

**NUCLEONS ON THE LIGHT CONE:
THEORY AND PHENOMENOLOGY OF BARYON
DISTRIBUTION AMPLITUDES**

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This is a short review of the theory and phenomenology of baryon distribution amplitudes, including recent applications to the studies of nucleon form factors at intermediate momentum transfers using the light-cone sum rule approach.

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1. Introduction

In the next generation of experiments in hadron physics there is a tendency to go for more and more exclusive channels. One main reason for this is that one has understood only in recent years how much one can learn from reactions like deeply virtual Compton scattering (DVCS) about the internal hadron structure and especially the spin structure. All future plans also call for very high luminosity and would therefore be perfectly suited for the investigation of exclusive and semi-exclusive reactions with and without polarization.

Main question which has to be addressed at this stage is whether studies of hard exclusive processes can be made fully quantitative. The classical theoretical framework for the calculation of hard exclusive processes in QCD is based on QCD factorization¹⁻³. This approach introduces a concept of hadron distribution amplitudes (DAs) as fundamental nonperturbative functions describing the hadron structure in rare parton configurations with a fixed number of Fock constituents at small transverse separation. DAs are ordered by increasing twist. For example, the leading-twist-2 meson DA $\phi_{2;P}$ describes the momentum distribution of the valence quarks in the meson P and is related to the meson's Bethe-

Salpeter wave function $\phi_{P,BS}$ by an integral over transverse momenta: $\phi_{2;P}(x, \mu) = Z_2(\mu) \int^{|k_\perp| < \mu} d^2 k_\perp \phi_{P,BS}(x, k_\perp)$. Here x is the quark momentum fraction, Z_2 is the renormalization factor (in the light-cone gauge) for the quark-field operators in the wave function, and μ denotes the renormalization scale. Higher-twist DAs are much more numerous and describe either contributions of “bad” components in the wave function, or contributions of transverse motion of quarks (antiquarks) in the leading-twist components, or contributions of higher Fock states with additional gluons and/or quark–antiquark pairs. Within the hard-rescattering picture, the corresponding contributions to the hard exclusive reactions are suppressed by a power (or powers) of the large momentum Q and usually have received less attention.

The distribution amplitudes are equally important and to a large extent complementary to conventional parton distributions which correspond to one-particle probability distributions for the parton momentum fraction in an *average* configuration. They are, however, much less studied: The direct experimental information is only available for the pion distribution amplitude and comes from the CLEO measurement⁴ of the $\gamma^* \gamma \pi$ transition form factor. In this talk I present a short review of the theoretical status of baryon DAs, mainly those of the nucleon. I describe the basic theoretical framework and discuss how the nucleon DAs can be related to the experimental measurements of form factors at accessible momentum transfers within the light-cone sum rule (LCSR) framework.

2. General framework

2.1. Definitions

Nucleon DAs are most conveniently defined as nucleon-to-vacuum transition matrix elements of nonlocal light-ray three-quark operators. In order to facilitate the power counting it is usually convenient to choose a light-like vector z_μ orthogonal to the large momentum q_μ involved in the problem: $q \cdot z = 0$, $z^2 = 0$. The nucleon momentum P_μ , $P^2 = m_N^2$ can be used to introduce the second light-like vector $p_\mu = P_\mu - \frac{1}{2} z_\mu \frac{m_N^2}{P \cdot z}$, $p^2 = 0$, so that $P \rightarrow p$ if the nucleon mass can be neglected, $m_N \rightarrow 0$.

The nucleon leading-twist-three DA $\Phi_N(x_i)$ can be defined as^{5–8}

$$\begin{aligned} \langle 0 | \varepsilon^{ijk} \left(u_i^\dagger(a_1 z) C \not{z} u_j^\dagger(a_2 z) \right) \not{z} d_k^\dagger(a_3 z) | N(P) \rangle = \\ = -\frac{1}{2} p z \not{z} N^\dagger(P) \int \mathcal{D}x e^{-ipz \sum x_i a_i} \Phi_N(x_i) \end{aligned} \quad (1)$$

where $\int \mathcal{D}x = \int_0^1 dx_1 dx_2 dx_3 \delta(1 - x_1 - x_2 - x_3)$, x_i correspond to quark momentum fractions, C is the charge-conjugation matrix, $N(P)$ is the Dirac spinor and the arrows correspond to the helicity projections $q^{\uparrow(\downarrow)} = (1 \pm \gamma_5)/2 q$.

