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Bloch-Nordsieck cancellations beyond logarithms in heavy particle decays

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Abstract

We investigate the one-loop radiative corrections to the semileptonic decay of a charged particle at finite gauge boson mass. Extending the Bloch-Nordsieck cancellation of infrared logarithms, the subsequent non-analytic terms are also found to vanish after eliminating the pole mass in favor of a mass defined at short distances. This observation justifies the operator product expansion for inclusive decays of heavy mesons and implies that infrared effects associated with the summation of the radiative corrections are suppressed by at least three powers of the mass of the heavy decaying particle.

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The problem of the infrared (IR) behavior of amplitudes is inherent to gauge theories since by construction they contain massless bosons. Historically, already the first studies of the problem revealed both complexities and simplicities. On the one hand, if a finite gauge boson mass λ is employed as IR regulator, the perturbative amplitudes contain logarithms in the mass, $\ln \lambda^2$, which apparently do not allow for the limit $\lambda \to 0$. On the other hand, as shown first by Bloch and Nordsieck [1], these singularities cancel if one considers inclusive processes. This summation over final states is an integral part of any calculation in gauge theories.

With the advent of QCD the problem of the IR behavior of amplitudes became even more acute since the effective coupling blows up in the infrared. Thus, if an amplitude is perturbatively sensitive to infrared momenta, it is contributed in fact by nonperturbative effects as well and cannot be evaluated reliably. From this point of view the famous prediction of QCD that the total cross section of e^+e^- into hadrons measures quark charges rests on the Bloch-Nordsieck (BN) cancellation. Indeed, all the IR logarithms disappear from the total cross section and it is determined by short distance physics. Technically, e^+e^- annihilation is treated within the operator product expansion (OPE). Then, at least naively, IR contributions are suppressed by four powers of the large scale. The BN type cancellations ensure that this counting is not violated by divergencies of perturbative expansions, explicit or through its divergence in large orders. In this sense there is a correspondence between IR cancellations and the OPE.

The example of e^+e^- annihilation into hadrons is still special since there are no colored particles in the initial state. In the general situation, it is known from the work of Kinoshita and Lee and Nauenberg (KLN) [2] that to remove all IR singularities, summation over degenerate initial states might also be needed. This summation does not necessarily reflect the actual experimental situation and in this sense some IR logarithms can survive.

The OPE has been applied more recently in the context of heavy quark physics and the inclusive decay widths of heavy hadrons in particular. In this case there is a charged particle in the initial state. Though it is generally accepted that with only one colored particle in the initial state, the BN theorem is sufficient to ensure the absence of *explicit* soft divergencies, it is not a priori clear whether one can rely on *extended* BN type cancellations in the sense of IR power counting, implying the validity of OPE, or whether one has to invoke the more general KLN cancellation, which would require modifications of the standard OPE. In particular, the standard OPE states that there are no corrections to the decay width linear in the inverse mass of the heavy quark. This conclusion is not trivial in view of the recent observation [3, 4] that the pole mass of a charged particle itself does receive such corrections. It has been argued, therefore, that the widths are to be expressed in terms of a (unphysical) quark mass normalized far off-mass-shell, which is infrared stable in the linear approximation [3].

In this Letter we test the counting of IR effects implied by the OPE by an explicit calculation of the IR sensitive contributions to the decay of a charged particle. The infrared contributions are singled out by non-analytic terms in the gauge boson mass, some of which are associated with higher dimensional operators. The ones which are not should disappear either after summation over final states or final and initial states. In the latter case the IR counting implied by the OPE is violated. The leading $\ln \lambda^2$

terms are the subject of the original BN cancellation [1], and we show that the BN mechanism extends to comprise the $\sqrt{\lambda^2}$ and $\lambda^2 \ln \lambda^2$ terms, if the physical (or pole) mass is eliminated in favor of a short distance mass. The cancellation of linear in λ terms is in accord with the arguments of Ref. [3]. Note that although we use the QCD terminology, present calculations are not sensitive to the non-abelian nature of QCD and our results are strictly speaking applicable to the abelian case only.

