Instanton-induced production of jets with large transverse momentum in QCD

I.I. Balitsky[a*] and V.M. Braun[b*]

[a] Physics Department, Penn State University, 104 Davey Lab.,University Park, PA 16802, USA

[b]Max-Planck-Institut für Physik, P.O.Box 40 12 12, Munich, Germany

We consider the instanton-induced cross section for production of a gluon jet with large transverse momentum in QCD and point out that Mueller's corrections corresponding to the rescattering of hard quanta are likely to remove contributions of large instantons, making this cross section well defined. Some speculations about possible phenomenological signatures are presented.

In recent years there is a revival of interest to instanton effects in gauge theories, inspired by the conjection [1] that in high-energy collisions such effects are enhanced enough to produce an observable violation of the baryon number. Although present theoretical arguments rather seem to disfavour such a strong enhancement [2], so that the instanton effects in the electroweak theory presumably remain far below the level at which they might become observable at future colliders, these studies have trigged an increasing interest to semiclassical effects in gauge theories at high energies in a more general context.

In [3,4] we have suggested to look for the instanton contributions in high-energy collisions in the QCD. In this case the coupling is not so small as in the electroweak theory, and the instanton contributions might be observable even if they remain exponentially supressed. The Ringwald's phenomenon — enhancement of the cross section through the dominance of final states with many gluons — allows one to hope that these cross sections may reach observable values, and similtaneously provides a good trigger for their observation, since a fireball of $\sim 2\pi/\alpha_s$ gluons is likely to produce an event with a very high density of particles in the final state.

The major difficulty for the identification of instanton effects in QCD is that in the generic situation they are not infra-red (IR) stable, typically involving a IR-divergent integral over the instanton sizes. There are speculations that in the true QCD vacuum this integration is effectively cut off at sizes of order $\rho \sim 1/600$ MeV, but this assertion is difficult to justify theoretically.

To be on a safer side, one should take special care to select contributions of small instantons, which implies going over to a certain hard process. This notion is applied to reactions in which hard scale is related either to a large virtuality of the external particle (photon or W,Z boson), or to a large momentum transfer. In perturbation theory this distinction is subtle: a large momentum transfer necessarily involves an exchange by a highly-virtual gluon (quark). Thus, perturbative description of these processes is similar, in both cases dynamics of small distances can be factorized from the large distance effects. It is this way that most of the QCD predictions arise.

This distinction is crucial, however, for the discussion of instanton effects. In the deep inelastic lepton-hadron scattering the hard scale is brought in by the virtuality Q^2 of the photon. In this case [3,4] the contribution of small instantons is distinguished by a non-trivial power dependence on Q, corresponding to a fractional twist, and can be disentangled from IR divergent contributions of large instantons, included in parton distributions. Thus, coefficient functions in front of parton distributions receive well-defined non-

*On leave of absence from St.Petersburg Nuclear Physics Institute, 188350 Gatchina, Russia

perturbative corrections

$$C(x,Q^2) \sim e^{-\frac{4\pi}{\alpha_s(\rho)}\left[1-\frac{3}{2}\left(\frac{1-x}{1+x}\right)^2+\dots\right]}\,, \qquad (1)$$

coming from instantons with the size of order $\rho \sim \pi/\alpha_s \cdot 1/Q$, which turn out to be of order $10^{-2} - 10^{-5}$ in the region of sufficiently large $Q^2 > 100$ GeV^2 and Bjorken variable $x > 0.3$, where the derivation of (1) is justified.

The situation proves to be essentially different in large momentum transfer reactions, from which we consider production of a gluon jet with large $q_\perp$ as a representative example. On physical grounds it is obvious that this cross section cannot be affected by large instantons. However, a semiclassical calculation fails in this case: to the accuracy to which (1) is derived, the instanton contribution is given by a power-like divergent integral, and contributions of small instantons $\rho \sim 1/q_\perp$ do not produce any non-trivial dependence on $q_\perp$. Indeed, to the semiclassical accuracy the effect of small instantons is to introduce new *point-like* multi-particle vertices, which do not involve any momentum transfer dependence. Thus, instanton-induced amplitudes do not decrease at large momentum transfers.