The definition in (1) is equivalent to the following representation for the three-quark component of the proton wave function ³

$$|p \uparrow\rangle = \int \frac{\mathcal{D}x \Phi_N(x_i)}{2\sqrt{24x_1x_2x_3}} \{ |u^\uparrow(x_1)u^\downarrow(x_2)d^\uparrow(x_3)\rangle - |u^\uparrow(x_1)d^\downarrow(x_2)u^\uparrow(x_3)\rangle \}, \quad (2)$$

where the standard relativistic normalization of spinors is implied. One often writes $\Phi_N(1, 2, 3) = V_1(1, 2, 3) - A_1(1, 2, 3)$, where $V_1(x_1, x_2, x_3) = V_1(x_2, x_1, x_3)$ and $A_1(x_1, x_2, x_3) = -A_1(x_2, x_1, x_3)$ correspond to the symmetric and the antisymmetric part of $\Phi_N(x_i)$ w.r.t. the interchange of the u-quark momenta, respectively. The definitions of the leading-twist DAs of other baryons of the octet can be found in Ref. 9.

For higher twists, there exists an important conceptual difference between mesons and baryons. For mesons, all effects of the transverse motion of quarks in the valence quark-antiquark state can be rewritten in terms of higher Fock state contributions by using QCD equations of motion (EOM). Since quark-antiquark-gluon admixture in meson wave functions turns out to be numerically small, the transverse momentum contributions are small as well, and the higher-twist contributions to hard exclusive reactions involving mesons are dominated in most cases by meson mass corrections, see e.g. Ref. 10. For baryons, EOM are not sufficient to eliminate higher-twist three-quark DAs in favor of the components with extra gluons, so that the former present genuine new degrees of freedom. A systematic classification of such contributions is carried out in Ref. 8. One finds that to the twist-four accuracy there exist three independent DAs:

$$\begin{aligned} \langle 0 | \varepsilon^{ijk} \left(u_i^\uparrow(a_1z) C \not{z} u_j^\downarrow(a_2z) \right) \not{z} d_k^\uparrow(a_3z) | N(P) \rangle &= \\ &= -\frac{1}{2} p z \not{z} N^\uparrow(P) \int \mathcal{D}x e^{-ipz \sum x_i a_i} \Phi_4(x_i), \\ \langle 0 | \varepsilon^{ijk} \left(u_i^\uparrow(a_1z) C \not{z} \gamma_\perp \not{z} u_j^\downarrow(a_2z) \right) \gamma_\perp \not{z} d_k^\downarrow(a_3z) | N(P) \rangle &= \\ &= -m_N p z \not{z} N^\uparrow(P) \int \mathcal{D}x e^{-ipz \sum x_i a_i} \Psi_4(x_i), \\ \langle 0 | \varepsilon^{ijk} \left(u_i^\uparrow(a_1z) C \not{z} \not{z} u_j^\uparrow(a_2z) \right) \not{z} d_k^\uparrow(a_3z) | N(P) \rangle &= \\ &= \frac{1}{2} m_N p z \not{z} N^\uparrow(P) \int \mathcal{D}x e^{-ipz \sum x_i a_i} \Xi_4(x_i). \quad (3) \end{aligned}$$

In addition, there exist three twist-5 and one twist-6 three-quark DA, which do not involve new parameters to this accuracy, and can be expressed in terms of twist-3,4 DAs.

Note that in the approach of Ref. 8 the higher-twist DAs are introduced as matrix elements of light-ray operators involving “minus” components of the quark field operators. All transverse degrees of freedom are eliminated. There exists an alternative approach¹¹ in which only “plus” components are involved, but the transverse momentum dependence is retained. Both techniques are probably equivalent but the precise connection has not been worked out yet.

2.2. Conformal expansion

A convenient tool to study DAs is provided by conformal expansion^{12–18}. The underlying idea is similar to partial-wave decomposition in quantum mechanics and allows one to separate transverse and longitudinal variables in the Bethe–Salpeter wave function. The dependence on transverse coordinates is traded for the scale dependence of the relevant operators and is governed by renormalization-group equations, the dependence on the longitudinal momentum fractions is described in terms of irreducible representations of the corresponding symmetry group, the collinear conformal group $SL(2, \mathbb{R})$. The conformal partial-wave expansion is explicitly consistent with the equations of motion since the latter are not renormalized. It thus makes maximum use of the symmetry of the theory to simplify the dynamics.

