

MPI-PhT/94-8  
TAUP-2137-94  
hep-ph/9402270

# Soft contribution to the pion form factor from light-cone QCD sum rules

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## Abstract:

We propose a simple method to calculate the pion form factor at not very large momentum transfers, which combines the technique of the QCD sum rules with the description of the pion in terms of the set of wave functions of increasing twist. This approach allows one to calculate the soft (end point) contribution to the form factor in a largely model-independent way. Our results confirm existing expectations that the soft contribution remains important at least up to the momentum transfers of order  $10 \text{ GeV}^2$ , and suggest that it comes from the region of relatively small transverse separations of order  $1 \text{ GeV}^{-1}$ .

*submitted to Physics Letters B*

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1. Hard exclusive processes in QCD [1] are attracting continuous interest for already two decades. In difference to inclusive reactions, like the deep inelastic scattering, exclusive processes are selective to the partonic content of the participating hadron. Apriori, one could consider two different possibilities to transfer a large momentum to a hadron. One is the so-called Feynman mechanism, in which large momentum transfer selects the configuration, in which one parton carries almost all the momentum of the hadron. The transverse size of the remaining “soft” cloud remains in this case arbitrary. The second is the hard rescattering mechanism, in which large momentum transfer selects configurations with a small transverse size and a minimal number of Fock constituents. In this case the momentum fraction carried by the interacting parton (quark) remains an average one. The hard rescattering mechanism involves a hard gluon exchange, and can be written in the factorized form [2].

It has been proved [2] that the hard rescattering mechanism is the leading one at asymptotically large  $Q^2$ , and yields the pion form factor

$$F_\pi(Q^2) = \frac{8}{9}\pi\alpha_s \frac{f_\pi^2}{Q^2} \left| \int_0^1 \frac{du}{1-u} \phi_\pi(u) \right|^2. \quad (1)$$

Here  $\phi_\pi(u)$  is the pion wave function of the leading twist, which describes the distribution of the valence pion constituents in the longitudinal momentum. Note that convergence of the integral in (1) requires that the pion wave function decreases at  $u \rightarrow 1$ , and the crucial point in establishing the formula in (1) was the proof [2] that at asymptotically large  $Q^2$  the wave function is given by the simple formula

$$\phi_\pi^{(as)}(u) = 6u(1-u). \quad (2)$$

The results in (1),(2) belong to the most important and most rigorously proved statements in QCD.

At large but finite momentum transfers there might be a number of corrections to the hard rescattering formula in (1), and till now there was a much more moderate progress in understanding whether available energies could be treated as asymptotic ones. An attempt to describe the data for the pion form factor starting from  $Q^2 \geq 3 \text{ GeV}^2$  by the contribution of hard rescattering alone, implies that the low energy pion wave function must be very different from its asymptotic form, an issue which has been put to fore and studied in detail by Chernyak and Zhitnitsky [3]. Using the QCD sum rule approach they have shown that the pion wave function at a low scale is wider than the asymptotic one, and proposed a model

$$\phi_\pi^{(CZ)}(u, \mu \sim 500 \text{ MeV}) = 30u(1-u)(2u-1)^2, \quad (3)$$

which has a peculiar “humped” profile, with a zero in the middle point, corresponding to the symmetric configuration where the quark and the antiquark carry equal momenta. However, there exists a number of arguments that force to doubt the assertion about the dominance of the hard-scattering mechanism in the region of available  $Q^2 \sim 1 - 10 \text{ GeV}^2$ . The well known point of Isgur and Llewellyn-Smith [4] is that the wave functions of the type

suggested in [3] strongly emphasize the contribution of the end-point region of large  $u$  in (1), where the virtuality of the gluon is not enough to justify the perturbative treatment. This contribution of large  $u$ ,  $1-u \sim m^2/Q^2$  (where  $m^2$  is a certain hadronic scale) corresponds to the Feynman mechanism to transfer the large momentum, and should be treated separately. At present, there is an increasing evidence that this contribution is numerically important up to very high  $Q^2$ , although it is down by an extra power  $1/Q^2$  in the asymptotics.

Using the QCD sum rule approach [5] it has been shown [6, 7] that the pion form factor at  $Q^2 \sim 1-2 \text{ GeV}^2$  is practically saturated by the Feynman-type contribution. Unfortunately, this method in its standard form cannot be applied for higher values of  $Q^2$ , since it involves the expansion in the contributions of vacuum condensates, which coefficients appear to be enhanced by increasing powers of  $Q^2$ . Thus, at sufficiently large momentum transfers the expansion breaks down. In attempt to cure this problem Radyushkin and collaborators [8, 9] have suggested to resum the series of power corrections by introducing the nonlocal extension of the concept of the vacuum condensates, which takes into account the final correlation length in the QCD vacuum. Results of [9] indicate the dominating role of the soft contribution at least up to  $Q^2 \sim 10 \text{ GeV}^2$ .

The approach of [9] clearly demonstrates the origin of difficulties in the standard QCD sum rule calculations, but may receive objections concerning its theoretical accuracy, since not all high-order contributions can consistently be taken into account in this way, and also the parametrization of the nonlocal condensates is essentially model-dependent. In this letter we suggest an alternative approach to the calculation of the pion form factor at not very large values of  $Q^2$ , which seems to be free from both the difficulties of the standard QCD sum rule method, and the ambiguities involved in its extension in [8, 9]. Our method essentially follows earlier works [10]–[15], where the QCD sum rule approach has been modified to incorporate the operator product expansion in powers of the deviation from the light-cone (in contrast to the short distance expansion in [5]).

