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Exclusive Radiative B-Decays in the Light-Cone QCD Sum Rule Approach

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Abstract

We carry out a detailed study of exclusive radiative rare B-decays in the framework of the QCD sum rules on the light cone, which combines the traditional QCD sum rule technique with the description of final state vector mesons in terms of the light-cone wave functions of increasing twist. Our calculation is restricted to the leading twist-two operators. The decays considered are: $B_{u,d} \to K^* + \gamma$, $B_{u,d} \to \rho + \gamma$, $B_d \to \omega + \gamma$ and the corresponding decays of the B_s mesons, $B_s \to \phi + \gamma$ and $B_s \to K^* + \gamma$. Based on our estimate of the transition form factor $F_1^{B\to K^*}(0) = 0.32 \pm 0.05$, we find for the branching ratio $BR(B \to K^* + \gamma) = (4.8 \pm 1.5) \times 10^{-5}$, which is in agreement with the observed value of $(4.5 \pm 1.5 \pm 0.9) \times 10^{-5}$ measured by the CLEO collaboration. We present detailed estimates for the ratios of the radiative decay form factors, which are then used to predict the rates for the exclusive radiative B-decays listed above. This in principle allows the extraction of the CKM matrix element $|V_{td}|$ from the penguin-dominated CKM-suppressed radiative decays when they are measured. We give a detailed discussion of the dependence of the radiative transition form factors on the b-quark mass and on the momentum transfer, as well as their interrelation with the CKM-suppressed semileptonic decay form factors in $B \to \rho + \ell + \nu_\ell$, which we also calculate in our approach.

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1 Introduction

Theoretical interest in rare B-decays lies in the first place in their potential role as precision tests of the quark sector of the Standard Model (SM). Interpreted within this framework, the eventual measurements of these decays will provide quantitative information about the top quark mass and more importantly about the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, V_{td}, V_{ts} and V_{tb}. In particular, the short-distance contribution to the CKM-suppressed rare B-decays $b \to d + \gamma$ and $b \to d + \ell^+\ell^-$ directly measures V_{td}. Together with improved measurements of the CKM matrix elements $|V_{cb}|$ and $|V_{ub}|$, this will determine the CKM unitarity triangle resulting from the constraint $\sum_i V_{id}V_{ib}^* = 0$, and thus, pin down the CP-violating phases in the Standard Model. These non-trivial constraints on the CKM unitarity and the importance of rare B-decays in this respect cannot be overemphasized. At the same time, rare B-decays induced by flavour changing neutral currents (FCNC) have the potential of providing one of the early hints for non-SM physics. It is, therefore, imperative to get estimates of these FCNC decays in the SM as reliable as possible and carry out an experimental physics programme sensitive to rare B-decays.

The experimental searches for the FCNC B-decays has already provided first dividends. The recent CLEO observation [1] of the rare decay mode $B \to K^\star + \gamma$ having a combined branching ratio $BR(B \to K^\star + \gamma) = (4.5 \pm 1.5 \pm 0.9) \times 10^{-5}$ and an improved upper limit on the inclusive branching ratio $BR(B \to X_s + \gamma) < 5.4 \times 10^{-4}$ (95% C.L.) [2] have been analysed in the SM context, as well as in several SM extensions. Within the SM, the resulting experimental measurements have been interpreted in terms of upper and lower limits on the CKM matrix element ratio $0.50 \leq |V_{ts}|/|V_{cb}| \leq 1.67$ (at 95% C.L.) [3].

On general grounds, the most quantitative tests of FCNC processes in B-decays are the ones involving inclusive measurements such as $B \to X_{d,s} + \gamma$ and the corresponding semileptonic and purely leptonic processes such as $B \to X_{d,s} + \ell^+\ell^-$ and $B_{d,s} \to \ell^+\ell^-$. Thus, the ratio of inclusive decay rates such as $BR(B \to X_d + \gamma)/BR(B \to X_s + \gamma)$ would provide one of the cleanest determinations of $|V_{td}|/|V_{ts}|$ [4]. However, inclusice branching ratios are rather difficult to measure. Exclusive decays such as $B \to \rho + \gamma,\ B \to \omega + \gamma$ and the semileptonic decays $B \to (\pi, \rho, \omega)\ell^+\ell^-$ are probably easier to measure, given a large number of B hadrons and a good electromagnetic detector and particle identification. The interpretation of these and the already measured decay $B \to K^* + \gamma$ necessarily needs reliable estimates of the decay form factors. The decay $B \to K^* + \gamma$ has received quite a bit of theoretical attention [5]–[15].

The aim of this paper is to estimate a number of exclusive radiative decays. These include the CKM-allowed decays, $B_{u,d} \to K^* + \gamma$ and $B_s \to \phi + \gamma$, and the CKM-suppressed decays, $B_{u,d} \to \rho + \gamma,\ B_d \to \omega + \gamma$ and $B_s \to K^* + \gamma$. While the effective Hamiltonian approach allows to include short-distance QCD corrections at scales $\mu^2 \geq m_b^2$, estimates of the hadronic matrix elements of the relevant operators necessarily require some non-perturbative technique. We use here QCD sum rules, which have proved to be a powerful tool for such purposes, and in their classical form have been introduced in the pioneering papers by Shifman, Vainshtein and Zakharov [16]. In this paper, we use a modification of the QCD sum rule approach, which we refer to as QCD sum rules on the light cone.

This technique has been developed originally for light quark systems in [17, 18]. The application for the heavy meson decays was first suggested and studied in [19].

In this approach, the ideas of duality and matching between the parton and hadron descriptions, intrinsic to the QCD sum rule framework, are combined with the specific techniques used in the studies of the hard exclusive processes in QCD [20, 21]. In contrast to the standard sum rule method for the evaluation of form factors [22, 23], in which the hadrons in the initial and final state are treated in a symmetric way, we resort to the QCD sum rule treatment of the initial B-meson only, and describe the outgoing light vector meson by the set of its wave functions of increasing twist. Hence the operator product expansion in the light-cone approach is governed by the twist of the operators rather than by their dimension, and the vacuum expectation values of local operators are replaced by the light-cone hadron wave functions. A physical motivation of the light-cone sum rule approach in the present context is an obvious asymmetry of the participating heavy initial and light final state mesons. The advantage of this formulation is that it allows to incorporate additional information about high-energy asymptotics of correlation functions in QCD, which is accumulated in the wave functions. The high energy behaviour of these functions is related to the (approximate) conformal invariance of QCD, and many properties and results, following from this approximate invariance and obtained previously, can be advantageously used in the present context as well. Existing applications of the light-cone sum rules include the calculations of the amplitudes for the radiative decay $\Sigma \to p\gamma$ [17], nucleon magnetic moments [18], the strong couplings $g_{\pi NN}$ and $g_{\rho\omega\pi}$ [18], the semileptonic B-meson [19, 24] and D-meson decay amplitudes [25]. In all these cases the results have been encouraging.

The principal theoretical result of this paper is a sum rule for the electromagnetic penguin form factor appearing in the decays $B \to V + \gamma$, where V is a vector meson. We derive the corresponding sum rule in the traditional QCD sum rule approach also and comment on similar existing sum rules in the literature [6]–[8]. We argue that the sum rules derived in the light-cone approach are more reliable and we calculate the radiative transition form factors for a number of exclusive radiative B-decays in this framework. The main numerical results of our work can be summarized as follows:

$$\begin{array}{rclrcl} F_1^{B\to K^*} & = & 0.32 \pm 0.05 & \qquad F_1^{B\to(\rho,\omega)} & = & 0.24 \pm 0.04 \\ F_1^{B_s\to\phi} & = & 0.29 \pm 0.05 & \qquad F_1^{B_s\to K^*} & = & 0.20 \pm 0.04 \end{array} \tag{1}$$

where the form factor F_1 is defined below (eq. (9)). The above estimate for the decay form factor $F_1^{B\to K^*}$ can be combined with the QCD-improved inclusive radiative branching ratio $BR(B \to X_s + \gamma) = (3.0 \pm 1.2) \times 10^{-4}$ [3] to yield $BR(B \to K^* + \gamma) = (4.8 \pm 1.5) \times 10^{-5}$. This agrees fairly well with the corresponding branching ratio measured by the CLEO collaboration, posted as $(4.5 \pm 1.5 \pm 0.9) \times 10^{-5}$ [1]. Together with the eventual measurements of the CKM-suppressed radiative decays $B_{u,d} \to \rho + \gamma,\ B_d \to \omega + \gamma$ and $B_s \to K^* + \gamma$, the remaining form factors in (1) can then be used to determine the CKM parameters.

While the main thrust of the paper is on providing theoretical estimates of the exclusive radiative B-decay rates, we also discuss a number of other, related, theoretical and phenomenological issues. In particular, we discuss in detail the heavy-quark-mass dependence of the radiative B-decay form factors, and derive the sum rules in the heavy quark limit. A light-cone QCD sum rule for the semileptonic decay $B \to \rho + \ell\nu_\ell$ is also derived and the relation between radiative and semileptonic form factors is studied.

The paper is organized as follows. In section 2, we review the effective Hamiltonian for radiative B-decays, dominated by the electromagnetic penguins, and we work out explicitly the dependence of their rates on the CKM matrix element parameters. A number of relations involving exclusive and inclusive radiative B-decays is given in this section. In section 3, we describe the method of the QCD sum rules on the light cone and derive our general sum rule for the form factors of the electromagnetic penguin operators entering in the decays $B \to V + \gamma$. The wave functions of the vector mesons which are needed to evaluate the sum rule are discussed in section 4, and the numerical estimates of the form factors are presented in section 5. Section 6 includes the discussion of the b-quark mass dependence of the form factors, and the heavy quark limit. In section 7 we discuss the q^2 dependence and the interrelation of radiative and semileptonic B-decay form factors in the QCD sum rule approach. Section 8 contains a summary and some concluding remarks, and the impact of measuring the decays $B_d \to \omega + \gamma$, $B_{u,d} \to \rho + \gamma$ and $B_s \to K^* + \gamma$ in determining the CKM matrix element $|V_{td}|$ is underlined. The traditional three-point QCD sum rule for the radiative B-decay $B \to K^* + \gamma$ is derived and discussed in the appendix.

2 Effective Hamiltonian for inclusive and exclusive radiative B-decays

In this section we review the theoretical framework within the Standard Model of electroweak interactions for inclusive and exclusive radiative B-decays dominated by the electromagnetic penguins. The inclusive decays are grouped as $B \to X_f + \gamma$, where we use the flavour of the light quark $f = s, d$ in the transition $b \to f$ to characterize the hadronic system recoiling against the photon. Including lowest-order QCD corrections and gluon bremsstrahlung, these decays are described at the parton level by the transitions $b \to f\gamma$ and $b \to f\gamma g$. In calculating the inclusive decay widths, we shall follow the work reported in [3, 4, 11]. The exclusive decays that are the principal concern of this work are grouped in an analogous way into $b \to s$ transitions:

- $B_u \to K^* + \gamma, \quad B_d \to K^* + \gamma,$
- $B_s \to \phi + \gamma,$

which we shall also term CKM-allowed, according to the dominant CKM matrix element dependence of their decay rates, and $b \to d$ transitions:

- $B_d \to \rho + \gamma, \quad B_d \to \omega + \gamma, \quad B_u \to \rho + \gamma,$
- $B_s \to K^* + \gamma,$

which will be called CKM-suppressed.

The framework to incorporate (perturbative) short-distance QCD corrections in a systematic way is that of an effective low energy theory with five quarks. It is obtained by integrating out the heavier degrees of freedom, i.e. the top quark and $W^\pm$ bosons. Since the running of α_s between m_t (present estimates $m_t = 164 \pm 27$ GeV [26]) and m_W is not very significant, and since m_W/m_t would not be a good expansion parameter, it is a reasonable approximation to integrate out both the top quark and the $W^\pm$ at the same scale.

Before using the unitarity properties of the CKM matrix, the effective Hamiltonian relevant for the processes $b \to f\gamma$ and $b \to f\gamma g$ has the form

$$\begin{aligned} H_{eff}^{(b\to f)} &= -\frac{4G_F}{\sqrt{2}} \left(V_{tb}V_{tf}^* \sum_{j=3}^{8} C_j(\mu)\, O_j(\mu) \right. \\ &+ \left. V_{cb}V_{cf}^* \sum_{j=1}^{8} C_j'(\mu)\, O_j'(\mu) + V_{ub}V_{uf}^* \sum_{j=1}^{8} C_j''(\mu)\, O_j''(\mu) \right) \quad , \end{aligned} \tag{2}$$

where G_F is the Fermi coupling constant and j runs through a complete set of operators with dimension up to six; the $C_j(\mu)$ are their Wilson coefficients evaluated at the scale μ.

We recall here that $O_1^{\prime(\prime)}$ and $O_2^{\prime(\prime)}$ represent the colour-singlet and colour-octet four-fermion operators, respectively, obtained from the SM charged current Lagrangian written in the charge retention form; in particular,

$$O_2' = (\bar{c}_{L\alpha}\gamma^\mu b_{L\alpha})(\bar{f}_{L\beta}\gamma_\mu c_{L\beta}) \quad \text{and} \quad O_2'' = (\bar{u}_{L\alpha}\gamma^\mu b_{L\alpha})(\bar{f}_{L\beta}\gamma_\mu u_{L\beta}) \ , \tag{3}$$

where α and β are SU(3) colour indices. The remaining operators are the same for all three CKM prefactors, i.e. $O_j' = O_j'' = O_j$ for $j \neq 1, 2$. At tree level, the only contribution to $b \to f\gamma$ comes from the magnetic moment operator

$$O_7 = \frac{e}{16\pi^2}\, \bar{f}\, \sigma^{\mu\nu}\, (m_b R + m_f L)\, b\ F_{\mu\nu} \ , \tag{4}$$

where $L, R = (1 \mp \gamma_5)/2$, and e is the QED coupling constant. The four-fermion operators $O_3, ..., O_6$ and the QCD magnetic moment operator O_8, which is the gluonic counterpart of O_7, arise from penguin diagrams in the full theory (before integrating out W and t). These operators enter indirectly in $b \to f\gamma$ decays due to operator mixing and through (virtual and bremsstrahlung) gluon corrections.

Taking into account the unitarity of the CKM matrix, only two terms in the combinations $\lambda_i \equiv V_{ib}V_{is}^\star$ — or in the combinations $\xi_i \equiv V_{ib}V_{id}^\star$ in the case of CKM-suppressed transitions — are independent. For $b \to s$ transitions, one finds $|\lambda_u| \ll |\lambda_c|, |\lambda_t|$; therefore, when neglecting terms proportional to λ_u, the effective Hamiltonian (2) can be brought into a form proportional to $\lambda_t \approx -\lambda_c$:

$$H_{eff}^{(b\to s)} = -\frac{4G_F}{\sqrt{2}}\, \lambda_t \sum_{j=1}^{8} C_j(\mu)\, O_j(\mu) \quad . \tag{5}$$

For $b \to d$ transitions, where ξ_u, ξ_c and ξ_t are all of the same order of magnitude, it is most convenient to choose ξ_u and ξ_c as independent CKM factors during the matching for the Wilson coefficients at $\mu = m_W$ and during the renormalization group evolution to $\mu \approx m_b$. Since terms of order $O(m_u^2/m_W^2)$ and $O(m_c^2/m_W^2)$ (or $O(m_u^2/m_t^2)$ and $O(m_c^2/m_t^2)$)

are thereby neglected, the terms proportional to ξ_u and ξ_c are multiplied by just the same coefficient functions, i.e. $C'_j(\mu) = C''_j(\mu)$ (although the corresponding operators are, of course, different, see (3)). Finally, exploiting again $\xi_u + \xi_c = -\xi_t$, one ends up with

$$H_{eff}^{(b\to d)} = -\frac{4G_F}{\sqrt{2}} \left(\xi_t \sum_{j=3}^{8} C_j(\mu)\, O_j(\mu) - \sum_{j=1}^{2} C_j(\mu) \left\{ \xi_c O'_j(\mu) + \xi_u O''_j(\mu) \right\} \right) \quad , \qquad (6)$$

where the Wilson coefficients $C_j(\mu)$ are precisely the same functions as in (5). Details of the full operator basis, the matching of the Wilson coefficients C_j at $\mu \approx m_W$, and the complete leading-logarithmic renormalization group evolution to $\mu \ll m_W$ can be found in the literature [28].

In estimates of the inclusive branching ratio for $BR(B \to X_s + \gamma)$ one has to include the contribution of the QCD bremsstrahlung process $b \to s + \gamma + g$ and the virtual corrections to $b \to s + \gamma$, both calculated in $O(\alpha\alpha_s)$ in ref. [11]. Expressed in terms of the inclusive semileptonic branching ratio $BR(B \to X\ell\nu_\ell)$, one finds:

$$BR(B \to X_s + \gamma) = 6\frac{\alpha}{\pi}\frac{|\lambda_t|^2}{|V_{cb}|^2}\frac{|C_7(x_t, m_b)|^2 K(x_t, m_b)}{g(m_c/m_b)(1 - 2/3\frac{\alpha_s}{\pi} f(m_c/m_b))} \times BR(B \to X\ell\nu_\ell) \ , \qquad (7)$$

where $x_t = m_t^2/m_W^2$ indicates the explicit m_t-dependence in $C_7(x_t, m_b) \equiv C_7(m_b)$, and $g(r) = 1 - 8r^2 + 8r^6 - r^8 - 24r^4 \ln(r)$ is the phase-space function for $\Gamma(b \to c + \ell\nu_\ell)$. The function $f(r)$ accounts for QCD corrections to the semileptonic decay and can be found, for example, in ref. [32]. It is a slowly varying function of r, and for a typical quark mass ratio of $r = 0.35 \pm 0.05$, it has the value $f(r) = 2.37 \mp 0.13$. The contributions from the decays $b \to u + \ \ell\ \nu_\ell$ have been neglected in the denominator in (7) since they are numerically inessential ($|V_{ub}| \ll |V_{cb}|$). For the semileptonic branching ratio, the measured value $BR(B \to X\ell\nu_\ell) \simeq 11\%$ will be used.

