called “gap opening” may result in the portfolio value to drop considerably bellow the floor before any transactions can be initiated to transfer funds to the risk-free asset. In such a case the transfer of funds away from the risky asset may no longer guarantee the minimum portfolio value required, as the proceeds invested at the risk-free rate may no longer yield a sufficient amount.

4.3. Protective Put Strategy

The basic approach to implement a portfolio insurance strategy is the use of put options to create a portfolio with an asymmetric payoff profile. This strategy commonly referred to as the protective put strategy, guarantees a floor value to the portfolio at the end of the investment period, which is known with certainty at the time of the implementation of the strategy.[54] In figure 9 this approach is illustrated with a single stock and a corresponding put option on the stock.

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[53] An illustration of the path dependency of the stop-loss strategy based on a binomial model is presented in Rubinstein (1988), p.14

[54] See Bruns/Meyer-Bullerdiek (2008), p. 385

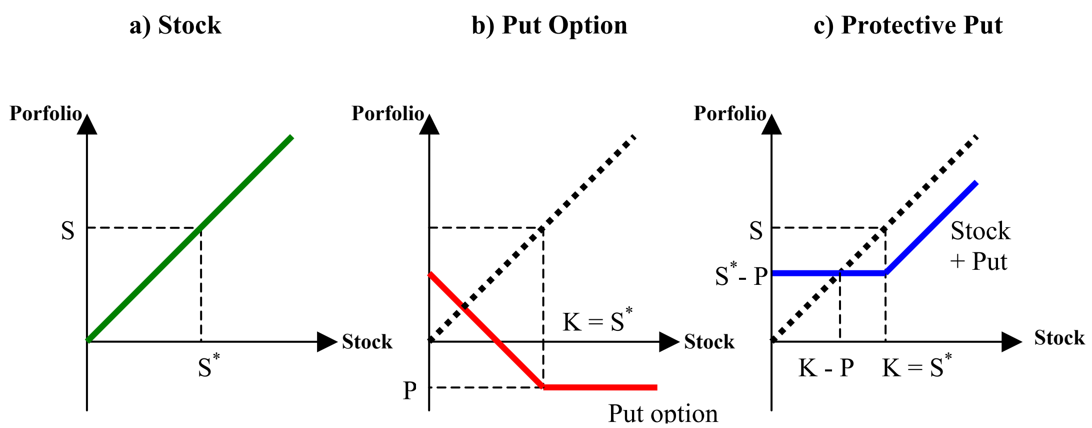

Figure 11: Payoff profiles (based on Clarke/Harindra/McMurran (1998), p. 351 and Faber (2007), p. 25)

The value of an investment in a stock with an initial value S^* moves linearly with the value of the stock as depicted in graph a. The second graph shows the relationship between the value of a put option, with a strike price K equal to the initial value of the stock, and the underlying stock. The intrinsic value of a put option, calculated as max(K-S, 0) is zero if the stock moves above the strike price, and (K-S) if the stock falls below the strike price. Thus, the value of the put option as depicted in graph b results from its intrinsic value minus the option premium P, which is the cost of acquiring the put. [55]

The value of a portfolio consisting of both the stock and the corresponding put option, with a strike price equal to the purchasing price of the stock is depicted in graph c. The dashed line shows the value of a simple stock investment. The solid line depicts the value of the protective put strategy, which exhibits the properties of portfolio insurance. When the value of the stock falls below the strike price the put option increases in value and compensates for the loss in the stock position. Thus, the protective put offers a floor to the portfolio value equal to the strike price minus the option premium, which is (K-P). On the other hand the protective put offers unlimited upside potential, which however, is always lower than the pure stock investment by the amount of the option premium P. The protective put becomes more advantageous than the simple stock investment only from the point on where the stock value falls below (K-P).[56]

[55] See Clarke/Harindra/McMurran (1998), p. 350

[56] See Clarke/Harindra/McMurran (1998), p. 351

The following table summarizes the resulting payoff-structure of the protective put strategy at maturity:

	S < K	S > K
Stock value	S	S
Intrinsic value (Put)	K – S	0
Option Premium	– P	– P
Net Payoff	K – P	S – P

with S… Stock value
K… Strike price
P… Option premium

Table 1: Payoff structure of the protective put strategy at maturity (based on Clarke/Harindra/McMurran (1998), p. 350 and Faber (2007), p. 23)

As already mentioned, the protective put offers a floor to the portfolio value of (K-P). For the illustration in figure 9 the strike price was chosen to equal the purchase price of the stock. This implies that when at expiration the stock value equals its initial purchase price, the protective put strategy results in a loss to the initial portfolio value, as the cost of the put option is incurred. However, by choosing a put option with a higher strike price than the purchase price of the stock, the investor can increase the floor level at the cost of a lower upside potential. In fact, choosing a put option with a strike price K = S + P, the investor can guarantee the entire initial investment.[57] On the other hand, by choosing a put option with a lower strike price than the initial purchase price of the stock, the investor trades off lower downside protection, by a reduced floor, against higher upside participation, by reducing the cost of the insurance.[58]

Whether the portfolio value stays above the floor over the entire investment period or is guaranteed only to be above the floor at the end of that period, depends on whether European or American options are used for the implementation of the strategy. With European options, which can be exercised only at maturity, the portfolio value may fall below the floor over the course of the investment period as the payoff from the put option can only be realized at the end of the period. With American options, which can be exercised at any point in time until maturity, the portfolio value will stay above the floor for the entire investment period. However, as American options offer the same features as European options but in addition can be exercised at any time until maturity, they are more expensive. [59] An investor who usually cares about a minimum portfolio

[57] See Clarke/Arnott (1987), p.47: An insured floor return of zero and no dividend payments are assumed.
[58] See Abken (1987), p. 5
[59] See Rubinstein (1988), p.17

value at the end of his investment horizon does not require the additional feature of the American option, thus he does not have to incur this additional cost.[60]

So far the protective put strategy has been illustrated with a single stock and a corresponding put option on the stock. When implementing a protective put strategy for an entire portfolio consisting of a variety of stocks, some additional considerations have do be made. In case of a well-diversified portfolio consisting of stocks underlying a broad market index such as the S&P 500, exchange traded put options on the S&P 500 can be utilized to implement a protective put strategy.[61] However, if the portfolio composition deviates significantly from the stock index, a protective put strategy using index options might exhibit a considerable tracking error. The alternative approach of buying put options on each stock in the portfolio separately is not equivalent to acquiring a single put option on the entire portfolio. Due to the effect of diversification a portfolio of stocks exhibits lower volatility than each individual stock by itself, which has an effect on the price of the option.[62] Everything remaining equal, a higher volatility results in a higher price of the option.[63] Thus, implementing the protective put strategy with put options on each individual stock in the portfolio might result in higher costs than using a single put option on the entire portfolio.

Another issue accompanying portfolio insurance strategies using options is the possible mismatch between the investor's investment horizon and the maturity of the options. Investors usually require insurance over a longer period than the maturities offered by most exchange traded options. This problem however, has been somewhat alleviated since the emergence of long-term traded options with maturities of several years.[64] To overcome the problem of options with maturities shorter than the investment horizon, a strategy of rolling over put options can be implemented. In case of a one-year horizon and the availability of only three-month put options, the strategy involves the initial purchase of a three-month put option. As soon as the option expires another three-month option is purchased.[65] This strategy of rolling over put options also guarantees a

[60] See O'Brian (1988), p. 40

[61] See Abken (1987), p. 4

[62] See Bruns/Meyer-Bullerdiek (2008), p. 386

[63] For a detailed description of the properties of stock options, including the effect of volatility on the price of an option see Hull (2005), chapter 9.

[64] See Bruns/Meyer-Bullerdiek (2008), p. 386

[65] See Bookstaber/Langsam (1988), p.21

prespecified floor at the end of the investment horizon. However, it suffers from path dependency and usually results in higher costs.[66]

4.3.1. *Assessment of the Protective Put Strategy*

The protective put strategy, as a combination of an equity portfolio and a corresponding put option, can fully guarantee a floor value at the end of the investment horizon, which is equal to the strike price minus the option premium. The floor value can be adjusted arbitrarily by choosing a corresponding strike price, which, however, is subject to the tradeoff between downside protection and upside participation. In case of a portfolio with a similar composition as a broad market index and the availability of long-term index put options with the same maturity as the investment horizon, the total cost of the insurance are known in advance at the time of implementation.

Some uncertainty with respect to costs may emerge if options with the required maturity are not available. This requires rolling over short term put options, which introduces some path dependency to the strategy and usually increases the cost of the insurance. Another problem may arise if a corresponding put option on the entire portfolio is not available. The alternative of purchasing separate put options on each stock in the portfolio is not equivalent as it might entail higher costs of protection. Option features such as the contract size, strike prizes of traded options, which might not correspond to the desired floor and position limits on traded options might pose additional constraints to the implementation of portfolio insurance using options.[67]

4.4. Portfolio Insurance using Call Options

A portfolio insurance strategy with similar properties as the protective put strategy can be realized with call options as well. This follows directly from the put-call parity condition:[68]

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[66] See Rubinstein (1988), p.18
[67] See O'Brian (1988), p. 40
[68] See Hull (2005), p.212

$$C + Ke^{-rT} = P + S$$

C... option premium for a call
K... strike price
r... risk - free rate
T... time to maturity of the option
P... option premium for a put
S... stock price

The put-call parity implies that a portfolio consisting of one stock and one put option yields the same amount over a respective investment period as a portfolio consisting of one call option on the stock and cash in the amount of the present value of the strike price, which is invested at the risk-free rate. Thus, purchasing call options on the desired stock portfolio and investing a corresponding amount in the risk-free asset achieves a similar result as the protective put.[69] Figure 10 illustrates the resulting payoff profile in a similar way as with the protective put option.

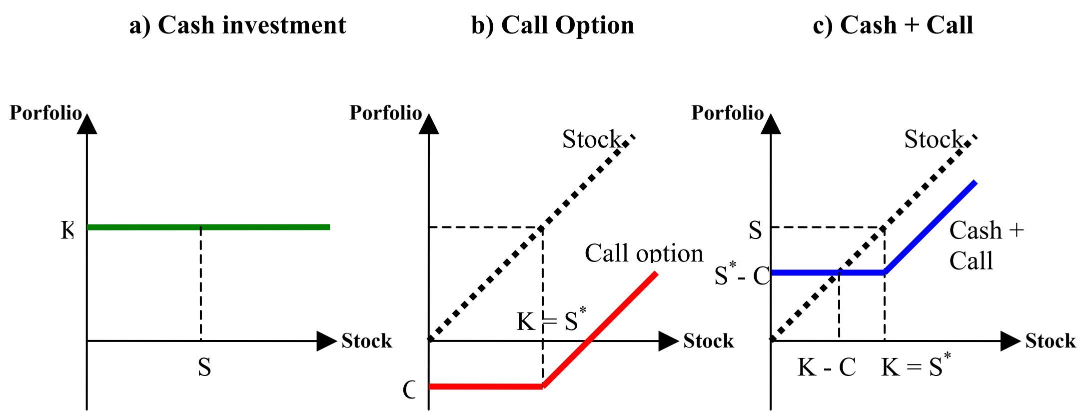

Figure 12: Payoff profiles (based on Bruns/Meyer-Bullerdiek (2008), p. 391)

The first graph shows the value of the investment in the risk-free asset, which is independent of the movement in the stock value. The second graph shows the relationship between the value of a call option with a strike price K equal to the initial value of the stock and the underlying stock. The intrinsic value of a call option calculated as max(S-K, 0) is (S-K) if the stock moves above the strike price and zero if it moves below the strike price. Thus, the value of the call option as depicted in graph b

[69] See Hagen (2002), p.121

results from its intrinsic value minus the option premium C, which is the cost of acquiring the call. The third graph depicts the relationship between the value of the stock and a portfolio consisting of a call option and an investment in the risk-free asset. The payoff profile looks similar to the one of the protective put and exhibits the same properties of portfolio insurance as the protective put strategy.[70] The initial investment in the risk-free asset of Ke^{-rT} is equal to the present value of the strike price and guarantees the floor to the portfolio, which amounts to (K-C).