To construct the conformal expansion for an arbitrary multiparticle distribution, one first has to decompose each constituent field into components with fixed Lorentz-spin projection onto the light-cone. Each such component has conformal spin $j = \frac{1}{2}(l+s)$, where l is the canonical dimension and s the (Lorentz-) spin projection. In particular, $l = 3/2$ for quarks and $l = 2$ for gluons. The quark field is decomposed as $\psi_+ \equiv \Lambda_+ \psi$ and $\psi_- = \Lambda_- \psi$ with spin projection operators $\Lambda_+ = \frac{\not{v} \not{z}}{2pz}$ and $\Lambda_- = \frac{\not{z} \not{v}}{2pz}$, corresponding to $s = +1/2$ and $s = -1/2$, respectively. Note that the “minus” components of quark fields that contribute to higher-twist DAs correspond to the negative spin projection and thus lower conformal spin. The three-particle states built of quark with definite Lorentz-spin projection can be expanded in irreducible representations of $SL(2, \mathbb{R})$ with increasing conformal spin. The explicit expression for the DA with the lowest possible conformal spin $j = j_1 + j_2 + j_3$, the so-called asymptotic DA, is

$$\phi_{as}(x_1, x_2, x_3) = \frac{\Gamma(2j_1 + 2j_2 + 2j_3)}{\Gamma(2j_1)\Gamma(2j_2)\Gamma(2j_3)} x_1^{2j_1-1} x_2^{2j_2-1} x_3^{2j_3-1}. \quad (4)$$

For the leading twist DAs $j_1 = j_2 = j_3 = 1$ reproducing the familiar result

$$\Phi_N^{\text{as}}(x_1, x_2, x_3) = 120x_1x_2x_3. \quad (5)$$

The nucleon DA can be expanded in the sum over irreducible representations with higher spin $N + 3$. For example for leading twist

$$\Phi_N(x_i) = \Phi_N^{\text{as}}(x_i) \sum_{N=0}^{\infty} \sum_{n=0}^N \varphi_{N,n}(\mu) \Psi_{N,n}^{(12)3}(x_i) \quad (6)$$

where ⁷

$$\Psi_{N,n}^{(12)3}(x_i) = (x_1 + x_2)^n P_{N-n}^{(2n+3,1)}(x_3 - x_1 - x_2) C_n^{3/2} \left(\frac{x_1 - x_2}{x_1 + x_2} \right) \quad (7)$$

where the constraint $\sum_{k=1}^3 x_k = 1$ is implied and $C_n^{3/2}(x)$ and $P_n^{\alpha,\beta}(x)$ are Gegenbauer and Jacoby polynomials, respectively. The superscript (12)3 stands for the order in which the conformal spins of the three quarks are summed to form the total spin $N + 3$: First the u -quark spins are summed to the total spin $n + 2$, and then the d -quark spin is added. This order is of course arbitrary; the functions $\Psi_{N,n}^{(12)3}$ and e.g. $\Psi_{N,n}^{1(23)}$ are related with each other through the Racah $6j$ symbols of the $SL(2)$ group, see Ref. 7 and Appendix B in Ref. 19 for details. The basis functions in (7) are mutually orthogonal w.r.t. the conformal scalar product (4) and are more convenient than Appell polynomials used in earlier studies ^{20,12,14,21,15,22,23}.

The explicit expression for the conformal expansion of the leading-twist proton DA (1) to the next-to-leading conformal spin accuracy ($N = 0, 1$) reads

$$\Phi_N(x_i, \mu) = 120x_1x_2x_3 [\phi_3^0 + (x_1 - x_2)\phi_3^- + \phi_3^+(1 - 3x_3)] , \quad (8)$$

and the twist-4 DAs (3) to the same accuracy are given by

$$\begin{aligned} \Phi_4(x_i) &= 24x_1x_2 [\phi_4^0 + \phi_4^-(x_1 - x_2) + \phi_4^+(1 - 5x_3)] , \\ \Psi_4(x_i) &= 24x_1x_3 [\psi_4^0 + \psi_4^-(x_1 - x_3) + \psi_4^+(1 - 5x_2)] , \\ \Xi_4(x_i) &= 24x_2x_3 [\xi_4^0 + \xi_4^-(x_2 - x_3) + \xi_4^+(1 - 5x_1)] , \end{aligned} \quad (9)$$

The twelve coefficients $\phi_3^0 \dots \xi_4^+$ can be expressed in terms of eight independent non-perturbative parameters $f_N, \lambda_1, \lambda_2, f_1^u, f_1^d, f_2^d, A_1^u, V_1^d$ corresponding to matrix elements of local operators. One obtains ⁸

$$\phi_3^0 = f_N, \quad \phi_3^- = \frac{21}{2} f_N A_1^u, \quad \phi_3^+ = \frac{7}{2} (1 - 3V_1^d) \quad (10)$$