The inclusive decay width for $B \to X_s + \gamma$ is dominantly contributed by the magnetic moment term $C_7(x_t, \mu)O_7(\mu)$, hence the rationale of factoring out this coefficient in the expression for $BR(B \to X_s + \gamma)$ in Eq. (7) above. Including $O(\alpha_s)$ corrections brings to the fore other operators with their specific Wilson coefficients. The effect of these additional terms can be expressed in terms of the function $K(x_t, m_b)$, which is a K-factor in the sense of QCD corrections. The function $K(x_t, m_b)$ has been computed in [3] taking into account the dominant corrections from C_2 and C_8 (the coefficients of other operators are considerably smaller [28]). The resulting branching ratio is[‡] [3]:

$$BR(B \to X_s + \gamma) = (3.0 \pm 0.5) \times 10^{-4} \qquad (8)$$

for m_t in the range 100 GeV $\leq\ m_t\ \leq$ 200 GeV. This is to be contrasted with the present upper limit on the exclusive radiative decay, $BR(B \to X_s + \gamma) < 5.4 \times 10^{-4}$ (at 90% C.L.) [2].

It has been argued in ref. [3] that the scale dependence of $C_7(x_t, \mu)$ is very pronounced in the presently available leading-logarithmic approximation. This inherent uncertainty of the present theoretical framework has to be borne in mind in a quantitative discussion

[‡]obtained by setting the CKM matrix element ratio $|\lambda_t|^2/|V_{cb}|^2$ to 1, according to our present knowledge of the CKM matrix (readily seen, e.g., in the Wolfenstein representation [27]).

of the QCD-improved decay rates. In ref. [3] this uncertainty has been estimated by varying the scale parameter μ in the range $m_b/2 \leq \mu \leq 2m_b$ and introduces an additional theoretical error of $\pm 1 \times 10^{-4}$ in the above estimates for $BR(B \to X_s + \gamma)$.

The case of inclusive $B \to X_d + \gamma$ decays is somewhat more complicated with respect to the factorization of the CKM parameters. When evaluating the one-loop matrix elements for the parton process $b \to s\gamma g$, at least the four-quark operators O_2' and O_2'' should be kept since their coefficients are of order unity (they contribute in penguin diagrams emitting both a gluon and a photon). These contributions involve a non-trivial dependence on the CKM matrix elements through terms proportional to ξ_u and ξ_c, see eq. (6). Explicit expressions for this dependence on the ρ and η parameters in the Wolfenstein representation of the CKM matrix have been evaluated in ref. [4].

In the following we shall focus our attention on *exclusive* decays, like $B_{u,d} \to K^* + \gamma$ and $B_s \to \phi + \gamma$, and the corresponding CKM-suppressed modes listed at the beginning of this section. At tree level only the magnetic moment operator O_7 of eq. (4) contributes to the transition amplitudes. For a generic radiative decay $B \to V + \gamma$ one defines a transition form factor $F_1(q^2)$ as:

$$\langle V, \lambda | \bar{f} \sigma_{\mu\nu} q^\nu b | B \rangle = i \epsilon_{\mu\nu\rho\sigma} e^{*(\lambda)\nu} p_B^\rho p_V^\sigma 2 F_1^{B \to V}(q^2) \ , \tag{9}$$

where V is a vector meson ($V = \rho, \omega, K^*$ or ϕ) with the polarization vector $e^{(\lambda)}$; and B is the generic B-meson B_u, B_d or B_s. The vectors p_B, p_V and $q = p_B - p_V$ denote the four-momenta of the initial B-meson and the outgoing vector meson and photon, respectively.

In (9) it is understood that the operator is evaluated at the scale $\mu = m_b$, and all large logarithms, $\ln(m_W/m_b)$ and $\ln(m_t/m_b)$, are included in the coefficient function $C_7(\mu = m_b)$. The b-quark mass, which has been factored out, should be identified with the pole mass, although the complete two-loop treatment of the coefficient function is needed to make this identification meaningful.

In evaluating the hadronic matrix elements, one may consider the b-quark mass as a large parameter, and try to collect logarithms, corresponding to the so-called hybrid anomalous dimension [35]. At zero-recoil, $q^2_{\max} = (m_B - m_V)^2 \simeq 19\ \mathrm{GeV}^2$, this treatment is simple and the answer is obtained in the framework of the heavy quark effective theory. However, in this case the extrapolation to the physical point $q^2 = 0$ introduces a large uncertainty. We take a different approach, and use the QCD sum rules to calculate the form factors directly at the physical point $q^2 = 0$ and for the finite value of the b-quark mass. The price to pay is that the treatment of hybrid logarithms becomes complicated, and we shall ignore them in this paper.

With the above definition, the exclusive decay widths are given by ($B = B_u$ or B_d):

$$\Gamma(B \to K^\star + \gamma) = \frac{\alpha}{32\pi^4} G_F^2 |\lambda_t|^2 |F_1^{B \to K^*}(0)|^2 C_7(x_t, m_b)^2 (m_b^2 + m_s^2) \frac{(m_B^2 - m_{K^*}^2)^3}{m_B^3}, \tag{10}$$

and the analogous expression for $\Gamma(B_s \to \phi + \gamma)$. The branching ratios of these exclusive decays can again be written in terms of the inclusive semileptonic branching ratio:

$$BR(B \to K^\star + \gamma) = 6 \frac{\alpha}{\pi} \frac{|\lambda_t|^2}{|V_{cb}|^2} \frac{|C_7(x_t, m_b)|^2 |F_1^{B \to K^*}(0)|^2}{g(m_c/m_b)(1 - 2/3 \frac{\alpha_s}{\pi} f(m_c/m_b))} \frac{(1 - m_{K^*}{}^2/m_B^2)^3}{(1 - m_s^2/m_b^2)^3} \times (11\%). \tag{11}$$

A good quantity to test the model dependence of the form factors for the exclusive decay is the ratio of the exclusive-to-inclusive radiative decay widths ($B = B_u$ or B_d):

$$R(K^*/X_s) \equiv \frac{\Gamma(B \to K^* + \gamma)}{\Gamma(B \to X_s + \gamma)} = \frac{(1 - {m_{K^*}}^2/m_B^2)^3}{(1 - m_s^2/m_b^2)^3} \frac{m_B^3}{m_b^3} \frac{|F_1^{B_i \to K^*}(0)|^2}{K(x_t, m_b)} . \qquad (12)$$

Note that $K(x_t, m_b) \simeq 0.83$ is almost independent of m_t (for the range 100 GeV $\leq m_t \leq$ 200 GeV) [3]. The exclusive-to-inclusive ratio involving B_s-decays is defined in an analogous way.

Since the same short-distance-corrected coefficient function $C_7(m_b)$ enters in the Hamiltonian for CKM-allowed (5) and CKM-suppressed (6) modes, the QCD scaling is identical for the two-body decays $b \to s + \gamma$ and $b \to d + \gamma$, and it does not affect the ratio of the decay widths $\Gamma(b \to d + \gamma)/\Gamma(b \to s + \gamma)$. The same applies for the exclusive decays such as $B \to \rho + \gamma$ and $B \to K^* + \gamma$, in which case the CKM factors factorize in the decay amplitudes.

From this observation a number of relations between the exclusive decay rates follow in the Standard Model [4]. This is exemplified by the decay rates for $B_{u,d} \to \rho + \gamma$ and $B_{u,d} \to K^* + \gamma$:

$$\frac{\Gamma(B_{u,d} \to \rho + \gamma)}{\Gamma(B_{u,d} \to K^* + \gamma)} = \frac{|V_{td}|^2}{|V_{ts}|^2} \frac{|F_1^{B \to \rho}(0)|^2}{|F_1^{B \to K^*}(0)|^2} \Phi_{u,d} , \qquad (13)$$

where $\Phi_{u,d}$ is a phase-space factor:

$$\Phi_{u,d} = \frac{(m_b^2 + m_d^2)}{(m_b^2 + m_s^2)} \frac{(m_{B_{u,d}}^2 - m_\rho^2)^3}{(m_{B_{u,d}}^2 - m_{K^*}^2)^3}. \qquad (14)$$

The ratio (13) depends only on the CKM matrix elements and the ratio of form factors, while it is independent of the top quark mass (and of the renormalization scale μ).

Note that the decay width $\Gamma(B_d \to \omega + \gamma)$ is expected to be equal to the decay width $\Gamma(B_d \to \rho + \gamma)$, apart from the minor difference in the phase-space factors Φ. This follows from the assumption that the quark wave functions for ω and ρ are described by the isoscalar and isovector combinations, $|\omega\rangle = 1/\sqrt{2}(\bar{u}u + \bar{d}d)$ and $|\rho\rangle = 1/\sqrt{2}(\bar{u}u - \bar{d}d)$, respectively.

The CKM-suppressed exclusive decay width $\Gamma(B_s \to K^* + \gamma)$ can be related to the CKM-allowed decay width $\Gamma(B_d \to K^* + \gamma)$ by a relation similar to the one given in eq. (13). However, one expects a substantial difference in the form factors $F_1^{B_d \to K^*}(0)$ and $F_1^{B_s \to K^*}(0)$ due to the exchange of the roles of s and d quarks in the wave function of the K^* in the two decay modes.

3 QCD sum rules on the light cone

The aim of this and the following two sections is to calculate transition form factors, governing the radiative B-decays $B \to V + \gamma$, as defined in (9). Our approach is very close to the calculation of the semileptonic $B \to \pi e \nu$ form factor in [19, 24].

To derive the sum rule, we consider the correlation function

$$i \int dx\, e^{iqx} \langle V(p, \lambda) | T\{\bar{\psi}(x) \sigma_{\mu\nu} q^\nu b(x) \bar{b}(0) i\gamma_5 \psi(0)\} | 0 \rangle = i\epsilon_{\mu\nu\rho\sigma} e^{*(\lambda)\nu} q^\rho p^\sigma T((p+q)^2) \quad (15)$$

at $q^2 = 0$, $p^2 = m_V^2$, and at Euclidean $m_b^2 - (p+q)^2$ of order several GeV^2. Hereafter we use ψ as a generic notation for the field of the light quark. Writing down the dispersion relation in $(p+q)^2$, we can separate the contribution of the B-meson as the pole contribution to the invariant function $T((p+q)^2)$:

$$T((p+q)^2) = \frac{f_B m_B^2}{m_b + m_q} \frac{2F_1(0)}{m_B^2 - (p+q)^2} + \dots , \tag{16}$$

where the dots stand for contributions of higher-mass resonances and the continuum. The B-meson decay constant is defined in the usual way,

$$\langle 0|\bar{\psi}\gamma_\mu\gamma_5 b|B(p)\rangle = ip_\mu f_B , \tag{17}$$

and m_B, m_b and m_q are the B-meson, b-quark and light-quark masses, respectively.

The virtuality of the heavy quark in the correlation function under consideration is large, of order $m_b^2 - (p+q)^2$, and one can use the perturbative expansion of its propagator in the external field of slowly varying fluctuations inside the vector meson. The leading contribution corresponds to the diagram shown in Fig. 1a, and equals

$$\int dx\, e^{iqx} \int \frac{dk}{(2\pi)^4}\, e^{-ikx} \frac{q_\nu}{m_b^2 - k^2} \langle V(p,\lambda)|T\{\bar{\psi}(x)\sigma_{\mu\nu}(m_b + \not k) i\gamma_5 \psi(0)\}|0\rangle. \tag{18}$$

To the leading-twist accuracy, taking into account gluon corrections like the one in Fig. 1b produces the path-ordered gauge factor in between the remaining quark fields in (18), which we do not show for brevity.

Thus, in general we are left with matrix elements of gauge-invariant non-local operators, sandwiched in between the vacuum and the meson state. These matrix elements define the light-cone meson wave functions, which have received a lot of attention in the past decade. Following Chernyak and Zhitnitsky [36], we define

$$\langle 0|\bar{\psi}(0)\sigma_{\mu\nu}\psi(x)|V(p,\lambda)\rangle = i(e_\mu^{(\lambda)} p_\nu - e_\nu^{(\lambda)} p_\mu) f_V^\perp \int_0^1 du\, e^{-iupx} \phi_\perp(u,\mu^2). \tag{19}$$

Likewise,

$$\begin{aligned}
\langle 0|\bar{\psi}(0)\gamma_\mu\psi(x)|V(p,\lambda)\rangle &= p_\mu \frac{(e^{(\lambda)}x)}{(px)} f_V m_V \int_0^1 du\, e^{-iupx} \phi_\parallel(u,\mu^2) \\
&\quad + \left(e_\mu^{(\lambda)} - p_\mu \frac{(e^{(\lambda)}x)}{(px)}\right) f_V m_V \int_0^1 du\, e^{-iupx} g_\perp^{(v)}(u,\mu^2), \qquad (20)\\
\langle 0|\bar{\psi}(0)\gamma_\mu\gamma_5\psi(x)|V(p,\lambda)\rangle &= -\frac{1}{4}\epsilon_{\mu\nu\rho\sigma} e^{(\lambda)\nu} p^\rho x^\sigma f_V m_V \int_0^1 du\, e^{-iupx} g_\perp^{(a)}(u,\mu^2). \qquad (21)
\end{aligned}$$

The functions $\phi_\perp(u,\mu^2)$ and $\phi_\parallel(u,\mu^2)$ give the leading-twist distributions in the fraction of total momentum carried by the quark in transversely and longitudinally polarized mesons, respectively. The functions $g_\perp^{(v)}(u,\mu^2)$ and $g_\perp^{(a)}(u,\mu^2)$ are discussed in detail below. The normalization is chosen in such a way that for all four distributions $f = \phi_\perp, \phi_\parallel, g_\perp^{(v)}, g_\perp^{(a)}$, we have:

$$\int_0^1 du\, f(u) = 1.$$

In the matrix elements of non-local operators on the l.h.s. of (19)–(21) the separations are assumed to be light-like, i.e. $x^2 = 0$. Regularization of UV divergences that arise in the process of the extraction of the leading $x^2 \to 0$ behaviour produces a non-trivial scale dependence of the wave functions, which can be found by renormalization group methods [20, 21]. The scale in (18) is fixed by the actual light cone separation, $\mu^2 \sim x^{-2} \sim m_b^2 - (p+q)^2$.

Putting eqs. (18)–(21) together, we obtain

$$\begin{aligned} T((p+q)^2) &= \int_0^1 du \frac{1}{m_b^2 + \bar{u} u m_V^2 - u(p+q)^2} \left[m_b f_V^\perp \phi_\perp(u) + u m_V f_V g_\perp^{(v)}(u) \right. \\ &\left. + \frac{1}{4} m_V f_V g_\perp^{(a)} \right] + \frac{1}{4} \int_0^1 du \frac{m_b^2 - u^2 m_V^2}{(m_b^2 + \bar{u} u m_V^2 - u(p+q)^2)^2} m_V f_V g_\perp^{(a)}(u) \,. \end{aligned} \tag{22}$$

where $\bar{u} = 1 - u$. The expression in (22) has the form of a dispersion integral in $(p+q)^2$, which can be made explicit by introducing the squared mass of the intermediate state $s = m_b^2/u + \bar{u} m_V^2$ as the integration variable, instead of the Feynman parameter u. The basic assumption of the QCD sum rule approach is that the contribution of the B-meson corresponds in this dispersion integral to the contribution of intermediate states with masses smaller than a certain threshold $s < s_0$ (duality interval). Making the Borel transformation $1/(s - (p+q)^2) \to \exp(-s/t)$ and equating the result to the B-meson contribution in (16), we arrive at the sum rule

$$\begin{aligned} &\frac{f_B m_B^2}{m_b + m_q} 2F_1(0) e^{-(m_B^2 - m_b^2)/t} = \\ &= \int_0^1 du \frac{1}{u} \exp\left[-\frac{\bar{u}}{t} \left(\frac{m_b^2}{u} + m_V^2 \right) \right] \theta \left[s_0 - \frac{m_b^2}{u} - \bar{u} m_V^2 \right] \left\{ m_b f_V^\perp \phi_\perp(u, \mu^2 = t) \right. \\ &\left. + u m_V f_V g_\perp^{(v)}(u, \mu^2 = t) + \frac{m_b^2 - u^2 m_V^2 + ut}{4ut} m_V f_V g_\perp^{(a)}(u, \mu^2 = t) \right\}, \end{aligned} \tag{23}$$

which should be satisfied for values of the Borel parameter t of order several GeV^2. To the leading logarithmic accuracy, the scale in the wave functions coincides with the Borel parameter. Principal input in this sum rule are the vector meson wave functions, which contain non-trivial information about the dynamics at large distances, and which we are going to discuss now.