All problems and contract specific constraints accompanying the previously introduced protective put strategy, such as the mismatch of option maturity and investment horizon and the availability of options corresponding to the investor's stock portfolio, apply in the same way to a portfolio insurance strategy using call options and the risk-free asset.[71]

5. Dynamic Strategies

5.1. Synthetic Put Strategy

As mentioned earlier in this thesis, it was Leland in the seventies who recognized that insurance on a portfolio was equivalent to having a put option on the entire portfolio.[72] However, at the time when Leland developed the idea of portfolio insurance, index options didn't exist and a direct realization of his idea was not possible. Even today the earlier mentioned constraints implied by options, such as contract size, available maturities and strike prices can make portfolio insurance with put options troublesome.[73]

Modern option pricing theory builds on the insight that any option payoff can be replicated through a dynamic strategy involving the underlying stock and a cash position. This insight, which allowed Leland to realize his idea of portfolio insurance,

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70 See Bruns/Meyer-Bullerdiek (2008), p. 390
71 See Bruns/Meyer-Bullerdiek (2008), p. 392
72 See Leland/Rubinstein (1988), p. 1
73 See Abken (1987), p.7 and Leland/Rubinstein (1988), p. 1

was laid out in an influential paper on option pricing by Black and Scholes in 1973.[74] Using an arbitrage argument they showed that over a short period of time a call option could be perfectly hedged with a short position in the underlying stock, which they used to price the option. Leland reversed this argument to achieve an option-like payoff using a dynamic strategy involving the underlying stock and a riskless asset.[75] Later on Leland and Rubinstein published their own paper describing how to replicate options by dynamically adjusting positions in stocks and cash.[76]

The basic approach for constructing a replicating portfolio, which is systematically adjusted to replicate the payoff from an option position, is based on the option pricing formulas developed by Black and Scholes.[77] The pricing formulas from the Black-Scholes model for options on non-dividend paying stock are presented below:[78]

$$call = S_0 * N(d_1) - K * e^{-rT} * N(d_2)$$
$$put = Ke^{-rT} N(-d_2) - S_0 N(-d_1)$$

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}; \quad d_2 = d_1 - \sigma\sqrt{T}$$

$$delta_{call} = N(d_1)$$
$$delta_{put} = -N(-d_1) = N(d_1) - 1$$

S_0... stock price at time 0
K... strike price
r... risk - free rate
σ... volatility of the stock
T... time to maturity of the option
N(x)... cumulative probability distribution for a standardized normal distribution

From all the required input parameters, volatility is the only one that cannot be observed in the market and needs to be estimated. This is a critical point when using the Black-Scholes model as the volatility can have a significant impact on the pricing of an option and thus affect portfolio insurance strategies relying on this model.[79]

An important parameter, which is the key indicator for any option replication strategy is the delta. It measures the effect of a price change in the underlying stock on the price of the option. The creation of a synthetic option requires that at any point in time, the

[74] See Abken (1987), p.7 and Black/Scholes (1973)
[75] See Leland/Rubinstein (1988), p. 2
[76] See Leland/Rubinstein (1981)
[77] See Bennnga (1990), p. 21
[78] See Hull (2005), p. 295, p.346
[79] See Hull (2005), p. 300 and Rendleman/O’Brien (1990), p. 61

replicating portfolio has the same delta as the option it is to replicate.[80] Figure 10 shows the graphical interpretation of the delta of a put option, which is the slope of the option price curve at a particular stock price. As depicted in the second graph the delta of a put option can take on values between -1 and 0.

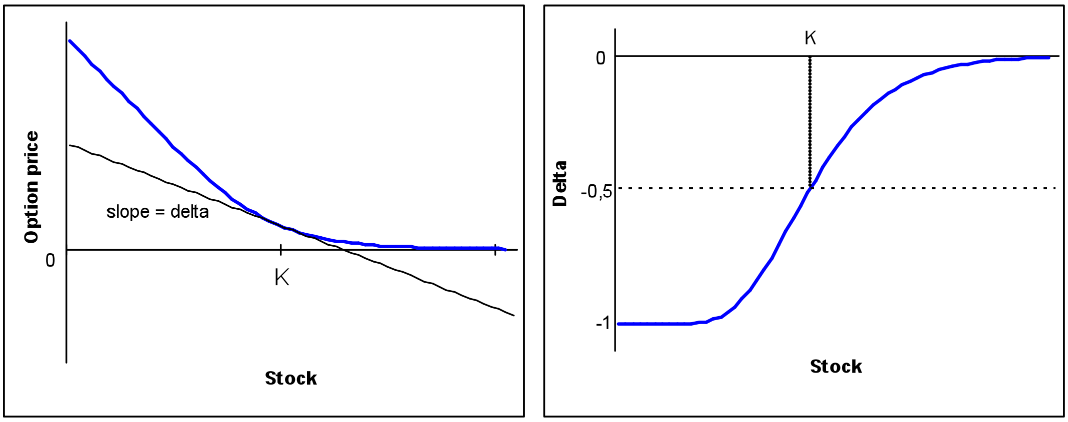

Figure 13: Delta of a put option (see Hull (2005), pp. 345/346)

The synthetic put strategy involves the replication of a portfolio consisting of a risky asset and a put option by investing into a portfolio consisting of the same risky asset and a risk-free asset. The allocation between those two assets is adjusted systematically over the respective period and is determined by the delta, the value of which is a function of time and of the portfolio value.[81]

The total value of a portfolio consisting of a risky asset and a corresponding put option is the stock price at the time of the purchase and the cost of the put, or S_t+P_t. Using the Black-Scholes model this can be stated as follows:[82]

$$\begin{aligned} S_t + P_t &= S_t + \left[Ke^{-r(T-t)}N(-d_2) - S_t N(-d_1)\right] \\ &= S_t\left[1 - N(-d_1)\right] + Ke^{-r(T-t)}N(-d_2) \\ &= S_t N(d_1) + Ke^{-r(T-t)}N(-d_2) \end{aligned}$$

The replication of a portfolio consisting of stocks and corresponding options involves the following positions:[83]

[80] See Hull (2005), p. 344, p. 364
[81] See Annaert/Van Osselaer/Verstreate (2007), p. 7
[82] See Hagen (2002), p. 123 and Benninga (1990), p. 29

	long position in stocks :	S_t
Synthetic Put option	short position in stocks :	$-N(-d_1)S_t$
	long position in riskfree asset :	$Ke^{-r(T-t)}N(-d_2)$
	Net positions	
	long position in stocks :	$N(d_1)S_t$
	long position in riskfree asset :	$K * e^{-r(T-t)} * N(-d_2)$

When creating a synthetic put the delta indicates the amount of the underlying stock which is to be sold. Netting out short and long positions the synthetic put strategy results in a long position in the risky asset and an investment in the risk-free asset. The fractions of funds, which are to be allocated to the risky asset and to the riskless asset at each time t are determined as:[84]

$$\omega_{risky} = \frac{S_t N(d_1)}{S_t N(d_1) + Ke^{-r(T-t)}N(-d_2)}$$

$$\omega_{riskf\,ree} = \frac{Ke^{-r(T-t)}N(-d_2)}{S_t N(d_1) + Ke^{-r(T-t)}N(-d_2)}$$

The strike price at the initiation of the strategy, which corresponds with the required floor, is set accordingly: [85]

$$K = (1 + r_{Floor}) * (S + P) - D$$

r_{Floor} ...required floor - return

D ...Dividend paid before maturity

The synthetic put and the protective put strategy will have exactly the same payoff only if the replicating portfolio is being revised continuously. In practice a continuous rebalancing of the portfolio is not possible, resulting in replication errors, which may be gains or losses. This is of particular relevance in the presence of jumps in stock prices, which give no opportunity for a timely rebalancing of the portfolio. Thus, some

[83] See Hagen (2002), p. 123
[84] See Hagen (2002), p. 123 and Benninga (1990), p. 29
[85] See Clarke/Arnott (1987), p.47

uncertainty accompanies the synthetic put strategy with respect to cost and actual protection.[86]

As the creation of a synthetic put does not require the purchase of an option but simply relies on a systematically rebalanced asset allocation, it might appear that this form of insurance has no cost. However the initial investment involves the allocation of funds between the risky asset and the risk-free asset. As not all funds are invested into the risky asset the upside potential of this portfolio is reduced. By the time of expiration of the strategy the opportunity cost in the form of the reduced upside capture will amount to the initial value of the put option.[87]

Figure 14 illustrates this fact showing the payoff from a simulated synthetic put strategy with 10000 possible paths for the index.[88] Whenever the index performs poorly the synthetic put strategy proves to yield the desired floor value in the amount of the initial investment. In times when the index performs strongly reaching values above the floor level, the synthetic put strategy is able to capture the upside of the index. However, the terminal value of the insured portfolio is always below the value of the index, which is the implicit cost of a portfolio insurance strategy.

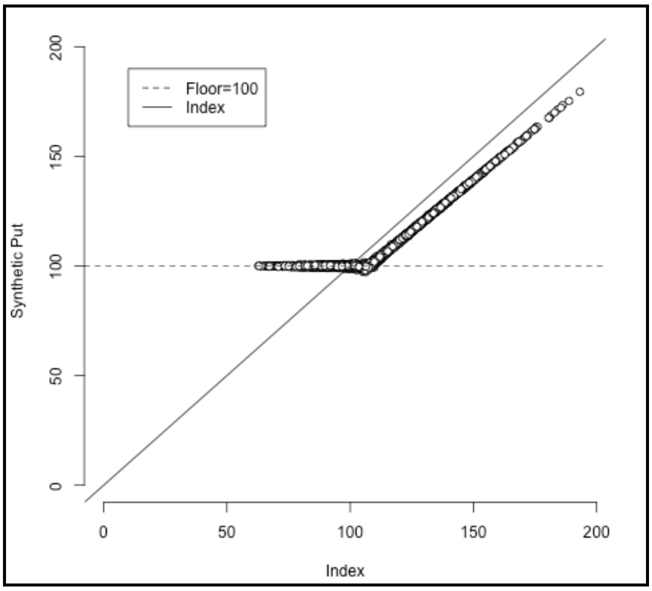

Figure 14: Payoff-diagram, synthetic put (simulation result)

86 See Abken (1987), p. 10
87 See Abken (1987), p. 9 and Leland/Rubinstein (1981), p. 297
88 Details on the simulation-method employed together with further results will be presented in the last chapter.