For definiteness, we consider the decay of a B meson. To lowest order in the weak coupling, the QCD dynamics is contained in the matrix element of the forward scattering operator of the product of two weak hadronic currents between B-meson states

$$T^{\mu\nu}(p,q) = i \int \mathrm{d}^4 x \, e^{iqx} \langle B(p)|T\left\{J_h^{\mu\dagger}(x) J_h^\nu(0)\right\}|B(p)\rangle,$$

where an average over initial state polarizations is understood. The differential inclusive width is then given by

$$\mathrm{d}\Gamma = \frac{G_F^2 |V_{qb}|^2}{2} \frac{\mathrm{d}^3 k_{\bar{\nu}}}{(2\pi)^3 2k_{\bar{\nu}}^0} \frac{\mathrm{d}^3 k_l}{(2\pi)^3 2k_l^0} L_{\mu\nu} \cdot 2 \operatorname{Im}\left[\frac{1}{2p^0} T^{\mu\nu}(p,q)\right]. \tag{1}$$

The lepton tensor $L_{\mu\nu}$ contains the lepton momenta k_l, $k_{\bar{\nu}}$ only and the imaginary part is taken in $p \cdot q$, where $q = k_l + k_{\bar{\nu}}$. Assuming validity of the OPE a systematic expansion in powers of the heavy quark mass can be performed for the lepton spectrum and results in the prediction [5]

$$\Gamma_B = \Gamma_0 \left[1 + \sum_{n=0}^{\infty} r_n \alpha(m_b)^{n+1} + \frac{\mu_K - 3\mu_G}{2m_b^2} + O\left(\frac{1}{m_b^3}\right)\right] \tag{2}$$

for the total width. Here m_b is the *pole* quark mass, $\Gamma_0 = (G_F^2 |V_{qb}|^2 m_b^5)/(192\pi^3)$ the tree decay width for the free quark and the first order radiative correction is known explicitly [6]. For simplicity, the final quark q is taken massless.

A striking feature of Eq. (2) is that corrections to the free quark decay are suppressed by *two* powers of the quark mass. They can be parametrized by

$$\mu_K = \frac{1}{2m_B}\langle B|\bar{b}(iD_\perp)^2 b|B\rangle \qquad \mu_G = \frac{g}{4m_B}\langle B|\bar{b}i\sigma_{\mu\nu}G^{\mu\nu}b|B\rangle, \tag{3}$$

where m_B is the meson mass and we ignore the radiative corrections to these terms. The crucial assumption to arrive at this conclusion is the OPE in the kinematic region where the energy release into the hadronic final state is large and the decaying quark is almost on-shell. In addition, radiative corrections to the leading operator must be unambiguous to this accuracy. The near-mass-shell condition for the decaying quark is a new ingredient compared to the familiar expansions of the same current product on the light cone or at short distances. In particular, the pole mass which provides the overall normalization in Eq. (3) is intrinsically ambiguous by an amount of order Λ_{QCD} [3, 4]. The perturbative series that relates the pole mass to a mass defined at short distances, e.g. the $\overline{MS}$-mass,

$$m_b = m_b^{\overline{MS}} \left(1 + \sum_{n=0}^{\infty} c_n \alpha^{n+1} \right), \tag{4}$$

exhibits a strong divergence as n increases, which leads to an IR renormalon pole in its Borel transform at $t = -1/(2\beta_0)$ with t the Borel parameter (to be defined below) and β_0 the first coefficient of the β-function. Truncating the series at its optimal order leaves an uncertainty of order Λ_{QCD}, assuming $m_b^{\overline{MS}}$ as given.

The important question is whether the heavy quark expansion which relies on the heavy quark being near mass-shell – a notoriously IR-singular point – captures *all* IR effects *after* the effects associated with the definition of mass have been accounted for. To clarify this point, we have investigated the IR structure of the asymptotic behavior of the radiative corrections to the leading term in Eq. (2). Through the appearance of IR renormalons the asymptotic behavior signals the presence of power corrections, which should be added as explicit nonperturbative corrections. These explicit corrections may – and often do – turn out numerically larger than higher order perturbative corrections, which are neglected. Any indication of such terms of order $1/m_b$ threatens the validity of the OPE for inclusive decays.