4 Wave functions of the vector mesons

It is known that the decomposition of the leading-twist meson wave functions in terms of conformal invariant operators allows one to diagonalize the mixing matrix at one-loop order. Thus, the (approximate) conformal invariance of QCD implies that the coefficients in the expansion of leading-twist wave functions in the series of Gegenbauer polynomials [37] are renormalized multiplicatively to the leading logarithmic accuracy. In our case

$$\phi_\perp(u, \mu) = 6u(1-u) \left[1 + a_1(\mu)\xi + a_2(\mu) \left(\xi^2 - \frac{1}{5} \right) + a_3(\mu) \left(\frac{7}{3}\xi^3 - \xi \right) + \ldots \right],$$

$$a_n(\mu) \;=\; a_n(\mu_0)\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\gamma_n/b}, \tag{24}$$

where we have introduced the shorthand notation $\xi = 2u-1$. Here $b = (11/3)N_c - (2/3)n_f$. The anomalous dimensions turn out to be [38]

$$\gamma_n = C_F\left(1 + 4\sum_{j=2}^{n+1} 1/j\right), \tag{25}$$

where $C_F = (N_c^2-1)/(2N_c)$. The coefficients in the Gegenbauer expansion (24) at a low scale, $a_n(\mu_0)$, should be determined by a certain non-perturbative approach, or taken from experiment. At present, most of the existing information comes from the QCD sum rules. Following [36], we use the model wave functions for ρ, K^* and ϕ mesons at the scale $\mu_0^2 = 1$ GeV^2, corresponding to the following choice of the parameters in (24):

$$\begin{gathered} a_2^{(\rho)} = -1.25, \\ a_1^{(K^*)} = 0.75, \quad a_2^{(K^*)} = -2, \quad a_3^{(K^*)} = 0.75, \\ a_2^{(\phi)} = -2.5. \end{gathered} \tag{26}$$

Note that in the case of ρ and ϕ mesons only a_2 is non-zero, and the wave function is symmetric under the interchange $u \leftrightarrow 1-u$. In the case of a K^*-meson, it is necessary to specify that the variable u is the fraction of the total momentum carried by the strange quark. The decay constants appearing in (19) are [36]:

$$\begin{aligned} f_\rho^\perp &= 200\,\mathrm{MeV}, \\ f_{K^*}^\perp &= 210\,\mathrm{MeV}, \\ f_\phi^\perp &= 230\,\mathrm{MeV}. \end{aligned} \tag{27}$$

As mentioned above, the correct normalization point in the sum rules is of the order of the typical Borel parameter, which for B-meson decays is of order $\mu^2 \sim m_B^2 - m_b^2 \sim 5$ GeV^2. Using $\Lambda_{\overline{MS}} = 225$ MeV in the renormalization group rescaling factors given in (24), we obtain at this scale

$$\begin{gathered} a_2^{(\rho)} = -0.85, \\ a_1^{(K^*)} = 0.57, \quad a_2^{(K^*)} = -1.35, \quad a_3^{(K^*)} = 0.46, \\ a_2^{(\phi)} = -1.7. \end{gathered} \tag{28}$$

The wave functions corresponding to the above parametrization are shown in Fig. 2. As a general effect of the rescaling, the wave functions become somewhat wider and closer to the asymptotic expression $6u(1-u)$. In the case of the K^*-meson, the wave function becomes both wider and more symmetric, so that scaling violation affects the region $u < 1/2$ only (see Fig. 2b). The integration over u in the sum rule in (23) is restricted to the interval of rather large momentum fractions, carried by the light quark involved in the electromagnetic penguin operator. For realistic values of parameters, see below, one has $u > 0.65$–0.7. According to the analysis in [36], the K^* wave function turns out to be in this region very close to the asymptotic expression $6u(1-u)$. In the case of the decay $B_s \to K^*\gamma$, the role of the strange and non-strange quarks in the K^*-meson is reversed, corresponding to the formal substitution $u \to 1-u$ in the wave function. Thus, the region of small values $u < 0.3 - 0.35$ in Fig. 2b becomes appropriate, where deviations from the asymptotic form are large and the effect of rescaling is important. This results in a significant difference between the form factors in the decays $B_d \to K^* + \gamma$ and $B_s \to K^* + \gamma$.

The conformal expansion of the wave functions $g_\perp^{(v)}(u,\mu^2)$ and $g_\perp^{(a)}(u,\mu^2)$ is somewhat more involved and can be obtained using the approach of [39]. To this end we define new wave functions, which contain quarks with fixed spin projections on the light cone:

$$
\begin{aligned}
\langle 0|\bar\psi(0)\, \not{z}\gamma_\mu\, \not{p}\psi(x)|V(p,\lambda)\rangle &= -e_\mu^\lambda m_V f_V (px)\int_0^1 du\, e^{-ipxu} g^{\uparrow\downarrow}(u,\mu^2), \\
\langle 0|\bar\psi(0)\, \not{p}\gamma_\mu\, \not{z}\psi(x)|V(p,\lambda)\rangle &= -e_\mu^\lambda m_V f_V (px)\int_0^1 du\, e^{-ipxu} g^{\downarrow\uparrow}(u,\mu^2).
\end{aligned} \tag{29}
$$

The wave functions $g^{\uparrow\downarrow}(u)$ and $g^{\downarrow\uparrow}(u)$ can be expanded in terms of irreducible representations of the collinear conformal group as [40, 41, 39]:

$$
\begin{aligned}
g^{\uparrow\downarrow}(u) &= 2(1-u)\left[1 + c_1^{\uparrow\downarrow} P_1^{(1,0)}(\xi) + c_2^{\uparrow\downarrow} P_2^{(1,0)}(\xi) + \ldots\right], \\
g^{\downarrow\uparrow}(u) &= 2u\left[1 + c_1^{\downarrow\uparrow} P_1^{(0,1)}(\xi) + c_2^{\downarrow\uparrow} P_2^{(0,1)}(\xi) + \ldots\right],
\end{aligned} \tag{30}
$$

where $P_k^{(l,m)}(\xi)$ are Jacobi polynomials [37]. The normalization in (29) follows from the standard definition of vector decay constants

$$
\langle 0|\bar\psi\gamma_\mu\psi|V(p,\lambda)\rangle = e_\mu^\lambda m_V f_V. \tag{31}
$$

The numerical values determined from QCD sum rules are [16, 36]

$$
\begin{aligned}
f_\rho &= 200\,\mathrm{MeV}, \\
f_{K^*} &= 210\,\mathrm{MeV}, \\
f_\phi &= 230\,\mathrm{MeV}.
\end{aligned} \tag{32}
$$

Note that these couplings, to the accuracy of the existing QCD sum rule calculations, coincide with the couplings in (27).

Neglecting $SU(3)$-breaking effects related to the difference of the quark and antiquark masses in the mesons, one has

$$
c_k^{\uparrow\downarrow} = (-1)^k c_k^{\downarrow\uparrow} \equiv c_k. \tag{33}
$$

Finally, the coefficient c_1 is actually fixed by the equations of motion, which allow one to reduce the matrix element

$$
\langle 0|\bar\psi\gamma_\mu\gamma_5(i\overleftrightarrow{D}_\nu)\psi|V(p,\lambda)\rangle = iA\epsilon_{\mu\nu\rho\sigma}e^\rho p^\sigma \tag{34}
$$

to the matrix element in (31). Thus, we find $A = -(1/2)f_V m_V$ [42] and

$$
c_1 = -1/2\,. \tag{35}
$$

On the other hand, one has the obvious relations:

$$
\begin{aligned}
g_\perp^{(v)}(u) &= \frac{1}{2}[g^{\uparrow\downarrow}(u) + g^{\uparrow\downarrow}(u)]\,, \\
\frac{d}{du}g_\perp^{(a)}(u) &= 2[g^{\uparrow\downarrow}(u) - g^{\uparrow\downarrow}(u)]\,.
\end{aligned}
\tag{36}
$$

Using the identities [37]

$$
\begin{aligned}
(1+\xi)P_k^{(0,1)}(\xi) + (1-\xi)P_k^{(1,0)}(\xi) = 2C_k^{1/2}(\xi)\,, \\
(1+\xi)P_k^{(0,1)}(\xi) - (1-\xi)P_k^{(1,0)}(\xi) = 2C_{(k+1)}^{1/2}(\xi)\,, \\
\frac{d}{d\xi}(1-\xi^2)C_k^{3/2}(\xi) = -(k+1)(k+2)C_{k+1}^{1/2}(\xi)\,,
\end{aligned}
\tag{37}
$$

where $C_k^\lambda(\xi)$ are Gegenbauer polynomials, we arrive at the expansions:

$$
\begin{aligned}
g_\perp^{(v)}(u) &= \sum_{k=2n}(c_k - c_{k-1})C_k^{1/2}(\xi)\,, \\
g_\perp^{(a)}(u) &= 2(1-\xi^2)\sum_{k=2n}\frac{c_k - c_{k+1}}{(k+1)(k+2)}C_k^{3/2}(\xi)\,.
\end{aligned}
\tag{38}
$$

The coefficients in front of each Gegenbauer polynomial come from the operators with two neighbouring conformal spins. According to a general result [40], the operators with different conformal spin do not mix under renormalization to leading logarithmic accuracy. This is a major simplification but does not guarantee multiplicative renormalization in the present case because of the existence of three-particle antiquark–quark–gluon operators with the same twist and conformal spin (see [39]).

Further insight in the structure of the wave functions $g_\perp^{(v)}(u)$ and $g_\perp^{(a)}(u)$ can be obtained by using the equations of motion. Note that both these wave functions correspond to transverse spin distributions, and involve precisely the same operators (apart from a missing γ_5 in the case of $g_\perp^{(v)}(u)$) as the ones involved in the operator product expansion for the structure function $g_2(x,Q^2)$ of polarized deep inelastic lepton–nucleon scattering. The difference is indeed in the matrix elements – which involve the (polarized) nucleons with equal momenta in the latter case and the vacuum and the vector meson state in the present situation. Thus, similar to the case of the structure function g_2, the wave functions $g_\perp^{(v)}(u)$ and $g_\perp^{(a)}(u)$ contain contributions coming from both operators of twist 2 and twist 3. In close analogy to deep inelastic scattering, the twist-2 contributions to the "transverse" wave functions $g_\perp^{(v)}$, $g_\perp^{(a)}$ can be expressed in terms of the leading-twist "longitudinal" wave function $\phi_\parallel(u,\mu^2)$ defined in (21):

$$
\begin{aligned}
g_\perp^{(v),\mathrm{twist-2}}(u) &= \frac{1}{2}\left[\int_0^u dv\frac{\phi_\parallel(v)}{\bar v} + \int_u^1 dv\frac{\phi_\parallel(v)}{v}\right] \\
\frac{d}{du}g_\perp^{(a),\mathrm{twist-2}}(u) &= 2\left[-\int_0^u dv\frac{\phi_\parallel(v)}{\bar v} + \int_u^1 dv\frac{\phi_\parallel(v)}{v}\right]
\end{aligned}
\tag{39}
$$

which is the analogue of the Wandzura-Wilczek relations [43] between the structure functions g_2 and g_1. The remaining twist-3 contributions to $g_\perp^{(v)}$, $g_\perp^{(a)}$ can be written in terms of three-particle antiquark–gluon–quark wave functions of transversely polarized vector mesons, considered in [42, 36]. The derivation of (39) and the relations for twist-3 contributions will be given elsewhere.

In this paper, we do not take into account contributions of twist 3 which come from the gluon-exchange diagram in Fig. 1b, and to this accuracy we need to keep the twist-2 contributions to the wave functions $g_\perp^{(v)}$, $g_\perp^{(a)}$ only. In anology with a similar situation in the case of the structure function g_2, one should expect that corrections to the asymptotic wave function $\phi_\parallel(u) = 6u(1-u)$, coming from twist-2 operators involving gluons [36] are of the same order of magnitude as neglected contributions of twist 3. Thus, for consistency, we must neglect the gluon corrections to $g_\perp^{(v)}$, $g_\perp^{(a)}$ altogether, which amounts to putting to zero all the coefficients in (30) except the first two, $c_0 = 1$ and $c_1 = -1/2$. Thus, we obtain, finally

$$\begin{aligned} g_\perp^{(v)}(u) &= 1 + \frac{1}{2}C_2^{1/2}(\xi) = \frac{3}{4}(1+\xi^2)\,, \\ g_\perp^{(a)}(u) &= \frac{3}{2}(1-\xi^2) = 6u(1-u)\,, \end{aligned} \tag{40}$$

which we use in the numerical analysis. It is easy to check that these expressions are exactly the ones that follow from (39) with the asymptotic expression for the longitudinal distribution amplitude $\phi_\parallel(u) = 6u(1-u)$. The same expressions for the transverse wave functions in (40) have been found in [42] by a different method. The identification of these contributions as being of twist 3 in ref. [42] is, however, an error.

5 Evaluation of the sum rules

To complete the list of entries which appear in the sum rule (23) we must specify the value of the pole b-quark mass m_b and the continuum threshold s_0. We vary these parameters in the limits

$$\begin{aligned} m_b &= 4.6\text{–}4.8\,\mathrm{GeV}\,, \\ s_0^{(B)} &= 33\text{–}35\,\mathrm{GeV}^2\,, \end{aligned} \tag{41}$$

and for the continuum threshold in the B_s-channel, we take

$$s_0^{B_s} - s_0^B = m_{B_s}^2 - m_B^2 \simeq 1\ \ \mathrm{GeV}^2, \tag{42}$$

making use of the present world average $m_{B_s} = 5373.2 \pm 4.2\,\mathrm{MeV}$ [44, 45, 46].

The value of the B-meson decay constant f_B has received a lot of attention recently. Within the QCD sum rule approach, its value is strongly correlated with the values of the b-quark mass and the continuum threshold (see [47] and references cited therein). For consistency, we substitute the value of f_B in (23) by the square root of the corresponding QCD sum rule:

$$\begin{aligned} &\frac{f_B^2 m_B^4}{m_b^2} e^{-(m_B^2-m_b^2)/t} \\ &= \frac{3}{8\pi^2}\int_{m_b^2}^{s_0} sds\, e^{-(s-m_b^2)/t}(1-m_b^2/s)^2 - m_b\langle\bar{q}q\rangle_{\mu^2=t} - m_b\langle\bar{q}\sigma gGq\rangle_{\mu^2=t}\frac{1}{2t}\left(1-\frac{m_b^2}{2t}\right)\,, \end{aligned} \tag{43}$$

in which we discarded the radiative $O(\alpha_s)$ corrections, as they are not taken into account in the sum rule (23) either, and we also neglected numerically insignificant contributions of the four-quark operators and of the gluon condensate.[§] The sum rule in (43) is evaluated at precisely the same values of m_b and s_0 as in (23), and using the standard values of the vacuum condensates $\langle \bar{q}q \rangle = -(240 \text{ MeV})^3$ and $\langle \bar{q}\sigma gGq \rangle / \langle \bar{q}q \rangle = 0.8 \text{ GeV}^2$ (at the low scale $\mu_0^2 = 1 \text{ GeV}^2$), which have been used in the QCD sum rule analysis of the wave functions in [36]. The renormalization group evolution is given by

$$\begin{aligned} \langle \bar{q}q \rangle_{\mu^2} &= \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{-4/b} \langle \bar{q}q \rangle_{\mu_0^2} \,, \\ \langle \bar{q}\sigma gGq \rangle_{\mu^2} &= \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{2/(3b)} \langle \bar{q}\sigma gGq \rangle_{\mu_0^2} \,. \end{aligned} \tag{44}$$

For the decay constant of the B_s-meson, we use the sum rule in (43), and the ratio

$$f_{B_s}/f_{B_d} = 1.15 \pm 0.05, \tag{45}$$

which is in agreement with most existing estimates [8, 51].

The stability plots for the form factors as functions of the Borel parameter are shown in Fig. 3. The stability is in all cases good, and the variations of the continuum threshold within the limits (41) induce uncertainties in the values of form factors within less than 3%. Using the region in the Borel parameter $6 \text{ GeV}^2 < t < 9 \text{ GeV}^2$, we extract the values of form factors

$$\begin{aligned} F_1^{B\to K^*} &= 0.32 \pm 0.05 \\ F_1^{B\to(\rho,\omega)} &= 0.24 \pm 0.04 \\ F_1^{B_s\to\phi} &= 0.29 \pm 0.05 \\ F_1^{B_s\to K^*} &= 0.20 \pm 0.04 \end{aligned} \tag{46}$$

The given error mainly comes from the uncertainty in the b-quark mass and the dependence on the Borel parameter. In addition, we have included an uncertainty of 20% for the parameters (26) of the wave functions. This leads for all decays to an error of order $\pm(0.01\text{–}0.015)$ for F_1. For all decays except $B_s \to K^*\gamma$, the sum rule is dominated by the contribution of the wave function $\phi_\perp$, and the average value of the momentum fraction u under the integral in (23) is $\langle u \rangle \simeq 0.8$.