5.1.1. *Assessment of the Synthetic Put Strategy*

The possibility of creating synthetic options with a dynamically adjusted portfolio of the underlying stock and the risk-free asset enabled the implementation of portfolio insurance at a time when index options were not available. It also allows insuring a risky portfolio with a synthetic put when traded put options don't offer the desired strike price, don't have maturities which match the investor's investment horizon or pose other limitations to the implementation of a portfolio insurance strategy. Under orderly market conditions the strategy of systematically rebalancing a portfolio of stocks and the risk-free asset, will result in the same payoff as a strategy involving the purchase of corresponding outright options.[89]

However, the synthetic put strategy has its drawbacks, which need to be considered. A continuous rebalancing of the replicating portfolio, as required for the equivalence with a protective put strategy, is not feasible in practice. Particularly in the presence of price jumps this might pose some risk to the reliability of the insured floor.[90]

The strategy requires the estimation of the stock's volatility, which introduces another source of uncertainty. Underestimating the volatility will result in less insurance than actually needed, whereas overestimation may lead to more insurance than actually required. This may result in uncertainty for an investor pursuing a portfolio insurance strategy, with respect to the actual protection and the forgone gains.[91]

Another critical point is the time constraint of this strategy. At maturity, a portfolio following a synthetic put strategy will be either fully invested in the risky asset, in case the synthetic put expires out of the money, or it will be fully invested in the risk-free asset, in case the synthetic put expires in the money. If the investor wants to roll over his insurance strategy a radical reallocation of his portfolio will be necessary, even though the investor's risk perception or market conditions didn't change significantly.[92]

[89] See O'Brien (1988), p. 40
[90] See O'Brien (1988), p. 41
[91] See Rendleman/O'Brien (1990), p. 61
[92] See Black/Rouhani (1989), p. 699

5.2. Modified Stop-Loss Strategy

One of the main disadvantages of the static stop-loss strategy is its strong path dependency. Should the portfolio, which initially is invested entirely into the risky asset, fall below the present value of the prespecified floor, all funds are transferred to the riskless asset so as to guarantee a floor value at the end of the investment horizon. Once the funds were transferred no participation in the subsequent market development is possible, even if a recovery followed and a portfolio value above the floor level would be attainable.

Bird, Dennis and Tippett suggested a modification to the stop-loss approach, which allows for a participation in a recovery of the market after the floor has been breached. To reduce the path dependency the funds are moved more gradually between the risky asset and the risk-free asset. The rebalancing of the portfolio is determined by two factors:[93]

1. Securing a minimum portfolio value at the end of the investment horizon in the amount of the initial funds invested.
2. The assumption that the value of the risky asset will not move over the remaining investment horizon.

In the following, the modified stop-loss approach will be illustrated with an example.[94] The initial funds amount to 100.000,- which also constitutes the floor value to be insured over the investment period of one year. A risk-free rate of 10% is assumed. According to the assumption that the value of the risky asset does not change over the year, all funds are initially invested in the risky asset at an index level of 100. Should the stock index move, however, the portfolio allocation needs to be adjusted. The adjustment is made according to the following rule:[95]

[93] See Bird/Dennis/Tippett (1988), p. 35
[94] See Bird/Dennis/Tippett (1988), pp. 35 - 36
[95] See Hagen (2002), p. 1114

$$P_0 = F_T = A_t + B_t e^{r(T-t)} - \Delta + \Delta e^{r(T-t)}$$

A_t ... value of risky asset at time t
B_t ... value of riskfree asset at time t
P_0 ... initial portfolio value
r... risk - free rate
F_T ... floor value at maturity
Δ... amount of funds to be shifted from risky asset to riskfree asset

After the first quarter the stock index drops to 98 resulting in a portfolio value of 98.000,-. According to the formula above, an amount of 25.679,- is to be transferred from the risky asset to the risk-free asset:

$$100.000 = 98.000 - \Delta + \Delta e^{0,075} \rightarrow \Delta = 25.679$$

1.Quarter	Risky Asset	Risk-free Asset	Total value	Final Value at T
before	98.000	0	98.000	98.000
after	72.321	25.679	98.000	100.000

The amount of funds remaining in the risky asset is 72.321,-. Under the assumption of no further movement in the stock index, the portfolio consisting of the risky asset and the risk-free investment, which by the end of the year will accumulate interest and yield 25.679*$e^{0,075}$=27.679, sums up to a final value which equals the required floor of 100.000,-. At the end of the second quarter the index drops further to a level of 95.123,- resulting in a decline in value of the risky asset to 70.198,-. Again, additional funds in the amount of 41.410,- are transferred to the risk-free asset:

$$100.000 = 70.198 + 25.679e^{0,05} - \Delta + \Delta e^{0,05} \rightarrow \Delta = 41.410$$

2.Quarter	Risky Asset	Risk-free Asset	Total value	Final Value at T
before	70.198	26.329	96.527	97.877
after	28.788	67.739	96.527	100.000

After rebalancing, the portfolio consists of 28.788,- invested in the risky asset and 67.739,- invested in the risk-free asset. Over the course of the third quarter the index level rises to a level of 98.000,- resulting in an increase in value of the risky asset to 29.659,-. This time funds in the amount of 34.396,- are transferred away from the risk-free asset to the risky asset.

$$100.000 = 29.659 + 67.739e^{0,025} - \Delta + \Delta e^{0,025} \rightarrow \Delta = -34.394$$

3.Quarter	Risky Asset	Risk-free Asset	Total value	Final Value at T
before	29.658	69.454	99.112	102.673
after	64.052	35.060	99.112	100.000

In the final quarter the stock index increases further to a level of 110,517 resulting in an increase in value of the risky asset to 72.233,-. The risk-free investment has accumulated interest and yielded 35.060*$e^{0,025}$=35.948,- which together with the risky asset amounts to a final portfolio value of 108.181,-.

4.Quarter	Risky Asset	Risk-free Asset	Total value
after	72.233	35.948	108.181

Table 2 compares the modified stop-loss strategy with the conventional stop-loss approach. The improvement is evident from the terminal value of the portfolio. The table illustrates again the path-dependency of the conventional stop-loss strategy, where no participation in the recovery of the stock index is possible once the floor has been breached and all funds been transferred to the risk-free asset.

			Stop-loss		Modified Stop-loss	
	Index	PV of Floor	Risky	Risk-free	Risky	Risk-free
	100	90.484	100.000	0	100.000	0
1. Quarter	98	92.774	98.000	0	72.321	25679
2. Quarter	95,123	95.123	0	95.123	28.788	67739
3. Quarter	98	97.531	0	97.531	64.052	35060
4. Quarter	110,52	100.000	0	100.000	72.233	35948
		Total Value	100.000		108.181	

Table 2: Stop-Loss vs. Modified Stop-Loss (See Bird/Dennis/Tippett (1988), p. 36)

5.2.1. *Assessment of the Modified Stop-Loss Strategy*

As illustrated by the example, the modified stop-loss strategy is an improvement over the static stop loss as it moves funds more gradually between the risky and the risk-free asset. Another advantage is that it is not based on a particular pricing model and does not require the estimation of the volatility, as it is the case with the synthetic put strategy. However, other simulations have shown that the modified stop loss offers less protection against negative returns than the synthetic put and is also inferior with respect to path dependency.[96]

5.3. Constant Proportion Portfolio Insurance – CPPI

The CPPI strategy was introduced in papers by Perold and by Black and Jones. Similar to the dynamic strategies introduced previously, the CPPI-strategy comprises a set of trading rules, which systematically shift funds between a risky asset and a risk-free asset in order to guarantee the desired floor while preserving the upside potential of a portfolio. The basic framework for the CPPI strategy is presented below:[97]

- **Floor (F):** The floor value corresponds to the minimum portfolio value, which is to be maintained over the entire investment horizon. It has to be lower than the initial funds and grows at the riskless rate over the course of the investment horizon.[98]
- **Cushion (C):** The cushion is calculated as the difference between the current portfolio value (P) and the floor.
- **Exposure (E):** The exposure represents the portion of the portfolio allocated to the risky asset.
- **Multiplier (M):** The multiplier takes on values larger than 1 and determines the riskiness of the CPPI strategy.

The portion of the portfolio to be allocated to the risky asset is determined by the cushion and the multiplier in the following way:

$$\begin{array}{ll} \text{Cushion:} & C = \max(P - F\ ;\ 0) \\ \text{Exposure:} & E = C * M = \max(P - F; 0) * M \end{array}$$

The reciprocal value of the multiplier indicates by how much the funds invested in the risky asset can fall, before the portfolio undershoots the floor value:[99]

96 See Bird/Dennis/Tippett (1988), p. 40
97 See Bossert/Burzin (2002), p. 139
98 See Perold/Sharpe (1995), p. 154
99 See Bruns/Meyer-Bullerdiek (2008), p. 150

$$\frac{1}{M} = \frac{P - F}{E}; \text{ for } P - F \geq 0$$

A short example will illustrate the basic mechanism of the CPPI strategy.[100] We consider an initial portfolio value of €1m with a desired minimum portfolio value of €900.000,- and a multiplier of 2. Based on these parameters the initial asset allocation is determined as:

$$\begin{array}{lll} t_0: & \text{Cushion} & \Rightarrow 1.000.000 - 900.000 = 100.000 \\ & \text{Exposure} & \Rightarrow 100.000 * 2 = 200.000 \\ & \text{Risk - free} & \Rightarrow 1.000.000 - 200.000 = 800.000 \end{array}$$

Thus, the exposure of €200.000,- is the amount invested in the risky asset at initiation. The remainder of €800.000,- is allocated to the risk-free asset.[101] In the following period the stock market increases by 10% and so do the funds invested in the risky asset, yielding an €220.000,-. The overall portfolio value increases to €1.020.000 and so does the cushion, as the floor value remains the same. The multiplier again determines the exposure:

$$\begin{array}{lll} t_1: & \text{Cushion} & \Rightarrow 1.020.000 - 900.000 = 120.000 \\ & \text{Exposure} & \Rightarrow 120.000 * 2 = 240.000 \\ & \text{Risk - free} & \Rightarrow 1.020.000 - 240.000 = 780.000 \end{array}$$

In the following period the stock market experiences a decline of 15% which diminishes the funds invested in the risky asset by the same rate to an amount of €204.000,-. As a consequence the overall portfolio value decreases to €984.000,- resulting in a lower cushion than in the previous period. Thus, the exposure is adjusted again by applying the multiplier, which results in selling a portion of the risky asset to reduce the exposure adequately:

$$\begin{array}{lll} t_2: & \text{Cushion} & \Rightarrow 984.000 - 900.000 = 84.000 \\ & \text{Exposure} & \Rightarrow 84.000 * 2 = 168.000 \\ & \text{Risk - free} & \Rightarrow 984.000 - 168.000 = 816.000 \end{array}$$

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[100] Based on Bruns/Meyer-Bullerdiek (2008), p. 150.