Conceptually, it is easy to realize what is missing: to obtain a sensible result one must take into account an (exponentially small) overlap between the initial state, which involves a few hard quanta, with the semiclassical final state [5]. This necessarily involves taking into account quantum corrections to semiclassical amplitudes in the instanton background, the study of which has been pioneered by Mueller [6], see [2] for a review and further references. We demonstrate that the "Mueller's corrections" indeed remove contributions of large instantons to the jet production with large $q_\perp$, making the non-perturbative contribution to this cross section well defined.

Mueller finds [6] that to the one-loop accuracy the asymptotics of the gluon propagator in the instanton background takes the factorized form

$$G(p,q) \simeq A_I(p)A_I(q)\frac{\alpha_s}{8\pi}(pq)\ln(pq)\,, \qquad (2)$$

assuming $pq \gg 1/\rho^2$, $p^2 = q^2 = 0$. From this, it is possible to derive that the corresponding quantum correction to the instanton-induced amplitude acquires the factor

$$\exp\left\{-\frac{\alpha_s}{8\pi}M(p_i)\right\}\,, \qquad (3)$$

where

$$M(p_i) = \sum_{i<j}(p_i\cdot p_j)\ln(p_i\cdot p_j)\,. \qquad (4)$$

The summation goes over all the ingoing and outgoing particles. To $O(\alpha_s)$ accuracy this formula follows directly from (2), provided $(p_i \cdot p_j) \gg 1/\rho^2$, while the exponentiation of this result is a plausible conjecture beyond, see [2]. Our strategy here is to take the quantum correction in (3) for granted, and evaluate its effect on the jet production. We note in passing that in the deep inelastic scattering the quantum correction (3) is necessary to cancel the ambiguity in the $\bar{I}I$ interaction. In this case the instanton size determined from the saddle-point equations is of order $\rho \sim (4\pi)/\alpha_s \cdot 1/Q \cdot 1/\xi^2$ where $\xi \simeq (R/\rho)^2$ [3], and the $\bar{I}I$ separation R is fixed by the Bjorken x alias by the initial energy. At large $\xi \gg 1$ one has, generally, $M \sim Q^2$ and thus the expression under the exponent in (3) is of order $\pi/\alpha_s \cdot 1/\xi^4$, same as ambiguities in $U_{\rm int}^{\bar{I}I}$.

The Mueller's factor M (4) involves momenta of hard particles (a few), which we denote by p_i, and soft particles ($n \sim (4\pi)/\alpha_s U_{\rm int}^{\bar{I}I}$), denoted by k_i. Hard particles are the two colliding partons (gluons) and the final state gluons with large momentum, which are resolved as jets. We consider the inclusive cross section, summing over all soft particles and integrating over their phase space.

It is possible to prove that at large ξ the soft momenta appearing in M can be substituted by

$$k_i \to \frac{E = \sum k_i}{n}\,. \qquad (5)$$

The derivation of (5) will be given in [7]. The dependence on n actually cancels, and M is expressed to this accuracy entirely in terms of hard momenta. For the back-to-back production of a pair of gluon jets $g+g \to g+g+X$ we find

$$M = 2\ln 2(p-q)^2 + 4pq\ln 2\,G(\theta)\,, \qquad (6)$$

$$\ln 2\,G(\theta) = -\sin^2\frac{\theta}{2}\ln\sin^2\frac{\theta}{2} - \cos^2\frac{\theta}{2}\ln\cos^2\frac{\theta}{2}\,, \qquad (7)$$

where p, q are the momenta of ingoing and outgoing gluons in c.m. frame, respectively, and θ is the angle between them. The instanton-induced cross section to the exponential accuracy reads

$$\sigma_I \sim \int dR\, d\rho\, e^{ER - \frac{4\pi}{\alpha_s} S(\xi) - \frac{\alpha_s}{4\pi} M(p,q)\rho^2} . \qquad (8)$$

Here $S(\xi)$ is the QCD action on the $\bar{I}I$ configuration, and $E = 2(p-q)$ is the energy transferred to the instanton (and released in soft particle emission). The integral is taken by the saddle-point method. Neglecting for simplicity the running of the QCD coupling and taking into account the dipole interaction term in the expansion of the action $S(\xi) = 1 - 6/\xi^2 + \ldots$, one gets the saddle-point values for ρ and R from the equations

$$\frac{\alpha_s}{\pi}\rho = \frac{2E}{M}\sqrt{\xi}\,, \quad \frac{E^2}{M} = \frac{48}{\xi^3}\,. \qquad (9)$$