2. The idea is to combine the standard technique for the study of hard exclusive processes and the QCD sum rule method. To this purpose, we consider the correlation function

$$T_{\mu\nu}(p, q) = i \int dx \exp(iqx) \langle 0 | T \{ j_\mu^5(0) j_\nu^{em}(x) \} | \pi^+(p) \rangle, \quad (4)$$

where  $j_\mu^5 = \bar{d}\gamma_\mu\gamma_5 u$  and  $j_\nu^{em} = e_u \bar{u}\gamma_\nu u + e_d \bar{d}\gamma_\nu d$  is the electromagnetic current. At large Euclidian momenta  $(p-q)^2$  and  $q^2$  this correlation function can be calculated in QCD, in the precise analogy with the calculation of the  $\pi\gamma^*\gamma^*$  form factor (for different virtualities of photons), [3]. The leading contribution is written in terms of the pion wave function of the leading twist

$$T_{\mu\nu}(p, q) = 2i f_\pi p_\mu p_\nu \int_0^1 du \frac{u\phi_\pi(u)}{(1-u)Q^2 - u(q-p)^2} + \dots, \quad (5)$$

where  $f_\pi$  is the pion decay constant,  $Q^2 = -q^2$ , and the ellipses stand for the contributions of other Lorentz structures. On the other hand, the dispersion relation over  $(p-q)^2$  relates

the correlation function (4) to the pion form factor:

$$T_{\mu\nu} = if_\pi(p-q)_\mu \frac{1}{m_\pi^2 - (p-q)^2} 2F_\pi(q^2)p_\nu + \dots, \quad (6)$$

where the dots stand for higher resonances and the continuum contributions. Matching between the representations in (5) and (6) at no-so-large Euclidian  $-(q-p)^2 \sim 1 \text{ GeV}^2$ , we obtain a sum rule for the pion form factor.

To this end, we use the standard concept of duality, which tells that the pion occupies the “region of duality” in the invariant mass of the  $\bar{q}q$  pair, up to a certain threshold  $s_0 \sim 0.7 - 0.8 \text{ GeV}^2$ . Note that the formula in (5) can be rewritten as the dispersion relation in  $(p-q)^2$ , with  $s = (1-u)Q^2/u$  being the mass of the intermediate state. To pick up the contribution of the pion, we cut the dispersion integral at  $s = s_0$ , which translates to a low bound for the integral over  $u$ :  $u_{min} = Q^2/(s_0 + Q^2)$ . Following [5], we also use the Borel transformation to convert the power suppression of higher mass contributions in the dispersion integral to the exponential suppression

$$\begin{aligned} \frac{u}{(1-u)Q^2 - u(q-p)^2} &\rightarrow \exp\left\{-\frac{(1-u)Q^2}{uM^2}\right\}, \\ \frac{1}{m_\pi^2 - (q-p)^2} &\rightarrow \exp\left\{-\frac{m_\pi^2}{M^2}\right\}, \end{aligned} \quad (7)$$

where  $M^2$  is the new variable (the Borel parameter). In what follows we put the pion mass to zero.

Thus, the sum rule arises

$$F_\pi(Q^2) = \int_0^1 du \phi_\pi(u) \exp\left\{-\frac{(1-u)Q^2}{uM^2}\right\} \Theta\left(u - \frac{Q^2}{s_0 + Q^2}\right), \quad (8)$$

which should be satisfied at the values of the Borel parameter  $M^2$  of order  $1 \text{ GeV}^2$ . The pion wave function in (8) should be taken at a low normalization scale, of order of the Borel parameter.

In what follows we shall complement the sum rules in (8) by contributions of higher twist. Before doing this, and before going over to the numerical analysis, let us study the behavior of the sum rule in the limit of large momentum transfers  $Q^2 \rightarrow \infty$ .

Because of the  $\Theta$ -function, the integration region in (8) is restricted to values  $1 - u < s_0/(s_0 + Q^2) \rightarrow 0$ . Thus, the form factor is sensitive to the wave function in the highly asymmetrical configuration, where the scattered quark carries almost all the pion momentum. According to the general analysis in [2, 3] the behavior of the pion wave function in this region coincides with the asymptotic behavior in (2),  $\phi_\pi(u) \xrightarrow{u \rightarrow 1} 1 - u$ . Thus, asymptotically, the sum rule in (8) yields

$$F_\pi(Q^2) \sim \frac{\phi'_\pi(0)}{Q^4} \int_0^{s_0} s^2 ds e^{-s/M^2}, \quad (9)$$

where  $\phi'_\pi(0) = (d/du)\phi_\pi(u)|_{u=0}$ . (We have made use of the symmetry  $\phi_\pi(u) = \phi_\pi(1-u)$ ).

For the asymptotic wave function (2)  $\phi'_\pi(0) = 6$ . For the model suggested by Chernyak and Zhitnitsky (3)  $\phi'^{(CZ)}_\pi(0) = 30$ , and thus it gives rise to a much bigger soft contribution to the form factor. This is in agreement with the observation by Isgur and Llewellyn-Smith [4] that “realistic” model wave functions, which allow for the description of the data on pion form factor at virtualities of order a few GeV by the hard rescattering mechanism alone, in fact greatly enhance the soft contribution coming from the end point region.

