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Optimal spin current pattern for fast domain wall propagation in nanowires

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Abstract – One of the important issues in nanomagnetism is to lower the current needed for a technologically useful domain wall (DW) propagation speed. Based on the modified Landau-Lifshitz-Gilbert (LLG) equation with both Slonczewski spin-transfer torque and the field-like torque, we derive an optimal temporally and spatially varying spin current pattern for fast DW propagation along nanowires. Under such conditions, the DW velocity in biaxial wires can be enhanced as much as tens of times higher than that achieved in experiments so far. Moreover, the fast variation of spin polarization can efficiently help DW depinning. Possible experimental realizations are discussed.

Fast magnetic domain wall (DW) propagation along nanowires by means of electrical currents is presently under intensive study in nanomagnetism experimentally [1–5] and theoretically [6–8]. In addition to the technological interest such as race track memory [1], DW dynamics is also an interesting fundamental problem. The dynamics of a single DW can be qualitatively understood from one-dimensional (1D) analytical models [9] that predict a rigid-body propagation below the Walker breakdown and an oscillatory motion above it [9,10]. The latter process is connected with a series of complicated cyclic transformations of the DW structure and a drastic reduction of the average DW velocity. The Walker limit is thus the maximum velocity at which DW can propagate in magnetic nanowires without changing its inner structure. From a technological point of view, such a limit seems to represent a major obstacle since the fidelity of data transmission may depend on preserving the DW structure while the utility requires speeding up the DW velocity adequately. Various efforts have been made to overcome this limit through the geometry design. For instance, Lewis *et al.* [11] proposed a chirality filter consisting of a cross-shaped trap to preserve the DW structure. Yan *et al.* [12] demonstrated the removal of Walker limit via a micromagnetic study on the current-driven DW motion in cylindrical Permalloy nanowires. Our focus is to find a way to increase the velocity-current slope below the Walker breakdown.

A DW propagates under a spin-polarized current through angular-momentum transfer from conduction electrons to the local magnetization, known as the spin-transfer torque (STT) [13], which is different from the magnetic-field–driven DW propagation originated from the energy dissipation [10,14,15]. Two configurations have been studied so far. One is the mostly studied case in which current is along the wire axis [1–5]. The STT exerted in this configuration is very small because the angle between the current spin polarization direction and local magnetization is very small everywhere. Very recently, an alternative setup where the spin current is injected perpendicular to the wire is proposed [16] and experimentally realized [17]. The STT is much larger in this perpendicular configuration. Generally speaking, two types of spin torques exist: the Slonczewski torque [13] (a-term) $\mathbf{T}_a=-\gamma\frac{a_J}{M_s}\mathbf{M}\times(\mathbf{M}\times\mathbf{s})$ and the field-like torque [18,19] (b-term) $\mathbf{T}_b=-\gamma b_J\mathbf{M}\times\mathbf{s}$, where $\gamma=|e|/m_e$, $\mathbf{M}$, $M_s=|\mathbf{M}|$, and $\mathbf{s}$ are the gyromagnetic

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ratio, the magnetization of the magnet, the saturation magnetization, and the spin polarization direction of itinerant electrons, respectively. $a_J = Pj_e\hbar/2d|e|M_s$ and $b_J = \beta a_J$ [13,19] depend on the current density j_e and spin polarization P, where d is the thickness of the free magnetic layer and β is a small dimensionless parameter that describes the relative strength of the field-like torque to the Slonczewski torque and typically ranges from 0 to 0.5 [19,20]. In the case of a recent proposal [16] where a constant spin current is injected perpendicular to the wire with biaxial anisotropy and the spin polarization is along the wire axis, the a-term is incapable of generating a sustained DW motion, unless a very large current is used, while the b-term can induce a DW propagation. However, the b-term is much smaller than a-term for usual magnetic materials [19,20]. Thus, it leads to a large switching current requirement in order to reach a technologically useful DW propagation velocity, but a large current could damage a device or affects its performance. We show that the problem can be figured out if one uses an optimal temporally and spatially varying spin current pattern.

