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# Instanton-induced contributions of fractional twist to the cross section of hard gluon-gluon scattering in QCD

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We study the instanton-induced cross section of hard gluon scattering in deep-inelastic kinematics. We find that the structure functions may possess numerically large nonperturbative contributions, which are related to Ringwald-type processes with the violation of chirality, and correspond to the exponential correction to the coefficient function in front of the parton distribution of the leading twist. Instanton-induced contributions are well defined for the structure function at the fixed value of the Bjorken scaling variable, and are probably absent in the moments. At intermediate virtualities of the order of $Q^2 \sim (30-50\ \mathrm{GeV})^2$ these essentially nonperturbative contributions of fractional twist reach large values, of order $10^0-10^{-2}$ of the perturbative cross section, and remain at the same time under theoretical control.

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## I. INTRODUCTION

The exciting possibility of observing baryon-number violation has generated a lot of recent discussion of instanton-induced cross sections at high energies in electroweak theory and beyond. The possibility of instanton-induced baryon-number violation has been mentioned already by 't Hooft [1], but was not taken seriously at that time because of the huge semiclassical suppression factor accompanying such a transition: $\exp(-16\pi^2/g_W^2) \sim 10^{-160}$. The present interest in this problem has been triggered by the observation [2] that strong semiclassical suppression can be compensated in high-energy collisions by the large probability of emission of accompanying gauge $W$ bosons. The point is that the amplitude of the emission of each additional $W$ in the presence of a strong nonperturbative instanton field is enhanced by the large factor $1/g$. As a result, the total cross section of baryon-number violation is written to exponential accuracy as

$$\sigma_{\mathrm{BNV}} \sim \exp\left[-\frac{4\pi}{\alpha_W}F(\epsilon)\right], \tag{1.1}$$

where $F(\epsilon)$ is a certain function of energy, $\epsilon = E/E_{\mathrm{sph}}$, in units of the so-call sphaleron mass $E_{\mathrm{sph}} = m_W/\alpha_W$. This "holy grail" of functions is calculable at small energies and has the form $F(\epsilon) = 1 - \frac{1}{2}(\frac{3}{2}\epsilon)^{4/3} + \cdots$, so that the cross section grows exponentially with energy. The main disputed question is whether $F(\epsilon)$ falls to zero at energies of order of the sphaleron mass or the decrease stops at a certain finite value $F(\epsilon) \to \mathrm{const} > 0$ [3,4], and baryon-number violation remains exponentially suppressed.

Much of this discussion has direct relevance to instanton effects in QCD, in which case the Ringwald-type contribution would correspond to processes with a violation of chirality. Although violation of chirality is not so interesting by itself as the change in the $B-L$ number, instanton-induced contributions might have a clear experimental signature, since they are expected to produce peculiar events with a large number of particles in the final state produced by a pointlike classical source. In any case, the possibility of having well-defined and large nonperturbative contributions to hard-scattering cross sections in QCD deserves serious study. Since the value of the strong coupling is much larger than that of the weak coupling, instanton-induced effects may show up at much smaller energies and may be observable even in the case that they are exponentially suppressed.

A remarkable property of electroweak theory is that integrations over the instanton size turn out to be convergent because of the Higgs component of the instanton field. In the case of QCD, the situation is more complicated, and the fate of the integral over the instanton size depends on the particular problem in question. In this paper we consider the instanton-induced contribution to the cross section of hard gluon-gluon scattering, which is dominated by the distances close to the light cone $x^2=0$. As is well known, the cross sections of "hard" processes, and of deep-inelastic scattering in particular, can be written in a factorized form as the product of coefficient functions, which include all the singularities at $x^2 \to 0$, and parton distribution functions. The coefficient functions can be evaluated order by order in perturbation theory, while the parton distributions accumulate all information

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about the dynamics of large distances and can be taken from experiment. The dependence of the parton distribution functions on the virtuality is governed by renormalization-group equations. Contributions of parton distributions of leading twist determine the asymptotical behavior of the cross sections at $Q^2 \to \infty$, resulting in Bjorken-scaling-law modulo logarithmic corrections, whereas the distributions of higher twists produce power corrections, which are down by integer powers of $Q^2$.

Instantons in QCD may contribute to parton distributions in a similar way as they contribute to the vacuum energy. Such contributions are generically given by divergent integrals over the instanton size, so that instantons tend to overlap strongly and to be "melted." The actual importance of instanton-induced contributions of this type has been disputed for more than a decade [5]. We have nothing to add to this discussion.

The objective of this paper is rather to study instanton-induced contributions to the coefficient functions. It has been noted [6,7] that instantons may give rise to contributions which have a peculiar behavior at short distances and are absent within the operator-product expansion. They change the analytical structure of the light-cone expansion, producing terms of *fractional* twist:

$$\sim \exp\{-S_0/\alpha_s(x^2)\} \sim (x^2)^{(b/4\pi)S_0} .$$

In this paper we demonstrate that Ringwald-type instanton contributions indeed lead to well-defined nonperturbative contributions of fractional twist to structure functions of the deep-inelastic scattering at the fixed values of the Bjorken $x$ (but not to the moments). An explicit calculation suggests that at not very large values of $Q^2 \sim (20\text{–}50\ \text{GeV})^2$ and at $x \sim \frac{1}{3}$ the instanton-induced contributions are large, of the same order as the perturbative ones, and remain at the same time under theoretical control. These contributions are given by convergent integrals over the instanton size and correspond to the exponential correction to the coefficient function in front of the parton distribution of the leading twist.

The presentation is organized as follows. In Sec. II we calculate the instanton contribution to the cross section of deep-inelastic gluon-gluon scattering, taking into account all the preexponential factors, and argue that this cross section may become large in the region where the calculation seems to be under control. In Sec. III we rederive the same result in coordinate space and show that the Ringwald-type cross section corresponds to the nonperturbative exponential correction $\sim \exp\{-S/\alpha_s(x^2)\}$ to the coefficient function in front of the gluon distribution. Section IV contains a summary and several concluding remarks concerning possible generalizations to more practical cases of deep-inelastic lepton scattering, jet production, etc. We note that one should be very careful in the separation of the instanton effects in question, since at least in certain cases they turn out to be completely canceled.