[101] For the purpose of illustration, interest payments from the risk-free asset will be neglected.

Figure 15 visualizes each step of the CPPI strategy as presented in the example. It illustrates the trading rule of the CPPI strategy, which is to keep the portfolio's exposure to the risky asset a constant multiple of the cushion.[102] Thus, it is buying stocks as they rise and selling stocks as they fall.[103]

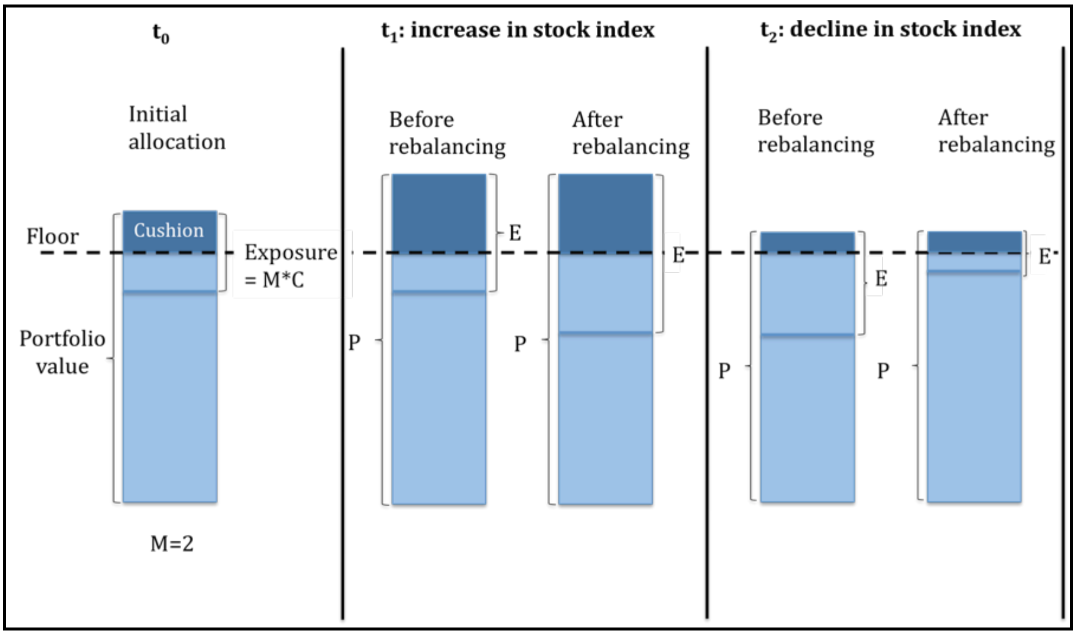

Figure 15: Rebalancing under the CPPI strategy

To illustrate the properties of the CPPI strategy in more detail the above example will be extended to 10 periods.[104] Without any restrictions the CPPI strategy can result in short positions in either the risk-free asset or the risky asset. In practice the CPPI strategy is used with restrictions constraining it to only long positions in either of the assets.[105] The CPPI model used in the following examples underlies those restrictions. The strategy will be analyzed under three different scenarios, one with a constant decline in the stock index, one with a constant increase in the stock index and a crash scenario with a recovery of the stock index.

Table 3 shows the performance of the CPPI strategy during a constant decline in the stock index. The strategy systematically reduces the exposure to the risky asset as the stock market declines. In period 9, when the investment in the risky asset declines

[102] See Perold/Sharpe (1995), p. 154
[103] See Perold/Sharpe (1995), p. 155
[104] Based on Bossert/Burzin (2002), p. 142/146 and Meyer-Bullerdiek/Schulz (2004), p. 56/57/64
[105] See Benninga (1990), p.22

further, resulting in a portfolio value equal to the floor of €900.000, the cushion reduces to zero and all funds are transferred to the risk-free asset. Thus, the CPPI strategy is able to deliver the desired floor value during declining stock markets.[106]

t	Stock index	CPPI	Floor	Cushion	Exposure (t-1)	Riskfree (t-1)	Exposure (t)	Riskfree (t)
0	100%	1.000.000	900.000	100.000	200.000	800.000	200.000	800.000
1	90%	980.000	900.000	80.000	180.000	800.000	160.000	820.000
2	80%	962.222	900.000	62.222	142.222	820.000	124.444	837.778
3	70%	946.667	900.000	46.667	108.889	837.778	93.333	853.333
4	60%	933.333	900.000	33.333	80.000	853.333	66.667	866.667
5	50%	922.222	900.000	22.222	55.556	866.667	44.444	877.778
6	40%	913.333	900.000	13.333	35.556	877.778	26.667	886.667
7	30%	906.667	900.000	6.667	20.000	886.667	13.333	893.333
8	20%	902.222	900.000	2.222	8.889	893.333	4.444	897.778
9	10%	900.000	900.000	0	2.222	897.778	0	900.000
10	0%	900.000	900.000	0	0	900.000	0	900.000

Table 3: CPPI strategy with a multiple m=2 during a constant decline in the stock index

The performance of the respective CPPI strategy during a constantly rising stock market is illustrated in table 4. As the stock market rises, the strategy systematically increases the exposure to the risky asset, while reducing the investment in the risk-free asset. The additional exposure allows for an increased participation in consecutive upward movements of the market. [107]

t	Stock index	CPPI	Floor	Cushion	Exposure (t-1)	Riskfree (t-1)	Exposure (t)	Riskfree (t)
0	100%	1.000.000	900.000	100.000	200.000	800.000	200.000	800.000
1	110%	1.020.000	900.000	120.000	220.000	800.000	240.000	780.000
2	120%	1.041.818	900.000	141.818	261.818	780.000	283.636	758.182
3	130%	1.065.455	900.000	165.455	307.273	758.182	330.909	734.545
4	140%	1.090.909	900.000	190.909	356.364	734.545	381.818	709.091
5	150%	1.118.182	900.000	218.182	409.091	709.091	436.364	681.818
6	160%	1.147.273	900.000	247.273	465.455	681.818	494.545	652.727
7	170%	1.178.182	900.000	278.182	525.455	652.727	556.364	621.818
8	180%	1.210.909	900.000	310.909	589.091	621.818	621.818	589.091
9	190%	1.245.455	900.000	345.455	656.364	589.091	690.909	554.545
10	200%	1.281.818	900.000	381.818	727.273	554.545	763.636	518.182

Table 4: CPPI strategy with a multiple m=2 during a constant increase in the stock index

The less desired property of the CPPI strategy, which is its path dependency, is revealed when not continuous market developments are considered, as illustrated in table 5. Initially, the stock market increases strongly resulting in the CPPI strategy increasing its

[106] See Bruns/Meyer-Bullerdiek (2008), p. 151
[107] See Bruns/Meyer-Bullerdiek (2008), p. 152

exposure to the risky asset and reducing the investment in the risk-free asset. In period 4 the stock market crashes suddenly and falls by 50%. The portion of the portfolio invested in the risky asset, which has increased over the first three periods, is fully hit by the crash and declines in value by 50%. Now, the overall portfolio value equals the floor value leaving no cushion for investments in the risky asset. All funds are transferred to the risk-free asset, so as to guarantee the floor value. However, no further participation in the following recovery is possible, as no cushion is left to allow an investment in the risky asset, which again prevents the build up of any cushion in the following periods.[108]

t	Stock index	CPPI	Floor	Cushion	Exposure (t-1)	Riskfree (t-1)	Exposure (t)	Riskfree (t)
0	100%	1.000.000	900.000	100.000	200.000	800.000	200.000	800.000
1	115%	1.030.000	900.000	130.000	230.000	800.000	260.000	770.000
2	125%	1.052.609	900.000	152.609	282.609	770.000	305.217	747.391
3	140%	1.089.235	900.000	189.235	341.843	747.391	378.470	710.765
4	70%	900.000	900.000	0	189.235	710.765	0	900.000
5	80%	900.000	900.000	0	0	900.000	0	900.000
6	90%	900.000	900.000	0	0	900.000	0	900.000
7	100%	900.000	900.000	0	0	900.000	0	900.000
8	110%	900.000	900.000	0	0	900.000	0	900.000
9	120%	900.000	900.000	0	0	900.000	0	900.000
10	130%	900.000	900.000	0	0	900.000	0	900.000

Table 5: CPPI strategy with a multiple m=2 during a crash scenario with a recovery of the stock index

As illustrated under the third scenario, the portfolio value of a CPPI strategy at the end of the investment horizon does not depend solely on the final value of the underlying stock index. It is determined as well by the path taken over the respective time-period, which makes the CPPI strategy path-dependent.[109] The path-dependency also manifests itself in the fact that the strategy may result in the portfolio being fully invested in either the risky asset or the risk-free asset. At a point where the portfolio is fully invested in the risky asset any further appreciation of the stock index will have no impact on the portfolio composition. However, if the price path occurs at a point where the portfolio is only partially invested in the risky asset, any further appreciation of the stock-index will change the portfolio composition.[110]

[108] See Black/Perold (1992), p. 414
[109] See Bossert/Burzin (2002), p. 148
[110] See Bookstaber/Langsam (1988), p. 19/20

Having illustrated the properties of the CPPI strategy under different market conditions, we need to examine the effect of the multiplier. As it is with the floor value, the investor chooses the multiplier at initiation of the strategy, corresponding to his risk preferences.[111] Figure 16 shows the effect of three different multipliers on the performance of a CPPI strategy under the previously considered scenarios.

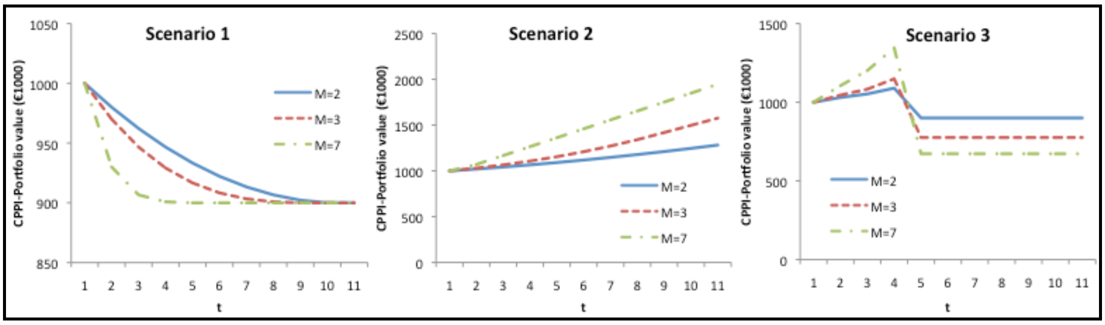

Figure 16: Performance of CPPI-strategies with multipliers 2, 3 and 7

Increasing the multiplier from the initial value of 2 to 3 and 7, results in the portfolio value moving faster towards the floor value during sustained decreases in the stock index. Similarly, the higher the multiplier the higher is the portfolios participation in sustained upward movements of the market. The results under the third scenario confirm these observations and in addition illustrate another effect of the choice of the multiplier. As already mentioned in the beginning of this chapter, the reciprocal value of the multiplier indicates by how much the funds invested in the risky asset can fall, before the portfolio undershoots the floor value. When choosing a multiplier of 2, the stock market can experience a maximum decline of 50% in order to retain the desired floor. In period 4 of the third scenario the market crashes by 50% and the CPPI strategy with a multiplier of 2 delivers the desired floor value as expected. The strategies involving higher multipliers of 3 and 7 can sustain market declines of only 33% and 14%, respectively and thus fall below the floor value of €900.000 in the third scenario.