The uncertainties in the values of input parameters tend to be reduced in the form factor ratios, see Fig. 4, and we obtain

$$\frac{F_1^{B\to(\rho,\omega)}}{F_1^{B\to K^*}} = 0.76 \pm 0.06. \tag{47}$$

[§] The sum rule in (43) yields $f_B \sim 120$–160 MeV, which is lower than the value preferred at present, typically $f_B \sim 180$ MeV (see [47] for a review). This smaller value is an artefact of neglecting the radiative corrections, which are numerically large. One can hope that these corrections will be reduced significantly by taking the ratio of the sum rules. By direct calculations in the limit $m_b \to \infty$, it has been shown on examples of increasing complexity (the Isgur-Wise function [48, 49] and the kinetic energy operator [50]) that radiative corrections are indeed very strongly reduced by considering the sum rule ratios, and we conform to this practice here.

which is not sensitive to the value of the B-meson decay constant. Similarly, we have

$$\frac{F_1^{B_s\to K^*}}{F_1^{B_s\to\phi}} = 0.66 \pm 0.09 \tag{48}$$

and

$$\frac{F_1^{B_s\to K^*}}{F_1^{B\to K^*}} = 0.60 \pm 0.12. \tag{49}$$

The errors given for these ratios are dominated by uncertainties in the $SU(3)$-breaking in the wave functions, while the quark-mass dependence is greatly reduced. The small value of the last ratio is due to the asymmetry of the K^* wave function – the strange quark carries a larger fraction of the meson momentum (see Fig. 2). It should be noted that errors given here reflect uncertainties in the input parameters, but do not include a possible "theoretical" uncertainty of the method itself. We estimate the actual accuracy of the calculation to be within 20–30% for the form factors and 10–20% for their ratios. To improve the accuracy, it is necessary to calculate corrections to the sum rule (23) coming from antiquark–gluon–quark components of the vector meson wave functions, $SU(3)$-breaking corrections to the wave functions $g_\perp^{(a,v)}$, and the perturbative QCD radiative corrections.

6 The b-quark mass dependence and the heavy quark limit

In order to facilitate the comparison of the QCD sum rule results with lattice calculations [9, 10], we have done the numerical analysis of the sum rules (23) for an arbitrary value of the b-quark mass, assuming the following parametrization suggested by the heavy quark effective theory [52]:

$$\begin{aligned} m_B &= m_b + \bar\Lambda\,, & \bar\Lambda &\simeq 500\text{–}700\ \text{MeV}\,,\\ s_0^B - m_b^2 &= 2m_b\omega_0\,, & \omega_0 &\simeq 1.0\text{–}1.2\ \text{GeV}\,,\\ t &= 2m_b\tau\,, & \tau &= 0.5\text{–}0.8\ \text{GeV}\,. \end{aligned} \tag{50}$$

Here $\bar\Lambda$ is the binding energy in the limit $m_b \to \infty$, and ω_0 is the continuum threshold. The variable τ has the meaning of the non-relativistic Borel parameter [53]–[57].

The results are shown in Fig. 5, where for the reason which is explained below we have plotted the value of the form factor multiplied by the meson mass to the power 3/2. In addition to the curves describing the quark-mass dependence in the heavy-quark parametrization (50), we also show our result for $F_1^{B\to K^*}$ given in (46). Furthermore, using the mass of the D-meson, instead of m_B, and the standard values of the parameters for D-mesons, $s_0^D = 6\ \text{GeV}^2$, $m_c = 1.4 \pm 0.05$ GeV, and $t = 1.5\text{–}3\ \text{GeV}^2$ [25], we obtain

$$F_1^{D\to K^*} = 0.85 \pm 0.13\ . \tag{51}$$

Although the number given in (51) is not directly physically relevant, it can facilitate the comparison with the present lattice data, which are collected mostly in the D-meson mass range.

The curves in Fig. 5 disagree slightly with our values for the form factors for B and D-mesons. For B-mesons, the reason is that in (41) we use a rather generous uncertainty in the b-quark mass. For D mesons, the omitted $O(1/m)$ corrections to the parameters given in (50) can already become significant.

Comparing the values given in (46) and (51), we conclude that in the region between the charm and beauty masses the form factor scales approximately as $F_1 \sim 1/m_B$. However, this power behaviour holds only for the sum rule results in the constrained interval of quark masses, and should not be confused with the theoretical behaviour in the $m_b \to \infty$ limit, which we are going to discuss now.

In the heavy b-quark limit the sum rule in (23) involves the integration over a small interval of u, of order $1-u \sim 1/m_b$. This condition has a simple interpretation: the spectator antiquark must recombine with the fast quark coming from the b-decay. Therefore, the form factor is determined by the "tail" of the vector meson wave function corresponding to a strongly asymmetric configuration where almost all the momentum is carried by one of the constituents. It has been proven [20, 21] that the behaviour of the wave functions in this region at sufficiently large virtualities must coincide with the perturbative behaviour, which can be obtained from the conformal properties of the corresponding operators. Quite rigorously, one obtains for the wave functions involved in (23): $\phi_\perp(u) \sim \phi_\parallel(u) \sim g_\perp^{(a)} \sim O(1-u)$, and $g_\perp^{(v)} \sim O(1)$ for $1-u \to 0$ (see (24) and (38)). It is easy to check that this behaviour is exactly the one needed to ensure that the three terms in (23) are of the same order in the limit $m_b \to \infty$, yielding the form factor $F_1(0) \sim O(m_b^{-3/2})$. The diagram with the hard gluon exchange produces the same power behaviour [19, 13], and is estimated to be small [13]. The large factors $\sim m_b$ in front of the wave functions $\phi_\perp$ and $g_\perp^{(a)}$ in (23) are compensated by the smallness of the wave functions $\sim 1/m_b$ in the relevant region $1-u \sim 1/m_b$.

In order to establish a connection to the heavy-quark expansion of the two-point sum rules [53]–[57], we introduce the scale-invariant (up to logarithms) form factor and B-decay constant

$$\begin{aligned} \widehat{F}_1(0) &= F_1(0) \cdot m_B^{3/2} \,, \\ \widehat{f}_B &= f_B \cdot \sqrt{m_B}\,, \end{aligned} \tag{52}$$

and rewrite the sum rule (23) in terms of the non-relativistic Borel parameter $\tau = t/(2m_b)$. The result reads as:

$$\widehat{f}_B \widehat{F}_1(0) e^{-\bar{\Lambda}/\tau} = f_V \int_0^{\omega_0} d\omega\, e^{-\omega/\tau} \left\{ -2\omega\phi_\perp'(1) + m_V g_\perp^{(v)}(1) - \frac{\omega}{4\tau} m_V (g_\perp^{(a)})'(1) \right\}\,, \tag{53}$$

where the new parameters $\bar{\Lambda}$ and ω_0 are defined in (50). The variable $\omega = m_b\bar{u}/2$ has the meaning of frequency, and is defined as $(p+q)^2 - m_b^2 = 2m_b\omega$ (see eq. (22)). For the wave functions, we obtain, to our accuracy:

$$\begin{aligned} -\phi_\perp'(1) \equiv -\frac{d}{du}\phi_\perp(1) &= 6[1 + a_1(\mu) + (4/5)a_2(\mu) + (4/3)a_3(\mu)]\,, \\ g_\perp^{(v)}(1) &= 3/2\,, \\ -(g_\perp^{(a)})'(1) &= 6\,, \end{aligned} \tag{54}$$

where $\mu \simeq 2\tau$ [57]. In the same limit, the sum rule for the B-decay constant becomes [53]–[57]:

$$\widehat{f}_B^2 e^{-\bar{\Lambda}/\tau} = \frac{3}{\pi^2}\int_0^{\omega_0} d\omega\, e^{-\omega/\tau}\omega^2 - \langle\bar{q}q\rangle_{\mu=2\tau} + \frac{1}{16\tau^2}\langle\bar{q}\sigma gGq\rangle_{\mu=2\tau}\,. \tag{55}$$

Using the sum rule in (55) to eliminate the coupling $\widehat{f}_B$ from (53), we obtain the invariant form factor for $B \to K^*\gamma$

$$\widehat{F}_1(0) \simeq 5 \tag{56}$$

(with $\mu^2 \sim 1\mathrm{GeV}^2$), which translates to the static limit of the decay form factor (at the same μ^2)

$$F_1(0) = \widehat{F}_1(0)/m_B^{3/2} \simeq 0.4\,. \tag{57}$$

This number appears to be 30% higher than the result in (46). The scale dependence reminds of possible logarithmic corrections which we have ignored. Note that the standard logarithmic factor corresponding to the hybrid anomalous dimension of the interpolating current of the B-meson cancels in the ratio of (53) and (55). However, additional corrections can appear, similar to the Sudakov-type corrections, which in the context of inclusive radiative decays $B \to X_s + \gamma$ have been discussed in [11]. A related discussion of the perturbative corrections produced by the hard gluon exchange can be found in [13]. Since these corrections may exponentiate, the power law in (57) should be taken with caution. A study of the perturbative two-loop corrections to the sum rule would be welcome, but is beyond the tasks of this paper.

The contributions of higher twist to the sum rule (23) do not spoil the existence of the heavy-quark expansion. In this case, again, explicit enhancement factors $\sim m_b^2/t \sim O(m_b)$ are compensated by the smallness of the wave functions. However, in contrast with the contributions of higher-dimension condensates to the sum rule in (55), the higher-twist contributions to (53) generally bring in factors (ω/τ), which indicates that the structure of the expansion is different. We hope to return to this question in the future.

Thus, we conclude that the light-cone sum rules, which we suggest in this paper, remain well-defined in the heavy b-quark limit, since all the contributions appear to have the same power behaviour at $m_b \to \infty$. The same conclusion has been drawn in the work [19] for the case of semileptonic $B \to \pi e\nu$ decays.

7 The $\mathbf{q^2}$-dependence and the interrelation of radiative and semileptonic B-decay form factors

Up to this point, we have been discussing the radiative decay form factor F_1 at $q^2 = 0$. Now we are in a position to calculate the q^2-dependence. The interest in the q^2-dependence is due to the fact that within most of the existing approaches the calculations are actually done for high values of q^2, close to the maximum possible value $q^2_{\max} = (m_B - m_{K^*})^2$, and then extrapolated to $q^2 = 0$ using plausible assumptions like the pole-dominance approximation. Within the QCD sum rule approach the q^2 dependence of form factors can be calculated in a wide interval of q^2, see e.g. [25, 24].

The QCD sum rule for the form factor F_1 for finite values of q^2 is obtained from the one given in (23) by trivial modifications. We obtain

$$\frac{f_B m_B^2}{m_b+m_q} 2F_1(q^2) e^{-(m_B^2-m_b^2)/t} =$$
$$= \int_0^1 du \frac{1}{u} \exp\left[-\frac{\bar{u}}{t}\left(\frac{m_b^2-q^2}{u}+m_V^2\right)\right] \theta\left[s_0 - \frac{m_b^2-\bar{u}q^2}{u} - \bar{u}m_V^2\right] \left\{ m_b f_V^\perp \phi_\perp(u) \right.$$
$$\left. + u m_V f_V g_\perp^{(v)}(u) + \frac{m_b^2+q^2-u^2m_V^2+ut}{4ut} m_V f_V g_\perp^{(a)}(u) \right\}. \tag{58}$$

The region of applicability of this sum rule is restricted by the requirement that the value of $q^2 - m_b^2$ be sufficiently less than zero, so that the correlation function (15) is evaluated in the Euclidean region. In order not to introduce an additional scale in the problem, we require that $q^2 - m_b^2 \leq (p+q)^2 - m_b^2$, which translates to the condition that $m_b^2 - q^2$ is of order of the typical Borel parameter $t \sim$ 5–8 GeV^2. Thus, we end up with an upper bound for the applicability of the QCD calculation $q^2 <$ 15–17 GeV^2, which is a few GeV^2 below the zero-recoil point. The results are shown in Fig. 6. It is seen that the stability of the sum rules becomes worse with the increase of q^2. The reason is that the omitted contributions of higher twists are suppressed by factors $1/(m_b^2 - u(p+q)^2)$ in the momentum space, which translates into suppression by powers of $1/(ut)$ after the Borel transformation. At $q^2 = 0$ the average value of u under the integral is of order 0.8, but it decreases to ~ 0.6 at $q^2 \simeq 15$ GeV^2. Thus, in order not to enhance the higher twist effects one should somewhat increase the value of the Borel parameter t at high q^2. This increase, however, potentially comes in conflict with the requirement that the Borel parameter t be small enough to allow to separate the B-meson contribution from the continuum. Thus, in general, the light-cone sum rules for large values of q^2 are less reliable than at $q^2 = 0$. The calculation shown in Fig. 6 was done using the rescaled Borel parameter $t \to t/\langle u\rangle(q^2)$, where $\langle u\rangle(q^2)$ is the average value of u under the integral. This allows for a certain improvement of the stability.

For the radiative decays discussed here, the sum rules do not contradict the pole-type behaviour in the region $0 \leq q^2 \leq 10$ GeV^2:

$$F_1(q^2) = \frac{F_1(0)}{1 - q^2/m_{\text{pole}}^2} \tag{59}$$

with the pole masses of order 3.8–4.2 GeV. A much better quality of the fit is provided, however, by the dipole formula

$$F_1(q^2) = \frac{F_1(0)}{(1 - q^2/m_{\text{dipole}}^2)^2} \tag{60}$$

with $m_{\text{dipole}} = 5.6$, 5.4, 5.2, and 4.9 GeV for Fig. 6a, b, c, and d, respectively.

In the case of $B \to K^*\gamma$ decays, an important result was derived by Isgur and Wise in [12] and by Burdman and Donoghue in [13], where it has been shown that in the heavy b-quark limit an exact relation exists between the form factor F_1 and the semileptonic form factors defined by:

$$\langle K^*(p',\lambda)|\bar{s}\gamma_\mu(1-\gamma_5)b|\bar{B}(p)\rangle = \frac{2V(q^2)}{m_B+m_{K^*}}\epsilon_{\mu\nu\rho\sigma}e^{*(\lambda)\nu}p^\rho p'^\sigma - i(m_B+m_{K^*})A_1(q^2)e^{*(\lambda)}_\mu + \dots \tag{61}$$

where the ellipses denote terms proportional to $(p+p')_\mu$ or q_μ. According to [12, 13]:

$$F_1^{B\to K^*}(q^2) = \frac{q^2+m_B^2-m_{K^*}^2}{2m_B}\frac{V(q^2)}{m_B+m_{K^*}} + \frac{m_B+m_{K^*}}{2m_B}A_1(q^2)\,, \tag{62}$$

as far as the value of q^2 is sufficiently close to the point $q^2_{\rm max} = (m_B - m_{K^*})^2 \simeq 19$ GeV^2. Similar relations can of course be derived for other radiative decays. The central problem in using predictions based on the heavy quark effective theory to the rare B-decays is whether this relation is satisfied for small values of q^2, as suggested in [12, 13], and what is the size of the power $1/m_b$ corrections. Our main result in this section is that the relation in (62) is fulfilled in the QCD sum rule approach discussed here to a good accuracy for finite b-quark masses and for all momentum transfers.

The sum rules for the semileptonic form factors entering (62) can be derived in a similar way, repeating the steps described in section 3. A simple calculation yields

$$\begin{aligned}
&\frac{f_Bm_B^2}{m_b+m_q}[m_B+m_V]A_1(q^2)e^{-(m_B^2-m_b^2)/t} = \\
&\quad = \int_0^1 du\frac{1}{u}\exp\left[-\frac{\bar{u}}{t}\left(\frac{m_b^2-q^2}{u}+m_V^2\right)\right]\theta\left[s_0-\frac{m_b^2-\bar{u}q^2}{u}-\bar{u}m_V^2\right] \\
&\qquad\times\left\{\frac{m_b^2-q^2+u^2m_V^2}{2u}f_V^\perp\phi_\perp(u)+m_bf_Vm_Vg_\perp^{(v)}(u)\right\}, \\
&\frac{f_Bm_B^2}{m_b+m_q}\frac{2V(q^2)}{m_B+m_V}e^{-(m_B^2-m_b^2)/t} = \qquad\qquad\qquad\qquad\qquad\qquad (63)\\
&\quad = \int_0^1 du\frac{1}{u}\exp\left[-\frac{\bar{u}}{t}\left(\frac{m_b^2-q^2}{u}+m_V^2\right)\right]\theta\left[s_0-\frac{m_b^2-\bar{u}q^2}{u}-\bar{u}m_V^2\right] \\
&\qquad\times\left\{f_V^\perp\phi_\perp(u)+\frac{m_bf_Vm_V}{2ut}g_\perp^{(a)}(u)\right\}. \qquad\qquad\qquad\qquad\qquad\qquad (64)
\end{aligned}$$

Note that the three sum rules in (58), (63), and (64) involve the same wave functions as specified in section 3.

It is now easy to check the Isgur-Wise relation by forming the ratio

$$R(q^2) = F_1(q^2)/F_1^{\rm IW}(q^2)\,, \tag{65}$$

where $F_1(q^2)$ is the form factor of the radiative decay according to (58), and $F_1^{\rm IW}(q^2)$ is the particular combination of the semileptonic form factors, which enters on the r.h.s. of (62), and which is evaluated by using to the sum rules (63) and (64). The ratio (65) is shown as a function of q^2 in Fig. 7 for the two decays $B\to K^*\gamma$ and $B\to\rho\gamma$, and is practically the same for the two remaining decays $B_s\to\phi\gamma$ and $B_s\to K^*\gamma$. We find that the relation between radiative and semileptonic form factors is satisfied with a high accuracy for all momentum transfers. For the physically relevant point $q^2 = 0$ a typical value of the ratio R is about 0.92–0.95. These predictions should be rather reliable, since many of the uncertainties in the individual decay rates are greatly reduced in the ratios.