We emphasize that the absence of IR renormalons can be stated as an extension of the BN cancellations. Such a relation might be envisaged, since the IR renormalons probe the IR behavior of Feynman amplitudes and can therefore be traced by an IR regulator. To establish a formal connection, we recall that the large-order behavior of radiative corrections to first approximation is generated by diagrams such as in Fig. 1, with a chain of fermion bubbles inserted in the gluon line. This procedure generates a gauge-invariant set of diagrams in the abelian theory. Denote by $\{r_n^f\}$ the series of perturbative corrections to B-decays ($\mu^+ e^-$-decays) generated in this way, and define the Borel transform of the series by $B[\{r_n^f\}](t) \equiv \sum_{n=0}^{\infty} r_n^f t^n/n!$. The fermion loop insertions are renormalized and each loop is proportional to $(-\beta_0)[\ln(-k^2/\mu^2) + C]$, where β_0 includes the fermionic contribution only, μ is a renormalization point and C a finite subtraction constant. Next call $r_0(\lambda)$ the one-loop radiative correction calculated with a finite gluon (photon) mass. To ensure the existence of the zero mass limit, we keep the standard gauge fixing and work with the propagator

$$-i\delta^{AB} \frac{1}{k^2 - \lambda^2 + i\epsilon} \left[g_{\mu\nu} - (1-\xi) \frac{k_\mu k_\nu}{k^2 - \xi\lambda^2 + i\epsilon} \right]. \tag{5}$$

Then, in the Landau gauge, $\xi = 0$, one finds the identity

$$r_0(\lambda) = \frac{1}{2\pi i} \int_{-1/2-i\infty}^{-1/2+i\infty} ds \, \Gamma(-s)\Gamma(1+s) \left(\frac{\lambda^2}{\mu^2} e^C \right)^s B[\{r_n^f\}](s) \tag{6}$$

with $s = -\beta_0 t$. To obtain this identity, one uses that the effective Borel-transformed propagator of the massless gauge boson is proportional to $1/(k^2)^{1+s}$ after summation over the fermion loops [7], whereas the propagator, Eq. (5), can be written in a Mellin representation,

3

$$\frac{1}{k^2 - \lambda^2} = \frac{1}{2\pi i} \frac{1}{k^2} \int_{-1/2-i\infty}^{-1/2+i\infty} \mathrm{d}s\, \Gamma(-s)\Gamma(1+s) \left(-\frac{\lambda^2}{k^2}\right)^s .$$

The significance of Eq. (6) rests on the observation that the coefficients in front of the $(\sqrt{\lambda^2})^{2n+1}$ and $\lambda^{2n} \ln \lambda^2$ terms of the expansion of $r_0(\lambda)$ in the small mass determine the residues of the IR renormalon poles of $B[\{r_n^f\}](t)$ at half-integer and integer multiples of $-1/\beta_0$, which in turn fixes the overall normalization of the large-order behavior of the series r_n^f. In particular, cancellations of terms in the expansion of $r_0(\lambda)$ imply the absence of the corresponding renormalons. In this unified framework, the BN cancellations of $\ln \lambda^2$ in physical processes appear simply as the absence of an IR renormalon pole at $s = 0$ [8]. Note that the analytic terms λ^{2n} in the expansion of $r_0(\lambda)$ are of purely kinematic origin and not related to renormalons. Since the Borel transform is gauge-independent for physical processes and the S-matrix elements of the abelian gauge theory are independent of ξ even in the presence of a mass term, it follows, that Eq. (6) holds in fact in any gauge.

Using the relation between IR renormalons and non-analytic in λ^2 terms we can rewrite the relation between the pole and a short-distance mass as

$$m_b \sim m_{SD} \left(1 - 2\tilde{\lambda}\right) \qquad \tilde{\lambda} \equiv C_F \alpha/(4\pi) \times \pi\lambda/m_b . \tag{7}$$

The presence of linear terms in this place is equivalent to the asymptotic behavior $c_n \stackrel{n \to \infty}{=} (C_F \mu e^{-5/6})/(\pi m_b^{\overline{MS}})(-2\beta_0)^n n!$ for the series in Eq. (4) obtained in [3, 4], which gives rise to the uncertainty of order Λ_{QCD}. For our purposes here we do not need to specify m_{SD} and a renormalization point any further.