Figure 17 shows a payoff-diagram for three CPPI-strategies to further illustrate the effect of the multiplier and is based on a simulation of 1000 possible paths. The higher the multiplier the faster the exposure increases, which allows to capture more of the market's upside. On the other hand, strategies using higher multipliers experience their

[111] See Bertrand/Prigent (2001), p. 2

portfolio value to move faster towards the floor under less favourable market developments. The opposite is observable for low values of the multiplier.

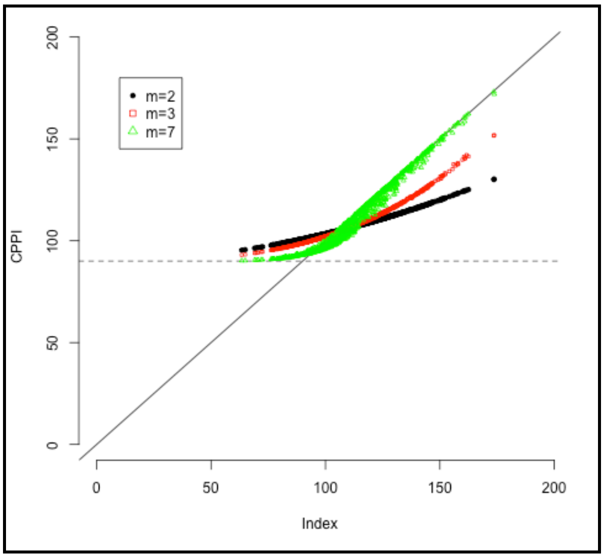

Figure 17: Payoff-diagram, CPPI (Floor=90, simulation results)

In general, the multiplier determines the aggressiveness of the CPPI strategy, with higher values allowing to capture more of the upside during rising markets and resulting in the portfolio value moving faster towards the floor value during sustained declines of the market.[112]

5.3.1. *Assessment of the CPPI Strategy*

The CPPI strategy offers a very simple decision rule approach to portfolio insurance, when compared to more complex option-based approaches such as the synthetic put strategy. [113] As illustrated under different scenarios, the strategy is able to secure a predetermined floor value under adverse market conditions while allowing for participation in rising markets. By choosing an appropriate floor value and multiplier the investor can determine the aggressiveness of the strategy. In addition to its simple realisation the CPPI approach has the advantage of being perpetual. Thus, at expiration

[112] See Black/Rouhani (1989), p. 703
[113] See Black/Perold (1992), p. 404

of the strategy no significant reallocation of funds is necessary, unlike the synthetic put strategy where this is the case.[114]

However, there are certain drawbacks, which need to be considered. Its path dependency introduces some uncertainty to the strategy, which is not desired from an investor's point of view. Moreover, the performance of the CPPI strategy depends strongly on the frequency of portfolio adjustments. The more often a rebalancing of the portfolio takes place, the more reliable the protection will be. On the other hand, increasing the number of portfolio adjustments will likewise increase transaction costs. The costs incurred by a CPPI-strategy making continuous adjustments will affect its performance significantly, implying a tradeoff between reliability and cost-effectiveness.[115]

6. Monte Carlo Analysis

The following analysis determines the impact of two popular portfolio insurance strategies on the shape of the return distribution. Specifically, the cost and reliability of the synthetic put and the CPPI model will be tested under different parameter settings. The analysis will employ Monte Carlo simulation, which by sampling a large number of paths for each strategy offers insight into the underlying return distribution.

The parameters for the simulation are the mean and standard deviation of the S&P500 daily log-returns for the period of 6th January 1961 to 8th May 2009, which are μ=8,85% and σ=15,85%.[116] A total of 10000 paths are generated, each with 250 log-normally distributed returns.[117] Thus, the time horizon for all analysed strategies is set to one year. To assess how different market environments affect the analysed strategies, several time series sets are generated using different volatility parameters. Furthermore, a risk-free rate of 5% is assumed.

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114 See Black/Rouhani (1989), p. 701

115 See Bossert/Burzin (2002), p. 151

116 Source: Datastream, based on S&P500 index data a total return index is calculated, where dividend payments from companies the index comprises are considered.

117 For a more detailed introduction to Monte Carlo simulation see Hull (2005), pp. 410-412.

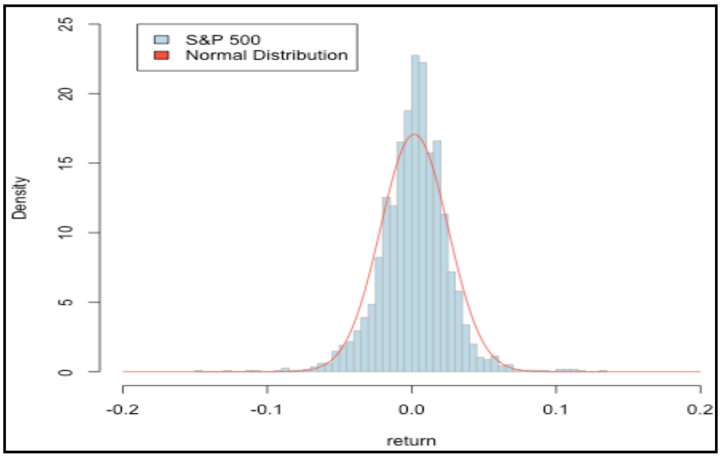

Figure 18: Frequency distribution of S&P 500 returns vs. normal distribution.

In the chapter on risk measurement it was shown that the distribution of S&P500 returns is very close to a normal distribution, as depicted in figure 18. A normal distribution is symmetric around its mean and is fully described by its mean and standard deviation. As portfolio insurance strategies aim to reshape the return distribution by cutting its negative tail, the resulting return distribution is asymmetric as it becomes skewed towards positive returns. Mean and standard deviation no longer suffice to describe the shape of such a distribution, thus additional sample statistics will be used to assess the resulting return distributions more accurately.

To illustrate the implicit cost of portfolio insurance strategies, the impact on the median return will be considered, which is reduced even more than the mean return.[118] The calculation of skewness, semi-variance and quartiles will give some insight into the shape of the resulting return distribution. The probability of returns below the floor-return as well as the expected shortfall[119] will be calculated to assess the reliability of the strategy in securing the required floor.

[118] See Clarke/Arnott (1988), p. 28

[119] In this context the expected shortfall is calculated as the expected value of the negative deviation from the floor-return, given the portfolio-return turns out lower than the floor return.

6.1. Synthetic Put

6.1.1. *Cost and Reliability at Different Floor-Levels*

The mechanics of the synthetic put have been described in detail in the chapter on dynamic strategies. Recalling from that chapter the investor has to provide several input-parameters to implement the strategy, among which is the strike price and the estimate of volatility. At initiation the adequate strike price needs to be determined, which has to be higher than the desired floor, as the implicit cost of the put has to be taken into account.[120] Table 6 illustrates the results of synthetic put strategies for five different floor levels. The last row of the table shows the minimum return corresponding to each floor level on a continuously compounded basis, in order to match the data of the simulation, which builds on the assumption of log-normally distributed returns.[121]

	Uninsured	Floor=80	Floor=85	Floor=90	Floor=95	Floor=100
Put price		0,24%	0,64%	1,48%	3,23%	7,09%
Mean	8,77%	8,67%	8,52%	8,27%	7,81%	7,01%
Median	8,73%	8,48%	8,08%	7,21%	5,41%	1,33%
std.	15,79%	15,45%	14,99%	14,11%	12,54%	9,72%
semi-std.(r_{target}=0%)	6,82%	6,30%	5,69%	4,57%	2,79%	0,19%
Skewness	0,01	0,14	0,28	0,49	0,84	1,49
Min.	-46,49%	-23,88%	-18,83%	-12,53%	-7,98%	-2,83%
1st qu.	-1,91%	-2,16%	-2,58%	-3,47%	-4,59%	0,01%
3rd qu.	19,51%	19,27%	18,89%	18,01%	16,20%	12,13%
Max.	65,86%	65,62%	65,22%	64,36%	62,57%	58,49%
Prob.($r<r_{Floor}$)	-	1,66%	3,51%	7,27%	13,73%	24,06%
Exp. Shortfall	-	0,35%	0,34%	0,32%	0,32%	0,28%
r_{Floor}	-	-22,31%	-16,25%	-10,54%	-5,13%	0,00%

Table 6: Results of the synthetic put strategy with different floor levels.

The first row shows the explicit price for a put option as a percentage of initial funds, as indicated by the Black-Scholes model. As expected, the higher the protection level the higher is the price of a put option to insure the portfolio. Even though the replication of a put option as part of the synthetic put strategy does not require the explicit expense of a put premium, the strategy incurs this cost implicitly. This is evident by the reduction in the mean return, which gets more pronounced the higher the level of protection. At a

[120] See Benninga (1990), p. 29; For the calculation of the Strike price see the chapter on the synthetic put strategy.
[121] See Hull (2005), p.281

floor level of 100 the mean return decreases from 8,77% of the uninsured portfolio to 7,01%. The cost incurred shows up even more in the dramatic reduction in the median return, which decreases from 8,73% to just 1,33%. Basically, the strategy foregoes 7,4% in return during positive years in exchanged for the protection in years with negative returns.[122] As the strategy shifts funds from the risky asset to the riskless asset in times when the stock market performs poorly, the risk measured as the standard deviation decreases as well with a higher floor. At a floor level of 100 the risk decreases even by 6,07% to 9,72%.

As already mentioned in the chapter on risk measurement, the standard deviation is a symmetric measure and as such not appropriate to describe an asymmetric distribution as generated by portfolio insurance strategies. The semi-variance measures only the negative deviation of returns below 0, which gives more insight into the shape of the left tail of the distribution.[123] It shows the synthetic put strategy is able to alter the return distribution as the semi-variance decreases from 6,82% of the uninsured portfolio to just 0,19% of an insured portfolio with a floor of 100. Thus, it proves to be effective in cutting the left tail and reducing the risk of negative returns. The skewness also confirms this property as it increases up to 1,49 indicating a higher probability of positive returns as compared to negative returns.