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for the leading twist, and

$$\begin{aligned}
\phi_4^0 &= \frac{1}{2}(\lambda_1 + f_N), & \xi_4^0 &= \frac{1}{6}\lambda_2, & \psi_4^0 &= \frac{1}{2}(f_N - \lambda_1) \\
\phi_4^- &= \frac{5}{4}(\lambda_1(1 - 2f_1^d - 4f_1^u) + f_N(2A_1^u - 1)), \\
\phi_4^+ &= \frac{1}{4}(\lambda_1(3 - 10f_1^d) - f_N(10V_1^d - 3)), \\
\psi_4^- &= -\frac{5}{4}(\lambda_1(2 - 7f_1^d + f_1^u) + f_N(A_1^u + 3V_1^d - 2)), \\
\psi_4^+ &= -\frac{1}{4}(\lambda_1(-2 + 5f_1^d + 5f_1^u) + f_N(2 + 5A_1^u - 5V_1^d)), \\
\xi_4^- &= \frac{5}{16}\lambda_2(4 - 15f_2^d), & \xi_4^+ &= \frac{1}{16}\lambda_2(4 - 15f_2^d).
\end{aligned} \tag{11}$$

for the twist-four DAs, respectively. Note that the truncation of the conformal expansion at first order tacitly implies an assumption that this expansion is well convergent at least as a distribution in mathematical sense: after convolution with a smooth test function.

2.3. Scale dependence and Complete Integrability

The scale dependence of the nonperturbative coefficients $\varphi_{N,n}(\mu)$ in (6) is obtained by the diagonalization of the mixing matrix for the three-quark operators

$$B_{k_1, k_2, k_3} = (D_+^{k_1} q)(D_+^{k_2} q)(D_+^{k_3} q); \quad k_1 + k_2 + k_3 = N \tag{12}$$

As well known, conformal symmetry allows one to resolve the mixing with operators containing total derivatives^{12–18}. In particular, the coefficients $\varphi_{N,n}(\mu)$ with different values of N (related to the total conformal spin $J = N + 3$) do not mix with each other by the one-loop evolution. The conformal symmetry is not sufficient, however, to solve the evolution equations: the coefficients $\varphi_{N,n}(\mu)$ with the same N but different n do mix, producing a nontrivial spectrum of anomalous dimensions, see Fig. 1. The corresponding multiplicatively renormalizeable contributions to the DA are given by linear combinations of the conformal polynomials

$$P_{N,q}(x_i) = \sum_{n=0}^N c_{N,n}^{(q)} \Psi_{N,n}^{(12)3}(x_i) \tag{13}$$

with the coefficients $c_{N,n}^{(q)}$ and anomalous dimensions $\gamma_{N,q}$ that have to be found by the diagonalization of the mixing matrix.

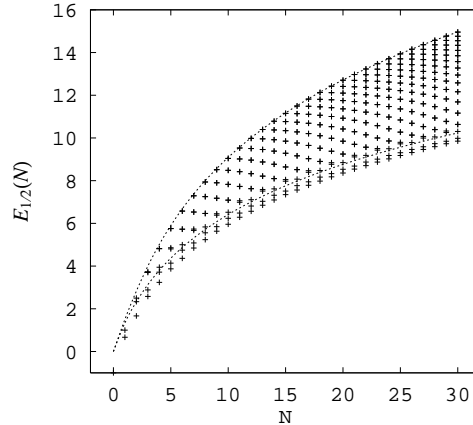


Fig. 1. The spectrum of anomalous dimensions $\gamma_N \equiv (1 + 1/N_c)E_N + 3/2C_F$ for the baryon distribution amplitudes with helicity $\lambda = 1/2$. The lines of the largest and the smallest eigenvalues for $\lambda = 3/2$ are indicated by dots for comparison.

It turns out ²⁴ that the index q that enumerates the solutions can be identified with an eigenvalue of a certain conserved charge. The physical interpretation is that one is able to find a new ‘hidden’ quantum number that distinguishes between partonic components in the proton with different scale dependence.