**3.** Now we are in a position to take into account the corrections to the sum rule in (8), induced by the pion wave functions of the nonleading twist 4. These corrections correspond to the contributions to the correlation function in (4), which are suppressed by an extra power of the large momentum  $(p-q)^2$  or  $q^2$ . Physically, these corrections have two sources. One of them is the transverse momentum of quarks in the leading order diagram in Fig. 2a. The second source are contributions of higher Fock states in the pion wave function — containing gluon fields in addition to the quark and antiquark ones. In fact, these two effects are indistinguishable thanks to the equations of motion, which allow one to eliminate all transverse degrees of freedom at the cost of introducing the higher Fock components. In this letter we cannot give all the details of the calculation, which becomes rather technical. A suitable formalism for the operator product expansion of the correlation functions beyond the leading twist has been developed in [16].

The complete contribution of the diagram in Fig. 1a to the twist-4 accuracy reads

$$\Pi_{(2a)} = 2ip_\mu p_\nu f_\pi \int_0^1 du \left\{ -\frac{u\phi_\pi(u)}{(q-up)^2} - 4u \frac{g_1(u) + G_2(u)}{(q-up)^4} + 2u^2 \frac{g_2(u)}{(q-up)^4} \right\} + \dots \quad (10)$$

where we have introduced the pion wave functions of twist 2 and 4 defined by the matrix element [17]

$$\begin{aligned} \langle 0|\bar{d}(0)\gamma_\mu\gamma_5 u(x)|\pi(p)\rangle &= if_\pi p_\mu \int_0^1 du e^{-iupx} (\phi_\pi(u) + x^2 g_1(u) + O(x^4)) \\ &+ f_\pi (x_\mu - \frac{x^2 p_\mu}{px}) \int_0^1 du e^{-iupx} g_2(u) + \dots, \end{aligned} \quad (11)$$

and all logarithmic dependence on  $x^2$  is implicitly included in the wave functions. The function  $G_2$  in (10) is defined as  $g_2(u) = -(d/du)G_2(u)$ . The diagram shown in Fig. 1b produces an additional contribution ( $\alpha_3 = 1 - \alpha_1 - \alpha_2$ )

$$\begin{aligned} \Pi_{(2b)} &= 2if_\pi \int_0^1 \frac{u du}{(q-up)^4} \int_0^u d\alpha_1 \int_0^{\bar{u}} d\alpha_2 \left\{ \frac{\Psi_\parallel + 2\Psi_\perp}{\alpha_3} \right. \\ &\left. + \frac{1-2u+\alpha_1-\alpha_2}{\alpha_3^2} (\Phi_\parallel + 2\Phi_\perp) \right\}, \end{aligned} \quad (12)$$

where we have followed [17] in the definition of three-particle wave functions of twist 4:

$$\langle 0|\bar{d}(-x)\gamma_\mu\gamma_5 g G_{\alpha\beta}(vx)u(x)|\pi(p)\rangle = p_\mu (p_\alpha x_\beta - p_\beta x_\alpha) \frac{1}{px} f_\pi \int D\alpha \Phi_\parallel(\alpha_i) e^{-ipx(\alpha_1 - \alpha_2 + v\alpha_3)}$$

$$+ \left[ p_\beta \left( \delta_{\alpha\mu} - \frac{x_\alpha p_\mu}{px} \right) - p_\alpha \left( \delta_{\beta\mu} - \frac{x_\beta p_\mu}{px} \right) \right] f_\pi \int D\alpha \Phi_\perp(\alpha_i) e^{-ipx(\alpha_1 - \alpha_2 + v\alpha_3)}. \quad (13)$$

The wave functions  $\Psi_\parallel$  and  $\Psi_\perp$  are defined similar to (13), with the substitution  $(\gamma_5 g G_{\alpha\beta}) \rightarrow (ig\tilde{G}_{\alpha\beta})$ .

A systematic study of the higher twist wave functions has been done in the work [17], and makes use of the expansion in representations of the collinear conformal group  $SO(2,1)$ , which is a subgroup of full conformal group acting on the light-cone. The asymptotic wave functions are defined as contributions of operators with the lowest conformal spin. The set of wave functions suggested in [17] includes also the corrections corresponding to the operators with the next-to-leading conformal spin, which numerical values are calculated by the QCD sum rule method. The results for twist 4 wave functions are (hereafter  $\bar{u} = 1 - u$ ):

$$\begin{aligned} \Phi_\parallel(\alpha_i) &= 120\varepsilon\delta^2(\alpha_1 - \alpha_2)\alpha_1\alpha_2\alpha_3, \\ \Psi_\parallel(\alpha_i) &= -120\delta^2\alpha_1\alpha_2\alpha_3\left[\frac{1}{3} + \varepsilon(1 - 3\alpha_3)\right], \\ \Phi_\perp(\alpha_i) &= 30\delta^2(\alpha_1 - \alpha_2)\alpha_3^2\left[\frac{1}{3} + 2\varepsilon(1 - 2\alpha_3)\right], \\ \Psi_\perp(\alpha_i) &= 30\delta^2\alpha_3^2(1 - \alpha_3)\left[\frac{1}{3} + 2\varepsilon(1 - 2\alpha_3)\right], \\ g_1(u) &= \frac{25}{6}\delta^2\bar{u}^2u^2 + \varepsilon\delta^2[\bar{u}u(2 + 13\bar{u}u) \\ &\quad + 10u^3(2 - 3u + \frac{6}{5}u^2)\ln u + 10\bar{u}^3(2 - 3\bar{u} + \frac{6}{5}\bar{u}^2)\ln \bar{u}], \\ g_2(u) &= \frac{10}{3}\delta^2\bar{u}u(u - \bar{u}), \\ G_2(u) &= \frac{5}{3}\delta^2u^2\bar{u}^2, \\ \delta^2 &\simeq 0.2 \text{ GeV}^2, \quad \varepsilon \simeq 0.5. \end{aligned} \quad (14)$$