## II. HARD GLUON-GLUON SCATTERING

We consider the simplest possible example, namely, the total cross section of the deep-inelastic scattering of a virtual gluon with momentum $q$, $Q^2 = -q^2 > 0$, from a (almost) real gluon with momentum $p$, $p^2 \to -0$ (see Fig. 1). To this purpose we concentrate on the Green's function

$$A = \frac{1}{N} \int \mathcal{D}A\, \mathcal{D}\bar{\psi}\mathcal{D}\psi \operatorname{Tr}[A_\mu(q)A_\nu(-q)] \times \operatorname{Tr}[A_\alpha(p)A_\beta(-p)]e^{iS} \tag{2.1}$$

and amputate the four gluon legs at the very end of the calculation. As noted by Zakharov [8], because of the optical theorem, we can first evaluate the amplitude in (2.1) in the Euclidean region and then calculate the cross section by continuing the result analytically to positive energies $E^2 = (p+q)^2$ and taking the imaginary part. We are interested in the contribution to the functional integral in (2.1) coming from the vicinity of the instanton–anti-instanton configuration and, to semiclassical accuracy, can write it as

$$\begin{aligned} A^{I\bar{I}} = \int dU \int \frac{d\rho_1}{\rho_1^5} d(\rho_1) \int \frac{d\rho_2}{\rho_2^5} d(\rho_2) \\ \times \int d^4R \operatorname{Tr}[A_\mu^{\bar{I}}(q)A_\nu^{I}(-q)] \\ \times \operatorname{Tr}[A_\alpha^{\bar{I}}(p)A_\beta^{I}(-p)]Z^{n_f} \\ \times \exp\left[-\frac{16\pi^2}{g^2}S^{I\bar{I}}\right] . \end{aligned} \tag{2.2}$$

Here we denote by $A^I$ the instanton of size $\rho_2$ located at the origin and by $A^{\bar{I}}$ the anti-instanton of size $\rho_1$ standing at the point $R$. The integrations go over the sizes of $I$ and $\bar{I}$, their separation $R$, and their relative orientation $U$. The explicit expression for the instanton density is

$$d(\rho) = \frac{C_1}{(N_c-1)!(N_c-2)!}\left[\frac{8\pi^2}{g^2(\rho)}\right]^{2N_c} \times \exp\{-N_cC_2 + n_fC_3\} , \tag{2.3}$$

where $C_1 = 0.466$, $C_2 = 1.54$, and $C_3 = 0.153$ in the modified minimal substraction ($\overline{\text{MS}}$) scheme. Note that for convenience we do not write down the exponential factor $\exp\{8\pi^2/g^2(\rho)\}$ in Eq. (2.3), but rather include it in the action evaluated on the $I\bar{I}$ configuration. If the $I\bar{I}$ separation is large compared to instanton sizes, the action $S^{I\bar{I}}$ is equal to the sum of individual actions of the instanton and anti-instanton. In our normalization, $S^{I\bar{I}}(R \to \infty) = 1$. As the separation decreases, the instantons begin to interact, and the $I\bar{I}$ configuration no longer satisfies the classical equations of motion. Thus the expansion around such a field should be done with care. A

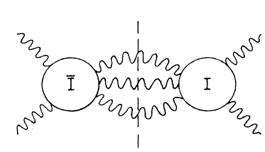


FIG. 1. Instanton-induced contribution to the cross section of hard gluon-gluon scattering.

systematical approach to the semiclassical expansion of the functional integral around the $I\bar{I}$ configuration is provided by the valley method [9]. The main idea is that by choosing a specific "valley" form of the field one is able to minimize the violation of equations of motion to an extent that a saddle-point evaluation of the functional integral becomes possible. We do not dwell here on the motivation and technics of the valley method. A detailed discussion can be found in [10–12].

Following the valley approach, we take into account the $I\bar{I}$ interaction and the distortion of instantons caused by it using the particular form of the action $S^{I\bar{I}}=S(\xi,U)$, which corresponds to the conformal instanton-anti-instanton valley [10]. Because of the conformal invariance of QCD (on the tree level), the action on such a configuration depends on one dimensionless parameter only $\xi$, which is called the conformal parameter:

$$\xi=\frac{R^2+\rho_1^2+\rho_2^2}{\rho_1\rho_2} . \tag{2.4}$$

The explicit expression for the action for the case of the maximum attractive $I\bar{I}$ orientation has been found in [13,14]:

$$\begin{aligned}\hat{S}(\xi)&=3\left[\frac{6\eta^2-14}{(\eta-1/\eta)^2}-\frac{17}{3}\right.\\&\quad\left.+\ln(\eta)\left[\frac{(5/\eta-\eta)(\eta+1/\eta)^2}{(\eta-1/\eta)^3}+1\right]\right]\\&=\left[1-\frac{6}{\xi^2}+O(\ln(\xi)/\xi^4)\right],\\ \eta&=\tfrac{1}{2}[\xi+(\xi^2-4)^{1/2}] .\end{aligned} \tag{2.5}$$

The action in (2.5) is a monotonously rising function of the $I\bar{I}$ separation and varies from zero at $\xi=1$ to unity at $\xi\to\infty$. As it should be, the classical action does not depend on the common scale of $I$ and $\bar{I}$; this dependence reveals itself on the quantum level only and comes from the argument of $g^2$. In what follows we assume for simplicity that the dependence of the $I\bar{I}$ interaction on the relative orientation follows that of the dipole-dipole interaction:

$$S(\xi,U)=1-\frac{1}{\xi^2}(2\,\mathrm{Tr}\mathcal{O}\,\mathrm{Tr}\mathcal{O}^\dagger-\mathrm{Tr}\mathcal{O}\mathcal{O}^\dagger)+\cdots , \tag{2.6}$$

where $\mathcal{O}$ is the $2\times2$ matrix standing in the upper left corner of the $N_c\times N_c$ matrix of the relative $I\bar{I}$ orientation. Finally, the factor $Z$ in (2.2) comes from the convolution integral of quark zero modes:

$$Z=a^\dagger a=\frac{4}{\xi^3}\mathrm{Tr}\mathcal{O}\,\mathrm{Tr}\mathcal{O}^\dagger . \tag{2.7}$$

To semiclassical accuracy we can neglect the distortion of instanton fields in front of the exponent in (2.2). The required Fourier transforms equal, respectively,

$$\begin{aligned}A_\nu^I(-q)&=-\frac{i}{g}\frac{4\pi^2}{q^4}(\sigma_\nu\bar{q}-q_\nu)\\&\quad\times\{2+\rho_2^2q^2\mathbf{K}_2[\rho_2(-q^2)^{1/2}]\} ,\\ A_\mu^{\bar{I}}(q)&=\frac{i}{g}\frac{4\pi^2}{q^4}U(\bar{\sigma}_\mu q-q_\mu)\\&\quad\times\bar{U}\{2+\rho_1^2q^2\mathbf{K}_2[\rho_1(-q^2)^{1/2}]\} .\end{aligned} \tag{2.8}$$