In this letter, we find the optimal spatiotemporal spin current pattern for fast DW propagation whose speed can be enhanced as much as tens of times in biaxial wire. Possible experimental realizations of the spin current pattern are also discussed.

The internal magnetic energy of a nanowire along the z-axis can be expressed as

$$U[\mathbf{M}] = \int \mathrm{d}^3x \left(\frac{J}{2}\left[(\nabla\theta)^2 + \sin^2\theta\,(\nabla\phi)^2\right] + w(\theta,\phi)\right), \tag{1}$$

where θ is the polar angle and ϕ is the azimuthal angle of the local magnetization $\mathbf{m} = \mathbf{M}/M_s$. J and w are the exchange energy constant and magnetic anisotropic energy, respectively. The dynamics of $\mathbf{M}$ is governed by the modified Landau-Lifshitz-Gilbert (LLG) equation [13,16] with both the Slonczewski torque and the field-like torque:

$$\frac{\partial \mathbf{M}}{\partial t} = -\gamma \mathbf{M}\times\mathbf{H}_{eff} + \frac{\alpha}{M_s}\mathbf{M}\times\frac{\partial \mathbf{M}}{\partial t} + \mathbf{T}_a + \mathbf{T}_b, \tag{2}$$

here $\mathbf{H}_{eff} = -\frac{1}{\mu_0}\delta U/\delta\mathbf{M}$ is the effective magnetic field and α is the phenomenological Gilbert damping constant.

Consider a biaxial anisotropy $w(\theta,\phi) = -\frac{K}{2}m_z^2 + \frac{K_\perp}{2}m_x^2$, the effective field takes the form of $\mathbf{H}_{eff} = \frac{1}{\mu_0 M_s}(Km_z\hat{z} - K_\perp m_x\hat{x}) + \frac{J}{\mu_0 M_s^2}\frac{\partial^2\mathbf{M}}{\partial z^2}$. Here K and $K_\perp$ describe energetic anisotropies along easy $\hat{z}$-axis and hard $\hat{x}$-axis, respectively. We assume that all local spins lie in a fixed plane called DW plane, *i.e.*, $\phi(z,t) = \phi(t)$, which should be checked self-consistently late. In the spherical coordinates, eq. (2) becomes

$$\dot{\theta} + \alpha\sin\theta\dot{\phi} = \gamma a_J(s_\theta + \beta s_\phi) + \frac{\gamma K_\perp}{2\mu_0 M_s}\sin\theta\sin 2\phi, \tag{3}$$

$$\sin\theta\dot{\phi} - \alpha\dot{\theta} = \gamma a_J(s_\phi - \beta s_\theta) - \gamma\frac{J\frac{\partial^2\theta}{\partial z^2} - \frac{\sin 2\theta}{2}\left(K + K_\perp\cos^2\phi\right)}{\mu_0 M_s}, \tag{4}$$

where s_r, s_θ, and s_ϕ are the components of the unit spin vector $\mathbf{s}$ along $\mathbf{m}$ (playing the role of $\hat{r}$), $\hat{\theta}$, and $\hat{\phi}$ defined in terms of $\mathbf{m}$, respectively. The DW profile satisfies $J\frac{\partial^2\theta}{\partial z^2} - \frac{\sin 2\theta}{2}\left(K + K_\perp\cos^2\phi\right) = 0$ with the boundary condition of $\theta = 0$ and π at distance. One obtains the famous Walker's DW motion profile $\tan\frac{\theta}{2} = \exp\left(\frac{z - X(t)}{\Delta}\right)$, in which $X(t)$ is the position of the DW center and $\Delta = \sqrt{\frac{J}{K + K_\perp\cos^2\phi}}$ is the DW width resulting from the balance of anisotropy energy and exchange energy [9]. These assumptions are valid under sufficiently low current density which will be demonstrated later. Substituting this DW profile into eqs. (3) and (4), we have

$$-\frac{\dot{X}}{\Delta} + \alpha\dot{\phi} = \gamma\frac{a_J(s_\theta + \beta s_\phi)}{\sin\theta} + \frac{\gamma K_\perp}{2\mu_0 M_s}\sin 2\phi, \tag{5}$$

$$\alpha\frac{\dot{X}}{\Delta} + \dot{\phi} = \gamma\frac{a_J(s_\phi - \beta s_\theta)}{\sin\theta}. \tag{6}$$