To check the infrared effects in the decay widths we have calculated the radiative corrections to the total width and the lepton spectrum keeping the gluon mass finite (and the final quark massless). The Mellin representation of the gluon propagator is useful technically, since often it turns out to be easier to obtain the relevant terms in the expansion in λ by closing the contour in the right plane and picking out the relevant residues rather than to calculate the integrals exactly and then expand in the mass. The resulting expressions for the diagrams of Fig. 1 in $4 - 2\epsilon$ dimensions are ("$\sim$" means: the $1/\epsilon$, $\ln \lambda^2$, $\sqrt{\lambda^2}$ and $\lambda^2 \ln \lambda^2$ terms of the l.h.s. equal the r.h.s.):

$$T_{(b)}^{\mu\nu}(p,q) \sim T_{(a)}^{\mu\nu}(p,q) \frac{C_F \alpha}{4\pi} \left[-\frac{\xi}{\epsilon} - (3 - \xi) \ln \frac{\lambda^2}{m_b^2} + 3\pi \frac{\lambda}{m_b}\right]$$

$$T_{(c)}^{\mu\nu}(p,q) \sim T_{(a)}^{\mu\nu}(p,q) \frac{C_F \alpha}{4\pi} \left[-\frac{\xi}{\epsilon} + \left(2 - \frac{1 - \xi^2}{2}\right) \frac{\lambda^2}{p_q^2 + i\epsilon} \ln \left(-\frac{\lambda^2}{p_q^2}\right)\right]$$

$$T_{(d)}^{\mu\nu}(p,q) \sim T_{(a)}^{\mu\nu}(p,q) \frac{C_F \alpha}{4\pi} \left[\frac{2\xi}{\epsilon} - 4\pi\omega \frac{\lambda}{m_b}\right] - \frac{1}{p_q^2 + i\epsilon} \operatorname{tr}\left(\not{p}\Gamma^\mu \not{p}\Gamma^\nu\right) \frac{C_F \alpha}{4\pi} \left[2\pi \frac{\lambda}{m_b}\right]$$

$$+ T_{(a)}^{\mu\nu}(p,q) \frac{C_F \alpha}{4\pi} \left[1 - \xi^2\right] \frac{\lambda^2}{p_q^2 + i\epsilon} \ln \left(-\frac{\lambda^2}{p_q^2}\right)$$

$$-\frac{1}{p_q^2+i\epsilon}\,\frac{C_F\alpha}{4\pi}\left[\frac{m_b^2}{p_q^2+i\epsilon}\,\mathrm{tr}\,(\not{p}_q\Gamma^\mu\,\not{p}_q\Gamma^\nu)-\frac{1}{2}m_b^2\,\mathrm{tr}\,(\gamma_\tau\Gamma^\mu\gamma^\tau\Gamma^\nu)\right.$$
$$\left.+\left(\omega^2-2\omega-\frac{m_b^2}{p_q^2+i\epsilon}\right)\mathrm{tr}\,(\not{p}\Gamma^\mu\,\not{p}_q\Gamma^\nu)-(\omega-2)\,\mathrm{tr}\,(\not{p}\Gamma^\mu\,\not{p}\Gamma^\nu)\right]\frac{\lambda^2}{m_b^2}\,\ln\frac{\lambda^2}{m_b^2}$$

$$
\begin{aligned}
T_{(e)}^{\mu\nu}(p,q)\ \sim\ & T_{(a)}^{\mu\nu}(p,q)\,\frac{C_F\alpha}{4\pi}\left[(3-\xi)\ln\frac{\lambda^2}{m_b^2}+\pi(2\omega-3)\frac{\lambda}{m_b}\right] \\
& -\frac{1}{p_q^2+i\epsilon}\,\mathrm{tr}\,(\not{p}\Gamma^\mu\,\not{p}\Gamma^\nu)\,\frac{C_F\alpha}{4\pi}\left[-\pi\frac{\lambda}{m_b}\right] \\
& +T_{(a)}^{\mu\nu}(p,q)\,\frac{C_F\alpha}{4\pi}\left[-\frac{1-\xi^2}{2}\right]\frac{\lambda^2}{p_q^2+i\epsilon}\,\ln\left(-\frac{\lambda^2}{p_q^2}\right) \\
& -\frac{1}{p_q^2+i\epsilon}\,\frac{C_F\alpha}{4\pi}\left[-\frac{m_b^2}{p_q^2+i\epsilon}\,\mathrm{tr}\,(\not{p}_q\Gamma^\mu\,\not{p}_q\Gamma^\nu)+\frac{1}{2}m_b^2\,\mathrm{tr}\,(\gamma_\tau\Gamma^\mu\gamma^\tau\Gamma^\nu)\right. \\
& \left.-(\omega^2-2\omega)\,\mathrm{tr}\,(\not{p}\Gamma^\mu\,\not{p}_q\Gamma^\nu)+(\omega-2)\,\mathrm{tr}\,(\not{p}\Gamma^\mu\,\not{p}\Gamma^\nu)\right]\frac{\lambda^2}{m_b^2}\,\ln\frac{\lambda^2}{m_b^2}
\end{aligned}
\tag{8}
$$