The quartiles give further insight into the shape of the resulting return distribution. By looking at the minimum and maximum values the synthetic put again proves to deliver the desired insurance properties. In accordance with the desired floor level the downside of the distribution is effectively eliminated while at the same time the ability to participate in years of high returns is retained. However, the inner quartiles reveal that the probability mass somewhat shifts to the left, resulting in a higher probability of moderate returns as the insurance level rises.[124] Where in case of the uninsured portfolio 50% of returns lie in the upper range between 8,73% and 65,86%, the range shifts to 1,33% and 58,49% for the synthetic put portfolio with a floor of 100. With 25% of the uninsured portfolio's returns lying in the range between -1,91% to 8,73% it has a larger probability of returns above the risk-free rate of 5% in the lower half of its distribution than the insured portfolio. This has 25% of its returns fall into the moderate range

[122] See Clarke/Arnott (1988), p.30
[123] Under this definition the semi-variance is the LPM2 with a target rate of 0.
[124] See Clarke/Arnott (1988), p.30

between 0,01% and 1,33%. The benefit of the insured portfolio is revealed in the first quartile, where 25% of returns fall in the range of -2,83% to 0,01%, compared to the range of -46,49% to -1,91% in case of the uninsured portfolio. Figure 19 illustrates the change in the shape of the return distribution resulting from a synthetic put strategy and contrasts it to a normal distribution with a mean and standard deviation of the S&P500 returns.

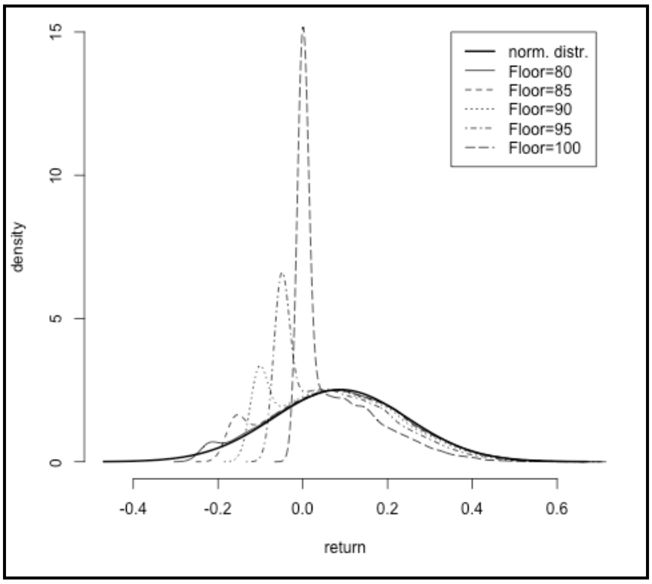

Figure 19: Normal distribution (S&P500 parameters) vs. Synthetic Put (various floors)

Having shown that the synthetic put strategy is able to effectively alter the shape of the return distribution and cut its left tail, the strategy's reliability in securing the prespecified floor is to be tested. The probability of returns below the floor-return rises with the level of protection up to 24% for a floor of 100, which might raise doubts about the reliability of the strategy. However, the expected shortfall, calculated as the expected deviation below the floor-return given the return turns out lower than the floor-return, is very low. Ranging between 0,28% and 0,35% it can be considered as a "protection error" which arises at times when the synthetic put is at the money while approaching expiration. Recalling from the chapter on the synthetic put, when the strategy expires in the money all funds will be invested in the riskless asset. If it expires out of the money all funds will be invested in the risky asset. An at the money synthetic put portfolio which is near expiration has a very high exposure sensitivity to the risky asset. [125] Every movement around the strike price triggers large shifts between the risky

[125] See Black/Rouhani (1989), p.699

and the risk-free asset, which might cause trading losses resulting in a terminal value marginally below the floor level. This however cannot be regarded as a failure of the synthetic put strategy, as the “protection error” is negligibly small as evidenced by the expected shortfall. A scatter-plot for the synthetic put with a floor of 100 is shown in figure 20.[126] It illustrates the strategies ability to deliver the desired insurance in adverse markets while preserving the ability to participate in upward markets.

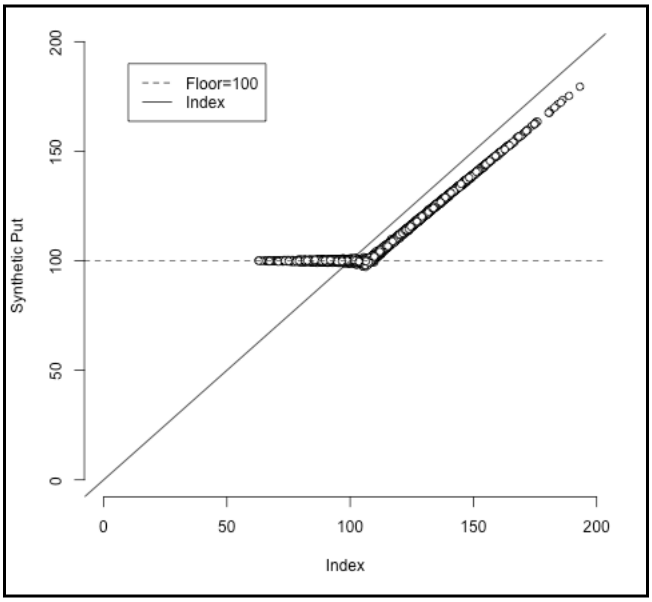

Figure 20: Scatter-plot of a synthetic put with a floor of 100

6.1.2. *Effect of Misestimating Volatility*

The second simulation assesses the impact of misestimating volatility on the cost and reliability of a synthetic put strategy. Volatility is the only parameter entering the Black-Scholes model, which is not observable in the market and thus needs to be estimated. Overestimating the volatility increases the price of a put option resulting in a higher insurance level than desired. Conversely, underestimating the volatility decreases the price of a put option leading to less insurance than required. For the simulation a market volatility of 20% is assumed and a floor level of 90 is chosen. Table 7 presents the results of synthetic put strategies with different estimates of market volatility.

[126] Scatter-plots for the other simulated synthetic put strategies are presented in the appendix.

Floor	90				
σ_{Market}	20%				
estimated σ_{Market}	**10%**	**15%**	**20%**	**25%**	**30%**
Put price	2,94%	6,47%	10,43%	14,75%	19,05%
Mean	8,20%	8,15%	8,10%	8,06%	8,02%
Median	6,15%	5,82%	5,70%	5,65%	5,63%
std.	17,96%	17,19%	16,53%	16,00%	15,48%
semi-std.(r_{target}**=0%)**	6,57%	5,92%	5,43%	5,06%	4,79%
Skewness	0,60	0,67	0,71	0,74	0,76
Min.	-18,45%	-14,85%	-13,20%	-12,07%	-12,46%
1st qu.	-9,56%	-8,81%	-7,83%	-6,94%	-6,17%
3rd qu.	20,70%	19,92%	19,27%	18,67%	18,17%
Max.	84,14%	83,05%	81,88%	80,69%	79,59%
Prob.($r<r_{Floor}$)	21,26%	19,25%	11,43%	4,37%	3,43%
Exp. Shortfall	2,24%	1,05%	0,39%	0,35%	0,53%

Table 7: Results of the synthetic put strategy for different estimates of volatility

When volatility is underestimated, the portion of funds allocated to the risky asset at each point in time is always larger as would be the case with an accurate estimate.[127] The higher exposure over the entire investment horizon explains the increase in the mean and median return when volatility is underestimated. With an estimate 10% below the actual market volatility, the mean increases slightly from 8,1% to 8,2% and the median from 5,7% to 6,15%. At the same time the risk increases from 16,53% to 17,96%. The left tail of the return distribution also extends further to the left as evidenced by a higher semi-variance and a lower skewness, implying an increased probability of negative returns.

The most important effect of underestimation however, is the increase in the probability of returns below the floor-return, accompanied by an increase in the expected shortfall. The probability increases from 11,43% when volatility is estimated accurately with 20% to a probability of 21,26% with an estimate of 10%. The expected shortfall increases likewise from 0,39% to 2,24%. Having considered an expected shortfall in the magnitude of 0,39% as an acceptable protection error, the almost doubling in the probability and the increase in the expected shortfall to 2,24%, which is about 20% of the required floor-return, clearly means a reduction in reliability of the synthetic put.[128]

Overestimating volatility has less severe consequences on the reliability of the synthetic put. It results in a lower portion of funds invested into the risky asset at each point in

[127] See Rendleman/O'Brien (1990), p. 64
[128] See Zhu/Kavee (1988), p.52

time, as would be the case with an accurate estimate. Hence, mean return decreases slightly from 8,1% to 8,02% and the median return from 5,7% to 5,63%, when the market volatility is overestimated by 10%. The return distribution gets more skewed towards positive returns as the semi-variance decreases and the skewness increases. The probability of returns below the floor-return reduces to just 3,43% with only a slight increase in the expected shortfall to 0,53%. Thus, overestimating the volatility results in opportunity costs for the synthetic put in form of a lower mean and median return, although the strategy's reliability in securing the floor is retained.

6.2. CPPI

6.2.1. *Varying the Level of Protection*

Unlike the synthetic put, the CPPI does not build on a complex mathematical model and therefore is easier to implement. The only parameters required from the investor are the desired floor and the multiplier. Table 8 presents the simulation-results for CPPI strategies with different floor-levels and a constant multiplier of 5.

m = 5	Uninsured	Floor=80	Floor=85	Floor=90	Floor=95
Mean	8,77%	8,54%	8,28%	7,82%	7,15%
Median	8,73%	7,85%	5,84%	4,38%	4,38%
std.	15,79%	15,18%	14,18%	12,22%	9,26%
semi-std.(r_{target}=0%)	6,82%	5,48%	4,14%	2,47%	0,88%
Skewness	0,92	0,35	0,63	1,03	1,56
Min.	-46,49%	-20,34%	-14,86%	-9,55%	-4,51%
1st qu.	-1,91%	-3,93%	-3,52%	-1,65%	0,56%
3rd qu.	19,51%	19,24%	18,25%	15,23%	10,85%
Max.	65,86%	65,86%	65,81%	64,34%	59,25%
Prob.($r<r_{Floor}$)	-	0,00%	0,00%	0,00%	0,00%
Exp. Shortfall	-	0,00%	0,00%	0,00%	0,00%

Table 8: CPPI strategy with different floor levels and a constant multiplier of 5.

As in the case of the synthetic put, choosing a higher protection level increases the cost of the insurance strategy as the mean return declines. With a mean return of 8,54% and a floor of 80, it declines to 7,15% when the floor is raised to 95. Again, the reduction in the median return is even more pronounced, as it declines from 7,85% at a floor of 80 to 4,38% at a floor of 95. As the initial cushion is reduced when higher floor levels are chosen, the exposure to the risky asset builds up more slowly resulting in a reduction of risk. The standard-deviation of the portfolio decreases from 15,18% to 9,26%.