Adding the higher-twist contributions in (10) and (12) to the sum rule in (8) we arrive at

$$\begin{aligned} F_\pi(Q^2) &= \int_0^1 du \exp\left[-\frac{\bar{u}Q^2}{uM^2}\right] \left\{ \phi_\pi(u) - \frac{4}{uM^2}(g_1(u) + G_2(u)) + \frac{2}{M^2}g_2(u) \right. \\ &\quad \left. + \frac{1}{uM^2} \int_0^u d\alpha_1 \int_0^{\bar{u}} d\alpha_2 \left[ \frac{\Psi_\parallel + 2\Psi_\perp}{\alpha_3} + \frac{1 - 2u + \alpha_1 - \alpha_2}{\alpha_3^2} (\Phi_\parallel + 2\Phi_\perp) \right] \right\} \Theta\left(u - \frac{Q^2}{s_0 + Q^2}\right), \end{aligned} \quad (15)$$

which presents our final result. Note that the higher-twist contributions are suppressed by a power of the Borel parameter  $M^2$ , as expected.

With the particular expressions (15) the integrals of the three-particle wave functions can be taken analytically, yielding

$$\begin{aligned} \int_0^u d\alpha_1 \int_0^{\bar{u}} d\alpha_2 \frac{\Psi_\parallel + 2\Psi_\perp}{\alpha_3} &= \frac{10}{3}\delta^2\bar{u}u(1 - 2\bar{u}u), \\ \int_0^u d\alpha_1 \int_0^{\bar{u}} d\alpha_2 \frac{1 - 2u + \alpha_1 - \alpha_2}{\alpha_3^2} (\Phi_\parallel + 2\Phi_\perp) &= -2g_1(u) - \frac{10}{3}\delta^2\bar{u}u\left(1 - \frac{15}{2}\bar{u}u\right). \end{aligned} \quad (16)$$

Note that the term  $\sim \bar{u}u$  in (16) cancel in the sum rule (16), and thus contributions of three-particle wave functions turn out to be of order  $1/Q^6$ , i.e. are suppressed by an additional power of  $1/Q^2$ . However, the twist 4 contributions still survive in the high- $Q^2$  limit due to the contribution of  $g_2$ , which yields the same asymptotic behavior  $\sim 1/Q^4$  as the leading twist contribution in (9).

4. We turn now to the numerical analysis. Apart from the wave functions, the sum rule in (16) depends on the value of the continuum threshold  $s_0$ , and on the Borel parameter  $M^2$ . We take  $s_0 = 0.7 - 0.8 \text{ GeV}^2$  [5] and vary  $M^2$  in the interval  $1 - 2 \text{ GeV}^2$ , which is the expected stability region. The results are shown in Fig.2. In Fig. 2a we plot the value of  $Q^2 F_\pi(Q^2)$  as a function of  $Q^2$  for  $s_0 = 0.7 \text{ GeV}^2$  and  $s_0 = 0.8 \text{ GeV}^2$  and for different choices of the leading twist pion wave function  $\phi_\pi(u)$ : asymptotic wave function (2) and the Chernyak-Zhitnitsky model (3). Since this model in fact refers to a substantially lower normalization point than the typical value of the Borel parameter in the sum rule, we give the results also for the Chernyak-Zhitnitsky wave function rescaled to  $\mu^2 \sim 1 - 2 \text{ GeV}^2$

$$\phi_\pi^{(CZ)}(u, \mu \sim 1 \text{ GeV}) = 6u(1-u) \left[ 1 + 0.44 C_2^{3/2} (2u-1) \right]. \quad (17)$$

The wave function in (17) corresponds to the value of the second moment  $\langle (2u-1)^2 \rangle = 0.35$  [12], which is to be compared to  $\langle (2u-1)^2 \rangle = 0.43$  for (3). We remind that for the asymptotical wave function  $\langle (2u-1)^2 \rangle = 0.2$ . The contribution of wave functions of twist 4 does not exceed 20%, and these wave functions are not far from their asymptotic form. Thus possible inaccuracy in the model wave functions in (15) does not have any noticeable effect. The stability of the sum rule (16) to the choice of the Borel parameter is illustrated in Fig. 2b for several values of  $Q^2$ .

It is seen that the soft contribution to the pion form factor clearly dominates at  $Q^2 \sim 1 - 3 \text{ GeV}^2$  and constitutes about 15–30% of the experimental value at  $Q^2 \simeq 10 \text{ GeV}^2$  (for the asymptotic wave function). For the Chernyak–Zhitnitsky model, the soft contribution increases substantially.