Here $\Gamma^\mu=\gamma_\mu(1-\gamma_5)$, $C_F=4/3$, $p_q=p-q$, $\omega=2(p\cdot p_q)/(p_q^2+i\epsilon)$ and the tree diagram is given by

$$T_{(a)}^{\mu\nu}(p,q)=-\frac{1}{2}\frac{1}{p_q^2+i\epsilon}\,\mathrm{tr}\,(\not{p}\Gamma^\mu\,\not{p}_q\Gamma^\nu)\,.$$

Diagram (b) is to be understood as an on-shell wavefunction renormalization and m_b is the pole mass. The ultraviolet (UV) divergent terms vanish in the sum of all diagrams as well as the logarithms in λ as implied by the BN theorem. In addition, the $\lambda^2\ln\lambda^2$ terms add to zero.

Note that the terms linear in λ do not yet cancel. The reason is the use of the pole mass which contains terms linear in λ, see Eq. (7). One may now eliminate the pole mass from the differential width with the result ($p_{SD}^2=m_{SD}^2$)

$$\frac{1}{2p^0}\,T^{\mu\nu}(p,q)\sim\frac{1}{2p_{SD}^0}\left(-\frac{1}{2}\right)\frac{1}{p_{q,SD}^2+i\epsilon}\,\mathrm{tr}\,(\not{p}_{SD}\Gamma^\mu\,\not{p}_{q,SD}\Gamma^\nu)$$

with *no* corrections linear in λ to first order in α. Since all dependence on m_b has been eliminated in Eq. (1), it is never reintroduced through the subsequent phase space integrations [9]. Integrating the differential width with the radiative corrections, Eq. (8), which has to be done with care for a massless final quark, gives

$$\Gamma_B\sim\frac{G_F^2|V_{qb}|^2m_b^5}{192\pi^3}\left(1+\tilde\lambda\{3_b+20_d-13_e\}\right).\tag{9}$$

The subscript indicates the diagram, from which the respective terms originate. This form makes the cancellation of linear terms, when m_b is replaced through Eq. (7), more explicit. Let us rephrase the meaning of Eq. (9) in the language of renormalons: The

presence of linear terms in the gluon mass implies an IR renormalon at $t = -1/(2\beta_0)$ in the asymptotic behavior of the radiative corrections to the free quark decay in Eq. (2). However, this divergent behavior is organized in such a way, that it exactly cancels against the IR renormalon in the series, Eq. (4), when the pole mass in the overall normalization of the width is replaced by a short-distance mass. This cancellation supports the validity of the OPE in the kinematic situation specified by a decaying hadron. Even though the initial state is non-trivial, to eliminate the non-analyticity one does not have to invoke a summation over degenerate initial states, which would go beyond an OPE treatment. Note that the disappearance of linear terms in the total width has already been concluded in Ref. [3] on basis of the applicability of the OPE [10].

As already stated above, the subsequent non-analytic term $\lambda^2 \ln \lambda^2$, which is related to an IR renormalon at $t = -1/\beta_0$, adds to zero in the sum of all contributions in Eq. (8). Note that this non-analyticity is not reintroduced when the pole mass is eliminated, since the mass shift, Eq. (7), does not contain such terms, when m_{SD} is a MS-like mass. Thus, IR effects associated with the summation of the radiative corrections to the free quark decay are suppressed by *three* powers of the heavy quark mass. This may be anticipated, since explicit nonperturbative corrections of order $1/m_b^2$ are present only due the spin interaction and the kinetic energy of the heavy quark (cf. Eq. (3)), both of which are zero for the free quark decay. The matrix elements between B states are not affected by an IR region in the Feynman diagrams for the free quark decay. This is clear for μ_G, the spin energy, which can be related to an observable, the mass difference of the vector and pseudoscalar mesons, to leading order, but less clear for the kinetic energy contribution, given the problematic aspects of the definition of a physical mass. Vanishing of the kinetic energy for a charged particle is a consequence of Lorentz symmetry, and is related to the absence of an IR renormalon in the pole mass at $t = -1/\beta_0$, observed in Ref. [4]. Combining this observation with the heavy quark expansion for the meson masses, it follows that the kinetic energy matrix element between B states is protected form ambiguities due to renormalons.