We use the standard notation $\sigma^\mu=(1,i\boldsymbol{\sigma})$, $\bar{\sigma}^\mu=(1,-i\boldsymbol{\sigma})$, and $x=x_\mu\sigma^\mu$, $\bar{x}=x_\mu\bar{\sigma}^\mu$. At large values of $\rho(-q^2)^{1/2}$, one can replace the Bessel functions in (2.8) by their asymptotical expressions at large values of the argument, which yields, e.g.,

$$A_\nu^I(-q)\simeq-\frac{i}{g}(\sigma_\nu\bar{q}-q_\nu)\left\{\frac{8\pi^2}{q^4}+(2\pi)^{5/2}\frac{\rho_2^2}{2q^2}(-\rho_2^2q^2)^{-1/4}e^{-\rho_2(-q^2)^{1/2}}\right\} . \tag{2.9}$$

Now comes the central point. The two terms in (2.9) correspond to two distinct types of contributions to the deep-inelastic scattering. The first term leads to a divergent integral over the instanton size, and its low-momentum part gives rise to the instanton contribution to matrix elements of local operators, which determine moments of the corresponding structure function. Since the integral over $\rho$ in this case is cut off at a certain IR scale, the dependence of this contribution on $Q^2$ is standard—a power of $Q^{-2}$, accompanied by logarithms of $Q^2$. The second term potentially gives rise to quite a different behavior. Because of the exponential suppression of the large instantons, this contribution is well defined and can be evaluated by the saddle-point method (see below). The saddle-point value of $\rho_1\sim\rho_2$ turns out to be of the order of $\rho_*Q\sim4\pi/g^2(\rho_*)\gg1$, so that the approximation in Eq. (2.9) is justified. The corresponding cross section is of the order of

$$\exp\left[-\frac{4\pi}{\alpha_s(\rho_*)}S_*\right]\sim(Q^2)^{-bS_*} ,$$

where $b=\frac{11}{3}N_c-\frac{2}{3}n_f$ and $S_*=\hat{S}(\xi_*)$ is the $I\bar{I}$ action (2.5) evaluated in the saddle point. Thus small-size instantons potentially produce terms of fractional power in $Q^{-2}$, which are absent within the operator expansion. The presence of convergent contributions in the integrals over the instanton size has been discussed in a similar context in [13].

In what follows we shall only be interested in these nontrivial contributions of fractional twist and, to this end, substitute in (2.2):

$$\begin{aligned}&\mathrm{Tr}[A_\mu^{\bar{I}}(q)A_\nu^I(-q)]\\&\quad=\frac{1}{g^2}\frac{\rho_1^2\rho_2^2}{4Q^4}(2\pi)^5(\rho_1^2\rho_2^2Q^4)^{-1/4}e^{iqR-(\rho_1+\rho_2)Q}\\&\quad\quad\times\mathrm{Tr}\{(\sigma_\nu\bar{q}-q_\nu)U(\bar{\sigma}_\mu q-q_\mu)\bar{U}\} .\end{aligned} \tag{2.10}$$

A similar factor coming from the soft gluons in (2.2) equals

$$\mathrm{Tr}[A^{\bar I}_\alpha(p)A^I_\beta(-p)] = \frac{1}{g^2}\frac{\rho_1^2\rho_2^2}{4p^4}(2\pi)^4 e^{ipR}\mathrm{Tr}\{(\sigma_\beta\bar p - p_\beta)U(\sigma_\alpha\bar p - p_\alpha)\bar U\} \ . \tag{2.11}$$

The remaining calculation is a routine repetition of tricks, well known in the context of calculations of instanton-induced cross sections of baryon-number violation in electroweak theory. However, in contrast to that practice, we keep trace of all the preexponential factors.

The key observation is that analytical continuation to Minkowski space and calculation of the imaginary part in $(p+q)^2$ actually reduce to the substitution of the oscillating factor $e^{i(q+p)R}$ in (2.10) and (2.11) by $e^{(q+p)R}=e^{ER_0}$ (in the c.m. frame). After this rotation all the integrals are done by the repeated application of the saddle-point method.

First of all, we integrate over the $I\bar I$ orientations. To this end we note that an arbitrary matrix $U\in SU(3)$ ($N_c=3$) can be parametrized as

$$U=\hat A e^{i\alpha\lambda_4}\hat B e^{i\beta\lambda_8} \ , \tag{2.12}$$

where $\hat A$ and $\hat B$ are $3\times 3$ matrices of the form

$$\hat A=\begin{bmatrix} A & 0\\ 0 & 1\end{bmatrix} ,\quad \hat B=\begin{bmatrix} B & 0\\ 0 & 1\end{bmatrix} ,\quad A,B\in SU(2) \ , \tag{2.13}$$

and $\lambda_4,\lambda_8$ are the usual Gell-Mann matrices. The measure $dU$ takes the form

$$dU=\mathrm{const}\times dA\, dB\, d\beta\, d\alpha \sin^3\alpha\cos\alpha \ , \tag{2.14}$$

where the constant is fixed by the condition $\int dU=1$. The action on the $I\bar I$ configuration depends on the product $AB$ only, which we parametrize in a usual way as $AB=a_\mu\sigma_\mu$, $\int d(AB)=\int d^4a\,\delta(a^2-1)$. Thus the orientation-dependent factor in (2.2) becomes

$$\int dU\, e^{\lambda(2\,\mathrm{Tr}\mathcal{O}\,\mathrm{Tr}\mathcal{O}^\dagger-\mathrm{Tr}\mathcal{O}\mathcal{O}^\dagger)} = \frac{4}{\pi}\int_0^1 da_0\,da_3\,\theta(1-a_0^2-a_3^2)2\int_0^\pi d\alpha|\cos\alpha|\sin^3\alpha\exp\left\{-2\lambda\left[(1-4a_0^2)\cos^4\frac{\alpha}{2}+(1-4a_3^2)\sin^4\frac{\alpha}{2}\right]\right\} \simeq \frac{1}{9\sqrt{\pi}}(2\lambda)^{-7/2}e^{6\lambda} \tag{2.15}$$

(for large values of $\lambda$), and the saddle point corresponds to the most attractive $I\bar I$ orientation. Taking $\lambda=(4\pi/\alpha_s)[\xi S'(\xi)/12]$, where the prime stands for the derivative over $\xi$, we end up with

$$\int dU\, e^{-(16\pi^2/g^2)S(\xi,U)} = \frac{1}{9\sqrt{\pi}}\left[\frac{3\alpha_s}{2\pi\xi S'(\xi)}\right]^{7/2} e^{-(16\pi^2/g^2)S(\xi)} \ . \tag{2.16}$$

Hereafter we drop the caret from the notation of the $I\bar I$ action in the most attractive orientation (2.5).