Within our approach, the hard gluon exchange contribution originates from the radiative correction to the contribution of the leading twist, see diagram in Fig. 1c. This contribution is not restricted to the end-point region, and thus has no power-like  $1/Q^2$  suppression. Its explicit calculation goes beyond the tasks of this letter. As a rough estimate, one can use the expression in (1), yielding  $Q^2 F_\pi(Q^2)_{\text{hard}} \simeq 0.15$  and  $Q^2 F_\pi(Q^2)_{\text{hard}} \simeq 0.3$  for the asymptotic wave function and the Chernyak–Zhitnitsky model, respectively. One sees that the full answer for the pion form factor, given by the sum of the soft and hard contributions, is likely to overshoot the data, if one uses the Chernyak–Zhitnitsky model.

Main lesson to be learnt from our calculation is that the soft contribution to the pion form factor decreases very slowly with  $Q^2$  and is important in the whole region of momentum transfers, which are available at present. This conclusion is in full agreement with the results of [4, 9], although our argumentation is different.

5. The method described above is quite general, and can be applied to different form factors as well. As a one more example, we calculate here the soft contribution to the

transition form factor  $\gamma\pi\rho$ . For the  $|\lambda| = 1$   $\rho$ -meson ( $\rho_\perp$  hereafter) the transition form factor is defined as

$$\langle \rho_\perp(p_1) | j_\mu^{em} | \pi(p_2) \rangle = \varepsilon_{\mu\nu\lambda\sigma} p_1^\lambda p_2^\sigma \varepsilon_\perp^\rho F_{\pi\rho}(Q^2), \quad (18)$$

where  $Q^2 = -(p_1 - p_2)^2$  and  $\varepsilon_\perp^\rho$  stands for the polarization vector of the  $\rho$ . This process is clearly due to non-leading twist effects [3] as long as it is related with the helicity flipping. As a result, the hard rescattering diagram yield the asymptotic behavior  $F_{\pi\rho}(Q^2) \sim 1/Q^4$  [3]. As we will show, the soft contribution exhibits the same asymptotic dependence  $1/Q^4$ , and is of the same order, therefore, as the hard contribution (up to the Sudakov suppression, which is unlikely to be important at moderate values of  $Q^2$ ).

We consider the correlation function

$$\begin{aligned} A(p, q) &= i \int dx e^{iqx} \langle 0 | T \{ \bar{d}(0) \sigma_{\xi\nu} u(0) j_\mu^{em}(x) \} | \pi(p) \rangle \\ &= (\varepsilon_{\mu\nu\lambda\sigma} q_\xi - \varepsilon_{\mu\xi\lambda\sigma} q_\nu) q_\lambda p_\sigma \Pi(q^2, (p - q)^2) + \dots \end{aligned} \quad (19)$$

which contains a contribution of interest of the  $\rho_\perp$ -meson

$$\Pi(q^2, (p - q)^2) = f_\rho^T \frac{F_{\pi\rho}(Q^2)}{m_\rho^2 - (p - q)^2}. \quad (20)$$

Here  $f_\rho^T$  is the  $\rho_\perp$ -meson decay constant:

$$\langle 0 | \bar{d}(0) \sigma_{\xi\nu} \gamma_5 u(x) | \pi(p) \rangle = i(\varepsilon_\xi^\perp p_\nu - \varepsilon_\nu^\perp p_\xi) f_\rho^T. \quad (21)$$

Calculation of the diagram in Fig.1a yields in this case

$$\Pi(q^2, (p - q)^2) = -(e_u + e_d) \frac{f_\pi m_\pi^2}{3(m_u + m_d)} \int_0^1 du \frac{\varphi_\sigma(u)}{(q - up)^4}, \quad (22)$$

where  $\varphi_\sigma(u)$  is the pion wave function of twist 3 [17]

$$\langle 0 | \bar{d}(0) \sigma_{\xi\nu} \gamma_5 u(x) | \pi(p) \rangle = \frac{if_\pi m_\pi^2}{6(m_u + m_d)} (p_\xi x_\nu - p_\nu x_\xi) \int_0^1 du e^{-iupx} \varphi_\sigma(u). \quad (23)$$

It has been shown in [17] that the wave function  $\varphi_\sigma(u)$  is close to its asymptotic form  $\varphi_\sigma(u) = 6u(1 - u)$ .

It is easy to check that the diagram of Fig.1b does not contribute to the Lorentz structure of interest. Then, to the twist 3 accuracy, we arrive at the very simple sum rule

$$F_{\pi\rho}(Q^2) = (e_u + e_d) \frac{2\langle \bar{\psi}\psi \rangle e^{m_\rho^2/M^2}}{3f_\pi f_\rho^T} \int_0^1 du e^{-\frac{uQ^2}{uM^2}} \frac{\varphi_\sigma(u)}{u^2} \Theta(u - \frac{Q^2}{s_0 + Q^2}), \quad (24)$$

in which we have replaced the factor appearing in the normalization of the wave function  $\varphi_\sigma$  (23) by the quark condensate  $\langle \bar{q}q \rangle \simeq -(250 \text{ MeV})^3$ . Following [5, 3] we use the values  $s_0 = 1.5 \text{ GeV}^2$  and  $f_\rho^T \simeq 200 \text{ MeV}$  for the continuum threshold in the  $\rho$ -meson channel, and



the  $\rho$ -meson coupling, respectively. The results are shown in Fig. 3. Using them we obtain an estimate for the  $\Psi \rightarrow \gamma \rightarrow \pi^0\omega$  decay rate ( $F_{\pi\omega}(Q^2) = 3F_{\pi\rho}(Q^2)$  due to the isospin symmetry)

$$Br(\Psi \rightarrow \gamma \rightarrow \pi^0\omega / \Psi \rightarrow e^+e^-) = \frac{9}{32}(M_\Psi F_{\pi\omega}(M_\Psi^2))^2 \simeq (4 \pm 2) \times 10^{-4}. \quad (25)$$

A relatively large error is due to a poor stability of the sum rule in this case. The number in (25) appears to be in good agreement to the experimental number  $(4.2 \pm 0.6) \times 10^{-4}$  [18].