The decline in semi-variance and the increase in skewness prove the CPPI's ability to alter the shape of the return distribution. The reduction in semi-variance to 0,88% and a skewness of 1,56 at a floor level of 95 reveal a lower risk of negative returns and a higher probability of positive returns as compared to negative returns. The minimum and maximum values confirm the CPPI's portfolio insurance properties, offering protection during down-markets and allowing participation during favourable market conditions.

With regard to reliability, the CPPI delivers full protection at every floor level, which is evidenced by the probability of returns below the floor-return being 0 in any case. Figure 21 illustrates the change in the shape of the return distribution resulting from a CPPI strategy and contrasts it to a normal distribution with a mean and standard deviation of the S&P500 returns.

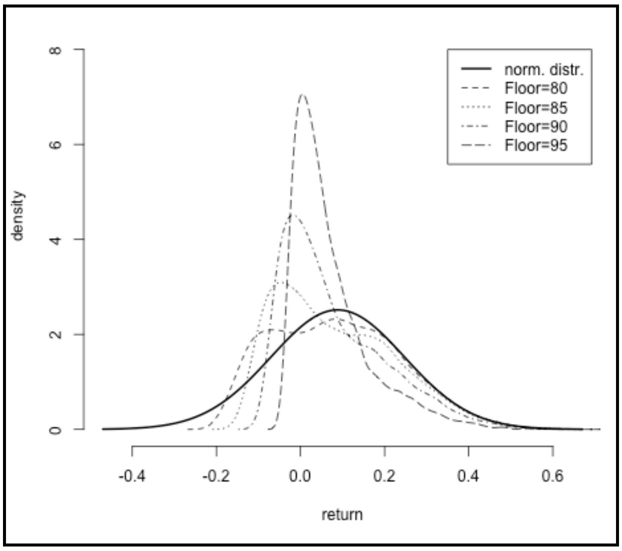

Figure 21: Normal distribution (S&P500 parameters) vs. CPPI (various floors)

6.2.2. *Varying the Multiplier*

To assess the impact of the multiplier, several CPPI strategies with different multipliers and a constant floor of 90 were simulated. Table 9 shows that the selection of higher multipliers indeed allows capturing more of the upside as the mean return increases.

Floor = 90

	Uninsured	m=2	m=3,5	m=5	m=7	m=10
Mean	8,77%	6,38%	7,26%	7,82%	8,13%	8,25%
Median	8,73%	5,72%	5,29%	4,38%	4,85%	6,61%
std.	15,79%	4,97%	9,17%	12,22%	13,96%	14,73%
semi-std.(r_{target}=0%)	6,82%	0,43%	1,42%	2,47%	3,58%	4,44%
Skewness	0,01	0,80	1,24	1,03	0,76	0,58
Min.	-46,49%	-4,85%	-8,09%	-9,55%	-10,26%	-10,50%
1st qu.	-1,91%	2,80%	0,63%	-1,65%	-3,94%	-5,72%
3rd qu.	19,51%	9,27%	11,81%	15,23%	18,02%	18,88%
Max.	65,86%	33,40%	57,95%	64,34%	65,86%	65,86%
Prob.($r<r_{Floor}$)	-	0,00%	0,00%	0,00%	0,00%	0,00%
Exp. Shortfall	-	0,00%	0,00%	0,00%	0,00%	0,00%

Table 9: Results of the CPPI strategy with different multipliers and a constant floor of 90.

Starting with a multiplier of 2 and a mean return of 6,38%, increasing the multiplier to 10 results in a higher mean return of 8,25%. The effect on the median return is somewhat ambiguous as it declines initially when the multiplier increases from 2 to 5. For multipliers above 5 the median return increases again to 6,61% with a multiplier of 10. A higher multiplier results in the exposure being built up faster, which increases the risk of the portfolio. The standard-deviation for a CPPI model increases from 4,97% with a multiplier of 2 to 14,73% with a multiplier of 10. Likewise, semi-variance increases, implying a higher probability of negative returns. The skewness offers a similar picture as the median, increasing initially when the multiplier rises from 2 to 3,5 and decreasing again for multipliers from 5 to 10.

Generally, the multiplier determines the aggressiveness of the CPPI strategy, where higher values accelerate the build up of exposure when markets move upward and make the portfolio move faster towards the floor in adverse markets.[129] With a high multiplier the portfolio is more likely to be fully invested in the risky asset. For instance, with a floor level of 90 and a multiplier of 10 all funds are invested in the risky asset already at initiation. Thus, with an increased probability to be fully invested, the CPPI portfolio will behave similar to an uninsured portfolio for values above the floor.[130] This explains the convergence of mean, median, skewness and standard-deviation towards the values of an uninsured portfolio as the multiplier is increased. In this simulation the selection of the multiplier did not affect the reliability of the strategy, which had a probability of 0 for returns below the floor-return for all tested multipliers.

[129] See Black/Rouhani (1989), p. 703
[130] See Black/Perold (1992), p. 414

6.2.3. *CPPI under Different Market Conditions*

The third simulation analyses the behaviour of the CPPI strategy under different market conditions. Specifically, the performance of a CPPI strategy with a constant floor of 90 and a constant multiplier of 5 is tested under different market volatilities. The results of this simulation are summarized in table 10.

Floor = 90
m = 5

realized σ_{Market}	10%	15,86%	20%	25%	30%
Mean	8,05%	7,82%	7,81%	7,76%	7,70%
Median	6,61%	4,38%	2,67%	0,67%	-1,87%
std.	8,08%	12,22%	14,93%	17,88%	20,87%
semi-std.(r_{target}=0%)	1,06%	2,47%	3,41%	4,40%	5,27%
Skewness	0,75	1,03	1,18	1,34	1,51
Min.	-7,21%	-9,55%	-10,30%	-10,49%	-10,51%
1st qu.	1,94%	-1,65%	-3,83%	-5,84%	-7,51%
3rd qu.	13,09%	15,23%	16,34%	17,33%	18,01%
Max.	52,88%	64,34%	82,69%	104,90%	110,60%
Prob.($r<r_{Floor}$)	-	0,00%	0,00%	0,00%	0,00%
Exp. Shortfall	-	0,00%	0,00%	0,00%	0,00%

Table 10: Results of the CPPI strategy under different volatility scenarios (Floor=90, m=5).

Even though the volatility is not required as a parameter for the CPPI model, it clearly affects the performance of the strategy. The higher the volatility of the market the higher is the cost of insurance. The mean return of 8,05% under a low market volatility of 10% decreases to 7,7% when market volatility is 30%. However, the reduction in the median return is more dramatic. With a median return of 6,61% when market volatility is 10%, it decreases to -1,87% when market volatility is 30%.

The skewness and quartiles hint at what happens to the CPPI's return distribution under different market conditions. The skewness increases clearly from 0,75 to 1,51 implying a higher asymmetry in volatile markets. The minimum values prove the CPPI's reliability to secure the prespecified floor even in highly volatile markets.[131] However, the inner quartiles reveal a large shift of the probability mass to the left, implying a high probability of modest returns under high market volatility. Whereas in case of a

[131] This is not generally valid and might be different with other CPPI-parameters. Specifically, a higher multiplier might result in the portfolio value falling below the floor under high volatility. See the chapter on the CPPI strategy.

volatility of 10%, half of the returns lie in the range of -7,21% to 6,61%, under a volatility of 30% the range shifts to -10,51% and -1,87%. This implies that more than half of all returns of the CPPI strategy are negative when market volatility is high, although not falling below the floor-return.

7. Conclusion

The recent financial market crisis which started out in 2007 and climaxed in the bankruptcy of Lehman Brothers in 2008 causing all major stock markets to plummet, reminded investors once again of the riskiness of stock investments and how unpredictable market movements are. In face of these events, strategies, which allow investors to protect their portfolios when markets move downward while retaining the ability to participate in favourable market movements, might appear to be more relevant than ever.

The aim of this thesis has been to offer an overview of portfolio insurance strategies and to analyse the cost and reliability of two widespread dynamic strategies, the synthetic put and the CPPI. Dynamic strategies are particularly interesting as they offer the desired properties by systematically shifting funds between a risky asset and a risk-free asset. This is done according to a simple rule, as in the case of the CPPI, or according to a complex mathematical model as in the case of the synthetic put.

Even though all strategies presented in this thesis are able to secure a floor to the portfolio value under adverse market-conditions and retain the ability to capture the upside, their drawbacks were discussed as well. Some strategies exhibit a certain degree of path dependency, as in the case of the stop-loss, which is not desired from an investor's point of view as it introduces some uncertainty to the terminal value. The synthetic put for instance, requires an estimate of volatility, which can impair the reliability of protection when underestimated.

To analyse the performance of the synthetic put and the CPPI in more detail, a Monte Carlo simulation was implemented which allowed deducing the return distribution of each strategy. It illustrated the effect of portfolio insurance strategies, which is to alter the shape of the return distribution by cutting its left tail. Both strategies proved to be effective in creating asymmetric return distributions, which were skewed toward positive returns. The simulation also offered insight into the implicit cost of both strategies, which showed up as a reduction in the mean and median return. The higher the level of protection the higher this cost turned out to be. When the volatility was estimated accurately the synthetic put strategy reliably secured the floor return, although in certain cases some "protection error" occurred, which was negligible in its magnitude. Underestimation of volatility has proven to impair the reliability of the strategy. The CPPI strategy was 100% reliable under the tested parameter settings.

Despite the favourable properties of portfolio insurance strategies they are not undisputed. Due to their trading pattern, which is to systematically sell stocks as they fall and purchase stocks as they rise, critics contest portfolio insurance strategies would amplify swings in the market. In fact, portfolio insurance received the blame for having exaggerated the stock market crash of October 1987, where the Dow Jones dropped by 22,6% on a single day.

Nevertheless, portfolio insurance when implemented with its limitations in mind can be a valuable asset-management strategy, as it is testified through the adoption by large funds and pension plans.