The saddle-point evaluation of integrals over $\mathbf{R}$ and over $\delta=\rho_1-\rho_2$ is elementary and produces the factors

$$\left[\frac{\alpha_s\rho_1\rho_2}{4S'(\xi)}\right]^{3/2}$$

and

$$\rho\left[\frac{\alpha_s}{S'(\xi)(\xi+2)}\right]^{1/2} ,$$

respectively. The saddle points are $\mathbf{R}=0$ and $\delta=0$. The remaining integral over the common size of the instantons $\rho$ and over their separation $R_0$ reads (apart from the factors listed above)

$$\int \rho\, d\rho\int dR_0 \exp\left\{-2Q\rho+ER_0-\frac{4\pi}{\alpha_s(\rho)}S(\xi)\right\} . \tag{2.17}$$

It is convenient to use the size of the instanton $\rho$ and the conformal parameter $\xi=2+R_0^2/\rho^2$ as the two independent variables. The saddle-point equations take the form

$$\begin{aligned} 2Q\rho_* &= \frac{8\pi}{\alpha_s(\rho_*)}(\xi_*-2)S'(\xi_*)+2bS(\xi_*) \ ,\\ E\rho_* &= -\frac{8\pi}{\alpha_s(\rho_*)}\sqrt{\xi_*-2}S'(\xi_*) \ , \end{aligned} \tag{2.18}$$

and the determinant of the matrix of second derivatives equals

$$\rho_*^2 D=\left[\frac{4\pi}{\alpha_s(\rho)}+2b\right]^2[S'(\xi_*)]^2 - \frac{8\pi b}{\alpha_s(\rho_*)}S(\xi_*)\left\{\frac{S'(\xi_*)}{2(\xi_*-2)}+S''(\xi_*)\right\} . \tag{2.19}$$

Note that we take into account the running of the QCD coupling, which is important numerically, but strictly speaking is beyond our semiclassical accuracy, since the corresponding corrections are formally of the order of $O(b\alpha_s)$. Thus the second term in the first of the saddle-point equations in (2.18), which is proportional to the first coefficient $b$ of the Gell-Mann–Low function, should only provide a moderate correction to the first term, which is proportional to the derivative of the $I\bar I$ action. Since $S'(\xi)\simeq 12/\xi^2$ at large values of the conformal parameter, this means that our calculation is valid for not too small values $\xi_* \leq 4.5$, i.e., for large enough energies only.

Collecting everything, averaging over transverse polarizations of the hard gluons and over polarizations of the soft gluons, and adding a factor of 2 in order to take into account a symmetrical contribution with the instanton replaced by the anti-instanton, we obtain the final answer ($n_f = 3$)

$$2E^2\sigma_\perp = C_1^2 e^{6(C_3-C_2)} \frac{\rho_*^3}{(\xi_*^2-4)^{1/2}} \frac{\alpha_s(\rho_*)^{-17/2}\pi^{16}}{\xi_*^{25/2} S'(\xi_*)^{11/2}} \left[\frac{3}{2}\right]^{3/2} 2^{23}(\rho_*^2 D)^{-1/2}$$
$$\times \exp\left[-\left[\frac{4\pi}{\alpha_s(\rho_*)}+2b\right] S(\xi_*)\right] \frac{1}{Q}\frac{1}{16}\left\{\frac{(E^2+Q^2)^2}{E^2}\right\}\left\{\frac{E^4+Q^4}{E^2}\right\}, \tag{2.20}$$

expressed in terms of the saddle-point values $\rho_*$ and $\xi_*$. The saddle-point equations in (2.18) are easily solved numerically. Before we proceed to the results, several remarks are in order, concerning the region of validity of the above derivation.

The first restriction comes from the requirement that the saddle-point value of the instanton size should be sufficiently small in order that the instanton density in (2.3) is not distorted by presence of large-scale nonperturbative fluctuations in the QCD vacuum. The effect of the latter has been estimated in [15], with the result that the interaction with large-scale fluctuations is controlled by the parameter

$$\delta = \frac{\pi^4 \rho_*^4}{8\alpha_s^2(\rho_*)} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle, \tag{2.21}$$

which should be sufficiently less than unity in order that the results in (2.3) is justified: $d_{\text{eff}}(\rho) \simeq d(\rho)[1+\delta]$. Here $\langle(\alpha_s/\pi)G^2\rangle \simeq 0.012\ \text{GeV}^4$ is the gluon condensate. In practice, this means that the instanton in the QCD vacuum is well defined starting from $\rho^{-1} \geq 1$ GeV, and for $\rho^{-1} \geq 2$ GeV the interaction with large-scale fluctuations (instantons) can already be neglected. From the saddle-point equations in (2.18), it follows readily that $Q\rho_* \sim 4\pi/\alpha_s(\rho_*)$, and starting from the virtualities $Q$ of order 10–20 GeV, our derivation is from this side justified.

The second requirement is that the instantons be distorted not too strongly by the interaction with each other, so that the valley approach for the construction of the distorted $I\bar{I}$ configuration still makes sense. It is natural to assume that one can trust the expression in (2.5) for the valley action, until $S(\xi_*)$ decreases not more than by one-half compared to the value $S=1$ for infinitely large $I\bar{I}$ separations. In other words, we demand that the action on the $I\bar{I}$ valley should be larger than the action of a single instanton. From the explicit expression in (2.5), we find that the value $S(\xi_*)=\frac{1}{2}$ corresponds to the value of the conformal parameter $\xi_* = R^2/\rho^2+2 \simeq 2.95$, i.e., to $R \simeq \rho$. On the other hand, neglecting the running of the coupling (which only produces a small correction), one easily obtains, from the saddle-point equations,

$$\frac{R_*}{\rho_*} \simeq \frac{2Q}{E}. \tag{2.22}$$

Expressing the right-hand side (RHS) in terms of the Bjorken scaling variable

$$x = \frac{Q^2}{2pq} = \frac{Q^2}{E^2+Q^2}, \tag{2.23}$$

we see that the overlapping of instantons increases with the decrease in $x$, and we are allowed to consider sufficiently large values only, $x \geq 0.2$.