To explain this result, we have to introduce the so-called Hamiltonian approach ²⁵, in which the evolution kernels are rewritten in terms of the $SL(2)$ generators. It is instructive to consider two cases separately, corresponding to helicity $\lambda = 3/2$ and $\lambda = 1/2$ operators related to the evolution of the Δ -isobar DA and the nucleon, respectively. The corresponding evolution kernels can be written in the following compact form ^{24,7}:

$$H_{3/2} = 2 \left(1 + \frac{1}{N_c} \right) \sum_{i < k} \left[\psi(J_{ik}) - \psi(2) \right] + \frac{3}{2} C_F, \quad (14)$$

$$H_{1/2} = H_{3/2} - 2 \left(1 + \frac{1}{N_c} \right) \left[\frac{1}{J_{12}(J_{12} - 1)} + \frac{1}{J_{23}(J_{23} - 1)} \right]. \quad (15)$$

Here $\psi(x)$ is the logarithmic derivative of the Γ -function and J_{ik} , $i, k = 1, 2, 3$ are defined in terms of the two-particle Casimir operators of the $SL(2, R)$ group

$$J_{ik}(J_{ik} - 1) = L_{ik}^2 \equiv (\vec{L}_i + \vec{L}_k)^2, \quad (16)$$

with \vec{L}_i being the group generators acting on the i -th quark, which have to

be taken in the adjoint representation ⁷:

$$\begin{aligned} L_{k,0}P(x_i) &= (x_k\partial_k + 1)P(x_i), \\ L_{k,+}P(x_i) &= -x_kP(x_i), \\ L_{k,-}P(x_i) &= (x_k\partial_k^2 + 2\partial_k)P(x_i). \end{aligned} \quad (17)$$

Solution of the evolution equations corresponds in this language to solution of the Schrödinger equation

$$HP_{N,q}(x_i) = \gamma_{N,q}P_{N,q}(X_i) \quad (18)$$

with $\gamma_{N,q}$ being the anomalous dimensions. The $SL(2, R)$ invariance of the evolution equations implies that the generators of conformal transformations commute with the ‘Hamiltonians’

$$[H, L^2] = [H, L_\alpha] = 0, \quad (19)$$

where $L^2 = (\vec{L}_1 + \vec{L}_2 + \vec{L}_3)^2$ and $L_\alpha = L_{1,\alpha} + L_{3,\alpha} + L_{3,\alpha}$, so that the polynomials $P_{N,q}(x_i)$ corresponding to multiplicatively renormalizable operators can be chosen simultaneously to be eigenfunctions of L^2 and L_0 :

$$L^2P_{N,q} = (N+3)(N+2)P_{N,q}, \quad L_0P_{N,q} = (N+3)P_{N,q}, \quad L_-P_{N,q} = 0. \quad (20)$$

The third condition in (20) ensures that the operators do not contain overall total derivatives.

Main finding of Ref. 24 is that the Hamiltonian $H_{3/2}$ possesses an additional integral of motion (conserved charge):

$$Q = \frac{i}{2}[L_{12}^2, L_{23}^2] = i(\partial_1 - \partial_2)(\partial_2 - \partial_3)(\partial_3 - \partial_1)x_1x_2x_3, \quad [H_{3/2}, Q] = 0 \quad (21)$$

The evolution equation for baryon distribution functions with maximum helicity is, therefore, completely integrable. The premium is that instead of solving a Schrödinger equation with a complicated nonlocal Hamiltonian, it is sufficient to solve a much simpler equation

$$QP_{N,q}(x_i) = qP_{N,q}(x_i). \quad (22)$$

Once the eigenfunctions are found, the eigenvalues of the Hamiltonian (anomalous dimensions) are obtained as algebraic functions of N, q .

The Hamiltonian in (14) is known as the Hamiltonian of the so-called $XXX_{s=-1}$ Heisenberg spin magnet. The same Hamiltonian was encountered before in the interactions between reggeized gluons in QCD ^{26,27}.

The equation in (22) cannot be solved exactly, but a wealth of analytic results can be obtained by means of the $1/N$ expansion ²⁸. One general

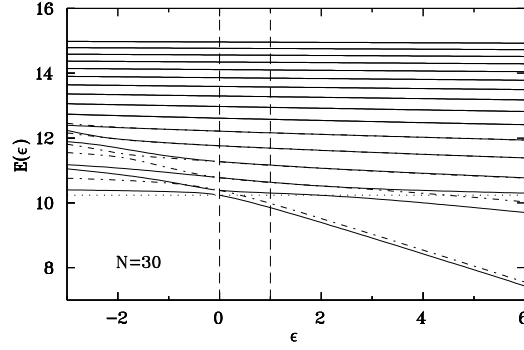


Fig. 2. The flow of energy eigenvalues for the Hamiltonian $H(\epsilon)$ for $N = 30$. The solid and the dash-dotted curves show the parity-even and parity-odd levels, respectively. The two vertical dashed lines indicate $H_{3/2} \equiv H(\epsilon = 0)$ and $H_{1/2} \equiv H(\epsilon = 1)$, respectively (up to the color factors). The horizontal dotted line shows position of the unperturbed ‘ground state’ given by Eq. (24).