Since the semileptonic decay $B \to K^* + \ell + \nu_\ell$ does not occur in nature, tests of the Isgur-Wise relation must take into account the correction for the $SU(3)$ breaking between $B \to K^*$ and $B \to \rho$ semileptonic form factors. From our sum rule calculation we get

$$\begin{aligned} A_1(0)^{B\to\rho}/A_1(0)^{B\to K^*} &= 0.76 \pm 0.05\,, \\ V(0)^{B\to\rho}/V(0)^{B\to K^*} &= 0.73 \pm 0.05\,. \end{aligned} \tag{66}$$

As a by-product of our analysis, we have derived the q^2 dependence of the semileptonic decay form factors, V and A_1, which is shown in Fig. 8 for the transition $B \to \rho + \ell + \nu_\ell$. In Table 1 the values for $V(q^2 = 0)$ and $A_1(q^2 = 0)$ are compared with the corresponding results from other approaches [58]–[62]. We find a satisfactory agreement with the quark models, while our results are somewhat lower than previous QCD sum rule estimates, which were done using the traditional approach. In agreement with [58] we obtain a steeper increase of the form factor $V(q^2)$ with q^2, compared to $A_1(q^2)$, although values of the slopes come out to be different.

Predictions of the traditional sum rules should be more reliable at the values $m_b^2 - q^2 = O(m_b)$ than at $q^2 = 0$, since in this region they do not suffer from large contributions of the operators of high dimension, see the detailed discussion in the appendix. In the light-cone sum rules the situation is opposite, so that our results are to a large extent complementary to the calculation in [58, 60]. It is worth while to note that the difference in predictions is minimal at intermediate $q^2 \sim$ 5–10 GeV^2, i.e. exactly in the region where both the approaches should work well.

Table 1: Form factors for the semileptonic decay $B \to \rho + \ell + \nu_\ell$

Reference	$A_1(0)^{B\to\rho}$	$V(0)^{B\to\rho}$
This paper	0.24 ± 0.04	0.28 ± 0.06
[58][a]	0.5 ± 0.1	0.6 ± 0.2
[59][a]	0.35 ± 0.16	0.47 ± 0.14
[60][b]	0.28	0.33
[61][b]	0.05	0.27
[62][c]	0.21	1.04

a: QCD sum rules
b: Quark models
c: HQET + chiral perturbation theory

8 Summary and Conclusions

Motivated by the discovery of the electromagnetic penguins through the decay $B \to K^* + \gamma$, we have presented an alternative method of calculating the transition form factors in this and related decays using the approach of QCD sum rules on the light cone. Our calculations give $F_1^{B\to K^*} = 0.32 \pm 0.05$; combined with the estimates of the inclusive branching ratio $BR(B \to X_s + \gamma) = (3.0 \pm 1.2) \times 10^{-4}$ in the Standard Model [3], this yields $BR(B \to K^* + \gamma) = (4.8 \pm 1.5) \times 10^{-5}$, which is in reasonably good agreement with the observed branching ratio $(4.5 \pm 1.5 \pm 0.9) \times 10^{-5}$ measured by the CLEO collaboration.

We use the same method to calculate the related form factors in the decays $B_{u,d} \to \rho + \gamma$, $B_d \to \omega + \gamma$, $B_s \to \phi + \gamma$ and $B_s \to K^* + \gamma$. These decays can be related to the already discussed and measured decay $B_{u,d} \to K^* + \gamma$. To this end, we have evaluated in detail the numerical effect of the $SU(3)$-breaking in the form factors. This fixes the decay width $\Gamma(B_s \to \phi + \gamma)$, which involves the same CKM matrix element $|V_{ts}|$ as the decay width $\Gamma(B_{u,d} \to K^* + \gamma)$. Since the form factors for the two decay modes are comparable in size, see Fig. 3 and eq. (46), and since the lifetimes of the B_u, B_d and B_s mesons are expected to be very similar — an expectation which is consistent with present experiments — we estimate that $BR(B_s \to \phi + \gamma) \simeq BR(B_{u,d} \to K^* + \gamma)$.

More interesting, however, are the CKM-suppressed decay modes discussed above. A measurement of both the CKM-allowed and CKM-suppressed radiative decays $B \to V + \gamma$ would give direct information on the CKM matrix element ratio $|V_{td}|/|V_{ts}|$. The form factors needed for such determination are plotted in Fig. 4, showing the ratios of $F_1^{B\to\rho}/F_1^{B\to K^*}$, $F_1^{B_s\to K^*}/F_1^{B_s\to\phi}$ and $F_1^{B_s\to K^*}/F_1^{B_{u,d}\to K^*}$. The numerical results for the form factor ratios are given in eqs. (47)–(49). The ratios of the branching ratios $BR(B \to \omega + \gamma)/BR(B \to K^* + \gamma)$ and $BR(B_s \to K^* + \gamma)/BR(B \to K^* + \gamma)$ as functions of $|V_{td}|/|V_{ts}|$ are shown in Fig. 9 ($B = B_u$ or B_d.). As already stressed, these ratios are practically independent of m_t and of the QCD scale parameter μ. The dependence of the ratios on the various parameters endemic to the QCD sum rule approach also becomes rather mild, as opposed to the predictions for the partial decay widths themselves. Therefore, the two curves shown in Fig. 9 should give a fair representation of the residual theoretical error, and they allow a rather clean determination of the ratio of the CKM matrix elements $|V_{td}|/|V_{ts}|$ as and when the required data for the ratios of branching ratios become available.

It should be pointed out here that similar relations have been derived for the ratios of the inclusive branching ratios $BR(B \to X_d + \gamma)/BR(B \to X_s + \gamma)$ in [4], and for the ratios of the B^0–$\overline{B^0}$ mixing parameters x_d and x_s [67]. In particular, the Standard Model predicts:

$$\frac{x_d}{x_s} = \frac{\tau_{B_d} M_{B_d} \left(f_{B_d}^2 B_{B_d}\right)}{\tau_{B_s} M_{B_s} \left(f_{B_s}^2 B_{B_s}\right)} \left|\frac{V_{td}}{V_{ts}}\right|^2 , \tag{67}$$

where B_{B_d} and B_{B_s} are the hadronic bag constants. Since all dependence on the t-quark mass and the QCD renormalization of the box amplitudes drops out, we are left with the square of the ratio of CKM matrix elements, multiplied by a factor that reflects $SU(3)_{\text{flavour}}$-breaking effects. With the known values of x_d and $BR(B \to K^* + \gamma)$, a measurement of x_s and of the CKM-suppressed radiative decays would provide valuable quantitative tests on the CKM matrix and, in particular, its unitarity.

In order to obtain an estimate of the absolute branching ratios for the CKM-suppressed decays, we use the constraints on the matrix element $|V_{td}|$ that already exist from the measurement of the mixing ratio x_d. The present value $x_d = 0.71 \pm 0.07$ [45] yields the model dependent bound $0.005 < |V_{td}| < 0.012$ [45]. This in turn gives

$$0.10 \leq \frac{|V_{td}|}{|V_{ts}|} \leq 0.33, \tag{68}$$

where we have used the CKM unitarity constraint and recent data giving $|V_{ts}| \simeq |V_{cb}| = 0.042 \pm 0.005$ [33]. We can combine this bound with Fig. 9 to predict:

$$0.01 \leq \frac{BR(B \to \omega + \gamma)}{BR(B \to K^* + \gamma)} \leq 0.10, \tag{69}$$

with the ratio $BR(B \to \rho + \gamma)/BR(B \to K^* + \gamma)$ bounded likewise. While this gives a ball-park estimate for the branching ratios in question, it is clear from this numerical exercise that a measurement of the above ratio within a factor 2 would enormously reduce the present (model-dependent) uncertainty on the CKM matrix element ratio $|V_{td}|/|V_{ts}|$.

The main theoretical content of this paper is in pointing out and providing general arguments for the light-cone QCD sum rule approach to be a more adequate tool for the study of heavy meson decay form factors compared to the traditional approach. The principal input in the light-cone sum rules are the wave functions of vector mesons, and the accuracy of our calculation can be substantially improved by taking into account the wave functions of twist 3 and twist 4. For the case of the π-meson, a complete set of wave functions to this accuracy is given in [39]. For vector mesons, this problem is not solved yet, and is of acute practical interest. Some new results concerning the interrelation of the transverse and longitudinal wave functions of vector mesons are given in section 4.

We have also studied the heavy-quark mass dependence of F_1 in the region $m_c \leq m_Q \leq m_b$. This is useful for comparison with lattice-QCD results where the heavy quark systems at present are calculated in the vicinity of m_c and extrapolated to m_b. We find that the transition form factor F_1 scales approximately in this region as $F_1 \sim 1/m_b$, and has a value $F_1 = 0.85 \pm 0.13$ for m_B equal to the D-meson mass, i.e. for the (unphysical) process $D \to K^* + \gamma$.

Apart from the phenomenologically interesting value of the form factor $F_1^{B\to V}$ at the on-shell point $q^2 = 0$, we have investigated the behaviour of F_1 as a function of q^2. The case $q^2 \neq 0$ contributes to the decays $B \to V + \ell^+\ell^-$ via off-shell photons. In addition, we have derived the light-cone QCD sum rules for the form factors $A_1(q^2)$ and $V(q^2)$, describing the semileptonic decay $B \to \rho + \ell\,\nu_\ell$. It will be interesting to compare them with data on the semileptonic decay $B \to \rho + \ell\,\nu_\ell$, as and when these become available. Another use of this sum rules is to check the relation between the radiative decay form factor $F_1^{B\to K^*}$ and the semileptonic form factors in $B \to \rho + \ell\,\nu_\ell$, which was derived in [12, 13] using the heavy-quark symmetry and for q^2 near the kinematic point $q^2_{\rm max}$. We have shown that the ratio $F_1(q^2)/F_1^{IW}(q^2)$ evaluated in the QCD sum rule approach remains very close to unity in the complete kinematic range of q^2 which is available in the CKM-suppressed semileptonic decays.

In summary, we have given a number of predictions here in the crucial area of exclusive radiative and semileptonic B-decays, involving the CKM-suppressed and CKM-allowed transitions using QCD sum rules on the light-cone. These predictions are specific to this approach and it will be instructive to confront them with data. The existing data on $BR(B \to K^* + \gamma)$ are reasonably well explained, and we argue that our results allow to quantify the ratios of CKM-allowed and CKM-suppressed exclusive radiative B-decays. This would prove useful in determining the CKM ratio $|V_{td}|/|V_{ts}|$ in a fairly model-independent way.

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Appendix

Three-point sum rule

In this appendix we compare our approach, for the particular case of $B \to K^*\gamma$ decays, with the results of the standard QCD sum rule analysis [6, 8, 7]. Here, the starting point is the operator product expansion for the three-point function involving the penguin operator and the interpolating currents for both the B-meson and the K^*-meson

$$i^2 \int dx\, dy\, e^{ipx+iqy} \langle 0|T\{[\bar{d}\gamma_\alpha s](x)[\bar{s}\sigma_{\mu\nu}q_\nu b](y)[\bar{b}i\gamma_5 d](0)\}|0\rangle \tag{A.1}$$

in which the contribution of interest is identified with the one having poles at $p^2 = m_{K^*}^2$ and $(p+q)^2 = m_B^2$. The standard treatment, details of which we are not going to present here, leads to the following sum rule:

$$\begin{aligned}
&\frac{f_{K^*} m_{K^*} f_B m_B^2}{m_b} 2F_1^{B\to K^*}(0) e^{-m_{K^*}^2/t_1} e^{-(m_B^2-m_b^2)/t_2} = \\
&= \frac{3}{4\pi^2}\int_0^{s_0^K} ds_1\, e^{-s_1/t_1} \int_{m_b^2}^{s_0^B} ds_2 e^{-(s_2-m_b^2)/t_2} \theta(s_2 - s_1 - m_b^2)\rho(s_1,s_2) - (m_s+m_b)\langle\bar{d}d\rangle \\
&\quad + \frac{1}{12}\langle\bar{d}\sigma g G d\rangle \left\{ \frac{3m_b^2(m_s+m_b)}{t_2^2} + \frac{2m_b^2(2m_b+m_s)}{t_1 t_2} + \frac{6m_b+8m_s}{t_2} - \frac{2m_s}{t_1} \right\},
\end{aligned} \tag{A.2}$$

where $\rho(s_1, s_2)$ is the spectral density of the triangle diagram in Fig. 10a

$$\rho(s_1,s_2) = \frac{m_b^4 s_1}{(s_2-s_1)^3} + m_b m_s \frac{s_2 - s_1 - m_b^2}{(s_2-s_1)^2}\,, \tag{A.3}$$

and we have calculated also the contribution of the quark condensate in Fig. 10b, and of the mixed condensate in Fig. 10c (a typical graph is shown). The contributions of the gluon condensate and of the four-quark condensate are negligible [7]. In contrast with ref. [8], we give the answer with $O(m_s)$ corrections included. To the accuracy that these corrections are put to zero, our sum rule agrees with the one given in [8], but for the sign in front of the third term in the contribution of the mixed condensate. Numerically, the difference is negligible. Note that in contrast to the light-cone sum rules we have now two Borel parameters, t_1 for the K^*-meson and t_2 for the B-meson, which should be taken two times larger than the Borel parameters in the corresponding two-point sum rules (cf. [25]). The quantity $s_0^K = 1.7$ GeV^2 is the continuum threshold in the sum rules for the K^*-meson, and m_s is the mass of the strange quark, which is of order $m_s = 150$ MeV. The other entries have been specified in the text.

A numerical treatment of the sum rule in (A.2) yields values of the form factor of order

$$F_1^{B\to K^*}(0) \simeq 0.5\text{–}0.6 \tag{A.4}$$

which is in agreement with the analysis in [6, 7]. The lower value ~ 0.38 quoted in [7] was obtained by the rescaling of a similar value ~ 0.55 from the scale of the typical Borel parameter to the scale $\mu = m_b$, using the large anomalous dimension of the penguin operator (4). We have not understood the reasoning for such a rescaling, since the renormalization group treatment of the effective Hamiltonian in (2) makes sense at scales $\mu > m_b$ only. A somewhat smaller value obtained in ref. [8] is due to a smaller value of the quark condensate and due to a higher value $f_B = 180$ MeV used in [8].

The value in (A.4) appears to be considerably larger than our result in (46). We are going to claim, however, that the sum rule in (A.2) is ill-behaved in the limit of heavy b-quark, and is less reliable.

An inspection of the sum rule in (A.2) shows that various terms in it have a different behaviour in the limit $m_b \to \infty$. One easily finds that the perturbation theory contribution to (A.2) yields the form factor $F_1(0) \sim O(m_b^{-3/2})$, the quark condensate produces a contribution of order $m_b^{1/2}$, and the mixed condensate contribution is of order $m_b^{3/2}$. Thus, the sum rule (A.2) blows up in the limit $m_b \to \infty$.

This problem has actually been known for a long time, and was originally found in the QCD sum rule calculations of the pion form factor [22, 23]. In technical terms, the problem appears because the operator product expansion for three-point functions involves the dimensionless parameter $(m^2 - q^2)/t$, increasing powers of which multiply the contributions of local operators of higher dimensions. In the case of light quark systems, $m^2 = 0$ and the Borel parameter is of order 1 GeV^2. Thus the expansion breaks down at sufficiently large values of q^2. In the case of heavy–light mesons $t \sim O(m_b)$, and for $q^2 = 0$ the operator product expansion involves increasing powers of the heavy-quark mass. In the present case, we observe a considerable enhancement of the contribution of the mixed condensate, and conclude that higher-order corrections, e.g. proportional to the dimension-7 condensate $\langle \bar{\psi} g^2 G^2 \psi \rangle$, can influence the result significantly. It has been claimed recently [63] that this condensate is much larger than its factorized value.