Notice s_r does not contribute to the dynamics of DW at all, the efficient way of using current in driving DW motion should align spin polarization as

$$s_r = 0, \qquad s_\theta = \cos\eta, \qquad s_\phi = \sin\eta, \tag{7}$$

with η being the optimization parameter. Furthermore, to ensure the spatial independence of $\dot{X}$ and ϕ, the above equations require a_J to be proportional to $\sin\theta$, so we let $a_J = A_J\sin\theta = A_J\,\mathrm{sech}\left(\frac{z - X(t)}{\Delta}\right)$ with a constant A_J. Thus, we have

$$\dot{X} = \gamma\Delta\frac{\alpha a'_J - b'_J}{1 + \alpha^2} - \Delta\frac{\gamma K_\perp}{2\mu_0 M_s(1 + \alpha^2)}\sin 2\phi, \tag{8}$$

$$\dot{\phi} = \gamma\frac{a'_J + \alpha b'_J}{1 + \alpha^2} + \frac{\alpha\gamma K_\perp}{2\mu_0 M_s(1 + \alpha^2)}\sin 2\phi, \tag{9}$$

where $a'_J(\eta) = A_J(\sin\eta - \beta\cos\eta)$ and $b'_J(\eta) = A_J(\cos\eta + \beta\sin\eta)$. The time dependence of the DW width Δ is neglected in the derivations of eqs. (8) and (9). Equations (8) and (9) become exact when the DW undergoes a rigid-body propagation with $\phi(t) = \phi_0 = \text{const}$. Equations (8) and (9) describe the DW propagation speed and the DW plane precession velocity, and for rigid-body solutions, they give

$$\frac{\alpha K_\perp}{2\mu_0 M_s}\sin 2\phi_0(\eta) = -\left(a'_J + \alpha b'_J\right), \tag{10}$$

$$\Delta(\eta) = \sqrt{\frac{J}{K + K_\perp\cos^2\phi_0(\eta)}}, \tag{11}$$

$$\dot{X}(\eta) = \gamma\frac{a'_J}{\alpha}\Delta(\eta). \tag{12}$$

The spin current pattern is described by η. A different value yields different canted angle, DW width and propagation velocity. From eq. (10), it is straightforward to show that the assumption of rigid-body motion is valid under the condition of $A_J\sqrt{(1+\alpha^2)(1+\beta^2)} \leqslant \frac{\alpha K_\perp}{2\mu_0 M_s}$. Using the materials parameters of Permalloy: $M_s = 8.6\times10^5$ A/m, $K_\perp = 8\times10^5$ J/m^3 [1], α ranging from 0.01 to 0.5 and the field-like parameter β ranging from 0 to 1, we find that our proposed condition can be satisfied if we choose a reasonable value $A_J = 25$ Oe [16], which corresponds to a peak current density of 6×10^{10} A/m^2 when $d = 3$ nm and $P = 0.32$. Before finding the optimized spin current pattern for maximal velocity, let us first consider two special cases. One case is $\eta = \pi$. It gives the velocity $u_1 = \gamma\frac{\beta A_J}{\alpha}\Delta(\pi)$, which is equal to the velocity achieved in one recent experiment [17] where a uniform spin current $j_e = 2d|e|M_s A_J/P\hbar$ is injected perpendicular to the nanowire with electron spin polarization along the z-axis, *i.e.*, $s_r = \cos\theta$, $s_\theta = -\sin\theta$, and $s_\phi = 0$. It shows again that the Slonczewski torque is incapable of generating sustained DW propagation in a biaxial wire while the field-like torque can. However, the velocity is rather small because of $\beta \ll 1$ for usual materials. The DW velocity can be greatly enhanced if the a-term is used. This is the case of $\eta = \frac{\pi}{2}$. It gives the velocity $u_2 = \gamma\frac{A_J}{\alpha}\Delta\left(\frac{\pi}{2}\right)$. In typical materials [20], $\beta \sim 0.1$, so that the velocity can be 10 times larger than u_1. One can see that the DW propagation velocity is greatly enhanced under a modification of the spin polarization and spatially varying current density pattern.