consequence of complete integrability is that all anomalous dimensions are double degenerate except for the lowest ones for each *even* N , corresponding to the solution with $q = 0$. The corresponding eigenfunctions have a very simple form ⁷

$$x_1 x_2 x_3 P_{N,q=0}^{\lambda=3/2}(x_i) = x_1(1-x_1)C_{N+1}^{3/2}(1-2x_1) + x_2(1-x_2)C_{N+1}^{3/2}(1-2x_2) + x_3(1-x_3)C_{N+1}^{3/2}(1-2x_3) \quad (23)$$

and the anomalous dimension is equal to

$$\gamma_{N,q=0} = (1 + 1/N_c) \left[4\psi(N+3) + 4\gamma_E - 6 \right] + 3/2C_F. \quad (24)$$

The asymptotic expansions for the charge q and the anomalous dimensions at large N are available to the order $1/N^8$ ^{28,7} and give very accurate results.

The additional term in $H_{1/2}$ (the nucleon) spoils integrability but can be considered as a (calculable) small correction for all of the spectrum except for two lowest levels ⁷. To illustrate this, consider the flow of energy levels for the Hamiltonian $H(\epsilon) = \sum_{i < k} \left[\psi(J_{ik}) - \psi(2) \right] - \epsilon \left[1/L_{12}^2 + 1/L_{23}^2 \right]$ (cf. (14), (15)) as a function of an auxiliary parameter ϵ , see Fig. 2. It is seen that the two lowest levels decouple from the rest of the spectrum and are separated from it by a finite mass gap. As shown in Ref. 7, this phenomenon can be interpreted as binding of the two quarks with opposite helicity and forming a scalar ‘diquark’. The effective Hamiltonian for the low-lying levels can

be constructed and turns out to be a generalization of the famous Kroning-Penney problem for a particle in a δ -function type periodic potential. The value of the mass gap between the lowest and the next-to-lowest anomalous dimensions at $N \rightarrow \infty$ can be calculated combining the small- ϵ and the large- ϵ expansions and is equal to $\Delta\gamma = 0.32 \cdot (1 + 1/N_c)$ in agreement with the direct numerical calculations. The corresponding contributions to the nucleon DA are of the form ⁷

$$P_{N,q=0}^{\lambda=1/2}(x_i) \stackrel{\ln N \rightarrow \infty}{=} P_N^{(1,3)}(1 - 2x_3) \pm P_N^{(1,3)}(1 - 2x_1), \quad (25)$$

where $P_N^{(1,3)}(x)$ are Jacobi polynomials.

The approach based on complete integrability can be used to obtain parts of the two-loop evolution kernels for baryon operators beyond the leading order ²⁹, but a complete calculation to the two-loop accuracy is so far absent.

3. Nonperturbative parameters

To the leading-order accuracy in the conformal spin expansion, the leading-twist-3 DA involves one, f_N , and the twist-4 DAs two, λ_1 and λ_2 , nonperturbative parameters. To the next-to-leading accuracy in the conformal spin there are two additional parameters for twist-3, A_1^u and V_1^d , and three parameters for twist-4, f_1^u , f_1^d and f_2^d , cf. Eqs. (10),(11). The number of parameters proliferates rapidly if higher spins are included, and their estimates become increasingly complicated and unreliable. Hence I stop at the first nontrivial order and summarize the existing estimates in Table 1 and Table 2 for the leading and the higher twist DAs, respectively.

Most of the estimates are obtained using QCD sum rules. The quoted numbers correspond to the sum rules to the leading order accuracy in the QCD coupling. The NLO radiative corrections are known for λ_1 ^{30,31} but not for other cases, to my knowledge. The effect of such corrections can be substantial, see e.g. Ref. 32.

The calculations presented in Refs. 6, 9, 33, 34 make use of the same sum rule and are, therefore, not entirely independent. The errors are difficult to quantify, but are probably of the size of the spread in the quoted values. The result for f_N is expected to be rather reliable although the quoted error might well be underestimated. The parameter λ_1 is also well known to QCD sum rule practitioners and corresponds to the nucleon coupling to the so-called Ioffe current ³⁷. Note that although an overall sign in the couplings $f_N, \lambda_{1,2}$ is arbitrary and can be readjusted by the phase factor in

Table 1. Parameters of the leading-twist nucleon distribution amplitude (8), (10) at the scale 1 GeV. The constant f_N is given in units of 10^{-3} GeV^2