Now comes the central point. The function $G(\theta)$ varies between 0 and 1, with a minimum value $G = 0$ at $\theta = 0$ and $\theta = \pi$, and a maximum $G = 1$ at $\theta = \pi/2$. Consider first the collinear jet production, $\theta = 0, \pi$. Hence $M = 2\ln 2(p-q)^2 = E^2/2 \cdot \ln 2$ is of order of the energy transferred to the instanton. From the second of the saddle-point equations in (9) one finds then $\xi^3 = 24 \ln 2$, independent on the external momenta. Thus, in this case the cross section is defined by the region of $R \sim \rho$ where instantons interact strongly and the calculation is not justified (parametrically). On the other hand, consider jets with large transverse momentum, $\theta = \pi/2$. Then $M = 2\ln 2(p^2 + q^2) \simeq s \ln 2$, where $s = 4p^2$ is the total energy, and substituting this to the saddle-point equation we find $\xi^3 = 48 \ln 2 \cdot s/E^2$. Keeping $E^2 \ll s$ (which means that momenta of gluon jets are close to momenta of coliding gluons) we get $R \gg \rho$ and the calculation is under control. Note that we get $\alpha_s/4\pi \cdot \rho^2 M \sim \pi/\alpha_s \cdot 1/\xi^2$, which indicates that the Mueller's correction now contributes on the leading $1/\xi^2$ level, same as the dipole interaction.

The same effect is observed in the instanton-induced production of monojets $g + g \to g + X$, in which case for $\theta = \pi/2$ in the c.m. frame we obtain

$$M = s\ln 2 - (s/2)\ln(1 + E^2/s) + (E^2/2)\ln 2 - (E^2/2)\ln(1 + s/E^2)\,. \qquad (10)$$

The cross section is again given by the integral in (8), and the saddle-point values in the limit $E^2/s \ll 1$ are

$$\xi^3 = 48 \ln 2\, s/E^2\,, \quad \sqrt{s}\,\rho = \frac{4\pi}{\alpha_s} \cdot \frac{\sqrt{3\ln 2}}{\xi}\,. \qquad (11)$$

Thus, at least in this academic limit, the calculation is under control. This example can be interesting phenomenologically, since it has a clear signature and smaller perturbative background. Typical numbers are as follows: in gluon-gluon collisions with $\sqrt{s} = 400$ GeV, one could look for production of a monojet with $q_\perp > 180$ GeV, balanced by $n_g \sim 10{-}15$ gluons and $2n_f$ quarks with transverse momenta of order $\rho^{-1} \sim 10$ GeV each. The cross section is difficult to estimate, but is expected to be of the same order or larger than in the deep inelastic scattering [4].

To summarise, we have shown that Mueller's corrections are likely to cut off the IR divergent integrals over the instanton size in the process of gluon jet production with large transverse momentum, indicating that their role is more important than usually believed. In general, one may thus conjecture that in *any* hard process there is a well-defined nonperturbative contribution due to small instantons with the size of order $\rho \sim 1/(Q\alpha_s(Q)$, where Q is the corresponding hard scale. A search for instanton-induced effects in large $q_\perp$ reactions may be most fruitful because of larger rates and smaller backgrounds.

REFERENCES

1. A. Ringwald, Nucl. Phys. B330 (1990) 1.
2. M.P. Mattis, Phys. Rep. 214 (1992) 159; P.G. Tinyakov, Int. J. Mod. Phys. A8 (1993) 1993.
3. I.I. Balitsky and V.M. Braun, Phys. Rev. D47 (1993) 1879.
4. I.I. Balitsky and V.M. Braun, Phys. Lett. B314 (1993) 237.
5. T. Banks et al., Nucl. Phys. B347 (1990) 581.
6. A.H. Mueller, Nucl. Phys. B348 (1991) 310; Nucl. Phys. B353 (1991) 44.
7. I.I. Balitsky and V.M. Braun, in preparation