In physical terms, the reason of the difficulty with the traditional three-point sum rules in the $m_b \to \infty$ limit is that the operator product expansion does not take into account the effect of the finite correlation length between the quarks in the physical vacuum. A possible remedy, suggested in [64], is to use the expansion in non-local condensates, which take into account this effect in a model-dependent way, see e.g. [65]. Following Radyushkin, we write down the non-local quark condensate in the Borel representation:

$$\langle \bar{d}(x) d(0) \rangle = \langle \bar{d} d \rangle \int_0^\infty d\nu \, e^{\nu x^2/4} f(\nu). \tag{A.5}$$

Moments of the function $f(\nu)$ are determined by vacuum expectation values of local operators

$$\begin{aligned} \int_0^\infty d\nu \, f(\nu) &= 1 \,, \\ \int_0^\infty d\nu \, \nu f(\nu) &= \frac{1}{4} \langle \bar{d} \sigma \mathrm{g} G d \rangle / \langle \bar{d} d \rangle \simeq 0.2 \text{ GeV}^2 \,, \end{aligned} \tag{A.6}$$

and similar relations for higher moments. An effect in the sum rule (A.2) is that in the term proportional to the quark condensate one should make a replacement

$$\langle \bar{d}d\rangle \to \langle \bar{d}d\rangle \int_0^t d\nu\, f(\nu) \exp\left\{-\frac{m_b^2}{t_2}\frac{\nu}{t-\nu}\right\} \tag{A.7}$$

where $t \equiv t_1t_2/(t_1+t_2)$, and discard the contribution of the mixed quark–gluon condensate, since its contribution is partly included in (A.5). In the limit of the heavy b-quark mass the integral over ν in (A.7) is dominated by contributions of small ν; thus the behaviour of $f(\nu)$ at $\nu \to 0$ is of crucial importance. Note that no information about this behaviour can be extracted from known values of a few first moments. In a confining theory such as QCD one should expect that correlation functions decrease exponentially at large distances in Euclidean space,

$$\langle \bar{d}(x)d(0)\rangle \overset{x^2\to-\infty}{\sim} \exp\{-a\sqrt{-x^2}\}\,, \tag{A.8}$$

where a is the correlation length. Existence of a finite correlation length in the physical vacuum is a part of a common wisdom about confinement and a starting point of many existing models of the vacuum (see e.g. [66]). It is easy to show that the finite correlation length implies quite generally the exponential behaviour of the function $f(\nu)$ at small ν:

$$f(\nu) \sim e^{-a^2/\nu}. \tag{A.9}$$

Inserting this behaviour in (A.7) and taking the limit $m_b \to \infty$ we obtain to the exponential accuracy the contribution of the quark condensate to be

$$\sim \exp\{-2am_b/\sqrt{t_1t_2}\}\,. \tag{A.10}$$

Since $t_1 = O(1)$ and $t_2 = O(m_b)$, this result means that the contribution of the quark condensate decreases as $\exp\{-\sqrt{m_b}\}$. Hence, the perturbative contribution dominates the sum rule in this limit, yielding the same behaviour $F_1(0) \sim m_b^{-3/2}$ which is intrinsic to the light-cone sum rules in the text. Note that this result is a quite general consequence of the existence of a finite correlation length in the QCD vacuum. It is actually not surprising, since the power behaviour of exclusive process with large momentum transfers [20, 21] is given by perturbative diagrams. In the present case, the diagrams with hard gluon exchange yield the same power behaviour as the ones considered here, see [19].

To summarize, we argue that the light-cone sum rule written in (23) remains well-defined in the limit of the heavy b-quark mass, similar to the traditional QCD sum rules for the two-point functions, which can be written directly in the heavy-quark effective theory [57]. The heavy-quark limit does not exist, however, in the standard sum rules for three-point functions, as the one in (A.2). This property of the light-cone sum rules argues in favour of our approach.

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Figure captions

Fig. 1 The leading contribution (a) and the gluon correction (b) to the correlation function in (15).

Fig. 2 The leading-twist wave function $\phi_\perp$ (19) from the sum rule analysis in [36] at the scale $\mu^2 = 5$ GeV^2 for (a) ρ- (solid line) and ϕ- (dashed dotted line) mesons and (b) for the K^*-meson. The wave function for K^* is also shown at the low scale $\mu^2 = 1$ GeV^2 (dashed line). The dotted lines correspond to the asymptotic wave function $\phi_\perp^{\rm as} = 6u(1-u)$.

Fig. 3 Stability plots for the sum rule in (23) as a function of the Borel parameter for $B \to K^*\gamma$, $B \to \rho\gamma$, $B_s \to \phi\gamma$, and $B_s \to K^*\gamma$, see a, b, c and d, respectively. Dashed lines correspond to $s_0^B = 35$ GeV^2 and dotted lines to $s_0^B = 33$ GeV^2 (with $s_0^{B_s}$ given in (42)). For each value of s_0 curves are shown for $m_b = 4.6$ GeV (upper curve), $m_b = 4.7$ GeV (middle) and $m_b = 4.8$ GeV (lowest curve).

Fig. 4 Ratios of the form factors $F_1(0)$ for the processes (a) $B \to \rho\gamma$ and $B \to K^*\gamma$, (b) $B_s \to K^*\gamma$ and $B_s \to \phi\gamma$, and (c) $B_s \to K^*\gamma$ and $B \to K^*\gamma$. Dashed lines correspond to $s_0^B = 35$ GeV^2 and dotted lines to $s_0^B = 33$ GeV^2 (with $s_0^{B_s}$ given in (42)). The corresponding upper curves are for $m_b = 4.6$ GeV and the lower ones for $m_b = 4.8$ GeV.

Fig. 5 Dependence of the form factor $F_1(0)$ on the quark mass. The various parameters are scaled according to (50). Solid lines correspond to $\tau = 1.0$ GeV and the dotted lines to $\tau = 0.6$ GeV. The corresponding upper curves are for $\bar\Lambda = 600\,$MeV with $\omega_0 = 1.2\,$GeV, and the lower ones for $\bar\Lambda = 500\,$MeV with $\omega_0 = 1.0\,$GeV. For comparison the results corresponding to the parameters discussed in section 5 are indicated by vertical bars.

Fig. 6 Momentum dependence of the form factors $F_1(q^2)$ for (a) $B \to K^*\gamma$, (b) $B \to \rho\gamma$, (c) $B_s \to \phi\gamma$, and (d) $B_s \to K^*\gamma$. Dotted lines correspond to $t = 5\,{\rm GeV}^2/\langle u\rangle$ and dashed lines to $t = 8\,{\rm GeV}^2/\langle u\rangle$. The corresponding upper curves are for $m_b = 4.6$ GeV and the lower ones for $m_b = 4.8$ GeV.

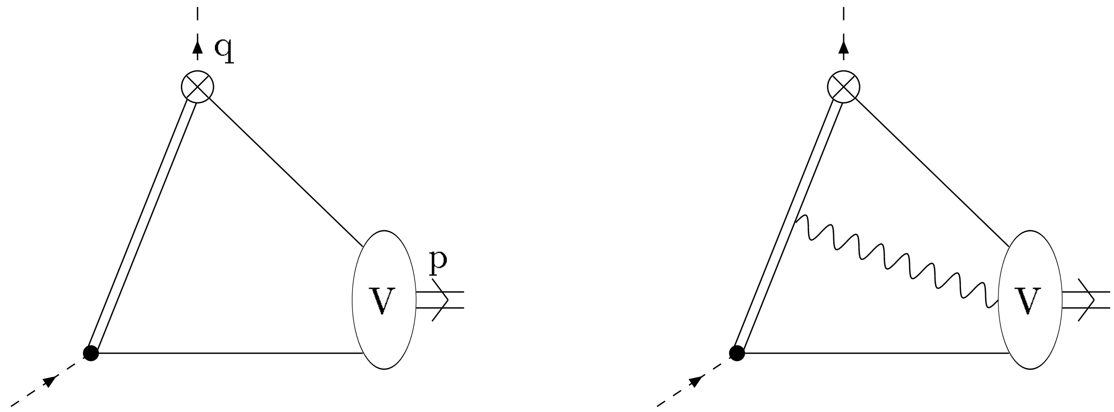
Fig. 7 Momentum dependence of the ratio of radiative and semileptonic form factors (a) $B \to K^*\gamma$ and (b) $B \to \rho\gamma$. Dotted lines correspond to $t = 5$ GeV$^2/\langle u\rangle$ and dashed lines to $t = 8$ GeV$^2/\langle u\rangle$. The corresponding upper curves are for $m_b = 4.8$ GeV and the lower ones for $m_b = 4.6$ GeV.

Fig. 8 Momentum dependence of the form factors $A_1(q^2)$ (a) and $V(q^2)$ (b) for $B \to \rho\gamma$. Dotted lines correspond to $t = 5$ GeV$^2/\langle u\rangle$ and dashed lines to $t = 8$ GeV$^2/\langle u\rangle$. The corresponding upper curves are for $m_b = 4.6$ GeV and the lower ones for $m_b = 4.8$ GeV.

Fig. 9 Ratio of the branching ratios $BR(B \to \omega\gamma)/BR(B \to K^*\gamma)$, where $B = B_u$ or B_d, and $BR(B_s \to K^*\gamma)/BR(B \to K^*\gamma)$, as a function of the CKM matrix elements $|V_{td}|/|V_{ts}|$. The two lines correspond to the errors given in (47).

Fig. 10 Contributions of perturbation theory and of vacuum condensates to the sum rule in (A.2).

Fig. 11Y Stability plot for the sum rule in (A.2)



a

b

Fig. 1

Fig. 2a

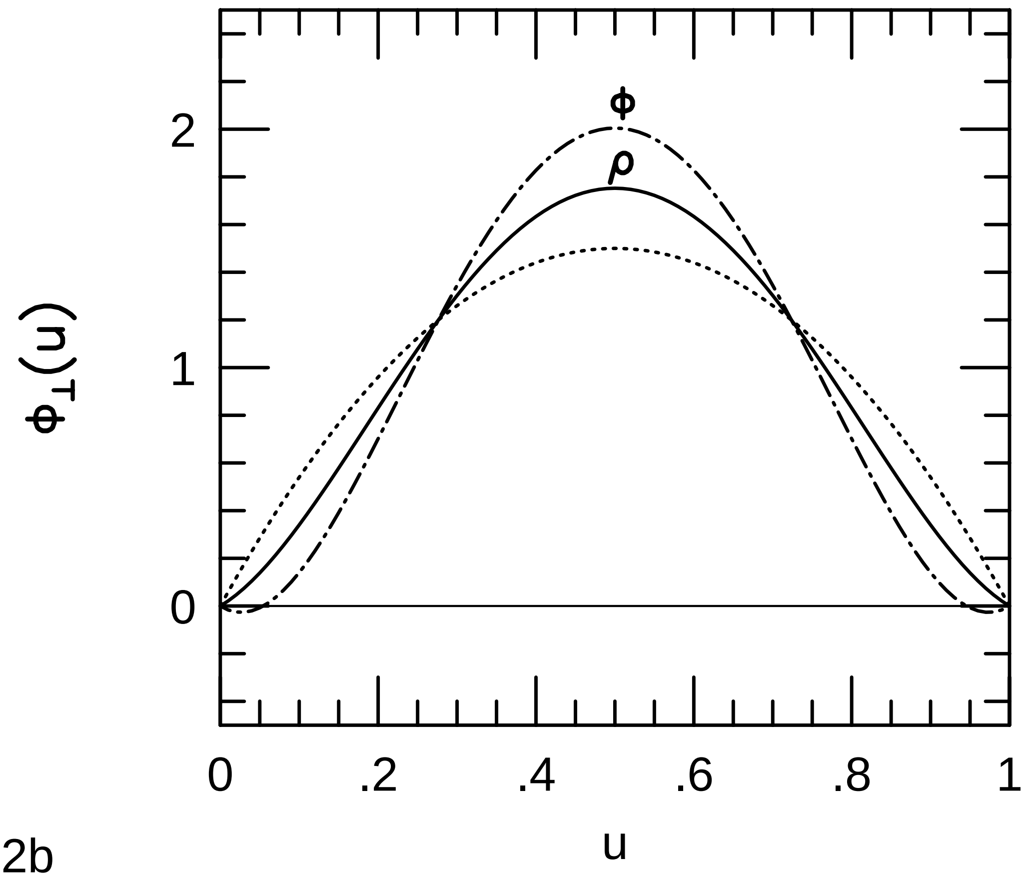
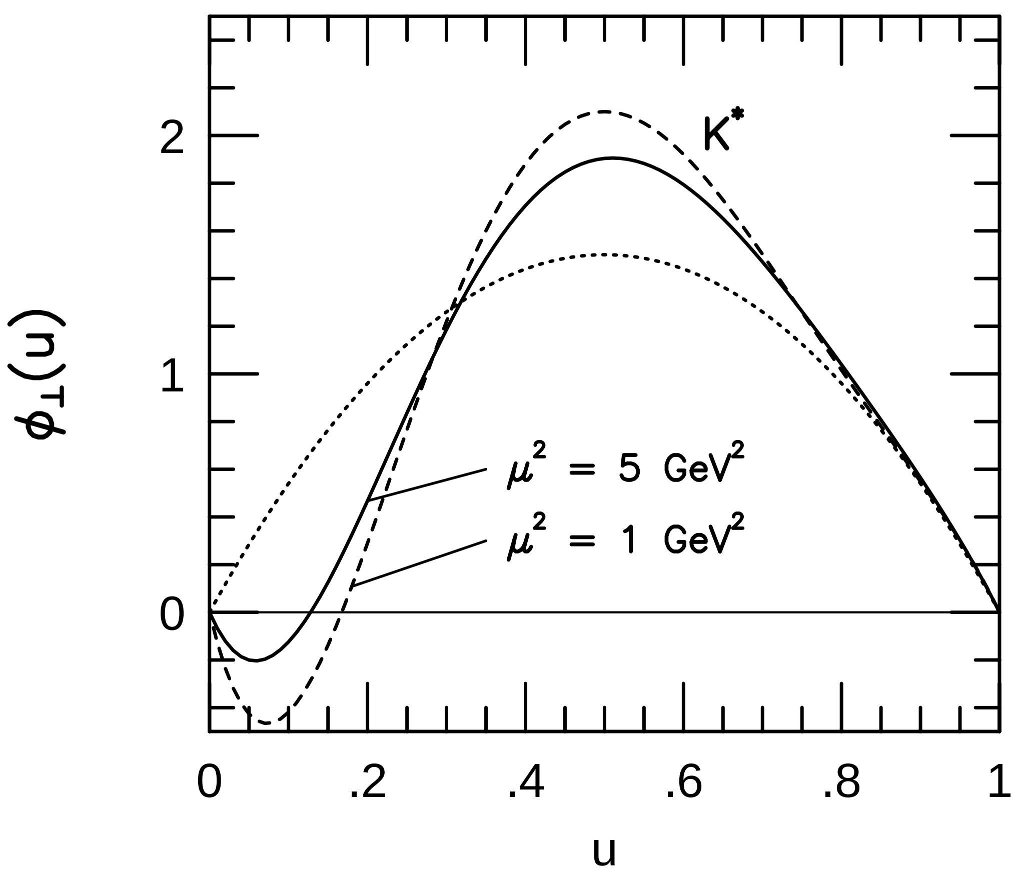

φ
ρ
2
1
0
0
.2
.4
.6
.8
1
u
$\phi_\perp(u)$

Fig. 2b


K^*
$\mu^2 = 5\ \mathrm{GeV}^2$
$\mu^2 = 1\ \mathrm{GeV}^2$
2
1
0
0
.2
.4
.6
.8
1
u
$\phi_\perp(u)$

This figure "fig1-3.png" is available in "png" format from:

http://arXiv.org/ps/hep-ph/9401277v1

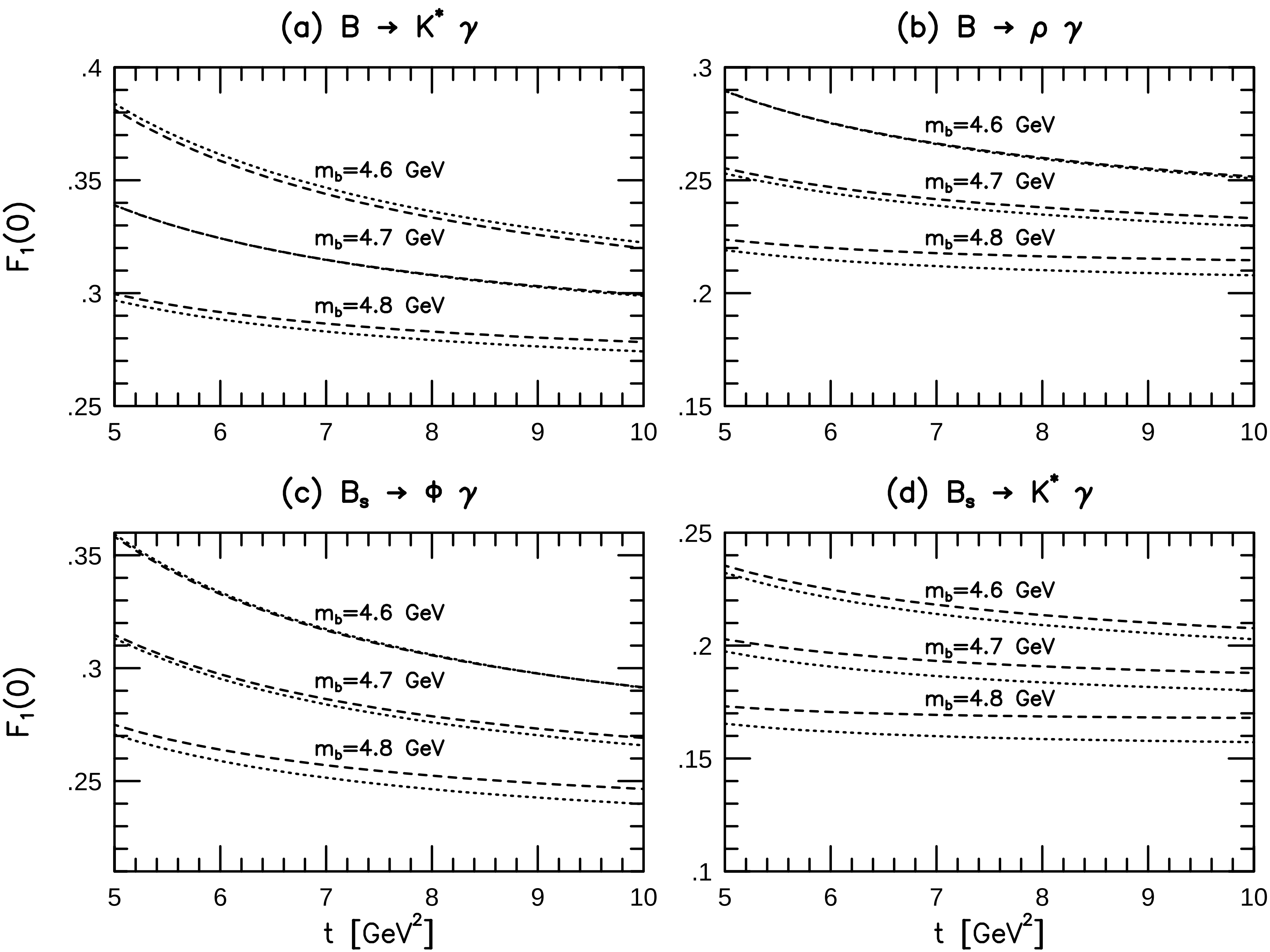

(a) B → K* γ
(b) B → ρ γ
(c) B_s → φ γ
(d) B_s → K* γ
m_b=4.6 GeV
m_b=4.7 GeV
m_b=4.8 GeV
$F_1(0)$
t [GeV^2]
.4
.35
.3
.25
.2
.15
.1
5
6
7
8
9
10