With these preliminary remarks in mind, we now turn to the numerical results. The cross section in (2.20) is plotted in Fig. 2 as a function of the Bjorken variable $x$ for different values of $Q$ in the interval 20–100 GeV (solid curves). The dashed curves show the lines with constant $\xi_*$. The corresponding values of the action are $S(\xi_*=3)=0.516$ and $S(\xi_*=3.4)=0.609$, so that roughly one half of the $I\bar{I}$ action is absorbed by the interaction. The same results are shown in Fig. 3 at the two-dimensional plane in the variables $E$ and $Q$. The solid curves show the lines with a constant cross section, and the shaded area corresponds to the region $\xi_* < 3$, where the calculation is not reliable. The saddle-point values of the instanton size $\rho_*$ are shown in Fig. 4 in dependence

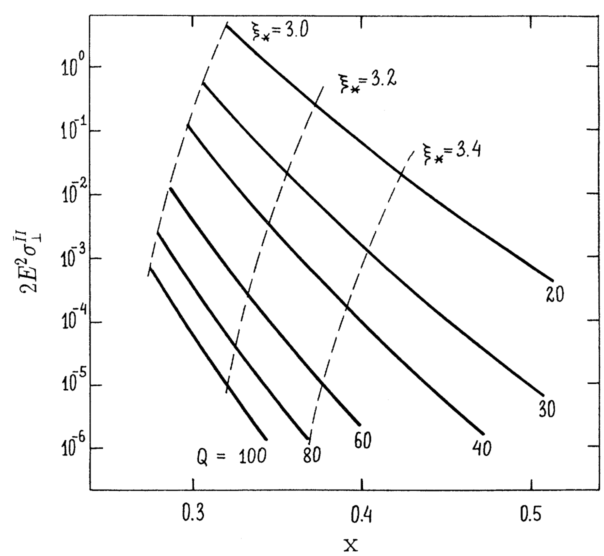


FIG. 2. Cross section in (2.20) as a function of Bjorken $x=Q^2/(2pq)$ for different values of $Q$ (solid curves). The dashed curves show the lines with the constant saddle-point value of the conformal parameter $\xi_*$.

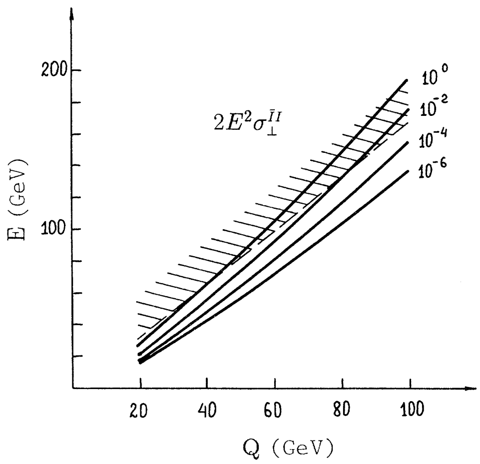


FIG. 3. Cross section in (2.20) as a function of $E$ and $Q$. The solid curves show the lines with the constant cross section. The dashed area denotes the region of parameters where $S(\xi_*)<\frac{1}{2}$ and the calculation is not reliable.

on $Q$. The dependence on the energy, that is, on the saddle-point value of $\xi_*$, is very weak, and the given values of $\rho_*$ correspond to the interval $3<\xi_*<4$.

The main conclusion which we would like to draw from the above calculation is that in the interval of virtualities, $Q\sim 20-50$ GeV, and at the values $x\sim 0.3-0.4$ the instanton-induced cross section becomes large, of the order of $10^0-10^{-2}$ of the cross section in perturbation theory (e.g., induced by the four-gluon vertex). At the same time, the size of the relevant instantons is small, $\rho_*<1\ \text{GeV}^{-1}$, and they overlap (interact) not too strongly, $R_*\geq\rho_*$. The typical value of the coupling at the saddle point is $\alpha_s(\rho_s)\simeq 0.2$, so that at $S(\xi_*)\simeq\frac{1}{2}$ the semiclassical factor $\exp\{[-4\pi/\alpha_s(\rho_*)]S(\xi_*)\}$ amounts $10^{-14}$. What happens is that this smallness is compensated by the large factor $(2\pi/\alpha_s)^{4N_c}$, which is due to a large number of instanton zero modes. The light-cone structure of this contribution is analyzed in the following section. We find that it corresponds to the contribution of a fractional twist $\sim(x^2)^{(4\pi/\alpha_s)S(\xi_*)}$, which is formally missing within the operator-product expansion, but can be incorporated in this framework as the nonperturbative $\sim\exp[-1/\alpha_s(x^2)]$ contribution to the coefficient function in front of the contribution of the gluon distribution function of the leading twist.

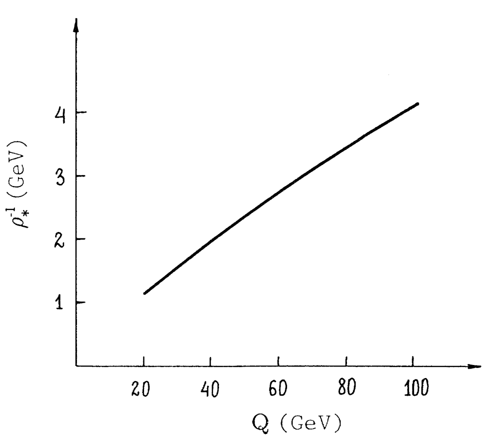


FIG. 4. Saddle-point value of the instanton size $\rho_*$ as a function of $Q$ and for $\xi_*\sim 3-4$. The dependence on $\xi_*$ is actually very weak.

Taken at face value, our result in (2.20) yields an exponentially large cross section, which violates the unitarity bound if continued to sufficiently small values of $\xi_*<3$ (higher energies). An obvious correction, which has, however to, be taken into account in this region, is that the instanton (anti-instanton) fields in (2.2) and (2.8)–(2.11) must be replaced by the $I\bar{I}$ valley [10]. In addition, there are quantum "soft-hard" and "hard-hard" corrections [16] to the semiclassical amplitude in (2.2). We expect that the answer will become explicitly unitary if all these corrections are taken into account. The situation may be quite different in this respect from that in electroweak theory, in which case the unitarization corrections due to multi-instantons [3] probably stop the exponential growth of the instanton-induced cross section at about one half of the 't Hooft suppression. The reason is that the pointlike instanton-induced vertex is only supported in QCD by the external virtually, so that the general arguments for the "premature unitarization" given in [3] may be not applicable. The multi-instanton contributions of the Magiorre-Shifman type are absent in our situation, since the virtuality of external gluons will cut off the integration over the size of the first and last instantons in the chain only. The remaining ones will tend to become large, and the interaction with them will become negligible. The question obviously deserves further study. We note, however, that in any case the effect of multi-instantons is expected to become important at $S(\xi)\leq\frac{1}{2}$, i.e., in the region which we do not consider here.