The maximal velocity $\dot{X}_{\max} = \dot{X}(\eta^*)$ at the optimal parameter η^* can be easily found numerically from eqs. (10), (11), and (12). The factor $\lambda = \dot{X}_{\max}/u_1 = \frac{\sin\eta^* - \beta\cos\eta^*}{\beta}\frac{\Delta(\eta^*)}{\Delta(\pi)}$ measures the velocity enhancement. The β^{-1}-dependence of λ for various damping coefficients and typical magnetic parameters is shown in fig. 1(a). It is approximately linear, and insensitive to the damping parameter α. The parameter η^* as a function of β^{-1} for $\alpha = 0.01$ is also plotted in fig. 1(a). η^* decreases with β^{-1} and is saturated for large β^{-1}. We find that η^* is not sensitive to α (the variation is less than 0.1% for $\alpha \in [0.01, 0.5]$, not shown in the figure). Figure 1(b) is the plot of the spatial distribution of s_x, s_y, s_z and a_J for the optimized spin current pattern around the DW center. We note that s_x, s_y, and s_z vary only near the DW center, and approach fixed values away from the DW. A large perpendicular component s_y is required to achieve a large DW velocity. The reason is that the perpendicular spin component induces a large effective field $\mathbf{H}_a = \frac{a_J}{M_s}\mathbf{M}\times\mathbf{s}$. Thus, the DW moves under the Slonczewski torque with a large component along the wire axis. This finding agrees with the common wisdom that the angle between spin polarization and local magnetization should be large in order to increase the STT. It is also very interesting that the current density is finite only near the DW center while it becomes zero at distance, which should greatly lower the energy consumption.

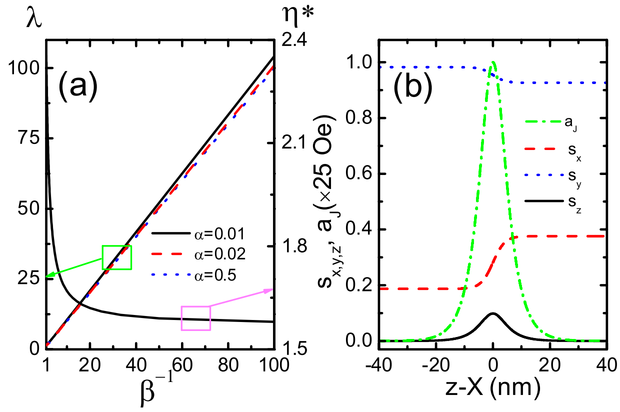

Fig. 1: (Color online) (a) DW velocity enhancement factor λ *vs.* β^{-1} at different damping coefficients and optimized parameter η^* *vs.* β^{-1} for $\alpha = 0.01$. (b) The spatial distribution of x, y, z components of the optimal spin polarization pattern and current density pattern for the maximal DW velocity for $\alpha = 0.01$ and $\beta = 0.1$. The other parameters are using the materials parameters of Permalloy: $M_s = 8.6\times10^5$ A/m, $J = 1.3\times10^{-11}$ J/m, $K = 500$ J/m^3, $K_\perp = 8\times10^5$ J/m^3 [1], and a reasonable value $A_J = 25$ Oe according to ref. [16].

As a comparison, let us consider an ideal wire with uniaxial anisotropy, *i.e.*, $K_\perp = 0$. In ref. [21], we find the velocity enhancement factor $\lambda = \sqrt{1+\left(\frac{\alpha-\beta}{1+\alpha\beta}\right)^2}$, which is rather inconspicuous even if the optimal spin current is used since both α and β are far less than 1 in usual magnetic materials. The physical reason lies in that the a-term is capable of generating a sustained DW motion in uniaxial wire, which is different from the role of the a-term in the biaxial case. This demonstrates that our optimal spin current pattern has especial significance in real magnetic nanowires.