Method	f_N	V_1^d	A_1^u	Ref.
asymptotic	–	1/3	0	
QCDSR	5.3 ± 0.5	0.220	0.480	[6]
QCDSR	5.0 ± 0.3	0.229	0.387	[9]
QCDSR	5.1 ± 0.3	0.240	0.340	[33]
QCDSR	–	0.236	0.490	[34]
Model	–	0.310	0.071	[35]
LCSR	–	0.300	0.130	[36]

Table 2. Parameters of the twist-four nucleon distribution amplitudes (9), (11) at the scale 1 GeV. The constants λ_1 and λ_2 are given in units of 10^{-3} GeV^2

Method	λ_1	λ_2	f_1^d	f_2^d	f_1^u	Ref.
asymptotic	–	–	3/10	4/15	1/10	
QCDSR	-27 ± 5	54 ± 19	0.40 ± 0.05	0.22 ± 0.05	0.07 ± 0.05	[36]
LCSR	–	–	0.33	0.25	0.09	[36]

the nucleon wave function, the relative sign is physical and important for the applications.

Alternatively, there exists a phenomenological model for the leading-twist DA ³⁵ which was obtained by modelling the soft contribution to electromagnetic form factors by a convolution of light-cone wave functions. Estimates of the higher-twist DAs in the same technique are not available. Finally, I quote the parameters obtained in Ref. 36 from the fit of the light-cone sum rules to the experimental data on the nucleon form factors. This approach will be explained below. In future, one should expect that a few lowest order parameters in the conformal expansion of baryon DAs can be calculated on the lattice, cf. Ref. 38. Main technical problem on this way seems to be the necessity to use nonperturbative renormalization of three-quark operators.

4. Light-Cone Sum Rules

Main problem that does not allow to extract the information on baryon DAs from experiment is that nature does not provide us with point-like three-quark currents. Baryon number conservation implies that physical processes always involve baryons in pairs. Hence one has to deal with the convolution of two baryon DAs, and also the so-called “soft” contributions to the form factors which cannot expressed in terms of DAs prove to be numerically significant at present energies. A (partial) remedy is suggested by

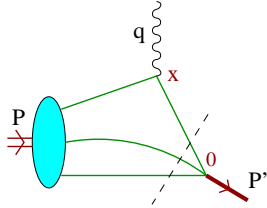


Fig. 3. Schematic structure of the light-cone sum rule for baryon form factors.

the approach known as light-cone sum rules (LCSRs) ^{39–41}. This technique is attractive because in LCSRs “soft” contributions to the form factors are calculated in terms of the same DAs that enter the pQCD calculation and there is no double counting. Thus, the LCSRs provide one with the most direct relation of the hadron form factors and distribution amplitudes that is available at present, with no other nonperturbative parameters.

The basic object of the LCSR approach is the correlation function

$$\int dx e^{-iqx} \langle 0 | T \{ \eta(0) j(x) \} | N(P) \rangle$$

in which j represents the electromagnetic (or weak) probe and η is a suitable operator with nucleon quantum numbers. The other (in this example, initial state) nucleon is explicitly represented by its state vector $|N(P)\rangle$, see a schematic representation in Fig. 3. When both the momentum transfer Q^2 and the momentum $(P')^2 = (P - q)^2$ flowing in the η vertex are large and negative, the asymptotics of the correlation function is governed by the light-cone kinematics $x^2 \rightarrow 0$ and can be studied using the operator product expansion (OPE) $T\{\eta(0)j(x)\} \sim \sum C_i(x)\mathcal{O}_i(0)$ on the light-cone $x^2 = 0$. The x^2 -singularity of a particular perturbatively calculable short-distance factor $C_i(x)$ is determined by the twist of the relevant composite operator \mathcal{O}_i , whose matrix element $\langle 0 | \mathcal{O}_i(0) | N(P) \rangle$ is given by an appropriate moment of the nucleon DA. Next, one can represent the answer in form of the dispersion integral in $(P')^2$ and define the nucleon contribution by the cut-off in the quark-antiquark invariant mass, the so-called interval of duality s_0 (or continuum threshold). The main role of the interval of duality is that it does not allow large momenta $|k^2| > s_0$ to flow through the η -vertex; to the lowest order $O(\alpha_s^0)$ one obtains a purely soft contribution to the form factor as a sum of terms ordered by twist of the relevant operators and hence including both the leading- and the higher-twist nucleon DAs. Note that, in difference to the hard mechanism, the contribution of higher-twist DAs is only suppressed by powers of $|(P')^2| \sim 1 - 2 \text{ GeV}^2$ (which is trans-