## III. LIGHT-CONE EXPANSION

The successful application of QCD to "hard" processes is based on a set of factorization theorems [17], which provide the possibility of separating contributions of quarks and gluons with small momenta in a finite number of parton distribution functions, which can be taken from experiment. Divergences that arise because of the extraction of the asymptotic behavior give rise to renormalization of parton distributions.

In coordinate space this subtraction procedure corresponds to picking up the leading singularity of the amplitude at $x^2\to 0$. All singularities on the light cone are included in coefficient functions in front of nonlocal operators, the matrix elements of which define the parton distribution functions. To illustrate the general structure and the factorization of singular and analytical contributions on the light cone, let us consider for a moment a simple example of scalar $g\phi^3$ theory in $d=6$ dimensions and calculate the $O(g^2)$ contribution to the matrix element of the $T$ product of two scalar fields $\bar{\phi}(x)\phi(0)$, shown in Fig. 5. A straightforward calculation yields [18]

$$\langle p|\bar{\phi}(x)\phi(0)|p\rangle = \frac{g^2}{4(2\pi)^{d/2}}\int_0^1 \bar{u}\, du\, e^{iupx}\left[\frac{m_u^2}{-x^2+i\epsilon}\right]^{(d-6)/4} \mathbf{K}_{d/2-3}\{[m_u^2(-x^2+i\epsilon)]^{1/2}\}\ , \tag{3.1}$$

where $m_u^2 = m^2(1-\bar{u}u)$. The expansion of $\mathbf{K}_{d/2-3}$ in powers of the deviation from light cone produces a series of terms of the form

$$\langle p|\bar{\phi}(x)\phi(0)|p\rangle = \frac{g^2}{64\pi^{d/2}}\int_0^1 \bar{u}\, du\, e^{iupx}\left\{\frac{\Gamma(3-d/2)}{(m_u^2/4)^{3-d/2}}\sum_{n=0}^{\infty}\frac{\Gamma(d/2-2)}{n!\Gamma(n+d/2-2)}(-x^2 m_u^2/4)^n \right.$$
$$\left. + \frac{\Gamma(d/2-3)}{(-x^2+i\epsilon)^{d/2-3}}\sum_{n=0}^{\infty}\frac{\Gamma(4-d/2)}{n!\Gamma(n+4-d/2)}(-x^2 m_u^2/4)^n\right\}\ . \tag{3.2}$$

The two types of contributions in (3.2) correspond to the two distinct regions of momentum flow in the diagram in Fig. 5. Contributions of high virtualities $k^2>\mu^2$ are included in the "hard" part, which is singular at $x^2\to 0$, while the analytic in the $x^2$ part contains contributions of low momenta $k^2<\mu^2$. Note that each series in (3.2) contains pole singularities at $d\to 6$, which cancel in their sum. We apply the minimal subtraction ($\overline{\text{MS}}$) to separate singular and analytic parts at $d=6$ with the result

$$\langle p|\bar{\phi}(x)\phi(0)|p\rangle = \langle p|\bar{\phi}(x)\phi(0)|p\rangle_{\text{sing}}^{\mu^2} + \langle p|\bar{\phi}(x)\phi(0)|p\rangle_{\text{anal}}^{\mu^2}\ , \tag{3.3}$$

$$\langle p|\bar{\phi}(x)\phi(0)|p\rangle_{\text{sing}}^{\mu^2} = \frac{g^2}{64\pi^3}\int_0^1 \bar{u}\, du\, e^{iupx}\sum_{n=0}^{\infty}\frac{(-m_u^2x^2/4)^n}{(n!)^2}\left[\ln\frac{4}{-x^2\mu^2}+\psi(n+1)-C\right]\ , \tag{3.4}$$

$$\langle p|\bar{\phi}(x)\phi(0)|p\rangle_{\text{anal}}^{\mu^2} = \frac{g^2}{64\pi^3}\int_0^1 \bar{u}\, du\, e^{iupx}\sum_{n=0}^{\infty}\frac{(-m_u^2x^2/4)^n}{(n!)^2}\left[\ln\frac{\mu^2}{m_u^2}+\psi(n+1)+C\right]\ , \tag{3.5}$$

where $\psi(z)=(d/dz)\ln\Gamma(z)$ and $C$ is the Euler constant. With certain reservations the analytic part (3.5) can be interpreted as the $O(g^2)$ contribution to the parton distribution functions of increasing twist (multiplied by the tree-level coefficient function), while the singular part gives one-loop corrections to the coefficient functions, multiplied by the tree-level contributions to the parton distributions.

The aim of this section is to find a proper place for the instanton-induced cross section in (2.20) in the context of a similar expansion on the light cone of the matrix element over the one-gluon states of the product of two gluon fields near the light cone, $\langle p|T[A_\mu^I(x)A_\nu^I(0)]|p\rangle$. The calculation turns out to be more difficult than in momentum space, and to its end we take into account the dipole-dipole interaction term in the action only: $S^{I\bar{I}}=1-6\rho_1^2\rho_2^2/R^4+\cdots$. We tacitly assume that the integration over the relative $I\bar{I}$ orientations is done by the saddle-point method, and we can consider the $I\bar{I}$ pair in the most attractive configuration. It turns out to be convenient to expand the interaction term as

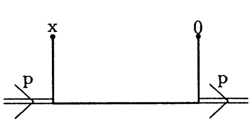


FIG. 5. Typical contribution to the matrix element of the nonlocal operator $\bar{\phi}(x)\phi(0)$ in scalar theory.

$$\exp\left[\frac{24\pi}{\alpha_s}\frac{\rho_1^2\rho_2^2}{R^4}\right] = \sum_{n=0}^{\infty}\frac{1}{n!}(6b)^n\left[\frac{\rho_1^2\rho_2^2}{R^4}\right]^n\left[\ln\frac{1}{\rho_1\rho_2\Lambda^2}\right]^n\ . \tag{3.6}$$