The contribution to this form factor of the hard rescattering has been calculated by Chernyak and Zhitnitsky [3], using the leading twist pion wave function in (3), and three-particle wave functions of the  $\rho$ -meson of nonleading twist. It has the same functional dependence  $\sim 1/Q^4$ , and approximately the same numerical value as the soft contribution which we have calculated here. A simple patching them together would yield the branching ratio  $\Psi \rightarrow \pi^0\omega$  several times above the data. To our opinion, the hard contribution to this decay given in [3] is strongly overestimated.

6. In this letter we have suggested a simple method to calculate the pion form factor in the region of intermediate momentum transfers, which is essentially a hybrid of the standard QCD sum rule approach and the conventional expansion in terms of the pion wave functions. Its value is in the possibility to estimate the soft (end point) contribution to the form factor in a model independent way, which is a problem of acute interest. The main advantage compared to the standard QCD sum rule calculation [6, 7] is that the “light-cone sum rules” suggested in this paper remain well-defined in the limit  $Q^2 \rightarrow \infty$ , and is related to the fact that the parameter of the expansion in our sum rules is the twist of relevant operators, but not their dimension as in the standard sum rules. In this way contributions of various local operators are resummed in the set of wave functions of increasing twist, the end-point behavior of which is known from general arguments. In effect, explicit factors  $\sim Q^2$  which may appear in the calculation of higher twist contributions will be compensated by factors  $\sim 1/Q^2$  originating from a more fast decrease of higher twist wave functions at  $u \rightarrow 1$  compared to the leading twist ones. The physical reason for disappearance of the contributions enhanced by powers of  $Q^2$  is in our approach the same as in the calculation involving the nonlocal condensates in [9]. However, our method is practically model-independent.

Our main result is the calculation of the soft contribution to the form factor, which turns out to be large at least up to  $Q^2 \sim 10 \text{ GeV}^2$ . In agreement to [4] we find that this contribution depends strongly on the shape of the pion wave function. Patching together the contribution of hard rescattering and the soft contribution, we find that the model by Chernyak and Zhitnitsky is likely to overshoot the data. Combining this result with the criticism in [8, 12], we conclude that there is increasing evidence, coming from different calculations, that the true low energy pion wave function is not that much different from its asymptotic form, as proposed in [3].

On the evidence of an impressive calculation of Sudakov-type double-logarithmic corrections to the contribution of the hard rescattering, it has been argued in [19] that the

end-point contribution to the pion form factor is strongly suppressed already at moderate  $Q^2 \sim 10 \text{ GeV}^2$ . Radiative corrections to the correlation function in (4) can involve logarithms of the type  $\ln(q^2/(q-p)^2)$ , but not  $\ln(Q^2/\Lambda_{QCD}^2)$ , since it is IR-protected. Hence the corrections to the sum rule can only be accompanied by logs like  $\ln(Q^2/s_0)$ ,  $\ln(Q^2/M^2)$  which never become large (at moderate momentum transfers). Thus, the Sudakov exponential suppression is not likely to occur in our sum rules. To our opinion, the significance of Sudakov corrections is overestimated in [19]. The reason is that the effective transverse momentum, generated by the Sudakov suppression, should be compared not to the Compton wave length of the pion, of order  $1/200 \text{ MeV}$ , but to the average transverse separation of the quark and antiquark in the particular configuration which dominates the soft contribution to the form factor. Note in this respect, that the quark-antiquark separation in a “free” pion does not have any physical meaning beyond the leading twist accuracy, since effects of transverse degrees of freedom can be rewritten in terms of higher Fock components in the wave function thanks to the equations of motion. For the particular hard process, however, the question of relevant transverse distances is well-defined. In the case of the correlation function in (4), the characteristic transverse separation between the quark and the antiquark is given by the deviation from the light-cone  $x^2 \sim (q-up)^2$ , as can easily be checked by an explicit calculation in light-cone coordinates in the position space. After the Borel transformation,  $(q-up)^2$  goes into  $uM^2$ , so that the characteristic transverse separations yielding the form factor in Fig. 2 are of the order

$$x_{\perp}^2 \sim \frac{1}{uM^2}. \quad (26)$$

Note that  $1/(uM^2)$  is exactly the expansion parameter in our calculation, which controls the size of higher-twist corrections, and in the working region of the sum rule is of order  $1/s_0 \sim (0.2-0.3 \text{ fm})^2$ . To our opinion, it is this scale rather than  $\Lambda_{QCD}$  which should serve as the IR cutoff in the calculation in [19]. Since the average value of  $u$  under the integral in the sum rule increases slightly with  $Q^2$ , one may speculate that the relevant transverse size and the importance of higher twist contributions are slightly decreasing.

Thus, the soft contribution to the pion form factor comes from configurations with a much smaller transverse size than the pion electromagnetic radius  $\sim 0.65 \text{ fm}$ , which is dominated by contributions of multiparton states. At distances  $\sim 0.2-0.3 \text{ fm}$  the strong coupling is already not large, and the application of perturbation theory to the calculation of the contribution of the hard gluon exchange can be justified. However, this contribution must be complemented by the contribution of Feynman type, coming from the end-point region. Our conclusions essentially support the picture described in [20].