We conclude that the suppression of infrared effects implied by the OPE is indeed valid for heavy particle decays. Phenomenologically, the subtleties associated with the definition of mass are only of secondary importance, as long as one does not attempt a determination of the (unphysical) mass parameter. One may always sacrifice one measurement to predict another quantity and use any mass parameter in intermediate steps, in particular the pole mass, which has convenient properties within perturbation theory in low orders. The prime question concerns the IR properties of the widths, after the IR properties of the mass parameter have been abstracted. The presented cancellation is an important step in proving that inclusive decay rates of heavy quarks are infrared stable (in an extended sense) and can be reliably calculated within perturbation theory. However, our result applies strictly speaking to the abelian case. It is not obvious that degenerate initial states are equally unimportant in the non-abelian case as they are in the abelian. To prove this is a challenge yet to be met.

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[8] Some care is required to disentangle the IR pole at $s = 0$ from an UV pole at the same position, related to renormalization.

[9] A subtle point is that, when taking the imaginary part in $p \cdot q$, one has to insert $\Theta(p^0 - q^0)$ according to the Cutkosky rules. Shifting the mass according to Eq. (7) produces a term proportional to $\tilde{\lambda}\,\delta(p_{SD}^0 - q^0)$, which is not compensated by the radiative corrections, Eq. (8), but does not contribute to the lepton spectrum and the total width.

[10] The arguments of Ref. [3] tacitly assume that the Coulomb gauge is used. In this particular gauge, diagram (e) and the wavefunction renormalization (b), not considered in Ref. [3], do not contribute to linear terms (A. Vainshtein, private communication), leaving only diagram (d) to cancel the mass shift. Note that in covariant gauges the linear terms are gauge-independent for each diagram individually.

Figure for: " Bloch-Nordsieck cancellations beyond logarithms in heavy particle decays" by M.Beneke, V.M.Braun and V.I.Zakharov.
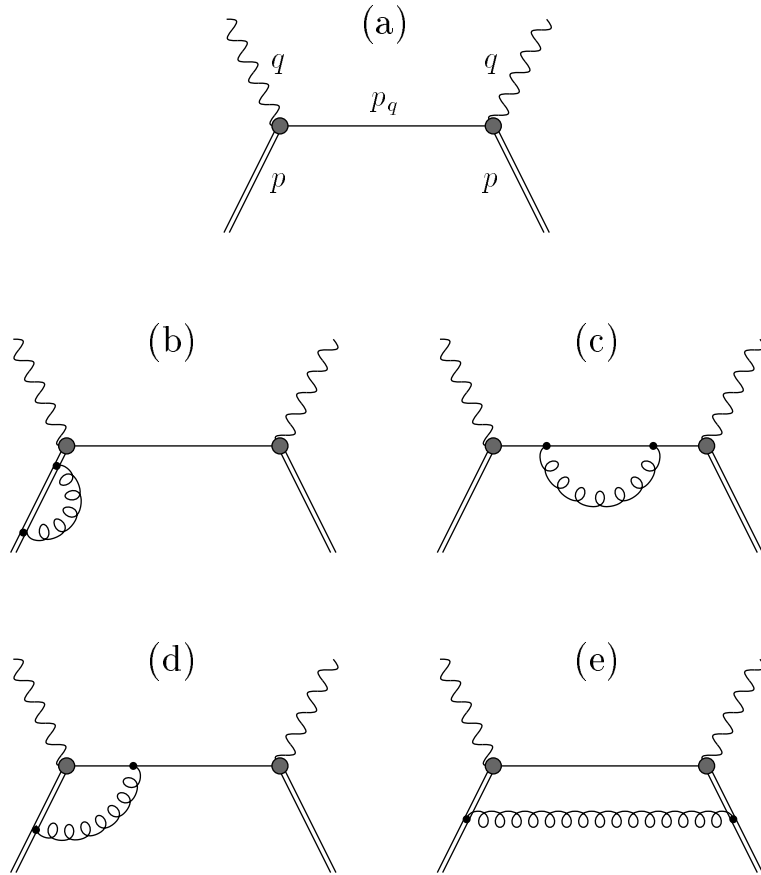

Fig. 1

Fig.1 (a) Tree diagram and (b)–(e) radiative corrections to the free quark decay. Diagrams (b) and (d) are counted twice.