Replacing $n\to n+\epsilon$, we can take into account the logarithmic factors coming from the running of the coupling, if we pick up the $n$th term of the expansion in $\epsilon$ at the very end of the calculation. In each separate term in the expansion, the dependences on $\rho_1$ and $\rho_2$ factorize from each other, and the corresponding integrals are understood in the sense of analytical continuation. For example, we write

$$\int_0^\infty d\rho\frac{(\rho^2)^\alpha}{x^2+\rho^2} = \tfrac{1}{2}(x^2)^{\alpha-1/2}\Gamma(\tfrac{1}{2}+\alpha)\Gamma(\tfrac{1}{2}-\alpha)\ . \tag{3.7}$$

Here the denominator comes from the (anti-)instanton field, which we take to be in the regular gauge. As above, the justification for taking the convergent pieces of the integrals in $\rho$ is that these terms produce a different structure of singularities on the light cone. The integrals over the total translation of the $I\bar{I}$ configuration and over their separation are elementary. Finally, we note that the sum over $n$ in (3.6) is dominated by contributions of large $n\sim 4\pi/\alpha_s(x^2)$, so that we can replace the summation by the integration and evaluate the integral by the saddle-point method. After some algebra we get the final result, in which we keep the trace on the singular on the light-cone factors only:

$$\langle p|T[A^{\bar{I}}_{\mu}(x)A^{I}_{\nu}(0)]|p\rangle$$
$$=\frac{(px)^2}{x^2}\int_0^1 du\, e^{iupx}[g_{\mu\nu}x^2F_1(u)+x_\mu x_\nu F_2(u)]$$
$$\times\left[\frac{4\pi}{\alpha_s(x^2)}\right]^{15/2-b[1-(3/8)\bar{u}^2/u^2]}$$
$$\times\exp\left\{-\frac{4\pi}{\alpha_s(x^2)}\left[1-\frac{3}{8}\frac{\bar{u}^2}{u^2}\right]\right\}. \quad (3.8)$$

The particular form of the functions $F_1(u)$ and $F_2(u)$ is not important for our purposes. The expression in (3.8) can be rewritten in a form resembling the answer for the cross section in (2.20) by introducing the variables

$$\xi_*=4u/\bar{u}\ ,$$
$$\rho_*^2=\bar{u}x^2\frac{4}{\xi_*^2}\frac{4\pi}{\alpha_s(\rho_*^2)}\ , \quad (3.9)$$

in terms of which the $x^2$-dependent factor in (3.8) becomes

$$[4\pi/\alpha_s(\rho_*)]^{15/2}\exp\left[-\frac{4\pi}{\alpha_s(\rho_*)}S(\xi_*)\right].$$

Note that the effective value for the instanton radius $\rho_*$ is different from the saddle-point value in momentum space.

Thus the instantons indeed induce contributions of fractional twist $\sim(x^2)^{bS(\xi_*)^{-1}}$ to the light-cone expansion of the cross section of gluon-gluon scattering, and the power of $x^2$ depends on the value of the Bjorken variable $u$. Note that our calculation is only valid for sufficiently small values of $u$ where the running factor $3u^2/8\bar{u}^2$ in the exponent in (3.8) is sufficiently smaller than unity. This means that *moments* of the structure functions cannot be calculated in this way and presumably are free from the contributions of fractional twist. On the other hand, making the Fourier transformation and taking the imaginary part, we obtain

$$\frac{1}{\pi}\mathrm{Im}\,i\int d^4x\, e^{iqx}\langle p|T[a^{\bar{I}}_{\mu}(x)A^{I}_{\nu}(0)]|p\rangle$$
$$\sim\int_{x_B}^1 du\, F(u)\left[\frac{4\pi}{\alpha_s(\rho_*)}\right]^{15/2}$$
$$\times\exp\left[-\frac{4\pi}{\alpha_s(\rho_*)}S(\xi_*)\right]\frac{\sin[bS(\xi_*)]}{(u-x_B)^{bS(\xi_*)}}\ , \quad (3.10)$$

where the integral should be understood as the principal value and $x_B$ is the Bjorken variable $Q^2/(2pq)$. We see that the cross section at the fixed value of the Bjorken $x$ receives a contribution from the region of large $u$ only, where our derivation is justified.

Returning to Eq. (3.8), we note that this expression defines a singular (on the light cone) contribution to the amplitude to gluon-gluon scattering, which is infrared stable. It is easy to demonstrate that by adding soft gluons, e.g., calculating three-or four-gluon matrix elements of the $T$ product of gluon currents, one introduces extra powers of $x^2$ (extra powers of $1/Q^2$ in momentum space), just because the instanton field is explicitly proportional to the square of the instanton size $\rho^2\sim x^2$. Thus the calculated $I\bar{I}$ contribution is related to the two-particle gluon distribution of the leading twist. Its interpretation in the context of the light-cone expansion in (3.2)–(3.5) can be done as follows. To the leading-twist accuracy, one arrives within perturbation theory at a factorized expression, which we write schematically as

$$\langle p|T[A^{\bar{I}}_{\mu}(x)A^{I}_{\nu}(0)]|p\rangle$$
$$=\frac{1}{x^2}C[px,\alpha_s(x^2)]\otimes F_G(px,p^2,\mu^2=1/x^2)\ . \quad (3.11)$$

An overall factor of $1/x^2$ may be written from dimensional arguments. What happens is that instantons induce nonperturbative contributions to the coefficient functions in front of the leading-twist parton distributions:

$$C\sim c_0+c_1\alpha_s(x^2)+c_2\alpha_s^2(x^2)+\cdots+e^{-S/\alpha_s(x^2)}$$
$$=C_{\mathrm{pert}}+C_{\mathrm{nonpert}}\ . \quad (3.12)$$

The separation of powers and logarithms in $x^2$ becomes meaningless beyond perturbation theory, and the relation between the twist and power asymptotics at $Q^2\to\infty$ is lost.

As is well known, the separation of perturbative and nonperturbative pieces in the amplitudes is ambiguous because of the asymptotical nature of the perturbation series [19]. However, the contribution calculated in this paper seems to be well defined, at least to semiclassical accuracy. We would like to note that the instanton-induced [20] asymptotics of the coefficients in front of high powers of $\alpha_s$ is related to the $I\bar{I}$ contribution of a different type [21], in which both instanton and anti-instanton appear to be to one and the same side of the cut (i.e., either both in the initial or both in the final amplitude) and which produces a singularity in the complex plane of the Borel transform of the coupling at the value of the Borel parameter corresponding to the doubled instanton action. In addition, the usual contributions of increasing twist create a series of (infrared) renormalon singularities. The Ringwald-type subprocess considered in this paper is not related to the asymptotics of coefficients of perturbation theory, but probably reflects the uncertainty in the summation of perturbation series [22]. It produces a purely real contribution to the cross section which must somehow be reproduced by the dispersion relation in the coupling constant. We do not see in our calculation the shift with the energy of the instanton-induced singularity in the Borel plane of the coupling, as suggested in [23].