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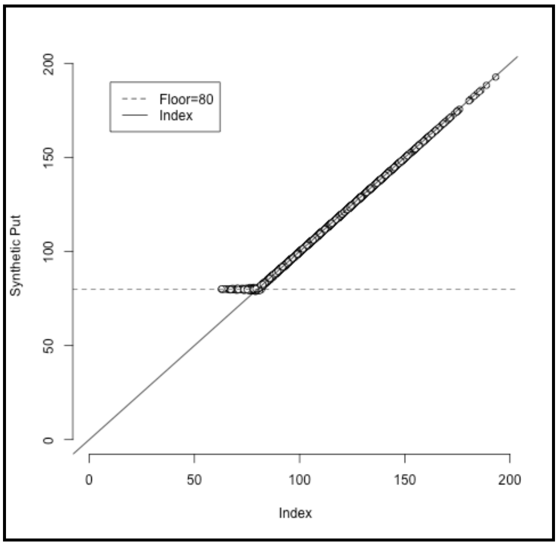
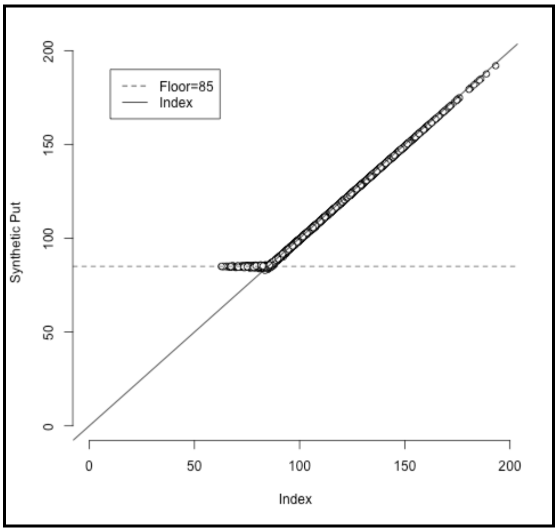
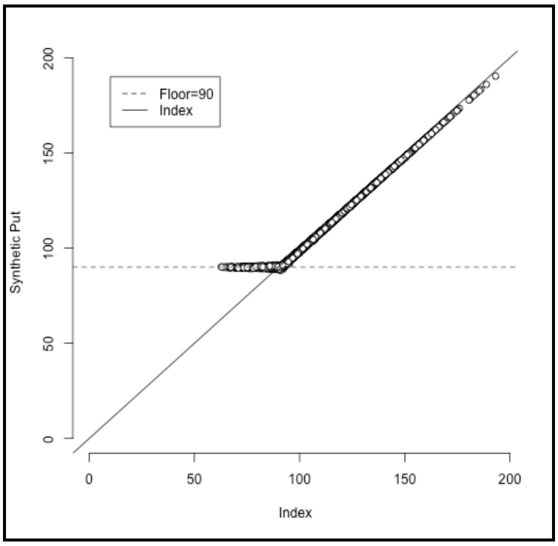
Appendix A: Scatter-plots of Synthetic Put Strategies with Different Floor-Levels:



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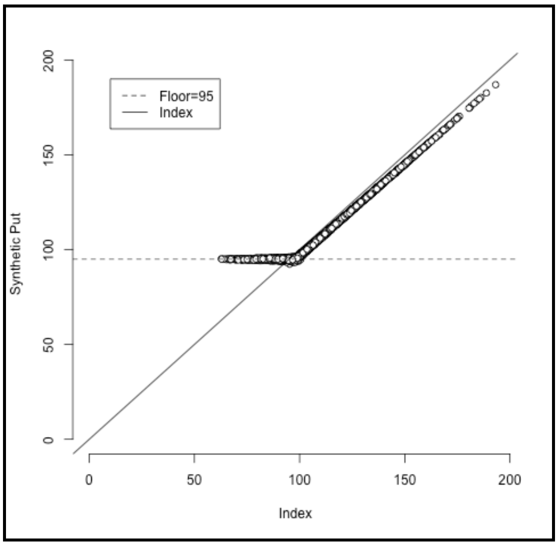
Synthetic Put
Index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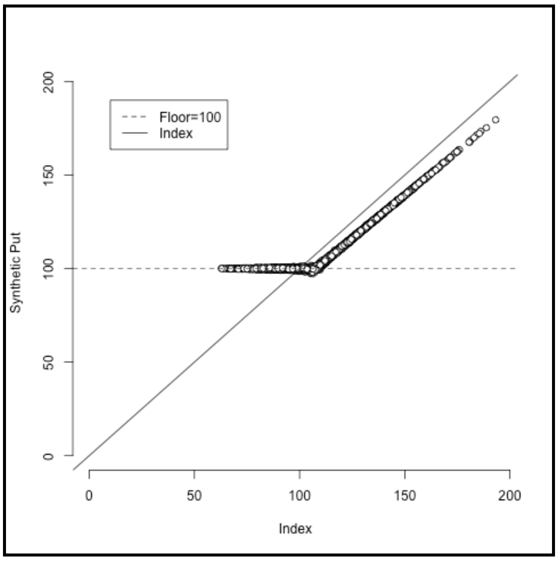
Synthetic Put
Index


Floor=100
Index
Synthetic Put
Index

Appendix B: R-Code

The simulation was implemented in r, a free statistical package available at http://www.r-project.org/. Apart from the functions already implemented in r, the author developed additional functions for the Monte Carlo simulation, the synthetic put and the CPPI. Those functions are presented in this appendix.

Monte Carlo Simulation

- Function for the simulation of returns

```
mc.sim=function(mu,sigma,time.length,time.steps,paths){

	dt=time.length/time.steps;
	a=array(0,c(time.steps,paths));

	for (i in 1:paths) {
		j=1;
		for (j in 1:time.steps) {
			a[j,i]=mu*dt+sigma*sqrt(dt)*rnorm(1,0,1);
			j=j+1;
		}
		i=i+1;
	}
	a
}
```

- Function for the calculation of holding-period returns of an array with simulated returns.

```
mc.hpr=function(sim.array){
	s=1:length(sim.array[1,])

	for(i in 1:length(sim.array[1,])){
		s[i]=sum(sim.array[,i])
	}
	s
}
```

Synthetic Put

- Function for the simulation of a synthetic put strategy

```
sput=function(K,S,T,sigma,r,mc,freq){

        dt=1/freq;
        rows=freq*T;


        P=array(0,c(rows+1,length(mc[1,])));

        for (i in 1:length(mc[1,])){

            P[1,i]=100;
            Index=S;

            for (j in 2:(length(P[,1]))){

                y=j-1;

                d1=(log(Index/K)+(r+sigma^2/2)*(T-
                y*dt))/(sigma*sqrt(T-y*dt));
                d2=d1-sigma*sqrt(T-y*dt);

                N1=pnorm(d1,0,1);
                N2=pnorm(-d2,0,1);

                wRisky=(Index*N1)/(Index*N1+K*exp(-r*(T-
                y*dt))*N2);
                wRF=1-wRisky;

                P[j,i]=wRisky*P[y,i]*exp(mc[y,i]) +
                wRF*P[y,i]*exp(r*dt);
                Index=Index*exp(mc[y,i]);
            }
        }
        P;
}
```

CPPI

- Function for the simulation of a CPPI strategy

```
cppi=function(Floor, m, r, mc){

# calculates a cppi-model starting with a portfolio-value of 100. Input-
parameters are the Floor, the multiplier, the risk-free rate, and an array with
returns coming from a monte-carlo simulation

        P=array(0,c((length(mc[,1])+1),length(mc[1,])));
        T=1
        dt=T/length(mc[,1]);

        for (i in 1:length(mc[1,])){

            P[1,i]=100;
            F=Floor*exp(-r*T);

            for (j in 1:(length(mc[,1]))){

                if (P[j,i]>F) {
                    C=P[j,i]-F;
                    E=C*m;
                    RF=P[j,i]-E;
                    if (E>P[j,i]){
                        P[j+1,i]=P[j,i]*exp(mc[j,i]);
                    }
                    else {

                P[j+1,i]=E*exp(mc[j,i])+RF*exp(r/250);
                    }
                }
                else {
                    P[j+1,i]=P[j,i]*exp(r*dt);
                }
                    F=Floor*exp(-r*(T-j*dt));
            }
        }
        P;
```

Abstract (Deutsch)

Diese Magisterarbeit stellt mehrere Wertsicherungs-Strategien vor und präsentiert die Ergebnisse einer Monte Carlo-Analyse zur Beurteilung der impliziten Kosten und der Zuverlässigkeit von zwei weit verbreiteten Strategien. Im ersten Teil der Arbeit wird die Nachfrage nach solchen Strategien erläutert und deren Entstehung dokumentiert. Im nachfolgenden Kapitel werden verschiedene Risko-Maße vorgestellt, welche später in der Simulation verwendet werden. Die einzelnen Strategien werden entsprechend ihrer Kategorisierung in statische und dynamische Strategie, beschrieben. Die vorgestellten statischen Strategien sind 1) Buy&Hold, 2) Stop-Loss, 3) Protective Put und 4) deren Equivalent mit Einsatz von Call-Optionen. Die vorgestellten dynamischen Strategien sind 1) Synthetic Put, 2) Modified Stop-Loss und 3) Constant Proportion Portfolio Insurance (CPPI).

Der Hauptteil dieser Arbeit ist eine detaillierte Analyse der Synthetic Put und CPPI Strategien unter Einsatz der Monte-Carlo-Simulation. Das Ziel ist eine Beurteilung der impliziten Kosten und der Zuverlässigkeit beider Strategien durch das Betrachten der gesamten Wahrscheinlichkeitsverteilung der Renditen der abgesicherten Portfolios. Die Simulation zeigt, dass beide Strategien in der Lage sind eine asymmetrische Wahrscheinlichketsverteilung zu generieren welche eine Schiefe in Richtung positiver Renditen hat. Somit ermöglichen es beide einen Portfolio-Mindestwert zu sichern und gleichzeitg die Partizipation an steigenden Märkten zu erlauben. Allerdings verdeutlicht die Simulation auch die Kosten einer solchen Absicherung, welche sich durch eine Verringerung im Mittelwert und Median der Portfolio-Renditen niederschlägt. Die Synthetic Put Strategie hat sich als zuverlässig erwiesen, wenn die Schätzung der Volatilität genau ist. Es ergibt sich lediglich ein kleiner "Abischerungs-Fehler" wenn der Synthetic Put kurz vor auslaufen am Geld steht, welcher jedoch in der Höhe vernachlässigbar ist. Wird die Volatilität unterschätzt, so schafft es die Strategie nicht den gewünschten Mindestwert zu sichern. Die CPPI Strategie hat sich unter allen getesteten Parametern als zuverlässig erwiesen. Mit steigendem Multiplikator stieg auch der Mittelwert und Median der Portfolio-Renditen an. Jedoch zeigte sich auch eine deutliche Verschiebung der Wahrscheinlichkeitsmasse in Rendite-Bereiche weit unter dem risikolosen Zinssatz.

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09/1987-06/1991: Primary School, Vienna, Austria

Extra-curricular activities

10/2007 – 05/2009: **Portfoliomanagement-Programme**
Cooperation between the Vienna University of Economics and Business Administration and the ZZ Vermögensverwaltung GmbH
Training in portfoliomanagement, research and analysis of financial data.

Work Experience

08/2009-09/2009: Accenture
Internship-Project Management
Reporting and controlling tasks to track the progress of the project. Customizing and developing excel-tools (VBA-programming).

07/2008-09/2008: Deutsche Bank AG, Frankfurt am Main
Internship-Global Markets
Macro Research

02/2005-12/2007: Raiffeisen Informatik GmbH
Network Operations Center
Providing server and network support for customers and in-house equipment.

07/2004-08/2004: Austrian Energy Agency
Internship
Research and preparation of reports on feed-in tariffs and support mechanisms for renewable energy in CEE.

02/2001 - 06/2004: AXA Versicherung
working student

Language Skills: German, English, Polish
IT-Skills: MS-Word, MS-Powerpoint , MS-Excel (incl. VBA), Datastream