Fig. 4a

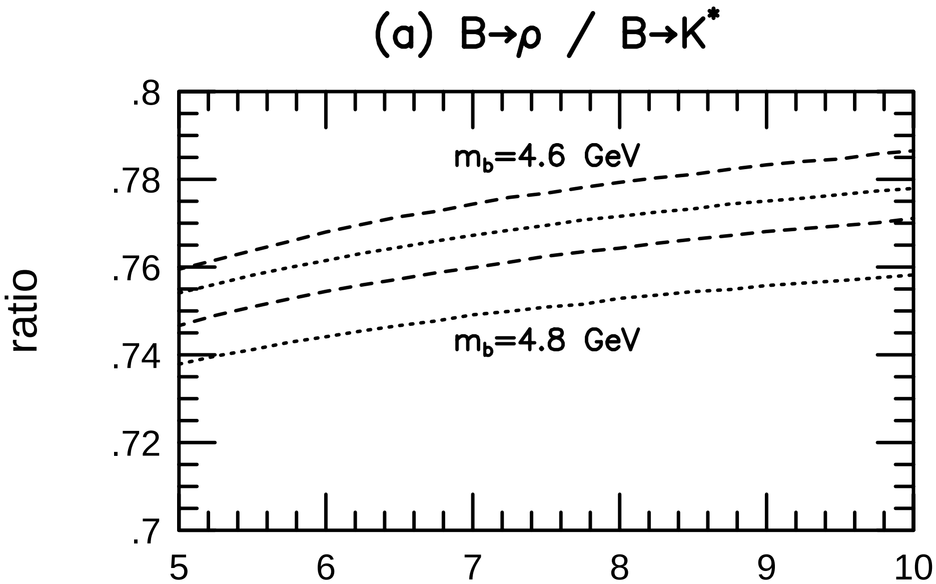


Fig. 4b

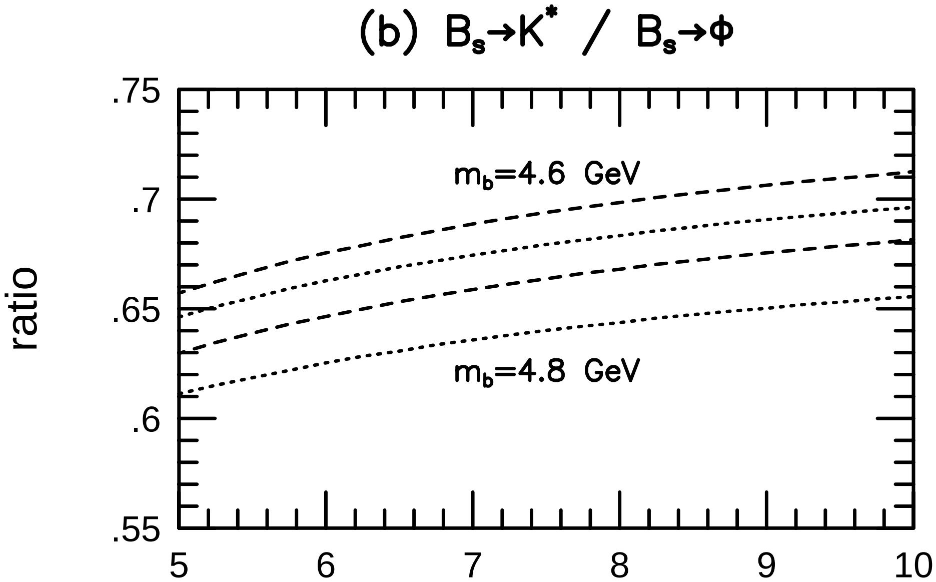


Fig. 4c

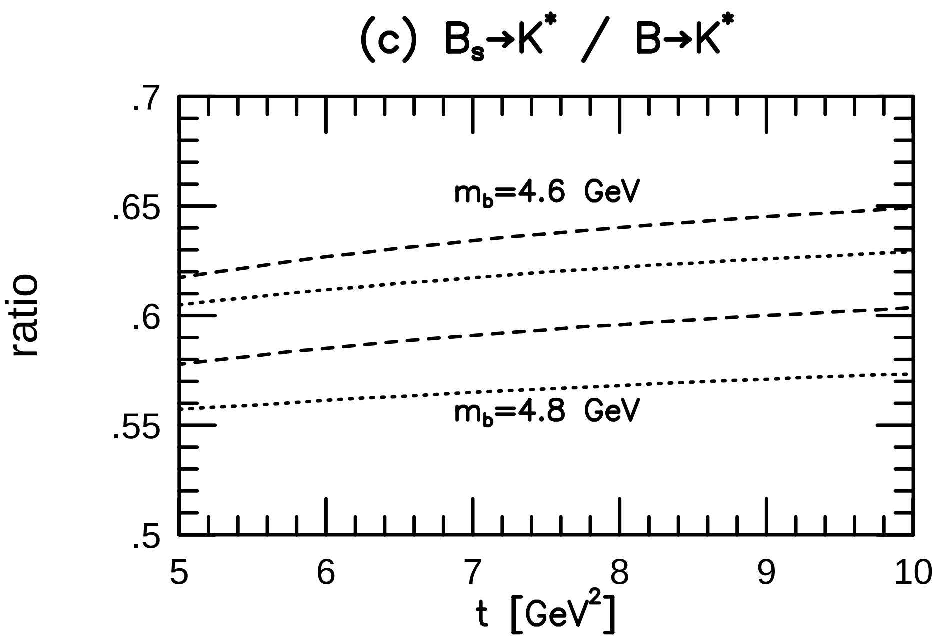
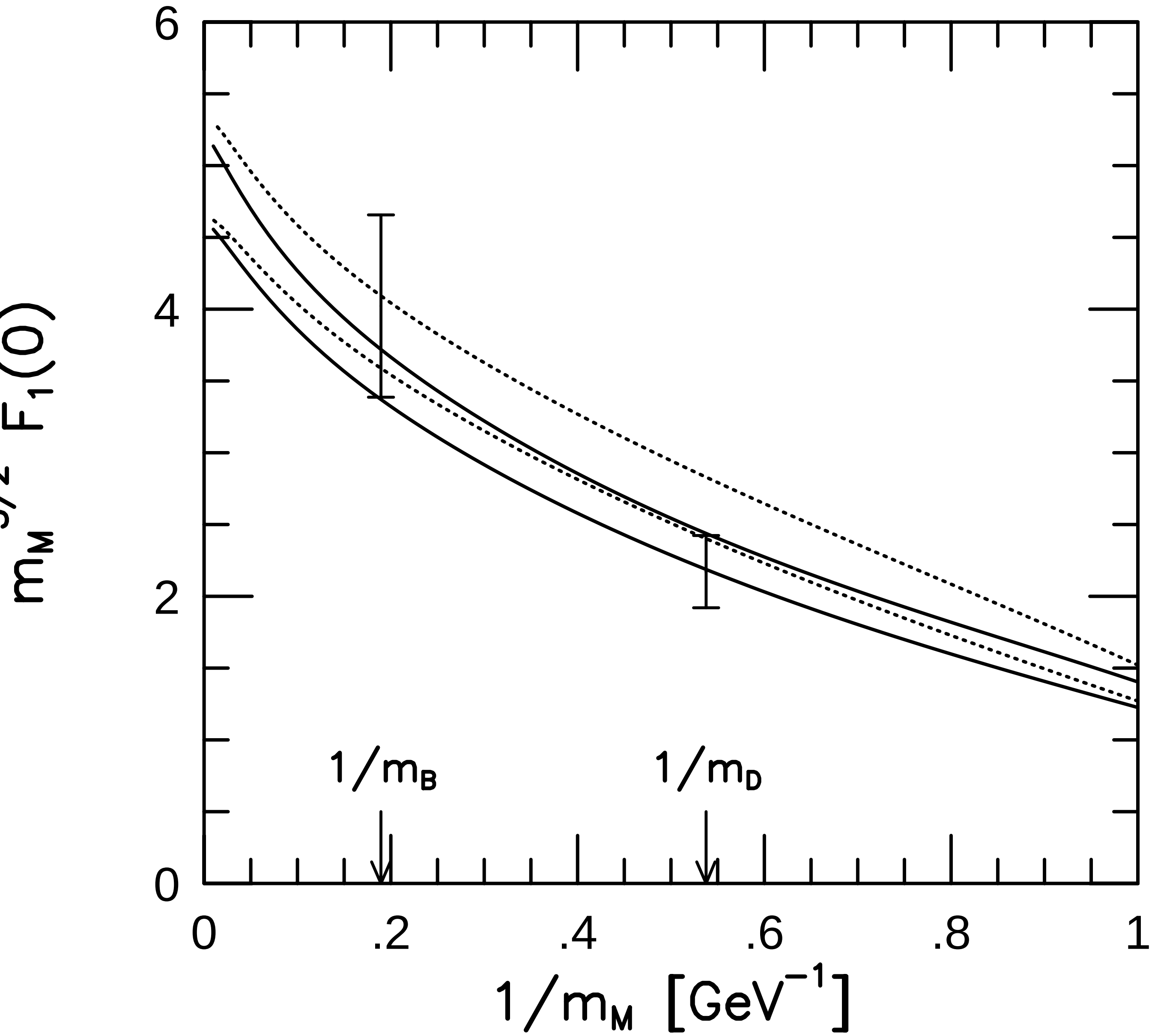
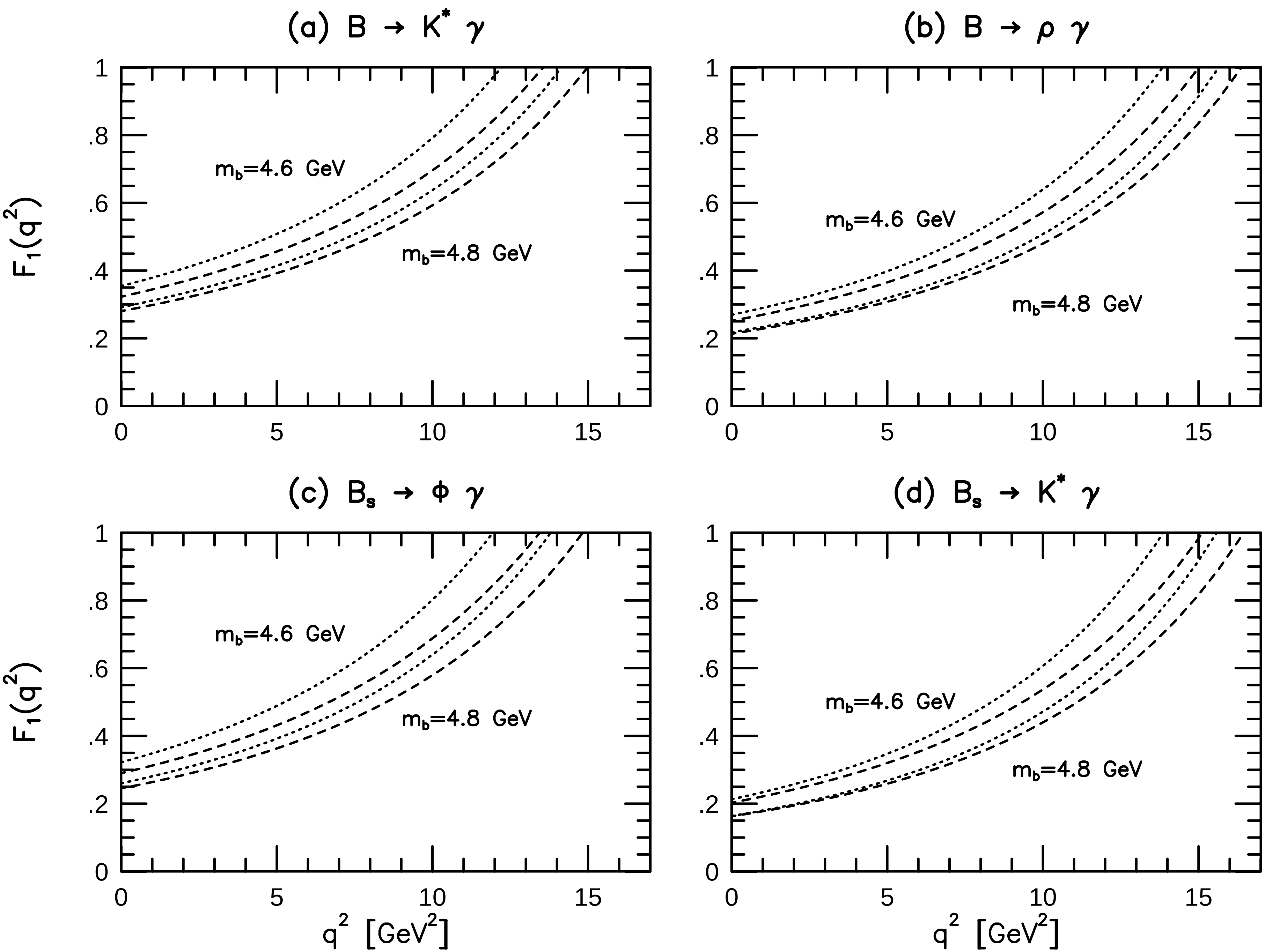


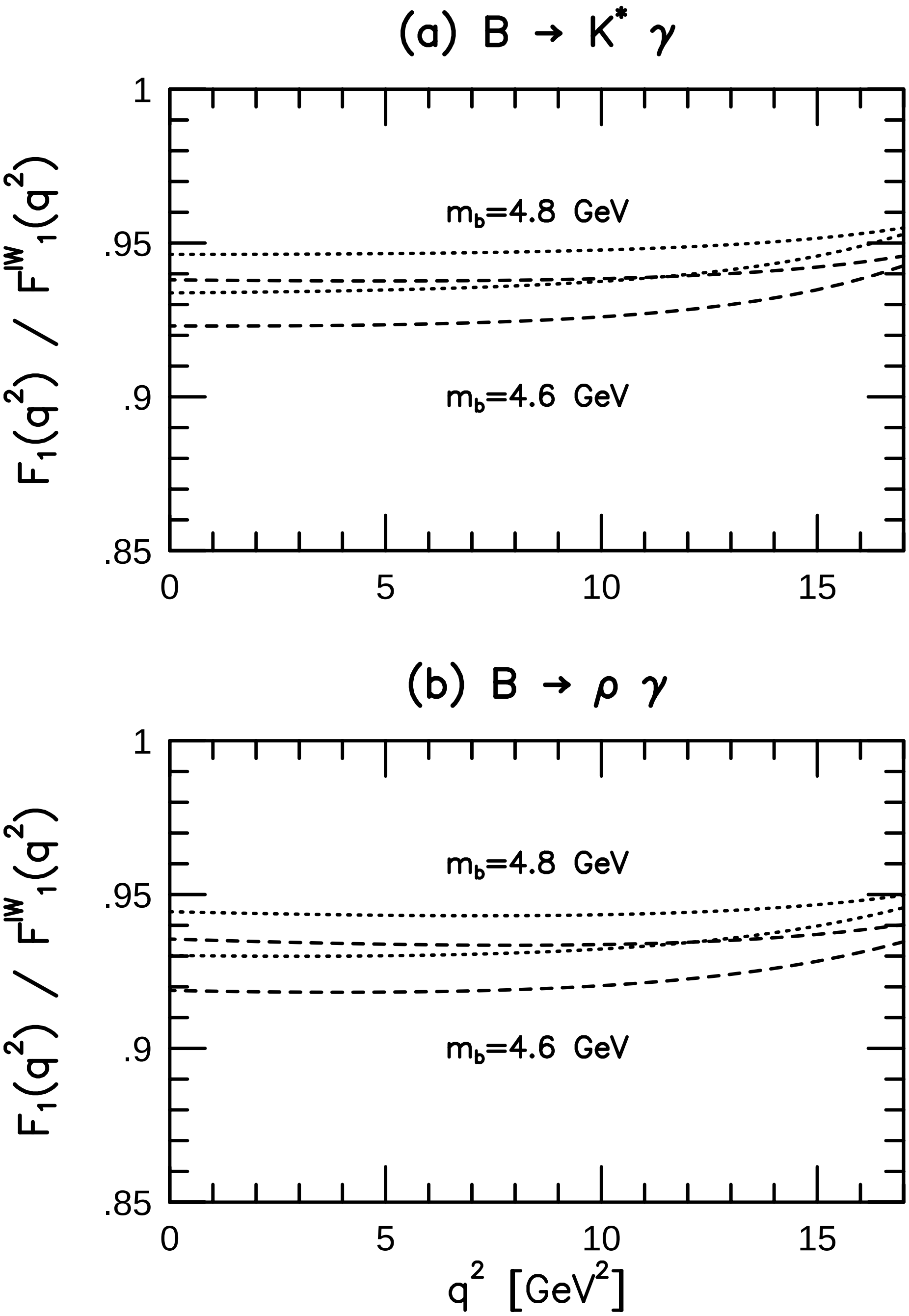
Fig. 5





(a) B → K* γ
(b) B → ρ γ
(c) B_s → φ γ
(d) B_s → K* γ
m_b=4.6 GeV
m_b=4.8 GeV
F_1(q^2)
q^2 [GeV^2]
0
.2
.4
.6
.8
1
5
10
15