## IV. CONCLUSIONS

We have calculated in this paper the instanton-induced cross section of hard gluon-gluon scattering and found that it is rather large. This calculation suggests that the coefficient function in front of the gluon distribution of the leading twist may possess significant nonperturbative contributions coming from instantons of sufficiently small size. These contributions seem to be well defined for the structure function at the fixed value of the Bjorken scaling variable and are probably absent in the moments. The key point is in the separation of finite contributions from the divergent integral over the instanton size, which is justified by producing in this way the terms with a new analytical structure on the light cone. This singularity actually corresponds to a saddle-point evaluation of the contributions of $\rho^2 \sim x^2$ in the functional integral. The uncertainties in the separation of finite and divergent contributions, which would arise beyond the saddle-point approximation, are likely to be related to the uncertainties in the summation of the perturbation series.

These conclusions most likely are not an artifact of working with a "bad" object which is gauge variant. Indeed, the gauge invariance may be restored by adding the gauge exponential factors or, equally, by going over to the light-cone gauge. The expressions for the relevant gauge factors are rather cumbersome (see [12]), and we have not succeeded in obtaining a closed expression taking them into account. The exponential terms in the instanton field in momentum space, which have been crucial for the present analysis, are due to the pole singularity of the field in coordinate space $1/(\rho^2+x^2)$. The effect of the gauge factors is that the pole is replaced by the cut, but its position is not changed. Thus we believe that adding the gauge factors will not change the qualitative features of the results.

However, the generalization to deep-inelastic lepton scattering may turn out to be not immediate. The simplest way to produce a hard gluon is by a large-angle scattering of a quark [see Fig. 6(a)]. However, in this case the Ringwald-type contribution considered in this paper turns out to be exactly canceled by multiple gluon exchanges. Indeed, the cross section of the type shown in Fig. 6(a) is proportional to the propagator of the quark on mass shell at the instanton background. By an explicit calculation, one can verify that this propagator is proportional to a power of $(p_1-p_2)^2=q^2$ and does not contain any exponential terms of the type (2.9). This cancellation is probably accidental, since the gluon propagator does contain exponential terms in the same kinematics. Thus the cross section for the production of the gluon jet with large $p_\perp$ may contain nonperturbative contributions in question. The cross section for deep-inelastic lepton scattering may possess the nonperturbative contributions shown in Fig. 6(b). We have checked that the quark propagators indeed contain necessary exponential corrections in the required kinematics and that the nonperturbative contribution comes from the subprocess with small momenta of outgoing quarks. This study is in progress, and the results will be considered in a separate publication.

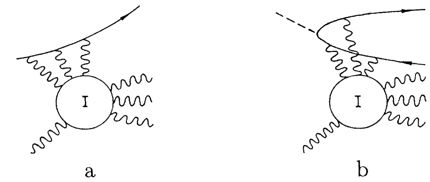


FIG. 6. Instanton-induced cross section (a) for the production of the quark jet and (b) for deep-inelastic lepton-gluon scattering.

Another possibility for obtaining a well-defined and non-negligible effect due to small instantons might be to consider a cross section with a fixed number of jets in the final state at high energy [24]. It may also be interesting to consider the contributions of instantons to the parton distributions, which in a general situation are given by divergent integrals over the instanton size. However, if multi-instantons are absent and if the instanton-induced cross section is only restricted by the unitary bound at high energies, this process should be considered as the main source of the large transverse energy events, which seem to disagree with the experiment [24].

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[1] G. 't Hooft, Phys. Rev. D **14**, 3432 (1976).
[2] A. Ringwald, Nucl. Phys. **B330**, 1 (1990).
[3] V. I. Zakharov, Nucl. Phys. **B353**, 683 (1991); G. Veneziano, Mod. Phys. Lett. A **7**, 1661 (1992).
[4] M. Maggiore and M. Shifman, Nucl. Phys. **B365**, 161 (1991); **B371**, 177 (1992).
[5] D. I. Dyakonov and V. Petrov, Nucl. Phys. **B245**, 259 (1984); E. V. Shuryak, *ibid.* **B302**, 559 (1988); **B302**, 574 (1988); **B302**, 599 (1988); **B302**, 621 (1988).
[6] V. A. Novikov, M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B174**, 378 (1980).
[7] M. S. Dubovikov and A. V. Smilga, Nucl. Phys. **B185**, 109 (1981).
[8] V. I. Zakharov, Report No. TPI-MINN-90/7-T, 1990 (unpublished); Nucl. Phys. **B371**, 637 (1992).
[9] I. I. Balitsky and A. V. Yung, Phys. Lett. **168B**, 113 (1986).
[10] A. V. Yung, Nucl. Phys. **B297**, 47 (1988).
[11] V. V. Khoze and A. Ringwald, Nucl. Phys. **B355**, 351 (1991).
[12] I. I. Balitsky and V. M. Braun, Nucl. Phys. **B380**, 51 (1992).
[13] V. V. Khoze and A. Ringwald, Phys. Lett. B **259**, 106 (1991).
[14] J. Verbaarschot, Nucl. Phys. **B362**, 33 (1991).
[15] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov,

Nucl. Phys. **B165**, 45 (1980).
[16] A. H. Mueller, Nucl. Phys. **B348**, 310 (1991); **B353**, 44 (1991).
[17] J. C. Collins, in *Perturbative Quantum Chromodynamics,* edited by A. H. Mueller (World Scientific, Singapore, 1989).
[18] I. I. Balitsky and V. M. Braun, Nucl. Phys. **B361**, 93 (1991).
[19] J. Zinn-Justin, Phys. Rep. **70**, 109 (1981).
[20] L. N. Lipatov, Zh. Eksp. Teor. Fiz. **71**, 2010 (1976) [Sov. Phys. JETP **44**, 1055 (1976)]; **72**, 411 (1977) [**45**, 216 (1977)].
[21] I. I. Balitsky, Phys. Lett. B **273**, 282 (1991).
[22] V. I. Zakharov, Nucl. Phys. **B385**, 452 (1992).
[23] M. Maggiore and M. Shifman, Report No. TPI-MINN-92/19-T, 1992 (unpublished).
[24] I. I. Balitsky and M. G. Ryskin, Phys. Lett. B **296**, 185 (1992).