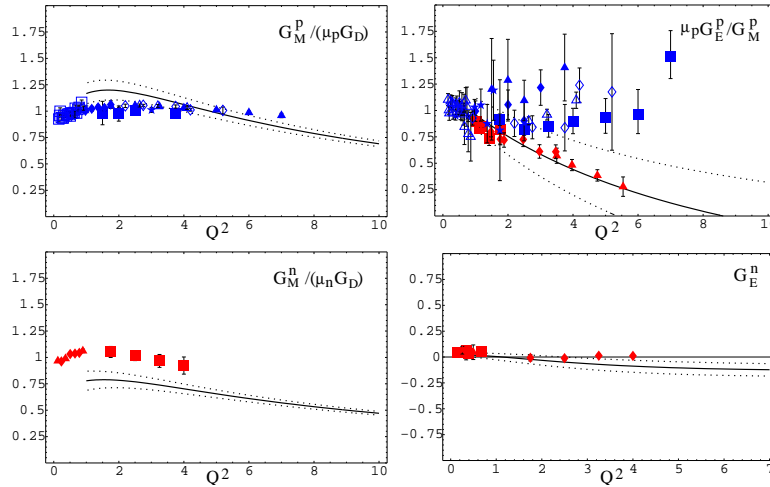


Fig. 4. LCSR results (solid curves) for the electromagnetic form factors of the nucleon, obtained using the model of the nucleon DAs with parameters from Tables 1,2. The dotted curves show the effect of the variation of the ratio f_N/λ_1 by 30%. For the identification of the data points and details of the calculation see Ref. 36.

lated to the suppression by powers of the Borel parameter after applying the usual QCD sum rule machinery), but not by powers of Q^2 . This feature is in agreement with the common wisdom that soft contributions are not constrained to small transverse separations.

The LCSR expansion also contains terms generating the asymptotic pQCD contributions. They appear at proper order in α_s , i.e., in the $O(\alpha_s)$ term for the pion form factor, at the α_s^2 order for the nucleon form factors, etc. In the pion case, it was explicitly demonstrated^{42,43} that the contribution of hard rescattering is correctly reproduced in the LCSR approach as a part of the $O(\alpha_s)$ correction. It should be noted that the diagrams of LCSR that contain the “hard” pQCD contributions also possess “soft” parts, i.e., one should perform a separation of “hard” and “soft” terms inside each diagram. As a result, the distinction between “hard” and “soft” contributions appears to be scale- and scheme-dependent⁴². During the last years there have been numerous applications of LCSRs to mesons, see Refs. 44, 45 for a review. Following the work Ref. 46 nucleon form factors were further considered in this framework in Refs. 47, 48, 49, 36 and the weak decays $\Lambda_b \rightarrow p l \nu_e$, $\Lambda_c \rightarrow \Lambda l \nu_e$ in Refs. 50, 51. The generalization to the $N\gamma\Delta$ transition form factor was worked out in Ref. 52.

The net outcome of these studies is that all nucleon form factors (with an

exception of the magnetic $N\Delta\gamma$ transition) can be reproduced to roughly 20% accuracy by using the parameters of the proton DA summarized in Tables 1,2 above, which are roughly in the middle of the range between asymptotic DAs and the QCD sum rule predictions, see Fig. 4. This conclusion is preliminary, however. More studies are needed and in particular radiative corrections to the sum rules have to be calculated.

5. Conclusions

Baryon distribution amplitudes are fundamental nonperturbative functions describing the hadron structure in configurations with a fixed number of Fock constituents at small transverse separation. They are equally important and to a large extent complementary to conventional parton distributions which correspond to one-particle probability distributions for the parton momentum fraction in an average configuration. The theory of baryon DAs has reached a certain degree of maturity. In particular, their scale dependence is well understood and reveals a beautiful hidden symmetry of QCD which is not seen at the level of the QCD Lagrangian. The basic tool to describe DAs is provided by the conformal expansion combined with EOM (for higher twists) that allows one to obtain parameterizations with the minimum number of nonperturbative parameters. There are indications that the conformal expansion is converging sufficiently rapidly so that only a few terms are needed for most of the practical purposes. A qualitative picture inspired by the QCD sum rule calculations⁶ seems to be that the valence quark with the spin parallel to that of the proton carries most of its momentum. It is timely to make this picture quantitative; combination of LCSRs and lattice calculations should allow one to determine momentum fractions carried by the three valence quarks with 5-7% precision within a few years. Further progress will depend decisively on whether studies of hard exclusive processes can be made fully quantitative. High quality data are needed in the $Q^2 \sim 10 \text{ GeV}^2$ range, and one has to develop a consistent theoretical framework for the treatment of end-point contributions.

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