We would like to digress from the main subject here, and to present our observation of the possible effect of the temporal variation of current polarization on DW depinning. Following the same analysis of refs. [22,23] on current-induced DW depinning, one can show that the time-dependent polarization of a current contributes an extra depinning force. In their analysis [22,23], a DW is treated as a quasiparticle [24], with mass $m_{\mathrm{w}} = \frac{2S\mu_0^2 M_s^2}{\Delta\gamma^2 K_\perp}$ trapped in a pinning potential E with a pinning force $F_{\mathrm{pin}} = -\frac{\mathrm{d}E}{\mathrm{d}X}$ whose explicit form is not important for the following analysis. Here S is the cross-section of the wire. Thus for small ϕ and according to the approximation in refs. [22,23], eqs. (8) and (9) can be decoupled and become

$$\frac{F}{m_{\mathrm{w}}} = \ddot{X} = \frac{\alpha\gamma K_\perp}{(1+\alpha^2)\mu_0 M_s}\dot{X} - \frac{1}{m_{\mathrm{w}}}\frac{\mathrm{d}E}{\mathrm{d}X} - \Delta\frac{\gamma^2 K_\perp}{\mu_0 M_s}\frac{a_J'}{1+\alpha^2} + \gamma\Delta\frac{a_J' + \alpha b_J'}{1+\alpha^2}\dot{\eta}, \qquad (13)$$

where the temporal variation of DW width is neglected. The contributions of the current-induced acceleration are the last two terms. The force F on the DW depends not only on the current density but also on the time derivative of the polarization $\dot{\eta}$. When $\dot{\eta}$ is large enough (greater than $\frac{\gamma K_\perp}{\mu_0 M_s} \sim 10^{12}\ \mathrm{s}^{-1}$ for Permalloy), the $\dot{\eta}$-term can dominate the depinning.

Interestingly enough, a very recent experiment generated a spin-polarized current perpendicular to a nanowire, demonstrating that such a current can indeed be used to manipulate DW motion [17]. To implement the strategy presented here, it is still a technological challenge to generate a required spatiotemporally dependent spin-polarized current. Some theoretical proposals for generating a designed current pattern can be found in the literature. For instance, using a magnetic scanning tunneling microscopic (STM) tip above a magnetic nanowire to produce localized spin-polarized current was already proposed by Tao *et al.* [25] and Delgado *et al.* [26]. Experimentally, Ziegler *et al.* [27] recently demonstrated a control of spin-polarized current in a STM by single-atom transfer. Although generating the optimal spin current pattern is beyond the present technology, there is no reason to believe that such a challenge cannot be met. Thus the theoretical results will be relevant when the generation of an arbitrary spin-polarized current pattern becomes true.

The spin pumping effect is neglected because the DW-motion–induced current [28], $\langle j_z \rangle = \frac{\hbar}{eL}(\sigma_\uparrow - \sigma_\downarrow)\frac{\xi \dot{X}}{\Delta}$ is much smaller than the applied external spin-polarized current, where ξ is the nonadiabaticity parameter typically around 0.01 [5], L is the length of the nanowire, $\sigma_\uparrow$ and $\sigma_\downarrow$ denote the conductivities of the majority and minority electrons, respectively. Using [17] $L \sim 5\,\mu\mathrm{m}$, $\Delta \sim 50\,\mathrm{nm}$, the DW velocity $\dot{X} \sim 800\,\mathrm{m/s}$, and a typical conductivity $\sigma_\uparrow \sim 10^6\ \Omega^{-1}\,\mathrm{m}^{-1}$, the pumped electric current density is less than $10^5\,\mathrm{A/m^2}$, which is much smaller than the applied current of the order of 10^{10}–$10^{12}\,\mathrm{A/m^2}$ in usual experiments [17].

To conclude, we propose an optimal spin current pattern for high DW propagation velocity without Walker breakdown in magnetic nanowires. In uniaxial wires this enhancement is of modest size, while in biaxial wires a factor of a few tens can be achieved. The nature of the ultrafast switching-time of the spin degree of freedom proves to be a novel way to improve the efficiency of DW motion against the pinning. We expect our proposal will stimulate and also possibly guide future experiments.

* * *

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