Fig. 7



(a) B → K* γ
m_b=4.8 GeV
m_b=4.6 GeV
F_1(q^2) / F^IW_1(q^2)
1
.95
.9
.85
0
5
10
15
(b) B → ρ γ
m_b=4.8 GeV
m_b=4.6 GeV
F_1(q^2) / F^IW_1(q^2)
1
.95
.9
.85
0
5
10
15
q^2 [GeV^2]

Fig. 8a

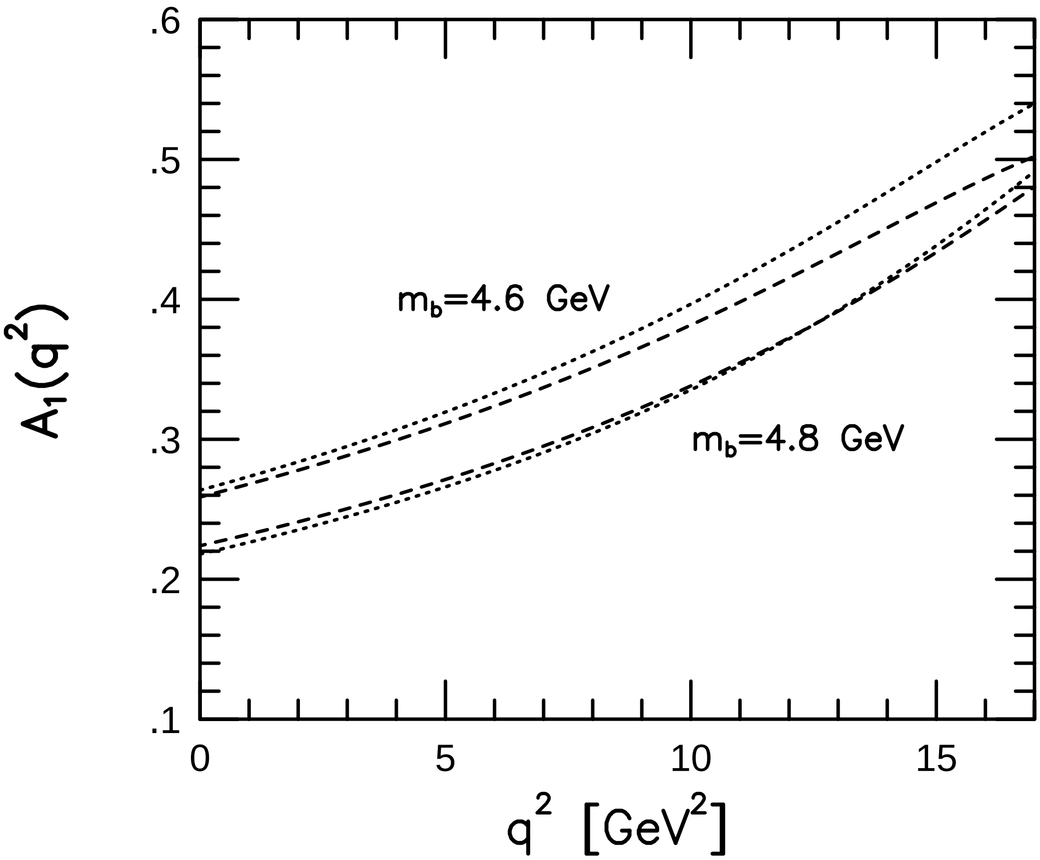

.6
.5
.4
.3
.2
.1
0
5
10
15
$A_1(q^2)$
q^2 [GeV^2]
m_b=4.6 GeV
m_b=4.8 GeV

Fig. 8b

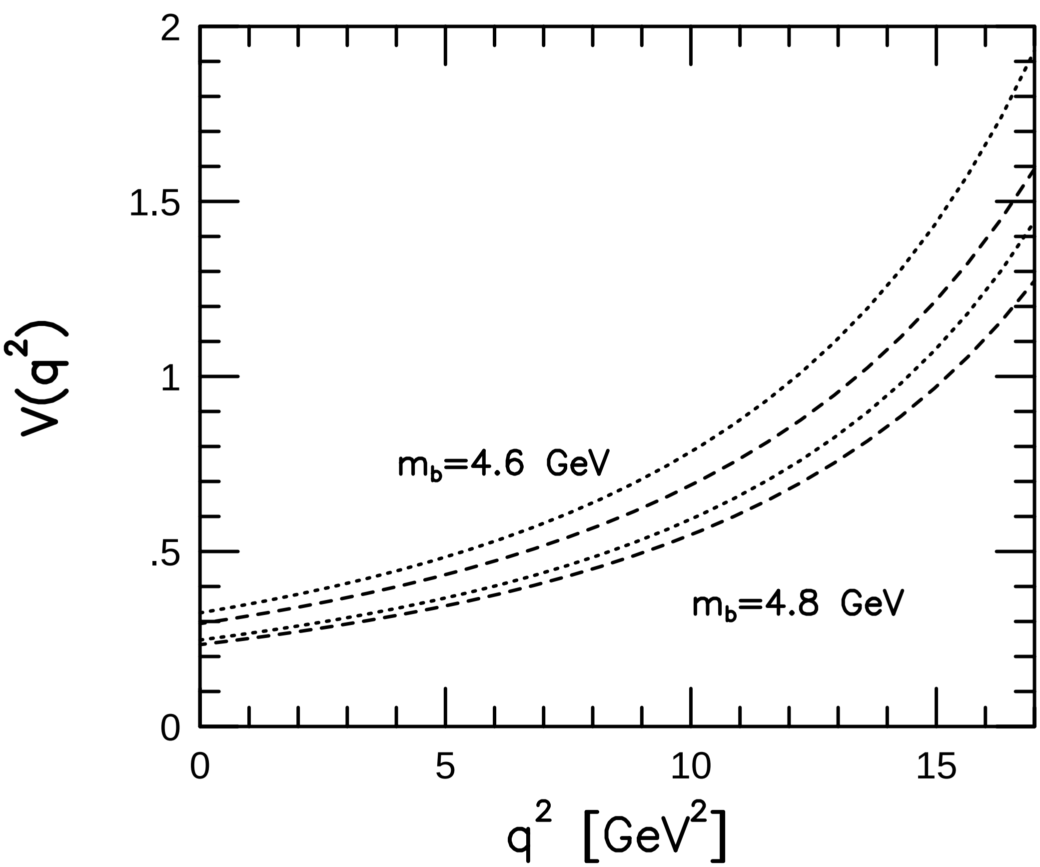
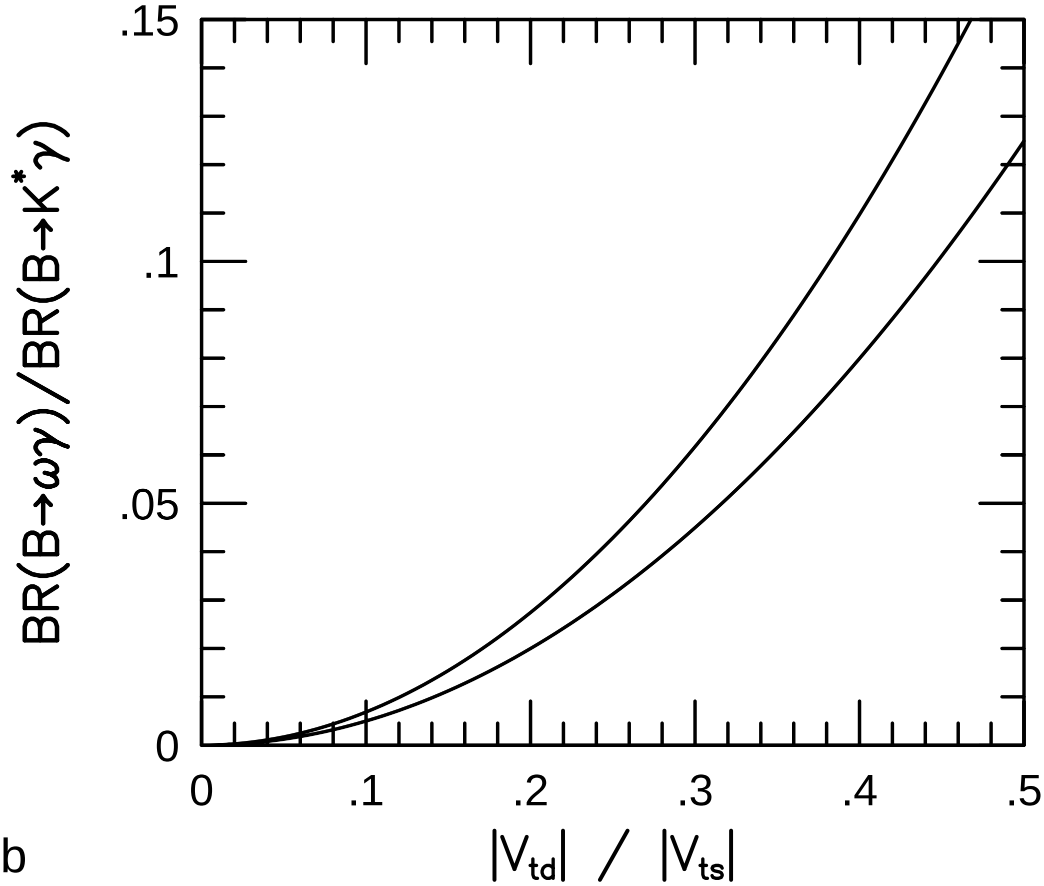
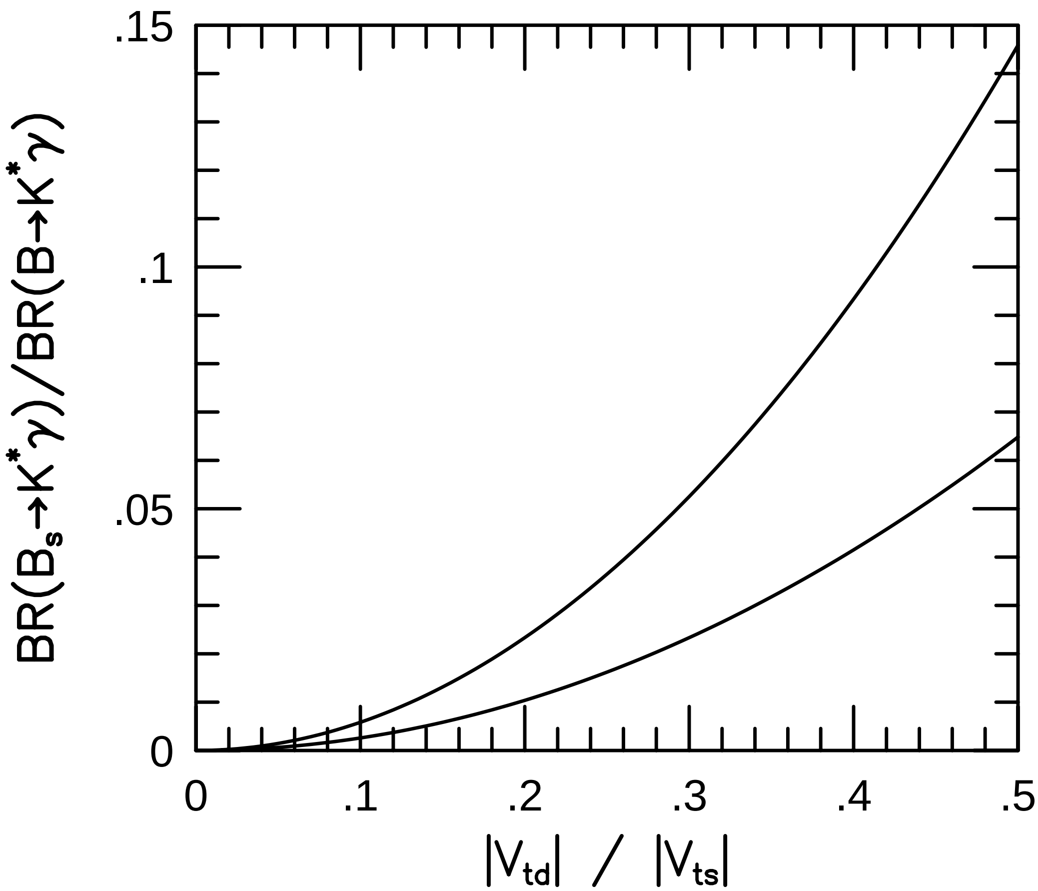

2
1.5
1
.5
0
0
5
10
15
$V(q^2)$
q^2 [GeV^2]
m_b=4.6 GeV
m_b=4.8 GeV

Fig. 9a



.15
.1
.05
0
0
.1
.2
.3
.4
.5
BR(B→ωγ)/BR(B→K*γ)
|V_td| / |V_ts|

Fig. 9b



.15
.1
.05
0
0
.1
.2
.3
.4
.5
BR(B_s→K*γ)/BR(B→K*γ)
|V_td| / |V_ts|

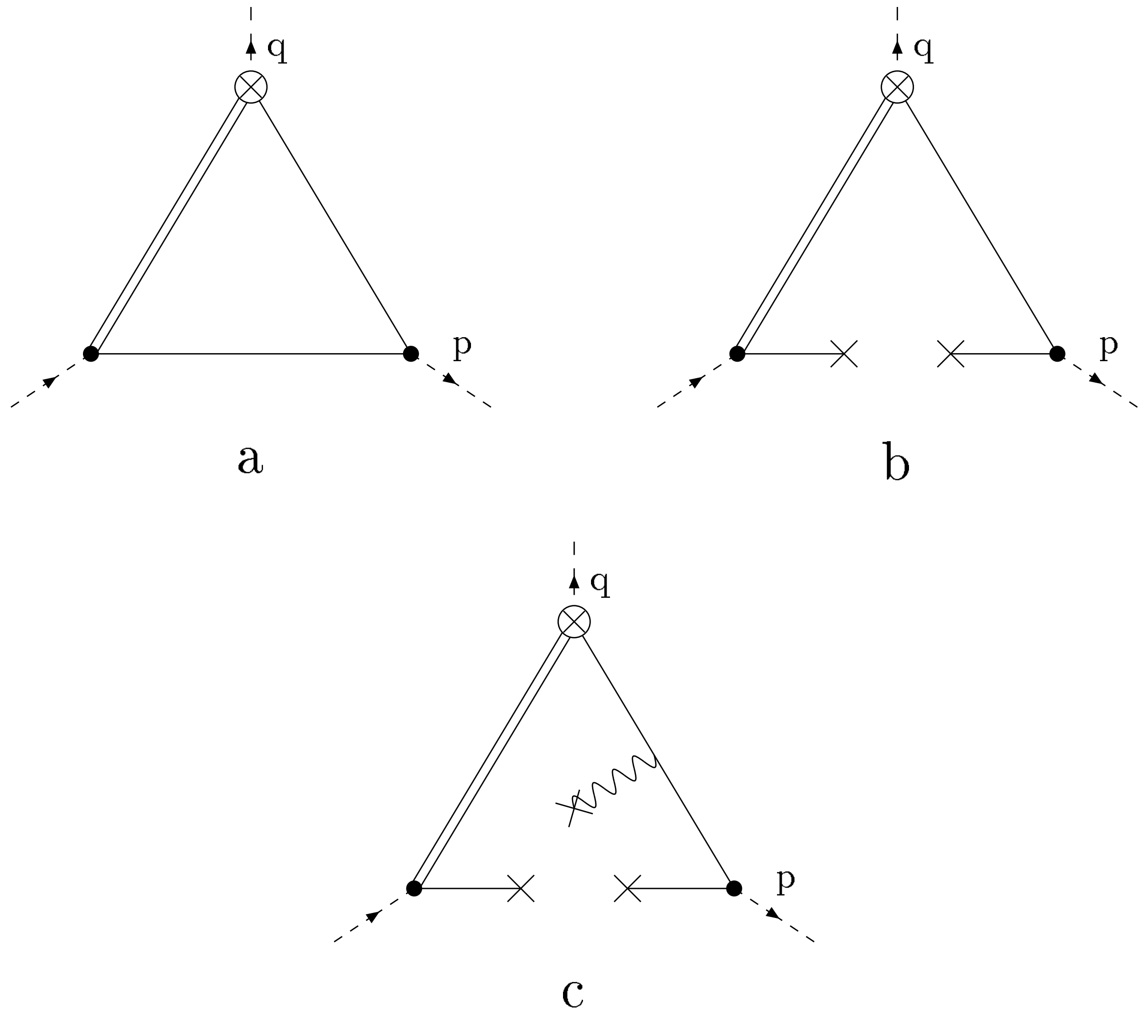


Fig. 10