We gratefully acknowledge stimulating discussions with L.L. Frankfurt and A.V. Radyushkin on the number of subjects related to this study. Our special thanks are due to V.M. Belyaev for pointing out an error in the preliminary version of this paper.

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## Captions

**Fig. 1** Leading contributions to the expansion of the correlation function in (4) in powers of the deviation from the light-cone.

**Fig. 2** The soft (end point) contribution to the pion electromagnetic form factor for the asymptotic pion wave function and for the Chernyak–Zhitnitsky model as a function of  $Q^2$  (a) and in dependence on the Borel parameter  $M^2$  in the sum rule (b). The solid and dashed curves in Fig. 2a correspond to the calculation with  $s_0 = 0.8$  and  $s_0 = 0.7$  GeV<sup>2</sup>, respectively, and the value of the Borel parameter  $M^2 = 1.5$  GeV<sup>2</sup>. The curves in Fig. 2b are calculated using  $s_0 = 0.8$  GeV<sup>2</sup> and the asymptotical wave function. Among the pairs of curves marked “CZ” the upper ones correspond to the calculation using the Chernyak-Zhitnitsky wave function at the scale  $\mu = 500$  MeV, and the lower ones at  $\mu = 1$  GeV, see (3) and (17), respectively.

**Fig. 3** The transition form factor  $\gamma\rho\pi$  as a function of the momentum transfer.

Figure 1: Leading contributions to the expansion of the correlation function in (4) in powers of the deviation from the light-cone.

Figure 2: The soft (end point) contribution to the pion electromagnetic form factor for the asymptotic pion wave function and for the Chernyak–Zhitnitsky model as a function of  $Q^2$  (a) and in dependence on the Borel parameter  $M^2$  in the sum rule (b). The solid and dashed curves in Fig. 2a correspond to the calculation with  $s_0 = 0.8$  and  $s_0 = 0.7$   $\text{GeV}^2$ , respectively, and the value of the Borel parameter  $M^2 = 1.5$   $\text{GeV}^2$ . The curves in Fig. 2b are calculated using  $s_0 = 0.8$   $\text{GeV}^2$  and the asymptotical wave function. Among the pairs of curves marked “CZ” the upper ones correspond to the calculation using the Chernyak-Zhitnitsky wave function at the scale  $\mu = 500$  MeV, and the lower ones at  $\mu = 1$  GeV, see (3) and (17), respectively.

Figure 3: The transition form factor  $\gamma\rho\pi$  as a function of the momentum transfer.

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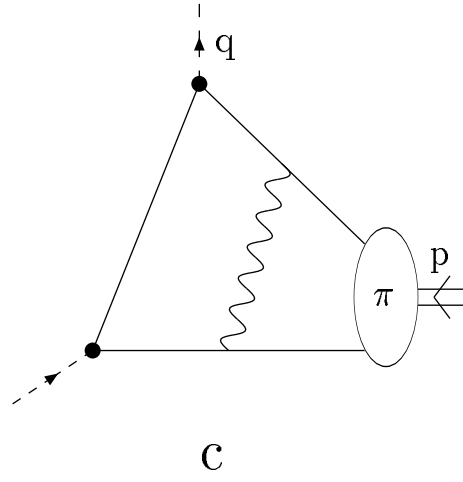
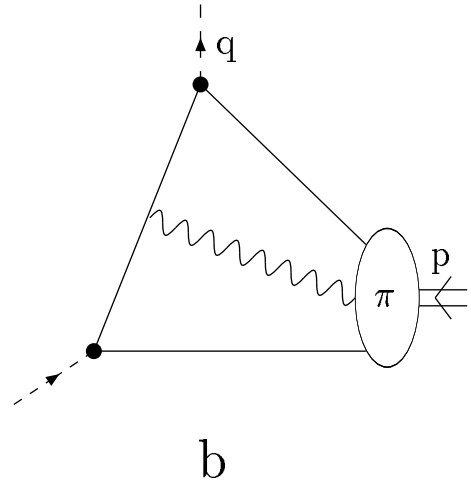
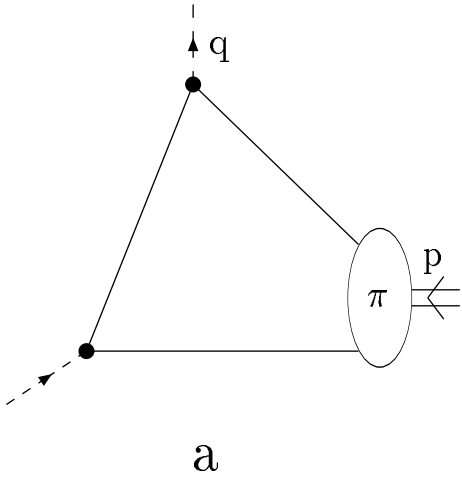


Fig. 1

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### Soft (End Point) Contribution to Pion Form Factor

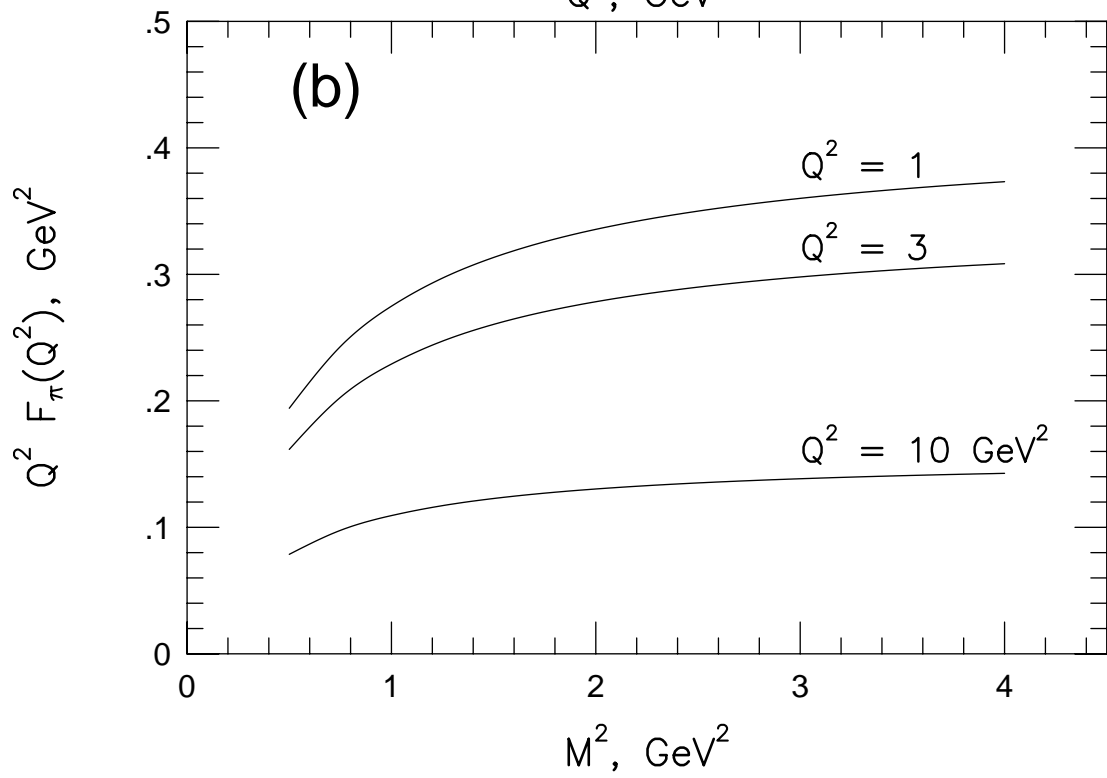
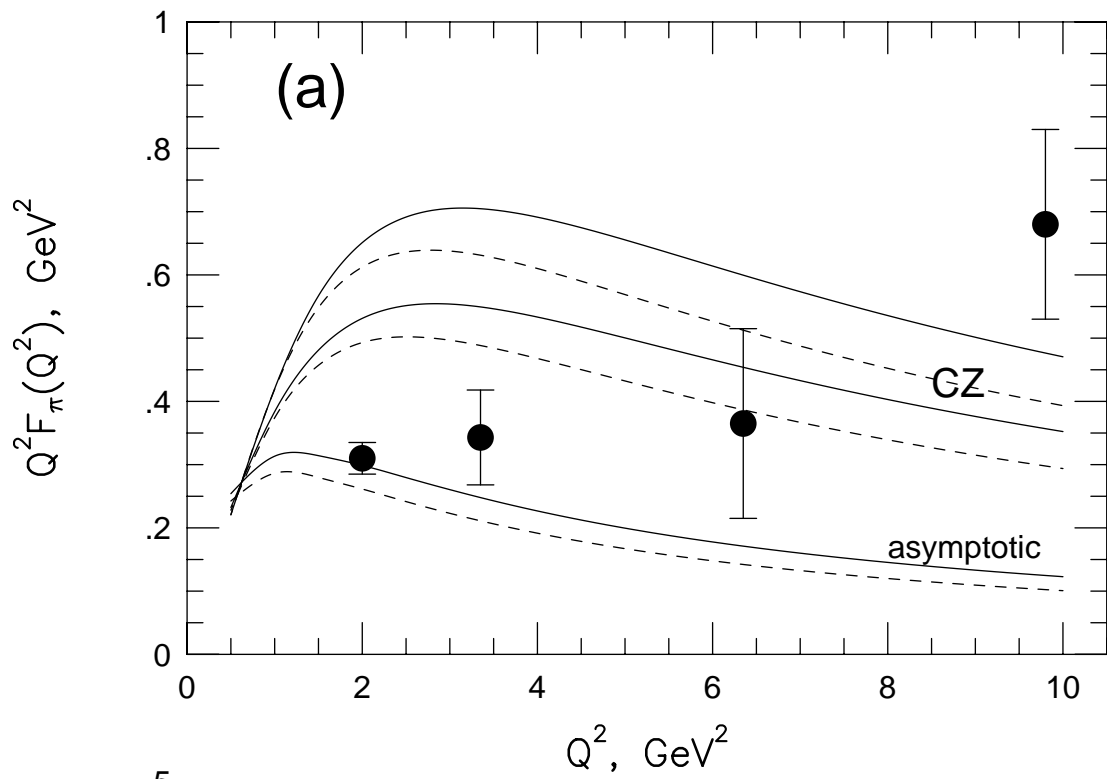


Fig. 3: The  $\gamma\pi\rho$  Form Factor

